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Organizing hospitals into networks: An hierarchical and multiproduct model with application to the Portuguese health system

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ABSTRACT

Health care planners in countries with a system based on a National Health Service have to make decisions upon where to locate and how to organize hospital services, so as to pursue geographic equity and efficiency in the delivery of health care. Previous methods for analysing hospital networks have not always adequately taken into account the hierarchical and multiproduct nature of hospital networks. This study develops a hierarchical multiproduct mathematical programming model to define location and supply of hospital services that maximizes patients' accessibility to hospitals. The model: a) considers inpatient care, external consultations and emergency care as hospital products; b) departs from a two-tiered hospital hierarchical system; c) and allows for two way referrals of patients between hospitals at different levels of the network. A mixed integer and linear program (MILP) formulation is developed, which is implemented in the generic algebraic modelling system, GAMS and solved through the use of a commercial Branch and Bound solver (CPLEX). As main results, it is obtained crucial information for planning, such as referral networks, hospital catchment's areas, and the structure of hospital supply. The model is applied to a case study of the Portuguese NHS that includes the Lisbon and Tagus Valley, Alentejo and Algarve Administrative Health Regions. Due to the complexity of the problem, a solution strategy involving a multi-stage solution decision is used. The model appears as highly demanding in terms of data available and calibration of parameters, but the

results are robust and indicate which changes could potentially improve the current hospital network.

Subject areas: decision analysis, health care, hospital networks, planning systems, supply chain.

INTRODUCTION

Health care systems in most countries attempt to maximize populations' health, equity, efficiency and quality, while minimizing health care spending. In order to pursue these objectives, and to plan resources, health care systems based on a National Health Service (NHS) require information to support the following decisions: where to locate hospitals so as to pursue equity in access to patients? Which is the optimal structure of hospital production that minimizes costs of delivering health care? How to define referral networks between hospitals? How to define hospital catchment's populations? How to organize a rational network of services?

In the decision of locating hospitals and organizing hospital networks, there are well-known trade-offs in literature with respect to pursuing some of the above objectives, such as between equity, and efficiency and costs. For example, increasing geographic equity in access might imply building small hospitals close to populations, which translates into inefficiencies in scale and into higher costs. On the other hand, the high cost of some medical equipment and low availability of high skilled human resources (such as specialized doctors) might imply that supply of services is delivered to large populations, which might have a negative impact in geographic equity of access.

Available published studies on hospital location have addressed these issues and have shown different approaches. The main methods used are spatial interaction models, entropy models, simulation and mathematical programming models (Ballou, 2004; Oliveira and Bevan, 2006), being the last one the most common. Mathematical programming involves the optimization of an objective function that represents the purpose of the model, subject to a set of constraints that reflect the characteristics of the system. The advantages of this approach are the flexibility of the objective function that can portray diverse objectives; the possibility of defining multiple objectives; the modelling of different problem characteristics and the recover of additionally information making use of extra constraints; and the possibility of providing a global solution (Oliveira and Bevan, 2006). These advantages overlap the computational difficulties observed when these models are applied to real problems.

The health system, like many others, for instance the education system, the postal services and the bank services, etc are organized in an hierarchical structure with various levels of supply that provide different services. In these cases the planner has to decide upon where to locate the facilities of each level while taking into account potential interactions between them. The facilities can be classified as *successively inclusive*, where the higher levels offer all the lower levels services additionally to theirs, and *successively exclusive*, when the facility only ensure their level services (Narula, 1986).

The hierarchical models can be classified in two large sets: the p-median models and the covering models. For the first, the objective is to minimize the total travel distance to reach the facilities, aiming at maximizing the patients' accessibility to services. In the covering models, it is established a standard distance that can't be exceeded. In this way, equal opportunity of access is promoted among the population; these models impose a more rigid structure, and can be infeasible for areas with small populations and low accessibilities. In the health sector these models are more adequate for analysing emergency care. In the present study we choose the mini-sum type (being a p-median type of model) because it allows for modelling equity of access and efficiency issues.

Within our knowledge, the first location model applied to the health system belongs to Gould and Lienbach (Rahman and Smith, 2000) who, in 1966, developed a p-median model and applied it to the health system of Guatemala. Since then, many other authors had been involved with this area. Rahman and Smith (Rahman and Smith, 2000) present a complete survey where they alert for the computational difficulties associated with this type of models and emphasise the frequent use of heuristic methods to surpass them.

The first hierarchical model analysed in the scope of this work belongs to Calvo and Marks and was developed in 1973 (Narula, 1986). The authors considered that patients could be separated in three groups according to their needs for different services. The model was formulated in linear programming and solved by Weaver and Church (Narula, 1986) using a heuristic procedure. Narula and Ogbu (Narula, 1986) tried to improve this model by adding questions related to the patients' rules for using different services. They have assumed that patients did not know the type of care they needed, so there would be a proportion of patients transferred to higher levels in order to receive specialised care. Up to our knowledge, this was a first attempt to capture the rules of a gatekeeping system.

Morre and ReVelle (Morre and ReVelle, 1982) considered the health care system characteristics by using a covering model applied to the Honduras' health system. The authors included different covering distances for patients attending the various types of health care units assuming that populations are attended in a higher level unit even if this implies a longer journey.

Recently, Galvão et al. (2002) have applied a three-level hierarchical model for the delivery of perinatal care in the municipality of Rio de Janeiro. They classified general patients, mothers and babies into three categories of risk and considered three main types of facilities that cooperate in a successively inclusive hierarchy. The model was formulated as a mixed integer linear programming where the continuous variables established the flows and the dummy ones indicate the most favourable location for the different type of facilities. In order to solve the model, four types of relaxations and heuristics were developed and compared. We dedicate special attention to the second type, due to its importance to this study. It was named as the "3 p-median" heuristic that consists in the sequentially location of the level 1 facilities, then the level 2 and finally the level 3. This procedure is only possible because the number of facilities is predefined as an input to the location of level 2 and level 3 facilities. After the location, a vertex substitution is done (that is, one location is substituted by another) in an iterative procedure that allows the exploitation of other locations and different configurations of the network.

In Portugal, there have been few location studies. We mention the work of Oliveira and Bevan (2006) that analysed the redistribution of the current hospital supply through the definition and comparison of three alternative models. The three models have used different objective functions representing alternative definitions of equity of access and utilization, and different constraints representing different institutional characteristics of the system and alternative assumptions on the behaviour of patients when using hospital services. These models captured the different roles of higher and lower level hospitals indirectly through the use of constraints on a proportion of patients accessing central hospitals or through a higher capacity of central hospitals to attract patients. Nonetheless, none of these models accounts explicitly for the administrative hierarchy of hospitals or for the flows of patients between hospitals.

Within our knowledge, and until now, hospitals have always been taken as single product facilities, with inpatient care being the main service, and hospital flows having only considered ascendant ways in the hierarchy. For many countries (including Portugal) it is crucial to develop models to inform the creation of hospital networks that consider the multiproduct nature of hospital facilities, as well as ascendant and descendent flows. In the Portuguese reality, the importance of these unconsidered aspects is raising. In Portugal, the emergency care is currently being reorganized so as to adequately consider population density and accessibility criteria, and there is an increasing recognition of the importance of the network of emergency services in improving efficiency in the system (Health Ministry, 2007). A new network of long-term care for patients that have already been treated, and require care from tertiary hospitals or from hospitals at a lower level of the network is also being developed (Health Ministry, 2003a). Finally, there have been several redesigns of the referral networks between district and central hospitals for emergency services and other medical specialties.

The aim of this study is to develop a decision support tool that addresses these modelling issues: the multiproduct structure of hospital production, the articulation between different services and units, and ascendant and descendent flows of patients in the hierarchy.

We propose a multiproduct and hierarchical optimization model that particularly informs the question: where to locate and how to organize hospital networks, so as to maximize equity in access while accounting for some efficiency issues. The model considers the institutional context of a health system based on a NHS that plans public hospital supply.

The model is generic and might be adapted to health systems with different characteristics. Its application to the Portuguese health care system is studied along this paper. Due to the problem complexity a solution strategy is developed. This is characterized by a two stage approach. In a first stage the multiproduct model is solved for a single product. The results of this stage define the network structure that is taken as fixed at the second stage of the solution where the multiproduct model is then solved. The results of the real case-study are analysed and a discussion on the usefulness of the model for health care decision makers is made.

In this paper, we start by briefly describing background information of the Portuguese health care system (common to the health system of many other countries) which is required to develop the optimization model. Secondly, we present and characterize the multiproduct hierarchical model that we have developed. Thirdly, we apply the model to the Portuguese health care system and discuss results. Within this section the adopted solution strategy is characterized. Finally we present concluding remarks.

HEALTH SYSTEM BACKGROUND INFORMATION

The Portuguese health care system is based on a NHS structure, funded by public taxation and with nearly free access in the point of use and universal coverage. A key objective of the political system is to achieve equality among the citizens on the access to health care, despite their economic condition or geographic distribution, and also to guarantee the equity in the distribution of health care resources (Health Ministry, 1990). The supply of health care services is dominated by a set of public providers that should cooperate in an integrated network in order to take advantage of existing synergies and benefit from economies of scale. Key health care providers include primary care centers and hospitals that are the subject of the developed models.

The public hospital system operates in practice as a centralised system with a hierarchical nature. Hospitals activity includes the diagnosis, treatment and rehabilitation, and the hospital system is organized in four administrative types of hospitals (from lower to higher technological complexity, from small to large catchment populations, and from basic to specialized care): level one, district, specialized and central hospitals. Level one hospitals are small units mainly located in smaller cities and offer basic services. District hospitals include small and medium hospitals located in larger cities and that offer more specialties. Specialized hospitals offer specific specialties, are located in three urban centres (Lisbon, Coimbra and Porto) and have large catchment populations. Central hospitals are located also in those urban centres and provide both basic services to local populations and specialized care to large catchment areas. Consequently level one and district hospitals refer patients to central and specialized hospitals.

Hospitals provide three main types of services: external consultations, emergency care and inpatient care. Access to hospital services depends upon referral from primary care centres or emergency care, as a gatekeeping system applies. Access to external consultations also requires a register and making an appointment. Emergency care is intended for situations of a sudden risk of collapse of one or more of the body vital functions (Health Ministry, 2006). As mentioned earlier, this service is a key entry in the system, and available evidence points for excessive and inadequate use in Portugal (Health Ministry, 2001). This is partly explained by a quicker access to medical exams and to consultations through emergency care and by inadequacies in the delivery of primary care. Inpatient care is essentially a hospital service (yet there are primary care centres with a small number of beds) and occurs when the patient needs to stay in the hospital for more them 24 hours. Hospital admission occurs after a patient has accessed primary care, emergency care or external consultations. Portuguese hospitals thus integrate a hierarchical network that cooperates in a gatekeeping system where the use of a higher level only is possible when there is a need for more specialised care. The inadequate use of emergency care creates organizational problems and inefficiencies in the system.

For most countries based on a NHS structure, there are two main levels in the hospital hierarchy: central hospitals and smaller scale hospitals (we name them district hospitals). Assuming this simplification, the hierarchical relationships and flows in hospital systems may be generally represented as in Figure 1. This representation is the underlying basis for the hierarchical and multiproduct model presented in the next section.

Insert Figure 1 here

One should note that this representation of the hospital system makes use of a key assumption: patients are represented as entering the hospital system directly, when they are referred in fact from primary care centres. This assumption is consistent also with the model application below in which population need is converted into expected utilization for hospital services. Figure 1 does not include all the possible flows between hospitals and services, but the most relevant ones, for which it is expected to exist available data in the Portuguese application (these flows could be slightly different for using the model in other countries).

When interpreting Figure 1, population from a certain population area needs to use three types of hospital services, provided in two levels of hospitals. The relation between levels of hospitals and types of services is as follows:

- D services: these are basic hospital services provided in District Hospitals (DH), and also provided in Central Hospitals (CH);

- C services: these are higher technology and specialized (including some high cost) services that are only provided in CH.

When a patient enters the hospital system, he has a predefined probability to be referred to another service within the hospital and/or to another hospital service (the other cases in which patients leave the system are not explicitly drawn in Figure 1). One should note that this representation considers: two way flows for inpatient care (ascendant and descendent flows between DH and CH) and multiple flows (for example, after emergency care in a DH, a patient might be admitted to inpatient care in the DH or to be referred to inpatient care in a CH).

DEVELOPMENT OF HIERARCHICAL MODELS

In this section we develop a multiproduct hierarchical mathematical programming model, which defines the optimal hospital network for a decision maker who wants to maximize patients' access to hospital services, while taking into account the population needs, the characteristics of the hospital system, and efficiency issues. The model is formulated as a mixed integer linear programming (MILP) model, where the decision variables are associated to the locations of the hospitals and the continuous variables are related to the flows of patients within the network.

We use a *p-median* type of model with an objective function that minimizes the total travel time for patients to use hospital services, and considers two hierarchical levels – central and district hospitals. The model defines as outputs the location and the structure of hospital production, with hospital production being disaggregated by service and by the population point that makes use of these services. The mathematical programming model structure and constraints capture the institutional characteristics of the system, such as the hierarchical levels of hospitals, the referral system between hospitals, and the flows between different hospital products; and the indirect but critical role of efficiency and cost issues, through the introduction of capacity constraints for different types of hospitals. The use of capacity constraints takes into account normative information from health care planners that provide indicative values of hospitals' minimum and maximum capacities; and evidence from literature that very large and very small hospitals are under diseconomies of scale, which translate into higher hospital costs (McGuire and Hugues, 2002).

The characteristics of the multiproduct hierarchical model can be summarised as follows –the model:

- Considers two levels in the hospital hierarchy: larger central hospitals and smaller district hospitals;
- Has a multiproduct flow structure where at least three products can be identified by the indexes: (1) inpatient care, (2) emergency care, and (3) external consultations;
- Considers two types of services: C type services and D type services in a successively inclusive hierarchy where C level services can only be provided in a central hospital while D level services can be provided in a district or in a central hospital;
- Considers as objective function the minimization of the total time for patients to access all hospital services. This objective function embodies a narrow definition of equity, as it tends to penalize patients from rural areas with low population density;
- Produces as outputs: the location of hospitals; the level and structure of hospital production; the 'optimal' referral between population points and hospital points, and between hospitals at different levels of the network; and hospital catchment's population areas for each service provided by each hospital;
- Makes use of information on patients' need for hospital services by converting population numbers into expected utilisation measures (in accordance to need indicators such as age and sex and geographic location), and utilisation measures into hospital capacity measures such as beds (in the case of inpatient care, through the use of the length of stay);
- Should be used with information on population points and potential hospital location points at the small area level.

We consider the following types of products and hospitals: central hospitals that provide level C and D services, and district hospitals that provide only level D services. Depending upon the size and range of products supplied, a hospital might be classified as district or central. In terms of demand there is a proportion (θ) of patients arriving at district hospitals that require level C services, and consequently will be referred to a central hospital. The model captures the relationship between services by decomposing the type of service flows, and by explicitly modeling both ascendant and descendent flows of services between different levels of the hierarchy. Depending on the structure of the health system, some of these ascendant and descendent flows might assume the zero value. In general terms, there might be flows within and between hospitals, and between products.

The model uses as basic notation: *i* stands for population demand point ($i \in I$); *j* and *k* stand for potential hospital locations ($j, k \in J$); *w*, *v* and *a* stand for hospital services.

The following set of **parameters** and variables were defined:

- $d1_{ij}$: travel time from population point *i* to hospital *j* (e.g. minutes);
- $d2_{jk}$: travel time from district hospital *j* to central hospital *k;*
- utl_i^w : population need from population point *i* for hospital service *w*;
- $perDC^{wv}$: share of demand transferred from service *w* in a district hospital to service *v* in a central hospital;
- $perCD^w$: share of demand for service *w* transferred from central to district hospital;
- $pertrans^{wv}$: share of demand transferred from service *w* to *v* in the same hospital (defined as a percentage);
- α: factor that differentiates travel times for patients that have already attended an hospital service;
- avt^w : average time spent in service *w* (relevant only for inpatient care);
- $cap\max DH^w$, $cap\min DH^w$, $cap\max CH^w$ and $cap\min CH^w$: Maximum and minimum capacities allowed for district and central hospitals respectively for service *w*.

In terms of **location and flow variables**, the model computes:

- the number of district hospitals that are located in *j* and provide service *w* (X_j^w) and similarly the number of central hospitals located in *j* (Y_j^w);
- the flows of patients between population points and hospitals, with fd_{ij}^w as the flow for service *w* between demand point *i* and district hospitals in *j*, and fc_{ij}^w as the flow of patients for service *w* from demand point *i* and central hospitals in *j*;
- the flows of patients of service *w* between district and central hospitals, with zdc_{jk}^{wv} as the flow of service *w* that is transferred (or referred) from a district

hospital *j* to central hospital *k*, and zcd_{kj}^{w} as the flow of patients for service *w* that is transferred (or referred) from central hospital *k* to district hospital *j*;

- the flows within hospitals, with td_{j}^{wv} as the flow from service *w* to service *v* within district hospital *j*, and tc_{k}^{wv} as the flow from service *w* to service *v* within central hospital *j*;
- and the hospitals capacity though the variables $cap\_X_{j}^{w}$ and $cap\_Y_{k}^{w}$ that stand for district and central hospitals capacity, respectively.

The location variables and the flow variables are defined in a range for ensuring integrality and nonnegativity (i.e., $cap\_X_{j}^{w}$, and $cap\_Y_{k}^{w}$). Integer variables are chosen for the location, instead of the dummy ones, because they allow for locating more them one hospital in a single population point, a feature that is crucial for locating hospitals in high density areas.

Most of the variables are schematically represented in Figure 2. The Mixed Integer Linear Programming model that we developed is based on this Figure.

Insert Figure 2 here.

The model minimizes the total travel time for patients to access hospital inpatient services, with the objective function being presented in Eq. [1]. The objective function includes four terms, each one representing: the demand-weighted travel time to reach district hospitals; the travel time to reach central hospitals; the travel time from patients transferred from district to central hospitals (ascendant flow); and the travel time for descendent flows in the hierarchy, i.e. for patients transferred from central to district hospitals. A factor α allows us to weight differently the patients that have already had a hospital admission; and the choice of weights for the terms of Eq. [1] entails value judgements on the importance of travel time for patients accessing different services ($0 \leq \alpha \leq 1$, these parameter might vary for different decision makers).

$$Min \quad z = \sum_{i \in I} \sum_{j \in J} \sum_{w \in W} d1_{ij} \times fd_{ij}^{w} + \sum_{i \in I} \sum_{k \in J} \sum_{w \in W} d1_{ik} \times fc_{ik}^{w} + \\ + \sum_{j \in J} \sum_{k \in J} \sum_{w \in W} \sum_{v \in W} \alpha \times d2_{jk} \times zdc_{jk}^{wv} + \sum_{j \in J} \sum_{k \in J} \sum_{w \in W} \alpha \times d2_{jk} \times zcd_{kj}^{w} \qquad [1]$$

The characteristics of the hospital system are captured by the following set of constrains.

Eq. [2] ensures that all the demand for hospital services is satisfied for each service. Demand is captured by the parameter utl_i^w which, as defined above, represents the need for the hospital service *w,* and should be the result of converting population numbers into hospitals admissions through indicators that capture populations' needs for hospital services (like age, gender and geographic location).

$$\sum_{j \in J} fd_{ij}^{w} + \sum_{k \in J} fc_{ik}^{w} = utl_i^{w} \qquad \forall\ i \in I,\ w \in W \qquad [2]$$

Eqs. [3] and [4] define the ascendant and descendent flows between hospitals in the network through the equality between the proportion of patients that will be transferred (exits) due to the need of services from the level above and the two possible ways to get in (entries): from population points and from other services at the same hospital. These constraints balance the flows from the different levels. Eq. [3] converts population demand for district hospitals into demand to be transferred (or referred) from district to central hospitals. Eq. [4] defines the descendent flows from central hospitals to district hospitals.

$$\left(\sum_{i \in I} fd_{ij}^{w} + \sum_{a \in W} td_j^{aw} \right) \times perDC^{wv} = \sum_{k \in J} zdc_{jk}^{wv} \qquad \forall\ j \in J,\ w\ v \in\ W \qquad [3]$$

$$\left(\sum_{i \in I} fc_{ij}^{w} + \sum_{a \in W} tc_k^{aw} \right) \times perCD^{w} = \sum_{k \in J} zcd_{kj}^{w} \qquad \forall\ k \in J,\ \ w \in W \qquad [4]$$

Eqs. [5] and [6] define the flows within district and central hospitals. For example, the percentage of patients that have used emergency services will require inpatient care.

$$\sum_{i \in I} fd_{ij}^{w} \times pertrans^{wv} = td_j^{wv} \qquad \forall\ j \in J,\ w\ v\ \in\ W \qquad [5]$$

$$\sum_{i \in I} fc_{ik}^{w} \times pertrans^{wv} = tc_k^{wv} \qquad \forall\ k \in J,\ \ w\ v \in\ W \qquad [6]$$

Eqs. [7] and [8] determine respectively the district and central hospitals capacity. For inpatient care, the capacity is measured in inpatient days which afterwards are converted in hospitals beds. For the other services (emergency care and external consultations), capacity is measured by the number of attendances.

$$cap\_X_j^w = \left(\sum_{i \in I} fd_{ij}^w + \sum_{v \in W} td_j^{vw} + \sum_{k \in J} zcd_{kj}^w \right) \times avt^w \quad \forall\ j \in J,\ w \in W \qquad [7]$$

$$cap\_Y_k^w = \left(\sum_{i \in I} fc_{ik}^w + \sum_{v \in W} tc_k^{vw} + \sum_{j \in J} \sum_{v \in W} zdc_{jk}^{vw} \right) \times avt^w \quad \forall\ k \in J,\ w \in W \qquad [8]$$

Eqs. [9] and [10] ensure that, for all services, the minimum capacity is reached and the maximum capacity is not exceeded, and also state that a service can only be obtained at population points where hospitals are located.

$$cap\min DH^w \times X_j^w \leq cap\_X_j^w \leq cap\max DH^w \times X_j^w \quad \forall\ j \in J \quad w \in W \qquad [9]$$

$$cap\min CH^w \times Y_k^w \leq cap\_Y_k^w \leq cap\max CH^w \times Y_k^w \quad \forall\ k \in J \quad w \in W \qquad [10]$$

Depending on the health care system, additional constraints might be used to model other decision maker preferences. For example, constraints might be built to impose that there is a maximum distance allowed for a patient to access a certain hospital or service. Eq. [11] illustrates this question for the district hospitals in service *w*. cob_{ij}^w is a binary matrix that defines if the population ($i \in I$) can be supplied by a certain hospital ($j \in J$) with respect to the following criterion: if an hospital is less then a predefined standard travel time from a population, it can deliver hospital services to that population, being cob_{ij}^w a parameter with the unit value; otherwise, it assumes the zero value. Eq. [11] states that patients' demand for hospital care from a population point can only be met by hospitals within a maximum travelling time.

$$fd_{ij}^w \leq cob_{ij}^w \quad \forall i \in I,\ j \in J \qquad [11]$$

We also consider the possibility in that some services can only be provided if that hospital location also delivers another service. Eq. [12] exemplifies the case in which emergency services *(w=2)* and external consultations *(w=3)* can only be served in hospitals where inpatient *(w=1)* is provided.

$$\forall\, j \in J \quad X_j^1 \geq X_j^2 \;\; and \quad X_j^1 \geq X_j^3 \qquad [12]$$

As a limiting case, we eventually impose that wherever a hospital is opened, the three types of services need to be provided –this case is captured by Eq.[13].

$$\forall\, j \in J \quad X_j^1 = X_j^2 = X_j^3 \qquad [13]$$

CASE-STUDY

We have applied the multiproduct hierarchical model to the Portuguese NHS. Portugal has an administrative division that allows for delimiting three independent and self-sufficient geographic areas in the health care system: North, Centre and South. The present study focus on the South region, which includes three Administrative Health Regions: Lisbon and Tagus Valley, Alentejo and Algarve and these regions are divided in seven health sub-regions (Faro, Beja, Portalegre, Évora, Setúbal, Santarém and Lisbon). The South region is divided in 109 small area units (the chosen geographic unit -*Concelhos*- that are equivalent to the English wards) which include both urban and rural areas. The urban areas benefit from improved physical accessibilities and higher geographic proximity to health services. In the last decades the rural areas have suffered a population decrease, their populations have been ageing at a higher rhythm in comparison to urban populations, and the Alentejo region has populations living in remote areas.

The current hospital network under study is composed by 12 general central hospitals and 6 specialized central hospitals, all of them located in Lisbon sub-region (and 17 located in the Lisbon small area unit); and by 14 district hospitals and 5 level one hospitals that are more evenly spread around the South region, like we show in Figure 3.

Insert Figure 3 here.

Data Analysis

The application of the model involved collecting two main types of data: data related to expected need (or demand) for hospital care services, including related estimates on the transfer of patients between services; and other parameters of the mathematical programming model.

To compute estimates of population need for inpatient care (utl_i) we used data from the Diagnostic Related Group (DRG) database system from 2003 that allowed us to compute the expected level of inpatient care utilization in accordance to age and gender –this data was used to estimate expected utilisation from a population living in each small area. Those estimates show that on average for each 1000 inhabitants, one expects 100 hospital admissions per year (for populations living in the South region). We have weighted population numbers from each small area by expected utilization in accordance to the age and gender structure of that population (e.g., this adjustment implies higher expected utilization in areas with older populations) like we illustrate in Figure 4.

Insert Figure 4 here.

DRG data was also used to estimate the two following parameters: 13% of patients admitted to inpatient care in a district hospital will be transferred/referred to a central hospital, and 1.24% of the patients admitted to inpatient care in central hospitals are referred back for inpatient care in district hospitals – this is the parameter capturing the reverse flow.

To predict the need for external consultations from a population area, we have used data from the General Directorate of Health, in particular, the figure on the total number of external consultations in 2003 (Health Ministry, 2003b). Making use of this data, we have quantified that: 57,9% of the population of each area needed D level external consultations (this figure might be interpreted as: for each 1000 inhabitants, one expects 579 for external consultations appointments per year); and that figure is 19,6% for level C care provided in CH; and 25,3% of patients accessing external consultations in DH need to be transferred to CH (this proportion was obtained by computing the ratio 19,6/(19,6+57,9)) for a further external consultation.

Need for emergency care has also made use of information from the General Directorate of Health (Health Ministry, 2003b). Due to the absence of more detailed information, we have assumed that this service is not differentiated in district and central hospitals, and there is an utilization of 58,4% for each population point (i.e., from each 1000 inhabitants, one expects 584 entries an emergency care unit).

The remaining flows between hospitals and within services were not considered in the Portuguese case study, given the lack of information to estimate those parameters and also because of the expected low magnitude of those flows (in comparison to other flows).

Resident population estimates at the small area level were taken from the last Portuguese National Institute of Statistics census (with 2001 reference) (Portuguese National Institute of Statistics, 2006).

We have estimated other parameters of the mathematical programming model as follows. For the α parameter, we used the value of 0,5 which means that the travelling time of one transferred/referred patient is worth half the value of the travelling time of the journey for a patient to directly enter an hospital. As underlined above, the α parameter is a value judgement for the decision maker.

We have computed the travelling distances between the centroids of the population points (e.g., centroids of the small area units) using an internet website that computes travelling time while taking into account roads accessibility and roads condition (ViaMichelin, 2006).

For inpatient care, we have assumed an average length of stay within hospital of 7.7 days for each hospital inpatient admission (Health Ministry, 2003b).

At last, we have had to estimate hospital capacities for different services. We have initially used the standard limits defined by the Portuguese Health Ministry (Comissão Técnica Interdepartamental, 2006) which allows for relatively small capacities for some hospitals in areas with low geographic accessibility. In our applications of the model, we have found out (as expected) that the total number of hospitals is related with the minimum and maximum capacities allowed, and lower minimum capacities allow for the opening of too many units. This fact associated with evidence on the existence of economies of scale have lead us to use minimum capacities of 200 and 500 beds for district and central hospitals, respectively, and maximum capacities of 500 and 1000 beds.

The parameters of our model application (including estimated values) are synthesised in Table 1. This table allows for a quick reading about each parameter.

Insert Table 1 here.

The impact of using these parameters (and related assumptions) in the model is crucial and should take into account the decision maker's knowledge of the system; and the use of key parameters should be subject to sensitivity analysis to observe the impact of parameters variations in the model results. We should be aware of the low quality of data for some of the estimated parameters, in particular on the information on transfers between district and central hospitals. We expect that there was an underreporting on the number of these transfers; and we have used utilisation data to estimate some parameters, which has meant that our estimates of demand/need for emergency care and external consultations are influenced by variables that we could not control for (such as on the influence of supply on demand for hospital services, and we also lacked information on waiting lists that capture unmet need for hospital services). Consequently, analysis of results of the model application should consider the use of these crude parameters.

Model Solution Strategy

The described model was implemented in the general algebraic modelling system GAMS (version 22.0) (McCarl, 2004), and solved though the branch and bound method making use of CPLEX (version 9.0).

The application of the model to the global problem described above has lead to a complex problem which was impossible to solve with our computer resources, due to the raised number of integer variables that impose harder search methods like branch and bound. Table 2 presents the results from running the model.

Insert Table 2 here.

Based on these results, we have defined a solution strategy that involved a multi-stage solution algorithm. This solution strategy is described as follows:

Stage 1 – The multiproduct model is solved for a single product. We chose inpatient care as that product because it is a key component for current spending and for investment on infrastructure and equipment. The results of this single product hierarchical model have provided a set of hospital locations which were used as input in the model run in Stage 2.

Stage 2 – The multiproduct hierarchical model was run using the set of locations provided in Stage 1. As results we obtained the network flows for the three products for the complete network.

We now describe in detail these stages and present the results from their application.

Stage 1 – Single Product Model Results

The first stage involved the adaptation of the multiproduct hierarquical model to a single service –inpatient care (*w=1*). To accomplish this purpose, some adaptations were needed to eliminate some flows of the mathematical formulation presented above. Namely, we have eliminated the variables that stand for flows between services (td_k^{wv} and tc_k^{wv}) and some constrains (Eqs. [5] and [6]); other constraints were simplified -that was the case for Eqs. [3], [7] and [8]; the descendent flows for inpatient care were also ignored, therefore, Eq. [4] was also eliminated.

In order to obtain a more efficient model, a coverage constraint like Eq. [11] was used. One should note that standard travelling distance should be carefully chosen. Low values turn the resolution faster because limit the number of hospitals that can serve the population, although they have a side effect in that if an excessive boundary is used, it might imply further hospitals location, and thus shape different results. When rural and urban areas co-exist, different cover distances should be tried in order to find a balance. In this study we have tested the effect of several covers and observed that with this restriction we can achieve, in a faster way, a lower gap.

We have also tested different branch and bound search methods available in CPLEX, being the *depth first* the one that produced faster results.

The single product hierarchical model computer statistics are presented in Table 3 and the results are illustrated in Figure 5.

Insert Table 3 here.
Insert Figure 5 here.

In Stage 1, the optimal network is composed by 23 district hospitals and 7 central hospitals. Hospitals location in the South region area is shown in Figure 5. Comparing these results with the current network, the model indicates that central hospitals that are currently located in Lisbon should be transferred to other population points, many of those to other small area units in the Lisbon Metropolitan area; and in accordance to geographic accessibility and need for inpatient care, a central hospitals should be located in the Algarve region, and in a location in the South of the Tagus river, nearby Alentejo (that is, in Palmela). We do not focus on a central hospital in the northern part of the South region because this area has access to some hospital supply in the Centre region that is outside the area included in this case study. These results suggest that higher geographic equity of access can be obtained with reductions in hospital supply in Lisbon and with reinforcements in hospital supply in the metropolitan areas and in other less urban areas. A surprising result is the Palmela location, given that this is a small area with low population numbers and density (in comparison to other geographic contiguous areas). Nonetheless, this location benefits from good accessibility from populations from Alentejo, and is surrounded by small areas with high population numbers, which seems to justify a central hospital. Results also show that if one wants to maximize geographic accessibility, many hospitals will have the minimum capacity. This might be interpreted as an equity-efficiency trade-off: given that there are sparce populations in the South region, in order to maximize equity in access, we should have small hospitals, which in fact might not respect optimal hospital size as defined by efficiency or costs criteria.

Figure **6** and Figure 7 present a set of indicators that help to analyse the results of the model: average travel time weighted by utilization; maximum travel time to reach a unit; and share of population whose travel time to reach a hospital exceeds 45 and 60 minutes. We present these indicators by health sub-region or region, in order to have an idea about variations across areas.

Insert Figure 5 here.
Insert Figure 6 here.
Insert Figure 7 here.

Despite the reduced average travel time (less than 45 minutes), a substantial proportion of the populations living in the Algarve and Alentejo regions needs to travel more than sixty minutes to access hospital services. Nevertheless, higher availability of primary care resources in these rural areas is expected partly to compensate these inequalities in access to hospital care.

Stage 2 - The Multiproduct Model Results

Using the hospital locations suggested by the single product model, in this second stage the multiproduct model is run using those locations as the possible ones, and produces information on the flows of patients for different hospital services and types of hospitals. The computational statistics are presented in Tables 4 and the associated results in Table 5.

Insert Table 4 here.
Insert Table 5 here.

Analysis of results shows again that the capacity (as measured by the number of beds) for many hospitals equals the minimum capacity allowed for inpatient care, and this is a consequence of the procedure used to achieve the solution. We also observe that some patients need to travel further (than potentially required) in order to respect the minimum capacity in the model. This is specially the case of some populations from one small area that might be allocated to different hospitals. This can lead that population from a small area might be using different hospitals for different services, which might not be an acceptable rule in a planning/referral system. In order to test the extent to which we can avoid this result, we have solved the model without the capacity constraints. The computer statistics are presented in Table 6 and the results of this model are presented in Table 7.

Insert Table 6 here.
Insert Table 7 here.

Without using the capacity constraints we observe an improvement in the value of the objective function by 6% (decrease), but yet there are some hospitals with lower capacities than the minimum capacity previously defined in areas with relatively good geographical access to hospital services (for example, Alenquer and Mafra).

We have found differences between current locations and the ones proposed by the two stage multiproduct hierarchical model. This result indicates that the current network might be improved so as to achieve greater geographic access to hospital services, for example by closing facilities and by moving current locations through moving new replacement hospitals to other areas. Depending on the country context, the model can be used both to analyse the creation of new facilities or the redistribution of current hospital facilities. E.g., these questions can be answered by changing the locations variables values and letting the model find out the optimal redistribution in an iterative process similar to the vertex substitution of Galvão et al. (Galvão et al., 2002).

SENSITIVITY ANALYSIS TO THE LOCATIONS

A two-step approach was used to achieve the solution previously presented for the multiproduct model. Nevertheless, that approach does not guarantee results' optimality. The global solution obtained by splitting the problem in two sub-problems –the location and the flow distribution (allocation), with separate optimizing models– leads to a global solution that strongly depends on the procedure used in stage one. We chose only a single product (inpatient care) to establish the locations, yet the introduction of other possible locations might result in different network configurations and even the single product model can produce alternative results when tested with different parameters. In this case study, sensitivity analysis was selected to test: whether slight improvements in the network can be achieved when considering other policy objectives and the current hospital network; and to test the robustness of results to changes in parameters. Sensitivity analysis was done through scenario analysis since the model includes a large number of integer variables and parameters.

Thus the selection of scenarios intended to capture the extent to which ‘fixed’ locations (in the first stage) were susceptible of being changed, and we have explored the impact of closing facilities and permuting locations on results. The starting point for scenario analysis is the result of the multiproduct model without capacity constraints described in the previous section (we name this scenario as **C0**). We have added some additional potential locations to **C0**, which we define below. The following set of scenarios were defined: closing of hospitals whose capacity is under 200 beds and which are located in areas with good accessibilities (scenarios **C1**-**C3**); adding potential locations corresponding to areas where there is currently hospital supply (scenarios **C4**-**C7** and **C10**); and additional locations corresponding to political decisions (**C8**-**C9**). Each scenario corresponded to the following case: **C1** as closing Alenquer; **C2** as closing Mafra**;** **C3** as closing, simultaneously, Alenquer and Mafra; **C4** as swaping Castelo de Vide by Portalegre; **C5** as swaping Montemor-o-Novo by Évora; **C6** as swaping Borba by Elvas; **C7** as swaping Entroncamento by Torres Novas; **C8** as closing Faro; **C9** as closing Seixal.

We agreed that a scenario is better than the previous one through analysing the results of the multiproduct model: when there were no significant losses for patients (as measured by the set of indicators earlier explained: average travel time weighted by utilization, maximum travel time to reach a unit and share of population whose travel time to reach a hospital exceeds 45 and 60 minutes) and there were other benefits for the health system (such as a decrease in the number of hospitals, and locations matching to areas with existing hospital supply), we considered that a scenario is better than the previous one. Thus, the analysis of those scenarios shows how alternative configurations of the network can be evaluated using the multi-product model, and considering a wider set of criteria that are important for planners and which complement the information of the objective function.

We have tested the described scenarios sequentially and, when a scenario is considered better than the previous, we introduce those changes incrementally and depart from that case. The results from the comparison of the various scenarios are presented in Tables 8 and 9 and in Figures 7 and 8.

Insert Table 8 here.
Insert Table 9 here.
Insert Figure 7 here.
Insert Figure 8 here.

We now briefly present the result from the scenario analysis. We should note that in the described scenarios, the value of the objective function slightly decreases, which constitutes evidence in that the solutions generated by the two stage approach are robust. However we consider that many of the scenarios improve the solution of the C0 scenario, being the other information on additional indicators and on current supply relevant for considering the optimal network. The information on those indicators is analysed together with information variations in the value of the objective function.

In the examined scenarios, the maximum value of the objective function is achieved in **C0**. We observed that in most cases the decreases in total accessibility (as measured by the objective function) were very small and acceptable, when we have tested permutation of hospital to nearby locations where there is already a hospital or when closing a small hospital one unit. Comparing the results from the sequential scenarios and using C0 as the basis for comparison: with **C1** and **C2** we notice that there are small and acceptable decreases in accessibility and acceptable raises in the capacity of the closer hospitals. In this way, the case of **C3** which combines **C1** and **C2** is the most preferred case because it allows for a smaller number of hospitals. When we add **C4** to **C3**, we found that this is a preferred scenario because it is an approximation to the current network that does not substantially improve the accessibilities and is therefore accepted. **C5** is similar to C4 although there is a small increase in the proportion of population whose distance exceeds 45 and 60 minutes. This scenario is considered better due to the existence of a unit in this location. In **C6** we observe significant increases in the travel time so that this location (Borba) is not accepted. We have accepted **C7** because there is currently a hospital in that location. In **C8** we once again test the closure of a hospital, but in this case we found out a raise of 200 beds in the capacity of the closest unit, because there is a lack of alternative hospitals nearby. As the service level can be compromised, we consider that this decision should be supported by a more specific study. Scenario **C9** is not harmful for patients and so we accept the scenario of closing the hospital. We remark that the solution obtained by the defined procedures does not nullify the optimal one because the conditions used to obtain and to evaluate them are different.

In the next section we show how the multiproduct model solution can be used as a departing point to consider other objectives for establishing a hospital emergency network.

ANALYSIS FOR EMERGENCY CARE

In Portugal, reorganization of the emergency care system has recently been at the top of the political agenda. The government has commissioned a technical study which has defined some accessibility criteria and goals to achieve in planning this service and has already introduced some changes. For example, the study indicated that the time distance between a patient and an emergency service should not exceed 60 minutes (Health Ministry, 2006a). Examining Figure 7, we found out that for some health sub-regions this principle is not respected (e.g., populations living in Faro, Beja, Évora and Portalegre). We do not aim at creating a model to optimize the emergency care network, but we have analysed how the multiproduct model solution might be changed to have into account the principle in that citizen should not be located at a time distance higher than 60 minutes to emergency care.

We depart from the last scenario accepted in the previous section (scenario **C9**) and have adopted a bottom up strategy and analysed which would be the additional locations so that all patients are within 60 minutes from the emergency service. This objective is achieved with the opening of three additional facilities that only supply emergency care, like we show in Figure 9.

Insert Figure 9 here.

CONCLUSION

We have proposed a decision analysis tool to help health care planners to decide upon the location and redistribution of hospital supply. The model developed holds the general hierarchical structure of a health system, respects the multiproduct nature of hospitals activity, and aims at promoting equity in access among citizens through the maximization of the geographic access for patients to reach hospital care services. The model detaches from previous published models because it considers the multiproduct nature of the hospital, the interactions between the various services and levels, as well as it allows for two-way flows in the hospital hierarchy.

The application of the model to a real network has resulted in a quite complex problem that we found difficult to optimize. The model was solved using a solution strategy defined within two stages: in the first, the locations were established trough the optimization of a simpler network that considered only a single product (inpatient care); in the second stage the allowed hospital location points were fixed and the model has produced the redistribution of the flows accounting for the multiple hospital services and articulation between services between and within hospitals.

We found out that the multiproduct model was highly demanding with respect to the required data to calibrate parameters, and that the model results were strongly dependent on the parameter values. In this mater, the planner knowledge of the system can be crucial to obtain suitable results.

Given that we did not reach an optimal solution, we have shown the usefulness of sensitivity analysis to analyse alternative solutions to the model that also took into account other policy objectives to health care planned and the current location of hospitals. The final analysis has shown that if some accessibility criteria are to be reached for emergency care, further emergency services should be provided for population points that did not have an emergency service within 60 minutes distance, and the model can be analysed so as to account for this objective.

This study might be developed so that other approaches are developed to solve the multiproduct model; and the model might be generalized so as to include further levels of health care services within the network, as well as other constraints that capture institutional characteristics of the system.

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TABLES

Table 1: Synthesis, definition and value of the parameters in use.

<table>
<tr><th>Parameter</th><th>Description</th><th colspan="2">Estimated value</th></tr>
<tr><td>d_{ij} $d2_{jk}$</td><td>Travel time between small areas</td><td colspan="2">Travel time matrix</td></tr>
<tr><td rowspan="3">utl_i^w</td><td rowspan="3">Expected need for service w</td><td>w=1 inpatient care</td><td>Taking into account age, gender and location.</td></tr>
<tr><td>w=2 emergency care</td><td>58,4%</td></tr>
<tr><td>w=3 external consultation</td><td>D Level: 57,9%
C Level: 19,6%</td></tr>
<tr><td>$perDC^{wv}$</td><td>Proportion of patients transferred/referred from service w in DH to service v in CH</td><td colspan="2"><table>
<tr><th></th><th></th><th>CH v = 1</th><th>CH v = 2</th><th>CH v = 3</th></tr>
<tr><td>DH</td><td>w = 1</td><td>13,0%</td><td>0,0%</td><td>0,0%</td></tr>
<tr><td>DH</td><td>w = 2</td><td>2,5%</td><td>0,0%</td><td>0,0%</td></tr>
<tr><td>DH</td><td>w = 3</td><td>0,0%</td><td>0,0%</td><td>25,3%</td></tr>
</table></td></tr>
<tr><td rowspan="3">$perCD^w$</td><td rowspan="3">Proportion of transferences between DH and CH for service w.</td><td>w=1 inpatient care</td><td>1,24%</td></tr>
<tr><td>w=2 emergency care</td><td>0%</td></tr>
<tr><td>w=3 external consultation</td><td>0%</td></tr>
<tr><td>$pertrans^{wv}$</td><td>Transferences between services w and v within the same unit.</td><td colspan="2"><table>
<tr><th></th><th>v = 1</th><th>v = 2</th><th>v = 3</th></tr>
<tr><td>w = 1</td><td>0,0%</td><td>0,0%</td><td>0,0%</td></tr>
<tr><td>w = 2</td><td>9,2%</td><td>0,0%</td><td>0,0%</td></tr>
<tr><td>w = 3</td><td>0,0%</td><td>0,0%</td><td>0,0%</td></tr>
</table></td></tr>
<tr><td rowspan="3">avt^w</td><td rowspan="3">Average time spent in service w</td><td>w=1 inpatient care</td><td>7,7 days (length of stay)</td></tr>
<tr><td>w=2 emergency care</td><td>1 occurrence</td></tr>
<tr><td>w=3 external consultation</td><td>1 attendance</td></tr>
<tr><td>$cap \max HD^w$
$cap \min HD^w$
$cap \max HC^w$</td><td>Minimum and maximum capacities allowed in DH and</td><td colspan="2"><table>
<tr><th></th><th>w = 1 DH</th><th>w = 1 CH</th><th>w = 2 DH</th><th>w = 2 CH</th><th>w = 3 DH</th><th>w = 3 CH</th></tr>
<tr><td>Mínimum</td><td>200</td><td>500</td><td>54750</td><td>54750</td><td>4000</td><td>60000</td></tr>
<tr><td>Máximum</td><td>500</td><td>1000</td><td>182500</td><td>-</td><td>300000</td><td>500000</td></tr>
</table></td></tr>
</table>

$cap \min HC^w$	CH.	

Table 2: Multiproduct model computational statistics. Pentium mobile 1.73 GHz, 0.98 GB RAM, CPLEX 9.0

Statistics	Multiproduct Model
Objective function	-
Number of restriction	5 560
Number of continuous variables	216 475
Number of integer variables	654
Number of iteration	-
CPU time in seconds	-

Table 3: Single product model computational statistics. Pentium mobile 1.73 GHz, 0.98 GB RAM, CPLEX 9.0

Statistics	Two-tier hierarchical model
Objective function	4 079 133 (Gap 0%)
Number of restriction	7392
Number of continuous variables	35 862
Number of integer variables	218
Number of iteration	2 687 057
CPU time in seconds	6 762

Table 4: Computer statistics of the multiproduct model with capacity restrictions. Pentium mobile 1.73 GHz, 0.98 GB RAM, CPLEX 9.0

Statistics	Multiproduct Model
Objective function	56 709 135 (Gap 0%)
Number of restriction	5 560
Number of continuous variables	216 475
Number of integer variables	0
Number of iteration	449
CPU time in seconds	4

Table 5: Results of the multiproduct model with capacity restrictions

		Impatient care	Emergency care	External consultation
Distric Hospitals	Faro	200	63.665	76.616
	Portimão	255	91.646	110.288
	Beja	200	84.951	97.015
	Castelo de Vide	200	54.750	41.959
	Borba	200	54.750	83.259
	Montemor o Novo	200	71.562	86.531
	Almada	201	103.571	124.639
	Barreiro	200	54.750	61.234
	Moita	200	77.039	92.711
	Santiago do Cacém	200	55.184	66.408
	Seixal	200	96.775	116.460
	Setúbal	200	82.510	117.413
	Abrantes	200	54.750	60.302
	Entroncamento	200	64.502	89.765
	Tomar	200	63.527	76.449
	Alenquer	200	54.750	40.000
	Amadora	306	113.262	136.301
	Cascais	297	109.920	132.279
	Mafra	200	54.750	42.127
	Odivelas	208	86.197	103.731
	Oeiras	283	104.410	125.649
	Torres Vedras	200	70.491	84.830
	Vila Franca de Xira	200	91.016	142.335
	Total	4951	1.758.728	2.108.301
Central Hospitals	Loulé	500	104.455	202.339
	Palmela	546	54.750	210.297
	Santarém	500	91.357	177.864
	Lisboa	1065	363.639	500.932
	Loures	551	120.865	266.787
	Sintra	783	234.254	349.856
	Total	3945	969.320	1.708.074

Table 6: Computer statistics for the multiproduct model without capacity restrictions. Pentium mobile 1.73 GHz, 0.98 GB RAM, CPLEX 9.0

Statistics	Multiproduct Model
Objective function	54 403 107 (Gap 0%)
Number of restriction	4 252
Number of continuous variables	21 6475
Number of integer variables	0
Number of iteration	320
CPU time in seconds	4

Table 7: Results of the multiproduct model without capacity restrictions

		Impatient care	Emergency care	External consultation
District hospitals	Faro	198	63.665	76.616
	Portimão	279	91.646	110.288
	Beja	227	86.340	97.015
	Castelo de Vide	95	34.868	41.959
	Borba	189	69.184	83.259
	Montemor o Novo	193	70.173	86.531
	Almada	246	103.571	124.639
	Barreiro	121	50.884	61.234
	Moita	183	77.039	92.711
	Santiago do Cacém	136	55.184	66.408
	Seixal	230	96.775	116.460
	Setúbal	233	97.566	117.413
	Abrantes	136	50.108	60.302
	Entroncamento	202	74.592	89.765
	Tomar	172	63.527	76.449
	Alenquer	84	30.981	37.283
	Amadora	327	113.262	136.301
	Cascais	297	109.920	132.279
	Mafra	95	35.007	42.127
	Odivelas	238	86.197	103.731
	Oeiras	283	104.410	125.649
	Torres Vedras	191	70.491	84.830
	Vila Franca de Xira	347	127.199	145.052
	Total	4700	1.762.589	2.108.301
Central hospital	Loulé	534	104.455	202.339
	Palmela	543	43.560	210.297
	Santarém	443	91.357	177.864
	Lisboa	1071	363.639	469.143
	Loures	746	128.194	298.576
	Sintra	830	234.254	349.856
	Total	4168	965.459	1.708.074

Table 8: Comparison of the average and maximum travel time in each scenario

Average travel time

	Algarve	Beja	Portalegre	Évora	Setúbal	Santarém	Lisboa
C0	15,16	37,16	28,97	23,41	4,38	11,32	**0,00**
C1	15,16	37,16	28,97	23,41	4,38	11,32	**1,17**
C2	15,16	37,16	28,97	23,41	4,38	11,32	**1,56**
C3	15,16	37,16	**28,97**	23,41	4,38	11,32	**1,89**
C4	15,16	**37,16**	**25,33**	**23,41**	4,38	11,32	1,89
C5	15,16	**36,88**	**25,33**	**16,81**	4,38	11,32	1,89
C6	15,16	36,88	**21,53**	**23,51**	4,38	11,32	1,89
C7	**15,16**	36,88	**25,33**	**16,81**	4,38	11,60	1,89
C8	**18,93**	36,88	25,33	16,81	**4,38**	11,60	1,89
C9	15,16	36,88	25,33	16,81	**8,57**	11,60	1,89

Maximum travel time

	Algarve	Beja	Portalegre	Évora	Setúbal	Santarém	Lisboa
C0	83	97	51	84	39	35	29
C1	83	97	51	84	39	35	29
C2	83	97	51	84	39	35	29
C3	83	97	51	84	39	35	29
C4	83	**97**	51	**84**	39	35	29
C5	83	**100**	**51**	**61**	39	35	29
C6	83	100	**69**	61	39	35	29
C7	83	100	51	61	39	35	29
C8	83	100	51	61	39	35	29
C9	83	100	51	61	39	35	29

Table 9: Percentage of utilization, whose distance exceeds the 45 and 60 minutes.

% Above 45 minutes

	Algarve	Beja	Portalegre	Évora	Setúbal	Santarém	Lisboa
C0	5,50	43,65	4,09	5,09	0,00	0,00	0,00
C1	5,50	43,65	4,09	5,09	0,00	0,00	0,00
C2	5,50	43,65	4,09	5,09	0,00	0,00	0,00
C3	5,50	43,65	4,09	5,09	0,00	0,00	0,00
C4	5,50	43,65	4,09	5,09	0,00	0,00	0,00
C5	5,50	41,98	4,09	5,19	0,00	0,00	0,00
C6	5,50	41,98	7,03	5,19	0,00	0,00	0,00
C7	5,50	41,98	4,09	5,19	0,00	0,00	0,00
C8	5,50	41,98	4,09	5,19	0,00	0,00	0,00
C9	5,50	41,98	4,09	5,19	0,00	0,00	0,00

% Above 60 minute

	Algarve	Beja	Portalegre	Évora	Setúbal	Santarém	Lisboa
C0	0,95	18,31	0,00	1,86	0,00	0,00	0,00
C1	0,95	18,31	0,00	1,86	0,00	0,00	0,00
C2	0,95	18,31	0,00	1,86	0,00	0,00	0,00
C3	0,95	18,31	0,00	1,86	0,00	0,00	0,00
C4	0,95	18,31	0,00	1,86	0,00	0,00	0,00
C5	0,95	17,39	0,00	3,33	0,00	0,00	0,00
C6	0,95	17,39	4,09	3,33	0,00	0,00	0,00
C7	0,95	17,39	0,00	3,33	0,00	0,00	0,00
C8	0,95	17,39	0,00	3,33	0,00	0,00	0,00
C9	0,95	17,39	0,00	3,33	0,00	0,00	0,00

Table 10: Hospitals' capacity in each scenario (* means two units).

		Capacity in beds									
		C0	C1	C2	C3	C4	C5	C6	C7	C8	C9
District Hospitals	Faro	198	198	198	198	198	198	198	198	-	198
	Portimão	279	279	279	279	279	279	279	279	285	279
	Beja	227	227	227	227	227	224	224	224	224	224
	Castelo de Vide	95	95	95	95	-	-	-	-	-	-
	Elvas	-	-	-	-	-	-	141	-	-	-
	Portalegre	-	-	-	-	101	101	108	101	101	101
	Borba	189	189	189	189	183	183	-	183	183	183
	Évora	-	-	-	-	-	176	209	176	176	176
	Montemor o Novo	193	193	193	193	193	-	-	-	-	-
	Almada	246	246	246	246	246	246	246	246	246	476
	Barreiro	121	121	121	121	121	121	121	121	121	121
	Moita	183	183	183	183	183	183	183	183	183	183
	Montijo	-	-	-	-	-	-	-	-	-	-
	Santiago do Cacém	136	136	136	136	136	136	136	136	136	136
	Seixal	230	230	230	230	230	230	230	230	230	-
	Setúbal	233	233	233	233	233	234	234	234	234	234
	Abrantes	136	136	136	136	136	136	139	156	156	156
	Entroncamento	202	202	202	202	202	202	202	-	-	-
	Tomar	172	172	172	172	172	172	172	172	172	172
	Torres Novas	-	-	-	-	-	-	-	182	182	182
	Alenquer	84	-	84	-	-	-	-	-	-	-
	Amadora	327	327	327	327	327	327	327	327	327	327
	Cascais	297	297	297	297	297	297	297	297	297	297
	Mafra	95	95	-	-	-	-	-	-	-	-
	Odivelas	238	238	239	239	239	239	239	239	239	239
	Oeiras	283	283	283	283	283	283	283	283	283	283
	Torres Vedras	191	206	191	206	206	206	206	206	206	206
	Vila Franca de Xira	347	416	347	416	416	416	416	416	416	416
Central Hospitals	Loulé	534	534	534	534	534	533	533	533	668	533
	Palmela	543	543	543	543	541	556	554	556	556	475
	Santarém	443	443	443	443	445	445	448	445	445	445
	Lisboa*	1.071	1.071	1.071	1.162	1.162	1.162	1.162	1.162	1.162	1.243
	Loures	746	746	810	718	718	718	718	718	718	718
	Sintra	830	830	830	830	830	830	830	830	830	830

FIGURES

Figure 1: Hospitals flow scheme.

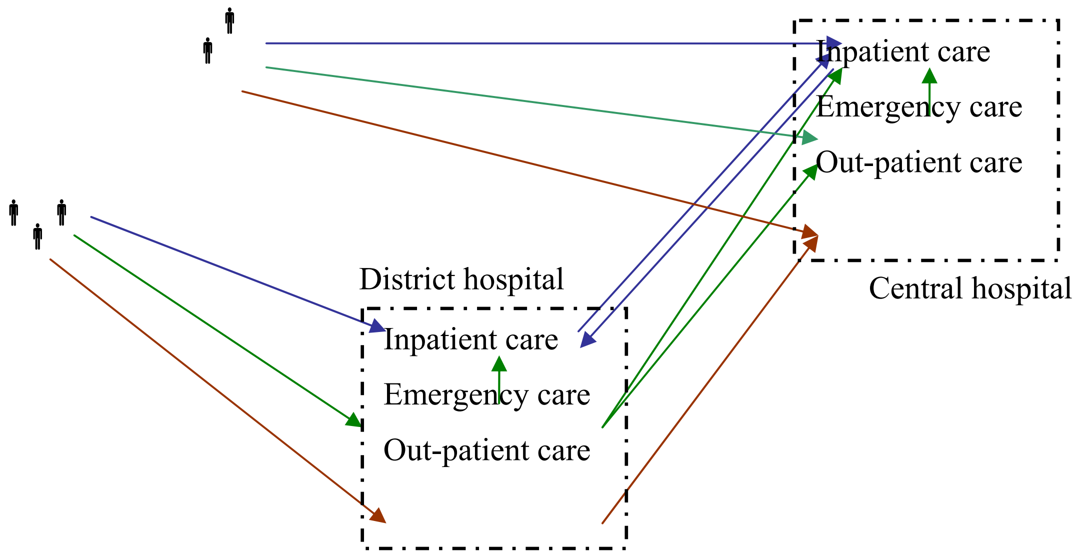


Figure 2: Hospitals flow scheme considered in the Portuguese case.

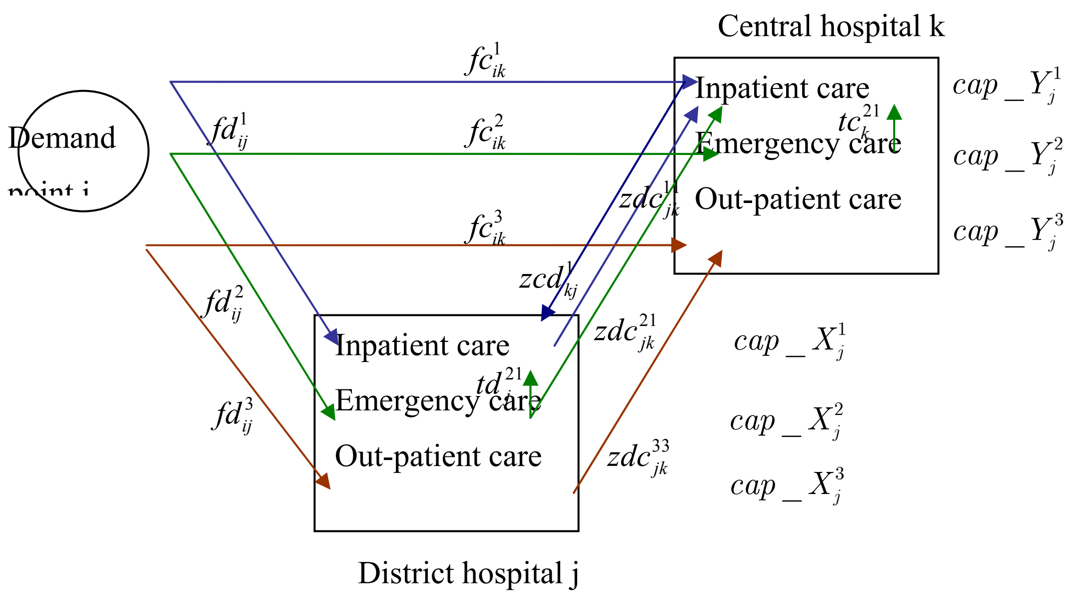


Figure 3: Current distribution of the hospitals in the South of Portugal.

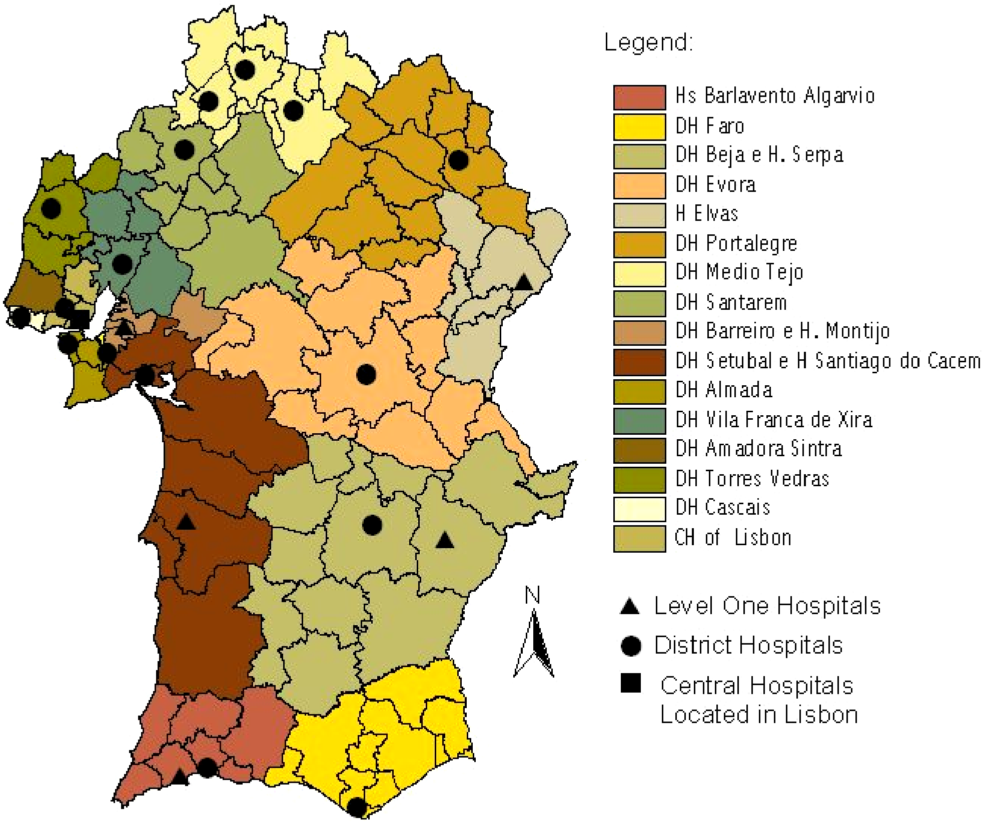


Figure 4: Percentages used in the estimation of the needs for the inpatient care.

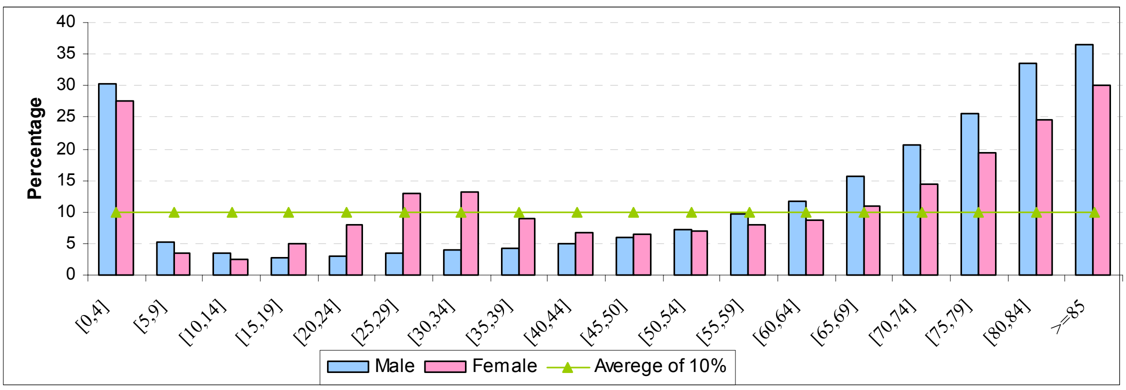
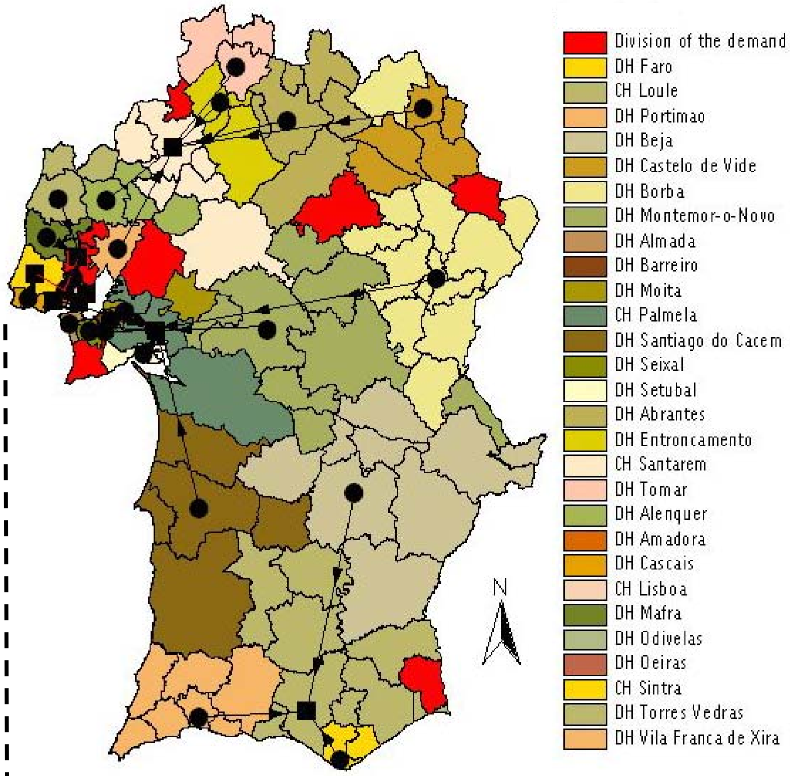


Figure 5: Synthesis of the results for the single product model

Faro	200
Portimão	282
Beja	225
Castelo de Vide	241
Borba	200
Montemor o Novo	200
Almada	318
Barreiro	200
Moita	206
Santiago do Cacém	200
Seixal	277
Setúbal	222
Abrantes	200
Entroncamento	200
Tomar	207
Alenquer	200
Amadora	333
Cascais	333
Mafra	200
Odivelas	248
Oeiras	315
Torres Vedras	200
Vila Franca de Xira	231
	5439

(23 units)

a) DH capacity

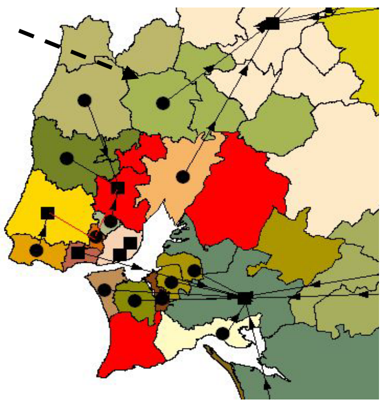


c) Spatial distribution of the hospitals and the population allocation.

Loulé	500
Palmela	500
Santarém	500
Lisboa	1291
Loures	500
Sintra	786
	4077

(7 units)

b) CH capacity



d) Amplifying Lisbon and the closed wards

Figure 6: Results on average and maximum travel time to reach an hospital.

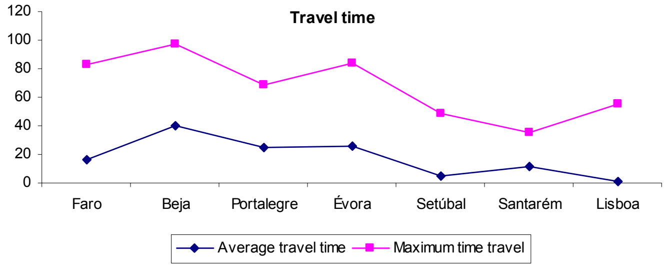


Figure 7: Share of population whose distance exceeds 45 and 60 minutes.

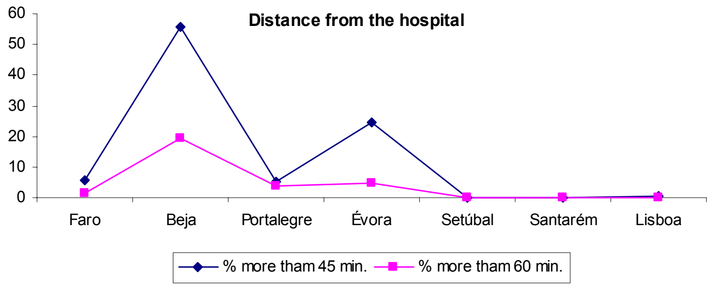


Figure 8: Results on the objective function and number of units to open in each scenario.

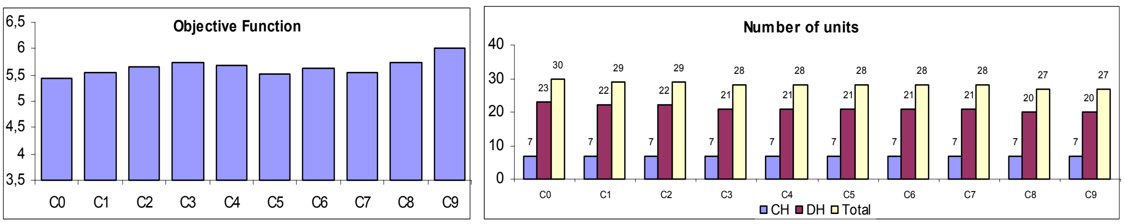


Figure 9: Results for the emergency care.