Mergers of producers of complements: how autonomous markets change the price effects

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Abstract

We analyze the price effects of mergers to monopoly between producers of complementary goods when there exists a fraction of consumers that value only one of the components. We show that customers are more likely to face a price decrease for the composite good under this setting than when such consumers do not exist.

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1 Introduction

Although most mergers occur between firms selling substitute goods, it is worth analyzing what changes when the goods involved are complements. When merging firms sell complements, mergers may be beneficial to consumers, contrary to what generally happens when goods are substitutes.[1] The intuition is that by lowering the price of one product, sales of the complement(s) are also expanded, an effect which is internalized when the owner is the same ("vertical" integration).[2]

The complements case has been dealt with in the literature mainly by considering products that are combined to yield a composite good. Consumers are interested in composite goods and not in just one of their components: for example, hardware plus software, ATMs plus bank cards, computers plus operating systems, cars plus gasoline or gasoil, and so on. However, there are many cases in which one of the components does not need to be purchased in conjunction with the other, such as computers which may operate by themselves, whereas printers need to be complemented by computers. Other examples are operating systems and internet browsers or media players, mobile phones and phone calls. These goods may be combined to create a composite good, or one of them may be consumed separately. We call these *asymmetric* complements.

Previous literature has focused mainly on *symmetric* complements. This is

[1] Mergers between producers of complements are typically considered as benign, except for the possibilities of foreclosure and bundling (see, for instance, Rey and Tirole, 2005).

[2] The word "vertical" is employed here bearing in mind that it does not refer to the usual integration upstream or downstream of production.

the case of Gaudet and Salant (1992), Economides and Salop (1992) or Kim and Shin (2002), which have different characteristics as to the number of components that make up the composite good and as to the number of competitors selling each component. Figure 1 helps to understand these differences. These papers consider price competing firms and look at the total price that must be paid to purchase a composite good. However, the market structure differs considerably. In Economides and Salop (1992) there are two differentiated brands of each of the two components needed to create the composite good, which may thus be combined in four different ways; Kim and Shin (2002) require three components for the composite good, with duopoly competition for the first component only, leading to two substitute composite goods. Gaudet and Salant (1992), in turn, use a framework in which each component is produced by a single firm, and hence there is only one composite good.

One important result in Economides and Salop (1992) is that the price of the composite good is lower under joint ownership, i.e., following a merger involving all four firms, than under independent ownership if and only if the four composite goods are distant substitutes. As compared with independent ownership, joint ownership has two opposite sign effects: on the one hand, it involves horizontal integration between producers of a given component, which pushes prices upwards; on the other hand, it involves "vertical" integration between producers of different components, which pushes prices downwards. The balance between the horizontal and the "vertical" effects is shown to depend on the degree of substitutability between composites. When these are close sub-

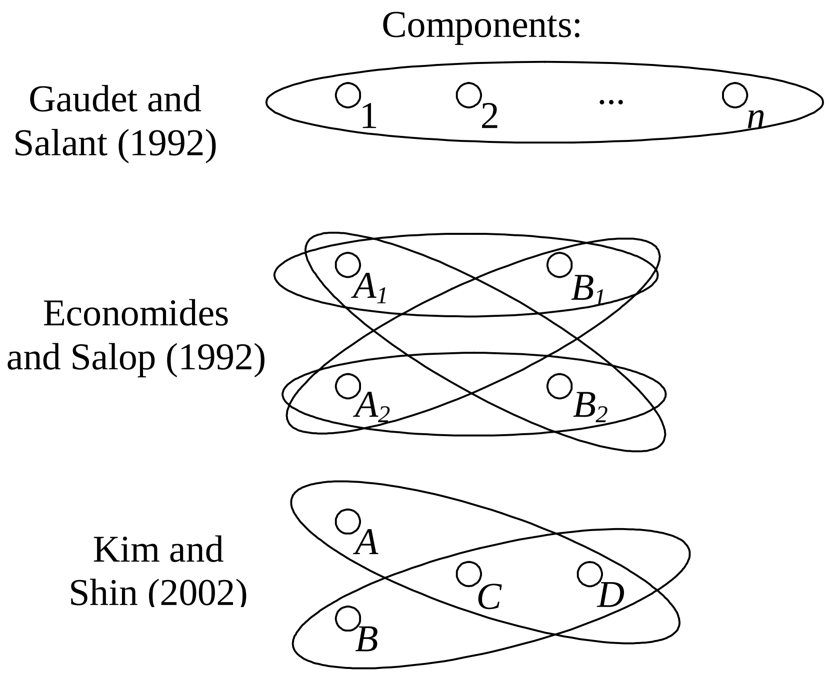

Figure 1: Number of components that make up the composite good and number of competitors for each component. Circles represent componets and elliptic forms represent the composite good.

stitutes, horizontal effects dominate and joint ownership leads to higher prices when compared with independent ownership.

In this paper, we analyze merger activity with firms selling *asymmetric* complements. To make the results comparable, we use a setting as similar as possible to Economides and Salop (1992). In addition to the market for the composite good, we admit the existence of a group of consumers that is solely interested in a single component produced, at the outset, by two competing firms. This group of consumers, which we refer to as an *autonomous market*, is assumed not to be distinguishable from the rest and therefore price discrimination is not possible. In this context, a merger involving all firms would have an additional horizontal effect as compared with Economides and Salop and one would hence expect it to be more likely to increase prices. However, we show that for some parameter values such merger may reduce composite good prices, whereas they would increase in the absence of the consumers who care only for one of the components. The reason is that the existence of this group pushes prices up before the merger, as compared with the situation in which they do not exist, therefore giving room for a price decrease after the merger.

As in all the papers cited above we abstain from considering the possibility of bundling.[3]

The paper is structured as follows. Section 2 presents the basic model and derives the equilibria. The results are presented in Section 3. Finally, the last section concludes. All proofs are presented in the appendix.

[3] For a thorough exposition on the economics of bundling see Kuhn, Stillman and Caffarra (2004).

2 The model

We assume the existence of a composite good, made up of two components, A and B. Each component is produced at zero marginal cost by two competitors: firms A_1 and A_2 produce component A and firms B_1 and B_2 produce component B. Let p_i denote the price of component A set by firm A_i, $i = 1, 2$ and q_j denote the price of component B set by firm $B_j, j = 1, 2$ and define $s_{ij} = p_i + q_j$ as the price of the composite good. We assume that there are two types of consumers. Some consumers have a demand for the composite good, while others only value component A.[4] The first group of consumers has four different composite goods available, as in Economides and Salop: 11, 12, 21, 22, where the first number refers to the firm producing component A and the second one to the identity of component B's producer. Consumers regard these alternatives as four simmetrically differentiated products and we assume that there is a market demand given by

$$D_{ij}(s_{ij}, s_{ii}, s_{ji}, s_{jj}) = a - b.s_{ij} + c.s_{ii} + c.s_{ji} + c.s_{jj} \text{ with } i, j = 1, 2 \text{ and } b > 3c > 0.$$

The second group of consumers has two alternatives available: it may purchase component A from producer A_1 or A_2. Market demand for each of these

[4] In Gabszewicz et al (2001) the joint consumption of two complemetary goods has a higher value than the addition of utilities when they are consumed in isolation. If high enough, this difference in valuation is a motive for some consumers to always prefer the composite good.

firms is given by

$$D_{A_i}(p_i, p_j) = f - g.p_i + h.p_j \text{ with } i, j = 1, 2 \text{ and } i \neq j \text{ and } g > h > 0.$$

Before proceeding we make the following assumptions regarding the parameterization of the autonomous market:

$g \in G := \left[(b - 3c)\, \frac{f}{a}, +\infty\right]$

$h \in H := \left[g - \frac{g(b-2c)+(b-c)(3b-7c)}{b-c} \frac{f}{a}, g - (b - 3c)\, \frac{f}{a}\right] \cap [0, +\infty]$

It is easy to check that G is non-empty for all parameter values and that if g belongs to G, H is also non-empty for all other parameter values. Additionally, any h in H is smaller than g. In the appendix we show that these assumptions ensure that all prices and quantities are positive under all ownership structures.[5] Throughout the remainder of the paper we will assume that $g \in G$ and $h \in H$.

The following subsections present the equilibrium prices for three alternative market structures: independent ownership, partial integration and joint ownership. Note that if $f = g = h = 0$ we obtain the same results as Economides and Salop, with $c = d = e$. Quantities are presented in the appendix.

2.1 Independent ownership structure

The independent ownership structure refers to the case in which all four firms are operated independently, maximizing own profits. We denote this case with

[5] Note that we cannot allow q_j to be negative because this would preclude the existence of the autonomous market, since consumers just interested in good A would choose to buy the composite instead.

superscript I. Objective functions are

$$\begin{aligned} Max_{p_i}\ \pi_{A_i} &= p_i(D_{ii} + D_{ij} + D_{A_i}), \qquad i = 1,2 \\ Max_{q_j}\ \pi_{B_j} &= q_j(D_{ij} + D_{jj}), \qquad j = 1,2 \end{aligned}$$

It is easy to show that the equilibrium prices are the following:

$$\begin{aligned} p_i^I &= \frac{a\,(b-c) + f\,(b-2c)}{(b-c)\,(3b-7c) + (2g-h)\,(b-2c)},\ i = 1,2; \\ q_j^I &= \frac{2a\,(b-c) + a\,(2g-h) - f\,(b-3c)}{2\,(b-c)\,(3b-7c) + 2\,(2g-h)\,(b-2c)},\ j = 1,2; \\ s_{ij}^I &= \frac{1}{2}\frac{4a\,(b-c) + a\,(2g-h) + f\,(b-c)}{(b-c)\,(3b-7c) + (2g-h)\,(b-2c)},\ i,j = 1,2 \text{ and } i \neq j. \end{aligned}$$

2.2 Partial integration

The partial integration scenario refers to the ownership structures resulting from a merger between A_1 and B_1 and another merger between A_2 and B_2.[6] In this case, two firms produce the composite good as a whole but we consider that it is still possible to purchase the components separately. We denote this case with superscript P. Objective functions are

$$\begin{aligned} Max_{p_1,q_1}\ \pi_{A_1} + \pi_{B_1} &= p_1(D_{11} + D_{12} + D_{A_1}) + q_1(D_{11} + D_{21}) \\ Max_{p_2,q_2}\ \pi_{A_2} + \pi_{B_2} &= p_2(D_{21} + D_{22} + D_{A_2}) + q_2(D_{12} + D_{22}) \end{aligned}$$

[6] These mergers are equivalent to one comprising A_1 and B_2, and the other A_2 and B_1.

The equilibrium prices are given by:

$$
\begin{aligned}
p_i^P &= \frac{4f(b-2c)+2a(b+c)}{(7b-17c)(b+c)+4(2g-h)(b-2c)},\ i=1,2; \\
q_j^P &= \frac{2a(b+c)+2a(2g-h)-3f(b-3c)}{(7b-17c)(b+c)+4(2g-h)(b-2c)},\ j=1,2; \\
s_{ij}^P &= \frac{4a(b+c)+2a(2g-h)+f(b+c)}{(7b-17c)(b+c)+4(2g-h)(b-2c)},\ i,j=1,2 \text{ and } i\neq j.
\end{aligned}
$$

Note that when moving from the independent structure to the partial integration case we are only in the presence of "vertical" integration. As in Economides and Salop, partial integration leads inequivocally to a price decrease.

Lemma 1: *The price for the composite good s_{ij} is always lower in the partial integration structure than under independent ownership, i.e., $s_{ij}^P < s_{ij}^I$.* ■

2.3 Joint ownership structure

The joint ownership structure refers to the merger between the four firms. We denote this case with superscript J. The objective function is

$$Max_{p_1,p_2,q_1,q_2}\ \pi_{A_1}+\pi_{A_2}+\pi_{B_1}+\pi_{B_2}$$

with the expressions for π_k given before. Profit maximizing prices are the following:

$$\begin{aligned} p_i^J &= \frac{f}{2(g-h)}, \ i = 1,2; \\ q_j^J &= \frac{a(g-h) - f(b-3c)}{2(g-h)(b-3c)}, \ j = 1,2; \\ s_{ij}^J &= \frac{a}{2(b-3c)}, \ i,j = 1,2 \text{ and } i \neq j. \end{aligned}$$

When moving from the partial ownership to the joint ownership structure there are both "vertical" and horizontal effects. The monopolist sets its monopoly prices in the market for the composite good and in the autonomous market (s_{ij}^J and p_i^J) and then adjusts q_j^J as the difference between the two.

3 Results

In this section we compare the price of the composite good under the different ownership structures and show that, for some parameter values, a merger between the four firms may reduce this price in circumstances under which it would increase in the absence of the autonomous market for component A.

Lemma 1 leads us to the existence of only three possible price rankings across the three ownership structures: $s_{ij}^I > s_{ij}^P > s_{ij}^J$, $s_{ij}^I > s_{ij}^J > s_{ij}^P$ or $s_{ij}^J > s_{ij}^I > s_{ij}^P$. The following Lemma establishes for which parameter values each of these price orderings occurs.

Lemma 2: *Let $g \in G$ and $h \in H$:*

(i) If $g < \frac{(b+c)(b-7c)}{4c}$*, then* $s_{ij}^I > s_{ij}^P > s_{ij}^J$*;*

(ii) If $\frac{(b+c)(b-7c)}{4c} < g < \frac{(b-c)(b-5c)}{c}$*, then either* $s_{ij}^I > s_{ij}^P > s_{ij}^J$ *or* $s_{ij}^I > s_{ij}^J > s_{ij}^P$*;*

(iii) If $g > \frac{(b-c)(b-5c)}{c}$*, then either* $s_{ij}^I > s_{ij}^P > s_{ij}^J$*,* $s_{ij}^I > s_{ij}^J > s_{ij}^P$ *or* $s_{ij}^J > s_{ij}^I > s_{ij}^P$*.* ■

Let us now focus on the price impact of the merger to monopoly when compared to independent ownership.

Recall from Economides and Salop that if (i) $b < 5c$, then $s_{ij}^J > s_{ij}^I > s_{ij}^P$, (ii) if $5c < b < 7c$, then $s_{ij}^I > s_{ij}^J > s_{ij}^P$ and (iii) if $b > 7c$, then $s_{ij}^I > s_{ij}^P > s_{ij}^J$. Thus, only if $b < 5c$ will the joint ownership structure yield a higher price than when all firms are independent. When $b < 5c$ all composite goods are close substitutes. Therefore the joint ownership structure, leading to a monopolist seller of close substitutes, results in the highest equilibrium prices despite the existence of "vertical" effects.

The inclusion of some consumers that only value component A would seem to enhance the horizontal effects of a merger between the four firms. Indeed, these consumers care only for one component that prior to the merger was sold by two independent firms and that after the merger has its price set by the same owner.

Nevertheless, as we will show in the next Proposition and Corollary, the inclusion of these consumers may actually change the price ranking between the independent and the joint ownership structures for some values of $b < 5c$, so that full integration decreases the price for the composite good as compared with the no-merger situation despite the stronger horizontal effects.

Proposition 1: *Assume that $g \in G$ and $h \in H$. The joint ownership case may yield lower equilibrium prices for the composite good than the independent ownership case if and only if* $\frac{f}{a} > -\frac{(b-c)(b-5c)}{(b-3c)^2}$ *and* $g < \frac{(b-c)(b-5c)}{c} + \frac{b-2c}{c}(b-3c)\frac{f}{a}$. ■

Under Economides and Salop's joint owership increases the composite good's price when compared to independent ownership when $b \in [3c, 5c]$. The next Corollary shows that we may obtain the opposite result for some values of b in this interval. We admit that $f/a < 1$ to keep the structure of the model as close to Economides and Salop's as possible, by introducing a second market that is less important in terms of (autonomous) demand than the one for the composite good.

Corollary 1: *Assume that $g \in G$, $h \in H$,* and $f/a < 1$. *The joint ownership case may yield lower equilibrium prices for the composite good than the independent ownership case if and only if (i)* $b > \left(3+\sqrt{2}\right)c$ and (ii) $g < \frac{(b-c)(b-5c)}{c} + \frac{b-2c}{c}(b-3c)\frac{f}{a}$. ■

In our setting, provided that $f/a < 1$ and that condition (ii) in Corollary 1 is verified, we obtain that joint owership clearly increases the composite good's price when compared to independent ownership only when $b \in \left[3c, (3+\sqrt{2})c\right]$.

This amounts to a reduction close to 30% in the magnitude of Economides and Salop's interval.

The intuition is simple: the existence of a group of consumers that is solely interested in component A pushes the price of this component upwards in the independent ownership case, as compared with the Economides and Salop setup, therefore giving room for a price fall after the merger. These consumers exert a negative externality upon the buyers of the composite good before the merger. After the merger of all firms, the owner sets the monopoly price for the composite good and for the autonomous one, and this externality disappears.

Nevertheless, the effect resulting from the existence of the autonomous market is not sufficiently strong for joint ownership to yield the lowest price for the composite good across all market structures considered when composites are close substitutes. We show this in the next Lemma.

Lemma 3: *It is impossible to obtain the price ranking $s_{ij}^I > s_{ij}^P > s_{ij}^J$ with $f/a < 1$ and $b < 5c$.* ■

As we noted, the result in Corollary 1 was obtained for $f/a < 1$, that is, for relatively small demand for component A when compared to that of the composite good. Clearly, if we allow for higher values of f/a, which is realistic for some of the examples given in the Introduction, it becomes even easier to obtain $s_{ij}^J < s_{ij}^I$.

4 Conclusions

Economides and Salop (1992) have illustrated that monopolizing mergers do not necessarily imply a price increase, even when they do not lead to cost reductions. Complete integration is a way to capture all the “vertical” connections between products and may prove beneficial for consumers as long as composite goods are not close susbstitutes. We used the same setting, but allowed for the existence of a group of consumers that is interested in only one of the components. Despite the stronger horizontal effects present, we have showed that monopolizing mergers may reduce the price of the composite good in more circumstances regarding the substitutability of composites than in Economides and Salop’s paper. The reason is that full integration eliminates the negative externality exerted by the buyers in the autonomous market upon the composite good consumers. This result has policy implications for the analysis of proposed acquisitions involving both horizontal and “vertical” effects.

Appendix

We start by showing that prices and quantities are positive for all ownership structures if and only if $g \in G$ and $h \in H$.

All prices p_i are positive by simple inspection of the expressions. As $s_{ij} = p_i + q_j$, it suffices to show that all q_j are positive.

All prices q_j are positive if and only if

$$\begin{aligned} h &< 2\,(b-c)+2g-\frac{f}{a}\,(b-3c) \quad (1)\\ h &< b+c+2g-\frac{3}{2}\frac{f}{a}\,(b-3c) \quad (2)\\ \text{and } h &< g-\frac{f}{a}\,(b-3c) \quad (3)\end{aligned}$$

It is clear that (3) and the assumptions that $b > 3c$ and $g > 0$ imply (1). Condition (2) can be re-written as

$$h < b+c+g-\frac{1}{2}\frac{f}{a}\,(b-3c)+g-\frac{f}{a}\,(b-3c)$$

which is implied by (3) and by the assumptions that b and c are positive and $g \in G$.

We now turn to output. In the case of independent owenership, quantities are given by

$$\begin{aligned} D^I_{ij} &= a-(b-3c)\frac{1}{2}\frac{4a(b-c)+a(2g-h)+f(b-c)}{(b-c)\,(3b-7c)+(2g-h)(b-2c)},\ i,j=1,2 \text{ and } i\neq j;\\ D^I_{Ai} &= f-(g-h)\,\frac{a\,(b-c)+f\,(b-2c)}{(b-c)\,(3b-7c)+(2g-h)\,(b-2c)},\ i=1,2.\end{aligned}$$

Under partial integration, we have

$$\begin{aligned} D_{ij}^{P} &= a-(b-3c)\left(\frac{4a\,(b+c)+2a\,(2g-h)+f\,(b+c)}{(7b-17c)\,(b+c)+4\,(2g-h)\,(b-2c)}\right),\ i,j=1,2,\ i\neq j; \\ D_{Ai}^{P} &= f-(g-h)\frac{4f\,(b-2c)+2a\,(b+c)}{(7b-17c)\,(b+c)+4\,(2g-h)\,(b-2c)},\ i=1,2. \end{aligned}$$

Finally, with joint ownership quantities are

$$\begin{aligned} D_{ij}^{J} &= \frac{a}{2},\ i,j=1,2 \text{ and } i\neq j. \\ D_{Ai}^{J} &= \frac{f}{2},\ i=1,2. \end{aligned}$$

The quantities D_{ij} are positive if

$$\begin{aligned} h &< 2(b-c)+2g-\frac{f}{a}\,(b-3c) && (4) \\ \text{and } h &< 2g+\frac{(3b-5c)\,(b+c)}{2\,(b-c)}-\frac{(b-3c)\,(b+c)}{2\,(b-c)}\frac{f}{a} && (5) \end{aligned}$$

Note that $h<g-\frac{f}{a}\,(b-3c)$ clearly implies (4). It is easy to check that with $b>3c$ we always have

$$g-\frac{f}{a}\,(b-3c)<2g+\frac{(3b-5c)\,(b+c)}{2\,(b-c)}-\frac{(b-3c)\,(b+c)}{2\,(b-c)}\frac{f}{a}$$

Hence, $h<g-\frac{f}{a}\,(b-3c)$ also implies (5).

Quantities D_{Ai} are positive if and only if

$$\begin{aligned} f-(g-h)\frac{a(b-c)+f(b-2c)}{(b-c)(3b-7c)+(2g-h)(b-2c)} &> 0 \\ \text{and } f-(g-h)\frac{4f(b-2c)+2a(b+c)}{(7b-17c)(b+c)+4(2g-h)(b-2c)} &> 0 \end{aligned}$$

These conditions are equivalent to

$$h > g-\frac{g(b-2c)+(b-c)(3b-7c)}{b-c}\frac{f}{a} \quad (6)$$

$$\text{and } h > g-\frac{4g(b-2c)+(7b-17c)(b+c)}{2(b+c)}\frac{f}{a} \quad (7)$$

Note that (6) implies (7) if

$$g-\frac{g(b-2c)+(b-c)(3b-7c)}{b-c}\frac{f}{a} > g-\frac{4g(b-2c)+(7b-17c)(b+c)}{2(b+c)}\frac{f}{a}$$

This is equivalent to

$$-2g(b-2c)-b^2+c^2<0$$

which is always true.

Recall that all prices q_j are positive if and only if $h<g-\frac{f}{a}(b-3c)$. This condition implies that all quantities D_{ij} are positive. In turn quantities D_{Ai} are positive if and only if $h>g-\frac{g(b-2c)+(b-c)(3b-7c)}{b-c}\frac{f}{a}$. Thus, all prices and outputs are positive if and only if $h\in H$.

Lemma 1: Condition $s_{ij}^P < s_{ij}^I$ is equivalent to

$$\frac{4a(b+c)+2a(2g-h)+f(b+c)}{(7b-17c)(b+c)+4(2g-h)(b-2c)} < \frac{1}{2}\frac{4a(b-c)+a(2g-h)+f(b-c)}{(b-c)(3b-7c)+(2g-h)(b-2c)}$$

which can be simplified to

$$a(2g-h)(3b-c)+4a(b-c)(b+c)+2f(2g-h)(b-2c)+\left(b^2-c^2\right)>0 \quad (8)$$

Inspection of (8) reveals that all terms are positive, given that $b > 3c$ and $g > h$, which completes the demonstration. ■

Lemma 2: Start by defining

$$\begin{aligned} z &:= \frac{(b-c)(b-5c)}{c} + \frac{(b-c)(b-3c)}{c}\frac{f}{a} \\ y &:= \frac{(b+c)(b-7c)}{4c} + \frac{(b+c)(b-3c)}{2c}\frac{f}{a} \end{aligned}$$

and noting that[7]

$s_{ij}^J > s_{ij}^I > s_{ij}^P$ if and only if $0 < h < 2g - z$;

$s_{ij}^I > s_{ij}^J > s_{ij}^P$ if and only if $\max\{2g-z, 0\} < h < 2g - y$;

$s_{ij}^I > s_{ij}^P > s_{ij}^J$ if and only if $h > 2g - y$.

(i) Let $g < \frac{(b+c)(b-7c)}{4c}$. Note that this implies $b > 7c$. As $h > g - \frac{g(b-2c)+(b-c)(3b-7c)}{b-c}\frac{f}{a}$

[7] Clearly, $z > y$ because $z - y = \frac{3}{4}\frac{(b-3c)^2}{c} + \frac{1}{2}\frac{(b-3c)}{c}^2\frac{f}{a} > 0$.

we have $h > 2g - y$ (i.e., $s_{ij}^I > s_{ij}^P > s_{ij}^J$) if

$$g - \frac{g(b-2c) + (b-c)(3b-7c)}{b-c}\frac{f}{a} > 2g - \frac{(b-7c)(b+c)}{4c} - \frac{(b-3c)(b+c)}{2c}\frac{f}{a}$$

This can be simplified to

$$(b-c)(b-7c)(b+c) - 4c(b-c)g + 2\left((b-c)\left(b^2 - 8bc + 11c^2\right) - 2c(b-2c)g\right)\frac{f}{a} > 0$$

The lefthand side of this inequality is clearly decreasing in g and, evaluated at the maximum admissible value for g in this case ($\frac{(b+c)(b-7c)}{4c}$) yields

$$(b-3c)^2(b-4c)\frac{f}{a} > 0$$

which is true as $b > 7c$.

(ii) Let $\frac{(b+c)(b-7c)}{4c} < g < \frac{(b-c)(b-5c)}{c}$. Note that this implies $b > 5c$. We will show that in this case the price ranking $s_{ij}^J > s_{ij}^I > s_{ij}^P$ is impossible. As $h > g - \frac{g(b-2c)+(b-c)(3b-7c)}{b-c}\frac{f}{a}$ we have $h > 2g - z$ (which excludes $s_{ij}^J > s_{ij}^I > s_{ij}^P$) if

$$g - \frac{g(b-2c) + (b-c)(3b-7c)}{b-c}\frac{f}{a} > 2g - \frac{(b-c)(b-5c)}{c} - \frac{(b-c)(b-3c)}{c}\frac{f}{a}$$

This can be simplified to

$$g < \frac{(b-c)(b-5c)}{c}$$

which is true, so the price ordering $s_{ij}^J > s_{ij}^I > s_{ij}^P$ is not possible. None of the other rankings can be excluded under these conditions for the parameters.

(iii) No price ranking can be excluded under these parameter conditions. ■

Proposition 1: From Lemma 2, the joint ownership case yields lower equilibrium prices for the composite good than the independent ownership case if and only if

$$h > \max\left\{2g - \frac{(b-c)(b-5c)}{c} - \frac{(b-c)(b-3c)}{c}\frac{f}{a}, 0\right\} \tag{9}$$

(i) Assume initially that $\frac{f}{a} > -\frac{(b-c)(b-5c)}{(b-3c)^2}$.Then $\frac{(b-c)(b-5c)}{2c} + \frac{b-c}{2c}(b-3c)\frac{f}{a} > (b-3c)\frac{f}{a}$. Two things may happen, for a given g belonging to G:

(ia) $g > \frac{(b-c)(b-5c)}{2c} + \frac{b-c}{2c}(b-3c)\frac{f}{a} > (b-3c)\frac{f}{a}$.

(ib) $\frac{(b-c)(b-5c)}{2c} + \frac{b-c}{2c}(b-3c)\frac{f}{a} > g > (b-3c)\frac{f}{a}$.

Let $g > \frac{(b-c)(b-5c)}{2c} + \frac{b-c}{2c}(b-3c)\frac{f}{a} > (b-3c)\frac{f}{a}$. This implies that (9) is equivalent to $h > 2g - \frac{(b-c)(b-5c)}{c} - \frac{(b-c)(b-3c)}{c}\frac{f}{a}$. There are values for h belonging to H such that condition (9) holds if and only if

$$g - (b-3c)\frac{f}{a} > 2g - \frac{(b-c)(b-5c)}{c} - \frac{(b-c)(b-3c)}{c}\frac{f}{a}$$

which is equivalent to

$$g < \frac{(b-c)(b-5c)}{c} + \frac{b-2c}{c}(b-3c)\frac{f}{a} \tag{10}$$

Thus, there are some values for h, belonging to H and verifying (9), provided that

$$\frac{(b-c)(b-5c)}{2c}+\frac{b-c}{2c}(b-3c)\frac{f}{a}<g<\frac{(b-c)(b-5c)}{c}+\frac{b-2c}{c}(b-3c)\frac{f}{a}.$$

Let $\frac{(b-c)(b-5c)}{2c}+\frac{b-c}{2c}(b-3c)\frac{f}{a}>g>(b-3c)\frac{f}{a}$. Then, (9) is equivalent to $h>0$. Clearly, there are values for h belonging to H such that condition (9) holds if $g\in G$.

Summing up, if $\frac{f}{a}>-\frac{(b-c)(b-5c)}{(b-3c)^2}$ there are values for h in H that verify (9) if $(b-3c)\frac{f}{a}<g<\frac{(b-c)(b-5c)}{c}+\frac{b-2c}{c}(b-3c)\frac{f}{a}$.

(ii) Assume now that $\frac{f}{a}<-\frac{(b-c)(b-5c)}{(b-3c)^2}$. Then $(b-3c)\frac{f}{a}>\frac{(b-c)(b-5c)}{2c}+\frac{b-c}{2c}(b-3c)\frac{f}{a}$ and $g>(b-3c)\frac{f}{a}>\frac{(b-c)(b-5c)}{2c}+\frac{b-c}{2c}(b-3c)\frac{f}{a}$. This implies that (9) is equivalent to $h>2g-\frac{(b-c)(b-5c)}{c}-\frac{(b-c)(b-3c)}{c}\frac{f}{a}$. There are values for h belonging to H such that condition (9) holds if and only if (10) holds. This is compatible with g belonging to G if and only if

$$\frac{(b-c)(b-5c)}{c}+\frac{b-2c}{c}(b-3c)\frac{f}{a}>(b-3c)\frac{f}{a}\Leftrightarrow\frac{f}{a}>-\frac{(b-c)(b-5c)}{(b-3c)^2}\qquad(11)$$

which is impossible. ■

Corollary 1: From Proposition 1 above, the joint ownership case may yield lower equilibrium prices for the composite good than the independent ownership case only if $\frac{f}{a}>-\frac{(b-c)(b-5c)}{(b-3c)^2}$. Assume that $g\in G$, $h\in H$, and $f/a<1$. Let $r=b/c$. Then $-\frac{(b-c)(b-5c)}{(b-3c)^2}=\left(-\frac{(r-1)(r-5)}{(r-3)^2}\right)$ with $r>3$. This is a strictly decreasing (convex) function in r with a zero at $r=5$ and $\lim_{r\to\infty}\left(-\frac{(r-1)(r-5)}{(r-3)^2}\right)=$

-1. We have that $-\frac{(b-c)(b-5c)}{(b-3c)^2} = 1 \Leftrightarrow b = \left(3+\sqrt{2}\right) c$ ■

Lemma 3: From Lemma 2, start by noting that $s_{ij}^I > s_{ij}^P > s_{ij}^J$ if and only if

$$h > 2g - \frac{(b-7c)(b+c)}{4c} - \frac{(b-3c)(b+c)}{2c}\frac{f}{a}.$$

We now show that this condition is incompatible with $h \in H$. The two conditions cannot verify simultaneously if

$$2g - \frac{(b-7c)(b+c)}{4c} - \frac{(b-3c)(b+c)}{2c}\frac{f}{a} > g - (b-3c)\frac{f}{a}$$

which is equivalent to

$$g > \frac{(b-7c)(b+c)}{4c} + \frac{(b-3c)(b-c)}{2c}\frac{f}{a}$$

We now show that

$$(b-3c)\frac{f}{a} > \frac{(b-7c)(b+c)}{4c} + \frac{(b-3c)(b-c)}{2c}\frac{f}{a}$$

This inequality is equivalent to

$$-\frac{(b-3c)^2}{2c}\frac{f}{a} - \frac{(b-7c)(b+c)}{4c} > 0$$

As $f/a<1$, we have that

$$-\frac{(b-3c)^2}{2c}\frac{f}{a}-\frac{(b-7c)(b+c)}{4c}>-\frac{(b-3c)^2}{2c}1-\frac{(b-7c)(b+c)}{4c}$$

Given that

$$-\frac{(b-3c)^2}{2c}1-\frac{(b-7c)(b+c)}{4c}>0\Leftrightarrow\left(3-\frac{4}{3}\sqrt{3}\right)c<b<\left(\frac{4\sqrt{3}}{3}+3\right)c$$

is always true for $3c<b<5c$, we may conclude that

$$g>(b-3c)\frac{f}{a}>\frac{(b-7c)(b+c)}{4c}+\frac{(b-3c)(b-c)}{2c}\frac{f}{a}$$

which completes the proof. ■

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