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Electron plasma waves and plasma resonances

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Abstract
In 1929 Tonks and Langmuir predicted of the existence of electron plasma waves in an infinite, uniform plasma. The more realistic laboratory environment of non-uniform and bounded plasmas frustrated early experiments. Meanwhile Landau predicted that electron plasma waves in a uniform collisionless plasma would appear to be damped. Subsequent experimental work verified this and revealed the curious phenomenon of plasma wave echoes. Electron plasma wave theory, extended to finite plasmas, has been confirmed by various experiments. Nonlinear phenomena, such as particle trapping, emerge at large amplitude. The use of electron plasma waves to determine electron density and electron temperature has not proved as convenient as other methods.

1. Introduction

In a ground-breaking paper in 1929 Tonks and Langmuir [1], amongst other things, derived a dispersion relation for longitudinal electrostatic waves in an unbounded, uniform plasma of density n with mobile electrons, mass m and immobile ions

$$\omega^2 = \omega_{\rm pe}^2 + k^2\left(\frac{3}{2}\frac{\kappa T}{m}\right), \qquad (1)$$

where ω is the wave frequency, $\omega_{\rm pe}$ is the electron plasma (angular) frequency given by $\omega_{\rm pe}^2 = ne^2/m\varepsilon_0$, κ is Boltzmann's constant and $T_{\rm e}$ is the electron temperature. Equivalently,

$$\omega^2 = \omega_{\rm pe}^2\left(1 + \frac{3}{2}k^2\lambda_{\rm De}^2\right),$$

where k is the wave number $2\pi/\lambda$ and λ is the wavelength; $\lambda_{\rm De}$ is the electron Debye length given by $\lambda_{\rm De}^2 = \varepsilon_0\kappa T_{\rm e}/ne^2$, the characteristic electrostatic scale length in plasmas. These longitudinal electrostatic waves are known as either 'electron plasma' or 'Langmuir' waves. Evidently in unbounded plasmas they occur only for frequencies above the electron plasma frequency. Interestingly, Tonks and Langmuir indicate in a footnote that they were in correspondence with J J Thomson in relation to their work.

Experimentally at the time it was difficult to obtain an obviously uniform plasma and so experiment lagged behind theory. Nevertheless, Tonks [2] did carry out measurements using a Lecher wire transmission line (twin parallel wires) loaded by a low-pressure positive column of a gas discharge. These measurements threw up more questions than they answered and were essentially ignored until the 1960s. Much of the difficulty can be seen in retrospect to have arisen from the complications of plasma boundaries that introduce non-uniformity to the plasma and modify the electrical environment.

The Tonks and Langmuir paper [1] not only suggested that oscillations were probably associated with electron plasma waves but also revealed at lower frequencies what are now known as ion waves. Ion waves are essentially confined to frequencies below the ion plasma frequency $\omega_{\rm pi}$, where the ion mass M now determines the dynamics so that $\omega_{\rm pi}^2 = ne^2/M\varepsilon_0$. This topic is considered elsewhere in this special issue [3].

2. Developments in theory and experiments

Further theoretical development occurred in 1946 when Landau [4], using the kinetic Vlasov equation [5], showed that electron plasma waves were expected to lose amplitude with an apparent damping factor that increased rapidly with ω, and effectively only existed for $k\lambda_{\rm De} < 1$ which is equivalent to $\omega < 1.6\omega_{\rm pe}$. Understanding the mechanism by which a wave could be damped in a *collisionless* plasma led to much discussion in the literature but clarification was provided by in 1949 by Bohm and Gross [6]: in fact the wave is not diminished by an energy absorbing mechanism, rather, as the wave energy is dispersed in velocity space it loses its spatial coherence so the amplitude of the signal detected at increasing distances from a source is progressively diminished. Six years later the inclusion of collisions with the background gas by Bhatnagar *et al* [7] meant that the theory could cope with both *collisional* damping and Landau (apparent) damping occurring simultaneously.

In terms of experiments in low-pressure positive columns, it was not until 1968 that Tutter [8] reported measurements of waves propagating along argon discharges, mapping out the dispersion relation. Results correlated closely with the form of equation (1), once the drift of the electrons had been taken into account. Waves propagating in either direction, i.e. anode directed or cathode directed were Doppler shifted by electron drift. However the situation had been transformed in 1963 when Malmberg and Wharton [9] reported results on a specially designed uniform plasma verifying the essential features of Landau's theory. Their work was later confirmed in the 1970s and extended in a series of papers by Franklin *et al* [10–13]. However that work was done with plasmas generated in so-called Q(quiescent plasma)-machines with the plasmas magnetically confined, electrons generated thermionically, and ions produced by a surface ionization mechanism. The understanding of electron plasma waves propagating in this environment of bounded, magnetized plasma required further insights: depending on the strength and direction of the magnetic field the Langmuir wave extends beyond the simple cut-off dispersion described by equation (1) and in general the waves are no longer purely electrostatic. It should be noted that most textbooks on this topic confine themselves to uniform plasmas.

3. Resonances

During the 1960s non-uniform plasmas were subject to an intense period of activity following the work by Dattner [14] that revealed a rich spectrum of resonances on 'scattering' microwaves from a low-pressure positive column. We recognize with hindsight that these features had been observed earlier by Tonks [2], but the technology was not then available to bring sufficient understanding since the typical electron plasma frequencies were around or above 100 MHz. Still these plasma boundary effects are generally referred to as Tonks–Dattner resonances, acknowledging Tonks's preliminary contribution. The theoretical resolution of the nature and cause of these resonances came in papers by Crawford [15], and by Nickel *et al* [16]. They modelled the propagation of electron plasma waves travelling radially across a warm, non-uniform plasma column excited by an external agency. The waves propagate between the plasma boundary where they are excited and the radial position where the wave frequency matches the local plasma frequency. At this point the wave is reflected and at a specific frequency a standing wave can be established. Energy is absorbed at the resonance, being ultimately dispersed through collisions. The former paper showed the essential features, but it was the latter that obtained excellent agreement between experiment and theory both for the principal electrostatic resonant modes and for the subsidiary resonances arising from radial propagation of electron plasma waves within the column. Contemporary experimental work varying the dielectric environment was able to show that the principal resonance was electrostatic in nature while the subsidiaries were essentially standing electron plasma waves (Bryant and Franklin [17]).

The early work by Tonks (1931) has been the subject of interest over the years in different geometries, for instance in the 1960s there was a lot of work done on probes excited by radio-frequencies and the information that could be obtained in such a way [18]. A recent paper covering much of the work on plasma resonances from Tonks to the present day is Czarnetzki *et al* [19]. Various designs of electrical resonance probe have been developed in recent years as an alternative to Langmuir probes [20].

4. Using Langmuir waves as a diagnostic

Given that the various aspects of electron plasma waves were well understood by the1970s the stage was set to apply that knowledge as a diagnostic tool in plasmas by exciting electron plasma waves within them and examining their dispersion. Langmuir and his colleagues, had electrostatic probes as their principal diagnostic, and since then such probes have been developed in a variety of ways as described elsewhere this special issue. There remains a need to complement electric probes, and electron wave diagnostics has proved a useful aid, albeit in a limited range of circumstance.

The excitation of Langmuir waves is usually a simple matter and can be achieved by introducing into a plasma a high frequency signal on a suitably designed electric probe. Franklin *et al* [10] used a 2 mm long, 0.2 mm diameter bare platinum wire, fed directly from a 50 Ω coaxial transmission line. Detection can be achieved on a similar probe that can be moved to generate an interference pattern by mixing the detected signal with the input signal. This arrangement enables the wavelength of the waves at fixed frequency to be obtained, on an essentially directional basis.

The ability to carry out such measurements was facilitated by the use of 'box-car' techniques linking generator and detector. An alternative way to determine the wavelength is to measure the propagation speed using a fixed transmitter and a fixed detector with a tone-burst signal based on a nominal wave frequency [3]. In infinite plasma the lower limit for transmission allows one to determine the electron plasma frequency, ω_{pe}, from which the electron density is readily obtained since $n = (\omega_{pe})^2 m\varepsilon_0/e^2$. In Q-machines however the Langmuir wave mode is not cut-off owing to the effects of the finite size of the plasma. More generally, by making wavelength measurements for a range of frequencies one can map out the dispersion relation, $\omega(k)$. From this, models with appropriate boundary conditions allow the determination of the electron density and the temperature T_e. If the plasma itself is varying temporally, provided the timescale of the variation is much slower than the wave period, the method remains effective.

As a general laboratory plasma diagnostic Langmuir waves have not proved anywhere near as useful as Langmuir probes. However, in a variety of remote physical situations—from plasmas in planetary atmospheres such as ionospheric plasmas, to plasmas within and between stars plasmas as well as dusty plasmas [21–24]—Langmuir's electron plasma waves are an important means of plasma characterization. The waves arise here through various mechanisms such as streaming instabilities and nonlinear electromagnetic wave phenomena so in these plasmas there generally is a rich spectrum of electron plasma waves to detect directly or else to interact with through other waves.

5. Electron plasma wave echoes

The recognition that some physical systems under 'collisionless' conditions exhibit a 'memory' was demonstrated first in the phenomenon of nuclear spin echoes [25]. The realization that there was a plasma analogue led quickly to models and to experiments. This work of the late 1960s is described in Gould *et al* [26], O'Neil and Gould [27] and Malmberg *et al* [28]. In essence they excited continuous electron plasma waves with antennae at locations separated by a distance L, excited at different frequencies, ω_1 and ω_2 ($>\omega_1$). Both were chosen far enough above the electron plasma frequency so that the waves were heavily Landau damped in a distance much less than the separation. The echo appeared, as expected by theory at a distance $L\omega_1/(\omega_2 - \omega_1)$ from the first antennae at a frequency $\omega_2 - \omega_1$. The appearance of these echoes is a striking demonstration of the true mechanism behind Landau damping, showing that it is not dissipative—as had been anticipated by Vlasov, though in practice the magnitude of the echo is decreased by collisions within the plasma.

6. Nonlinear electron plasma waves

A literature search today relating to Langmuir waves leads into the field of turbulence in plasmas, particularly associated with fusion plasmas. Initially in the references already given above [10–13], interest was in deviation from so-called linear Landau damping. The process of the nonlinear transfer of energy out of electron plasma waves by the generation of other waves of a lower frequency is often also referred to as wave decay.

At the same time new instabilities were being discovered, and an important example is the so-called sideband instability involving initially the trapping of electrons by a large amplitude wave and then de-trapping as the wave amplitude decreases [29, 30].

Another phenomenon that occupied a lot of attention was so-called quasi-linear damping. Herein a test wave was shown to interact with electrons near the phase velocity and then to modify the electron velocity distribution [31]. The main theoretical effort in the 1970s therefore went into trying to describe what went on in a plasma where the Langmuir waves grew to such a magnitude that they interacted back into the plasma from which they had grown, and modified it. There is a rich literature on this subject notably developed by Vedenov [32] and by Tsytovich [33, 34] especially in connection with geophysical and astrophysical plasmas. The topic has grown from simple one-dimensional considerations to full-blown three-dimensional work. The term 'Langmuir turbulence' appears to have emerged at roughly the same time in the literature in both the Soviet Union and the United States.

7. Conclusion

The study of electron plasma (Langmuir) waves dates back to the origins of the word plasma. These waves are ubiquitous in plasmas though under many circumstances they do little more than contribute to the noise level—just about any disturbance excites them. Nevertheless, careful study of electron plasma waves has led to major insights in plasma physics relating to Landau damping and a variety of nonlinear phenomena including turbulence. We suggest that if Irving Langmuir were alive today he might be agreeably surprised by how his name is currently being remembered for his work in a field that he essentially invented, but in a totally different regime. We also suggest that another employee of GE, namely Lewi Tonks, if he were alive today he too would draw great satisfaction from that fact that his work continues to inspire technological progress. For further reading on the topics in this paper see Vandenplas [35] and Franklin [36].

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