Formability limits by wrinkling in sheet metal forming

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Abstract
This paper presents a new combined experimental and theoretical methodology for determining the formability limits by wrinkling in sheet metal forming. The methodology is based on the utilization of rectangular test specimens clamped along its narrower sides and compressed lengthwise and is aimed at replicating the physics behind the occurrence of wrinkling in deformation regions submitted to in-plane compression along one direction. The methodology draws from a previous development in the field of flexible roll forming, and the overall objectives are to enhance and improve its methods and procedures and to provide a new level of understanding on the onset of wrinkling in sheet metal forming. Experimentation and finite element modelling of cylindrical deep-drawing without blank holder combined with the utilization of the space of effective strain vs. stress triaxiality are employed to discuss the applicability and validity of the new proposed methodology for determining the formability limits by wrinkling.

Keywords
Sheet metal forming, wrinkling, deep-drawing, experimentation, finite element method

Introduction
The formability limits in sheet metal forming set the amount of deformation that can be attained without failure by necking, fracture or wrinkling. Figure 1 provides a schematic representation of these limits in the principal strain space based on an original figure made by Marciniak that was recently revised by Martins et al., as the result of new phenomenological and experimental evidence on the formability limits by fracture.

The onset of necking is the most widely used formability limit in sheet metal forming, and it is generally plotted as a ‘V-shaped’ curve, designated as the forming-limit curve (FLC). The FLC indicates the amount of deformation to be imparted to sheet metal parts for which aesthetics difficulties and incipient fracture derived from localized zones of thinning are likely to occur.

The onset of fracture consists of a pair of curves (designated as the fracture loci) that intersect at the upper right-hand part of the second quadrant and delimit the strain loading conditions above which sheet metal parts fail by cracking opening modes I (by tension) and II (by in-plane shear) of fracture mechanics. The fracture loci and the circumstances leading to fracture in sheet metal forming are comprehensively discussed in a recent state-of-the-art review published by Silva et al.

The methodology for determining the formability limits by wrinkling is not as consolidated as those utilized in necking and fracture. The reason for this is attributed to the fact that wrinkling is influenced by a significantly wider and more diverse range of parameters that include the mechanical properties of the material, the geometry of the sheet metal part, the contact conditions imparted by the tools and the applied level of stresses and strains.

The idea behind the existence of a locus of in-plane strains delimiting the onset of wrinkling in sheet metal forming is attributed to Havranek, who proposed the concept of the wrinkling-limit curve (WLC) after measuring the circumferential and radial strains on the unsupported region of conical cups. The WLC is situated in the lower left-hand of the second quadrant of the principal strain space (Figure 1) but its exact location is commonly determined in a case-by-case basis depending on the regions of the sheet metal forming processes to be analysed.
Abe et al.,\(^7\) for example, proposed the utilization of a conical cup drawing test performed with a flat-bottomed punch having a much smaller diameter than that of the die opening to investigate the onset of wrinkling at the wall and flange of circular blanks. Narayanasamy and Sowerby\(^8\) utilized the same conical cup drawing test with two different flat-bottomed and hemispherical-ended punches and predicted the onset of wrinkling in partially draw cups by combining the theories of plasticity and structural stability. They confirmed the existence of a WLC corresponding to critical values \(d_{\text{1}}\) and \(d_{\text{2}}\) of the strain loading path for which wrinkling occurs.

The practical applicability of WLCs is sometimes questioned because of problems related to uniqueness and existence. In fact, different strain-paths are known to change the position of the WLCs and modifications in existing strain-paths to include a compressive in-plane strain component may eventually give rise to wrinkling anywhere in the principal strain space.\(^9\)

However, because typical strain-paths under which wrinkling is likely to occur in sheet metal forming do not vary significantly,\(^10\) it is relevant to carry out the experimental characterization of the WLCs under such representative loading conditions, so that wrinkling can be prevented.

Yoshida et al.\(^11\) proposed the utilization of a buckling test consisting of a flat square sheet clamped at opposite corners and pulled in a tensile test frame along the diagonal direction, as an attempt to propose a characterization procedure that is independent from specific sheet metal forming processes. The test was employed by various authors\(^12\) to investigate the influence of mechanical properties such as the yield strength, the strain hardening exponent and the anisotropy coefficient on the occurrence of wrinkling but its utilization may be limited by cracking of the specimen prior to wrinkling.

In addition to the above mentioned difficulties, the Yoshida buckling test may be further limited by the small levels of strain that are reached in the test, which are considerably below the typical strain values found in real sheet metal parts. As a result of this, Cao et al.\(^13\) proposed a modification of the Yoshida buckling test (designated as the ‘contact buckling test’) to include a support of the blank in order to replicate the curvature of the sheet and the contact with the tool, which are known to delay the occurrence of wrinkling.

The difficulty in combining the broad and diverse range of parameters that influence the onset of wrinkling into a universal testing methodology justifies the development of alternative procedures that are valid for specific sheet metal forming processes. For example, Kim and Yang\(^14\) proposed an energy-based criterion to determine the onset of wrinkling for various sheet metal forming processes such as cylindrical, spherical and elliptical cup deep-drawing. Sivasankaran et al.\(^15\) proposed the utilization of artificial neural networks to predict the onset of wrinkling and the position of the WLC in the principal strain space.

This paper is focused on a new testing methodology to determine the onset of wrinkling in sheet metal forming that circumvents the aforementioned problems related to the occurrence of cracking prior to wrinkling. The development has its origins in the work of Kasaei et al.\(^16\) to determine the process window of flexible roll forming and makes use of a rectangular sheet clamped at two opposite edges and compressed in a double-action tool frame along its longitudinal direction.

The aim and objectives of the presentation are threefold. First, to introduce the experimental setup of the new testing methodology and to explain its advantage in replicating the physics behind the occurrence of wrinkling in sheet metal forming regions that generally account for in-plane compression along one direction. Second, to combine experimental and finite element results to provide a new level of understanding of the WLC. Finally, to compare the experimental results and observations of wrinkling obtained from cylindrical deep-drawing without blank holder against the WLC built upon the new proposed testing methodology. This last objective is crucial to evaluate the potential of the methodology for successfully predicting the occurrence of wrinkling in sheet metal forming.

**Experimentation**

**Mechanical characterization of the material**

The work was carried out in aluminium AA1050 sheets with various thicknesses that were annealed at 345°C during 3h prior to testing. The mechanical characterization of the material was performed by tensile testing on specimens that were cut out from
the annealed sheets at 0° and 90° degrees with respect to the rolling direction. The values obtained for the modulus of elasticity $E$, the yield strength $\sigma_Y$, the ultimate tensile strength $\sigma_{UTS}$, the elongation at break $A$ and the anisotropy coefficient $r$ are given in Table 1. The stress–strain response was approximated by the following Ludwik–Hollomon’s equation

$$\sigma = 151.2 \varepsilon^{0.3} \text{(MPa)}$$ (1)

The results obtained for the anisotropy coefficients $r_0 \equiv r_{90} \equiv 1$ allow concluding that the annealed aluminium sheets behave nearly isotropically.

**Characterization of the onset of wrinkling**

The characterization of the WLC was performed by means of rectangular test specimens clamped along its narrower sides and compressed lengthwise (Figure 2(a)). The utilization of compressive rather than tensile loading as in case of the Yoshida buckling test is aimed at replicating the physics behind the occurrence of wrinkling in deformation regions undertaking in-plane compression in one direction. This is schematically illustrated in the stress-state analogy between the rectangular test specimen and the edge of the flange in cylindrical deep-drawing that is shown in Figure 2(b). The outer flange element $E$ plotted in dark grey is subjected to circumferential compressive loading and may be considered as ‘clamped’ along its radial edges due to deformation constraints along the circumferential direction.

In connection to this, it is worth noting that the utilization of alternative buckling tests combining tensile and compression loading will not replicate the stress-state at the edge but, somewhere, inside the flange of the cylindrical cup. Therefore, they are not suitable to reproduce the limiting strain conditions because the superposition of tensile stresses will minimize the tendency to wrinkle.

The testing apparatus developed by the authors is schematically pictured in Figure 2(a) and consists primarily of two parts: the frame and the self-centring clamping system. The frame is built upon a double-action cam sliding mechanism that converts the vertical movement of the actuator (A) into the horizontal movement of the holders (H) containing the left and right clamping elements towards each other. The horizontal movement of the holders (H) produces the compression loads that are applied on both clamped sides $w_0$ of the test specimens. The self-centring
clamping system is a simplified version of that used by Kasaei et al. and consists of two clamping shoes (S) and two clamping elements (C). The clamping shoes (S) serve to attach the clamping elements (C) to the holders (H). The clamping elements (C) provide the fixed end constraints of the rectangular test specimens and ensure its self-alignment in order to prevent deformation by pure bending.

As seen in Figure 2(a), the rectangular test specimens subject to in-plane compression develop plastic buckling in the centre along the width direction. The evolution of the in-plane strain field during the test required electrochemical etching a grid of interlaced circles of 2 mm initial diameter on the surface of the sheets and measuring the lengths of the major and minor axes of the ellipses that resulted from plastic deformation of the original grid at different instants of time (refer also to Figure 6(b)). The selected instants of time correspond to progressively smaller distances between the two opposite holders (H) containing the left and right clamping elements.

The in-plane strain pairs \( \varepsilon_1, \varepsilon_2 \) resulting from the above mentioned procedure are obtained at the grid points located at the outer surface of the buckled test specimens by means of conventional circle grid analysis, as follows

\[
\varepsilon_1 = \ln \left( \frac{a}{\text{d}} \right) \quad \varepsilon_2 = \ln \left( \frac{b}{\text{d}} \right)
\]

The deformed length \( l \) of the specimens and the amplitude \( h \) of the plastic buckled waves (Figure 2(a)) also need to be measured for further plots and calculations that will be addressed later in the presentation.

Table 2 presents a summary of the rectangular specimens that were used in the characterization of the onset of wrinkling by means of the new proposed testing methodology.

<table>
<thead>
<tr>
<th>Thickness ( t_0 ) (mm)</th>
<th>Length ( l_0 ) (mm)</th>
<th>Width ( w_0 ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3, 4</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

The geometry of the tool and circular blanks are summarized in Table 3.

### Theoretical background

#### Finite element modelling

The procedure to characterize the WLC by means of the new proposed methodology combines experimental data and finite element simulation. Finite element analysis was carried out with the computer program LS-Dyna, and the rectangular test specimens were discretized by means of 4450 shell elements with five integration points across the sheet thickness (Figure 3(a)). The rectangular test specimens were simulated as elastic-plastic deformable objects and their mechanical properties are supplied in equation (1) and Table 1. The clamping elements were modelled as rigid objects, and the application of compressive loading was accomplished by imposing a constant velocity at the constrained regions of the specimens. The desired amount of total displacement was accomplished by means of an updated Lagrangian formulation that made use of a succession of displacement increments each of one corresponding to \( 10^{-6} \) mm in the longitudinal direction. The overall central processing unit (CPU) time for a typical analysis consisting of a finite element model similar to that shown in Figure 3(a) was 6 h on a computer equipped with an Intel® Core™ i7 CPU (2.93 GHz) processor and making use of four physical cores.

#### Methodology to determine the onset of wrinkling

The combined experimental and numerical methodology to determine the onset of wrinkling is
summarized in Figure 3(b) to (d). The methodology is valid for a specific sheet thickness and first requires ensuring compatibility between the experimental and numerically predicted evolutions of the compression force with displacement (Figure 3(b)). Second, it is necessary to calculate the critical strain pairs \((\varepsilon_1^{\text{crit}}, \varepsilon_2^{\text{crit}})\) at the onset of instability for the entire set of test specimens and to plot them in the principal strain space in order to define the WLC (Figure 3(c)). Finally, it is necessary to transform the WLC from the principal strain space into the space of effective strain \(\tilde{\varepsilon}\) vs. stress triaxiality \(\sigma_m/\tilde{\sigma}\) (Figure 3(d)).

The first procedure is necessary because the onset of wrinkling is not always attained simultaneously along the entire width \(w\) of the specimens. This is particularly noticeable for specimens with small thicknesses and is attributed to the clamping conditions and to the straightness and perpendicularity tolerances in both opposite and adjacent sides of the specimens. In fact, these two issues may prevent users from obtaining a perfect alignment of the specimens and may even cause spurious localized deformations by pure bending (Figure 3(b)).

These localized deformations are observed at the beginning of the tests performed with thinner specimens and are responsible for triggering the onset of wrinkling for smaller values of forces and displacements than those obtained if the specimens were perfectly flat. As a consequence, it is necessary to introduce a quantity designated as the ‘initial effective width’ \(w_0^e\) to adjust the experimental settings and observations to the finite element simulative conditions, as follows (Figure 3(b))

\[
\frac{w_0^e}{w_0} = \frac{F_{\text{exp}}}{F_{\text{fem}}},
\]

where \(F_{\text{exp}}\) and \(F_{\text{fem}}\) are the experimental and finite element predicted critical forces at the onset of plastic instability.\(^{16}\)

The second procedure utilizes the finite element evolution of the principal strains with the ratio \(h/l\) of the amplitude \(h\) to the length \(l\) of the deformed test specimens (hereafter designated as the ‘normalized amplitude of plastic instability waves’) to determine the critical strain pairs \((\varepsilon_1^{\text{crit}}, \varepsilon_2^{\text{crit}})\) at the instant of time when the critical force \(F_{cr}\) is attained and wrinkling starts \((h > 0\text{mm}, \text{Figure 3(c)})\). The critical strain pairs for test specimens with different initial lengths \(l_0\) are then employed to construct the WLC in the principal strain space (Figure 3(d)).

In addition to finite element estimates, it is also important to obtain experimental values of the principal strains by circle grid analysis at selected instants of time, corresponding to progressively smaller distances between the two opposite holders (H) containing the left and right clamping elements. These experimental values were taken after the onset of wrinkling and are mainly utilized to double-check the finite element simulations and to ensure the overall quality of the predicted WLCs.

As will be seen in the following section, the WLCs can subsequently be transformed from the principal strain space into the space of effective strain \(\tilde{\varepsilon}\) vs. stress triaxiality \(\sigma_m/\tilde{\sigma}\)

**Representation of the WLC in the space of effective strain vs. stress triaxiality**

The transformation of the WLCs from the principal strain space into the space of effective strain \(\tilde{\varepsilon}\) vs. stress triaxiality \(\sigma_m/\tilde{\sigma}\) (Figure 3(d)) is based on the application of Hill’s 48 yield plasticity criterion under plane stress conditions \(\sigma_3 = 0\)

\[
\tilde{\sigma} = \sqrt{\sigma_1^2 + \sigma_2^2 - \frac{2r}{(1 + r)} \sigma_1 \sigma_2}
\]

*Table 3. Geometry of the tool and circular blanks that were utilized in the cylindrical deep-drawing tests.*

<table>
<thead>
<tr>
<th>Thickness (t_0) (mm)</th>
<th>Diameter (D_0) (mm)</th>
</tr>
</thead>
<tbody>
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<td>1, 2, 3, 4</td>
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</table>

\(T\)able 3. Geometry of the tool and circular blanks that were utilized in the cylindrical deep-drawing tests.

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In fact, by writing the constitutive equations associated to Hill’s 48 criterion that relate the major $d\varepsilon_1$ and minor $d\varepsilon_2$ in-plane strain increments with the applied stresses as follows

$$d\varepsilon_1 = \frac{d\delta}{\sigma_1} \left[ \frac{1}{1 + r} \right] (\sigma_1 + r (\sigma_1 - \sigma_2))$$

$$d\varepsilon_2 = \frac{d\delta}{\sigma_2} \left[ \frac{1}{1 + r} \right] (\sigma_2 + r (\sigma_2 - \sigma_1))$$

where the effective strain increment $d\delta$ is obtained from

$$d\delta = \frac{1 + r}{\sqrt{(1 + 2r)}} \sqrt{d\varepsilon_1^2 + d\varepsilon_2^2 + \frac{2r}{(1 + r)} d\varepsilon_1 d\varepsilon_2}$$

It is possible to establish a relation between the stress triaxiality $\sigma_m/\sigma$ and the slope $\beta = d\varepsilon_1/d\varepsilon_2$ corresponding to the proportional strain loading path of

Figure 3. Summary of the methodology utilized to characterize the onset of wrinkling: (a) finite element model of the rectangular test specimen before and after compression, (b) utilization of the initial effective width $w_0^c$ to ensure compatibility between the experimental and the finite element predicted evolution of the force-displacement curves, (c) utilization of the finite element estimates to determine the critical strain pair at the onset of wrinkling and (d) schematic representation of the wrinkling-limit curve in the principal strain space and in the space of effective strain vs. stress triaxiality.
any arbitrary critical strain pair lying on top of the WLCs (refer, for example, to point ‘A’ in Figure 3(d))

\[
\frac{\sigma_m}{\sigma} = \frac{\sqrt{1 + 2r}}{3} \frac{(1 + \beta)}{\sqrt{1 + \frac{2r}{\beta + \beta^2}}}
\]  \hspace{1cm} (7)

The above equation allows transforming the WLCs from the principal strain space into the space of effective strain vs. stress triaxiality that will be later utilized in the analysis of wrinkling in cylindrical deep-drawing.

**Results and discussion**

**Evolution of the force with displacement**

Figure 4 shows the evolution of the force with displacement for selected rectangular specimens taken from the entire set of testing conditions listed in Table 2. As seen, the force increases steeply up to a peak value corresponding to the critical instability force \( F_{cr} \) after which starts to decrease at a much lower rate with increasing displacement. The value of the critical instability force \( F_{cr} \) and the amount of displacement corresponding to the onset of plastic instability are found to increase as the initial length \( l_0 \) decreases (Figure 4(a)) and the initial thickness \( t_0 \) increases (Figure 4(b)).

The particular evolution of the force with displacement obtained for the test specimen with \( l_0 = 20 \text{ mm} \) and \( t_0 = 4 \text{ mm} \) (Figure 4(a)) corresponds to thickening without the occurrence of plastic instability. This mode of deformation is similar to that of pure in-plane compression but later change direction towards smaller slopes as the ratio \( h_0/t_0 \) of the initial length \( h_0 \) to the initial thickness \( t_0 \) of the rectangular test specimens decreases.

The change in direction of the WLCs is identified by means of point ‘B’ in case of the rectangular test specimens with \( t_0 = 4 \text{ mm} \) (Figure 7(a)) and corresponds to a change in material flow towards an increasing amount of thickening before wrinkling. This change in material flow is caused by the reduction of the initial length \( h_0 \) available to trigger and propagate the plastic instability waves and may prevent the occurrence of wrinkling in situations like that of the rectangular test specimen with \( l_0 = 20 \text{ mm} \) and \( t_0 = 4 \text{ mm} \) in Figure 4(a). As mentioned in previous section ‘Evolution of the force with displacement’, the mode of deformation of this particular test specimen is typical of sheet-bulk metal forming processes and, therefore, it will not provide a critical strain pair to be included in the definition of the WLC corresponding to \( t_0 = 4 \text{ mm} \).

The representation of the WLCs in the space of effective strain \( \bar{\varepsilon} \) vs. stress triaxiality \( \sigma_m/\sigma \) (Figure 7(b)) results from transformation of the results plotted in the principal strain space (Figure 7(a)) by means of the analytical procedure that was previously described in section ‘Representation of the WLC in the space of effective strain vs. stress triaxiality’. Point ‘B’ illustrates the result of the transformation between the two spaces.

The space of effective strain \( \bar{\varepsilon} \) vs. stress triaxiality \( \sigma_m/\sigma \) will be utilized in the following section of the paper to compare the experimental results and observations of wrinkling obtained from cylindrical deep-drawing without blank holder against the WLC built upon the new proposed testing methodology.

**Assessment by cylindrical deep-drawing**

Figure 8 shows a typical finite element model of the cylindrical deep-drawing test cases that are listed in Table 3. As in case of the rectangular test specimens,
the circular blanks were discretized with 11972 shell elements with five integration points across the sheet thickness. The blanks were simulated as elastic–plastic deformable objects and its mechanical properties are given in equation (1) and Table 1. The punch and die were modelled as rigid objects and discretized by means of spatial triangular and quadrilateral elements.

Figure 9 shows the finite element predicted loading paths in the space of effective strain vs. stress triaxiality for circular blanks with 1, 3 and 4 mm thickness. The loading paths were determined for points located at the edge of the flanges and include valleys and hills in case of wrinkled cylindrical cups. The experimental values are plotted as squares (solid squares for valleys and open squares for hills) for the circular blanks with 3 and 4 mm thickness and were obtained after transformation of the circle grid measurements by means of equations (6) and (7).

As seen in Figure 9(a), the finite element evolution of the loading path for the grid points located at the edge of the flange evolves under pure compression up to point 'B1' where the WLC corresponding to \( t_0 = 1 \) mm changes direction. The cause for this change in direction was already explained in section ‘Determination of the onset of wrinkling’ (refer to Figure 7) and what is now relevant to understand is if and how it will influence the loading path of the selected grid points located at the edge of the flange.

In fact, after crossing point ‘B1’, the edge of the flange is no longer capable of undertaking thickening without wrinkling and, therefore, its loading path bifurcates into the two extreme loading path conditions of the grid points located at the bottom of the valleys and at the top of the hills. The bottom of the valleys continues to be thick and this is the reason why its loading paths continue to evolve under pure...
compression. In contrast, the top of the hills starts to thin and its corresponding loading paths develop towards the right side of the space of effective strain vs. stress triaxiality. Both strain loading paths are the result of bending caused by wrinkling during which the neutral plane moves towards the inside of the bending regions.

The crossing point 'B1' can also be understood from another perspective by referring to the analogy between the new proposed test and cylindrical deep-drawing in Figure 2(b). In fact, because 'B1' corresponds to the maximum initial length \( l_{\text{max}} \) of a rectangular test specimen that undergoes wrinkling after previous thickening exclusively under pure compression loading, it can be utilized to estimate the total number of wrinkles \( n \) that will be triggered in the flange of a deep-drawing cup at the onset of wrinkling

\[
    n = \frac{\pi D_0}{l_{\text{max}}}
\]

For example, in case of a sheet with \( t_0 = 1\) mm, the maximum initial length \( l_{\text{max}} \approx 35\) mm (Figure 9(a)) and, therefore, the expected total number of wrinkles is 8. This value is identical to that observed in the cylindrical deep-drawing experiments.

Finally, in what concerns the remaining grid points located in between the bottom of the valleys and the top of the hills, it is worth noting that they are located on the loading envelope given by the dotted curve than is included in Figure 9(a), for at a particular instant of time. In other words, all possible values of stress triaxiality \( \sigma_{\text{eff}}/\bar{\sigma} \) are located within the two above-mentioned extreme loading paths.

The bifurcation of the loading path at the point where the WLC experiences a change in direction is also observed for the cylindrical deep-drawing tests performed with a sheet thickness \( t_0 = 3\) mm (Figure 9(b)). The point is now designated as 'B3' and the finite element predictions are in good agreement with the experimental values included as square markers.
The result shown in Figure 9(c) is somewhat different from those included in Figure 9(a) and (b) because the working range of the effective strain is now very close to the threshold point ‘B4’ where the WLC changes direction. As a result of this, the signs of bifurcation in the loading path are unnoticeable and no significant wrinkling is observed at the edge of the flanges. This is confirmed by observation of the photograph of the cylindrical cup included in Figure 9(c). The overall agreement with the experimental values is also good.

From what was mentioned above, it may be concluded that besides the relevance of determining the shape of the WLC by means of the new proposed
Figure 8. Finite element model utilized in the numerical simulation of the cylindrical deep-drawing without blank holder.

Figure 9. Assessment of the WLCs in the space of effective strain vs. stress triaxiality by means of cylindrical deep-drawing without blank holder using circular blanks with (a) $t_0 = 1$ mm, (b) $t_0 = 3$ mm and (c) $t_0 = 4$ mm.
testing methodology, it is of paramount importance to identify the critical strain pair at which the slope of the WLC changes towards an increasing thickening rate before wrinkling. In fact, because the edge of the flange in a deep-drawing operation is not capable of changing its loading path towards an increasing rate of thickening due to the stress boundary conditions, it will wrinkle and the loading path will bifurcate accordingly.

Conclusions

The new testing methodology based on the double-sided compression of a rectangular sheet along its longitudinal direction is capable of replicating the physics behind the occurrence of wrinkling in the sheet metal forming deformation regions submitted to in-plane compression along one direction. The onset of wrinkling is determined by combining circle grid analysis and finite element modelling of the rectangular test specimens and is plotted as the WLC in the principal strain space and in the space of effective strain vs. strain triaxiality. The WLC is made of two parts, an initial straight line and a final curved shape that matches the increasing rate of thickening before wrinkling. The transition point between the two parts of the WLC corresponds to the last admissible critical strain pair that is compatible with the stress boundary conditions that are commonly found at the edge of the flanges of deep-drawing cups. Thus, any loading path going beyond this transition point in a deep-drawing operation performed without blank holder will give rise to wrinkling and to bifurcation of the loading path into the two extreme conditions that are found at the bottom of the valleys and at the top of the hills. The initial length of a rectangular test specimen corresponding to the transition point can also be utilized to estimate the number of wrinkles that will be triggered in the flange of a deep-drawing cup at the onset of wrinkling.

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