Optimal Supply Chain Management with Detailed Scheduling

A.C.S. Amaro

Inst. Sup. Contabilidade e Administração, ISCAC

Quinta Agrícola, 3040 Coimbra

Portugal

and

A.P.F.D. Barbosa-Póvoa*

Centro de Estudos de Gestão do IST (CEG-IST)

DEG - Instituto Superior Técnico,

Av Rovisco Pais, 1049-001 Lisboa Codex

Portugal

(Submitted to European Journal of Operations Research, March 2005)

Abstract

In this paper a new model formulation is develop for industrial supply chains operation. This accounts for the supply chain structural and dynamic characteristics where different *topological*, *operational* and *marketing* characteristics (market supplies, demands and price levels) are considered in a single level formulation.

The formulation results into a Mixed Integer Linear Program (MILP) model which relies on the discretization of the time horizon of operation into intervals of equal duration.

As final solution the model provides a detailed supply chain operational plan where supplying, production, inventory and transportation are jointly scheduled so as to optimise a pre-defined economical or operational performance criterion.

Finally, the flexibility and applicability of the new formulation is validated through the solution of a practical example.

Keywords: supply chain management, scheduling, optimisation, discrete models, flexibility.

* Author to whom correspondence should be addressed: email: apovoa@ist.utl.pt

Introduction

Historically, the firms focussed on independent decision making processes supported by decoupled management strategies as a way of dealing with the existing organisational complexity. The geographical and functional requirements were then treated independently and supported by buffers of large inventories. However, in this way almost all management dependencies were disregarded leading to high cost decisions. Consequently, and in order to manage the multiple conflicts imposed by nowadays market requirements the firms started focussing their efforts on achieving global as well as integrated managing policies that would allow cost reduction and quality services improvement for a growing set of products (Thomas and Griffin, 1996).

Within this context integrated supply chain structures, SC, appear as an appropriated answer to this new firm's environment. The operational flexibility that can be reached by managing, in an integrated form, supply chain structures allows important improvements concerning both, economical and operational performance issues (Erengçu *et al*, 1999).

A former integrative approach, that accounts for the linkage between topological and functional supply chain constituents as material suppliers, production facilities, distribution services and costumers, has been proposed by Stevens (1989). Supply chain management, SCM, was defined based on the control of: (i) the feed forward flow of materials and (ii) the feedback flow of information. Following this integrative perspective a stream of research work focussing on the impact of materials and information flows integration between chain members emerged. New emphasis was attempt to the coordination strategies (Chandra and Fisher, 1994), the partnership concepts (Maloni and Benton, 1997) and more recently to information technology integration (Gunasekaran and Ngai, 2004).

As Maloni and Benton (1997) reports, despite the extensive conceptual literature, very few researchers have attempted more rigorous analytical approaches to SC issues, and the existing sparse rigorous contributions do not support the entire chain, since they have been limited to coordination of just some of the many SC functions. Vidal and Goetschalckx (1997) recognised common drawbacks in the strategic design of production-distribution systems and again the importance of integrated approaches incorporating operational details as well as uncertainty was emphasized.

Later on, Erengçu et al (1999) on its review of production/distribution planning models defined three different stages within the supply chain: supplier stage, production stage and distribution stage. The need for further research work on any of these stages was identified. This should address a large spectrum of aspects ranging from the role of the inventory till the need of more detailed analytical and simulations models that would considered the three stages in an integrated way.

At another perspective, the SC performance driven by the products variety was emphasised by the analytical approximation proposed by Thonemann and Bradley (2002). The requirement for further improvements concerning SC structures to which the model can result intractable, were identified.

Recently, Shah (2004) presented the state of the art in supply chains design and operation for the process industries and discusses challenges for the future. The importance of aspects such as; considerations of both business and physical processes, topological and operational complexity and scale, different information structures and cultures, strategic decisions uncertainties, sustainability and environment, amongst other, was identified.

Following this new trends, new simulation approaches, as Hung et al (2006), strives the integration of SC using object-oriented architecture to enabling flexible configurations, operational requirements and policies. As the authors' reports, simulation models are an important first step toward realistic optimisation. Although, future developments concerning the production scheduling, algorithms and transportation dynamics are required.

In conclusion, many of the research proposed so far covers a broad range of methodologies with an increasing level of complexity. However, a high number of formulations relied on several simplifications that reduce the problem dimension as well as its formal complexity so as to allow some near-optimal solutions to be achieved. Some space remains when trying to globally integrate SC multifunctional and geographical disperse structure while attempt to accurate operational details.

In this paper, the different stages of the supply chain are considered in an integrated form and a new model formulation for the detailed operation of the supply chain is developed. The proposed model has the advantage of combining the supply chain structure, process recipes and operational conditions as well as the transport operating policies and demand requirements into a single framework. This unambiguously represents the supply chain topology and its dynamic operability, translated by all the legal processing and transport operations defined for the set of handled materials. In terms of topology the model formulation allows the consideration of multiple sites with different geographical locations and accounts for the relationship existing amongst them (connectivity network). In terms of operability, different operating policies as well as production, storage, distribution and transportation requirements are allowed. Material requirements imposed by market conditions (customer demand) or pre-defined processing considerations are also explicitly integrated.

All problems are formulated as a Mixed Integer Linear Programming, MILP's model that is solved using a standard Branch and Bound procedure.

The rest of the paper is structured as follows. First, the supply chain structure is characterised and its operability is described. Then the model representation is presented followed by a detailed description of the mathematical formulation. The solution of a practical example is afterwards explored where the applicability and flexibility of the proposed model are shown. Finally, in the last section, some conclusions are drawn and some future improvements are discussed.

2. Problem Characteristics

A supply chain is a *master operational structure* that produces a defined set of *suitable materials* (intermediate and final products) so as to satisfy the demand of one or different *costumer markets* geographically disperse, using *internal* (production, storage and transport resources, etc) and *external resources* (raw-materials, utilities, etc), figure 1.

The *internal resources* define the *supply chain internal structure* (figure 1, central block) and establish its functional suitability, operational capacity and processing pre-conditions. Additionally, the *demand* requirements, defined by the costumers, determine the supply chain *operability levels* in terms of operations to be performed, resources usage and times schedules, as well as induce some backward demand into the external supplying market. At the same time, forward relations resulting from changes at the external resource supply can arise. Operability adjustments may then result accordingly and changes within the supply conditions may influence the achievable production levels and the fulfilment of the demand.

Figure 1 - Supply chain master characterization.

The global supply chain is then characterised by four main functions: (i) supply, (ii) production (iii) packing and (iv) distribution. These define the global chain operation that is guaranteed through the use of the installed resources (equipment and facilities) and transportation structures accordingly to their operational characteristics.

The installed resources can be grouped into *sites* (figure 2a) with different geographical locations. These are responsible for the fulfilment of a defined operational purpose based on a global master operational plan. The supply chain sites are grouped into clusters accordingly to their operational purposes, suitability, and functional similarity, figure 2a. Four types of clusters are considered; supplying, production, packing and distribution. The latter supplying a set of aggregated costumers regions – markets.

The transportation structures, on the other hand, ensure the mobility of material amongst the sites and into/from the external market (figure 2b). A transportation structure is a group of autonomous transport resource units (e.g. vehicles, trucks, train wagons, boat containers, etc) and is characterised by having the same or different capacity dimensions, a common functional similarity (materials suitability), an ownership criteria (in-house or contracted resources) and an operating mode (dedicated or shared mode). Concerning the ownership criteria, each supply chain site may have its own transportation capacity, using it in a dedicated mode to ensure the fulfilment of the material flows to its chain partners or, alternatively, it may contract the whole or a fraction of the required transport capacity to a third part logistics company, using it in a dedicated mode or sharing it with other partners.

Figure 2 - Supply chain: (a) internal structure and (b) connectivity network.

In conclusion the supply chains characteristics can be summarised as follows. Firstly, a supply chain is a master operational structure involving the linkage between multifunctional and geographically disperse facilities that are combined into a global network configuration. Secondly, the operation (transformation, storage and transport) of the supply chain instances (resources within sites and connectivity networks) depends on the resources installed, its capacity and suitability, as well as on the performance requirements defined for a given time horizon due to market considerations (customer demand) and/or other supplying and operational pre-condition criteria.

3. Problem Representation

The supply chain operability and its dynamic characteristics are described based on a new representation. This takes the advantage of some modelling concepts embedded within the *State Task Network*, *eSTN*, (Barbosa-Póvoa and Machietto, 1994) and in the *Flowpaths* (Amaro and Barbosa-Póvoa, 1998) representations.

The proposed representation focus on three main components: (i) *structure* – characterised by all the admissible sites and by the connectivity network that allows the materials to flow between them; (ii) *operation* – done through the occurrence of *transportation flows* and *general processing tasks*, and (iii) *materials* – specified by considering an adequate number of material states with an associated storage capacity and operating requirements.

In the structure two types of resources are considered: processing and transportation resources. The first ones are available at the supply chain sites and guarantee the processing tasks while the second ones ensure the transport of materials along the structure. It is important to note that the processing resources involve not only the common transformation resources (e.g reactor or facility) but also the storage resources (e.g. tank or warehouse).

In terms of operation two main events are considered: *tasks* and *transportation flows*. A task reports the linkage between an operation and a suitable resource *j* to execute it. Each resource is assigned to only one task at a time *t.* A *task* ***i***, operating in resource *j*, consumes a certain amount of input materials, $\boldsymbol{s} \in \overrightarrow{S_i}$ and produces another amount of output materials, $\boldsymbol{s} \in \overleftarrow{S_i}$, both defined by the task recipe, after $\boldsymbol{pt_i}$ time units, fixed operating time. A task may involve a chemical transformation of material, a physical transformation or a simple storage of material (for further details refer to the *equipment State Task Network - eSTN*, Barbosa-Póvoa and Macchietto, 1994).

A transportation flows l is characterised by *a material* to transport, a *transportation structure* (set of resources), a chain *path* and a fixed *transportation time (Flowpath representation*, Amaro and Barbosa-Póvoa, 1998). In here a major operational concept is the flows incompatibility. That is modelled through the definition of an incompatibility matrix, $\overline{\overline{IM}} = \left[IM_{l,l'}\right]_{l,l'=1,\ldots Nflows}$, covering all the pairs of transportation flows, ***l*** and ***l'***, characterising the transportation structures operation (*l, l'*=1,*...Nflows*). Therefore we have:

$$\forall\, l,l' \in \{1,\ldots Nflows\},\;\; IM_{l,l'} = \begin{cases} true & if \;\; \text{transportation flows } l \text{ and } l' \text{ are incompatible} \\ false & if \;\; \text{transportation flows } l \text{ and } l' \text{ are compatible} \end{cases}$$

Two transportation flows are incompatible if they share at a given time a common resource and the materials associated are incompatible or if they flow in opposite directions. All the resources available within each transportation structure have the same incompatibility set. The simultaneous operation of more then one transportation flow sharing a given resource requires flows compatibility. On the other hand, incompatible flows can only operate during the same time intervals if they are allocated to different resources.

For tasks and transportation flows a non pre-emptive mode of operation is allowed. Therefore, any of these instances once started cannot be interrupted.

Finally, a material state represents unambiguously the different materials and its possible locations within the global supply chain structure. If a single purpose suitable storage capacity is available at a certain state location the material may be stored. This represents a storage mode where no resource sharing between materials is allowed ($S_{s,t}$). If on the other hand the material state can be stored in a multipurpose resource a general processing task, as defined above, is used to model such situation ($Qp_{i,j,t}$).

In summary, the structure involves the supply chain constituents as: suppliers, plants, distribution sites, storage, transportation structures and their resources - *supply chain topology*. The operation describes the entire operating policies through a set of tasks and transportation flows. Additionally, by setting an appropriate number of *material states* it is possible to integrate the structure and the operation considering different operational pre-conditions and different market and supplying requirements.

4. Problem Formulation

Based on the problem characteristics and its model representation (section 2 and 3) a new mathematical formulation is developed for the scheduling of industrial supply chain structures.

The proposed formulation relies on a discretization of time where the scheduling horizon is divided into a number of elementary time steps of fixed length, Δ_t. All supply chain events are allowed to occur at the intervals boundaries and not between them. The completion time of each model event (transforming task or transportation flow) is defined as an integer multiple of the elementary time step Δ_t.

Supply chain events (*tasks, i,* and *flows, l*) and the associated instances (equipments resources, *j* and *v*, structures, $\pi \in \Pi$, etc) are characterised by a set of: (i) *binary* variables, $Y\lambda_{v,l,t}$ and $Yp_{i,j,t}$, defining the assignment of events ($i \in I$ and $l \in L$) to the suitable instances ($j \in J$ and $v \in \Gamma_\pi$,), at any time space *t*, of the scheduling domain, $t \in H$; (ii) *continuous or integer variables* that translate the assigned capacities or the occupation rate of instances by events, at any time *t* ($Q\lambda_{v,l,t}$, $Qp_{i,j,t}$, and $S_{s,t}$ for storage events, $j \in JStor \subseteq J$). Also general market supply and demand requirements are accounted by defining an appropriate number of *continuous*, (e.g. bulk materials) and *integer* variables (e.g. packed materials), $Recp_{s,t}$ *and* $Delv_{s,t}$.

The problem symbols details and definition (indices, sets parameters and variables) is provided at the nomenclature section.

A Mixed Integer Linear Programming (MILP), formulation is developed. This involves five types of linear constraints: 1.- *operability constraints*, 2.- *capacity constraints*, 3.- *general operating requirements* constraints, 4.- *material balance constraints*, and 5.- i*nitial and final bound constraints*. Also a linear objective function responsible for the evaluation of an economical criterion is defined.

4.1. Constraints

1.- Operability

The *operability constraints* include all the operating requirements arising from the assignment each model event (tasks and flows), independently of any capacity consideration for the involved instances. This set of constraints accounts for operational: (a) *incompatibility*, (b) *continuity* and (c) other *generic* conditions.

(a) ***Events Incompatibility:***

The incompatibility constraints are defined for the transportation flows and processing tasks and guarantee that incompatible occurrences of events do not occur:

Transportation Flows:

$$\sum_{t'=t}^{t-tt_l+1} Y\lambda_{v,l,t'} + \sum_{t'=t}^{t-tt_{l'}+1} Y\lambda_{v,l',t'} \leq 1 \qquad \forall \pi \in \Pi,\ \forall v \in \Gamma_\pi, \qquad (4.1)$$

$$\forall l,l' \in F_\pi : l' \succ l \wedge IM_{l,l'} = true, \forall t = 1,...H$$

The incompatibility matrix, $\overline{\overline{IM}}$, is symmetric ($IM_{l,l'}=IM_{l',l},\ \forall l,l' \in L$) and accordingly only the elements above the diagonal line ($IM_{l,l'}$: $l'>l$) are required for the incompatibility constraints generation ($F_\pi \subseteq L,\ \forall \pi \in \Pi$).

Processing Tasks:

$$\sum_{t'=t}^{t-pt_i+1} \sum_{i \in JI_j} Yp_{i,j,t'} \leq 1 \qquad \forall j \in JP,\ \forall t = 1,...H \qquad (4.2)$$

(b) ***Continuity***

Continuity guarantees that a non-interrupt mode of operation is observed for both model events: *flows* and *tasks*.

Mathematical relations are developed exclusively for the fully compatible flow events ($IM_{l,l'} = false,\ \forall l,\ l' \in F_\pi$), since for task events, *i*, and for flow events, *l*, having at least one incompatible flow, *l'*, defined over the same transportation structure π ($\exists l,\ l' \in F_\pi$:

$IM_{l,l'} = true$), the incompatibility constraints previously defined ensure both, the operational compatibility and the continuity relations.

Fully Compatible Flows

$$\sum_{t'=t}^{t-tt_l+1} Y\lambda_{v,l,t'} \leq 1 \qquad \forall \pi \in \Pi : \bigcap^{l \in F\pi} \left(¥_{\pi,l} \cup \{l\} \right) = F_\pi, \forall v \in \Gamma_\pi, \forall l \in F_\pi, \forall t = 1,...H \qquad (4.3)$$

Furthermore, compatible transport events, defined through the same transport structure, can only share a given transport instance if they are assigned within a common time frame:

$$\frac{1}{\vartheta_{l,\pi}} \sum_{l' \in ¥_{\pi,l}} \sum_{t'=t-1}^{t-tt_{l'}+1} Y\lambda_{v,l',t'} + Y\lambda_{v,l,t} \leq 1 \qquad \forall \pi, \forall v \in \Gamma_\pi, \forall l \in F_\pi : tt_l > \Delta_t, \forall t = 1,...H \qquad (4.4)$$

There, the parameter $\vartheta_{l,\pi} = \left| ¥_{\pi,l} \right|$, is introduced in order to account for the normalization of the double summation.
No set of constraints is required for those flow events, l, that spent a single time unit ($tt_l = \Delta t$), since this is formulation intrinsic.

2.- *Capacity Constraints*

The capacity constraints ensure the compromise between the assignment of events and the capacity bounds imposed by the associated instances.

Task events - these events may be assigned to instances characterized by a:
(i) *nominal capacity*

$$CJ_j \, \Phi_{i,j}^{\min} \, Yp_{i,j,t} \leq Qp_{i,j,t} \leq CJ_j \, \Phi_{i,j}^{\max} \, Yp_{i,j,t} \quad \forall j \in JB, \forall i \in JI_j, \forall t = 1,...H \qquad (4.5)$$

(ii) *processing rate*

$$CJ_j \, pt_i \, \Phi_{i,j}^{\min} \, Yp_{i,j,t} \leq Qp_{i,j,t} \leq CJ_j \, pt_i \, \Phi_{i,j}^{\max} \, Yp_{i,j,t} \qquad \forall j \in JP \setminus JB, \forall i \in JI_j, \forall t = 1,...H \qquad (4.6)$$

The capacity dimension parameter, CJ_j, is accordingly, multiplied by the working or processing period, pt_i, defined for the task event, i, while assigned to the suitable instance j.

Flow events

(i) Single events assignment:

$$Q\lambda_{v,l,t} \leq CV_v \; Y\lambda_{v,l,t} \qquad \forall \pi, \; \forall v \in \Gamma_\pi, \forall l \in F_\pi, \forall t = 1,... H \qquad (4.7)$$

(ii) Simultaneous allocation of compatible events to a single suitable instance.

$$\sum_{l \in F_\pi} Q\lambda_{v,l,t} \leq CV_v \qquad \forall \pi, \; \forall v \in \Gamma_\pi, \forall t = 1,... H \qquad (4.8)$$

Furthermore a lower bound limit may also be defined within the instance, v, operability.

$$\sum_{l' \in ¥_{\pi,l}} Q\lambda_{v,l',t} + Q\lambda_{v,l,t} \geq CV_v \; \Psi_v \; Y\lambda_{v,l,t} \qquad \forall \pi, \; \forall v \in \Gamma_\pi, \forall l \in F_\pi, \forall t = 1,...H \qquad (4.9)$$

The parameter Ψ_v represents the minimal percentage of the instance capacity that justifies any transport event assignment.

Storage Tasks events:

Each storage event, resource unit (vessels, tanks, containers, etc) or facility (warehouse, space floor, etc) is bounded by its usable capacity, $\partial_j^{use} CJ_j$:

$$\sum_{s \in KStor_j} S_{s,t} \leq \partial_j^{use} CJ_j \qquad \forall j \in JStor, \forall t = 1,...H+1 \qquad (4.10)$$

Based on the feasibility rates, $\varphi_{s,j}^{\min}$ and $\varphi_{s,j}^{\max}$, some upper and lower bounds can be defined for each storable state, s, while assigned to a suitable storage instance, j.

$$\varphi_{s,j}^{\min} \; CJ_j \; \delta_j^{use} \leq S_{s,t} \leq \delta_j^{use} \; CJ_j \; \varphi_{s,j}^{\max}$$
$$\forall j \in JStor, \forall s \in KStor_j, \forall t = 1,...H+1 \qquad (4.11)$$

3.- *General operating requirements*

The operating requirements involve: (1) *external delivers and receivers* that describe the requirements and pre-conditions governing the material flows between supply chain positions and the external market, (2) *contracted transportation structures operation* characterising the assignment of contracted resources to perform transport operations, and finally, (3) *scheduling time bound constraints* that translate the operational conditions of tasks and transports.

(1) *External Delivers and Receivers* – flows between chain sites and the external market.

(i) *Delivers*, $Delv_{s,t}$ - materials going from supply chain sites to the external market.

– *if* the transport of materials is performed by the external ordering customer:

$$\overleftarrow{D}_{s,t}^{Low} \leq Delv_{s,t} \leq \overleftarrow{D}_{s,t}^{UP} \qquad \forall s \in S, \forall t = 1,...H+1 \qquad (4.12)$$

– *if* the transport of materials is supported by the supply chain sourcing site:

$$\overleftarrow{D}_{s,t}^{Low} \leq \sum_{l \in \vec{S}L_s} Q\Lambda_{l,t} \leq \vec{D}_{s,t}^{Up} \qquad \forall s \in S_{out}, \ \forall t = 1,...H+1 \qquad (4.13)$$

$$Q\Lambda_{l,t} = \sum_{v \in \Gamma_\pi} Q\lambda_{v,l,t} \qquad \forall \pi, \ \forall l \in F_\pi, \forall t = 1,...H \qquad (4.14)$$

(ii) *Receivers*, $Recp_{s,t}$ - materials going from the external market into the supply chain sites.

- *Programmed Supplies* - minimal and maximal *market supply capacity bounds*:

$$\vec{R}_{s,t}^{Low} \leq \mathrm{Re}\, cp_{s,t} \leq \vec{R}_{s,t}^{UP} \qquad \forall s \in S, \forall t \in T\sup_s \qquad (4.15)$$

- *Non-Programmed Supplies* - if feasible, material receipts are based on a pre-defined number of discrete charges $NCh_{s,t'}$, of fixed dimension, QCh_s.

$$NCh_{s,t'} \geq \frac{\mathrm{Re}cp_{s,t'}}{QCh_s} \qquad \forall s \in S, \ \forall t' \notin T\sup_s \qquad (4.16)$$

(iii) *Contracted Transportation Structures Operation*

$$Yv_{v,t} - \frac{1}{\Xi_\pi} \sum_{l \in F_\pi} \sum_{t'=t}^{t-tt_l+1} Y\lambda_{v,l,t} \geq 0 \qquad \forall \pi \in \Pi \setminus \Pi_{own}, \forall v \in \Gamma_\pi, \forall t = 1,...H \quad (4.17)$$

(iv) *Scheduling time bound constraints* - *Scheduling periods, T* - set of discrete and independent time units (group of unitary time intervals as daily working timetables, a day of the week, a week of the month, etc).

$$\sum_{t'=t-1}^{t-tt_l+1} Y\lambda_{v,l,t'} \leq 0 \qquad \forall \pi, \ \forall v \in \Gamma_\pi, \forall l \in F_\pi : tt_l > \Delta_t, \forall t = \frac{T}{\Delta_t},...N_H\left(\frac{T}{\Delta_t}\right),\left(\frac{T}{\Delta_t}\right) \quad (4.18)$$

$$\sum_{t'=t-1}^{t-pt_i+1} Yp_{i,j,t'} \leq 0 \qquad \forall j \in JP, \ \forall i \in JI_j : pt_i > \Delta_t, \forall t = \frac{T}{\Delta_t},...N_H\left(\frac{T}{\Delta_t}\right),\left(\frac{T}{\Delta_t}\right) \quad (4.19)$$

These inequalities ensure that, no resource instance (*v* or *j*) is assigned to a feasible event (*l* or *i*), at a time *t* (within a tt_l-1 or pt_i-1 neighbourhood of an ending period), if the event conclusion is not within the schedule period time bounds ($T, 2T, \ldots N_H T$– going from *T* to *H*).

4.- *Material Balance Constraints*

The material balance constraints on each state *s* at every time interval *t* relate the amount of material in the state ($S_{s,t}$) with the existent in the previous time ($S_{s,t-1}$) and the amount being produced ($Qp_{i,j,t-pti}$), consumed ($Qp_{i,j,t}$) and transferred ($Q\Lambda_{l,t-ttl}$, $Q\Lambda_{l,t}$, $Delv_{s,t}$ and $Recp_{s,t}$) into/from the state.

$$S_{s,t} = S_{s,t-1} + \sum_{l \in \vec{SL}_s} Q\Lambda_{l,t-tt_l} - \sum_{l \in \overleftarrow{SL}_s} Q\Lambda_{l,t} + \sum_{j \in JP} \sum_{i \in JI_j \wedge i \in \vec{SI}_s} Qp_{i,j,t-pt_i} \vec{\alpha}_{s,i} \qquad (4.20)$$

$$- \sum_{j \in JP} \sum_{i \in JI_j \wedge i \in \overleftarrow{SI}_s} Qp_{i,j,t} \overleftarrow{\alpha}_{s,i} + \mathrm{Re}cp_{s,t} - Delv_{s,t} \qquad \forall s \in S, \forall t = 1,...H+1$$

Note that, accordingly to the measuring units used at the $Q\Lambda$ and *Qp* evaluations conversion factors may be required.

5.- *Initial and Final Bound*

The model formulation allows the possibility of considering initial and final conditions to the main model events (tasks and transport flows). Those operating relations are introduced by setting a specific value to the variables that describe the main occurrences on the supply chain operation.

$$Y\lambda_{v,l,t}\ \big|_{t=0} \ = \ Y\lambda_{v,l,t}\ \big|_{t=H+1} = 0 \qquad \forall \pi \in \Pi,\ \forall v \in \Gamma_\pi,\ \forall l \in F_\pi \qquad (4.23)$$

$$Yv_{v,t}\ \big|_{t=0} \ = \ Yv_{v,t}\ \big|_{t=H+1} = 0 \qquad \forall \pi \in \Pi,\ \forall v \in \Gamma_\pi \qquad (4.24)$$

$$Yp_{i,j,t}\ \big|_{t=0} \ = \ Yp_{i,j,t}\ \big|_{t=H+1} = 0 \qquad \forall j \in JP,\ \forall i \in JI_j \qquad (4.25)$$

These equality relations state that no event assignment is taken before the beginning of the scheduling horizon and no further assignments are feasible, at the end of time schedule.

$$Q\lambda_{v,l,t}\ \big|_{t=0} \ = \ Q\lambda_{v,l,t}\ \big|_{t=H+1} = 0 \qquad \forall \pi \in \Pi,\ \forall v \in \Gamma_\pi,\ \forall l \in F_\pi \qquad (4.26)$$

$$Q\Lambda_{l,t}\ \big|_{t=0} \ = \ Q\Lambda_{l,t}\ \big|_{t=H+1} = 0 \qquad \forall \pi \in \Pi,\ \forall l \in F_\pi \qquad (4.27)$$

$$Qp_{i,j,t}\ \big|_{t=0} \ = \ Qp_{i,j,t}\ \big|_{t=H+1} = 0 \qquad \forall j \in JP,\ \forall i \in JI_j \qquad (4.28)$$

$$S_{s,t}\ \big|_{t=0} \ = \ SI_{s,0} \qquad \forall s \in S' \qquad (4.29)$$

Also, some ending conditions reporting the final storage levels can be integrated.

4.2 Objective Function - *Profit Analyses*

The objective function is the maximisation of the net profit which is defined based on the: (1) material states income and costs and (2) supply chain event costs.

$$F.O.\ \ Max\ z : z = \underbrace{\frac{\text{materials}}{\text{assets}} + \text{Delivers} - \text{Receipts} - \text{Storage}}_{\text{material states}} \ - \ \underbrace{\frac{\text{In-house}}{\text{transport}} - \frac{\text{Contracted}}{\text{transport}} - \frac{\text{Processing}}{\text{operations}}}_{\text{supply chain events}} \qquad (4.30)$$

(1) *Material States*

(i) *Material assets* - material states produced but kept in storage, balanced at the ex-work or producing price, pw_s.

$$\sum_{s \in S'} pw_s \, (S_{s,H+1} - S_{s,0})$$

(ii) *Delivers Income*- Resulting from the materials flowing to the external market.

$$\underbrace{\sum_{s \in S \backslash S_{out}} \sum_{t=1}^{H+1} pw_s \; Delv_{s,t}}_{\text{transport ensured by customers}} + \underbrace{\sum_{s \in S_{out}} \sum_{t=1}^{H} \sum_{l \in \bar{IS}_s} pm_s \; Q\Lambda_{l,t}}_{\text{transport done by supply chain source site}}$$

(iii) *Receipts costs* – Due to the materials flowing into the supply chain sites

$$\underbrace{\underbrace{\sum_{s \in S_{raw}} \sum_{t=1}^{H+1} pw_s \operatorname{Re}cp_{s,t}}_{\text{programmed supplies}} + \underbrace{\sum_{s \in S_{raw}} \sum_{t \notin T\sup_s} CCh_s \; NCh_s}_{\text{non programmed supplies}}}_{\text{raw materials}} + \underbrace{\sum_{s \in S \backslash S_{raw}} \sum_{t=1}^{H+1} pw_s \, (1 + \Delta pw_s) \operatorname{Re}cp_{s,t}}_{\text{other material states}}$$

This involves raw material and other material states. For the former two different situations are considered: (i) *programmed supplies* evaluated at a source price (pw_s.) and (ii) *non-programmed* supplies have an add-on cost, (CCh_s) proportional to the number of discrete charges, of fixed dimension, (NCh_s) required. Other material states receipts, if feasible, are evaluated at a market retailing price, $pm_s=pw_s(1+\Delta pw_s)$.

(iv) *Storage costs*

The storage cost involves a fixed and a variable term. The former (*FSCj*) is independent of the instances usage and accounts for fixed expenses as equipment maintenance (vessels, tanks, etc) or storage capacity (warehouse spaces). Instead, the variable term depends on the amount of each material stored, $S_{s,t}$, as well as on the unitary storage cost, SC_s, defined for the storage of an unitary amount of a material state during a pre-defined storage period (T_s- a time interval, a day, a week, a month, etc).

Two different approaches can then be considered for the storage period:

(i) a time interval- the costs are balanced by the allocated amount of material states defined by the B.O.M.(4.30); or (ii) a set of time intervals- the amount of each material state stored during a period is evaluated based on the arithmetic mean of the stored amounts crossing the time intervals that defines the storage period (second option in the below equation).

$$\underbrace{\sum_{j\in JStor} FSC_j + \sum_{s\in S'} \sum_{t=1}^{H+1} SC_s\, S_{s,t}}_{if\, T_s=\Delta_t} \quad \text{or} \quad \underbrace{\sum_{j\in JStor} FSC_j + \sum_{s\in S'} \sum_{t=T_s^{'}}^{H+1\,(T_s)} \sum_{t'=t}^{t-T_s^{'}+1} SC_s \frac{S_{s,t}}{T_s^{'}}}_{if\, T_s\neq\Delta_t}$$

(2) Supply Chain Events

Three main types of costs are considered:

(i) *In-house transport* costs

$$\underbrace{\sum_{\pi\in\Pi_{owner}} \sum_{v\in\Gamma_\pi} \Bigg[\; FTC_v}_{\text{fixed costs}} + \underbrace{\sum_{t=1}^{H} \sum_{t'=t}^{t-tt_l+1} \sum_{l\in F_\pi} (VC_v^1 + \frac{VC_v^2}{\Xi_\pi})\, Y\lambda_{v,l,t} \Bigg]}_{\text{variable costs}}$$

(ii) *Contracted transport* costs

$$\underbrace{\sum_{\pi\in\Pi\backslash\Pi_{owner}} (\sum_{v\in\Gamma_\pi} \sum_{t=1}^{H} TC_v\, Yv_{v,t})}_{\text{VariableCosts - Equipmentusage}} + \underbrace{FTC_\pi\, CT_\pi}_{\text{Fixedcost of the contractedcapacity}}$$

(iii) *Processing* costs

$$\underbrace{\sum_{t=1}^{H} \sum_{j\in JP} \sum_{i\in JI_j} FC_j\, Yp_{i,j,t}}_{\text{Fixed Assignment Costs}} + \underbrace{\sum_{t=1}^{H} \sum_{j\in JP} \sum_{i\in JI_j} OC_j\, Qp_{i,j,t}}_{\text{Variable Operating Costs}}$$

In conclusion, the objective function (4.30) and the constraints (4.1) to (4.29) define the MILP formulation that models in detailed the operation of a supply chain. This was implemented in the GAMS® language and solved using a standard Branch and Bound (B & B) procedure (CPLEX©).

5. Example

A supply chain is considered where different blends for civil construction purposes as well as for wood related industries are produced on different facilities structures distributed geographical over different country locations. The supply chain structure is characterized by four types of *clusters* (Figure 2 (a)): (i) supplying (S1 to S5), (ii), production (I1 and I2),

(iii) packing and labelling (PL1 and PL2) and (iv) distribution (DC1 to DC10). These are linked by a *connectivity network* (figure 2 (b)) involving thirteen transportation structures (π=1,..13) that guarantee the materials flow along the chain.

Suppliers S1, S2 and S3 provide three classes of raw materials (*Ch_A, Ch_B, Ch_C*) to the plants, I1 and I2. Suppliers S4 and S5 supply, both packing sites, PL1 and PL2, respectively with the finishing additives (*AdF* and *AdR*) and with the required packing materials (*P50L* and *P5L* containers).

At plant I1, two preparing lines are installed, PP1 and PP2, that produce respectively the intermediate blend *IB*1, through the transformation of 55% *Ch_A* and 35% of *Ch_B*, and *IB*2, from the conversion of equal amounts of raw chemicals *Ch_A* and *Ch_C*, table 1.

Furthermore, plant I2 has a single production line, PP3, that produces the intermediate blend *IB*2 using the same amounts of *Ch_A* and *Ch_C*, table 1.

The final blends, *FB*1 to *FB*6, are obtained at the packing sites PL1 and PL2. Each one of these sites is characterised by two independent resource structures, that perform finishing operations followed by the suitable packing and labelling tasks, table 1.

At the PL1 site, the processing structure PLS1 produces 50 *u.v.* containers of *FB1* and *FB3* *FB*3, while PLS2 operates in a dedicated mode to *FB2,* producing 5 *u.v.* capacity containers. Formerly, a finishing task (corresponding to the addition of different percentages of an agent, *AdF/AdR*, into the sourced *IB* blends) is performed and it is then followed (with a non-waiting policy) by a suitable packing and label operation, table 1.

An equivalent situation arises at PL2 site, there a dedicate operation is observed for PLS3, that produces containers of 5 *u.v.* capacity of wood coating blend *FB*6. Instead, PLS4 structure produces 50 *u.v.* containers of concrete blend *FB*4 and wood infusion *FB*5. As it was reported for PL1 site, to reach each final blend, a non-waiting finishing tasks is performed before suitable packing and label operation, table 1.

Each one of the transforming operations mentioned (producing, finishing and packing) are characterised by a specific time duration (pt_i), and are processed within a suitable and finite capacity structure (mass, volume, etc., CJ_j). These data as well as the materials suitability's are shown in tables 1.

Table 1- Supply chain processing instances description.

Concerning the demand requirements, each packing and labelling site must fulfil a cluster of distribution sites located on its neighbourhood so as to guarantee the market requirements of a set of aggregated costumers or geographical positions (tables 2). PL1 supplies distribution sites, DC1 to DC6 located on the North of Portugal while PL2 supplies DC7 to DC10 distribution sites located at South of Portugal. The geographical positions supplied by each DC site have minimal weekly demand requirements as well as upper daily capacity absorption, defined as the maximal daily demand fulfil (table 2).

All the supply chain sites have their own storage instances accordingly to the process specificity and requirements. Dedicated and sharing policies are presented (table 2).

Table 2- Distribution sites characteristics and market demand requirements.

Concerning the flows of material, along the supply chain, these are ensured by a set of transportation structures defined within the connectivity network where 56 transport flows are defined (see Figure 3 a,b,c).

Each supply chain site may have in-house dedicated transport resources and it may also contract some transport capacity to an external company (contracted structures), table 3. In the latter a dedicated or sharing situation may arise between sites.

Each transportation structure has a limited capacity, resulting from the set of transport resources defined within the structure, table 3. Its usage is limited to the set of suitable materials that can be transported.

Figure 3 – Supply Chain structure characteristics- Transport Structures- (a) Suppliers to Plants; (b) Plants, additive and packing material supplier, S4 and S5, to PL sites and (c) PL to DC sites.

Table 3 – Supply chain transportation structures characteristics.

Furthermore, due to the materials proprieties and characteristic dimensions (bulk, packed, etc) some organization criteria were defined for the set of material states that may share a common transport instance (*AdF* / *AdR*, *P5L* /*P50L*, *FB1*/ *FB3*, or *FB4*/ *FB5*), tables 3.

Apart from the material suitability criteria, capacity requirements such as minimal and maximal equipment charges were considered. A percentage of 10% of the transportation capacity is defined as minimal charge for raw-materials and intermediate blends while a higher percentage (20%) exists for final blends. A full resource charge is allowed as an upper capacity bound.

Also, material storage operational pre-conditions were defined. At any scheduling time the stock levels of any storable material must reach a minimal bound of 5% of the storage capacity dedicated to that material state. Instead, at the beginning of the scheduling horizon each supply chain site has a minimal stock of every suitable product defined as 25% of the storage capacity dedicated to the material state. The same stock levels must be left at the end of the time horizon in order to allow the starting of the next scheduling period.

Concerning the material receipts, receipts of raw and packing materials from the external market to the supplier sites S1 to S5 are programmed to each day morning and charges are limited to 4000 *u.m* for raw chemicals, 80 *AdF*, 60 *AdR*, 500 *P50l* and 3000 *P5l* units per charge for additives and packing materials with an order expense of 150 €/ charge. On the other hand, any external receipt of material to the plants or packing sites is 35% more expensive than the equivalent amount fulfilled by a chain partner (S1 to S3 for the plants and S4, S5, I1 and I2 for the packing sites).

Finally, the economical values used are defined in tables 4. Each material has a storage cost, SC_s and two market values, the sourcing or production price, pw_s, and the retailing price, $pm_s = pw_s(1+\Delta pw_s)$. The latter with an incremental percentage relatively to the former, defined as 35% higher.

Concerning the model events (production, packing and transport operations) a fixed and a variable cost term is defined, table 5.

Table 4 - Production price and storage cost for the material states.

Table 5 - Fixed and Variable costs for the supply chain events (tasks and flows).

Having defined the case-study characteristics this is now solved for a scheduling horizon of a five days working week with 8 hours/day.

Three operational scenarios were studied: (i) ***Close Operation*** - only the sites defined within the *supplying cluster* are sourced by an outside supplier (external market) and no supply chain site, other than the DC sites, can perform delivers to the outside customers; (ii) ***Partially Open Operation*** - the delivers of white products (not packed intermediates, *IB1* and *IB2*) are allowed, within defined market capacity bounds, and every supply chain site may use external material supplies. For plants and packing sites external receipts will be performed at the retailing price, pm_s; (iii) ***Totally Opened Operation*** - equivalent to the previous one, but allowing all the supply chain sites to deliver their output materials to external customers (white and packed materials), accordingly to a defined market capacity.

The scheduling problem was solved, for these operational scenarios, considering the maximization of the supply chain global profit as the optimisation purpose. The results are shown in table 6 and figures 4 to 7.

Due to the lack of space, a single operational scenario (*Closed Operation*) will be detailed and some main remarks will be provided for the remaining two.

The optimal scheduling obtained for the closed operation scenario leads to a global profit of 514752,35 €/week (table 6). The costly operation is the external sourcing of materials (542702,15 €/week). This accounts for the programmed supplies of chemicals, additives and packing materials as well as for some further non-programmed sourcing operations performed at the chain suppliers (S1 to S5).

As a global behaviour, the transportation performances increase while saving at travelling times and handling of stocks, guaranteeing high service levels.

Particularly, the supplier sites are encouraged to decide on some contracted transportation capacities since a profitable operation results through the reduction of travelling times (materials sourced from the geographically closer supplier, S1 for I1 and S3 for I2) even if it requires costly transportation capacities (*e.g.* contracted structures, π=1,3 and 4, figure 4). A similar strategy, concerning the reduction of travelling times, is observed between plants and PL partners as well as between the packing and additive suppliers (S4 and S5) and the PL sites., figure 5.

Another important profit reduction results from the economical evaluation of the processing (producing and packing) operations (350362,77 €/week). This accounts for both, variable (255462,77 €/week) and fixed (94900,00 €/week) operational costs. The available structures operate at their upper capacity limits during almost all the week, since a profitable operational policy is achieved by splitting the variable costs amongst higher batch dimensions (figure 6).

The global performance observed suggest a very constrained production (reaching the minimal demand levels) for the less profitable final products *FB*3 within PL1 and *FB*4 within PL2. This arises because competition for a specific packing structure, PLS1 at PL1 and PLS4 at PL2, between *FB*1 and *FB*3 as well as between *FB*4 and *FB*5 is observed. Also, competition for *IB*1 blend exists between *FB*1 and *FB*2 at PL1 site and for *IB*2 blend, between *FB*5 and *FB*6, at PL2 site.

Concerning the transport costs (70402,00 €/week) they account for both, fixed and variable expenses. Due to the capacity limitations and profitability issues the transportation instances are usually used at their upper capacity limits, especially when the transport decisions involve higher operational costs (contracted structures, e.g. π=13, figure 8). Resource sharing is not a relevant working practice for any structure since the amounts required of each material state justifies almost the time a full transportation charge. However, the geographical dispersion of the DC sites and due date requirements enforce some final product transports, that do not require a full charge and consequently unused capacity is observed (figures 7 and 8).

The profitable transport operation between PL and DC sites is achieved through the use of the existing transportation structures (π=10 to 13) so as to ensure the of minimal demand requirements at the distribution sites (figure 7 and 8). This is done while combining the storage costs of final blends at PL and DC sites, with the minimal demand levels, due dates requirements and capacity bounds, observed for each DC market.

The storage costs (29985 €/week) summed up with the production levels are the main responsible for the lower storage levels observed around every site during the week. Almost all the material states exhibit a storage profile not far from the defined safety levels and even at the end of the schedule horizon only few material states cross the defined final storage bounds. As a result, the profit incomes achieved by the weekly existences ($\sum S_{s,H}$-$S_{s,0}$) is low (112061,11 €/week), table 6.

Finally, the delivers of final blends into the supply chain final costumers or aggregated positions are the main responsible for the global profit achieved. These are governed by the minimal distribution requirements for the tight due dates and less profitable final blends and by the production and transportation performances achievable at this constrained operational scenario.

As a final remark, it may be observed that the scheduling plan achieved involves a complex and combined set of cost and capacity decision factors, namely those arising from: (i) transforming (production, packing, etc), transportation and storage costs, capacities and suitability for each material and chain site; (ii) allocation of transport resources (typically internal structures to satisfy demand requirements and to overcome the fixed transportation costs; and, contracted transport structures to fulfil some non reached minimal demands); (iii) external market capacity (profit sources that prevent the increase of stock and overcome some transport limitations) and finally, (v) minimal material demands fixed by final costumers (enforces the material flows along the chain and outlines some production and packing requests) .

Moving from the closed to the **partially opened scenario** (table 6) a profit increase is observed. This is justified namely by: (i) *External Market Receipts* – as a result of the new sourcing conditions the raw-materials availability at the plant sites and of intermediate blends at the PL sites increase and some sourcing constraints observed at the closed operational scenario are overcome by performing direct supplies from the external market; (ii) *different production and packing schedules* – as a result of the increment of the materials availability at plants and PL sites. The intermediate blends production is almost the same and only a small increase on plant I1 production rate is observed. An equivalent situation arises for the PL sites ; *(iii) reschedule of final stock* - The storage costs increase slightly while moving to the partially opened scenario but, not as much as the week existences (table 6). This suggests a stock increment close to the final schedule time in order to increase material existences without incurring on further storage costs. (iv) *delivers of non packed materials to the external market*- in this scenario the intermediate blends produced at plant sites can go straight on to the external market costumers in order to reduce transportation costs and achieve a profitable operation, table 6.

For the **opened market scenario**, when the market constraints imposed to the final blends distribution are released the material flows to the external market customers increase and a global profitable operation (524130,71 €/week) is reached at the opened operational scenario, table 6. A global transportation reschedule is observed amongst chain partners as a result of the new market conditions. Also a better performance is achieved for the packing resources since the same fixed operating expenses were obtained (96600 €/week) for higher batch dimensions (variable costs- 261312,00 €/week). Accordingly some unitary profitable blends are reached, table 6.

The service levels to DC sites are satisfied at their minimal demand levels since a profitable operation is reached by delivering final blends directly to the external market customers instead of doing the materials transportation to the DC partners. These material flows to the outside customers go on from 25% of the produced final blends (for *FB*3) till 47,5% (for *FB*5 final blend). This distribution is performed while combining minimal demand requirements for each final blend with market daily capacity absorption and due dates.

These material delivers allow a transportation cost decrease, 65977,25 €/week (table 6) against 68142,25 €/week at partial opened scenario or 70402 €/week at the closed. Also, a better storage schedule at the chain sites that improve the capacity utilisation while reducing storage costs (28305,64 €/week) is observed.

In conclusion, the close market scenario is the one that produces the lower amounts of intermediate and final products, but at the same time, it is the one responsible for the higher global amount of materials transported amongst supply chain partners, from suppliers till DC site positions. On the opposite side, the opened market scenario is the most profitable one. This is due not only to the highly observed production and packing rates but also due to the combined and profitable policies between transportation and storage costs. These are enforced by the material flows amongst chain partners and from/into the external market environment.

Finally, in terms of model statistics table 7 reports the model dimension obtained for the three discussed scenarios. The models where solved using the GAMS® package coupled with the CPLEX® solver version 6.6.1 in a Pentium III. The optimal solutions were obtained for the LPs and CPU times reported while considering a maximal relative gap of 5%.

6. Conclusions

The inherent complexity typically found in supply chain structures requires detailed and robust models to handle all the requirements properly and effectively.

The formulation proposed allows the optimal scheduling of supply chains combining different structural and dynamic aspects within a single level formulation. Supply chain topology, feasible operability and processing pre-condition requirements, defined for each chain site, during the scheduling horizon are explicitly taken into account. Also transportations of materials are considered.

As final result the model provides a detailed operating plan at the production, storage and transportation levels where all processing, storage and transport occurrences are identified while satisfying pre-defined market requirements and guaranteeing a maximum plant profit.

A real case study was solved and good results obtained within a reasonable margin of optimality.

The drawback of the high generality of the proposed formulation is that the resulting MILPs may become hard to be solved. These is specially observed when a huge number of variables with an integer domain have to be considered, as in the solved case study, and also when a large number of hard constraints have to be satisfied for an also large number of binary variables (as incompatibility relations).

As future work the authors are exploring the model efficiency through some auxiliary model developments such as logical constraints and cutting planes amongst others.

Furthermore, and considering the model generality, the close loop supply chain situation is being studied and the model will be generalised accordingly.

Nomenclature:

CCh_s = unitary cost of each discrete charge of raw material state s, sourced by an external supplier at a non programmed supplying time (€/charge).

CJ_j = capacity of the processing resource *j*.

CV_v = capacity of transport resource *v* (volume, mass, etc).

CT_π = global transport capacity available in structure π ($\sum_{v \in \Gamma_\pi} CV_v$, volume, mass, etc).

$Delv_{s,t}$ = amount of material state *s* delivered by a specific supply chain site to external customers, at the beginning of time interval *t*.

$\overleftarrow{D}^{Low}_{s,t}$ / $\overleftarrow{D}^{Up}_{s,t}$ = minimal /maximal amount of material from state *s*, defined by external customers, at the beginning of time interval *t*. Minimal *market demand requirements*/ Maximal *market capacity absorption*.

F_π = { *l*: set of transportation flows *l* defined through the transportation structure π }

$FC_{i,j}$ = fixed operating cost incurred with the assignment of processing task *i* to instance *j* (€/batch; €/run).

FTC_v^{own} = fixed transport cost of an in-house resource *v*, (€/Δ_t).

FTC_π^{cont} = fixed transport cost defined for a contracted transportation structure accordingly with the contracted capacity, (€/(mass, volume, etc).

FSC_j = fixed storage cost of resource $j \in JStor$.

H – scheduling horizon. Number of unitary time intervals, Δ_t, characterising the operational time schedule.

$i \in I$ set of tasks characterising the operation of supply chain sites, *i=1,…NTask*.

$j \in J$ set of all processing and storage instances available at the supply chain sites, *j=1,…NJ*

JB = { $j \in JP$: set of *batch* processing resources}

JI_j = { *i*: set of processing tasks *i* that can be performed at resource *j* }

JP = {*j*: set of processing resources existing in the set of supply chain sites}

$JStor$ = {j: set of dedicated storage resources available in the set of supply chain sites}

$KStor_j$ = {s: set of material states s that can be stored in resource $j \in JStor$}

$l \in L$ set of transportation flows responsible for all the material transfers between supply chain sites, $l= l, \ldots Nflows$.

$OC_{i,j}$ = Size or dimension dependent operating cost of processing task i in resource j (€/(mass, volume, etc).

pm_s = unitary market price or retailing price for material state s (€/(mass, volume, etc)).

pt_i = processing time of a *batch* task i or of an operational *run* defined by continuous task i (completion or duration time).

pw_s = ex-work price or unitary producing price for material state s (€/(e.g.mass, volume).

QCh_s = fixed capacity defined to each non programmed discrete charge of raw material state s ((mass, volume, etc)/charge).

$Q\Lambda_{l,t}$ = global amount of material transported by flow l, through the set of suitable transportation resources, at the beginning of time interval t.

$Q\lambda_{v,l,t}$ = amount of material transported by flow l, while assigned to transport resource v, at the beginning of time interval t and during $\frac{tt_l}{\Delta_t}$ time intervals.

$Qp_{i,j,t}$ = amount of material processed by task i, using resource j, at the beginning of time interval t and during $\frac{pt_i}{\Delta_t}$ time intervals.

$s \in S$ set of all material states, material/location, existing in the supply chain $s=1, \ldots NSt$.

$S' \subseteq S$ = {s: set of material states that can be stored in a suitable instance, $j=1, \ldots JStor$}.

$\overleftarrow{S}_i$ / $\vec{S}_i$ = {s: set of *input*/ *output* material states required/ produced by task i, on a specific processing instance j}.

S_{raw} = {s: set of raw material states}.

S_{out} = {*s*: set of material states which external deliver is supported by the supply chain}.

SC_s = storage cost for material state $s \in S'$ (€/(amount . time).

$\vec{SI}_s / \bar{SI}_s$ = { *i*: set of processing tasks that produce/ consumes material into/from state *s*}.

$\bar{SL}_s$ / $\bar{SL}_s$ = { *l*: set of transportation flows *l* with *input/output* material in state *s*}.

$S_{s,t}$ = amount of material in state s (a material in a given location) available at the beginning of time interval *t*.

$Recp_{s,t}$ = amount of material state *s* received from the outside suppliers into a specific supply chain site.

$\vec{R}^{Low}_{s,t} / \vec{R}^{Up}_{s,t}$ = minimal/ maximal amount of material state *s*, demanded by/sourced to a specific supply chain site to/by the external market, at the beginning of time interval *t*. Minimal *sourcing requirements*/ Maximal *market supply capacity*.

tt_l = transportation time or travel duration for the transport flow *l*.

T – set of time intervals defining the scheduling operational period.

TC_v = transport cost defined for the assignment of a contracted transport resource *v* (€/Δ_t).

Ts – number of time intervals defined for each storage period.

Ts' – number of time units involved in each storage period, $Ts' = Ts + 1$.

$Tsup_s$ – discrete set of time units defined within the supplying program for material state *s*.

VC^1_v and VC^2_v = cost parameters defined to evaluate in-house transport resource *v*, *(€/Δ_t. l*).

$\Phi^{\min}_{i,j}$ / $\Phi^{\max}_{i,j}$ = minimal/ maximal utilisation factor of the processing resource *j*, by task *i*.

$\varphi^{\min}_{s,j} / \varphi^{\max}_{s,j}$ = minimal/ maximal percentage or proportion of the storage capacity defined for resource *j*, while dedicated to material state *s*.

$\vec{\alpha}_{i,s} / \bar{\alpha}_{i,s}$ = rate or proportion of material from state *s* undergoing/ leaving task *i*, accordingly with the task recipe.

δ^{use}_j = percentage of usable capacity defined for storage resource $j \in JStor$

Δpw_s = incremental cost percentage defined over the ex-work or over the sourcing price, to account respectively for the *out-door* fulfilments.

$\pi \in \Pi$ set of transportation structures, π, that ensures any material transport between supply chain sites, $\pi=1,\ldots N\Pi$.

Γ_π = { v: set of transport resources v, included in transportation structure π}

$\Pi_{own} \subseteq \Pi$ set of all in-house transportation structures.

$¥_{\pi,l}$ = { $l' \in F_\pi$: set of transportation flows l', defined through structure π, that are compatible with transportation flow l}

$\Lambda_\pi = \{¥_{\pi,l}, \forall l \in F_\pi$: set of all compatible transport flow sets defined through structure π $\}$

$\vartheta_{1,\pi} = \left|¥_{\pi,l}\right|$ = number of transportation flows l', defined through structure π, that are compatible with flow l.

$\Xi_\pi = \max_{\forall l \in F_\pi} \vartheta_{1,\pi} + 1$, maximum number of compatible transportation flows defined through the transportation structure π.

Ψ_v = utility factor for transportation resource v.

$$Y\lambda_{v,l,t} = \begin{cases} 1 & \text{\textit{if} the transportation flow } l \text{ is assigned to the transport equipment } v \text{ at the} \\ & \text{begining of time } t \\ 0 & \textit{otherwise} \end{cases}$$

$$Yp_{i,j,t} = \begin{cases} 1 & \text{\textit{if} processing task } i \text{ is assigned to equipment } j, \text{ at the begining of time } t \\ 0 & \textit{otherwise} \end{cases}$$

$$Y\nu_{v,t} = \begin{cases} 1 & \text{\textit{if} transport equipment } v \text{ is occupied at time } t, \text{ with the transport of some material(s)} \\ 0 & \textit{otherwise} \end{cases}$$

References

[1] Amaro, A.C.S. and Barbosa-Póvoa, A.P.F.D., 1998, Scheduling of industrial distribution manifolds with pre-condition, European Journal of Operation Research (EJOR), 199, 461-478.

[2] Barbosa-Póvoa, A.P.F.D., 1994, Detailed Design and Retrofit of Multipurpose Batch Plants, Ph.D. Thesis, Imperial College of Science, Technology and Medicine, University of London, London, U.K.

[3] Chandra, P. and M.L. Fisher, 1994, Coordination of production and distribution planning, European Journal of Operational Research (EJOR), 72, 503-517.

[4] Erenguç S., N.C. Simpson and A.J. Vakharia, 1999, Integrated production/distribution planning in supply chains: An invited review, European Journal of Operational Research (EJOR), 115, 219-236.

[5] Gunasekaran, A.and E.W.T Ngai, 2004, Information systems in supply chain integration and management, European Journal of Operational Research (EJOR), 159, 269-295.

[6] Hung, W.Y., N.J. Samsatli and N. Shah, 2006,Object-oriented dynamic supply chain modelling incorporated with production scheduling, European Journal of Operational Research (EJOR), 169, 1064-1076.

[7] Maloni, M.J. and W.G. Benton, 1997, Supply chain partnership: Opportunities for operations research, European Journal of Operational Research (EJOR), 101, 419-429.

[8] Shah, N (2004), Process Industries Supply Chains: Advances and Challenges, *Comp.-Aided Chem. Engng.*,18, Edts. A. Barbosa-Póvoa and H. Matos, Elsevier, 123- 138.

[9] Stevens, G.C., 1989, Integrating the supply chain, International Journal of Physical Distribution and Materials Management, 19 (8), 3-8.

[10] Thonemann, U.W. and J.R. Bradley, 2002, The effect of product variety on supply chain performance, European Journal of Operational Research (EJOR), 143, 548-569.

[11] Thomas D.J. and P.M. Griffin, 1996, Coordinated supply chain management, European Journal of Operational Research (EJOR), 94, 1-15.

[12] Vidal, C.J. and Goetschalckx M., 1997, Strategic production-distribution models: A critical review with emphasis on global supply chain models, European Journal of Operational Research (EJOR),143, 1-18.

Article Tables

Table 1- Supply chain processing instances description.

Instance, $j \in JP$ ╲ ***Site***	***I_1*** ***PP1***	 ***PP2***	***I_2*** ***PP3***	
Capacity, CJ_j^*	12500/ 16200	4250/ 4200	16500/ 16200	
Suitability, JI_j	*i1*	*i2*	*i3*	
Processing time, pt_i, *hr*	*4*	*2*	*4*	
Instance, $j \in JP$ ╲ ***Site***	***PL_1*** ***PLS1***	 ***PLS2***	***PL_2*** ***PLS3***	 ***PLS4***
Capacity, CJ_j^*	16000/ 16200	3200/4200	4000/ 4200	16000/ 16200
Suitability, JI_j	*i4, i6*	*i5*	*i9*	*i7, i8*
Processing time, pt_i, *hr*	4, 4	2	2	4, 4

* *u.v./u.m-* unit volume, *l*/ unit mass, *Kg*.

Table 2- Distribution sites characteristics and market demand requirements.

Storage Instance ╲ ***Site***	***DC_1*** 100 m^3	***DC_2*** 62,5 m^3	***DC_3*** 50 m^3	***DC_4*** 62,5 m^3	***DC_5*** 62,5 m^3	***DC_6*** 30 m^3
Capacity CJ_j, *(P50l/P5l)*[1]	2000/16000	1250/ 10000	1000/ 8000	1250/ 10000	1250/10000	600
Demand-Min.W/Max.D[2] *FB1* *FB2* *FB3*	 250/ 90 2000/ 900 250/ 60	 120 / 60 1800/ 900 150/ 80	 120/ 100 1200/ 1000 100/ 100	 180/ 90 1200/ 600 150/ 75	 200/ 90 2000/ 900 	 250/ 250
Max *days/week*	5	3 *(Mon/Wed/Fri)*	2 *(Tue/Thru)*	3 *(Mon/Wed/Fri)*	5	2 *(Tue/Thru)*
Storage Instance ╲ ***Site***	***DC_8*** 62,5 m^3	***DC_9*** 75m^3	***DC_{10}*** 100 m^3	***DC_{10}*** 100 m^3		
Capacity CJ_j, *50l/P5l*	1250/10000	1500/12000	2000/16000	2000/16000		
Demand-Min.W/Max.D[2] *FB4* *FB5* *FB6*	 150/ 100 150/ 100 1200/ 600	 250/ 100 250/ 100 2000/ 600	 360/ 250 3600/ 2400	 500/ 450 3000/ 2400		
Max *days/week*	3 *(Mon/Wed/Fri)*	5	3 *(Mon/Wed/Fri)*	2 *(Tue/Thru)*		

[1] *P5l* containers uses about 80% of the volumetric space unit (1 *P50l* ≡ 8 *P5l*); [2] W.- Weekly/ D.-Daily.

Table 3 – Supply chain transportation structures characteristics.

Suppliers S1 to S3	***π=1*** ***(contracted)***	***π=2*** ***(in-house)***	***π=3*** ***(contracted)***	***π=4*** ***(contracted)***	
Instance, *v*	*v1, v2*	*v3, v4*	*v5, v6*	*v7, v8, v9*	
Capacity, CV_v	3500, 3500 ^{m}	2000, 3000 ^{m}	4000, 6000 ^{v}	2500,2500,4000^m	
Supplier-Plants S4, S5 and I1,I2	***π=5*** ***(in-house)***	***π=6*** ***(in-house)***	***π=7*** ***(in-house)***	***π=8*** ***(contracted)***	***π=9*** ***(contracted)***
Instance, *v*	*v10, v11*	*v12,v13, v14*	*v15, v16, v17*	*v18, v19*	*v20, v21*
Capacity, $CV_v^{(1)}$	80 100 *AdF*	250,250,400 *P50L*	3500,3500,5000^m	4000,5000^m	5000,6000^m
Pack/labelling S4, S5 and I1,I2	***π=10*** ***(in-house)***	***π=11*** ***(in-house)***	***π=12*** ***(in-house)***	***π=13*** ***(contracted)***	
Instance, *v*	*v22, v23, v24,v25*	*v26, v27*	*v28, v29, v30*	*v31, v32*	
Capacity, $CV_v^{(2)}$	40,60,60,80*FB1*	600, 800 *FB2*	75, 75, 100 *FB4*	900, 1200 *FB6*	

(1) Volume Units: *P50L*= 8P*5L*, 1*AdR*= 2 *AdF*; (2) FB containers (weigh), 1 *FB1*=1,25 *FB3,* 1 *FB4* =1,20 *FB5*.

Table 4 - Production price and storage cost for the material states.

**Site S_i ** *State*	***1\Ch_A Ch_B***	***2\Ch_B Ch_C***	***3\Ch_A Ch_C***	***4\AdF AdR***	***5\ P50L P5L***	
Prod.Price, pw_s	0.42 0.28	0.5 0.75	0.50 0.75	34.5 134.5	1.25 1.75	
SC_s (€/u.m.day) 10^{-3}	2.52 1.68	1.68 4.20	2.52 4.20	0.207 0.816	0.0111 0.144	
FSC_j (€/st*.week)	180 120	200 200	200 200	300 200	240 560	
**Site ** *State*	***I1\ IB1 IB2***	***I2\ IB2***	***PL1\ FB1 FB2 FB3***		***PL2\ FB4 FB5 FB6***	
Prod. price, pw_s	0.90 1.50	1.50	175. 18. 180.		175. 180. 20.	
SC_s (€/u.m.day) 10^{-3}	5.40 9.00	9.00	1.05 0.108 1.08		1.05 1.08 0.108	
FSC_j (€/st*.week)	280 120	300	125 50 100		120 100 60	
**Site DC_i ** *State*	**1\FB1 FB2 FB3**	**2\FB1 FB2 FB3**	**3\FB1 FB2 FB3**	**4\FB1 FB2 FB3**	**5\FB1FB2**	**6\FB3**
SC_s (€/u.m.day)	1.05 0.108 1.08	1.05 0.108 1.08	1.05 0.108 1.08	1.05 0.108 1.08	1.05 0.108	1.08
FSC_j (€/st*.week)	126 105 119	84 126 140	108 102 90	102.4 115.2 102	144 156	250
**Site DC_i ** *State*	**7\FB4 FB5 FB6**	***8*\FB4 FB5 FB6**	***9*\ FB5 FB6**	**10\ FB5 FB*6***		
SC_s (€/u.m.day)	1.05 1.08 0.108	1.05 1.08 0.108	1.08 0.12	1.08 0.12		
FSC_j (€/st*.week)	128 128 144	144 144 112	160 240	240 160		

Table 5 - Fixed and variable costs for the supply chain events (tasks and flows).

Structure $j \in JP$	Fixed Costs, €	Var.Costs (€/u.m.)	***Structure*** $j \in JP$	Fixed Costs, €	Var.Costs (€/u.m.)	***Structure*** $j \in JP$	Fixed Costs, €	Var.Costs (€/u.m.)
I1 *PP1* ***1***	1200	0.18	***I1*** *PP2* ***2***	850	0.32	***I2*** *PP3* ***3***	1800	0.34
PL1 *PLS1* ***4*** ***6***	1700 2100	12.5 20.0	***PL1*** *PLS2* ***5*** ***PL2*** *PLS3* ***9***	300 400	2.0 2.4	***PL2*** *PLS4* ***7*** ***8***	1500 1750	18.5 17.5
Structure π	Fixed Costs, €	Var. Costs (€/charge*.Δt)	***Structure*** π	Fixed Costs, €	Var. Costs (€/charge*Δt)	***Structure*** π	Fixed Costs, €	Var. Costs (€/charge*Δt)
π=1 v_1, v_2		175, 175	π=6 $v_3, v_4,$ v_3	48,48 60	24,24 30	π=10 $v_{22}, v_{23},$ v_{24}, v_{25}	56,64 64,80	28,32 32,40
π=2 v_3, v_4	40,50	35,43.75	π=7 $v_{15}, v_{16} v_{17}$	60,60,72	53,53,63	π=11 v_{26}, v_{27}	72,84	36,73.5
π=3 v_5, v_6		200,275	**π=8** v_{18}, v_{19}		160,200	π=12 v_{28}, v_{29}, v_{30}	72,84,84	36,42,42
π=4 v_7, v_8, v_9		125,125,160	**π=9** v_{20}, v_{21}		190,220	**π=13** v_{31}, v_{32}		160,200
π=5 v_{10}, v_{11}	52,60	26,30						

* For in-house structures each transported material is balanced as a charge (to account for loading and handling costs) while for contracted structures those costs are implicitly contracted per travel or charge.

Table 6 – Economical values achieved for the computed operational scenarios.

Economic Issue (€/week)	**Closed**	**Partially**	**Opened**
Week Existences	112061,11	132941,83	99521,00
Delivers	1396144,00	1440299,30	1502400,00
Transport	70402,00	68142,25	65977,25
Production and Packing			
Variable	255462,77	259711,00	261312,00
Fixed	94900,00	96600,00	96600,00
External Receipts	542702,15	599259,15	625595,40
Storage	29985,83	31046,54	28305,64
Total *Cost issues*	993452,76	1054758,94	1077790,29
Total *Income issues*	1508205,11	1573241,13	1601921,00
Global Profit	514752,35	518482,19	524130,71

Table 7 - Computational statistics resulting for the computed operational scenarios.

Model Statistics		**Operational Scenario**	**Nº of Nodes**	**% of Optimality**	**CPU'sec**
Nº of Variable	14085	**Closed**	15226	2,07	5286
Nº of Integer Variables	9944	**Partial**	7387	4,04	2836
Nº of Constraints	34130	**Opened**	4017	4,14	1489

Article Figures

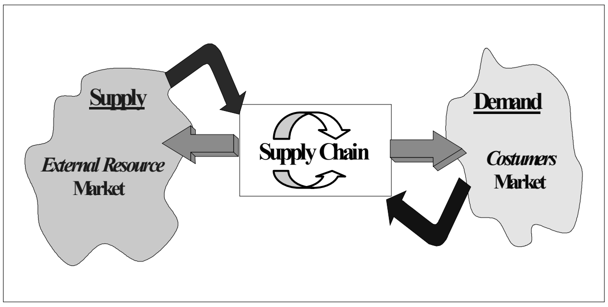


Figure 1 - Supply chain master characterization.

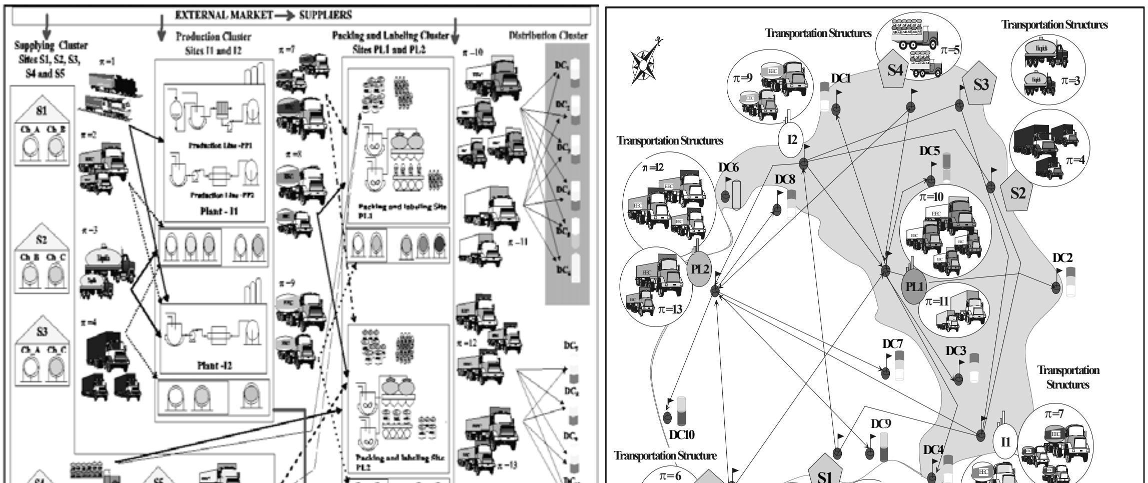


Figure 2 - Supply chain: (a) master structure and (b) connectivity network.

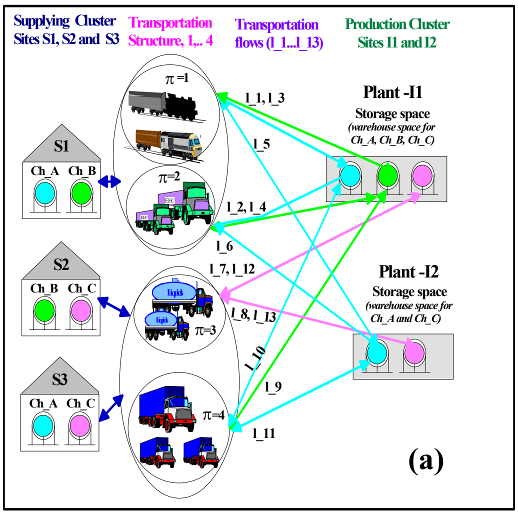
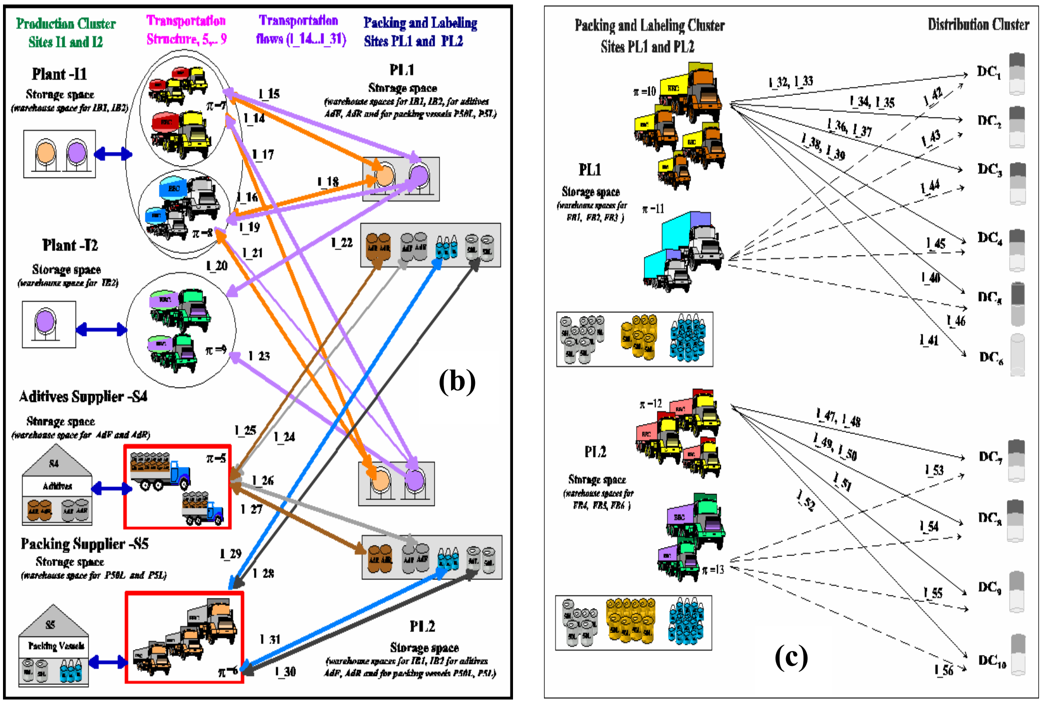


Figure 3 – Supply Chain structure characteristics- Transport Structures: (a) Suppliers to Plants; (b) Plants, additive and packing material supplier, S4 and S5, to PL sites and (c) PL to DC sites.



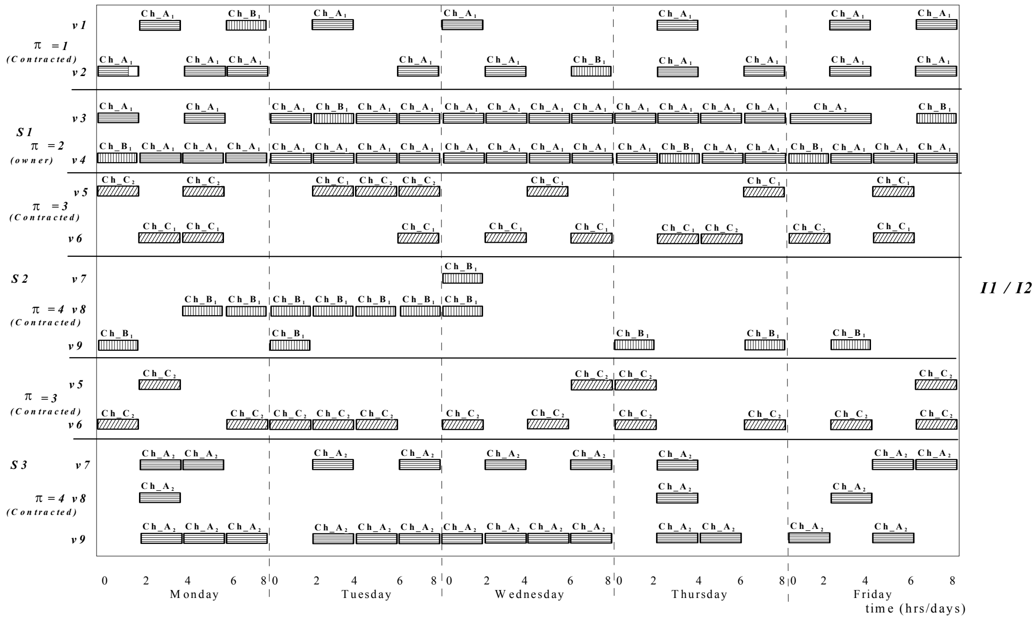


Figure 4- Scheduling results for the transportation structures connecting raw material suppliers, S1, S2 and S3, with the plant sites, I1 and I2, while considering a *closed* market scenario. Subscripts 1 and 2 reports the raw material destination: 1-to I1 and 2- to I2 plant.

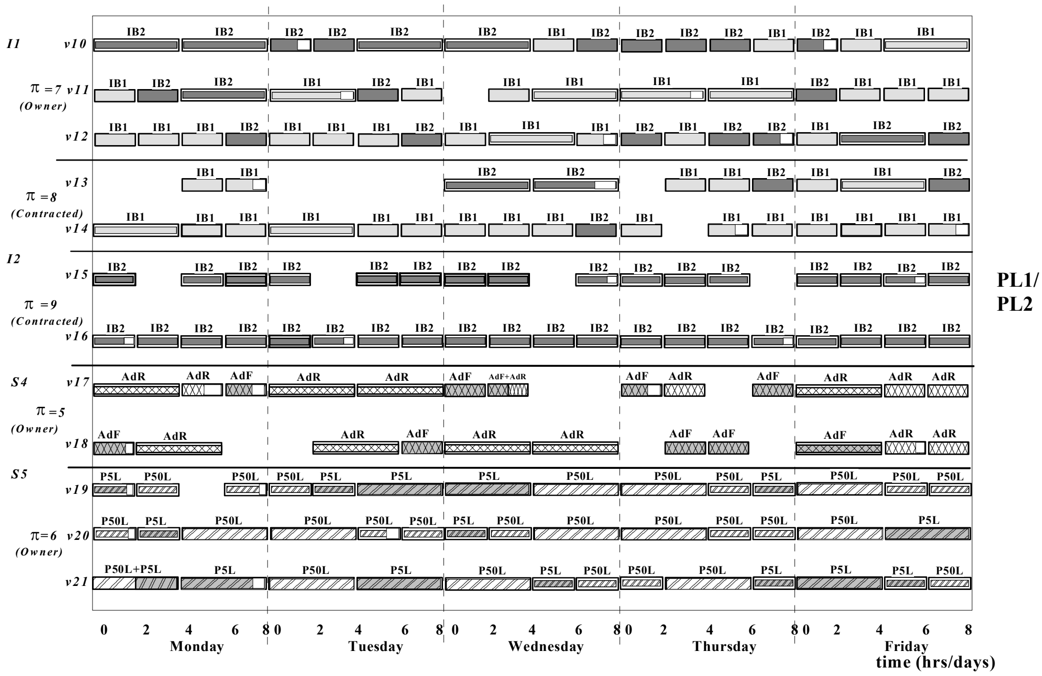


Figure 5- Scheduling results for the transportation structures connecting additive and packing suppliers, S4 and S5, as well as plant sites, I1 and I2, with the PL sites, while considering a *closed* operational scenario. Single border lines represent transportation flows to PL1 site while double border lines are used for flows to PL2 site.

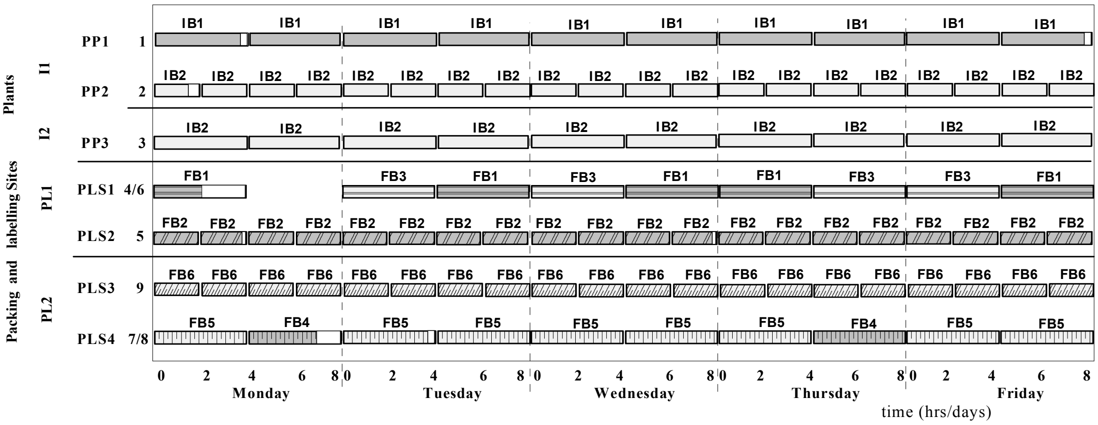


Figure 6– Scheduling results for the processing structures operation while considering the *closed* market operational scenario.

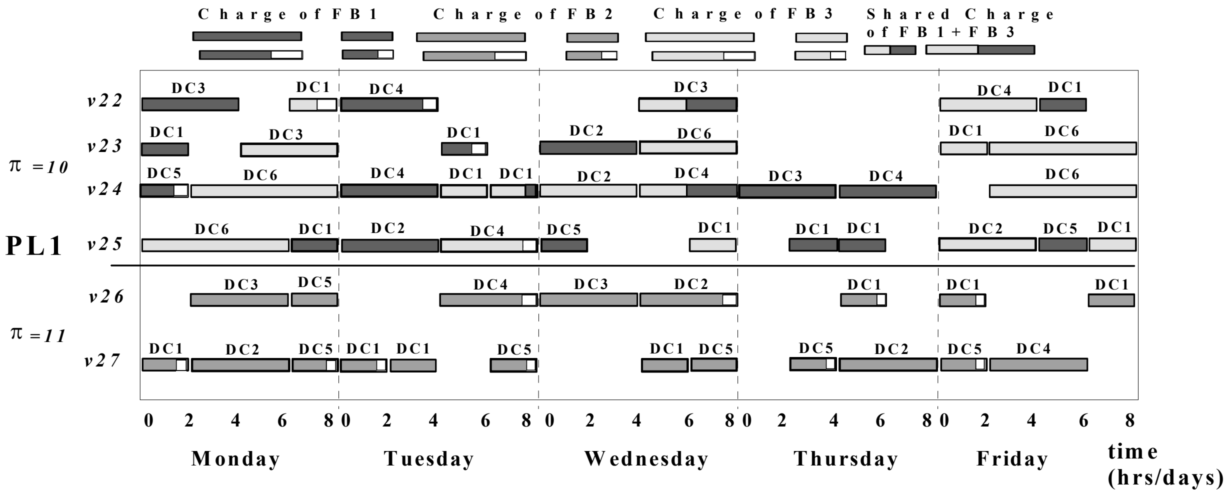


Figure 7- Scheduling results for the operation of transportation structures connecting PL1 site with the distribution sites, DC1 to DC6, while considering a *closed* market scenario.

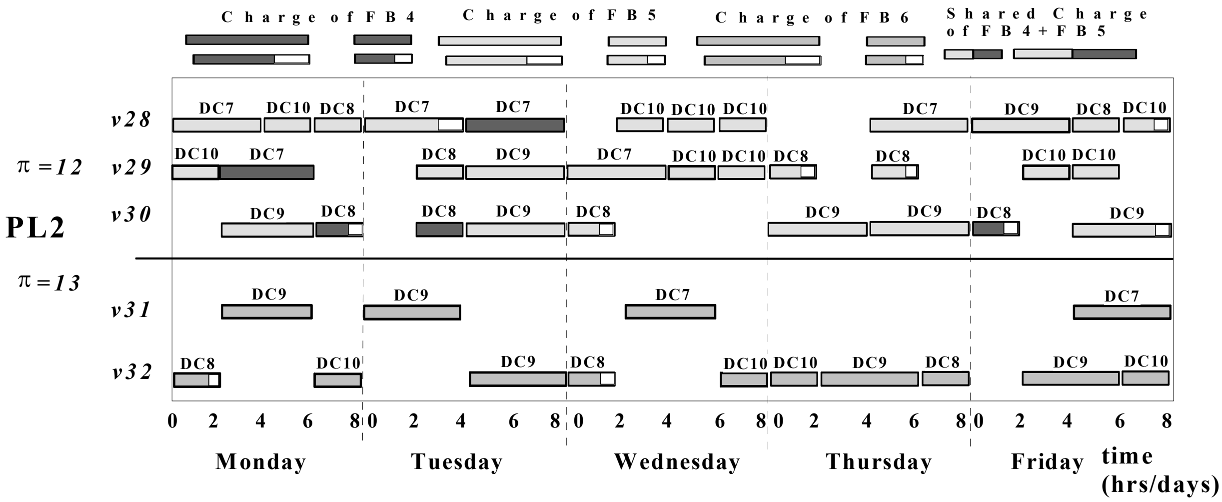


Figure 8- Scheduling results for the transportation structures operation connecting PL2 site with distribution sites, DC7 to DC10, while considering a *closed* market scenario.