

ELECTRE TRI-C: A MULTIPLE CRITERIA SORTING METHOD BASED ON CENTRAL REFERENCE ACTIONS

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Abstract

In this paper, we propose a new method within the ELECTRE framework. This method deals with sorting problems where the pre-defined and ordered categories are based on central reference actions instead of profile limits. We will call this method ELECTRE TRI-C. Therefore, the well-known method called up to now ELECTRE TRI based on profile limits, or boundary actions, will be designated here by ELECTRE TRI-B. After setting the interest of this new sorting method, we introduce the assumptions and structural requirements which seem natural to be fulfilled by the method. ELECTRE TRI-C provides two assignment rules: a descending rule and an ascending rule. These rules are quite similar to the pseudo-conjunctive rule (formerly called pessimistic) and the pseudo-disjunctive rule (formerly called optimistic) belonging to ELECTRE TRI-B. Therefore, there exist differences on the assignment results which will be outlined in this paper.

Keywords: Multiple Criteria Decision Aiding, Sorting, ELECTRE methods, Central Reference Actions

1 Introduction

In Multiple Criteria Decision Aiding (MCDA), the analyst can envisage the decisional analysis according to several perspectives, or problematics, which provide an idea of what is expected to be done with the object of a decision (i.e., the possible output) (Roy, 1996; Bouyssou et al., 2006). One of these problematics, the sorting problem, considers a set of categories $C_1, \dots, C_h, \dots, C_q$, which are defined *a priori* by those to whom the decision aiding is offered (i.e., the decision-makers).

The categories are defined in order to assign the objects of a decision (i.e., the actions). These actions can be credit demand files, patients waiting for treatment, risk zones, or R&D projects, among the many possibilities, and are subjected to treatment or analysis depending on the category to which they are assigned. A credit demand file can be accepted without additional information, accepted subject to additional information, sent to a particular department for further analysis, rejected under certain conditions, or rejected with no conditions at all. Or, based on a set of exams, a patient can be subject to a certain type of medical treatment among those defined *a priori* for the set of pathologies studied. Thus, the set of categories emerges naturally from the decision-aiding context through a process of interaction with the decision-makers.

Many sorting procedures have been proposed, such as the trichotomic segmentation procedure (Moscarola and Roy, 1977), N-TOMIC (Massaglia and Ostanello, 1991), filtering based procedures (Perny, 1998), PROAFTN (Belacel, 2000), TRINOMFC (Léger and Martel, 2002), multi-profile trichotomic procedure (Norese and Viale, 2002), IRIS (Dias et al., 2002b; Dias and Mousseau, 2003), PAIRCLAS (Doumpos and Zopounidis, 2004), SMAA-TRI (Tervonen et al., 2007), and a variant of ELECTRE TRI based on “central” profiles (Nemery, 2008). However, ELECTRE TRI (Yu, 1992; Roy and Bouyssou, 1993) is still currently one of the most completely formalized procedures available in MCDA for dealing with the sorting problem, with many applications having been analyzed with ELECTRE TRI over the last fifteen years (see Dimitras et al., 1995; Arondel and Girardin, 2000; Raju et al., 2000; Joerin et al., 2001; Georgopoulou et al., 2003; Merad et al., 2004; André and Roy, 2007).

ELECTRE TRI is a sorting procedure. Thus, the categories are assumed to be ordered from the “worst”, C_1 (e.g., the one that contains the worst actions, the lowest priority actions, the most risky actions) to the “best”, C_q , where $q \geq 2$ (e.g., the one that contains the best actions, the highest priority actions, the least risky actions). Profile limits are defined to mark the frontiers between two consecutive ordered categories. Each category is, therefore, bounded by a lower and an upper profile. These profile limits are defined through reference actions that can be realistic or unrealistic. These reference actions can be defined either by direct interaction with the decision-maker or by using an aggregation/disaggregation procedure in order to elicit the profile limits that allow a correct assignment of some actions previously introduced by the decision-maker to the categories (see Mousseau and Słowiński, 1998; Ngo The and Mousseau, 2002; Dias et al., 2002a;

Doumpos and Zopounidis, 2002; Mousseau and Dias, 2004; Doumpos and Zopounidis, 2007).

Defining reference actions is often a very hard task. This is particularly the case when the decision-maker has a fuzzy idea of the frontier between two consecutive categories. In order to improve the interaction with the decision-maker, the difficulties of setting profile limits led us to apprehend the categories by defining central reference actions, where the frontiers between two consecutive categories are not explicitly defined. The procedure proposed in this paper, designated ELECTRE TRI-C, is intended to achieve this goal. Therefore, the well-known method based on profile limits, or boundary actions, called up to now ELECTRE TRI will be designated here by ELECTRE TRI-B.

ELECTRE TRI-C is, therefore, a new assignment procedure. Like ELECTRE TRI-B, it assumes that each action to be considered for an assignment to a certain category is evaluated on a coherent family of criteria $g_1, \dots, g_j, \dots, g_n$. The assignment of an action only takes into account the intrinsic evaluation of this action on all the criteria and does not depend on nor influence the category to which another action should be assigned. The actions to be assigned are not compared to reference actions that define a lower and an upper bound of the category, but instead are compared to reference actions that we call “central”. To perform this new kind of actions comparison, the same outranking credibility indices used in ELECTRE TRI-B are used as they were originally defined (i.e., the same as in ELECTRE III; see, Roy and Bouyssou, 1993).

The rest of this paper is organized as follows. Section 2 introduces and reviews the concepts, definitions, and notation related to the outranking credibility indices. Section 3 is devoted to the proposed ELECTRE TRI-C method which contains the basic assumptions and the two assignment rules. Section 4 presents the properties of the ELECTRE TRI-C assignment rules. Section 5 presents an overview of ELECTRE TRI-B and a comparison between ELECTRE TRI-C and ELECTRE TRI-B. Section 6 provides a numerical example of ELECTRE TRI-C. Section 7 presents some additional results such as the comparison of the two ELECTRE TRI-C assignment rules. Finally, the last section offers our concluding remarks and some avenues for future research.

2 The credibility index: Definitions and notation

Let a_1, a_2, \dots denote the *potential actions*. The set of such actions, A , can be partially known *a priori*, and the actions can appear progressively during the decision aiding process. The objective is to assign the actions to a set of *ordered categories* $C_1, \dots, C_h, \dots, C_q$, with $q \geq 2$, the nature of which was provided in the previous section. Suppose that a coherent family F of n *criteria* $g_1, \dots, g_j, \dots, g_n$, with $n \geq 2$, has been defined in order to evaluate any action considered for assignment to a certain category.

Let us consider the criterion $g_j \in F$ and the two actions a and a' . Taking into account the preference direction of this criterion, the *advantage* of action a over action a' is defined

as follows:

$$\Omega_j(a, a') = \begin{cases} g_j(a) - g_j(a') & \text{if } g_j \text{ is to be maximized} \\ g_j(a') - g_j(a) & \text{if } g_j \text{ is to be minimized} \end{cases} \quad (2.1)$$

Below, each criterion g_j will be considered as a *pseudo-criterion*, which means that two thresholds are associated to g_j : an *indifference threshold*, q_j , and a *preference threshold*, p_j , such that $p_j \geq q_j \geq 0$. These thresholds are introduced in order to take into account the imperfect character of the data from the computation of the evaluations $g_j(a)$ as well as the arbitrariness that affects the definition of the criteria. Based on the definition of such thresholds, their values should be interpreted as follows:

- 1) $|\Omega_j(a, a')| \leq q_j$ represents a non-significant advantage of a over a' , meaning that a is *indifferent* to a' according to g_j , denoted aI_ja' .
- 2) $\Omega_j(a, a') > p_j$ represents a significant advantage of a over a' , meaning that a is *strictly preferred* to a' according to g_j , denoted aP_ja' .
- 3) $q_j < \Omega_j(a, a') \leq p_j$ represents an ambiguity zone. The advantage of a over a' is a little large to conclude about an indifference between a and a' , but this advantage is not enough to conclude about a strict preference in favour of a . This means that there is a hesitation between indifference and strict preference. In such a case, a is *weakly preferred* to a' , denoted aQ_ja' .

Let us notice that q_j can be null and/or equal to p_j .

The indifference and preference thresholds have been presented as constants. However, in practice, they can vary according to the evaluations $g_j(a)$. In order to simplify the formulae that are used in ELECTRE TRI-C, the possibility of using variable thresholds is ignored in what follows. Writing the formulae with variable thresholds introduces a complex formalization, as well as an additional distinction between direct and reverse thresholds (for more details, see Roy and Vincke, 1984 and Roy, 1996).

When using the outranking concept, the main idea is that an action a *outranks* an action a' according to the criterion g_j (denoted aS_ja') if a is judged *at least as good as* a' on the criterion g_j . This is true without ambiguity when $\Omega_j(a, a') \geq -q_j$. But, when $-p_j \leq \Omega_j(a, a') < -q_j$, the possibility of indifference between a and a' cannot be excluded. This indifference is less and less credible when $\Omega_j(a, a')$ moves closer to $-p_j$. From this point of view, the *credibility indices* $c_j(a, a')$, or the *partial concordance indices*, of an outranking of a over a' are defined as follows:

$$c_j(a, a') = \begin{cases} 0 & \text{if } \Omega_j(a, a') \leq -p_j \\ \frac{\Omega_j(a, a') + p_j}{p_j - q_j} & \text{if } -p_j < \Omega_j(a, a') < -q_j \\ 1 & \text{if } \Omega_j(a, a') \geq -q_j \end{cases} \quad (2.2)$$

Let us notice that despite the way the value of $c_j(a, a')$ is modeled within the “small range” $[-p_j, -q_j]$, this value is only related to the ordinal definition of the criterion g_j .

Finally, the credibility of the comprehensive outranking of a over a' , meaning that a may be judged at least as good as a' when taking all the criteria from F into account, is defined as follows. Let $\sigma(a, a')$ denote such a credibility index.

$$\sigma(a, a') = c(a, a') \prod_{j=1}^n T_j(a, a') \quad (2.3)$$

where

$$T_j(a, a') = \begin{cases} \frac{1 - d_j(a, a')}{1 - c(a, a')} & \text{if } d_j(a, a') > c(a, a') \\ 1 & \text{otherwise} \end{cases} \quad (2.4)$$

This credibility index aggregates a *comprehensive concordance index*, $c(a, a')$, and the *partial discordance indices*, $d_j(a, a')$. These two types of indices make use of two more parameters associated with each criterion g_j , $j = 1, \dots, n$: the *weights*, denoted w_j , where $w_j > 0$, and the *veto thresholds*, denoted v_j , such that $v_j \geq p_j$. In the following, assume without loss of generality that $\sum_{j=1}^n w_j = 1$.

$$c(a, a') = \sum_{j=1}^n w_j c_j(a, a') \quad (2.5)$$

$$d_j(a, a') = \begin{cases} 0 & \text{if } \Omega_j(a, a') \geq -p_j \\ \frac{\Omega_j(a, a') + p_j}{p_j - v_j} & \text{if } -v_j < \Omega_j(a, a') < -p_j \\ 1 & \text{if } \Omega_j(a, a') \leq -v_j \end{cases} \quad (2.6)$$

See, for example, (Roy, 1991; Yu, 1992; Roy and Bouyssou, 1993) for more details about the original formulae as well as their interpretations. Different variants, or extensions, for such indices have been proposed without changing the fundamental properties (see, for example, Mousseau and Dias, 2004; Figueira et al., 2006; Roy and Słowiński, 2008). The definitions and results presented in the next sections remain valid with these variants, or extensions.

3 Problem statement and assignment rules

The aim of this section is to present the ELECTRE TRI-C method, including the basic assumptions, the structural requirements, and the assignment rules.

3.1 Basic assumptions and structural requirements

Let b_h denote a *central reference action* introduced to characterize category C_h . Assume that the actions b_h , $h = 1, \dots, q$, have been defined through an interaction procedure with the decision-maker. Notice that C_1 is the worst category and C_q the best one, with $q \geq 2$.

Therefore, let $B = \{b_0, b_1, \dots, b_h, \dots, b_q, b_{q+1}\}$ denote the set of $(q + 2)$ reference actions, where b_0 and b_{q+1} are two particular reference actions defined as follows: $g_j(b_0)$ is the worst possible evaluation on criterion g_j , and $g_j(b_{q+1})$ is the best possible evaluation on the same criterion g_j , for all $g_j \in F$.

Consider two reference actions, b_h and b_{h+1} . According to the ordered character of the categories, it does not seem restrictive to assume that b_{h+1} strictly dominates b_h . Let $b_{h+1} \Delta_F b_h$ denote such a (strict) dominance relation, such that

$$\forall j, \Omega_j(b_{h+1}, b_h) \geq 0 \text{ and } \exists j, \Omega_j(b_{h+1}, b_h) > 0; h = 0, \dots, q \quad (3.7)$$

However, the intuition lead us to think that if $\Omega_j(b_{h+1}, b_h)$ is too small with respect to the indifference and preference thresholds, there is a risk of having a certain ambiguity on the assignment of some actions to categories C_h and C_{h+1} . It will be proved in Section 4 that this is true for ELECTRE TRI-C when the following condition is not verified.

Condition 1 (Strict separability)

The set of reference actions, B , fulfills the strict separability condition if and only if

$$\Omega_j(b_{h+1}, b_h) > p_j, j = 1, \dots, n; h = 0, \dots, q \quad (3.8)$$

If the strict separability condition holds, then $c_j(b_{h+1}, b_h) = 1$ and $c_j(b_h, b_{h+1}) = 0$, for all $g_j \in F$. Since there is no discordance on all criteria, $\sigma(b_{h+1}, b_h) = 1$ and $\sigma(b_h, b_{h+1}) = 0$.

For further analysis of the impact of the (strict) dominance relation and the strict separability condition on the results as well as the consistency of the assignment rules based on central reference actions, it seems natural to introduce the following structural requirements.

Definition 1 (Structural requirements)

- 1) *Conformity: Each central reference action, b_h , must be assigned to the category, C_h , $h = 1, \dots, q$.*
- 2) *Monotonicity: If an action a strictly dominates a' , then a is assigned to a category at least as good as the category a' is assigned to.*
- 3) *Homogeneity: Two actions must be assigned to the same category when they compare themselves in an identical manner with the reference actions.*
- 4) *Stability: After a modification of the set B by applying either a merging or a splitting procedure (see Definition 2), the non-adjacent categories to the modified ones will remain with the same actions as before the modification. More precisely:*
 - a) *After merging two consecutive categories, any action a previously assigned to the non-modified categories will remain in the same category. Moreover, the actions previously assigned to the merged categories will be assigned either to the new category or to one of the two adjacent categories.*

- b) After splitting the category C_h into two new consecutive categories, any action that was not previously assigned to C_{h+1} nor to C_{h-1} will remain in the same category. Furthermore, the actions previously assigned to the former category C_h will be assigned to one of the two new categories. Moreover, an action previously assigned to C_{h+1} will be assigned either to the same category or to the best of the two new categories. Similarly, an action previously assigned to C_{h-1} will be assigned either to the same category or to the worst of the two new categories.

Definition 2 (Basic modification procedures)

- 1) *Merging procedure:* The distinction between two consecutive categories, C_{h-1} and C_h , will be ignored by introducing a new central reference action, b'_h , such that $\Omega_j(b'_h, b_{h-1}) \geq 0$ and $\Omega_j(b_h, b'_h) \geq 0$, for all $g_j \in F$.
- 2) *Splitting procedure:* The category C_h will be split into two new consecutive categories by introducing two new central reference actions, b'_h and b''_h , such that b_{h+1} strictly dominates b''_h , b''_h strictly dominates b'_h , b'_h strictly dominates b_{h-1} , $\Omega_j(b''_h, b_h) \geq 0$, and $\Omega_j(b_h, b'_h) \geq 0$, for all $g_j \in F$.

It should be noticed that adding or removing a category are particular cases of these two basic procedures.

3.2 ELECTRE TRI-C assignment rules

As for the definition of the assignment rules, it is useful to introduce the concept of *slackness functions* as well as the related properties as follows.

Definition 3 (Slackness functions)

Let $\lambda \in [0.5, 1]$ denote the chosen majority level:

- 1) *Direct slackness function:* $\xi_h^+(a, \lambda) = \sigma(a, b_h) - \lambda$, $h = (q + 1), \dots, 0$.
- 2) *Reverse slackness function:* $\xi_h^-(a, \lambda) = \sigma(b_h, a) - \lambda$, $h = 0, \dots, (q + 1)$.

Proposition 1

- a) The direct slackness function does not decrease when moving from a given category to a worst one.
- b) The reverse slackness function does not decrease when moving from a given category to a best one.

Proof

- a) When moving from a given category to a worst one, the direct slackness function $\xi_h^+(a, \lambda)$ does not decrease because the credibility indices $\sigma(a, b_h)$ are non-increasing functions on the set B .
- b) When moving from a given category to a best one, the reverse slackness function $\xi_h^-(a, \lambda)$ does not decrease because the credibility indices $\sigma(b_h, a)$ are non-decreasing functions on the set B . \square

Therefore, two assignment rules for ELECTRE TRI-C are defined as follows.

Definition 4 (Descending assignment rule)

Choose a majority level λ ($0.5 \leq \lambda \leq 1$). Decrease h from $(q + 1)$ until the first value such that $\xi_h^+(a, \lambda) \geq 0$. If $\xi_h^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$, then assign action a to category C_h . Otherwise, assign a to C_{h+1} .

Taking into account that for all a , $\xi_{q+1}^+(a, \lambda) < 0$ and $\xi_0^+(a, \lambda) \geq 0$, according to Proposition 1.a there exists necessarily a value h such that $\xi_h^+(a, \lambda) \geq 0$ and $\xi_{h+1}^+(a, \lambda) < 0$. Thus, any action a is assigned to a *unique category* by the descending rule.

Definition 5 (Ascending assignment rule)

Choose a majority level λ ($0.5 \leq \lambda \leq 1$). Increase h from 0 until the first value such that $\xi_h^-(a, \lambda) \geq 0$. If $\xi_h^-(a, \lambda) \leq |\xi_{h-1}^-(a, \lambda)|$, then assign action a to category C_h . Otherwise, assign a to C_{h-1} .

Taking into account that for all a , $\xi_0^-(a, \lambda) < 0$ and $\xi_{q+1}^-(a, \lambda) \geq 0$, according to Proposition 1.b there exists necessarily a value h such that $\xi_h^-(a, \lambda) \geq 0$ and $\xi_{h-1}^-(a, \lambda) < 0$. Thus, any action a is assigned to a *unique category* by the ascending rule.

Notice that the assignment of a potential action a is *independent* from any others.

Remark 1 (A mirror equivalence)

Let F^* denote the coherent family of criteria g_j^* obtained from g_j through the inversion of the preference direction, for $j = 1, \dots, n$. Let $\sigma^*(a, b_h)$ and $\xi_h^{*+}(a, \lambda)$ denote the credibility indices and the direct slackness functions, respectively. It is trivial to verify that, for all a and b_h :

$$1) \sigma^*(a, b_h) = \sigma(b_h, a)$$

$$2) \xi_h^{*+}(a, \lambda) = \xi_h^-(a, \lambda)$$

When using the new family of criteria, F^* , the category C_1 becomes the best category and C_q the worst one. Let $C_h^* = C_{q+1-h}$, for $h = 1, \dots, q$, and $b_h^* = b_{q+1-h}$, for $h = 0, \dots, (q+1)$. When the ELECTRE TRI-C descending rule is applied to these new categories, the direct slackness function $\xi_h^{*+}(a, \lambda)$ is used as the reverse slackness function $\xi_h^-(a, \lambda)$ in the ELECTRE TRI-C ascending rule when applied to the initial categories.

This equivalence shows a way to replace the descending rule by the ascending rule. It will be referred in what follows as a “transposition in the mirror”.

4 Properties of the assignment rules

The aim of this section is to analyze the properties of the ELECTRE TRI-C assignment rules according to the conditions imposed to the set of reference actions, B .

Theorem 1

- a) The monotonicity, homogeneity, and stability requirements hold.
- b) If the strict separability condition is fulfilled, then the conformity requirement holds.

The following proof is done for the descending rule. It remains valid for the ascending rule by the transposition in the mirror (see Remark 1).

Proof

- a.1) *Monotonicity:* $a\Delta_F a' \Rightarrow \xi_k^+(a, \lambda) \geq \xi_k^+(a', \lambda)$, $k = q, \dots, 1$. Therefore, according to the descending rule if h is the first value of k such that $\xi_k^+(a, \lambda) \geq 0$ and h' the first value of k such that $\xi_k^+(a', \lambda) \geq 0$, then one necessarily has $h \geq h'$. If $h > h'$, then a is assigned either to the same category as a' is assigned to or to a better category. If $h = h'$, then the monotonicity would not be verified only when a is assigned to C_h and a' is assigned to C_{h+1} . Let us prove that this is impossible. Action a' is assigned to C_{h+1} if and only if $\xi_h^+(a', \lambda) > |\xi_{h+1}^+(a', \lambda)|$. Since $\xi_h^+(a', \lambda) = \sigma(a', b_h) - \lambda$ and $|\xi_{h+1}^+(a', \lambda)| = \lambda - \sigma(a', b_{h+1})$, then $\sigma(a', b_h) + \sigma(a', b_{h+1}) > 2\lambda$. Similarly, a is assigned to C_h if and only if $\sigma(a, b_h) + \sigma(a, b_{h+1}) \leq 2\lambda$. Therefore, $\sigma(a, b_h) + \sigma(a, b_{h+1}) \leq 2\lambda < \sigma(a', b_h) + \sigma(a', b_{h+1})$. This is impossible because $a\Delta_F a'$ means that $\sigma(a, b_h) \geq \sigma(a', b_h)$ and $\sigma(a, b_{h+1}) \geq \sigma(a', b_{h+1})$.
- a.2) *Homogeneity:* By definition two different actions, a and a' , are compared themselves in an identical manner with the reference actions if and only if the following conditions are verified: $\sigma(a, b_h) = \sigma(a', b_h)$ and $\sigma(b_h, a) = \sigma(b_h, a')$, for all $h = 1, \dots, q$. Therefore, the homogeneity condition is verified because the assignment of an action a to a category C_h by the descending rule only depends on $\sigma(a, b_h)$.

a.3) *Stability under a merging procedure:* Assume that the consecutive categories C_{h-1} and C_h are merged into only one category, denoted C'_h . Let b'_h denote the central reference action introduced to characterize the new category C'_h . From the conditions imposed to b'_h according to the merging procedure (Definition 2.1), the new set B' obtained from B by replacing b_h and b_{h-1} to b'_h leads to $b_{h+1}\Delta_F b'_h$ and $b'_h\Delta_F b_{h-2}$. According to the descending rule, we will prove successively that:

- 1) If a was previously assigned to category C_k , $k \geq (h+1)$, then a will be assigned to the same category, after modification.
- 2) If a was previously assigned to category C_k , $k \leq (h-2)$, then a will be assigned to the same category, after modification.
- 3) If a was previously assigned either to C_h or C_{h-1} , then a will be assigned either to the new category C'_h or to one of the two adjacent categories to the modified ones, i.e., C_{h-2} and C_{h+1} , after modification.

Let us prove these three cases:

- 1) Any action a such that $\xi_{h+1}^+(a, \lambda) \geq 0$ is assigned to one category C_k , $k \geq (h+1)$. It is clear that a merging procedure does not change the assignment of a . Thus, on the one hand, a was previously assigned to C_k , $k > (h+1)$ if and only if $\xi_{h+1}^+(a, \lambda) \geq 0$. On the other hand, a could have been assigned to C_{h+1} with $\xi_{h+1}^+(a, \lambda) < 0$ when $\xi_h^+(a, \lambda) > |\xi_{h+1}^+(a, \lambda)|$. Let us prove that according to these conditions a remains assigned to C_{h+1} after modification. Let $\xi_h^+(a, \lambda) = \sigma(a, b'_h) - \lambda$. After modification a will be assigned to C_{h+1} if and only if $\xi_h'^+(a, \lambda) > |\xi_{h+1}^+(a, \lambda)|$. This inequality is necessarily verified since $\xi_h'^+(a, \lambda) \geq \xi_h^+(a, \lambda)$ because $\Omega_j(b'_h, b_h) \geq 0 \Rightarrow \sigma(a, b'_h) \geq \sigma(a, b_h)$.
- 2) Any action a such that $\xi_{h-2}^+(a, \lambda) < 0$ is assigned to one category C_k , $k \leq (h-2)$. It is clear that a merging procedure does not change the assignment of a . Thus, on the one hand, a was previously assigned to C_k , $k < (h-2)$ if and only if $\xi_{h-2}^+(a, \lambda) < 0$. On the other hand, a could have been assigned to C_{h-2} with $\xi_{h-2}^+(a, \lambda) \geq 0$ when $\xi_{h-1}^+(a, \lambda) < 0$ and $\xi_{h-2}^+(a, \lambda) \leq |\xi_{h-1}^+(a, \lambda)|$. Let us prove that according to these conditions a remains assigned to C_{h-2} after modification. Since $\Omega_j(b'_h, b_{h-1}) \geq 0$ one obtains $\sigma(a, b'_h) \leq \sigma(a, b_{h-1})$. Thus, $\xi_h'^+(a, \lambda) \leq \xi_{h-1}^+(a, \lambda)$. This latter quantity being negative, then $|\xi_h'^+(a, \lambda)| \geq |\xi_{h-1}^+(a, \lambda)|$. Therefore, $\xi_{h-2}^+(a, \lambda) \leq |\xi_h'^+(a, \lambda)|$, which proves that a will remain assigned to category C_{h-2} , after modification.
- 3) From the two above paragraphs, only the actions previously assigned to the former categories C_{h-1} and C_h could be assigned to the new category C'_h after modification. But, nothing proves that all of those actions are assigned to the new category. Some of them can be assigned to one of the two adjacent categories (if they exist), C_{h-2} and C_{h+1} , after modification.

- i) Consider an action a previously assigned to C_{h-1} with $\xi_{h-1}^+(a, \lambda) < 0$ and $\xi_{h-2}^+(a, \lambda) > |\xi_{h-1}^+(a, \lambda)|$. From 2), according to these conditions, $|\xi_h'^+(a, \lambda)| \geq |\xi_{h-1}^+(a, \lambda)|$. Thus, an action a can verify the following condition: $|\xi_h'^+(a, \lambda)| > \xi_{h-2}^+(a, \lambda)$. In such a case, a will be assigned to C_{h-2} after modification.
- ii) Consider an action a previously assigned to C_h with $\xi_h^+(a, \lambda) \geq 0$ and $\xi_h^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$. Since $\Omega_j(b_h, b'_h) \geq 0$ one obtains $\sigma(a, b'_h) \geq \sigma(a, b_h)$. Thus, $\xi_h'^+(a, \lambda) \geq \xi_h^+(a, \lambda)$. This latter quantity being positive, the following condition can hold: $\xi_h'^+(a, \lambda) > |\xi_{h+1}^+(a, \lambda)|$. In such a case, a will be assigned to C_{h+1} after modification.

a.4) *Stability under a splitting procedure:* Assume that the category C_h is split into two new consecutive categories, denoted C'_h and C''_h . Let b'_h denote the central reference action introduced to characterize the worst of the two new categories, C'_h , and b''_h the central reference action introduced to characterize the best of the two new categories, C''_h . According to the descending rule, we will prove successively that:

- 1) If a was previously assigned to C_k , $k \neq \{h-1, h, h+1\}$, then a will be assigned to the same category, after modification.
- 2) If a was previously assigned to the former category C_h , then a will be assigned to one of the new categories, C'_h or C''_h , after modification.
- 3) If a was previously assigned to C_{h+1} , then a will be assigned either to the same category or to the best of the two new categories, C''_h , after modification.
- 4) If a was previously assigned to C_{h-1} , then a will be assigned either to the same category or to the worst of the two new categories, C'_h , after modification.

Let us prove these four cases:

- 1) The proof is similar to the first two cases of the merging procedure, when a was previously assigned either to category C_k , $k \geq (h+2)$ or to C_k , $k \leq (h-2)$.
- 2) Consider an action previously assigned to C_h . The following two cases must be analyzed:
 - i) $\xi_h^+(a, \lambda) \geq 0$ and $\xi_{h+1}^+(a, \lambda) < 0$, when $\xi_h^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$. Since $\Omega_j(b''_h, b_h) \geq 0$ one obtains $\xi_h''^+(a, \lambda) \leq \xi_h^+(a, \lambda)$. Since $\Omega_j(b_{h+1}, b''_h) \geq 0$ one obtains $\xi_{h+1}^+(a, \lambda) \leq \xi_h''^+(a, \lambda)$. Therefore, $\xi_{h+1}^+(a, \lambda) \leq \xi_h''^+(a, \lambda) \leq \xi_h^+(a, \lambda)$. When $\xi_h''^+(a, \lambda) \geq 0$, if $\xi_h''^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$, then a will be assigned to C''_h , after modification. Otherwise, if $\xi_h''^+(a, \lambda) < 0$, then a will be assigned at most to C''_h .
 - ii) $\xi_h^+(a, \lambda) < 0$ and $\xi_{h-1}^+(a, \lambda) \geq 0$, when $\xi_{h-1}^+(a, \lambda) > |\xi_h^+(a, \lambda)|$. Since $\Omega_j(b_h, b'_h) \geq 0$ one obtains $\xi_h'^+(a, \lambda) \geq \xi_h^+(a, \lambda)$. Since $\Omega_j(b'_h, b_{h-1}) \geq 0$ one obtains $\xi_h'^+(a, \lambda) \leq \xi_{h-1}^+(a, \lambda)$. Therefore, $\xi_h^+(a, \lambda) \leq \xi_h'^+(a, \lambda) \leq \xi_{h-1}^+(a, \lambda)$.

When $\xi_{h-1}^+(a, \lambda) < 0$, if $\xi_{h-1}^+(a, \lambda) > |\xi_h'^+(a, \lambda)|$, then a will be assigned to C'_h , after modification. Otherwise, if $\xi_h^+(a, \lambda) \geq 0$, then a will be assigned at least to C'_h .

3) Consider an action previously assigned to C_{h+1} . The following two cases must be analyzed:

- i) $\xi_{h+1}^+(a, \lambda) \geq 0$ and $\xi_{h+2}^+(a, \lambda) < 0$, when $\xi_{h+1}^+(a, \lambda) \leq |\xi_{h+2}^+(a, \lambda)|$. It is trivial to verify that in such a case, a will remain in the same category, after modification.
- ii) $\xi_{h+1}^+(a, \lambda) < 0$ and $\xi_h^+(a, \lambda) \geq 0$, when $\xi_h^+(a, \lambda) > |\xi_{h+1}^+(a, \lambda)|$. From 2.i) with the same conditions it was proved that $\xi_{h+1}^+(a, \lambda) \leq \xi_h''^+(a, \lambda) \leq \xi_h^+(a, \lambda)$. According to these inequalities the following condition holds: $\xi_h''^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$. In such a case, a will be assigned to category C''_h .

4) Consider an action previously assigned to C_{h-1} . The following two cases must be analyzed:

- i) $\xi_{h-1}^+(a, \lambda) \geq 0$ and $\xi_h^+(a, \lambda) < 0$, when $\xi_{h-1}^+(a, \lambda) \leq |\xi_h^+(a, \lambda)|$. From 2.ii) with the same conditions it was proved that $\xi_h^+(a, \lambda) \leq \xi_h'^+(a, \lambda) \leq \xi_{h-1}^+(a, \lambda)$. According to these inequalities, the following condition holds: $\xi_{h-1}^+(a, \lambda) > |\xi_h'^+(a, \lambda)|$. In such a case, a will be assigned to category C'_h .
- ii) $\xi_{h-1}^+(a, \lambda) < 0$ and $\xi_{h-2}^+(a, \lambda) \geq 0$, when $\xi_{h-2}^+(a, \lambda) > |\xi_{h-1}^+(a, \lambda)|$. It is trivial to verify that in such a case, a will remain in the same category, after modification.

b) *Conformity*: Assume that the strict separability condition holds. By construction of the credibility indices: $\sigma(b_h, b_h) = 1$, $\sigma(b_{h+1}, b_h) = 1$, and $\sigma(b_h, b_{h+1}) = 0$, for all $h = 0, \dots, q$. Therefore, from the direct slackness function (Definition 3.1) $\xi_h^+(b_h, \lambda) = 1 - \lambda \geq 0$ and $\xi_{h+1}^+(b_h, \lambda) = -\lambda < 0$. When applying the descending rule, each central reference action b_h is assigned to C_h if and only if $\xi_h^+(b_h, \lambda) \leq |\xi_{h+1}^+(b_h, \lambda)|$ which is equivalent to $1 - \lambda \leq \lambda$. This is true because $\lambda \geq \frac{1}{2}$. Thus, the conformity of the reference actions is always verified. \square

The decision-maker can have some good reasons to introduce a set of reference actions, B , that does not fulfill the strict separability condition, for certain ordered pairs (b_{h+1}, b_h) . As noticed in Section 3.1, the strict dominance condition is not enough to clearly separate the categories, when considering the possible minimum differences in the evaluations. In fact, the weak separability condition below defines the minimum differences in the evaluations which seems to us adequate to impose between two consecutive central reference actions.

Condition 2 (Weak separability)

The set of reference actions, B , fulfills the weak separability condition if and only if

$$\forall j, \Omega_j(b_{h+1}, b_h) \geq 0 \text{ and } \exists j, \Omega_j(b_{h+1}, b_h) > p_j; h = 0, \dots, q. \quad (4.9)$$

The weak separability condition allows the existence of some criteria g_j such that $\Omega_j(b_{h+1}, b_h) \leq p_j$. In such criteria, one obtains $c_j(b_{h+1}, b_h) = 1$ and $c_j(b_h, b_{h+1}) \leq 1$. Therefore, as there is no discordance, if the weak separability condition is fulfilled, then $\sigma(b_{h+1}, b_h) = 1$ and $\sigma(b_h, b_{h+1}) \leq 1$. As noticed in Section 3.1, when the strict separability condition is fulfilled, then $\sigma(b_{h+1}, b_h) = 1$ and $\sigma(b_h, b_{h+1}) = 0$.

According to Theorem 1, the conformity requirement holds if the strict separability condition is fulfilled. Let us analyze in which conditions such a requirement holds when the weak separability condition is fulfilled. Indeed, when the conformity requirement does not hold, the assignment model becomes inconsistent and, therefore, it is not able to support the assignment of the potential actions.

Theorem 2

If the weak separability condition is fulfilled, then there exists a compatible majority level, λ^c , for which the conformity requirement holds, whenever the chosen majority level $\lambda \geq \lambda^c$, such that

$$\lambda^c = \frac{1}{2} + \frac{1}{2} \max_{h=0, \dots, q} \left\{ \sigma(b_h, b_{h+1}) \right\} \quad (4.10)$$

The proof is done for the descending rule. It remains valid for the ascending rule by the transposition in the mirror (see Remark 1).

Proof According to the descending rule $\sigma(b_h, b_h) = 1 \Rightarrow \xi_h^+(b_h, \lambda) = 1 - \lambda \geq 0$ and b_h is assigned to a category at least as good as C_h . The central reference action b_h is assigned to C_h if and only if $\xi_{h+1}^+(b_h, \lambda) = \sigma(b_h, b_{h+1}) - \lambda < 0$ and $\xi_h^+(b_h, \lambda) \leq |\xi_{h+1}^+(b_h, \lambda)|$ which are equivalent to the following two inequalities:

$$\lambda > \sigma(b_h, b_{h+1}) \text{ and } 1 - \lambda \leq \lambda - \sigma(b_h, b_{h+1}) \quad (4.11)$$

We have shown above that with the weak separability condition one obtains $\sigma(b_h, b_{h+1}) < 1$. It follows that

$$\frac{1}{2} + \frac{1}{2} \sigma(b_h, b_{h+1}) \geq \sigma(b_h, b_{h+1}) \quad (4.12)$$

Therefore, if the second inequality of 4.11 is true, then the first one is also true, and consequently b_h is assigned to C_h . Taking all central reference actions b_h , $h = 1, \dots, q$, into account, the conformity requirement holds if and only if

$$\lambda \geq \frac{1}{2} + \frac{1}{2} \max_{h=0, \dots, q} \left\{ \sigma(b_h, b_{h+1}) \right\} \quad (4.13)$$

□

Let us analyze the impact of the two separability conditions on the two basic modification procedures defined for the stability requirement (Definition 2):

- 1) Let B denote a set of reference actions emerged from a practical situation with a compatible conformity majority level λ^c , and λ denote the chosen majority level such that $\lambda \geq \lambda^c$. The value of λ will remain at least as good as λ^c , where λ^c is the conformity majority level associated with B' obtained after applying a merging procedure (see a.3 in the Proof of Theorem 1).
- 2) The splitting procedure can provide different conclusions in comparison with the merging procedure (see a.4 in the Proof of Theorem 1). Indeed, after applying a splitting procedure, if $\lambda < \lambda^c$, then the conformity requirement does not hold. Therefore, one can easily prove that:

- according to the descending procedure, b'_h can be assigned to C''_h instead of C'_h .
- according to the ascending procedure, b''_h can be assigned to C'_h instead of C''_h .

Remark 2

When comparing an action a to the reference actions b_h , $h = 0, 1, \dots, (q + 1)$, it is also useful to analyze the set of reference actions, B , as follows:

- 1) *If $\xi_q^+(a, \lambda) \geq 0$, then the action a is assigned to C_q by the ELECTRE TRI-C descending rule. This means that the action a is judged “very good” in comparison to all central reference actions in B . Moreover, if the actions are systematically assigned to C_q , then a deeply analysis must be done in order to conclude about the under-evaluation of the set B , or to assume that all potential actions are really very good.*
- 2) *If $\xi_1^-(a, \lambda) \geq 0$, then the action a is assigned to C_1 by the ELECTRE TRI-C ascending rule. This means that the action a is judged “very poor” in comparison to all central reference actions in B . Moreover, if the actions are systematically assigned to C_1 , then a deeply analysis must be done in order to conclude about the over-evaluation of the set B , or to assume that all potential actions are really very poor.*
- 3) *From the two above cases, when a deeply analysis is required we can start to choose a different majority level, if possible, since the previous one has been chosen so high.*

5 Comparison with ELECTRE TRI-B

This section presents an overview of ELECTRE TRI-B and a comparison between the ELECTRE TRI-C and ELECTRE TRI-B assignment results.

5.1 An overview of ELECTRE TRI-B

According to (Yu, 1992) and (Roy and Bouyssou, 1993, p. 389-401), the assignment of an action a by ELECTRE TRI-B is based on pairwise comparisons between the action a and the profile limits which characterize the pre-defined and ordered categories. The set of such categories is denoted here $\widehat{C} = \{\widehat{C}_1, \dots, \widehat{C}_h, \dots, \widehat{C}_q\}$, where \widehat{C}_1 is the worst category and \widehat{C}_q is the best one, with $q \geq 2$. Each category \widehat{C}_h is defined by a lower profile limit, \hat{b}_{h-1} , and an upper profile limit, \hat{b}_h , such that $\hat{b}_h \Delta_F \hat{b}_{h-1}$, $h = 1, \dots, q$. Let $\widehat{B} = \{\hat{b}_0, \hat{b}_1, \dots, \hat{b}_h, \dots, \hat{b}_q\}$ denote the set of the $(q+1)$ profile limits. Furthermore, the role played by \hat{b}_0 and \hat{b}_q when using profile limits is the same as b_0 and b_{q+1} with central reference actions.

When using the slackness functions (Definition 3), the most well-known assignment rules of ELECTRE TRI-B (formerly called pessimistic and optimistic, respectively) are rewritten as follows. (Roy, 2002) showed that it would be suitable to replace pessimistic by pseudo-conjunctive and optimistic by pseudo-disjunctive.

Definition 6 (Pseudo-conjunctive assignment rule)

Choose a majority level λ ($0.5 \leq \lambda \leq 1$). Decrease h from q until the first value such that $\xi_{h-1}^+(a, \lambda) \geq 0$. Assign action a to category \widehat{C}_h .

Definition 7 (Pseudo-disjunctive assignment rule)

Choose a majority level λ ($0.5 \leq \lambda \leq 1$). Increase h from 0 until the first value such that $\xi_h^-(a, \lambda) \geq 0$ and $\xi_h^+(a, \lambda) < 0$. Assign action a to category \widehat{C}_h .

Based on the above ELECTRE TRI-B assignment rules, if an action a is assigned to category \widehat{C}_k by the pseudo-conjunctive rule and to \widehat{C}_h by the pseudo-disjunctive rule, then it was proved that $k \leq h$ (see Roy and Bouyssou, 1993, p. 395). Furthermore, the two assignment rules provide the same results if and only if there is no t such that $\xi_t^+(a, \lambda) < 0$ and $\xi_t^-(a, \lambda) < 0$ or there is at most one t such that $\xi_t^+(a, \lambda) \geq 0$ and $\xi_t^-(a, \lambda) \geq 0$.

5.2 Comparing the assignment results

When applying ELECTRE TRI-C, each category is characterized by a central reference action. In ELECTRE TRI-B, each category is bounded by a lower and an upper profile. Theorem 3 allows to compare the assignment results of ELECTRE TRI-C and ELECTRE TRI-B taking into account either the descending rule and the pseudo-conjunctive rule, respectively, or the ascending rule and the pseudo-disjunctive rule, respectively.

Theorem 3

- 1) Consider $(q + 2)$ reference actions defined to apply ELECTRE TRI-C with q categories. When such reference actions are used as profile limits of the $(q+1)$ categories in ELECTRE TRI-B,
 - a) if an action a is assigned to C_h by the ELECTRE TRI-C descending rule, then a is assigned to \hat{C}_h or \hat{C}_{h+1} by the ELECTRE TRI-B pseudo-conjunctive rule.
 - b) if an action a is assigned to C_t by the ELECTRE TRI-C ascending rule, then a is assigned to \hat{C}_k , with $k \geq t$, by the ELECTRE TRI-B pseudo-disjunctive rule.
- 2) Consider $(q + 1)$ profile limits defined to apply ELECTRE TRI-B with q categories. When such profile limits are used as reference actions of the $(q - 1)$ categories in ELECTRE TRI-C,
 - a) if an action a is assigned to \hat{C}_h by the ELECTRE TRI-B pseudo-conjunctive rule, then a is assigned to C_h or C_{h-1} by the ELECTRE TRI-C descending rule.
 - b) if an action a is assigned to \hat{C}_t by the ELECTRE TRI-B pseudo-disjunctive rule, then a is assigned to C_k , with $k \leq t$, by the ELECTRE TRI-C ascending rule.

Proof

- 1) Assume that the $(q + 2)$ reference actions were defined.
 - a) When applying the ELECTRE TRI-C descending rule (Definition 4), an action a is assigned to C_h if one of the two following cases holds. First, $\xi_h^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$, with $\xi_h^+(a, \lambda) \geq 0$ and $\xi_{h+1}^+(a, \lambda) < 0$. In such a case, a is assigned to \hat{C}_{h+1} according to the ELECTRE TRI-B pseudo-conjunctive rule (Definition 6). Second, $\xi_{h-1}^+(a, \lambda) > |\xi_h^+(a, \lambda)|$, with $\xi_{h-1}^+(a, \lambda) \geq 0$ and $\xi_h^+(a, \lambda) < 0$. In such a case, a is assigned to \hat{C}_h according to the ELECTRE TRI-B pseudo-conjunctive rule.
 - b) When applying the ELECTRE TRI-C ascending rule (Definition 5), an action a is assigned to C_t if one of the following two cases holds. First, $\xi_t^-(a, \lambda) \leq |\xi_{t-1}^-(a, \lambda)|$, with $\xi_t^-(a, \lambda) \geq 0$ and $\xi_{t-1}^-(a, \lambda) < 0$. In such a case, according to the ELECTRE TRI-B pseudo-disjunctive rule (Definition 7), on the one hand if $\xi_t^+(a, \lambda) < 0$, then a is assigned to \hat{C}_t , but on the other hand if there exist p central reference actions such that $\xi_{t+s}^+(a, \lambda) \geq 0$, $s = 0, \dots, (p - 1)$, then a is assigned to \hat{C}_{t+p} . Second, $\xi_{t+1}^-(a, \lambda) > |\xi_t^-(a, \lambda)|$, with $\xi_{t+1}^-(a, \lambda) \geq 0$ and $\xi_t^-(a, \lambda) < 0$. In such a case, according to the ELECTRE TRI-B pseudo-disjunctive rule, on the one hand if $\xi_{t+1}^+(a, \lambda) < 0$, then a is assigned to \hat{C}_{t+1} , but on the other hand if there exist p central reference actions such that $\xi_{t+s+1}^+(a, \lambda) \geq 0$, $s = 0, \dots, (p - 1)$, then a is assigned to \hat{C}_{t+p+1} .

2) Assume that the $(q + 1)$ profile limits were defined.

- a) According to the ELECTRE TRI-B pseudo-conjunctive rule (Definition 6), an action a is assigned to \hat{C}_h if, for the highest h , $\xi_{h-1}^+(a, \lambda) \geq 0$. Furthermore, $\xi_h^+(a, \lambda) < 0$. Thus, from the ELECTRE TRI-C descending rule (Definition 4), if $\xi_{h-1}^+(a, \lambda) \leq |\xi_h^+(a, \lambda)|$, then a is assigned to category C_{h-1} . Otherwise, a is assigned to C_h .
- b) According to the ELECTRE TRI-B pseudo-disjunctive rule (Definition 7), an action a is assigned to \hat{C}_t if, for the lowest t , $\xi_t^-(a, \lambda) \geq 0$ and $\xi_t^+(a, \lambda) < 0$. Thus, if $\xi_{t-1}^-(a, \lambda) < 0$, then, according to the ELECTRE TRI-C ascending rule (Definition 5), if $\xi_t^-(a, \lambda) \leq |\xi_{t-1}^-(a, \lambda)|$, then a is assigned to category C_t , or to C_{t-1} , otherwise. When there exist p profile limits such that $\xi_{t-s-1}^-(a, \lambda) \geq 0$ and $\xi_{t-s-1}^+(a, \lambda) \geq 0$, $s = 0, \dots, (p - 1)$, according to the ELECTRE TRI-C ascending rule, if $\xi_{t-s-1}^-(a, \lambda) \leq |\xi_{t-s-2}^-(a, \lambda)|$, then a is assigned to category C_{t-s-1} . Otherwise, a is assigned to C_{t-s-2} . \square

6 A numerical example

This section presents the assignment results provided by the ELECTRE TRI-C and ELECTRE TRI-B methods. This numerical example is based on a Case Study which concentrates on France's Lorraine region, where iron has been mined for more than a century. The underground mining tunnels have caused land subsidence, which led buildings to collapse. The object of this study was to make a partition of a piece of land into zones and assign such zones to pre-defined risk categories for decision concerning permanent surveillance (Merad et al., 2004).

Four categories have been defined to apply ELECTRE TRI-B according to a surveillance system that should be applied to the zones assigned to each category. The zones assigned to category \hat{C}_4 will be subject to a permanent monitoring system, the zones assigned to \hat{C}_3 will require a deeply investigation, the zones assigned to \hat{C}_2 will be subject to annually topographic surveys, and the zones assigned to \hat{C}_1 will need only topographic surveys. These risk categories are ordered and separated by three profile limits: \hat{b}_1 , \hat{b}_2 , and \hat{b}_3 . Since \hat{C}_4 is the highest risk category, profile \hat{b}_3 "displays a risk at least as high as" \hat{b}_2 for all criteria. Similarly, \hat{b}_2 "displays a risk at least as high as" \hat{b}_1 for all criteria.

The data for the numerical example are composed of 10 homogeneous zones (actions), a_1, \dots, a_{10} , which are evaluated on 10 criteria, g_1, \dots, g_{10} , and 3 profile limits which allow to define the 4 categories (see Tables 1, 2, and 3).

Table 1: Definition of the criteria

| Criteria | Description and preference direction | | Parameters | | |
|----------|--|------------|------------|-------|-------|
| | Short description | Direction | q_j | p_j | w_j |
| g_1 | Corrected mean stress applied on pillars | increasing | 0.05 | 0.1 | 5 |
| g_2 | Existence of fault | increasing | 0 | 0 | 1 |
| g_3 | Superimposition of pillars | increasing | 0 | 0 | 1 |
| g_4 | Size and regularity of pillars | increasing | 0 | 0 | 1 |
| g_5 | Sensitivity of rock to flooding | increasing | 0 | 0 | 5 |
| g_6 | Depth of the top mined layer | decreasing | 10 | 20 | 1 |
| g_7 | Maximum expected subsidence | increasing | 0.10 | 0.20 | 1 |
| g_8 | Expected surface deformation | increasing | 0.05 | 0.09 | 20 |
| g_9 | Zone extent | increasing | 0.5 | 1.0 | 1 |
| g_{10} | Vulnerability of building | increasing | 0 | 0 | 10 |

Source: Adapted from Merad et al., 2004.

Table 2: Potential actions

| Actions | g_1 | g_2 | g_3 | g_4 | g_5 | g_6 | g_7 | g_8 | g_9 | g_{10} |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| a_1 | 5.8 | 10 | 0 | 20 | 0 | 35 | 2.37 | 6.80 | 3.6 | 20 |
| a_2 | 4.8 | 0 | 40 | 0 | 0 | 70 | 1.28 | 1.83 | 0.2 | 10 |
| a_3 | 9.7 | 10 | 10 | 0 | 30 | 200 | 1.67 | 0.84 | 7.4 | 30 |
| a_4 | 10.4 | 10 | 10 | 10 | 30 | 203 | 1.68 | 0.83 | 9.0 | 20 |
| a_5 | 9.7 | 0 | 10 | 0 | 10 | 222 | 1.20 | 0.54 | 1.8 | 20 |
| a_6 | 9.8 | 10 | 0 | 20 | 0 | 50 | 1.27 | 2.54 | 6.7 | 20 |
| a_7 | 12.3 | 0 | 0 | 0 | 30 | 155 | 0.96 | 0.61 | 14.1 | 10 |
| a_8 | 11.2 | 10 | 0 | 0 | 30 | 180 | 0.71 | 0.39 | 6.4 | 20 |
| a_9 | 11.3 | 0 | 40 | 20 | 0 | 115 | 2.18 | 1.89 | 2.5 | 10 |
| a_{10} | 11.0 | 10 | 0 | 10 | 30 | 170 | 0.31 | 0.18 | 2.6 | 20 |

Source: Adapted from Merad et al., 2004.

Table 3: Profile limits or boundary actions

| | g_1 | g_2 | g_3 | g_4 | g_5 | g_6 | g_7 | g_8 | g_9 | g_{10} |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| \hat{b}_1 | 8.0 | 0 | 10 | 10 | 10 | 190 | 1.00 | 0.63 | 6.0 | 20 |
| \hat{b}_2 | 10.0 | 10 | 10 | 10 | 10 | 150 | 1.40 | 0.82 | 20.0 | 20 |
| \hat{b}_3 | 14.0 | 10 | 40 | 20 | 20 | 110 | 1.80 | 1.00 | 35.0 | 30 |

Source: Adapted from Merad et al., 2004.

In order to illustrate the two ELECTRE TRI-C assignment rules, assume that it is possible to obtain four central reference actions, b_h , one for each category, C_h , $h = 1, \dots, 4$. This is an alternative way to define the categories. Table 4 presents the proposed central reference actions. These actions were defined taking into account their position between two consecutive profile limits as well as the scale associated with each criterion as in Merad et al., 2004.

Table 4: Central reference actions

| Actions | g_1 | g_2 | g_3 | g_4 | g_5 | g_6 | g_7 | g_8 | g_9 | g_{10} |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| b_1 | 7.0 | 0 | 10 | 10 | 10 | 210 | 0.80 | 0.53 | 3.0 | 20 |
| b_2 | 9.0 | 10 | 10 | 10 | 10 | 170 | 1.20 | 0.725 | 13.0 | 20 |
| b_3 | 12.0 | 10 | 40 | 20 | 20 | 130 | 1.60 | 0.91 | 27.5 | 30 |
| b_4 | 16.0 | 10 | 40 | 20 | 30 | 90 | 2.00 | 1.10 | 37.5 | 40 |

Source: Adapted from Merad et al., 2004.

The credibility indices of the comprehensive outranking of the potential actions over the central reference actions, and *vice-versa*, are presented in Table 5. These indices and the chosen majority level are used to compute both assignment results of ELECTRE TRI-C and ELECTRE TRI-B methods.

Table 5: Outranking credibility (potential actions)

| Actions | $\sigma(a, b_h)$ | | | | $\sigma(b_h, a)$ | | | |
|----------|------------------|--------|--------|--------|------------------|--------|--------|--------|
| | b_1 | b_2 | b_3 | b_4 | b_1 | b_2 | b_3 | b_4 |
| a_1 | 0.7609 | 0.7391 | 0.5217 | 0.5217 | 0.4739 | 0.5000 | 0.5217 | 0.5217 |
| a_2 | 0.5217 | 0.5000 | 0.4783 | 0.4783 | 0.5000 | 0.5217 | 0.5435 | 0.5435 |
| a_3 | 0.9783 | 0.9348 | 0.5870 | 0.1304 | 0.0652 | 0.1087 | 0.8913 | 1.0000 |
| a_4 | 1.0000 | 0.9565 | 0.2609 | 0.1304 | 0.2826 | 0.3261 | 0.8913 | 1.0000 |
| a_5 | 0.9522 | 0.4783 | 0.0000 | 0.0000 | 0.8696 | 0.8913 | 1.0000 | 1.0000 |
| a_6 | 0.8696 | 0.8478 | 0.5000 | 0.5000 | 0.3478 | 0.4130 | 0.5435 | 0.5435 |
| a_7 | 0.7391 | 0.2609 | 0.2174 | 0.1087 | 0.4000 | 0.7500 | 0.7826 | 1.0000 |
| a_8 | 0.5217 | 0.4783 | 0.1304 | 0.1304 | 0.7174 | 0.7826 | 0.8913 | 1.0000 |
| a_9 | 0.6739 | 0.6304 | 0.5217 | 0.5000 | 0.3696 | 0.3696 | 0.5326 | 0.5478 |
| a_{10} | 0.5217 | 0.5000 | 0.1304 | 0.1304 | 0.7391 | 0.7826 | 0.8913 | 1.0000 |

The set of central reference actions (Table 4) does not fulfil the strict separability condition, but the weak separability condition is fulfilled since, for instance, $\Omega_j(b_2, b_1) = 0$, for all $j \in \{3, 4, 5, 10\}$. Moreover, there are some criteria in which the difference in the evaluations between two consecutive central reference actions is at least as good as the preference thresholds. The outranking credibility indices of the central reference actions over the same reference actions are presented in Table 6.

Table 6: Outranking credibility (central reference actions)

| $\sigma(b_h, b_t)$ | b_1 | b_2 | b_3 | b_4 |
|--------------------|--------|--------|--------|--------|
| b_1 | 1.0000 | 0.3696 | 0.0000 | 0.0000 |
| b_2 | 1.0000 | 1.0000 | 0.0217 | 0.0217 |
| b_3 | 1.0000 | 1.0000 | 1.0000 | 0.0652 |
| b_4 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

According to Theorem 2, the compatible majority level, λ^c , is computed from the credibility indices of the central reference actions (Table 6). For this numerical example, $\lambda^c = 0.69$. Consequently, we must choose a majority level within the range $[0.69, 1]$ in order to obtain a consistent assignment model. Let $\lambda = 0.70$ be the chosen majority level for this numerical example.

When using the four central reference actions presented in Table 4 as profile limits defining five categories according to Theorem 3, the ELECTRE TRI-C and ELECTRE TRI-B assignment results are the ones presented in Table 7. On the one hand the descending assignment results must be compared with the pseudo-conjunctive assignment results, and on the other hand the ascending assignment results must be compared with the pseudo-disjunctive assignment results.

Table 7: Comparing the assignment results

| Actions | ELECTRE TRI-C | | ELECTRE TRI-B | |
|----------|---------------|-----------|--------------------|--------------------|
| | Descending | Ascending | Pseudo-conjunctive | Pseudo-disjunctive |
| a_1 | C_2 | C_4 | \hat{C}_3 | \hat{C}_5 |
| a_2 | C_1 | C_4 | \hat{C}_1 | \hat{C}_5 |
| a_3 | C_3 | C_3 | \hat{C}_3 | \hat{C}_3 |
| a_4 | C_2 | C_3 | \hat{C}_3 | \hat{C}_3 |
| a_5 | C_2 | C_1 | \hat{C}_2 | \hat{C}_2 |
| a_6 | C_2 | C_4 | \hat{C}_3 | \hat{C}_5 |
| a_7 | C_1 | C_2 | \hat{C}_2 | \hat{C}_2 |
| a_8 | C_1 | C_2 | \hat{C}_1 | \hat{C}_1 |
| a_9 | C_1 | C_4 | \hat{C}_1 | \hat{C}_5 |
| a_{10} | C_1 | C_1 | \hat{C}_1 | \hat{C}_1 |

Note: $\lambda = 0.70$

Observe that when b_2 plays the role of a central reference action, the ELECTRE TRI-C descending rule leads to the assignment of actions a_1 , a_4 , a_5 , and a_6 to category C_2 , i.e. $C_2 = \{a_1, a_4, a_5, a_6\}$. If we now decide that b_2 plays the role of the upper profile limit of category \hat{C}_2 in ELECTRE TRI-B, $\hat{b}_2 = b_2$, the pseudo-conjunctive rule assigns the same four actions to two different but consecutive categories, i.e. a_5 is assigned to \hat{C}_2 and a_1 , a_4 , and a_6 are assigned to \hat{C}_3 . The actions initially assigned to C_2 are now shared between \hat{C}_2 and \hat{C}_3 , which clearly shows that b_2 plays the role of a central reference action in ELECTRE TRI-C.

7 Additional results

This section presents a comparison between the two ELECTRE TRI-C assignment rules and some particular results. Both results are based on the following definition.

Definition 8 (λ -binary relations)

Let λ be the chosen majority level and consider two actions a and a' .

- 1) λ -outranking: $aS^\lambda a' \Leftrightarrow \sigma(a, a') - \lambda \geq 0$
- 2) λ -indifference: $aI^\lambda a' \Leftrightarrow \sigma(a, a') - \lambda \geq 0 \wedge \sigma(a', a) - \lambda \geq 0$
- 3) λ -incomparability: $aR^\lambda a' \Leftrightarrow \sigma(a, a') - \lambda < 0 \wedge \sigma(a', a) - \lambda < 0$
- 4) λ -preference: $a \succ a' \Leftrightarrow \sigma(a, a') - \lambda \geq 0 \wedge \sigma(a', a) - \lambda < 0$

The credibility indices obtained when comparing an action a to the reference actions b_h are then compared to the chosen majority level as in the ELECTRE TRI-B method. Therefore, the result provided for ELECTRE TRI-B (Roy and Bouyssou, 1993, Rés. 6.3.1, p. 392) is still valid for ELECTRE TRI-C. According to this result, the comparison of an action a to the reference actions b_h provides one and only one of the three following cases:

- 1) There is no b_t such that $aR^\lambda b_t$ and there is a b_h such that $aI^\lambda b_h$. If b_h is not unique, then the reference actions which are λ -indifferent to the action a are consecutive.
- 2) There is no b_t such that $aI^\lambda b_t$ and there is a b_h such that $aR^\lambda b_h$. If b_h is not unique, then the reference actions which are λ -incomparable to the action a are consecutive.
- 3) There is no b_h such that $aI^\lambda b_h$ or $aR^\lambda b_h$.

Theorem 4 establishes a comparison between the two ELECTRE TRI-C assignment rules.

Theorem 4

- a) If an action a is λ -indifferent to at least one reference action, then a is assigned by the descending rule to a category at least as good as the one a is assigned to when using the ascending rule.
- b) If an action a is λ -incomparable to at least one reference action, then a is assigned by the descending rule to a category at most as good as the one a is assigned to when using the ascending rule.
- c) Otherwise, both rules assign the action a to the same category or to two different but consecutive categories.

Proof

- a) If an action a is λ -indifferent to at least one reference action, then the following case occurs: $a \succ b_0, a \succ b_1, \dots, a \succ b_t, aI^\lambda b_{t+1}, \dots, aI^\lambda b_s, b_{s+1} \succ a, \dots, b_{q+1} \succ a$, with $0 \leq t \leq (q-1)$ and $(t+1) \leq s \leq q$. According to the descending rule (Definition 4), the highest index h such that an action a is λ -indifferent to b_h is $h = s$. Thus, if $\xi_s^+(a, \lambda) \leq |\xi_{s+1}^+(a, \lambda)|$, then the action a is assigned to category C_s . Otherwise, a is assigned to C_{s+1} . According to the ascending rule (Definition 5), the lowest index h , such that an action a is λ -indifferent to b_h is $h = (t+1)$. Thus, if $\xi_{t+1}^-(a, \lambda) \leq |\xi_t^-(a, \lambda)|$, then the action a is assigned to category C_{t+1} . Otherwise, a is assigned to C_t . Consequently, the descending rule provides always a category at least as good as the one provided by the ascending rule because $t < (t+1) \leq s < (s+1)$.
- b) If an action a is λ -incomparable to at least one reference action, then the following case occurs: $a \succ b_0, a \succ b_1, \dots, a \succ b_t, aR^\lambda b_{t+1}, \dots, aR^\lambda b_s, b_{s+1} \succ a, \dots, b_{q+1} \succ a$, with $0 \leq t \leq (q-1)$ and $(t+1) \leq s \leq q$. According to the descending rule (Definition 4), the lowest index h , such that an action a is λ -incomparable to b_h is $h = (t+1)$. Thus, if $\xi_t^+(a, \lambda) \leq |\xi_{t+1}^+(a, \lambda)|$, then the action a is assigned to category C_t . Otherwise, a is assigned to C_{t+1} . According to the ascending rule (Definition 5), the highest index h , such that an action a is λ -incomparable to b_h is $h = s$. Thus, if $\xi_{s+1}^-(a, \lambda) \leq |\xi_s^-(a, \lambda)|$, then the action a is assigned to category C_{s+1} . Otherwise, a is assigned to C_s . Consequently, the descending rule provides always a category at most as good as the one provided by the ascending rule because $t < (t+1) \leq s < (s+1)$.
- c) If there is only λ -preference relations between an action a and all reference actions b_h , then the following case occurs: $a \succ b_0, a \succ b_1, \dots, a \succ b_t, b_{t+1} \succ a, \dots, b_{q+1} \succ a$, with $0 \leq t \leq q$. According to the descending rule (Definition 4), the highest index h , such that an action a is λ -preferred to b_h is $h = t$. Thus, if $\xi_t^+(a, \lambda) \leq |\xi_{t+1}^+(a, \lambda)|$, then the action a is assigned to category C_t . Otherwise, a is assigned to C_{t+1} . According to the ascending rule (Definition 5), the lowest index h , such that a reference action b_h is λ -preferred to an action a is $h = (t+1)$. Thus, if $\xi_{t+1}^-(a, \lambda) \leq |\xi_t^-(a, \lambda)|$, then the action a is assigned to category C_{t+1} . Otherwise, a is assigned to C_t . Consequently, both descending and ascending rules can provide either the same category (C_t or C_{t+1}) or the descending rule provides the category C_t and the ascending rule the category C_{t+1} or vice-versa. \square

The results expressed in this theorem are illustrated in Table 8 from the numerical example studied in Section 6.

Table 8: ELECTRE TRI-C assignment results

| Actions | b_1 | b_2 | b_3 | b_4 | Descending | Ascending |
|----------|-------------|-------------|-------------|-------------|------------|-----------|
| a_1 | \succ | \succ | R^λ | R^λ | C_2 | C_4 |
| a_2 | R^λ | R^λ | R^λ | R^λ | C_1 | C_4 |
| a_3 | \succ | \succ | \prec | \prec | C_3 | C_3 |
| a_4 | \succ | \succ | \prec | \prec | C_2 | C_3 |
| a_5 | I^λ | \prec | \prec | \prec | C_2 | C_1 |
| a_6 | \succ | \succ | R^λ | R^λ | C_2 | C_4 |
| a_7 | \succ | \prec | \prec | \prec | C_1 | C_2 |
| a_8 | \succ | \prec | \prec | \prec | C_1 | C_1 |
| a_9 | R^λ | R^λ | R^λ | R^λ | C_1 | C_4 |
| a_{10} | \succ | \prec | \prec | \prec | C_1 | C_1 |

Note: $\lambda = 0.70$

Observe that the action a_5 is λ -indifferent to b_1 . In such a case, a_5 is assigned to C_2 by the descending rule and to C_1 by the ascending rule. It clearly shows that it is possible that an action can be assigned to a better category by the descending rule than the ascending rule. We can also observe that there is a strong λ -incomparability between the actions a_2 and a_9 with respect to all central reference actions. In such cases, the descending rule assigns these two actions to C_1 , while the ascending rule assigns the same actions to C_4 .

Consider an action a and a central reference action b_h . The following proposition establishes the links between two λ -binary relations and ELECTRE TRI-C assignment results according to both the descending and the ascending rules.

Proposition 2

- a) If a λ -outranks b_h , then a is assigned at least to C_h by the descending rule.
- b) If b_h λ -outranks a , then a is assigned at most to C_h by the ascending rule.
- c) If a is λ -preferred to b_h , then a is assigned at least to C_h by both rules.
- d) If b_h is λ -preferred to a , then a is assigned at most to C_h by both rules.

Proof

- a) Assume that $aS^\lambda b_h$. Based on Definition 8.1 and Proposition 1.a, $aS^\lambda b_h \Rightarrow \xi_h^+(a, \lambda) \geq 0 \Rightarrow \xi_k^+(a, \lambda) \geq 0$, $k \leq h$. Thus, a is assigned to category C_t , such that $t \geq h$ because of the two following situations. First, when $\xi_{h+1}^+(a, \lambda) < 0$ if $\xi_h^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)|$, then a is assigned to C_h by the descending rule. Otherwise, a is assigned to C_{h+1} . Second, if $\xi_{h+1}^+(a, \lambda) \geq 0$, then a is necessarily assigned to a category at least as good as C_{h+1} .

- b) Assume that $b_h S^\lambda a$. Based on Definition 8.1 and Proposition 1.b, $b_h S^\lambda a \Rightarrow \xi_h^-(a, \lambda) \geq 0 \Rightarrow \xi_k^-(a, \lambda) \geq 0$, $k \geq h$. Thus, a is assigned to a category C_t , such that $t \leq h$ because of the two following situations. First, when $\xi_{h-1}^-(a, \lambda) < 0$ if $\xi_h^-(a, \lambda) \leq |\xi_{h-1}^-(a, \lambda)|$, then a is assigned to C_h by the ascending rule. Otherwise, a is assigned to C_{h-1} . Second, if $\xi_{h-1}^-(a, \lambda) \geq 0$, then a is necessarily assigned to a category at most as good as C_{h-1} .
- c) For the descending rule, the proof is similar to a) by using Definition 8.4 and Proposition 1.a. For the ascending rule, the proof is similar to b) by using Definition 8.4 and Proposition 1.b.
- d) For the descending rule, the proof is similar to a) by using Definition 8.4 and Proposition 1.a. For the ascending rule, the proof is similar to b) by using Definition 8.4 and Proposition 1.b. \square

8 Conclusions

This paper dealt with a new sorting method, called ELECTRE TRI-C, which categories are defined through central reference actions instead of profile limits. A comparison with ELECTRE TRI-B shows the main similarities of the two methods. Defining categories through central reference actions is, in our opinion, of the uttermost importance for modelling a wide variety of practical decision aiding situations dealing with the assignment of actions to pre-defined and ordered categories.

Central reference actions and profile limits are two alternative ways for defining ordered categories. These reference actions must be defined *a priori* to play an appropriate role. The procedures must preserve this role when assigning the potential actions to the categories. In several situations, it is more adequate to define the categories through an interaction process with the decision-maker by using central reference actions. Defining the categories through profile limits can be difficult when the frontier between the criteria used to delimit them is rather fuzzy in the mind of the decision-maker.

The two proposed assignment rules fulfill the structural properties of uniqueness, independence, conformity, monotonicity, homogeneity, and stability. When the set of reference actions does not fulfill the strict separability condition, but only a weak separability condition, a compatible majority level must be computed in order to obtain a consistent assignment model.

As for future research avenues, we intend to analyse a possible extension to multiple typical actions. Such an extension will allow us modelling a larger number of decision aiding situations in the field of sorting problems. When using the concept of central reference actions, we intend to study assignment procedures to deal with partially ordered categories and completely non-ordered categories. Furthermore, it is more difficult to introduce profile limits than central reference actions for the definition of partially ordered

categories. Currently there is no method incorporating the notion of category size in the assignment procedures to limit the number of actions that can be assigned to each category. In fact, we also intend to study this particular issue by introducing a notion of relative independence for characterizing a new sorting problematic. At the same time, we should focus our attention on the inference of some parameters through an aggregation-disaggregation procedure using central reference actions.

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References

- André, S. and Roy, B. (2007). Conception et mise en place d’un outil d’évaluation de la performance environnementale : Le cas des raffineries de la société TOTAL. *Journal of Decision Systems*, 16(3):335–367.
- Arondel, C. and Girardin, P. (2000). Sorting cropping systems on the basis of their impact on groundwater quality. *European Journal of Operational Research*, 127(3):467–482.
- Belacel, N. (2000). Multicriteria assignment method PROAFTN: Methodology and medical application. *European Journal of Operational Research*, 125(1):175–183.
- Bouyssou, D., Marchant, T., Pirlot, M., Tsoukiàs, A., and Vincke, P. (2006). *Evaluation and Decision Models with Multiple Criteria: Stepping Stones for the Analyst*, volume 86 of *International Series in Operations Research & Management Science*. Springer Science + Business Media, Inc., Boston.
- Dias, L. and Mousseau, V. (2003). IRIS: A DSS for multiple criteria sorting problems. *Journal of Multi-Criteria Decision Analysis*, 12(4-5):285 – 298.
- Dias, L., Mousseau, V., Figueira, J., and Clímaco, J. (2002a). An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI. *European Journal of Operational Research*, 138(2):332–348.

- Dias, L., Mousseau, V., Figueira, J., Clímaco, J., and Gomes da Silva, C. (2002b). IRIS 1.0 software. European Working Group “Multicriteria Aid for Decisions”. Newsletter, N. 5 (3), p. 4-6.
- Dimitras, A., Zopounidis, C., and Hurson, C. (1995). A multicriteria decision aid method for the assessment of business failure risk. *Foundations of Control Engineering*, 20(2):99–112.
- Doumpos, M. and Zopounidis, C. (2002). On the development of an outranking relation for ordinal classification problems: An experimental investigation of a new methodology. *Optimization Methods and Software*, 17(2):293–317.
- Doumpos, M. and Zopounidis, C. (2004). A multicriteria classification approach based on pairwise comparisons. *European Journal of Operational Research*, 158(2):378–389.
- Doumpos, M. and Zopounidis, C. (2007). Regularized estimation for preference disaggregation in multiple criteria decision making. *Computational Optimization and Applications*, 38(1):61–80.
- Figueira, J., Greco, S., and Roy, B. (2006). ELECTRE methods with interaction between criteria: An extension of the concordance index. Cahier du LAMSADE 242, Université Paris-Dauphine, Paris, France. Updated in Dec. 2007.
- Georgopoulou, E., Sarafidis, Y., Mirasgedis, S., Zaimi, S., and Lalas, D. (2003). A multiple criteria decision-aid approach in defining national priorities for greenhouse gases emissions reduction in the energy sector. *European Journal of Operational Research*, 146(1):199–215.
- Joerin, F., Thériault, M., and Musy, A. (2001). Using GIS and outranking multicriteria analysis for land-use suitability assessment. *International Journal of Geographical Information Science*, 15(2):153–174.
- Léger, J. and Martel, J. (2002). A multicriteria assignment procedure for a nominal sorting problematic. *European Journal of Operational Research*, 138(2):349–364.
- Massaglia, R. and Ostanello, A. (1991). N-TOMIC: A support system for multicriteria segmentation problems. In Korhonen, P., Lewandowski, A., and Wallenius, J., editors, *Multiple Criteria Decision Support: Proceedings, Helsinki, Finland, 1989*, volume 356 of *Lectures Notes in Economics and Mathematical Systems*, pages 167–174. Springer-Verlag, Berlin.
- Merad, M., Verdel, T., Roy, B., and Kouniali, S. (2004). Use of multi-criteria decision-aids for risk zoning and management of large area subjected to mining-induced hazards. *Tunnelling and Underground Space Technology*, 19:165–178.

- Moscarola, J. and Roy, B. (1977). Procédure automatique d'examen de dossiers fondée sur une segmentation trichotomique en présence de critères multiples. *R.A.I.R.O. Recherche Opérationnelle / Operations Research*, 11(2):145–173.
- Mousseau, V. and Dias, L. (2004). Valued outranking relations in ELECTRE providing manageable disaggregation procedures. *European Journal of Operational Research*, 156(2):467–482.
- Mousseau, V. and Słowiński, R. (1998). Inferring an ELECTRE TRI model from assignment examples. *Journal of Global Optimization*, 12(2):157–174.
- Nemery, P. (2008). Utilisation des relations de surclassement dans des problèmes de classification. Technical Report 2008/01, SMG, Université Libre de Bruxelles, Belgique.
- Ngo The, A. and Mousseau, V. (2002). Using assignment examples to infer category limits for the ELECTRE TRI method. *Journal of Multi-Criteria Decision Analysis*, 11(1):29–43.
- Norese, M. and Viale, S. (2002). A multi-profile sorting procedure in the public administration. *European Journal of Operational Research*, 138(2):365–379.
- Perny, P. (1998). Multicriteria filtering methods based on concordance and non-discordance principles. *Annals of Operations Research*, 80:137–165.
- Raju, K., Duckstein, L., and Arondel, C. (2000). Multicriterion analysis for sustainable water resources planning: A case study in Spain. *Water Resources Management*, 14(6):435–456.
- Roy, B. (1991). The outranking approach and the foundations of ELECTRE methods. *Theory and Decision*, 31(1):49–73.
- Roy, B. (1996). *Multicriteria Methodology for Decision Aiding*, volume 12 of *Nonconvex Optimization and Its Applications*. Kluwer Academic Publishers, Dordrecht.
- Roy, B. (2002). Présentation et interprétation de la méthode ELECTRE TRI pour affecter des zones dans des catégories de risque. Document du LAMSADE 124, Université Paris-Dauphine, Paris, France.
- Roy, B. and Bouyssou, D. (1993). *Aide Multicritère à la Décision : Méthodes et Cas*. Production et Techniques Quantitatives Appliquées à la Gestion. Economica, Paris, France.
- Roy, B. and Słowiński, R. (2008). Handling effects of reinforced preference and counter-veto in credibility of outranking. *European Journal of Operational Research*, 188(1):185–190.

- Roy, B. and Vincke, P. (1984). Relational systems of preference with one or more pseudo-criteria: Some new concepts and results. *Management Science*, 30(11):1323–1335.
- Tervonen, T., Figueira, J., Lahdelma, R., Almeida Dias, J., and Salminen, P. (2007). A stochastic method for robustness analysis in sorting problems. *European Journal of Operational Research*. In press.
- Yu, W. (1992). *Aide Multicritère à la Décision dans le Cadre de la Problématique du Tri : Concepts, Méthodes et Applications*. Thèse de Doctorat, LAMSADE, Université Paris-Dauphine, Paris, France.