A SURVEY ON STOCHASTIC MULTICRITERIA ACCEPTABILITY ANALYSIS METHODS

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Abstract

Stochastic Multicriteria Acceptability Analysis (SMAA) comprises a family of multiple criteria decision aiding (MCDA) methods for problems including incomplete, imprecise, and uncertain information. Methods of the family allow solving MCDA problems of various types. Even though the methods have been applied in the past in various real-life decision-making situations, the structure of a unified SMAA framework has not been studied. In this paper we describe the methods of the family and define a unified SMAA framework. We also point out the key points in the methodology for future research.

Keywords: *Stochastic Multicriteria Acceptability Analysis (SMAA); Multiple Criteria Decision Aiding (MCDA); Simulation.*

1 Introduction

Stochastic Multicriteria Acceptability Analysis (SMAA) is a recently developed family of Multiple Criteria Decision Aiding (MCDA) methods. The different SMAA methods can be used to handle the three main MCDA problem statements (Figueira et al., 2005): choosing, ranking, and sorting. The methodology considers these problem statements in a wider sense; for example, instead of resulting in a ranking, the SMAA-2 method provides probabilities for alternatives to obtain certain ranks. The methodology is based on an inverse analysis of the space of feasible parameter values. It allows ignorance on criteria measurements and preferences. One of the advantages of SMAA over most other MCDA methodologies is that it can be used without any preference information if such is not available.

We define ignorance divided into three subcategories: incompleteness, imprecision, and uncertainty (Smets, 1991). Incomplete information means that the value is missing. Imprecise information means that we have a value for the variable, but not with the required precision. These two subtypes are of objective type. Uncertainty instead is a subjective form of ignorance appearing when the observer is taken into account, and means that the observer gives complete and precise information, but is unreliable itself.

In this survey, we describe the methods and extensions of the SMAA family, and provide recommendations on which method to use in different MCDA contexts. We find the key points of the methodology by defining the SMAA framework. We describe some of the published SMAA applications for demonstrating real-world applicability and the practices involved in application of the methodology.

The rest of this paper is organized as follows: Section 2 describes the origins of the methodology. Section 3 contains a description of SMAA and SMAA-2, the methods that form a basis for the whole family. The extensions are presented in Section 4. The simulation technique used in the SMAA computations is described in Section 5. In Section 6 we briefly present three applications of SMAA, and provide references to other applications. The SMAA framework is defined and discussed in Section 7. We end this paper with conclusions in Section 8.

2 Origins of SMAA

There exists numerous MCDA methods that apply different approaches for tackling the difficulties encountered in real-life decision-making problems. One of the oldest and the most succesful ones is the utility function based approach. In this approach, the alternatives are evaluated based on utility scores that are derived using a function. The utility function based approach has been researched intensively and applied in various models (see, for example Figueira et al., 2005). Although the approach has a history of succesful applications, it had become apparent that the exact parameter values required by earlier methods of the approach were not sufficient in all decision-making situations. In some, the decision makers (DMs) might not want to reveal their preference model, and in others, the alternatives might have uncertain of imprecise values for criteria measurements. Therefore, new advances were needed for the approach to maintain its usefulness.

One way to overcome these weaknesses in the utility function based approach is to apply an inverse method. This means that instead of asking parameter values and giving an answer to the problem in question, the values resulting in different outcomes are described. The inverse method of SMAA includes computing multidimensional integrals over feasible parameter spaces in order to provide DMs with such descriptive measures. The method solves various problems encountered in the traditional approach by allowing to use parameters with ignorance on the values. For example, usually different weight elicitation techniques produce different values, and therefore deterministic weights are harder to justify than, for example weight intervals.

Before SMAA there were other inverse MCDA methods. Two the most important ones for the development of SMAA are the comparative hypervolume criterion and the overall compromise criterion.

2.1 Comparative hypervolume criterion

The first advance considered important for the SMAA methodology was the one by Charnetski (1973) and Charnetski and Soland (1978), who introduced the comparative hypervolume criterion. This method is based on computing, for each alternative, the volume of the multi-dimensional weight space that makes each alternative the most preferred one. It can handle preference information in form of linear constraints for weights, but is restricted to deterministic criteria measurements and an additive utility function. Rietveld (1980) and Rietveld and Ouwersloot (1992) presented similar methods for problems with ordinal criteria and ordinal preference information.

2.2 Overall compromise criterion

The overall compromise criterion by Bana e Costa (1986) is a method containing ideas that gave birth to the SMAA methodology. The method consists of calculating the amount of conflict between the preferences of different DMs in order to define a joint probability density function for the weight space. Although in theory it is very useful, in practice this method is rather limited as it can handle only 3 criteria. Nevertheless, it was an important background work for SMAA methods as the computation included the idea of integration over the weight space.

3 SMAA and SMAA-2

The discrete decision-making problem is defined to consist of a set of m alternatives (or actions in general) $X = \{x_1, \ldots, x_i, \ldots, x_m\}$, that are evaluated on the basis of a set of n criteria $\{g_1, \ldots, g_j, \ldots, g_n\}$. The evaluation of action x_i on criterion g_j is denoted by $g_j(x_i)$. The model considers multiple DMs, each having a preference structure representable with an individual weight vector w and a real-valued utility or value function $u(x_i, w)$ that has a commonly accepted shape. The most commonly used value function is the linear one:

$$u(x_i, w) = \sum_{j=1}^{n} g_j(x_i) w_j. \tag{1}$$

The weights are considered to be non-negative and normalized, therefore defining the feasible weight space:

$$W = \left\{ w \in R^n : w \geq 0 \text{ and } \sum_{j=1}^{n} w_j = 1 \right\}. \tag{2}$$

The feasible weight space of a 3-criteria problem with no preference information is illustrated in Figure 1.

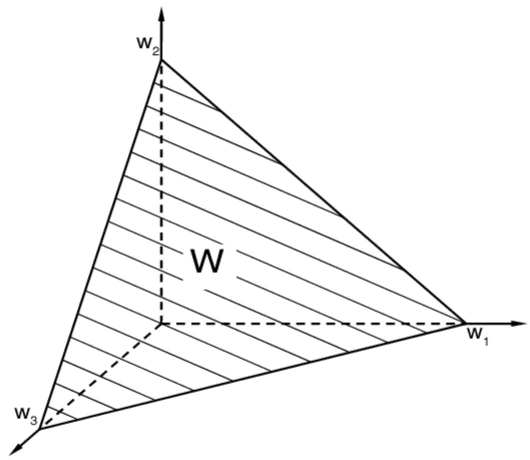

Figure 1: The feasible weight space of a 3-criteria problem.

The SMAA methods are developed for situations where neither criteria values nor weights or other parameters of the model are precisely known. Uncertain or imprecise criteria values are represented by stochastic variables ξ_{ij} (corresponding to the deterministic evaluations $g_j(x_i)$) with assumed or estimated joint probability function distribution and density function $f_\chi(\xi)$ in the space $\chi \subseteq R^{m \times n}$. Similarly, the DMs unknown or partially known preferences are represented by a weight distribution with a joint density function $f_W(w)$ in the feasible weight space W. Total lack of preference information on the weights is represented by a uniform weight distribution in W, that is:

$$f_W(w) = 1/\text{vol}(W). \tag{3}$$

As for the utility or value function based approaches, it should be noted here, that the weights are defined in the meaning of scale factors; the weights rescale the values of partial utility functions in such a way, that the full swing in the scaled function indicates the importance of the criterion (see Belton and Stewart, 2002, Sect. 5.4).

3.1 SMAA

The fundamental idea of SMAA is to calculate descriptive measures based on multidimensional integrals over stochastic parameter spaces. The original SMAA (Lahdelma et al., 1998) introduced three such measures: the acceptability index, the central weight vector, and the confidence factor. For this purpose, the set of favourable weights $W_i(\xi)$ is defined as follows:

$$W_i(\xi) = \{w \in W : u(\xi_i, w) \geq u(\xi_k, w), \forall k = 1, \ldots, m\}. \tag{4}$$

Any weight $w \in W_i(\xi)$ makes the overall utility of x_i greater than or equal to the utility of all other alternatives.

The descriptive measures of SMAA are computed based on Monte Carlo simulation. This means that they might contain errors, but the error margins are so small, that due to the nature of the problem they do not have to be taken into account (when the number of Monte Carlo iterations is large enough, see Section 5).

3.1.1 Acceptability index

Acceptability index describes the share of different weight valuations making an alternative the most preferred one. It is computed as a multidimensional integral over the criteria distributions and the favourable weight space as

$$a_i = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_i(\xi)} f_W(w)\, dw\, d\xi. \tag{5}$$

Acceptability indices can be used for classifying the alternatives into stochastically efficient ($a_i >> 0$) and inefficient ones (a_i zero or near-zero). A zero acceptability index means that an alternative is never considered the best with the assumed preference model. For stochastically efficient alternatives, the index measures the strength of the efficiency considering simultaneously the ignorance on the criteria measurements and the DMs' preferences.

Scaling of the criteria affects the acceptability indices. Scaling must therefore not be done abritrarily when trying to classify the alternatives on the basis of acceptability indices (Lahdelma and Salminen, 2001). For example, if the minimum and maximum criterion values are chosen as the corresponding scaling points, the possible introduction of a new alternative might change these values and therefore also the acceptability indices to a large extent (Bana e Costa, 1988).

3.1.2 Central weight vector

The central weight vector w_i^c is defined as the expected center of gravity of the favourable weight space. It is computed as a multidimensional integral over the criteria and weight distributions as

$$w_i^c = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_i(\xi)} f_W(w) w\, dw\, d\xi / a_i. \tag{6}$$

The central weight vector describes the preferences of a typical DM supporting this alternative with the assumed preference model. By presenting the central weight vectors to the DMs, an inverse approach for decision support can be applied: instead of eliciting preferences and building a solution to the problem, the DMs can learn what kind of preferences lead into which actions without providing any preference information.

3.1.3 Confidence factor

The confidence factor p_i^c is defined as the probability for an alternative to be the preferred one with the preferences expressed by its central weight vector. It is computed as a multidimensional integral over the criteria distributions as follows,

$$p_i^c = \int_{\xi \in \chi : u(\xi_i, w_i^c) \geq u(\xi_k, w_i^c)} f_\chi(\xi)\, d\xi. \tag{7}$$

Confidence factors can be calculated similarly for any given weight vectors. The confidence factors measure whether the criteria measurements are accurate enough to discern the efficient alternatives. If the problem formulation is to choose an alternative to realize, the ones with low confidence factors should not be chosen. If they are deemed as attractive ones, more accurate criteria data should be collected in order to make a reliable decision.

3.2 SMAA-2

The acceptability index of the original SMAA method was not designed for ranking of the alternatives, but instead for classifying them as more and less acceptable ones, from which the earlier ones should be taken into future consideration. SMAA-2 (Lahdelma and Salminen, 2001) extends SMAA by taking into account all ranks and provides five new descriptive measures: the rank acceptability index, three k-best rank-type measures, and the holistic acceptability index. These new measures provide DMs with more insight with the decision making problem. For defining the new measures, a ranking function is defined as follows:

$$rank(i, \xi, w) = 1 + \sum_{k=1}^{m} \rho\Big(u(\xi_k, w) > u(\xi_i, w)\Big), \tag{8}$$

where $\rho(true) = 1$ and $\rho(false) = 0$. Let us also define the sets of favourable rank weights $W_i^r(\xi)$ as follows,

$$W_i^r(\xi) = \{w \in W : rank(i, \xi, w) = r\}. \tag{9}$$

3.2.1 Rank acceptability index

Rank acceptability index is defined similarly to the acceptability index in (5), extending it to take into account the acceptability for a certain rank. The rank acceptability index b_i^r describes the share of parameter values granting alternative x_i rank r. It is computed as a multidimensional integral over the criteria distributions and the favourable rank weights as follows,

$$b_i^r = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, dw\, d\xi. \tag{10}$$

The most acceptable (best) alternatives are those with high acceptabilities for the best ranks. Evidently, the rank acceptability indices are within the range [0,1], where 0 indicates that the alternative will never obtain a given rank and 1 indicates that it will obtain the given rank always with any choice of weights. The first rank acceptability index b_i^1 is equal to the acceptability index a_i.

3.2.2 k-best rank indices

Rank acceptability indices are main indicators for the performance of alternatives. When the number of alternatives is large, it is sometimes appropriate to aggregate them in the early phase of the decision-making process to *k-best ranks (kbr) acceptabilities* as

$$a_i^k = \sum_{r=1}^{k} b_i^r. \tag{11}$$

The *kbr* acceptabilities can be used in an iterative process in which the weak alternatives are eliminated until a small group of alternatives reach sufficient acceptabilities.

The central weight vectors can also be extended in a similar way, to define the *central kbr weight vector* w_i^k as

$$w_i^k = \int_{\xi \in \chi} f_\chi(\xi) \sum_{r=1}^{k} \int_{w \in W_i^r(\xi)} f_W(w) w\, dw\, d\xi / a_i. \tag{12}$$

The *kbr* weight vector describes the preferences of a typical DM that assigns an alternative to one of the ranks from 1 to k. Also the confidence factors can be extended similarly, to define the *kbr confidence factor* p_i^k as

$$p_i^k = \int_{\xi \in \chi : rank(i,\xi,w_i^k) \geq k} f_\chi(\xi)\, d\xi. \tag{13}$$

3.2.3 Holistic acceptability index

The problem of comparing the alternatives in terms of their rank acceptabilities can be seen as a "second-order" multiple criteria decision aiding problem (Lahdelma and Salminen, 2001). The DMs attitude towards risk define the required magnitude of confidence factors and acceptability indices. The rank acceptability indices can be aggregated into holistic acceptability indices a_i^h as

$$a_i^h = \sum_r \alpha^r b_i^r, \tag{14}$$

where α^r are the so-called metaweights. There are numerous possible ways of choosing the metaweights (see Lahdelma and Salminen, 2001), the only constraints being that they should be non-negative, normalized and non-increasing when the rank increases. Using the holistic acceptability indices in the decision-making has its limitations, however. This "second-order" decision-making problem imposes an additional level of complexity to the indicators, and adds assumptions which the DMs might not realize.

In our opinion the holistic acceptability indices should only be used when there is no analyst available or when SMAA is used as an automated decision-making tool. However, in these cases it should be questioned if SMAA was an appropriate method to apply in the first place. The most appropriate use of the holistic acceptability indices could be in problems with a large amount of alternatives, to filter out alternatives that do not deserve attention from the DMs. Although in this type of problems the *kbr* acceptability indices might be more adequate.

3.3 Preference information

In most decision-making problems it is possible to elicit some, though probably imprecise and uncertain, preference information from the DMs. Although SMAA allows preference information to be represented with an arbitrary density function, usually it is easier to elicit the preferences as constraints for the weight space. Then the density function is defined with a uniform distribution in the restricted weight space W' as

$$f_{W'}(w) = \begin{cases} 1/\mathrm{vol}(W'), & \text{if } w \in W', \\ 0, & \text{if } w \in W \setminus W'. \end{cases} \tag{15}$$

In particular, SMAA-2 introduces the following types of constraints:

1. Intervals for weights ($w_j \in [w_j^{min}, w_j^{max}]$).
2. Intervals for weight ratios (trade-offs) ($w_j/w_k \in [w_{jk}^{min}, w_{jk}^{max}]$).
3. Linear inequality constraints for weights ($Aw \leq c$).
4. Nonlinear inequality constraints for weights ($f(w) \leq 0$).
5. Partial or complete ranking of the weights ($w_j > w_k$).

Figure 2 illustrates the feasible weight space of a 3-criteria problem with interval constraints for weight w_1. Figure 3 illustrates the feasible weight space of a 3-criteria problem with complete ranking of the weights.

When there are multiple DMs, the constraints have to be aggregated before applying. Possible non-interactive aggregation techniques include mathematical union, intersection, of averaging densities of the functions defining preferences of different DMs. There exists also a technique based on belief functions for eliciting and aggregating the preference information, see Tervonen et al. (2004b,c).

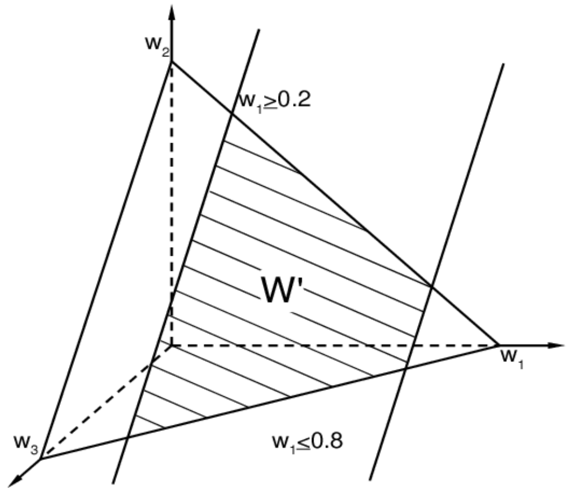

Figure 2: The feasible weight space of a 3-criteria problem with constraints on w_1.

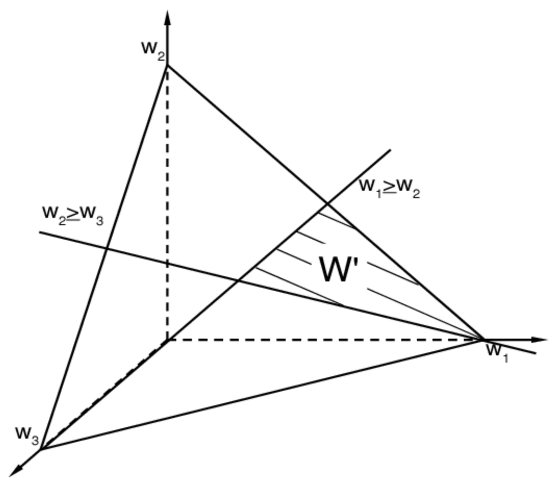

Figure 3: The feasible weight space of a 3-criteria problem with complete ranking of the weights.

4 Extensions

In this section we will describe the most important SMAA extensions for ordinal criteria measurements, dependent criteria, cross confidence factors, and those based on the outranking approach. There is also a variant of SMAA based on data envelopment analysis (SMAA-D). For description of it, we refer to Lahdelma and Salminen (2006b).

4.1 Ordinal criteria (SMAA-O)

SMAA-O (Lahdelma et al., 2003) extends SMAA to consider ordinal criteria measurements, meaning that the DMs have ranked the alternatives according to each (ordinal) criterion. In SMAA-O, the ordinal information is mapped to cardinal without forcing any specific mapping. This means that nothing is assumed about the weights of criteria ranks in the piecewise linear mapping.

The possibility of using ordinal measurements has its advantages. Usually the experts defining the criteria measurements can rank the alternatives with respect to each criterion faster than they can define cardinal measurements. Therefore, if ordinal measurements provide sufficient accuracy for the decision-making problem in question, savings can be obtained.

The ordinal criteria are measured by assigning for each alternative a rank level number $r_j = 1, \ldots, j^{max}$, where 1 is the best and j^{max} the worst rank level. Alternatives considered equally good are placed on the same rank level and the rank levels are numbered consecutively. On an ordinal scale, the scale intervals do not contain any information, and should be therefore treated as such without imposing any extra assumptions. However, some mapping can be assumed to underlie the ordinal information. In SMAA-O, all mappings that are consistent with the ordinal information are simulated numerically during the Monte Carlo iterations. This means generating random cardinal values for the corresponding ordinal criteria measurements in a way that preserves the ordinal rank information. Figure 4 illustrates a sample mapping generated in this way.

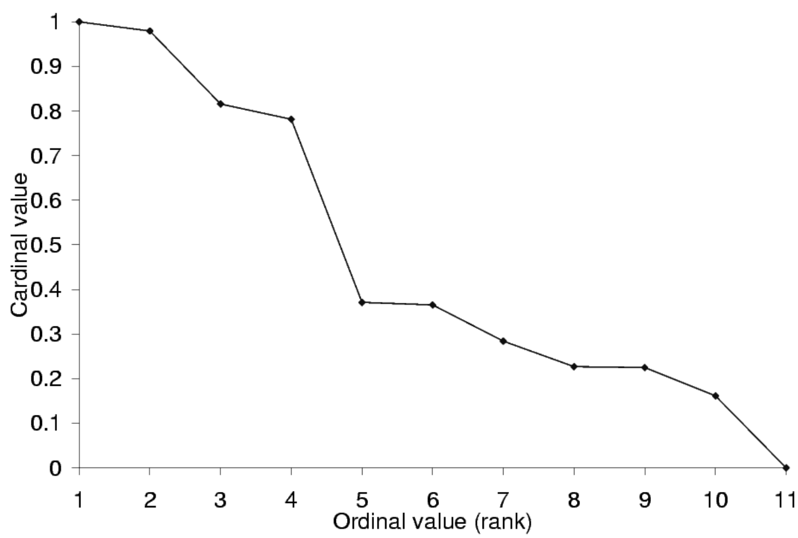

Figure 4: An sample ordinal to cardinal mapping of SMAA-O.

The SMAA methods can be used with any kind of value function jointly accepted by the DMs, but if we have an additive value function, the shape of the function can be considered unknown. In this case, the DMs partial value functions are simulated in the same way as the ordinal to cardinal mappings. However, the simulation is not necessary for the ordinal criteria, because the simulated cardinal values can be interpreted directly as partial values on a linear scale. Therefore, if the DMs accept an additive value function, it is not necessary for the DMs to agree on a common shape of the partial value functions for the ordinal criteria.

SMAA-O has been combined with the so-called SWOT methodology in the work of Kangas et al. (2003b). For an alternative technique for applying ordinal criteria in simulation-based approaches, see Leskinen et al. (2004).

4.2 Handling dependent criteria

In many real-life applications of SMAA the criteria measurements as well as their uncertainties are dependent, and by not considering them as such the results will contain bias (Lahdelma et al., 2004). SMAA allows using external sampling as a source for criteria measurements. This technique implicitly takes into account the dependencies. Another technique reported in the literature (Lahdelma et al., 2006) is to model the criteria with a multivariate Gaussian distribution. The multivariate Gaussian distribution between a vector of stochastic variables $[\lambda_1, \ldots, \lambda_L]^\perp$ is defined by the joint probability density function

$$f(\lambda_1, \ldots, \lambda_L) = \frac{1}{\sqrt{(2\pi)^L \det(\Lambda)}} e^{\frac{1}{2}(\lambda-\bar{\lambda})^\perp \Lambda^{-1}(\lambda-\bar{\lambda})}, \tag{16}$$

where $\bar{\lambda}$ is the vector of the expected values of the stochastic variables and Λ is the $L \times L$ covariance matrix,

$$\Lambda = \begin{pmatrix} \text{cov}(\lambda_1, \lambda_1) & \text{cov}(\lambda_1, \lambda_2) & \cdots & \text{cov}(\lambda_1, \lambda_L) \\ \text{cov}(\lambda_2, \lambda_1) & \text{cov}(\lambda_2, \lambda_2) & \cdots & \text{cov}(\lambda_2, \lambda_L) \\ \vdots & & \ddots & \\ \text{cov}(\lambda_L, \lambda_1) & \text{cov}(\lambda_L, \lambda_2) & \cdots & \text{cov}(\lambda_L, \lambda_L) \end{pmatrix}, \text{ where} \tag{17}$$

$$\text{cov}(\lambda_j, \lambda_k) = E\big((\lambda_j - \bar{\lambda}_j)(\lambda_k - \bar{\lambda}_k)\big). \tag{18}$$

Although the covariance matrix is reasonably compact presentation of the imprecision and dependancy information, it is more convenient to separate these two types into a vector of standard deviations σ and to an $L \times L$ correlation matrix ρ. The correlation matrix is computed from the covariance matrix with

$$\rho_{jk} = \frac{\text{cov}(\lambda_j, \lambda_k)}{\sigma(\lambda_j)\sigma(\lambda_k)}. \tag{19}$$

The correlation coefficients are within the range $[-1, 1]$ and measure how well a linear model $\lambda_j = a\lambda_k + b$ explains the dependancy of the variables.

Although it may be possible to determine the correlation of the variables "by hand", in practice in most applications it is too time consuming or even impossible. The multivariate Gaussian model is more suitable in applications, where there exists a simulation model or real-life process producing values for the criteria measurements (see Section 6.3). The mean of each criterion measurement is estimated from the sample y_{ij} by the sample mean with

$$\bar{\xi}_{ij} \cong \hat{\bar{y}}_{ij} = \sum_{k=1}^{K} x_{ij}^{k}/K. \tag{20}$$

An unbiased estimator for the covariance is calculated as the sample covariance with

$$\text{cov}(\xi_{ij}, \xi_{i'j'}) \cong \sum_{k=1}^{K} (y_{ij}^{k} - \hat{\bar{y}}_{ij})(y_{i'j'}^{k} - \hat{\bar{y}}_{i'j'})/(K-1). \tag{21}$$

The sample standard deviation is the square root of the sample variance:

$$\sigma(\lambda_{ij}) \cong \hat{\sigma}(y_{ij}) = \sqrt{\sum_{k=1}^{K} (y_{ij}^{k} - \hat{\bar{y}}_{ij})^2/(K-1)}. \tag{22}$$

Standard error of the sample mean, caused by the finite size of the sample, is calculated as

$$\sigma(\hat{\bar{y}}_{ij})/\sqrt{K}. \tag{23}$$

The sample correlation matrix is obtained by dividing the rows and columns of the sample covariance matrix by the sample standard deviations.

4.3 Cross confidence factors

SMAA has been developed for problems with ignorance on both the preferences and the criteria measurements. When the information is very imprecise, problems emerge because a large set of alternatives might seem acceptable as indicated by the acceptability indices. In this kind of situations, it would be desirable to obtain more precise information on the preferences of the DMs and on the criteria measurements, but it is not always possible due to limits with time and money.

One technique for improving the discrimination of a large set of efficient alternatives is to use *cross confidence factors* (Lahdelma and Salminen, 2006a). These descriptive measures are confidence factors computed for each alternative using each other's central weight vectors. The cross confidence factor for alternative x_i with respect to alternative x_k is computed as

$$p_i^k = \int_{\xi \in \chi : w_k^c \in W_i^1(\xi)} f_\chi(\xi)\, d\xi, \tag{24}$$

defined when the target alternative is efficient (and therefore has a central weight vector defined). The cross confidence factor is a probability for an alternative to obtain the first rank (considering the ignorance on the criteria measurements) when the central weight vector of the target alternative is chosen.

The cross confidence factors provide additional information in form of telling why the discrimination of alternatives is weak: an alternative that obtains a high cross confidence factor with respect to another is similar and because of that poorly discriminated. For identifying such alternatives the model defines reference sets, that are ordered stochastic sets of pairs $< a, p_i^k >$:

$$\{< i(k,r), p_{i(k,r)}^k > | r = 1, \ldots, m(k)\}, \tag{25}$$

where $m(k) \leq m$ determines the number of elements in the reference set and the index function $i(k,r)$ orders the elements by their cross confidence factors into descending order. This ordering makes it easy to quickly identify the most poorly discriminated alternatives. The reference sets can be visualized as column charts as shown in Figure 5.

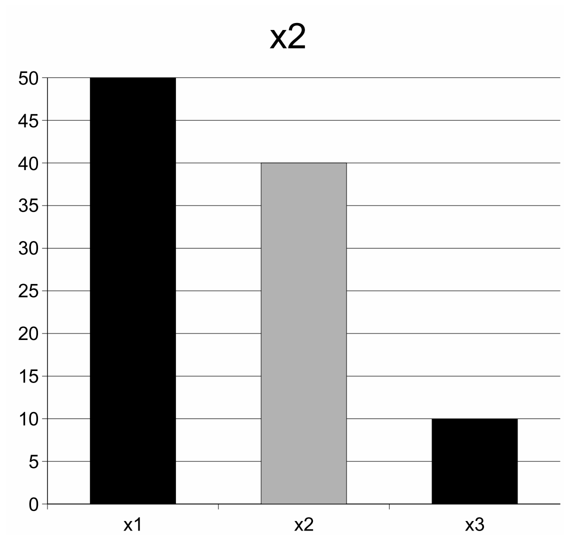

Figure 5: Sample cross confidence factors of alternative x2 in a 3-alternative problem.

4.4 Reference point approaches

Although the SMAA methods can be used without any information on the weights, it is preferable to try to elicit some information from the DMs. Rather than using weights, a more straightforward technique for representing the preferences is through reference points. With reference points, the DMs specify desirable or preferable values for each criterion instead of specifying trade-offs between criteria. Reference points model satisfying behaviour instead of trying to find optimal solutions, and can thus be more suitable in some decision-making contexts. There exists two reference-point based SMAA methods that we will describe next: SMAA-P and Ref-SMAA.

4.4.1 Prospect theory based variant (SMAA-P)

Prospect theory evaluates alternatives performances with respect to deviations from a reference alternative. This reference alternative is considered to be a 0-point in valuations of the DM. The gains with respect to the reference alternative are considered to be more important than the losses, and therefore the partial value functions are commonly S-shaped. In the part above the reference alternative (gain) the value function is concave (risk averse), and in the part under the reference alternative (loss) convex (risk seeking). Therefore when the desirable values represented by the reference point are reached, the DM is considered to be satisfied and further increases are according to risk-averse behaviour.

Prospect theory has its difficulties in finding the DMs' reference alternative, quantifying the tradeoffs for gains and losses, and combining preferences if there are multiple DMs. SMAA-P (Lahdelma and Salminen, 2003) tries to overcome these weaknesses by analyzing the sets of feasible values for the parameters defining the model. The DM's preferences are represented with additive piecewise linear difference functions as in prospect theory. This requires two separate weight sets to be defined: one for gains (w^+) and another for losses (w^-). Then the piecewise linear difference function for evaluating alternative x_i with respect to the reference alternative x_r is defined as

$$d(x_i, x_r, w^+, w^-) = \sum_{j=1}^{n} d_j(x_{ij}, x_{rj}, w_j^+, w_j^-), \tag{26}$$

where the partial difference functions are

$$d_j(x_{ij}, x_{rj}, w_j^+, w_j^-) = w_j^+ \max\{x_{ij} - x_{rj}, 0\} + w_j^- \min\{x_{ij} - x_{rj}, 0\}. \tag{27}$$

To be consistent with the intuition that losses are at least as important as equal gains, the weights are constrained with

$$0 \leq w^+ \leq w^-. \tag{28}$$

Loss and gain weights can be replaced by representing the preferences in terms of importance weights w_j and coefficients of loss aversion $s_j \geq 1$. This is the ratio between the loss and gain weights:

$$s_j = w_j^- / w_j^+. \tag{29}$$

The loss and gain weights are defined based on the loss aversion coefficient symmetrically around the importance weights as:

$$w_j = \sqrt{w_j^+ \cdot w_j^-}. \tag{30}$$

The original utility or value function is replaced in SMAA-P with a piecewise linear difference function including all preference information:

$$u(x_i, v) = d(x_i, w, x_r, s), \tag{31}$$

where $v = [w, x_r, s]$ is the *preference information vector*. The feasible preference information space V is defined as

$$V = W \times X_R \times S = \{v = [w, x_r, s] | w \in W, x_r \in X_R, s \in S\}, \tag{32}$$

where W is the feasible weight space, X_R is the feasible reference alternative space (should be defined to include at least all possible reference alternatives), and S is the feasible loss aversion coefficient space. The joint probability distribution of the preference information vector is defined as

$$f_V(v) = f_W(w) f_{X_R}(x_r) f_S(s). \tag{33}$$

The new value function (31) is used to map the stochastic criteria and preference distributions into value distributions as in SMAA-2. The stochastic sets of favourable rank weights are redefined to be stochastic sets of favourable rank preferences as

$$W_i^r(\xi) = \{v \in V | \text{rank}(i, \xi, v) = r\}. \tag{34}$$

The rank acceptability indices and the central weight vectors are computed as in SMAA-2, with the only difference being that the inner integration is done over the feasible preference information space instead of the feasible weight space. SMAA-P also defines additional descriptive measures. For more details on these, we refer to Lahdelma and Salminen (2003).

4.4.2 Reference point approach (Ref-SMAA)

The Ref-SMAA method (Lahdelma et al., 2005) (also called SMAA-A) allows to use reference points with multiple DMs by providing descriptive information about the sets of reference points that favour each alternative. An identical method (although with a simpler simulation model) has been presented by Durbach (2006).

Achievement functions are used for overcoming some weaknesses of the traditional goal programming, and used in Ref-SMAA for characterizing non-dominated solutions. These are solutions where none of the components can be improved without lowering the score of at least one of the others. An achievement function is a function $s_{\check{x}} : X \to R$, where $\check{x} \in R^k$ is an arbitrary reference point. The achievement function of Ref-SMAA can be selected in various ways, as for example:

$$s_{\check{x}}(x_i) = \min_{i=1,\ldots,k} \left[w_i(x_i - \check{x}_i) \right] + \mu \sum_{i=1}^{k} w_i(x_i - \check{x}_i), \tag{35}$$

where μ is a sufficiently small scalar and w is a fixed positive scaling vector. Usually, w_i is set to be equal to the inverse of the difference between the best and the worst value for each criterion.

Ref-SMAA operates on the basis of a set of favourable reference points for each alternative x_i, defined as:

$$\check{X}_i(\xi) = \{\check{x} \in \check{X} | s_{\check{x}}(\xi_i) \geq s_{\check{x}}(\xi_j),\ j = 1, \ldots, m\}. \tag{36}$$

Any reference point $\check{x} \in \check{X}_i(\xi)$ makes the overall preference of x_i greater than or equal to the preference of any other alternative. The feasible reference point space $\check{X}_i$ can be defined according to needs, for example as a convex combination of the reference points of all DMs. Similarly to the acceptability index (5), Ref-SMAA defines the reference acceptability index r_i, computed as a multidimensional integral over the criteria value distributions and the favourable reference point space as

$$r_i = \int_{\xi \in \chi} f_\chi(\xi) \int_{\check{x}_i \in \check{X}_i(\xi)} (\xi) f(\check{x})\, d\check{x} d\xi. \tag{37}$$

The *central reference point* $\check{x}_i$ is defined as the expected centre of gravity of the set of favourable reference points, computed as a multidimensional integral of the reference point vector $\check{x}$ over the criteria value distributions and the favourable reference point space as

$$\check{x}_i = \int_{\xi \in \chi} f_\chi(\xi) \int_{\check{x}_i \in \check{X}_i(\xi)} f(\check{x}) \check{x}\, d\check{x} d\xi / r_i. \tag{38}$$

All the descriptive measures of Ref-SMAA are related to reference points, and therefore the measures as well as the original alternatives all belong to the criterion space. For some decision makers this type of model might be easier to understand, as no artificial concepts such as weights are used.

4.5 Outranking based SMAA approaches

SMAA has been extended for using instead of value function (1) an outranking-based aggregation procedure for defining ranking of the alternatives. This and another approaches described in this section are based on using ELECTRE type pseudo-criteria. The pseudo-criteria are defined by using thresholds that are denoted as follows:

- $q_j(g_j(\cdot))$ is the *indifference threshold* for the criterion g_j,
- $p_j(g_j(\cdot))$ is the *preference threshold* for the criterion g_j, and
- $v_j(g_j(\cdot))$ is the *veto threshold* for the criterion g_j.

By using these thresholds a *concordance index* is defined. It is computed by considering individually for each criterion g_j the support it provides for the assertion of the outranking aS_jb, “alternative a is at least as good as alternative b”. The partial concordance index is a fuzzy index computed as follows, for all $j = 1, \ldots, n$:

$$c_j(a,b) = \begin{cases} 1, \text{ if } g_j(a) \geq g_j(b) - q_j(g_j(b)), \\ 0, \text{ if } g_j(a) < g_j(b) - p_j(g_j(b)), \\ \frac{g_j(a) + p_j\left(g_j(b)\right) - g_j(b)}{p_j\left(g_j(b)\right) - q_j\left(g_j(b)\right)}, \text{ otherwise.} \end{cases} \tag{39}$$

After computing the partial concordance indices, the comprehensive concordance index is calculated as follows,

$$c(a,b) = \sum_{j \in J} w_j c_j(a,b). \tag{40}$$

If the veto thresholds are used, also a *discordance index* can be defined. For more information on pseudo-criteria based models, see Roy and Bouyssou (1993).

4.5.1 Outranking aggregation procedure (SMAA-3)

SMAA-3 (Hokkanen et al., 1998) method is a variant of the original SMAA that applies, instead of the value function, ELECTRE type pseudo-criteria and "min in favor" choice procedure. According to this procedure, an alternative becomes the preferred one (not necessary unique) if the following set of constraints hold:

$$\begin{gathered}\min_{l=1,\ldots,m,l\neq i} c(x_i,x_l) \geq \min_{l=1,\ldots,m,l\neq k} c(x_k,x_l),\\ k=1,\ldots,m,k\neq i.\end{gathered} \tag{41}$$

Based on this the favourable weights of an alternative (4) are redefined as

$$\begin{aligned}W_i = \{w \in W : &\min_{l=1,\ldots,m,l\neq i} \sum_{j=1}^{n} w_j c_j(x_i,x_l)\\ &\geq \min_{l=1,\ldots,m,l\neq k} \sum_{j=1}^{n} w_j c_j(x_k,x_l),\\ &k=1,\ldots,k,k\neq i\}.\end{aligned} \tag{42}$$

The rest of the analysis is done as in SMAA, with the exception that the criteria measurements are considered to be deterministic (no integration over χ is done), and therefore no confidence factors are computed. It should be noted, that now the central weight vector can lie outside the space of favourable weights of an alternative, because this preference model is non-linear. Therefore, in this kind of (easily detectable) situations a favourable weight vector is chosen with a minimal distance to the central weight vector.

In the literature there exists simulation-tests of SMAA against SMAA-3, and in these tests the results of SMAA-3 were found to be quite unstable with respect to the indifference threshold (Lahdelma and Salminen, 2002). Therefore, when SMAA-3 is applied in practice, great care should be put into choosing the thresholds.

There exists also a variant of SMAA which applies the complete ELECTRE III procedure for producing a ranking. For more details on it, see Tervonen et al. (2004a).

4.5.2 SMAA-TRI

All the SMAA variants described until here are for ranking or choosing problem statements. ELECTRE TRI (Yu, 1992) is a method for sorting problem statements, and SMAA-TRI (Tervonen et al., 2005) extends it to allow ignorance on the parameter values.

ELECTRE TRI uses concordance and discordance indices for sorting the alternatives into predefined and ordered categories. Let us denote by $C = \{C_1,\ldots,C_h,\ldots,C_k\}$ the set of categories in ascending preference order (C_1 is the "worst" category). These categories are defined by upper and lower profiles, that are computationally equivalent to alternatives. The profiles are denoted as

$p_1, \ldots, p_h, \ldots, p_{k-1}$. Profile p_h is the upper limit of category C_h and the lower limit of category C_{h+1}. Notice that the profiles are strictly ordered, that is they have to satisfy

$$p_1 \,\Delta\, p_2 \,\Delta\, \ldots \,\Delta\, p_{k-2} \,\Delta\, p_{k-1}, \tag{43}$$

where Δ is the dominance relation ($p_1 \Delta p_2$ means that p_2 dominates p_1). This dominance relation needs to be interpreted in a wide sense, because the domination depends not only on the values of components of the two profiles, but also on the values of thresholds.

We will not describe the assignment procedure here, the interested reader should refer to Tervonen et al. (2005). For the assignment procedure an additional technical parameter, the lambda cutting level, has to be defined.

SMAA-TRI is developed for parameter stability analysis of ELECTRE TRI, and consists of analyzing finite spaces of arbitrarily distributed parameter values in order to describe for each alternative the share of parameter values that assign it to different categories. It analyzes the stability of weights, profiles, and the cutting level.

The input for ELECTRE TRI in SMAA-TRI is denoted as follows:

1. Uncertain or imprecise profiles are represented by stochastic variables ϕ_{hj} with a joint density function $f_\Phi(\phi)$ in the space $\Phi \subseteq R^{(k-1)\times n}$. The joint density function must be such that all possible profile combinations satisfy (43). Usually the category profiles are defined to be independently distributed, and in this case the distributions must not overlap. For example, if the profile values for a criterion are Gaussian distributed, the distributions must have tails cut off as shown by the horizontal lines in Figure 6.

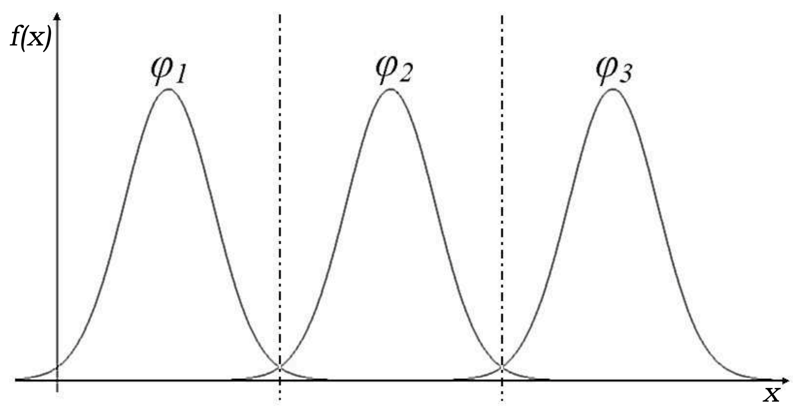

Figure 6: Probability distribution functions for three Gaussian distributed profile values (for a single criterion). The horizontal lines show where the tails of the distributions must be cut. (Tervonen et al., 2005)

2. The lambda cutting level is represented by a stochastic variable Λ with a density function $f_L(\Lambda)$ defined within the valid range [0.5,1].

3. The weights and criteria measurements are represented as in SMAA-2.
4. The data and other parameters of ELECTRE TRI are represented by the set $T = \{M, q, p, v\}$. These components are considered to have deterministic values.

SMAA-TRI produces category acceptability indices for all pairs of alternatives and categories. The category acceptability index π_i^h describes the share of possible parameter values that have an alternative x_i assigned to category C_h. Let us define a *categorization function* that evaluates the category index h to which an alternative x_i is assigned by ELECTRE TRI:

$$h = K(i, \Lambda, \phi, w, T), \tag{44}$$

and a category membership function

$$m_i^h(\lambda, \phi, w, T) = \begin{cases} 1, & \text{if } K(i, \Lambda, \phi, w, T) = h, \\ 0, & \text{otherwise,} \end{cases} \tag{45}$$

which is applied in computing the category acceptability index numerically as a multi-dimensional integral over the finite parameter spaces as

$$\pi_i^h = \int_{0.5}^{1} f_L(\Lambda) \int_{\phi \in \Phi} f_\Phi(\phi) \int_{w \in W} f_W(w) m_i^h(\Lambda, \phi, w, T)\, dw\, d\phi\, d\Lambda. \tag{46}$$

The category acceptability index measures the stability of the assignment, and it can be interpreted as a fuzzy measure or a probability for membership in the category. If the parameters are stable, the category acceptability indices for each alternative should be 1 for one category, and 0 for the others. In this case the assignments are said to be robust with respect to the imprecise parameters.

5 Simulation

The various distributions applied in the integrals of SMAA vary according to the application and can be arbitrarily complex. Usually the integrals have high dimensionality as well. The analytical integration techniques based on discretizing the distributions with respect to each dimension are infeasible, because the required effort depends exponentially on the number of iterations. Therefore, instead of trying to obtain exact values for the integrals, Monte Carlo simulation is applied to obtain sufficiently accurate approximations. In this section we address the simulation technique, accuracy of the computations, and the complexity issues. For description of the actual algorithms, we refer to Tervonen and Lahdelma (2006).

5.1 Simulation technique

Monte Carlo simulation is applied in computation of the integrals. For all the acceptability index-type measures, a similar technique is applied: in each iteration, measurements for the parameters (criteria measurements, weights, ...) are drawn from their corresponding joint distributions, and a ranking or a classification is built based on these values. After this, counters for corresponding ranks or classes with respect to the alternatives are increased. After a number of iterations, the indices are obtained by dividing the counters with the number of iterations. The central weights are computed in a similar fashion, by adding to the "sum of weight vectors" of the alternative obtaining the best rank the currently used weight vector. This vector is divided in the end component-wise by the number of iterations in order to obtain the central weight vector.

The weight generation is an important part of the simulation technique. If there is no preference information available, the n uniform distributed weights are generated as follows: first $n-1$ independent random numbers are generated from the uniform distribution within the range $[0,1]$, and sorted into ascending order ($q_1, \dots, q_{n-1}$). After that, 0 and 1 are inserted as the first (q_0) and last (q_n) numbers, respectively. The weights are then obtained as intervals between consecutive numbers ($w_j = q_j - q_{j-1}$) (Tervonen and Lahdelma, 2006).

If there exists preference information, the weight generation technique must be altered. In the case of complete ordinal preference information, the weights can simply be sorted according to the ranking. Lower bounds for weights can be handled by using a simple transformation technique. Let us illustrate this by re-examining Figure 2. By considering only the lower bound $w_1 \geq 0.2$, the feasible weight space is re-defined as one homomorphic with the original one. The lower bounded weights are defined by generating the random numbers from interval $[0, 1-s]$, where s is the sum of all lower bounds, and adding to them the corresponding lower bounds.

Upper bounds for weights cannot be handled by using a similar technique, but instead a simple rejection technique is applied, in which the weight vectors not satisfying the upper bounds are rejected. As can be seen in Figure 2, the tip of the simplex cut off by the upper bounds has relatively small area compared to the one of lower bounds. Therefore the increase in computational complexity due to upper bounds is relatively low. In addition, lower bounds might even render some of the upper bounds redundant. Consider for example a 3-criteria problem with lower bounds of 0.2 for all weights. The maximum value that any weight can obtain is $1 - 0.2 - 0.2 = 0.6$, and therefore all upper bounds higher than 0.6 are redundant. The amount of weights rejected due to upper bounds can be estimated in the following way: if we consider all weights to have a common upper bound w^{max}, the probability for the largest of the generated weights to exceed the upper bound is

$$
\begin{aligned}
P[max\{w_j\} > w^{max})] = n(1-w^{max})^{n-1} - \binom{n}{2}(1-2w^{max})^{n-1} \\
+ \cdots (-1)^{k-1}\binom{n}{k}(1-kw^{max})^{n-1} \cdots ,
\end{aligned}
\tag{47}
$$

where the series continues as long as $1 - kw^{max} > 0$ (David, 1970).

5.2 Accuracy of the computations

Accuracy of the computations can be calculated by considering the Monte Carlo simulations as point estimators for the descriptive measures. To achieve accuracy of A with 95% confidence for the rank acceptability indices, we need the following number of Monte Carlo iterations K:

$$K = \frac{1.96^2}{4A^2}. \tag{48}$$

For example, to achieve 95% confidence on error limits of ± 0.01 for the rank acceptability indices, we need to execute 9604 Monte Carlo iterations. The accuracy of confidence factors depends on the accuracy of central weight vectors in a complicated manner (Tervonen and Lahdelma, 2006), but if we disregard this source of error, the same equation for accuracy applies. The accuracy of the central weight vectors depends on the acceptability indices, and the required amount of iterations is calculated as follows:

$$K = \frac{1.96^2}{a_i 4A^2}. \tag{49}$$

It should be noted, that the accuracy of the computations does not depend on the dimensionality of the problem, but instead only on the number of iterations.

5.3 Complexity issues

The required number of Monte Carlo iterations in typical SMAA applications is fairly high, and therefore for having practical applicability the complexity of SMAA computations should not be too high with respect to the number of criteria and alternatives. The complexity of SMAA-2 and SMAA-O has been analyzed by Tervonen and Lahdelma (2006). The complexity of computing the acceptability indices and central weight vectors with independent criteria measurements and cardinal criteria is $O(K \cdot (n \log(n) + m \cdot n + m \log(m)))$. The complexity of computing the confidence factors is $O(K \cdot m^2 \cdot n)$. In these formulas K is the number of Monte Carlo iterations, m the number of alternatives, and n the number of criteria.

The usage of ordinal criteria adds to the complexity with a factor of $\log(m)$. In practice this has very little effect (Tervonen and Lahdelma, 2006). What has a larger impact to the running times is the handling of preference information. The formulas above suppose that there are no constraints on the weights, which in practice is usually not the case. As described in Section 5.1, lower bounds for weights do not affect the complexity of the weight generation, but upper bounds might have a great impact on the process.

6 Applications

SMAA was originally developed in conjunction with a real-life decision-making problem, and has been since applied in a variety of real-life cases. We will briefly present three different cases for illustrative purposes.

6.1 Infrastructure planning

In 1990, the city council of the capital of Finland, Helsinki, decided that a suburban area, Vuosaari, needed to be reserved for a general cargo harbor. The new city plan that contained the allocation of the land area was approved at 1992. After this, a planning process was initiated. According to the Finnish laws, Environmental Impact Assessment (EIA) needs to be done in this type of planning processes, taking into account also the opinions of residents of the affected areas. The actual decision process began with the construction of criteria taking into account environmental, sociological, as well as economical viewpoints. This leaded into definition of the following 11 criteria: sea, ground water, emissions into air, fauna, vegetation and flora, employment, recreational possibilities, landscape, and economy.

Different alternatives for developing the harbor area were constructed with different combinations of the naval navigation channel, roads, and railroads. A set of 24 alternatives was constructed in this way, and in accordance with the EIA legislation, also an additional alternative was added. This alternative considered improving existing facilities without constructing a new harbor (so-called zero-alternative). Thus, multiple criteria decision aiding was applied with a total of 25 alternative strategies.

The purpose of the EIA procedure is not to present a solution to the decision-making problem, but instead, to describe the effects of different possible actions. For the criteria considering environmental effects, determination of the values is usually objective and therefore uncertainties have to be taken into account. In decision-making processes such as this, it is often the case that the DMs are not willing to provide preference information. The original SMAA method that allows tackling problems with ignorance on the preference information and criteria measurements was developed in conjunction with this decision-making process.

After SMAA analysis, the results were presented to the DMs. Many different interest groups feared that their favourite alternative would not be among the most preferred ones, as they saw the method "too fair", taking into account all points of view. During the process, however, they understood that it is possible to choose almost any alternative based on certain preferences. In the end, the groups carried out real discussion about the values present in the process. This decision-making process leaded into a decision to build the Vuosaari harbour, and the first stone of the harbour was placed in 2003, after a long public political discussion.

This application has been described in detail by Hokkanen et al. (1999). For other applications of SMAA in infrastructure planning, see Hokkanen et al. (1998, 2000); Lahdelma et al. (2002).

6.2 Forest planning

Landspace ecological planning is an area where the time horizon is usually very long and the data concerning possible alternatives uncertain and imprecise. In Finland the state-owned forests cover nearly 9 million hectars and require landscape ecological plans. One of these plans was made for the Kivalo forest area located in the Finnish Lapland (Kangas et al., 2003a). In this study, 10 possible development stategies for a timeframe of 10 years were evaluated in terms of 2 cardinal and 3 ordinal criteria. The study demonstrated the applicability of SMAA-O in forest planning, especially as mixed (both ordinal and cardinal), imprecise and uncertain criteria measurements and vague preference information could be taken into account.

For other forest planning related studies, see Kangas and Kangas (2003); Kangas et al. (2005).

6.3 Elevator planning

Modern elevator systems in high-rise buildings consist of groups of elevators a with centralized control. The goal in elevator planning is to configure a suitable elevator group to be built, satisfying minimum requirements for the quality of service. In addition, it is desirable to use the least floor space possible and to minimize the cost of the system. During the planning, the measurements for some criteria can be estimated by experts. This is the case with, for example, price. For other criteria this is not possible, and simulation is required to determine the scores.

In the work by Tervonen et al. (2006), a realistic elevator planning problem of the above type is considered. It consists of a 20-floor buiding for which one of 10 possible elevator group configurations has to be chosen. The alternative configurations were analyzed using the KONE (one of worlds leading elevator manufacturers) Building Traffic Simulator. Based on the output of the simulator, the criteria values for performance-related criteria could be defined with a multivariate Gaussian distribution. The study presents an interesting application of the SMAA methodology: an area traditionally unconnected with MCDA is linked by using external simulation providing parameters for the distribution to be used. In this type of commercially linked studies in which the stakeholders are representatives of different companies, it is important that the interests of different groups of stakeholders are identified, as well as are compromise solutions. This was accomplished with SMAA in this case.

7 The framework

We define the SMAA framework for deciding a method to choose on a specific decision making context. The first question to ask is whether we are dealing with a ranking or sorting problem statements. If we are dealing with sorting problem, the only method of SMAA family we can use is SMAA-TRI. With ranking problems, we have to choose the type of preference model we have: whether it is based on weights or on reference points. If we have a weight-based model, we have to choose the type of aggregation procedure: utility (or value) function or outranking method. With the reference point approach we have to choose whether we want to use prospect theory (loss aversion model) or achievement functions in the aggregation. With all this information, we can choose whether to apply SMAA-2, SMAA-3, SMAA-P, or Ref-SMAA for the ranking problem. Depending on the method to apply, we obtain as output different descriptive measures that can some be used to derive “second-order” aggregate measures. Choosing of the method is presented as a decision-tree in Figure 7.

In the context of the framework, we should notice that all other methods than Ref-SMAA, which is based on reference points, can be used with arbitrary weight information. This means that we can apply them with no preference information at all, as well as with mixed information of ordinal and cardinal types. In practice the most useful ones are (partial) ordinal information and cardinal weight constraints. Complex weight constraints might be hard for the DMs to understand, and therefore by using more complex distributions the possibility for the information to contain uncertainty increases. If the DMs have problems understanding the underlying preference model, the achievement function based approach (Ref-SMAA) might be more suitable.

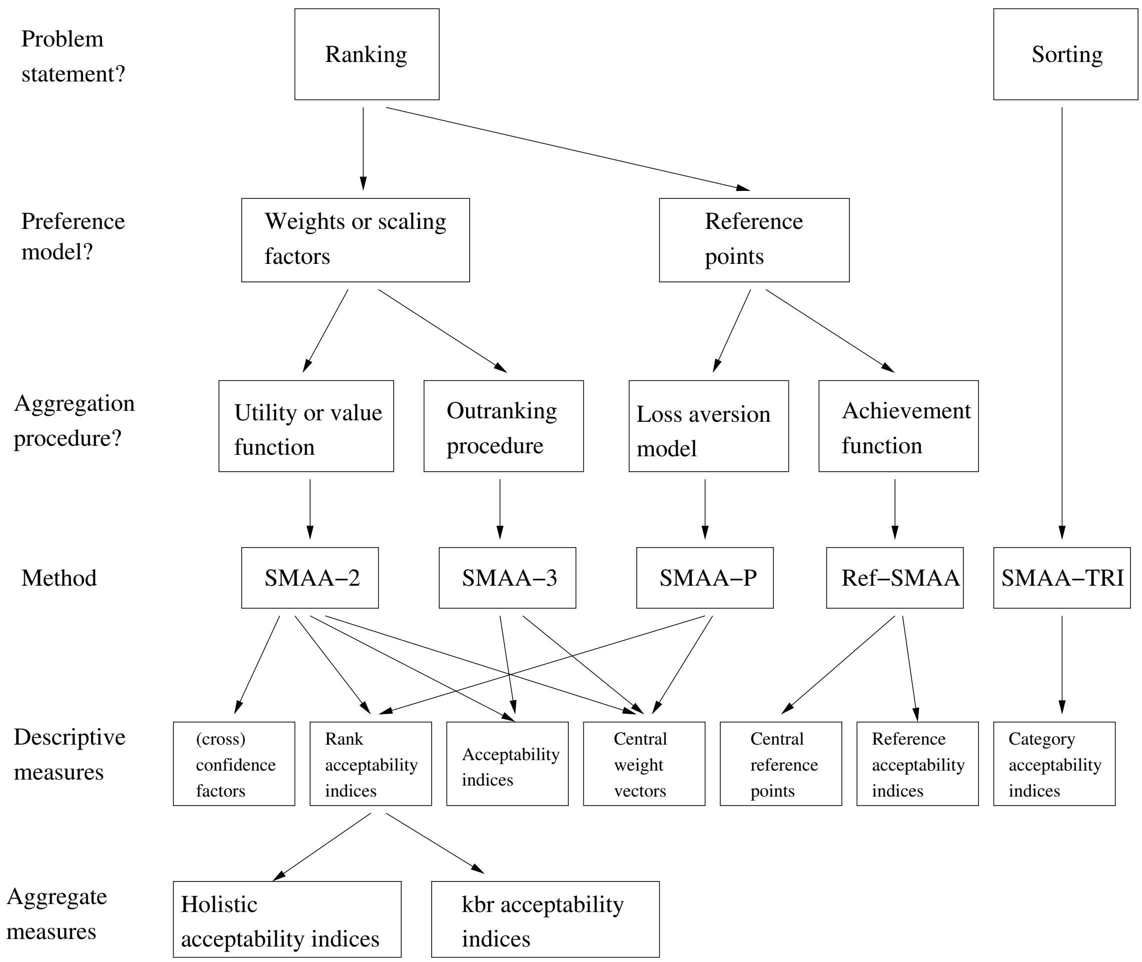

Figure 7: Decision-tree for choosing the SMAA variant.

The shortcoming of the utility-function based approach (SMAA-2) is that the scaling has large effect on the results, and the meaning of the weights is based on the scale. Therefore, if the shape of the utility function is hard to define, it might be more suitable to use SMAA-3 instead.

Arbitrarily distributed imprecise or uncertain criteria can be applied in all methods of the family except SMAA-3, that requires criteria measurements to have imprecision defined by the thresholds. It should be noted, that SMAA-O is not a stand-alone method, but rather a computational technique for handling ordinal criteria measurements. The possibility of using external sampling and the following generalisation to use SMAA with external methods can be considered a great advantage. For example, the approach applied in SMAA-TRI can probably be applied to other methods as well, to use them with ignorance on the parameter values in order to analyze the stability of the results.

One of the unsolved questions in SMAA is how to obtain aggregated measures based on the rank acceptability indices. The holistic acceptability indices can be used for this purpose, but they require meta-weights to be defined. This is an artificial concept with no connection to a cognitive decision-making process, and therefore their use is hard to justify. It might be, that to obtain more easily interpretable measures, we need to make models more complex. This would mean adding more parameters or using a more complex preference model. But on the other hand, the complexity introduced in this way brings new sources of ignorance. More research should be put on this subject.

8 Conclusions

SMAA is a recent methodology providing a general framework that has extensions to handle various specifities in multiple criteria decision aiding problems. In this paper, we presented the two basic methods, SMAA and SMAA-2, and the most important extensions of the methodology. The SMAA framework derived from these methods allows the decision analyst to choose the specific model to apply depending on the characteristics of the problem.

The SMAA framework allows to use methodology in a broad range of decision making contexts. Nevertheless, there exists unsolved questions, the most important being if we can develop aggregate measures that would help further in the decision making process. The holistic acceptability index is such, but its applicability in practice is questionable. Therefore, future research on the methodology should address this area. Other crucial need is a user-friendly and computationally efficient software implementing the methodology. There is currently available an open-source implementation of the basic methodology (by one of the authors, downloadable from http://monet.fe.uc.pt/thesessoftware/), but it lacks a graphical user interface. As the principles of SMAA are quite simple though the equations for computing the descriptive measures look complicated, we believe that a software with a graphical user interface would allow the methodology to be applied in every-day decision-aiding problems by users less adapted to the techniques of numerical computation.

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