Planning Bus Rapid Transit services: A case study in the Barcelona Metropolitan Area

Inês Miguel
ines.miguel@tecnico.ulisboa.pt

Instituto Superior Técnico, Universidade de Lisboa, Portugal

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Abstract

The current growth of the urban population in cities is leading to a mobility problem. The increase of private cars in the city centers is becoming unsustainable and a change in the efficiency and diversity of public transportation is needed. The buses are now seen as a viable and cost-effective alternative to this problem, namely the Bus Rapid Transit systems. Several methods are being studied and developed in order to cover as much area and demand as possible with the resources available. This work presents a mathematical programming model to design a bus network. The objective is to minimize the time an average user spends inside the network, by achieving a set of routes, with associated frequencies, that satisfy the demand predicted for each origin-destination pair. It is then applied to a case study in the Barcelona Metropolitan Area, where the impact of the implementation of a Bus Rapid Transit route in the existing network is studied. The results presented serve as a decision-making tool to improve the network, by returning the set of routes and associated frequencies that minimize the given objective function. A decrease of 8.75 minutes in the time spent by an average user inside the network is achieved, representing a reduction of 15% when compared to the current solution, and confirming the existence of room for improvement.

Keywords: Transport Network Design Problem; Bus Network Design; Bus Rapid Transit; Frequency Setting; Public Transportation

1. Introduction

The growth of the population living in urban areas, which is expected to reach 70% of the world's population by 2050 [1], together with the increasing concern about climate change, led governments and communities to understand that a change in urban mobility and in how people move inside and between cities is needed.

The Sustainable Development Goals (SDG), released by the United Nations General Assembly in 2015 [2], is a set of 17 global goals to reach a more sustainable and fairer world. To respond and comply with these goals, the European Commission (EC) released the European Green Deal in 2019, and in 2020 the Sustainable and Smart Mobility Strategy [3], both strategies that aim to transform Europe. Regarding the transportation sector, one of the objectives is to achieve a 90% reduction of greenhouse gas emissions by 2050 and to further increase the efficiency of the transport systems by strengthening the multimodal transport, along with the digitalization of these systems. These strategies suggest that the transport sector needs to go through a decarbonization process, both an increase in energy efficiency and a reduction in the number of private cars circulating need to happen. This can happen by creating ways that incentivize people to choose public transport, walking, or cycling over the use of private cars. As stated by the European Environmental Agency, buses are two times more energy-efficient than cars [1]. Therefore, along with a general improvement in the public transport, the bus networks should be reviewed and improved to accommodate the population's necessities, its new behaviors, and the needed decrease in emissions.

In order to do this, factors influencing bus networks' efficiency need to be studied, to achieve a more efficient and effective solution for each city/place. Changes in the frequencies, the creation or elimination of routes, the vehicle capacity for each route or the transition to low carbon emission vehicles are some of the options.

The Bus Rapid Transit (BRT) systems are also a possible solution that covers a lot of the topics discussed. Normally, the buses of these systems will operate in exclusive bus lanes, eliminating one of the sources of delays in this type of transportation: the mixed traffic. This, combined with a higher frequency of these buses and better traffic signal management, results in a reduction in the travel and waiting time spent by the users, increasing the service reliability and the satisfaction of the users. These systems are usually cheaper and quicker to implement when compared to other modes.

There has been a growth in the number of BRT systems implemented in the last decades: from 1992 to 2002, it was implemented in only 20 cities, contrasting with the 145 cities that implemented it from 2003 until 2020 [4]. One of the biggest problems of implementing these systems is often the resistance to change coming from the community. However, the COVID-19 pandemic brought a "window of opportunity" to implement such changes in urban mobility [5]. In fact, as a consequence of the several measures that were implemented in most countries, there is an opportunity for a new adaptation of the transport sector, as new routines are arising and people are more conscious and open to changes that may benefit them and the environment.

This work resulted from a challenge made by the Mobility Sector of the Barcelona Metropolitan Area (AMB), where a BRT route will be implemented. By providing data regarding the use of their current network, the objective is to contribute with inputs for the decision-makers of this entity regarding network modifications that may come from the introduction of a BRT system in this area.

2. State of the Art

2.1. Network Design in Transportation Systems

According to [6], a Urban Transportation Network Design Problem (UTNDP) can be described as a succession of decisions that leads to the planning of transit networks, being these decisions classified as strategical, tactical or operational. It usually involves minimizing or maximizing a set of objectives, while complying with specific constraints. The transit network design process includes steps such as network design, frequency setting, timetable development and crew and vehicle scheduling, as stated in [7].

However, as it is a very complex problem, with multiple variables at stake, it is normally divided into smaller problems, according to the number of modes or even the steps of the design in study. As an example, when more than one mode is considered for the analysis, one is faced with a MultiModal Network Design Problem (MMNDP). Efforts on this topic can be found on [8], [9], [10]. If only one mode is subject to analysis, then a new class of problems arise, the Transit Network Design Problem (TNDP).

As discussed in [11], the complexity of this kind of problems arises from the multi-objective nature of TNDP, since several variables and objectives can be explored, depending on the case under study. Some objectives that are usually presented in literature include the minimization of the time spent by the users in the network, maximization of the operators profit (by minimizing costs or the fleet size needed), limitations on the length of the routes, etc.

On [12], one can find an example of an analysis of a taxi network. The goal was to obtain a set of routes to be operated by this service, which has the particularity that a taxi can only leave when it is full. The objective of the mathematical model developed was to minimize the total travel time perceived by the users (including a penalty on the waiting time) while limiting the number and length of taxi lines and the transfer ratio (2 transfers allowed) so that the characteristic of having a high-frequency service could be achieved. After, the minimization of the fleet needed was also studied. It is seen that this model could be applied to any other transportation mode, such as bus, or rails, only adapting the constraints applied.

2.2. Network Design in Bus Networks

The Bus Transit Network Design Problem (BTNDP) is the class of problems that deal with the design of bus networks. Several approaches have been tried in the past decades regarding the optimization of bus networks. For example, in [13], three main topics are covered concerning route generation, network analysis accounting with demand prediction, and the frequency setting for the routes. It considers variable demand as an approximation to real-life cases, and the objective was to obtain the set of routes that optimize the network, according to the objective of minimizing the operator and users' cost and unsatisfied demand costs.

Cancela et al [14] presented a mathematical formulation for TNDP, that is used to solve a BTNDP. The objective was to obtain an optimal set of bus routes and respective operational frequencies that cover a given demand and comply with a given infrastructure of roads and bus stops. They formulated several aspects regarding the interest and behavior of the users and the interest of the operators. Some possible additions to the model such as bus capacity and transfers constraints are explored but not included in the analysis.

In [15], a model was developed regarding route design and frequency setting, including both user's and operator's costs. In this case, the fleet size was not an imposed value, but one of the variables they wanted to minimize, along with the time spent on the network. The network was represented by a graph, with nodes and arcs, similar to the approach presented in [14].

In [16], the existence of two typologies of vehicles (green and normal vehicles) is explored. However, instead of having them in the same network, they are treated as two different networks (two sub-networks) that should interact between them (multimodal) and complement each other. The green vehicles are limited, so a compromise between the two networks should happen.

Regarding the BRT systems, the research being developed involves normally the initial steps of this kind of systems, namely where and how to implement it [17].

In [18], it is presented a real case in Iran, where an existing network is improved. In this study, the authors took a different approach and consider two sub-networks, with faster buses, such as BRT systems, making the connection between the extremes of the city, while local buses connect the urban area of the city. They accounted for both the cost of the operators, with the purpose of minimizing the routes length, both for BRT and for the normal buses, and the users, with preference for the shortest paths, minimum transfers and higher use of BRT. The results allowed the authors to realize that, although an investment by the responsible authority is needed, an improvement in the current system should be in place. More recently, in [19], one can find a methodology to implement a BRT route in a city, namely how to choose the bus stops, routes and associated frequencies.

2.3. Target Contribution

The contribution of this work relies on the study of the implementation of a BRT route in an existing network, by developing and applying a mathematical model that is capable of capturing the main characteristics of the network. The results will support decision makers to understand what kind of changes need to take place, at a strategical and tactical level. They include a set of routes and respective operational frequencies that optimize the objective function in study, minimizing the travel time spent by an average user (on board time + waiting time). Since the BRT route was already designed, the focus of the work is not to give recommendations on where to place it, but to study the impact of the route in part of the network, as it will be in superposition with part of the existing routes.

This model is inspired in [14], but several modifications were implemented, as it will be discussed in sections 3 and 4. The addition of a constraint to account for the capacity of the buses, i.e., a restriction on the number of people allowed to be on board the bus in each path, as well as the allowance of one transfer, are part of the modifications made. At the same time, since the buses used for the BRT route will have different specifications and availability when compared to the normal buses, constraints regarding these aspects are also included.

3. Model Formulation

The mathematical model is an application adapted from a previous formulation presented in [14]. Several concepts need to be introduced.

Let a node be defined as a point in which users can enter or leave the network or transfer between routes (represented by n, n_1 and n_2). The connections between nodes are represented by arcs. To each arc, it will be associated a cost ($c_{n_1 n_2}$), that represents the on board time that users spent to go from one node to the other. A route can be characterized by a sequence of arcs, connecting the nodes that belong to that route (r, r_1 and r_2). Each route will have an associated frequency (f), expressed in buses per minute.

Figure 1 helps to understand better this notation. The points n_1 to n_5 represent nodes of the network. The lines between these nodes represent the arcs that connect them. In this example, nodes n_1, n_2 and n_3 form a route, as well as nodes n_3, n_4 and n_5. It would be possible to perform a transfer from route 1 to route 2, since node n_3 belongs to both routes.

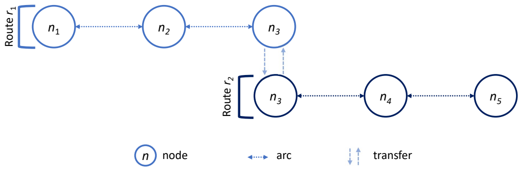

Figure 1: Representation of nodes and respective connection arcs.

An origin-destination (OD) pair (k) represents the connection between any two nodes (an origin node O_k and a destination node D_k) that do not need to be consecutive nor to belong to the same route. For each OD pair, it is associated a value for the expected demand (δ_k), representing the number of trips that must be satisfied per unit of time.

The number of buses available to perform all the services needed is given by the fleet size. Moreover, the bus capacity represents the number of users that are allowed in each bus at each arc.

As mentioned before, some modifications were applied to the mathematical formulation presented in [14]. Here, the model will be based on nodes and their relations, contrasting with the arcs' based formulation presented in [14]. This implies that, when referring to the travel arc between n_1 and n_2, it will be represented by (n_1, n_2) instead of a single variable a.

This work's model is an optimization problem that belongs to the class of MILP models. This class of mathematical programming models has mixed decision variables (i.e. continuous or integer decision variables), and the objective function and constraints have all linear relations with the decision variables.

On the other hand, as the objective function and all constraints considered in this problem are linear, it can be considered a linear model.

A minimization of an objective function wants to be achieved with this model. Starting from a set of nodes, arcs, and routes (i.e. by defining the graph of the network), a certain demand of passengers for each OD pair needs to be covered, along with the minimization of the time spent inside the network. Only one transfer is allowed for each connection between OD pairs. The expected solution for the problem will be the combination of routes and respective frequencies of operation that minimize the objective function, explained in subsection 3.2.

The constants used in this model included the number of nodes (NN), number of arcs (NA), number of routes (NR), number of OD pairs (NK), number of frequencies (NF), fleet size (B), large Number (M) and bus capacity (Ω). Different sets were defined, namely a set of nodes (N), a set of routes (R), a set for the arcs belonging to route r (A_r), a set of OD pairs (K), and a set of frequencies (Θ). There was the need to subdivide the set of nodes into a set of nodes of route r (N_r) and a set of terminal nodes of route r (N_r^T). The set of routes used as input to the model needs to be given either by expert knowledge or generated by a separate model.

It was still defined an array with the frequencies indexed by f (θ_f) and an array with the nodes where transfers can take place (σ). b_{n_k} was defined as being equal to δ_k if $n = O_k$, to -δ_k if $n = D_k$, and 0 otherwise.

3.1. Decision Variables

Several variables were left unknown so that they can be optimized. These decision variables are the flow between nodes n_1 and n_2 of route r from OD pair k ($\upsilon_{n_1n_2rk}$), the incoming flow at origin node n from OD pair k of route r (υ^+_{nrk}), the incoming flow at origin node n from OD pair k of route r at frequency θ_f (υ^+_{nrfk}), the outcoming flow at destination node n from OD pair k of route r (υ^-_{nrk}), the transfer flow between routes r_1 and r_2 at node n from OD pair k ($\upsilon'_{nr_1r_2k}$), the transfer flow between routes r_1 and r_2 at node n from OD pair k and route r_2 operated at frequency f ($\upsilon'_{nr_1r_2fk}$) and the waiting time multiplied by the demand of OD pair k at node n (w_{nk}). Moreover, the following binary decision variables are considered: x_r evaluates if route r is part of the solution, y_{rf} ensures if route r is operated at frequency f, $z_{n_1n_2r}$ if the arc (n_1,n_2) of route r is part of the solution and $\lambda_{n_1n_2r}$ if an empty bus is operating between nodes n_1 and n_2 from route r.

3.2. Objective Function

The objective fucntion being evaluated is given by:

$$\begin{aligned} Minimize: O = \sum_{k\in K}\Bigg[\sum_{r\in R}\sum_{(n_1,n_2)\in A_r} c_{n_1n_2}\upsilon_{n_1n_2rk} \\ + \sum_{n\in N} w_{nk}\Bigg] + \sum_{r\in R}\sum_{(n_1,n_2)\in A_r} c_{n_1n_2}\lambda_{n_1n_2r} \end{aligned} \tag{1}$$

The objective function, presented by equation 1, will consist of three parts related to: the on board travel time, obtained by multiplying the cost of traveling from node n_1 to node n_2 by the flow associated with that trip for a certain OD pair, and represented by $c_{n_1n_2}\upsilon_{n_1n_2rk}$; the waiting time, which accounts for the time the user will wait until entering the network and the time spent between transfers, and is given by w_{nk}; a penalization term, that indicates the cost of traveling between two nodes, which will only be considered in arcs where buses travel empty, thus being represented the multiplication of the cost of traveling between the two nodes and the binary that indicates if the bus is empty or not, $c_{n_1n_2}\lambda_{n_1n_2r}$.

3.3. Constraints

To restrict the solution space and simplify some of the variables, improving the computational time, several constraints were applied to the model.

$$\sum_{r\in R}\sum_{f\in\theta}\sum_{(n_1,n_2)\in A_r} 2\theta_f y_{rf} c_{n_1n_2} \leq B \tag{2}$$

Constraint 2 ensures that the maximum number of buses to operate the chosen routes at corresponding frequencies f does not exceed the fleet size of the operator.

$$\sum_{k\in K}\upsilon_{n_1n_2rk} \leq \sum_{f\in\theta} y_{rf}\theta_f\Omega \quad \forall \quad r\in R \tag{3}$$

Constraint 3 ensures that the capacity of each bus is not exceeded in any arc of a route, i.e. between two consecutive nodes of a route. The value of Ω is the bus capacity and it is an input.

$$\sum_{r\in R}\upsilon^+_{nrk} = b_{nk} \quad \forall \quad n\in N,\, k\in K : n = O_k \tag{4}$$

$$\sum_{r\in R}\upsilon^-_{nrk} = -b_{nk} \quad \forall \quad n\in N,\, k\in K : n = D_k \tag{5}$$

$$\begin{aligned} \upsilon^+_{nrk} = \sum_{n_2\in N:(n,n_2)\in A_r}\upsilon_{nn_2rk} \\ \forall \quad n\in N_r,\, r\in R,\, k\in K : n = O_k \end{aligned} \tag{6}$$

$$\begin{aligned} \upsilon^-_{nrk} = \sum_{n_1\in N:(n1,n)\in A_r}\upsilon_{n_1nrk} \\ \forall \quad n\in N_r,\, r\in R,\, k\in K : n = D_k \end{aligned} \tag{7}$$

$$\begin{aligned} \sum_{n_1\in N:(n_1,n)\in A_r}\upsilon_{n_1nrk} + \sum_{r_1\in R\setminus\{r\}:n\in N_{r_1}}\upsilon'_{nr_1rk} = \\ \sum_{n_2\in N:(n,n_2)\in A_r}\upsilon_{nn_2rk} + \sum_{r_1\in R\setminus\{r\}:n\in N_{r_2}}\upsilon'_{nrr_2k} \\ \forall \quad r\in R,\, n\in N\setminus\{N_r^T, O_k, D_k\} \end{aligned} \tag{8}$$

$$\begin{aligned} \upsilon_{n_1nrk} = \sum_{r_2\in R\setminus\{r\}:n\in N_{r_2}}\upsilon'_{nrr_2k} \\ \forall \quad r\in R,\, n\in N_r^T\setminus\{O_k,D_k\},\, n_1\in N\setminus\{D_k\}, \\ k\in K : (n_1,n)\in A_r \end{aligned} \tag{9}$$

$$\begin{aligned} \upsilon_{nn_2rk} = \sum_{r_1\in R\setminus\{r\}:n\in N_{r_1}}\upsilon'_{nr_1rk} \\ \forall \quad r\in R,\, n\in N_r^T\setminus\{O_k,D_k\},\, n_2\in N\setminus\{O_k\}, \\ k\in K : (n,n_2)\in A_r \end{aligned} \tag{10}$$

Constraints 4 to 10 guarantee the continuity of flows in each node, making sure that the number of users entering the node corresponds to the number of users leaving it, either for transfers, for continuity of the trip or leaving the network. There were three specific cases identified where constraint 8 was not ensured, namely in the nodes that are the origin or destination of a certain OD pair and the nodes that are terminal nodes of each route. For these three types of nodes, specific constraints were defined. For the origin nodes of the OD pair, constraints 4 and 6 were defined, while for the destination nodes, constraints 5 and 7 were considered. Regarding the terminal nodes, and excluding the ones that are origin or destination nodes, constraints 9 and 10 ensure that the flow entering the node will be transferred to other routes that share that node. Figure 2 schematizes the different flows mentioned before.

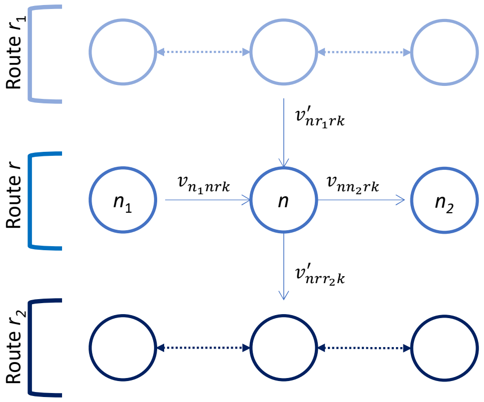

Figure 2: Incoming and outcoming flows of a generic node n.

$$\sum_{n_1 \in N:(n_1,n) \in A_r} \upsilon_{n_1nrk} = 0 \quad \forall \quad r \in R,\ n \in N_r,\ k \in K : n = O_k \tag{11}$$

$$\sum_{n_2 \in N:(n,n_2) \in A_r} \upsilon_{nn_2rk} = 0 \quad \forall \quad r \in R,\ n \in N_r,\ k \in K : n = D_k \tag{12}$$

Constraint 11 imposes that if n is an origin node, then within each route there will be no incoming flow from previous nodes. In line with this, constraint 12 imposes that if n is a destination node, then within each route there will be no outcoming flow for the following nodes.

$$\upsilon^{+}_{nrfk} \leq \theta_f w_{nk} \quad \forall \quad r \in R,\ n \in N_r, f \in \theta,\ k \in K : n = O_k \tag{13}$$

$$\sum_{r_1 \in R \setminus \{r\}: n \in N_{r_1}} \upsilon'_{nr_1r_2fk} \leq \theta_f w_{nk} \quad \forall \quad r_2 \in R,\ n \in N_{r_2} \setminus \{O_k, D_k\},\ f \in \theta,\ k \in K \tag{14}$$

Constraints 13 and 14 compute the waiting time that the users spend to enter the network and in transfers, respectively.

$$\upsilon_{n_1n_2rk} \leq \delta_k x_r \quad \forall \quad r \in R,\ (n_1,n_2) \in N_r,\ k \in K \tag{15}$$

$$\upsilon^{+}_{nrfk} \leq \delta_k y_{rf} \quad \forall \quad r \in R,\ n \in N_r,\ f \in \theta,\ k \in K : n = O_k \tag{16}$$

$$\upsilon'_{nr_1r_2fk} \leq \delta_k y_{r_2f} \quad \forall \quad (r_1,r_2) \in R,\ n \in (N_{r_1} \cap N_{r_2}),\ f \in \theta,\ k \in K : r_1 \neq r_2 \tag{17}$$

Constraints 15 to 17 ensure that users only use routes and frequencies that are part of the solution. Besides that, they establish the maximum limit for the respective flow (for example, constraint 15 limits the flow of a certain OD pair to the value of its demand).

$$\sum_{f \in \theta} y_{rf} = x_r \quad \forall \quad r \in R \tag{18}$$

$$\sum_{f \in \theta} \upsilon^{+}_{nrfk} = \upsilon^{+}_{nrk} \quad \forall \quad r \in R,\ n \in N,\ k \in K \tag{19}$$

$$\sum_{f \in \theta} \upsilon'_{nr_1r_2fk} = \upsilon'_{nr_1r_2k} \quad \forall \quad (r_1,r_2) \in R,\ n \in (N_{r_1} \cap N_{r_2}),\ f \in \theta,\ k \in K : r_1 \neq r_2 \tag{20}$$

Constraints 18 to 20 impose that only one frequency is associated with the frequency-dependent variables at each time, in order to ensure the compatibility between the different variables and decision variables. For example, constraint 18 ensures that if a route is part of the solution, then it will have a single frequency associated, and only one.

$$z_{n_1n_2r} = x_r \quad \forall \quad r \in R,\ (n_1,n_2) \in A_r \tag{21}$$

$$z_{n_1n_2r} \leq \lambda_{n_1n_2r} + M \sum_{k \in K} \upsilon_{n_1n_2rk} \quad \forall \quad r \in R,\ (n_1,n_2) \in N_r \tag{22}$$

Constraints 21 and 22 guarantee that if a bus is empty, then a penalization will enter in the objective function. If a route is active (i.e. it is part of the solution) and the flow between n_1 and n_2 is zero, then λ must be equal to 1, being considered in the objective function.

$$\upsilon_{n_1n_2rk} \geq 0 \quad \forall \quad r \in R,\ (n_1,n_2) \in N,\ k \in K \tag{23}$$

$$\upsilon^{+}_{nrk} \geq 0 \quad \forall \quad r \in R,\ n \in N,\ k \in K \tag{24}$$

$$\upsilon^{+}_{nrfk} \geq 0 \quad \forall \quad r \in R,\ n \in N,\ f \in \theta,\ k \in K \tag{25}$$

$$\upsilon^{-}_{nrk} \geq 0 \quad \forall \quad r \in R,\, n \in N,\, k \in K \tag{26}$$

$$\upsilon^{'}_{nr_1r_2k} \geq 0 \quad \forall \quad (r_1,r_2) \in R,\, n \in R,\, k \in K \tag{27}$$

$$\begin{gathered} \upsilon^{'}_{nr_1r_2fk} \geq 0 \\ \forall \quad (r_1,r_2) \in R,\, n \in N,\, f \in \theta,\, k \in K \end{gathered} \tag{28}$$

$$w_{nk} \geq 0 \quad \forall \quad n \in N,\, k \in K \tag{29}$$

$$x_r \in \{0,1\} \quad \forall \quad r \in R \tag{30}$$

$$z_{n_1n_2r} \in \{0,1\} \quad \forall \quad r \in R,\, (n_1,n_2) \in N \tag{31}$$

$$y_{rf} \in \{0,1\} \quad \forall \quad r \in R,\, f \in \theta \tag{32}$$

$$\lambda_{n_1n_2r} \in \{0,1\} \quad \forall \quad r \in R,\, (n_1,n_2) \in N \tag{33}$$

Constraints 23 to 33 ensure that the variables take values that are coherent with the formulation presented, namely that they are positive values, and in some cases can only take a value of 0 or 1, according to the previous definitions.

4. Application to the Barcelona Case Study

Following the example of several cities around the world, the mobility sector of AMB decided to introduce a BRT route between the major cities of the metropolitan area in order to improve the existing network.

The case study here presented is located on the Metropolitan Area of Barcelona, with special focus on the cities of Castelldefels, Gavá, Viladecans, Sant Boi de Llobregat, Cornellà de Llobregat, El Prat de Llobregat, L'Hospitalet de Llobregat and Barcelona.

Therefore, eight existing routes are included in the initial analysis, namely the routes L80, L81, L82, L85, L86, L94, L96, and L97 of AMB. These routes were chosen to simplify the analysis (as an analysis of the whole network would be computationally intractable) and because they are the most affected by the introduction of the BRT route, since parts of these routes are overlapped with the BRT route.

The introduction of the BRT line will allow for a more comfortable and faster trip, especially between the largest cities. In Figure 3, the reader may find the representation of existing routes on the map of Barcelona Area, as well as the proposed location for the BRT route, M8 (in red). Table 1 helps the interpretation of Figure 3 by presenting the cities of the two extremes of each route. For example, route L97 connects the cities of Castelldefels and Barcelona, as represented by the dark blue. It can be seen that the BRT route overlaps with several of the existing lines considered in this case study.

Figure 3: BRT introduction on the current network.

Table 1: Cities connected by the current network.

Route	City 1	City 2	Frequency [bus/min]
L80	Gavà	Barcelona	1/45
L81	Gavà	Barcelona	1/15
L82	Gavà	L'Hospitalet de Llobregat	1/20
L85	Gavà	L'Hospitalet de Llobregat	1/20
L86	Viladecans	Barcelona	1/15
L94	Castelldefels	Barcelona	1/30
L96	Castelldefels	Sant Boi de Llobregat	1/12
L97	Castelldefels	Barcelona	1/15

4.1. Data Preparation

4.1.1 Transforming the current network into a graph

In order to translate the current network design to a graph several steps were taken, and a simplification of the case study was made.

First, a reduction of nodes was performed using expert judgment from AMB technicians and considering key elements of the urban environment. All city centers were considered a node, followed by key locations, such as the proximity to shopping centers, hospitals, train stations, etc. Adding to this, more nodes were placed along the existing routes (considering, for example, the extreme of each route), the BRT route, so that each one could have a good coverage with nodes and a good distribution was achieved. A reduction to only 26 nodes was obtained. However, there was the need to had 3 more nodes to account for the new routes proposed by expert judgment.

By doing this, the number of points to be considered was reduced from more than 500 stops to 29 nodes. This final graph representation was verified with AMB technicians. Figure 4 presents the nodes used to describe the current network, along with their location on the map.

With all the nodes identified, the following step was to attribute each bus stop from the current network to a node of the graph. The criteria for this assignment

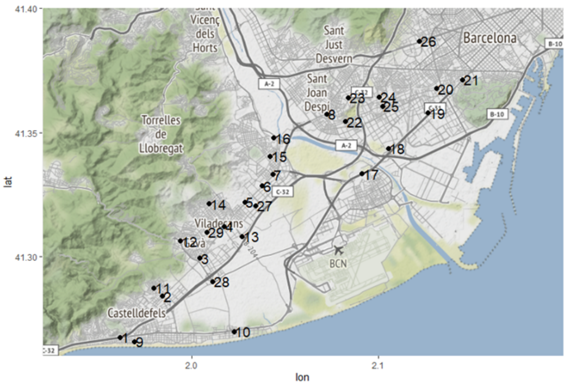

Figure 4: Nodes representation.

was the proximity between the stop and the node so that each node could aggregate the closest stops. By doing this, it was possible to define the current routes with the new nodes created by this simplification, as presented in Table 2. For instance, route L80 goes from node 12 to node 21 passing through nodes 3, 4, 13, 17, 18, and 19.

Table 2: current routes transformed into a nodes and arcs representation.

Route	Nodes representation
L80	12-3-4-13-17-18-19-21
L81	12-3-4-5-6-7-17-18-19-21
L82	12-3-4-5-6-7-8-22-25-20
L85	12-3-4-5-6-7-8-23-24-20
L86	14-4-5-6-7-17-18-19-21
L94	1-9-10-17-18-19-21
L96	16-15-7-6-5-4-3-2-11-9-1
L97	1-2-11-3-4-5-26

4.1.2 Cost Determination

With all the nodes and arcs defined, it was necessary to associate a cost to each connection. In this case, the cost is simply represented by the duration of the trip between two nodes, this is, the on board travel time of the arc. This time was obtained for each arc being considered in the case, by using the tool Google Maps which allows seeing the duration of the trip between two points.

4.1.3 Transfers

Transfers refer to the change of route by a user, in a certain node, which is common to both routes. In this model, transfers were only allowed to happen in certain nodes of the graph, namely in nodes 1 to 9 and 17.

Nodes 2 to 8 were chosen as they are part of the BRT route, and thus, it is expected that users will use them to connect between cities. Nodes 1 and 9 were also considered due to their strategic position that allows for the connection between some routes that do not have connections otherwise. The same happened with the selection of node 17, allowing for better flexibility between the routes. Transfers will penalize the time the user spends in the network, so it is not totally desirable that they happen, but at the same time, they allow for a more flexible network, with a better area coverage.

4.1.4 Definition of the OD Demand Matrix

Although the OD demand matrix is defined as an input in the mathematical model, it was necessary to obtain an estimation for it here.

This was made using both the data available from AMB and an estimation based on a trip generation/attraction model for each node. For this last estimation, a classification between 1 and 3 was attributed to each node, for both its attraction and generation, which was verified by the AMB technicians. On the classification scale, a score of 1 means a low generation/attraction, and a score of 3 is a high generation/attraction. A low generation means that in that node it is not expected to enter many users in the network, while a high generation means that the corresponding node is expected to have a high number of users entering the network. The same analogy can be made for the attraction scores, but this time it evaluates the receptivity of users in the node, meaning the number of users that leave the network in this node. This classification was made considering expert knowledge, the data given by AMB, and considering the different modes of transportation that exist in the node's area/zone.

Then, for each OD pair, the score related to the generation of the origin node and the score related to the attraction of the destination node were computed, and a weight was assigned to the OD pair. This, combined with real data provided by AMB, both regarding the demand of each route and the period of operation of the network (from 5 a.m. to 11 p.m. hence an 18h period), allowed the first estimation of the OD demand matrix.

Afterwards, the OD demand matrix was simplified, and several OD pairs were considered to have zero-demand, following several criteria: when there is another mode of transportation that makes the connection between the nodes of the OD pair more efficiently or when the OD pair was not connected directly or with only one transfer.

With these assumptions, it was possible to reach an OD demand matrix that satisfies all the conditions explained above, allowing for a more accurate comparison between cases, as it was used to run the model for all the scenarios, which will be presented in section 4.2.

4.2. Case Study Scenarios

Different scenarios were explored to study the impact of the BRT route on the network. These scenarios were differentiated by the pool of routes from which the model could choose the optimal solution. Three scenarios were considered: Current routes, Current routes + BRT, Current routes + BRT + Expert Knowledge routes. These are presented next, along with the specific characteristics of each case. The results expected for each case include the set of routes that are part of the solution of the optimization problem considered, along with the frequencies at which each route should be operated.

4.2.1 Current Network

In the first scenario studied, the set of routes considered by the algorithm involved only the routes of the current network, the ones presented in Figure 4.1.

For the first case (case 4.0), the real frequencies at which the routes are operating, presented in table 1, were imposed, serving this case as the baseline for the rest of the analysis.

In the following case (case 4.0.1), the operational frequency was left without any value assigned, so that it would be possible to conclude if the current network and frequencies were optimized. The set of frequencies that the routes could take included the values 1/12, 1/15, 1/20, 1/30, and 1/45 buses per minute.

Regarding the fleet size constraint, a total of 68 buses (with normal capacity, this is 50-65 people) were considered in this analysis. The same number was used in the following cases too.

4.2.2 BRT Introduction

In this scenario, the set of routes considered includes the current ones and the new BRT route. This last has an imposed frequency of operation of 1 bus every 10 minutes, a value that remained fixed in every case, and was established in a constraint.

Considering it, two cases were identified: one where the current routes had also their operational frequencies imposed (case 4.1.0) and other where these frequencies were considered as variables, and the model could find the best solution (case 4.1.1). The set of frequencies that the routes could take included the values 1/10, 1/12, 1/15, 1/20, 1/30, and 1/45 buses per minute.

Since the BRT route is operated by buses of a different typology, there was the need to include two new constraints in the model: one related to the fleet size, since a limited number of BRT buses will be available (in this case, 11), and another one considering the different bus capacity of this type of vehicle (100 people).

4.2.3 Expert Based Routes

The third scenario studied considered the current routes, the BRT route, and new routes suggested by experts. The new routes were obtained by asking experts on mobility in Barcelona what modifications they would like to propose to the existing network so that a more accurate and realistic response could be considered, that would serve better the community of this area.

The modifications proposed affect 3 routes: L82, L85, and L96.

In the current network, routes L82 and L85 already have similar paths, connecting Gavà to L'Hospitalet de Llobregat. As can be seen by Table 2, only the last nodes of these routes are different. The BRT route will also cover this part of the routes that are already overlapping. Then, the alteration proposed by the experts was to change the path of L82 at the end of Gavà, introducing a new section connecting Gavà and ViladeCans, in a more direct way, while changing the middle of the route so that it does not overlap with the BRT (node 27). For the first alteration, two possibilities were presented: one that would go more directly to Gavà (Route A: 12-29-4-27-7-8-22-25-20) and other that would link to the center of Viladecans (Route B: 12-14-4-27-7-8-22-25-20). At the same time, the route L85 would be reduced to connect the end of the BRT route to the city of L'Hospitalet de Llobregat, as before (L85 new: 8-23-24-20).

Regarding route L96, most of its trajectory will be overlapped by the BRT route, so two alternatives were proposed. The first one implies that the connection between Castelldefels and Sant Boi de Llobregat is assured directly, but having a different path from the BRT, covering another area that was not covered before, represented by node 28 (Route C: 1-9-11-2-28-13-5-6-7-15-16). The second alternative is to have route L96 divided into two different routes, each one at its extreme. (Route D - 1: 1-9-2-11 and Route D - 2: 13-5-6-7-15-16)

This scenario was then divided into several cases, in order to evaluate the impact of these alternatives in the network. Case 4.2.1 considers the pool of routes that include all the available routes described before, with no constraints regarding redundancy. In case 4.2.2, these constraints were added, to make sure that only 1 of the alternatives of each route would be part of the solution. In both cases, the frequencies were left unknown, and the set of frequencies that the routes could take included the values 1/10, 1/12, 1/15, 1/20, 1/30, and 1/45. These values are expressed in bus per minute.

4.3. Discussion of Results

Having presented the different scenarios, an analysis of the results obtained is made.

Before proceeding to the presentation and analysis of the results, it is important to introduce another concept. The unit of the objective function value obtained for each case is passenger.minute/minute of operation. In order to turn it into a more comprehensible value, it was divided by the total demand of the network, i.e., the sum of all b_k of the OD demand matrix (with a value of 41.35 passenger/minute). By doing this, one can obtain the time spent inside the network by an average user, making it easier to compare and understand better the average reductions of time that are being achieved with this optimization.

Case 4.0 was used as the baseline since it represents the current network. It obtained a value of 2306.51 for the objective function meaning that the average user will spend 55.8 minutes on the network. Then, case 4.0.1 was run having the frequency as a variable and not an input. It was concluded that the frequencies of the routes differ from the baseline case, showing that an optimization could be made to the network, even without the introduction of the BRT route, reducing the time the average user spends in the network to 51.6 minutes.

With the introduction of the BRT route, the two constraints explained before were added, and the pool of routes was changed.

Having the routes and respective frequencies as variables (excluding the obligation of having the BRT route with a frequency of 1 bus every 10 minutes as part of the solution), the optimized solution obtained includes routes BRT, L80, L82, L85, L86, L94, L96, and L97, with the respective frequencies 1/10, 1/15, 1/12, 1/15, 1/12, 1/20, 1/10, and 1/20. This solution allows for a reduction of 7.95 minutes in the time that an average user spends inside the network when compared to the baseline case, becoming 47.8 minutes.

For the last scenario, the pool of routes used included the current routes, the BRT route, and the alternative routes presented before. In case 4.2.0, a reduction of 8.8 minutes was reached, obtaining a time of 47.0 minutes spent in the network by an average user.

Although this solution represents a significant reduction in the time spent in the network, it did in fact include the three alternatives given for the L96 route, which would not be practical in a real-life situation, as identified by the AMB experts, given the redundancy it implies.

In order to overcome this problem, a restriction was added to the model, where only one route could be chosen between the current route and its alternatives. Case 4.2.1 was then considered. By doing this, an optimal solution was reached, including routes L80, L82, L86, L94, L96, L97, BRT, and the alternative proposed for the L85, at frequencies 1/10, 1/15, 1/10, 1/10, 1/20, 1/10, 1/15, and 1/15, respectively. A reduction of 8.4 minutes for each average user is achieved, a lower value than the case 4.2.0, as expected.

Table 3: Main results of the three scenarios presented.

Case	Objective function value	Time [min]
4.0 - Current routes	2306.51	55.8
4.0.1 - Current routes	2133.7	51.6
4.1.1 - Current routes + BRT	1977.45	47.8
4.2.0 - Current routes + BRT + Expert based routes	1945.02	47.0
4.2.1 - Current routes + BRT + Expert based routes	1959.13	47.4

Table 3 presents the main results obtained for each case. It is worth noting that a reduction of 15% in the time an average user spends in the network is achieved for the best (and realistic) solution. On the operator's side, it would imply the elimination of route L81, the addition of the BRT route, as imposed, and also the modification of route L85, which will be shortened and will guarantee the connection between the end of the BRT and the city of L'Hospitalet de Llobregat. The graphical representation of this optimal solution can be found in Figure 5.

Figure 5: Representation of the new network, including the BRT and the new L85 routes.

5. Conclusion

This work studied the redesign of an existing bus network due to the implementation of a BRT route, with the objective of minimizing the time spent by an average user spends in the network. In order to solve this optimization problem, an adaptation of the mathematical formulation of [14] was made and then applied to the case study of the Barcelona Metropolitan Area.

A third term was added to the objective function of the optimization problem in order to penalize the paths between nodes in which the buses travel empty, as this may be perceived as poor planning in the user's view. Furthermore, two constraints were added to the base formulation to account for the typology of vehicles used in the network, namely the number of available vehicles (fleet size) and their capacity. Since the buses operating the BRT route are different from the ones operating the rest of the network, two different types of buses were included in the analysis, for the same network.

Moreover, a step-by-step method to simplify a complex network into a graph that was used as an input for the model is presented. In the present case, this method relied on expert judgment, to simplify hundreds of bus stops into twenty-nine nodes, which allowed for a faster, but yet accurate analysis.

Finally, regarding the main goal of this work, it was possible to obtain a reduction of 15% in the travel time of an average user of the network. This is of extreme importance in the dynamic of a city since people can then apply this time in a more productive way, both in leisure or working time. The needed modifications in the network include the elimination of one route (L81), the shortening of route L85, in order to connect the end of the BRT route with the city of L'Hospitalet de Llobregat, and the introduction of the BRT route. Also, some frequency changes should be applied to the remaining routes. Thus, one can conclude that with simple modifications an optimization of the network can be achieved.

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