Planning Bus Rapid Transit services:
A case study in the Barcelona Metropolitan Area

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Declaração

Declaro que o presente documento é um trabalho original da minha autoria e que cumpre todos os requisitos do Código de Conduta e Boas Práticas da Universidade de Lisboa.

Aos meus pais,
por todo o amor.

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Abstract

The current growth of the urban population in cities is leading to a mobility problem. The increase of private cars in the city centers is becoming unsustainable and a change on the efficiency and diversity of public transportation is needed. The buses are now seen as a viable and cost-effective alternative to this problem, namely the Bus Rapid Transit systems. Several methods are being studied and developed in order to cover as much area and demand as possible with the resources available. This work presents a mathematical programming model to design a bus network. The objective is to minimize the time an average user spends inside the network, by achieving a set of routes, with associated frequencies, that satisfy the demand predicted for each origin-destination pair. It is then applied to a case study in the Barcelona Metropolitan Area, where the impact of the implementation of a Bus Rapid Transit route in the existing network is studied. The results presented serve as a decision-making tool to improve the network, by returning the set of routes and associated frequencies that minimize the given objective function. A decrease of 8.75 minutes in the time spent by an average user inside the network is achieved, representing a reduction of 15% when comparing to the current solution, and confirming the existence of room for improvement.

Keywords: Transport Network Design Problem; Bus Network Design; Bus Rapid Transit; Frequency Setting; Public Transportation

Resumo

O atual crescimento da população urbana está a conduzir a um problema de mobilidade. O aumento dos automovéis particulares nos centros das cidades está a tornar-se insustentável e é necessária uma mudança na eficiência e diversidade dos transportes públicos. Os autocarros são agora vistos como uma alternativa viável e rentável para este problema, nomeadamente os sistemas de *Bus Rapid Transit*. Vários métodos estão a ser estudados e desenvolvidos de modo a cobrir a maior área e procura possíveis com os recursos disponíveis. Este trabalho apresenta um modelo de programação matemática para o desenho de uma rede de autocarros. O objectivo é minimizar o tempo que um utilizador médio passa dentro da rede, alcançando um conjunto de rotas, com frequências associadas, que satisfaçam a procura prevista para cada par Origem-Destino. Posteriormente, o modelo é aplicado a um caso de estudo na Área Metropolitana de Barcelona, onde é estudado o impacto da implementação de uma rota de *Bus Rapid Transit* na rede atual. Os resultados apresentados servem como instrumento de tomada de decisão para melhorar a rede, sendo apresentados o conjunto de rotas e frequências associadas que minimizam a função objectivo dada. Obtém-se uma diminuição de 8.75 minutos no tempo gasto por um utilizador médio dentro da rede, o que representa uma redução de 15% quando comparado com o caso atual, mostrando que existe espaço para melhorias da rede.

Palavras-chave: Problema de Desenho da Rede de Transporte; Desenho de Rede de Autocarros; Autocarros de Trânsito Rápido; Definição de Frequências de Operação; Transportes Públicos

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List of Acronyms

AMB Barcelona Metropolitan Area

BRT Bus Rapid Transit

BTNDP Bus Transit Network Design Problem

EC European Comission

EU European Union

ITF International Transport Forum

LP Linear Programming

LRT Light Rail Transit

MILP Mixed-Integer Linear Programming

MMNDP MultiModal Network Design Problem

OD origin-destination

PSO Public Service Obligations

SDG Sustainable Development Goals

TNDP Transit Network Design Problem

UTNDP Urban Transportation Network Design Problem

Chapter 1

Introduction

This first chapter introduces the topic of transport network design, and the transportation sector's goal on achieving more sustainable mobility systems. These goals motivate the study of transit networks' design, in particular bus networks. An review on the airline network design is also provided. Afterwards, the purpose and objectives of this work are described, followed by a summary and an overview of the structure of this dissertation.

1.1 Motivation

The growth of the population living in urban areas, which is expected to reach 70% of the world's population by 2050 [1], together with the increasing concern about climate change, led governments and communities to understand that a change in urban mobility and in how people move inside and between cities is needed. The last 70 years of urban and mobility development led to unsustainable habits of mobility [1], for example, the rise in the number of private cars created an increase in traffic congestion and air and noise pollution, which are now more evidently revealing their consequences.

The Sustainable Development Goals (SDG), released by the United Nations General Assembly in 2015 [2], are a set of 17 global goals to reach a more sustainable and fairer world, in the areas present in Figure 1.1. The transportation sector contributes, on the one hand, directly to several targets, namely road safety, energy efficiency, sustainable infrastructure, urban access, and fossil fuel subsidies. On the other hand, it contributes indirectly to others, e.g. air pollution, sustainable cities, and climate change mitigation [3].

To respond and comply with these goals, in 2019, the European Comission (EC) released the European Green Deal, "a new growth strategy that aims to transform the European Union (EU) into a fair and prosperous society, with a modern, resource-efficient and competitive economy where there are no net emissions of greenhouse gases in 2050 and where economic growth is decoupled from resource use" [4]. One of the goals expressed in this strategy is the desire to achieve climate neutrality. Regarding the transportation sector, the aim is to achieve a 90% reduction of greenhouse gas emissions by 2050 and to further increase the efficiency of the transport systems by strengthening the multimodal transport, along with the digitalization of these systems, the production and deployment of sustainable alternative transport fuels and the revision of the price of transports to reflect the environmental and health impacts.

Figure 1.1: The Sustainable Development Goals [5].

In line with the European Green Deal, the Sustainable and Smart Mobility Strategy [6], released in 2020, reinforces the need and the goal for a more digital and greener EU transport system, leading to a more sustainable, smart, and resilient mobility [4]. In this strategy, specific metrics and milestones are described. By 2030, at least 100 European cities should be climate neutral, and scheduled collective travel of under 500 km should be carbon neutral within the EU. The following goal is, by 2035, to have zero-emission aircraft available in the market. Finally, by 2050, all cars, buses, vans, and heavy-duty vehicles should be zero-emission.

All these strategies suggest that the transport sector needs to go through a decarbonization process, along with the investment in sustainable technologies. Regarding decarbonization, both an increase in energy efficiency and a reduction in the number of private cars circulating need to happen. These can be achieved by providing better conditions for people to choose public transport, walking, or cycling as alternatives. As stated by the European Environmental Agency, buses are two times more energy-efficient than cars [1]. Therefore, along with a general improvement in the public transport, the bus networks should be reviewed and improved to accommodate the population's necessities, their new behaviors, and the needed decrease in emissions.

In order to do this, factors influencing bus networks' efficiency need to be studied, to achieve a more efficient and effective solution for each city/place. For instance, changes in the frequencies of the different routes that compose the bus network, adapted to the passenger demand, the creation or elimination of routes, the vehicle capacity for each route, or the transition to low carbon emission vehicles are some of the possible modifications that can be made. These aim to eliminate inefficient bus operations, and minimize the number of vehicles needed in operation, thus, decreasing the emissions produced and making the network more attractive to users that normally would travel on more polluting modes, such as private cars [7].

1.2 Bus Rapid Transit

The Bus Rapid Transit (BRT) systems are a possible solution that covers a lot of the topics discussed. According to the International Transport Forum, “a bus rapid transit system is a bus system with high speed, capacity, punctuality, and operating flexibility” [8]. Figure 1.2 shows an existing BRT system, operating in Istanbul. Usually, the buses of these systems will operate in exclusive bus lanes, eliminating one of the sources of delays in this type of transportation: the mixed traffic. This, combined with a higher frequency of these buses and better traffic signal management, results in a reduction in the travel and waiting time spent by the users, increasing the service’s reliability and the satisfaction of the users. Another advantage of this type of system, when compared for example with the Light Rail Transit (LRT), is the fact that it uses infrastructure that is normally already built, which reduces considerably the cost of implementation of such system, being typically 20 times cheaper than LRT and 10 up to 100 times cheaper than a metro system. Its deployment can also be quicker than other means of transportation. Moreover, as the BRT buses are usually more recent, with higher capacity and technologically more advanced, a decrease in the emissions is also expected.

Figure 1.2: Example of a BRT system (Istanbul) [9].

There has been a growth in the number of BRT systems implemented in the last decades: from 1992 to 2002, it was implemented in only 20 cities, contrasting with the 145 cities that implemented it from 2003 until 2020 [10]. One of the biggest problems of implementing these systems (and in general in implementing changes that involve society) is often the resistance to change that transport policymakers, planners, and governments face from the community. However, the COVID-19 pandemic brought a “window of opportunity” to implement such changes in urban mobility [11]. In fact, as a consequence of the several measures that were implemented in most countries, that led to habit changes, there is now an opportunity for a new adaptation of the transport sector, as new routines are arising and people are more conscious and open to changes that may benefit them and the environment. Besides that, digitalisation allows for a better understanding of the mobility behavior of the population (for example, the access to the data from the electronic cards required to travel), leading to a possible adaptation of the networks to better fit the necessities of the community.

This work also resulted from a challenge made by the Mobility Sector of the Barcelona Metropolitan Area (AMB), where a BRT route will be implemented. By providing data regarding the use of their current network, allowing for a better and more complete analysis, the objective is to provide inputs for the decision-makers of this entity regarding network modifications that may come from the introduction of a BRT system in this area. This case study is further explored in Chapter 4. According to [12], in the case of Barcelona, the adjustment of the network structure, along with the adaptation of routes and timetables, could contribute to a significant decrease in the carbon emissions (up to 50%), with little change in user costs or average travel times. Moreover, regarding the effect of the COVID-19 pandemic on the acceptance of changes in urban mobility, a study on the Spain territory found that 75% of the inquiries were receptive to restrictions on the use of private cars when returning to normality, and more than 90% agreed on the increase of space for cyclists and pedestrians on streets [13]. In general, Spanish people agree with the idea of a new urban hierarchy that privileges more sustainable transport modes and reduces the space available for cars. There was no distinction in the zones of Spain, but the conclusions of this study can also be applied to the case study presented in this dissertation. Hence, people in Barcelona are expected to be open to the changes that may result from this work.

1.3 Airlines Network Design

Although the scope of this thesis is to study the impact of the BRT route on the existing bus network, network design is also crucial in airline and air transportation systems. In fact, two main strategies are normally used by airline companies: i) the point-to-point network design, and ii) the hub-and-spoke network design. A detailed comparison between these two methods can be found in [14].

The point-to-point methodology consists of connecting an origin and a destination directly, without stops. This is the method used by most of the low-cost airline companies, such as Ryanair, Wizz, etc. In contrast, the hub-spoke methodology focuses on having some principal airports, named hubs, from where the main activity of the company starts or ends. Then, the other airports will be connected to this hub and the whole network is served with direct flights and transfers (that happen mainly in these hubs). A review on the different variations of this system can be found in [15]. This strategy needs fewer routes to satisfy the demand, but it will include more transfers, leading to an increase in the time the passenger spends in the network, along with the increase in the logistics needed, for example, regarding the transfer of luggage from one flight to the other, etc. The point-to-point methodology will need more routes to serve all the nodes with direct connections but will have fewer transfers. Figure 1.3 shows schematically the difference between the two methods.

Different characteristics of the network may influence the systems' choice, as studied in [16]. The distance between cities, the number of cities or the demand for each route are some of the aspects that should be taken into consideration.

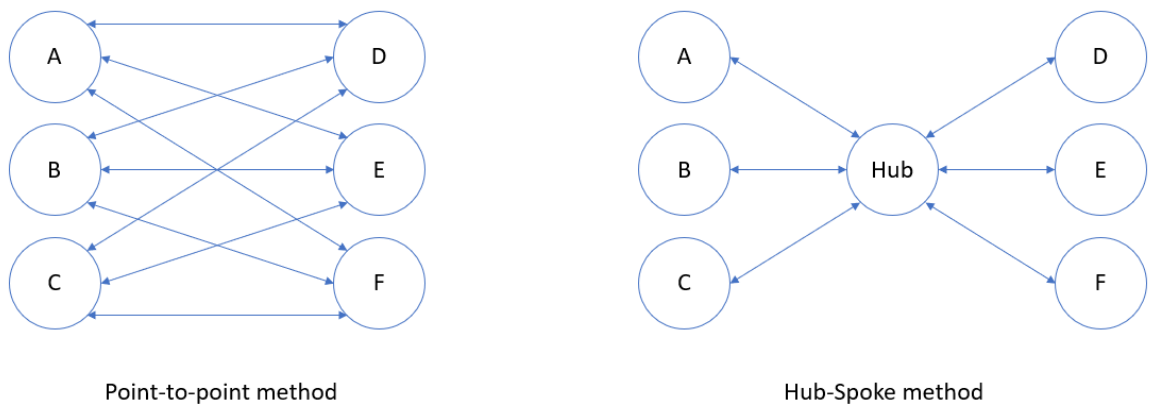

Figure 1.3: Point-to-point and Hub-spoke methodologies representation.

At the same time, the design of the network itself is influenced by several aspects, such as the objectives of the airline company: minimize the time the average user spends on the network (consider travel time, waiting time, and delays) or maximize the profit of the operator, by reducing the needed fleet size or its operational costs. As it is expected, this sector has tight profit margins due to its complexity, leading to a high need of optimization regarding the reduction of costs and the maximization of the use of the available resources. The ideal network is the balance between these objectives. The design of this kind of network is of extreme complexity and is a topic of research in several areas, namely economics, operations, and transportation systems. Moreover, as in other transportation systems, there has been an increasing concern about the reduction of emissions, and so efficient use of the resources is of extreme importance.

One of the major challenges when designing transport networks, especially in the aviation sector, is demand prediction. In order to obtain the most optimized network, it is important to have a good estimate of how many users will utilize it, so that the service provided can be in accordance to the expectations and needs of the community. The route design, the frequency setting and even the fleet assignment (including the type of vehicles that should be allocated to each route) are some of the steps of network design that can be better optimized if a good demand prediction is in place. These issues are studied in [17], [18] and [19].

Due to its relevance, the demand prediction has been widely addressed by several authors. In [20], the demand was studied as a function of the provided service and price of each route. Later, [21] added also the effect of aircraft size, including the seat availability, and the frequency of the service as factors that also impact the predicted demand in air transportation systems. [22] proposes a strategy that accounts for user behaviour in demand prediction, concluding that the available supply may influence the user's destination choice, then contributing for the so-called induced demand. More recently, [23] expanded this problem in order to estimate the demand as a function of the frequency setting and the network design itself, making it a more iterative process.

Nevertheless, in this specific sector, there are special cases where the objective is not to have the maximum profit possible or to serve the highest demand, but to provide a service to communities that do not fulfill the requirements of demand. This kind of networks are normally found in remote areas or specially in islands. In Europe, there is the European Union's Public Service Obligations (PSO) system, whose objective is to support airline companies to maintain routes that are critical to the economy of the areas they serve, but that are not commercially interesting. According to [24], the objective in these cases will be to minimize the costs for the company and the social costs as a whole. This type of service was also explored in [25] and [26], applied to case studies in the regions of Azores and Norway, respectively.

As it can be inferred from above, the process of designing both bus and airline networks presents several similarities. Regarding urban mobility, it is a complex process of designing a multimodal network, that may involve different transportation systems (bus, train, subway, etc) and where there may exist more than one way of doing the trajectory between two points. This will lead to a higher competition between the different transportation modes. When discussing the aviation sector, the competition between airlines is more relevant. The limitations of fleet size, the space in airports, the slots of time to fly, and even the weather are all factors that will affect the design of these networks. More aspects are covered by research on network design since this is an area in rapidly expansion, that needs a fast and better solution for demand prediction and route optimization. However, despite the differences here presented, for both cases, the mathematical models used are very similar, based on nodes and arcs, that can also be applied to the design of other transit networks.

1.4 Objectives

The main objective of this work is to study the potential impacts of the introduction of a BRT route in a pre-existing bus network. By doing this, an assessment of possible modifications on the network is made, to make it more efficient and adequate for the population. This will be done by formulating a mathematical programming model (described in detail in Chapter 3) for transit network design. Then, the model will be applied to a specific case study, in the Barcelona Metropolitan Area, and the results obtained will be presented and discussed. Finally, this work's conclusions are expected to aid decision-making regarding bus network design, specifically if any changes need to be done, and where they should be implemented.

1.5 Thesis Outline

In this first chapter, the reader found an introduction to the transport network design and this thesis' motivation and objectives. An introduction to the airline network design is also provided.

In Chapter 2, an overview of the literature regarding the Transit Network Design Problem (TNDP) is conducted, focusing then on the specific case of bus networks. The target contribution of this thesis is also explored.

In Chapter 3, the methodology is explored and the formulation of the mathematical model is described in detail, namely the objective function, the constraints used, and the inputs needed. An illustrative example is also presented and explored.

Chapter 4 focuses on the AMB case study, providing an overview of its context, as well as the conditions to apply the model developed in the previous chapter. Furthermore, different scenarios are built and explored, with the consequent presentation and discussion of the results achieved.

Lastly, in Chapter 5, the conclusions of this work are drawn along with the main limitations faced. Additionally, possible directions for future work are discussed.

Chapter 2

State of the Art

In this second chapter, a literature review is conducted on the previous research on transit network design, especially on bus networks. The target contribution of the present dissertation is also discussed.

2.1 Network Design in Transportation Systems

According to [27], a Urban Transportation Network Design Problem (UTNDP) can be described as a succession of decisions that leads to the planning of transit networks. These decisions can be classified as strategical, tactical or operational. The strategic level comprehends long-term decisions, including, for example, a survey on the infrastructure that is needed for the network, new routes, roads, etc. At a tactical level, the utilization of resources and infrastructures of the network is studied, having as an example the setting of frequencies for the routes or how the roads available are used. Regarding the operational level, it concerns the short term decisions, combining demand management or vehicle scheduling.

All these different examples foreshadow the complexity of the problem, that usually involves minimizing or maximizing a set of objectives, while complying with specific constraints. The transit network design process includes steps such as network design, frequency setting, timetable development and crew and vehicle scheduling, as stated in [28]. The solutions studied to solve this kind of problem can be applied to every transportation mode, be it bus, train or even airplanes.

However, as it is a very complex problem, with multiple variables at stake, it is normally divided into smaller problems, according to the number of modes or even the steps of the design in study. As an example, when more than one mode is considered for the analysis, one is faced with a MultiModal Network Design Problem (MMNDP). This kind of problem is important to combine the different modes operating, for example, on a city, so that the design of the network can be thought in a way that the different modes complement each other, instead of overlapping each other. Though, it can also be used to make decisions about only one of the modes, but considering the interaction with others. Efforts on this topic can be found on [29], [30], [31].

Believing that a lot of simplifications are considered when studying UTNDP, [32] presented a model that considered a multi-level and multi-mode method that includes three levels of routes: skeleton network, arterial network and feeder network. This would allow for faster trips between cities that are far away, but at the same time design routes that ensure the urban transit. For each layer, a transportation mode would be associated based on the city's features and the infrastructure existent. The objective for every level was the same: decrease the travel time for an average user.

Considering now that only one mode is subject to analysis, a new class of problems arise, the TNDP. It commonly encompasses the different steps presented before (route design, frequency setting, timetable development, etc).

As discussed in [33], the complexity of this kind of problems arises from the multi-objective nature of TNDP, since several variables and objectives can be explored, depending on the case under study. Some objectives that are usually presented in the literature include the minimization of the time spent by the users in the network, maximization of the operators profit (by minimizing costs or the fleet size needed), limitations on the length of the routes, etc. As explored in section 1.3, the demand prediction is of extreme importance (and complexity) in every transportation mode. For the formulation of mathematical models to solve this problem, it can be considered to have a fixed value, as it does not change during the process, or variable values, called elastic demand, when it is dependent on the other factors, such as travel time, etc. The assumption of the best approximation for this topic depends most times on the objective of the study and the accuracy required.

Several reviews on the approaches that have already been developed to try to tackle the TNDP and the common issues that arise from it are documented in [28], [27], [34].

On [35], one can find an example of an analysis to a taxi network. The goal was to obtain a set of routes to be operated by this service, which has the particularity that a taxi can only leave when it is full. The objective of the mathematical model developed was to minimize the total travel time perceived by the users (including a penalty on the waiting time), while limiting the number and length of taxi lines and the transfer ratio (2 transfers allowed), so that the characteristic of having a high frequency service could be achieved. Then, the minimization of the fleet needed was also studied. It is seen that this model could be applied to any other transportation mode, such as bus or rails, only adapting the constraints applied, confirming the versatility of these models.

2.2 Network Design in Bus Networks

The Bus Transit Network Design Problem (BTNDP) is the class of problems that deals with the design of bus networks. Several approaches have been tried in the past decades regarding the optimization of bus networks. It involves all the steps considered for the TNDP, but normally, and as a simplification, only some of these steps are taken into account at each time.

For example, in [36], three main topics are covered, with regard to the route generation, the network analysis accounting with demand prediction and the frequency setting for the routes. It considers variable demand as an approximation to real-life cases, and the objective was to obtain the set of routes that optimize the network, according to the objective of minimizing the operator and users' cost and unsatisfied demand costs.

Cancela et al [37] presented a mathematical formulation for TNDP, that is used to solve a BTNDP. The objective was to obtain an optimal set of bus routes and respective operational frequencies that cover a given demand and comply with an infrastructure of roads and bus stops. They formulated several aspects regarding the interest and behaviour of the users, and the interest of the operators. Some possible additions to the model such as bus capacity and transfers constraints are explored but not included in their analysis.

In [38], a model was developed regarding route design and frequency setting, including both users and operator's costs. In this case, the fleet size was not an imposed value, but one of the variables they wanted to minimize, along with the time spent in the network. The network was represented by a graph, with nodes and arcs, similar to the approach presented in [37].

The concern with air pollution derived from vehicles is increasing in the transportation sector, as explored in Chapter 1. Adding to this, the emergence of new and more environmental friendly vehicles is leading to a period of transition in the transportation sector, that needs to be addressed in the bus network design. For this, the appearance of techniques/models that can comply with two or more types of vehicles in the same network is of extreme importance. In [39], this problem is explored. However, instead of having two types of vehicles in the same network, the existence of these two types of vehicles is tackled as two different networks (two sub-networks) that should interact between them (multimodal) and complement each other. The green vehicles are limited, and so a compromise between the two networks should happen.

Regarding the BRT systems, a lot of information can be found in the literature about how to choose the location for a new BRT line or how to implement it in a city [40]. There are several studies available with specific case studies on the introduction of BRT routes in cities [41], [42], [43].

In [44], it is presented a real case in Iran, where an existing network is improved. In this study, the authors took a different approach and considered two sub-networks, with faster buses, such as BRT systems, making the connection between the extremes of the city, while local buses connect the urban area of the city. They accounted for both the cost of the operators, with the purpose of minimizing the routes length, both for BRT and for the normal buses, and the users, with preference for the shortest paths, minimum transfers and higher use of BRT. The results allowed the authors to realize that, although an investment by the responsible authority is needed, an improvement in the current system should be in place.

In the case of [45], the objective was to maximize the area covered with BRT, while satisfying a budget constraint. They used a Mixed-Integer Linear Programming (MILP) model, whose inputs included the demand matrix, a set of routes (obtained by a route generation module with several constraints inside) and the budget that was available. There were no constraints on the capacity of the vehicles, but a maximum value for the length of the routes was imposed. The results included the set of routes that optimize the objective function. In the case study that they presented, the goal was to find the best routes for new BRT routes to be installed in Isfahan's metropolitan road network, but it did not include an analysis on the impact that these routes will have on the existing network.

More recently, in [46], one can find a methodology to implement a BRT route in a city, namely how to choose the bus stops, routes and associated frequencies. They took into account the time the user spends in the network, including the waiting time for the first board, the on-board travel time and the waiting time in transfers and also the buses' capacities (3 types of buses are considered). Different scenarios, with different sets of routes, where studied and compared. This methodology was then applied to the real case study of BRT system of TransMilenio, in Bogotá, and proved to be able to improve the existing BRT network.

2.3 Target Contribution

The contribution of this work relies on the study of the implementation of a BRT route in an existing network, by developing and applying a mathematical model that is capable of capturing the main characteristics of the network.

The results will support decision makers to understand what kind of changes need to take place, at strategical and tactical levels. They include a set of routes and respective operational frequencies that optimize the objective function in study, minimizing the travel time spent by an average user (on-board time + waiting time). Since the BRT route was already designed, the focus of the work is not to give recommendations on where to place it, but to study the impact of the route in the network, as it will be in superposition with part of the existing routes.

This model is inspired in [37], but several modifications were implemented, as it will be discussed in Chapters 3 and 4. The addition of a constraint to account for the capacity of the buses, i.e., a restriction on the number of people allowed to be on-board of the bus in each path, as well as the allowance of one transfer, are part of the modifications made. At the same time, since the buses used for the BRT route will have different specifications and availability when compared to the normal buses, constraints regarding this difference in the typology of the vehicles will be included. In this case, the operator's cost is not modeled in the objective function, since it was not a constraint imposed by the case study. Contrasting to other previous works, the pool of routes to consider in the model included only the routes belonging to the existing network and, later, routes suggested by experts, not having used route generation algorithms to obtain new routes due to the complexity of the problem.

Table 2.1 summarizes the main aspects of the previous works done in the area of transit network design, allowing for a comparison with the contribution of the present work.

Table 2.1: Summary of the mentioned previous works

	Objectives	Multimode	# typologies of vehicles	#transfers	Vehicle capacity
Fan and Machemehl, 2008	Minimize: Total travel time Costs of operation Unsatisfied demand costs	no	1	2	yes
Beltran et al., 2009	Minimize: Transit users' costs Car users' costs Operator's costs	yes	2	not limited	yes
MirHassani et al., 2014	Minimize: Total travel time Fleet size Routes length	no	2	3	yes
Cancela et al., 2015	Minimize: Total travel time	no	1	-	-
Kermanshahi et al., 2015	Maximize: Coverage area respecting budget constraints	no	1	not defined	no
Babaei et al., 2016	Minimize: Total travel time Fleet size	no	1	2	yes
Triviño and García, 2021	Minimize: Total travel time	no	3	3	yes
Ahern et al., 2022	Minimize: Total travel time Fleet size	no	1	2+	yes
Present Work	Minimize: Total travel time	no	2 (different fleet sizes and capacities)	1	yes

Chapter 3

Model Description

In this chapter, the mathematical programming model to design a bus network is formulated. The problem under analysis is explored and the assumptions, inputs, decision variables, constraints, and objective function used to completely define the MILP model are presented. In order to verify the model, an illustrative example is introduced at the end of the chapter, including its description and the results obtained. The model here described was implemented in the FICO Xpress commercial software (version 8.11.1), with pre-processed data using RStudio software. The commercial solver was run in a Windows 10 Pro operating system in a computer with an Inter(R) Core(TM) i7-7700K CPU@ 4.20 GHz and 16GB of RAM memory.

3.1 Model Formulation

The mathematical model developed in this work is an adaptation from a previous formulation presented in [37]. It is then applied to a case study, which will be explored in Chapter 4, and will accommodate its particularities.

Before explaining the model in more detail, it is important to introduce several concepts that will be used in the next sections. The first step is to simplify the network under analysis into a graph. The components of the real network, such as bus stops and roads, are transformed into nodes and arcs, respectively. The network graph is then composed of nodes and their connections. The nodes represent the stops, i.e., the place where one assumes that users can enter or leave the bus, either to join or leave the network or to make a transfer between two routes. The connection between two nodes is called an arc, and it is assumed that users will be traveling inside the network when they are in an arc. Each arc of the network will have a cost associated, representing the on board time a passenger spends to go through it.

Let a route be defined by an ordered set of nodes that describes a certain path in which passengers can travel without any transfer. Each route will have an associated frequency that indicates the number of buses per unit of time that serve that route. Figure 3.1 helps to understand better this notation. The points n_1 to n_5 represent nodes of the network. The lines between these nodes represent the arcs that

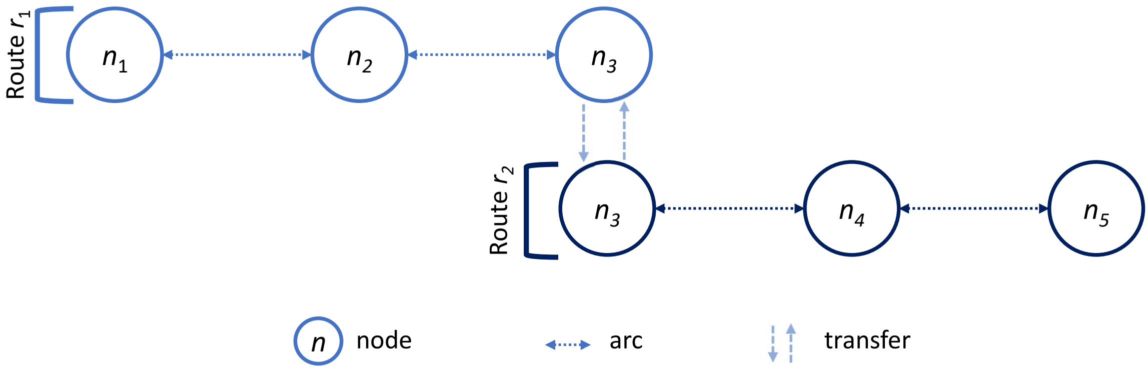

Figure 3.1: Representation of nodes and respective connection arcs.

connect them. In this example, nodes n_1, n_2 and n_3 form a route, as well as nodes n_3, n_4 and n_5. It would be possible to perform a transfer from route 1 to route 2 since node n_3 belongs to both routes.

An origin-destination (OD) pair represents the connection between any two nodes, that do not need to be consecutive nor to belong to the same route. To each OD pair is associated a value for its expected demand. This demand represents the number of trips that must be satisfied per unit of time, from the origin node to the destination node. The number of buses available to perform all the services needed is given by the fleet size. Moreover, the bus capacity represents the number of users that are allowed in each bus at each arc.

As mentioned before, the model that will be described below is based on the mathematical formulation presented in [37]. However, some modifications were applied. The formulation here developed is based on nodes and their relations, contrasting with the arcs-based formulation presented in [37]. This implies that, when referring to the travel arc between n_1 and n_2, it will be represented by (n_1,n_2) instead of a single variable a.

This thesis' model is an optimization problem that belongs to the class of MILP models. This class of mathematical programming models has mixed decision variables (i.e. continuous or discrete decision variables) that are included, with a linear correlation, in the objective function and constraints. For instance, there are binary decision variables that only take the value of one or zero. As an example, if a route is part of the solution to the problem, the correspondent decision variable will have a value of one, otherwise, it will be equal to zero. Besides that, the frequency assigned to each route will also be modeled through a binary variable. In summary, as the objective function and all constraints considered in this problem are linear, one is faced with a linear model.

As it is known, an optimization problem can be a minimization or a maximization problem, according to its objective. In this case, it is wanted to minimize of the objective function. Starting from a set of nodes, arcs, and routes (i.e. by defining the graph of the network), a certain demand of passengers for each OD pair needs to be covered, along with the minimization of the time an average user spends inside the network. Only one transfer is allowed for each connection between OD pairs. The expected solution for the problem will be the combination of routes and respective frequencies of operation that minimize the objective function, explained in subsection 3.1.3. As it would demand a lot of computational power to compute in a reasonable time all the possible routes between all the nodes considered and since this will be applied to a real case study, where constraints of infrastructure and capacity are imposed, the routes that the model can consider are limited and given as an input. Then, the optimal solution is explored in order to find the best set of routes that satisfy the conditions imposed and respect the constraints given.

In order to solve the optimization problem, the commercial software FICO Xpress was chosen. In this specific software, the MILP problems are solved by using a set of procedures, including branch-and-bound, cutting-plane methods, etc. The branch-and-bound method is used to find the optimal solution to an optimization problem, especially in a combinatory problem. Defining the search space as the admissible solutions to the problem, the method will calculate the first solution by Linear Programming (LP) relaxation. Then, for each node of the method, it will reach an upper and lower bound, which will serve as a comparison to exclude the solutions that are out of these bounds. The search space will be divided into smaller spaces (branching) with the goal of minimizing the objective function in each space. All values (both objective function and the bounds it should comply with, namely the best solution found until that point) are recorded and when the value of the objective function in one branch is higher than the value obtained in the bounds, then the operation in that branch is stopped as this space is no longer a candidate for obtaining the best solution. This process continues until a certain time is reached (given by the user) or if the optimal solution is found.

In the next subsections, the MILP model will be fully defined, namely by presenting:

- Constants - introduces the constants used;
- Sets - provides the sets considered in the model;
- Indexes - clarifies the notation used;
- Data Parameters - displays the variables that represent some graph specifications;
- Decision Variables - presents the variables that will be optimized;
- Objective Function - introduces the terms of the objective function;
- Constraints - presents the constraints used to limit the model and the solution space.

3.1.1 Inputs

Several inputs are introduced, namely: constants, sets, indexes and data parameters.

Constants

NN	Number of nodes
NA	Number of arcs
NR	Number of routes
NK	Number of OD pairs
NF	Number of frequencies
B	Fleet size
M	Large Number
Ω	Bus capacity

Sets

N	Set of nodes
N_r	Set of nodes of route r
N_r^T	Set of terminal nodes of route r
R	Set of routes
A_r	Set of arcs of route r
K	Set of OD pairs
Θ	Set of frequencies

The set of routes used as input to the model need to be given either by expert knowledge or generated by a separate model.

Indexes

n, n_1, n_2	Nodes
r, r_1, r_2	Routes
k	OD pair
f	Frequency

Data Parameters

$c_{n_1 n_2}$	Cost of travelling on arc linking nodes n_1 and n_2, i.e., the on board travel time
O_k	Origin node of OD pair k
D_k	Destination node of OD pair k
δ_k	Demand of OD pair k, i.e., the amount of trips associated with OD pair k
b_{n_k}	Equal to δ_k if n=O_k, -δ_k if n=D_k, and 0 otherwise
θ_f	Array of frequency values indexed by f
σ	Array of nodes where transfers can take place

3.1.2 Decision variables

As the objective of the model here presented is to optimize an objective function, there are variables whose values are left unknown so that they can be optimized. These decision variables are:

$\upsilon_{n_1n_2rk}$	Flow between nodes n_1 and n_2 of route r from OD pair k
υ^+_{nrk}	Incoming flow at origin node n from OD pair k of route r (n=O_k)
υ^+_{nrfk}	Incoming flow at origin node n from OD pair k of route r at frequency f (n=O_k)
υ^-_{nrk}	Outcoming flow at destination node n from OD pair k of route r (n=D_k)
$\upsilon^{'}_{nr_1r_2k}$	Transfer flow between routes r_1 and r_2 at node n from OD pair k
$\upsilon^{'}_{nr_1r_2fk}$	Transfer flow between routes r_1 and r_2 at node n from OD pair k and route r_2 operated at frequency f
w_{nk}	Waiting time multiplied by the demand of OD pair k at node n

Moreover, the following binary decision variables are considered:

$$x_r = \begin{cases} 1 & \text{if route } r \text{ is part of the solution,} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{rf} = \begin{cases} 1 & \text{if route } r \text{ is operated at frequency } f, \\ 0 & \text{otherwise} \end{cases}$$

$$z_{n_1n_2r} = \begin{cases} 1 & \text{if the arc } (n_1, n_2) \text{ of route } r \text{ is part of the solution,} \\ 0 & \text{otherwise} \end{cases}$$

$$\lambda_{n_1n_2r} = \begin{cases} 1 & \text{if an empty bus is operating between nodes } n_1 \text{ and } n_2 \text{ from route } r, \\ 0 & \text{otherwise.} \end{cases}$$

3.1.3 Objective Function

As presented before, the purpose of this model is to minimize an objective function, in this case, it represents the total time spent by the users on the network, modeling then the interest of the users.

$$Minimize : O = \sum_{k\in K} \left[\sum_{r\in R} \sum_{(n_1,n_2)\in A_r} c_{n_1n_2}\upsilon_{n_1n_2rk} \; + \sum_{n\in N} w_{nk} \right] \; + \sum_{r\in R} \sum_{(n_1,n_2)\in A_r} c_{n_1n_2}\lambda_{n_1n_2r} \quad (3.1)$$

The objective function, presented by equation 3.1, will consist of three parts, related to:

1. the on board travel time, obtained by multiplying the cost of traveling from node n_1 to node n_2 ($c_{n_1n_2}$) by the flow associated with that trip for a certain OD pair k ($\upsilon_{n_1n_2rk}$);
2. the waiting time, which accounts for the time the user will wait until entering the network and the

time spent between transfers, and is given by the decision variable w_{nk}. This value is obtained by multiplying the time the users will spend waiting at node n for buses serving their destination and the demand of the corresponding OD pair k. Although this value is not addressed directly in the model, it can be described as the inverse of the sum of the frequencies of the routes that serve that OD pair, i.e., the average time between buses serving the OD pair k at node n multiplied by the demand of the corresponding OD pair k.

3. a penalization term, that indicates the cost of travelling between two nodes, which will only be considered in arcs where buses travel empty, thus being represented by the multiplication of the cost of travelling between the two nodes, $c_{n_1n_2}$, and the binary decision variable that indicates if the bus is empty or not, $\lambda_{n_1n_2r}$.

Comparing this objective function to the one presented in [37], the difference remains in the third term added here. This term aims to incorporate in the objective function a problem that is normally faced in the day-to-day urban mobility: the operation of empty buses. These events are normally perceived by user's as bad planning on the side of the operator, and so they should be avoided. Thus, every time a passenger flow is zero between two nodes of any route, this term will add the cost of the arc to the value of the objective function, increasing its value. By penalizing this, it is expected that these connections are not considered for the optimal solution, and so a better use of the available resources is made.

3.1.4 Constraints

To restrict the solution space and simplify some of the variables, improving the computational time, several constraints were applied to the model.

$$\sum_{r\in R}\sum_{f\in\theta}\sum_{(n_1,n_2)\in A_r} 2\theta_f y_{rf} c_{n_1n_2} \leq B \tag{3.2}$$

Constraint 3.2 ensures that the maximum number of buses to operate the chosen routes at corresponding frequencies f does not exceed the fleet size available.

$$\sum_{k\in K} \upsilon_{n_1n_2rk} \leq \sum_{f\in\theta} y_{rf}\theta_f\Omega \quad \forall \quad r\in R, (n_1,n_2)\in A_r \tag{3.3}$$

Constraint 3.3 ensures that the capacity of each bus is not exceeded in any arc of a route, i.e., between two consecutive nodes of a route. The value of Ω is the bus capacity and it is given as an input. Although this constraint is presented in [37], it is not implemented in the model discussed by the authors. Thus, the introduction of this constraint in the model being developed in this thesis is a novelty.

$$\sum_{r\in R} \upsilon^{+}_{nrk} = b_{nk} \quad \forall \quad n\in N,\ k\in K : n = O_k \tag{3.4}$$

$$\sum_{r\in R} \upsilon^{-}_{nrk} = -b_{nk} \quad \forall \quad n\in N,\ k\in K : n = D_k \tag{3.5}$$

$$v^{+}_{nrk} = \sum_{n_2 \in N:(n,n_2)\in A_r} v_{nn_2rk} \quad \forall \quad n \in N_r,\ r \in R,\ k \in K : n = O_k \tag{3.6}$$

$$v^{-}_{nrk} = \sum_{n_1 \in N:(n1,n)\in A_r} v_{n_1nrk} \quad \forall \quad n \in N_r,\ r \in R,\ k \in K : n = D_k \tag{3.7}$$

$$\sum_{n_1 \in N:(n_1,n)\in A_r} v_{n_1nrk} + \sum_{r_1 \in R\setminus\{r\}:n\in N_{r_1}} v^{'}_{nr_1rk} = \sum_{n_2 \in N:(n,n_2)\in A_r} v_{nn_2rk} + \sum_{r_1 \in R\setminus\{r\}:n\in N_{r_2}} v^{'}_{nrr_2k}$$
$$\forall \quad r \in R,\ n \in N \setminus \{N^T_r, O_k, D_k\} \tag{3.8}$$

$$v_{n_1nrk} = \sum_{r_2 \in R\setminus\{r\}:n\in N_{r_2}} v^{'}_{nrr_2k} \quad \forall \quad r \in R,\ n \in N^T_r \setminus \{O_k, D_k\},\ n_1 \in N \setminus \{D_k\},$$
$$k \in K : (n_1, n) \in A_r \tag{3.9}$$

$$v_{nn_2rk} = \sum_{r_1 \in R\setminus\{r\}:n\in N_{r_1}} v^{'}_{nr_1rk} \quad \forall \quad r \in R,\ n \in N^T_r \setminus \{O_k, D_k\},\ n_2 \in N \setminus \{O_k\},$$
$$k \in K : (n, n_2) \in A_r \tag{3.10}$$

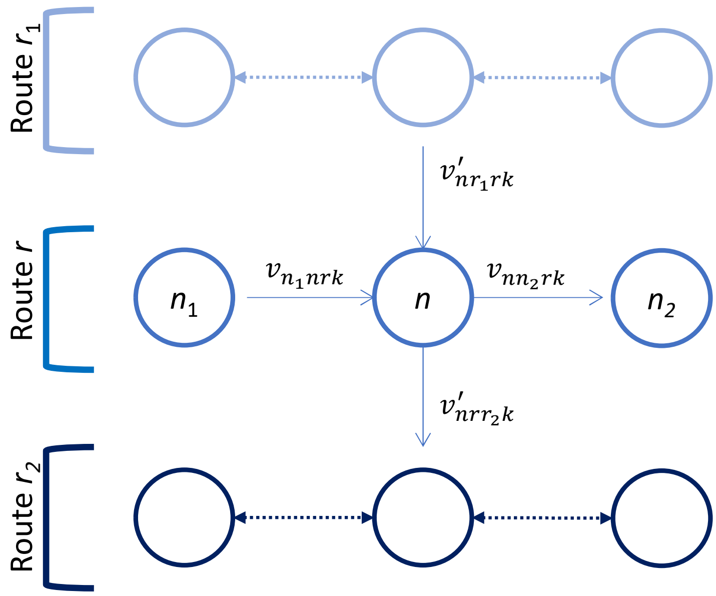

Figure 3.2: Incoming and outcoming flows of a generic node n.

Constraints 3.4 to 3.10 guarantee the continuity of flows in each node, making sure that the number of users entering the node corresponds to the number of users leaving it, either for transfers, for continuity of the trip or leaving the network. There were three specific cases identified where constraint 3.8 was not ensured, namely in the nodes that are the origin or destination of a certain OD pair k and the nodes that are terminal nodes of each route. For these three types of nodes, specific constraints were defined. For the origin nodes of the OD pair k, constraints 3.4 and 3.6 were defined, while for the destination nodes, constraints 3.5 and 3.7 were considered. Regarding the terminal nodes, and excluding the ones that are origin or destination nodes, constraints 3.9 and 3.10 ensure that the flow entering the node will be transferred to other routes that share that node. Figure 3.2 schematizes the different flows mentioned before.

$$\sum_{n_1 \in N:(n_1,n) \in A_r} \upsilon_{n_1 n r k} = 0 \quad \forall \quad r \in R,\ n \in N_r,\ k \in K : n = O_k \tag{3.11}$$

$$\sum_{n_2 \in N:(n,n_2) \in A_r} \upsilon_{n n_2 r k} = 0 \quad \forall \quad r \in R,\ n \in N_r,\ k \in K : n = D_k \tag{3.12}$$

Constraint 3.11 imposes that if n is an origin node, then within each route there will be no incoming flow from previous nodes. In line with this, constraint 3.12 imposes that if n is a destination node, then within each route there will be no outcoming flow for the following nodes.

$$\upsilon^{+}_{nrfk} \leq \theta_f w_{nk} \quad \forall \quad r \in R,\ n \in N_r, f \in \theta,\ k \in K : n = O_k \tag{3.13}$$

$$\sum_{r_1 \in R \setminus \{r\}: n \in N_{r_1}} \upsilon^{'}_{n r_1 r_2 f k} \leq \theta_f w_{nk} \quad \forall \quad r_2 \in R,\ n \in N_{r_2} \setminus \{O_k, D_k\},\ f \in \theta,\ k \in K \tag{3.14}$$

Constraints 3.13 and 3.14 compute the waiting time that the users spend to enter the network and in transfers, respectively.

$$\upsilon_{n_1 n_2 r k} \leq \delta_k x_r \quad \forall \quad r \in R,\ (n_1, n_2) \in N_r,\ k \in K \tag{3.15}$$

$$\upsilon^{+}_{nrfk} \leq \delta_k y_{rf} \quad \forall \quad r \in R,\ n \in N_r,\ f \in \theta,\ k \in K : n = O_k \tag{3.16}$$

$$\upsilon^{'}_{n r_1 r_2 f k} \leq \delta_k y_{r_2 f} \quad \forall \quad (r_1, r_2) \in R,\ n \in (N_{r_1} \cap N_{r_2}),\ f \in \theta,\ k \in K : r_1 \neq r_2 \tag{3.17}$$

Constraints 3.15 to 3.17 ensure that users only use routes and frequencies that are part of the solution. Besides that, they establish the maximum limit for the respective flow (for example, constraint 3.15 limits the flow of a certain OD pair to the value of its demand).

$$\sum_{f \in \theta} y_{rf} = x_r \quad \forall \quad r \in R \tag{3.18}$$

$$\sum_{f \in \theta} \upsilon^{+}_{nrfk} = \upsilon^{+}_{nrk} \quad \forall \quad r \in R,\ n \in N,\ k \in K \tag{3.19}$$

$$\sum_{f\in\theta} \upsilon^{'}_{nr_1r_2fk} = \upsilon^{'}_{nr_1r_2k} \quad \forall \quad (r_1, r_2) \in R,\ n \in (N_{r_1} \cap N_{r_2}),\ f \in \theta,\ k \in K : r_1 \neq r_2 \tag{3.20}$$

Constraints 3.18 to 3.20 impose that only one frequency is associated with the frequency-dependent variables at each time, in order to ensure the compatibility between the different variables and decision variables. For example, constraint 3.18 ensures that if a route is part of the solution, then it will have a single frequency associated, and only one.

$$z_{n_1n_2r} = x_r \quad \forall \quad r \in R,\ (n_1, n_2) \in A_r \tag{3.21}$$

$$z_{n_1n_2r} \leq \lambda_{n_1n_2r} \ + \ M \sum_{k\in K} \upsilon_{n_1n_2rk} \quad \forall \quad r \in R,\ (n_1, n_2) \in N_r \tag{3.22}$$

Constraints 3.21 and 3.22 guarantee that if a bus is empty, then a penalization will enter in the objective function. If a route is active (i.e., it is part of the solution) and the flow between n_1 and n_2 is zero, then λ must be equal to 1, being considered in the objective function.

$$\upsilon_{n_1n_2rk} \geq 0 \quad \forall \quad r \in R,\ (n_1, n_2) \in N,\ k \in K \tag{3.23}$$

$$\upsilon^{+}_{nrk} \geq 0 \quad \forall \quad r \in R,\ n \in N,\ k \in K \tag{3.24}$$

$$\upsilon^{+}_{nrfk} \geq 0 \quad \forall \quad r \in R,\ n \in N,\ f \in \theta,\ k \in K \tag{3.25}$$

$$\upsilon^{-}_{nrk} \geq 0 \quad \forall \quad r \in R,\ n \in N,\ k \in K \tag{3.26}$$

$$\upsilon^{'}_{nr_1r_2k} \geq 0 \quad \forall \quad (r_1, r_2) \in R,\ n \in R,\ k \in K \tag{3.27}$$

$$\upsilon^{'}_{nr_1r_2fk} \geq 0 \quad \forall \quad (r_1, r_2) \in R,\ n \in N,\ f \in \theta,\ k \in K \tag{3.28}$$

$$w_{nk} \geq 0 \quad \forall \quad n \in N,\ k \in K \tag{3.29}$$

$$x_r \in \{0, 1\} \quad \forall \quad r \in R \tag{3.30}$$

$$z_{n_1n_2r} \in \{0, 1\} \quad \forall \quad r \in R,\ (n_1, n_2) \in N \tag{3.31}$$

$$y_{rf} \in \{0, 1\} \quad \forall \quad r \in R,\ f \in \theta \tag{3.32}$$

$$\lambda_{n_1 n_2 r} \in \{0,1\} \quad \forall \quad r \in R,\ (n_1, n_2) \in N \tag{3.33}$$

Constraints 3.23 to 3.33 secure that the variables take values that are coherent with the formulation presented, namely that they are positive values, and in some cases can only take a value of 0 or 1, according to the previous definitions.

3.2 Illustrative Example

In order to verify the model presented in this work and give the reader a better understanding of the concepts introduced, an illustrative example is presented in this section. This example is similar to the one discussed in [37] so a comparison with the results obtained could be done.

The graph representing the network under analysis can be seen in Figure 3.3. It is composed of 8 nodes, 10 arcs, and 8 OD pairs. Here arises the first difference between the models and the two illustrative examples. Since the model developed in this thesis considers both ways of a route, contrasting to the one-way model presented in [37], the four OD pairs were doubled to count with the opposite path too. The demand of the OD pairs is presented in Table 3.1, expressed in trips per minute. The cost of each arc, i.e. the cost of the connection between two nodes, is represented in Figure 3.3 and it indicates the on board travel time, expressed in minutes. Regarding the frequencies, a set of three values was set up: 1/5, 1/30, and 1/60, being these expressed in buses per minute (a frequency of 1/5 means that a bus will operate that route every five minutes). In order to simplify the problem, and since the network is considered small, no transfers were allowed in this analysis.

As the model is developed to optimize the given problem, the set of routes given as an input to the problem included all the possible routes that can be generated with the present graph, only excluding the ones that did not have the origin or destination of the OD pairs as one of its endpoints. This does not exclude routes that could be part of the optimal solution since transfers are not being considered. With this method, shown in [37], a set of 79 routes was obtained. The fleet size used for the analysis consisted on 8 buses.

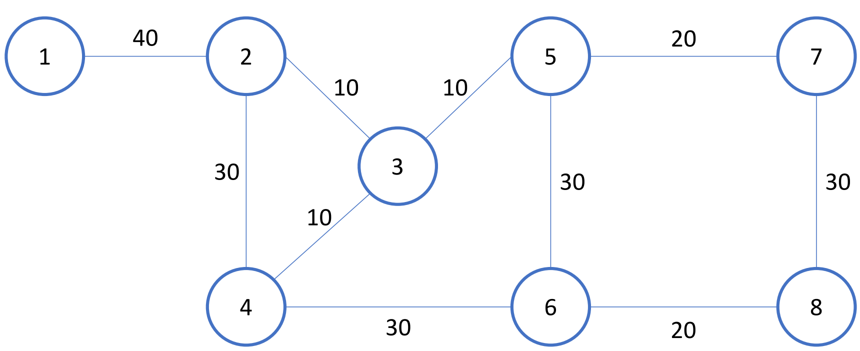

Figure 3.3: Graph of the network considered for the illustrative example.

Table 3.1: Demand matrix considered for the illustrative example.

O	D	Demand
3	1	1.0
1	3	1.0
3	8	2.0
8	3	2.0
4	7	0.5
7	4	0.5
6	1	0.5
1	6	0.5

The objective of this problem is then to obtain the set of routes that minimize the objective function while satisfying the demand. First, the example was run without the last term of the objective function, which penalizes the path where the buses operate empty, to verify the implementation of the model. It is important to refer that, since both ways of a route are being considered, the values for the objective function are expected to be twice the values shown in [37]. The results obtained are presented in Table 3.2. For a fleet size of 8 buses, without any restriction regarding the bus capacity, a value of 770 was obtained for the objective function, with routes 1-2-3-4, 1-2-3-4-6-8 and 4-3-5-7-8 being operated at a frequency of 1/60 buses per minute. Theses results are equivalent to the ones in [37], being then possible to conclude that the model was successfully implemented.

3.2.1 Introduction of bus capacity constraint

Another difference that separates both models is the introduction of a bus capacity constraint in the model. This constraint was also applied to this example, allowing at first for a capacity of 50 users per bus, since this is the capacity of a regular bus. Without further alterations on the demand, an unfeasible solution was obtained, meaning that this problem cannot be solved with 50-people buses.

Only with the increase of this capacity to 90 people was it possible to obtain a solution. The solution is composed of different routes from the obtained in the baseline case, as can be seen in Table 3.2, but the value of the objective function remained the same.

Table 3.2: Results obtained for different values of the bus capacity constraint.

# Buses	Bus capacity	Objective Function Value	Routes	Frequency [bus/min]	Computational Time [s]
8	-	770	1-2-3-4 1-2-3-4-6-8 4-3-5-7-8	1/60 1/60 1/60	2.0
8	50	-	-	-	-
8	90	770	1-2-3 1-2-3-5-6-8 4-3-5-7-8	1/60 1/60 1/60	2.4

3.2.2 Influence of the penalty of empty buses

As explained before, the objective function of the model here in study includes a term concerning the penalization of the paths where the buses circulate empty. In order to see the impact of this parameter in the study, it was added to the analysis done before.

Without considering the bus capacity constraint, the solution obtained changed again from the baseline case, but the value of the objective function remained unchanged. As it can be seen in table 3.3, the addition of this penalization in the objective function implied that the route 1-2-3-4, where 3-4 is a path where the bus operates empty, was not considered in this solution, being replaced by the route 1-2-3. This confirms a correct implementation of this third term.

Table 3.3: Results obtained with the term penalizing the paths with empty buses.

# Buses	Bus capacity	Objective Function value	Routes	Frequency [bus/min]	Computational Time [s]
8	-	770	1-2-3	1/60	3.4
			1-2-3-5-6-8	1/60	
			4-3-5-7-8	1/60	
8	90	770	1-2-3	1/60	4.1
			1-2-3-4-6-8	1/60	
			4-3-5-7-8	1/60	

3.2.3 Introduction of two types of buses

The case study that will be presented in Chapter 4 will need to consider two typologies of vehicles. This aspect was also addressed in this example, by dividing the 8 buses available into buses with different capacities, simulating two categories of buses. Thus, 4 buses with a capacity for 50/75/90 people (at each simulation) and 4 buses with a capacity for 100 people were chosen to try these constraints. The third term of the objective function was considered. Moreover, to present a case similar to the one that will be presented later, it was assumed that the route 1-2-3-4-6-8 is part of the optimal solution, and is operated by the buses with the 100 people capacity, simulating the BRT vehicles.

The results are presented in Table 3.4. It can be seen that depending on the bus capacity chosen, different results are obtained. By increasing the bus capacity of the buses that do not serve the special route, an approximation of the initial solution, and thus the minimization of the objective function, is achieved, but the computational time also increases, though slightly.

.

Table 3.4: Results obtained with the introduction of 2 types of buses.

# Buses	Bus capacity	Objective Function Value	Routes	Frequency [bus/min]	Computational Time [s]
4 4	50 100	850	1-2-3-4-6-8	1/60	0.2
			3-5-7-8	1/60	
			4-3-5-7	1/60	
4 4	75 100	800	1-2-3-4-6-8	1/60	0.2
			4-3-5-7	1/60	
			4-3-5-7-8	1/60	
4 4	90 100	770	1-2-3	1/60	0.4
			1-2-3-4-6-8	1/60	
			4-3-5-7-8	1/60	

With this, it can be concluded that the results presented in this illustrative example are in accordance with the ones obtained in [37], verifying the correct implementation of the model. Moreover, it was possible to confirm the addition of the new constraints and the third term in the objective function, since the results obtained with these are also in line with the expected solutions.

Chapter 4

Application to the Barcelona Case Study

After describing the mathematical formulation of the bus network design model, this fourth chapter will present its application to the Barcelona case study. First, an overview of the case study is provided, namely its context and the characteristics of the existing bus network. Then, the implementation of the model is detailed, including the data preparation needed and the different scenarios considered. Finally, a discussion on the results obtained is presented.

4.1 Context of the Barcelona Case Study

Following the example of several cities around the world, the mobility sector of AMB decided that a change was needed in the way urban and inter cities mobility was made. The introduction of a BRT route between the major cities of the metropolitan area contributed to this change, though it brought up the question of how the rest of the network would be impacted. One of the objectives of this thesis is then to provide a decision support tool on the changes that should be made to the network, along with an analysis of the optimization of at least a part of the network. Only some of the routes that are currently performed by the bus network will be considered, namely the most affected by the BRT.

The case study here presented is located on the Metropolitan Area of Barcelona, with special focus on the cities of Castelldefels, Gavá, Viladecans, Sant Boi de Llobregat, Cornellà de Llobregat, El Prat de Llobregat, L'Hospitalet de Llobregat and Barcelona. Some other cities are involved, as the routes in study pass them, but they are not considered as relevant in this analysis. Therefore, eight existing routes are included in the initial analysis, namely the routes L80, L81, L82, L85, L86, L94, L96, and L97 of AMB. These routes were chosen to simplify the analysis (as an analysis of the whole network would be computationally intractable) and because they are the most affected by the introduction of the BRT route since parts of these routes are overlapped with it. Data regarding these routes can be found at [47]. Moreover, the following figures, that present different representations of the case study, were generated using data from OpenStreetMap [48], that was subsequently processed using R.

In Figure 4.1, the reader may find the representation of these routes on the map of the Barcelona Area. It can be seen, for example, that route L97 connects the cities of Castelldefels and Barcelona, as represented by the dark blue route of the Figure 4.1. Table 4.1 helps the interpretation of Figure 4.1 by presenting the cities of the two extremes of each route.

Figure 4.1: Current network representation.

Table 4.1: Cities connected by the current network.

Route	City 1	City 2	Frequency [bus/min]
L80	Gavà	Barcelona	1/45
L81	Gavà	Barcelona	1/15
L82	Gavà	L'Hospitalet de Llobregat	1/20
L85	Gavà	L'Hospitalet de Llobregat	1/20
L86	Viladecans	Barcelona	1/15
L94	Castelldefels	Barcelona	1/30
L96	Castelldefels	Sant Boi de Llobregat	1/12
L97	Castelldefels	Barcelona	1/15

The introduction of the BRT route will allow for a more comfortable and faster trip, especially between the largest cities. This connection was presented by the AMB and the regional government in 2016, and in 2019 the infrastructure construction began. It is expected to be finished in early 2023 and that the first BRT buses start circulating in the middle of 2023. The proposed BRT route, M8, is represented in Figure 4.2, in red, and it can be seen that it overlaps with several of the existing routes considered in this case study. It will connect the Castelldefels to Cornellà de Llobregat.

Figure 4.2: BRT introduction on the current network.

Before proceeding to the simplification of the network, an analysis of the most used paths was conducted. AMB provided operational data that detailed the number of passengers that enter at a bus stop and exit at another, for every route. The results are presented in Figure 4.3 and express the number of passengers per day that use each path of each route. By analyzing this representation, one can conclude that routes L94 and L97 are the most used. Also, the path that corresponds to the location of the new BRT route are some of the busiest ones. Note that this flow data was collected on an ordinary day (no holiday, no strikes, or interruptions in the circulation), providing realistic flow data to describe a normal service day for the following analysis.

4.2 Data Preparation

To be able to apply the model described in Chapter 3, there is the need to prepare the inputs and simplify the existing network, since the number of nodes and arcs was considerably large without simplification. In this section, the process of preparation of the data available will be explained. Some of the inputs needed were the nodes and routes to be considered for the analysis, the cost of each arc, the OD

Figure 4.3: Most used paths of the current network.

demand matrix, and the nodes where transfers are allowed.

4.2.1 Transforming the current network into a graph

In order to translate the current network to a graph that allows a better analysis of the problem and the application of the optimization model, several steps were taken, and a simplification of the case study was made. This procedure is summarized in Figure 4.4.

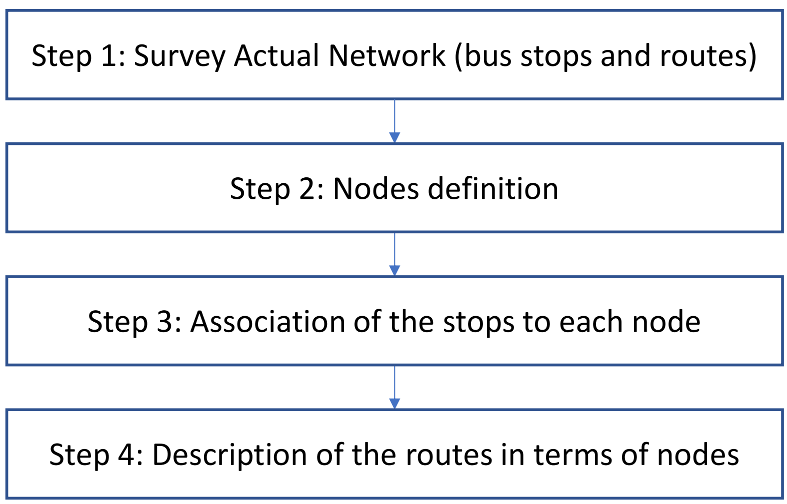

Figure 4.4: Diagram of the process of transforming the network into a graph.

First, from the information given by AMB and that is available on their website, it was possible to get all the stops that are part of the routes under analysis. Then, to simplify the network, as it would demand a large computational effort to run the model with all the stops considered, the need to reduce the number of points in the analysis appeared. Therefore, a reduction to nodes was performed using expert judgment from AMB technicians and considering key elements of the urban environment. First, all city centers were considered to be a node, followed by key locations, such as the proximity to shopping centers, hospitals, train stations, etc. Adding to this, more nodes were placed along the existing routes (considering, for example, the extreme of each route) and the BRT route, so that each route could be correctly represented and a good distribution of nodes along the network was achieved. A reduction to only 26 nodes was obtained. However, as it will be explained in the following sections, for the last scenario considered, new routes were proposed by expert judgment, and there was the need to add 3 more nodes to the existing graph. By doing this, the number of points to be considered was reduced from more than 500 stops to 29 nodes. This final graph representation was verified with AMB technicians. Figure 4.5 presents the nodes used to describe the current network, along with their location on the map.

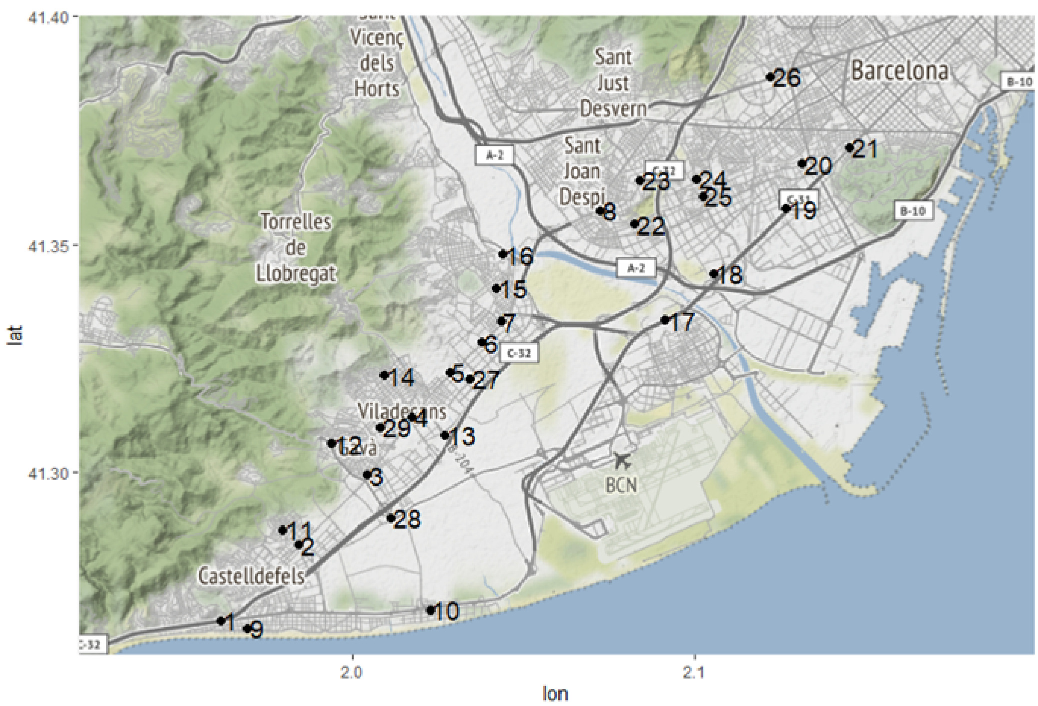

Figure 4.5: Nodes representation.

With all the nodes identified, the following step was to attribute each stop from the current network to a node of the graph. The criteria for this assignment was the proximity between the stop and the node so that each node could aggregate the closest stops. By doing this, it was possible to define the current routes with the new nodes created by this simplification. The identification of the possible connections between nodes, i.e., the arcs that represent the network, was also done. The result can be seen in Figure 4.6.

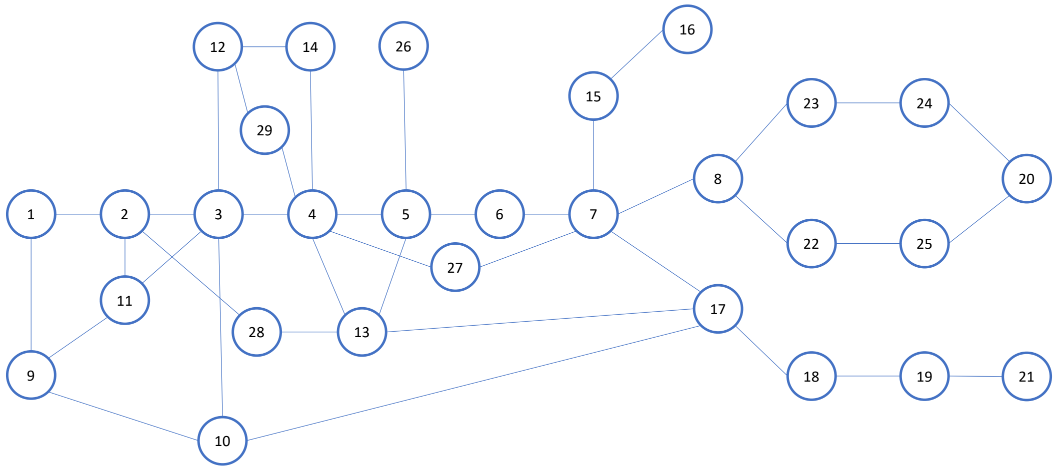

Figure 4.6: Graphical representation of the network.

In Table 4.2, one can find the definition of each route considering the nodes of the graph. For instance, route L80 goes from node 12 to node 21 passing through nodes 3, 4, 13, 17, 18, and 19. The process described allows for a simplification of a complex network, that can now be used as an input for the model in study.

Table 4.2: Current routes transformed into a nodes and arcs representation.

Route	Nodes representation
L80	12-3-4-13-17-18-19-21
L81	12-3-4-5-6-7-17-18-19-21
L82	12-3-4-5-6-7-8-22-25-20
L85	12-3-4-5-6-7-8-23-24-20
L86	14-4-5-6-7-17-18-19-21
L94	1-9-10-17-18-19-21
L96	16-15-7-6-5-4-3-2-11-9-1
L97	1-2-11-3-4-5-26

4.2.2 Cost determination

With all the nodes and arcs defined, it was necessary to associate a cost to each connection. In this case, the cost is simply represented by the duration of the trip between two nodes, this is, the on board travel time of the arc. This time was obtained for each arc by using the tool of Google Maps that allows to see the duration of a trip between two points.

4.2.3 Transfers

Transfers happen when a user wants to change from one route to another. They are performed in nodes, and the node where it happens should belong to both routes. In this model, transfers were only allowed in certain nodes of the graph, to decrease the complexity of the problem, and only one transfer is allowed. The group of nodes where these transfers can happen is the same for every scenario that will be presented afterwards, and it is composed of nodes 1 to 9 and node 17. Nodes 2 to 8 were chosen as they are part of the BRT route, and thus, it is expected that the users will use them to connect between cities. Nodes 1 and 9 were also considered due to their strategic position that allows for the connection between some routes that do not have connections otherwise. The same happened with the selection of node 17, allowing for better flexibility between routes.

On the one hand, transfers will penalize the time the user spends in the network, so it is not exactly desirable that they happen. On the other hand, they allow for a more flexible network, with better area coverage. Note that, as the network model focuses on a single mode (bus), more than one transfer would penalize the bus network against other alternatives, such as the private car or shared mobility solutions. The inclusion of more than one mode in a multimodal network model, and with more than one transfer, is left for further research.

4.2.4 Definition of the origin-destination demand matrix

Although the OD demand matrix is defined as an input in the mathematical model, it was necessary to obtain an estimation for it here. The demand prediction is crucial in any transport planning exercise, but it is hard to estimate in practice. In this case study, this was made using both the data available from AMB and an estimation based on a trip generation/attraction model for each node. For this last estimation, a classification between 1 and 3 was attributed to each node, for both its attraction and generation, which was verified by the AMB technicians. On the classification scale, a score of 1 means a low generation/attraction, and a score of 3 is a high generation/attraction. A low generation means that in that node it is not expected to enter many users in the network, while a high generation means that the corresponding node is expected to have a high number of users entering the network. The same analogy can be made for the attraction scores, but this time it evaluates the receptivity of users in the node, meaning the number of users that leave the network in this node. This classification was made considering expert knowledge, the data given by AMB and considering the different modes of transportation that exist in the node's area/zone. Table A.1 presents the scores used for this classification and can be found in Appendix A, along with weights for each combination of scores. The classification given to each node is stated in Table A.2. Then, for each OD pair, the score related to the generation of the origin node and the score related to the attraction of the destination node were computed, and a weight from Table A.1 was assigned to the OD pair. This, combined with real data provided by AMB, both regarding the demand of each route and the period of operation of the network (from 5 a.m. to 11 p.m. hence an 18h period), allowed the first estimation of the OD demand matrix.

Afterwards, the OD demand matrix was simplified, and several OD pairs were considered to have zero-demand. When there is another mode of transportation that makes the connection between the nodes of the OD pair more efficiently, it would not make sense to have a bus route covering that path, as it would imply a duplication of the service in the public transportation network. Then, it was considered that the user would choose the other mode of transport, leading to a zero-demand OD pair. For example, the connections between nodes 19 and 21 and nodes 20, 22, 23, 24, and 25 were not considered to have demand (except for those that have a direct route between them), since there is another network that connects them in a more direct way. Then, the important part was to make sure that the other cities would be connected to at least one of these nodes, so that passengers can arrive at them in a fast and comfortable way, and then continue their journey outside the bus network.

Besides that, it was necessary to evaluate which pairs of nodes were not connected with a single transfer (considering that transfers were only allowed in specific nodes). The demand for these OD pairs was also considered zero. By doing this, the OD demand matrix was once again simplified.

With all these assumptions, it was possible to reach an OD demand matrix that satisfies all the conditions explained above, allowing for a more accurate comparison between cases, as it was used to run the model for all the scenarios, which will be presented in section 4.3.

4.2.5 Route generation

Another input that is needed for the model is the pool of routes to be evaluated. It is common practice to obtain this pool through models or algorithms that return all the possible routes given a graph with nodes and corresponding arcs. The approach that is normally taken is the shortest path method. Starting from a graph of nodes and arcs, with the corresponding costs, the objective is to obtain the routes that allow for the shortest path between the nodes, either by minimizing the number of connections between two nodes or by minimizing the cost of the connection (for example, minimizing the time spent to make the connection). By doing this, one gets the pool of routes that includes all the shortest paths between the nodes of the graph, and it is used as the input to the mathematical formulation.

Though, in this specific case, and due to the complexity of the problem, this method was not used. Since the number of routes increases exponentially with the increase of the number of nodes and considering that in this case a set of 29 nodes is analyzed, along with 80 arcs and 502 OD pairs, it would be of extreme complexity to obtain a pool of routes connecting all this OD pairs with the shortest paths. Instead, the approach of considering only the current routes and the expert-based ones for the analysis was taken. It is also important to note that even with this restricted pool of routes, the computational problem was already complex.

4.3 Case Study Scenarios

Different scenarios were explored as a way of comparing different possibilities of routes' combinations. As explained in Chapter 3, the model presented in this thesis needs some inputs to be run, being one of them the set of routes given for the optimization problem. With this in mind, the different scenarios presented below will specifically differ in the set of routes considered by the model, to obtain the most optimized solution. The first one will only consider the current network so that a baseline result can be obtained and used to compare with the following ones. Then, the BRT route joins the set of routes, as well as the new type of vehicle that is used in this route, added as a constraint. Finally, the last case considered took into account the routes that belong to the current network, the BRT route, and other routes proposed by expert knowledge.

The results expected for each case include the set of routes that are part of the solution of the optimization problem considered, along with the frequencies at which each route should be operated. The frequency of each route can either be an input or a decision variable. To study different possible situations (combinations of routes), several cases were separated, and the model was run for each one.

Next, a description of the cases considered is presented. In section 4.4, the reader can find a table with the synthesis of the most relevant cases and results.

4.3.1 Current Network

In the first scenario studied, the set of routes considered by the algorithm involved only the routes of the current network, the ones presented in Figure 4.1.

For the first case (case 4.0), the real frequencies at which the routes are operating, presented in Table 4.1, were imposed, serving this case as the baseline for the rest of the analysis.

In the following case (case 4.0.1), the pool of routes in analysis included only the current routes, but this time their operational frequency was left without any assigned value so that it would be possible to conclude if the current network and frequencies were optimized. The set of frequencies that the routes could take included the values 1/12, 1/15, 1/20, 1/30, and 1/45 buses per minute. Regarding the fleet size constraint, a total of 68 buses (with normal capacity, this is 50-65 people) were considered in this analysis. The same number was used in the following cases too.

4.3.2 BRT Introduction

In this scenario, the set of routes considered includes the current ones and the new BRT route. This last has already an imposed frequency of operation of 1 bus every 10 minutes, a value that remained fixed in every case, and was established in a constraint.

Considering it, two cases were identified: one where the current routes had also their operational frequencies imposed (case 4.1.0) and the other where these frequencies were considered as variables, and the model could find the best solution (case 4.1.1). The set of frequencies that the routes could take included the values 1/10, 1/12, 1/15, 1/20, 1/30, and 1/45 buses per minute.

With the introduction of the BRT route, there was the need to introduce new variables that account for the differences that this route presents when compared to the normal ones. Starting with the buses, the ones doing the BRT service will be articulated buses, 18 meters long and with a capacity for 100 people, while the normal buses have a capacity of only 50-65 passengers, as stated before. To account for these differences, two new constraints were added to the model: one related to the fleet size, since a limited number of BRT buses are available (in this case, 11), and another one considering the different capacity of this type of vehicle.

4.3.3 Expert Based Routes

The third scenario studied considered the current routes, the BRT route, and new routes suggested by experts. The new routes were obtained by asking experts on mobility in Barcelona what modifications they would like to propose to the existing network so that a more accurate and realistic response, that would serve better the community of this area, could be studied. Considering that the BRT would allow for a more direct connection between the cities and since the existing routes have several sections overlapping the BRT route (and even between the existing routes), these new routes would focus more on the extreme sections of the BRT, complementing it and allowing for better coverage of the bus network in this area. The proposed modifications affect 3 routes: L82, L85, and L96.

In the current network, routes L82 and L85 already have similar paths, connecting Gavà to L'Hospitalet de Llobregat. As can be seen by Table 4.2, only the last nodes of these routes are different. The BRT route will also cover this part of the routes that are already overlapping. Thus, the alteration proposed by the experts was to change the path of L82 at the end of Gavà, introducing a new section connecting Gavà and ViladeCans in a more direct way, while changing the middle of the route so that it does not overlap with the BRT, including for that node 27. For the first alteration, two possibilities were presented: one that would go more directly to Gavà (route A) and the other that would link to the center of Viladecans (route B). At the same time, the route L85 would be reduced to connect the end of the BRT route to the city of L'Hospitalet de Llobregat, as before (route L85 new).

Regarding route L96, most of its trajectory will be overlapped by the BRT route, so two alternatives were proposed. The first one implies that the connection between Castelldefels and Sant Boi de Llobregat is assured directly, but has a different path from the BRT, covering another area that was not covered before, represented by node 28 (route C). The second alternative is to have route L96 divided into two different routes, each one at its extreme (routes D - 1 and D - 2). Table 4.3 summarizes the changes explained before.

This scenario was then divided into several cases, in order to evaluate the impact of these alternatives in the network. Case 4.2.1 considers the pool of routes that include all the available routes described before, with no constraints regarding redundancy. In case 4.2.2, these constraints were added, to make sure that only one of the alternatives of each route would be part of the solution. In both cases, the frequencies were left unknown, and the set of frequencies that the routes could take included the values 1/10, 1/12, 1/15, 1/20, 1/30, and 1/45 buses per minute.

Table 4.3: Alternatives presented by experts for routes L82, L85 and L96.

Route	**L82**	**L85**	**L96**
Current	12-3-4-5-6-7-8-22-25-20	12-3-4-5-6-7-8-23-24-20	1-9-11-2-3-4-5-6-7-15-16
Alternative 1	Route A 12-29-4-27-7-8-22-25-20	L85 new 8-23-24-20	Route C 1-9-11-2-28-13-5-6-7-15-16
Alternative 2	Route B 12-14-4-27-7-8-22-25-20	-	Route D - 1 1-9-2-11 Route D - 2 13-5-6-7-15-16

4.4 Discussion of Results

Having presented the different scenarios, an analysis of the results obtained is made. In the Appendix B, the reader can find the results of all the cases that were explored. In this section, only the main cases are compared.

Before proceeding to the presentation and analysis of the results, it is important to introduce another concept. The unit of the objective function value obtained for each case is passenger.minute/minute of operation. In order to turn it into a more comprehensible value, it was divided by the total demand of the network, i.e., the sum of all b_k of the OD demand matrix (with a value of 41.35 passenger/minute). By doing this, one can obtain the time spent inside the network by an average user, making it easier to compare and understand better the average reductions of time that are being achieved with this optimization.

Case 4.0 was used as the baseline since it represents the network that is currently in operation. Firstly, the objective function value obtained was 2306.51 passenger.minute/minute of operation. This means that the average user will spend 55.8 minutes in the network. Then, case 4.0.1 was run to analyze if the current network was effectively optimized, having the frequency as a variable and not an input. As it can be seen in Table 4.4, the frequencies of the routes differ from the baseline case, leading to the conclusion that even without the introduction of the BRT, an optimization could be made to the network. This optimization could reduce the time the average user spends in the network to 51.6 minutes.

Table 4.4: Results of the first scenario: current routes.

Case	Description	Routes	Frequency [bus/min]	Objective function value	Computational Time [s]	Optimal gap [%]
4.0 Current routes	Imposed frequencies for each route Bus capacity constraint applied: 63 people	L80	1/45	2306.51	1.0	$< 0.01\%$
		L81	1/15			
		L82	1/20			
		L85	1/20			
		L86	1/15			
		L94	1/30			
		L96	1/12			
		L97	1/15			
4.0.1 Current routes	No imposed frequencies for each route Bus capacity constraint applied: 63 people	L80	1/20	2133.7	606.9	$< 0.01\%$
		L81	1/45			
		L82	1/12			
		L85	1/15			
		L86	1/12			
		L94	1/20			
		L96	1/12			
		L97	1/20			

With the introduction of the BRT route, there was the need to add several constraints to the previous case, maintaining the general inputs. The set of routes available for the optimization was changed, since the BRT route needs to be included, and the constraints regarding the number of BRT buses available and their capacity were added. Having the routes and respective frequencies as variables (excluding the obligation of having the BRT route with a frequency of 1 bus every 10 minutes as part of the solution), the optimized solution obtained includes routes BRT, L80, L82, L85, L86, L94, L96 and L97, with the respective frequencies 1/10, 1/15, 1/12, 1/15, 1/12, 1/20, 1/10, and 1/20, as can be seen in Table 4.5. This solution allows for a reduction of 7.95 minutes in the time that an average user spends inside the network when compared to the baseline case, becoming 47.8 minutes.

Table 4.5: Results of the second scenario: current routes + BRT.

Case	Description	Routes	Frequency [bus/min]	Objective function value	Computational time [s]	Optimal gap [%]
4.1.1 Current routes + BRT	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT	1/10	1977.45	1757.9	$<$ 0.1%
		L80	1/15			
		L82	1/12			
		L85	1/15			
		L86	1/12			
		L94	1/20			
		L96	1/10			
		L97	1/20			

The set of routes used for the last scenario included the current routes, the BRT route, and the alternative routes presented before. At first, in case 4.2.1, no restrictions were imposed regarding the frequencies of the routes (except for the BRT) and it was allowed that more than one alternative for each route could be chosen. Thus, a reduction of 8.8 minutes was reached for this case, obtaining a time of 47.0 minutes spent in the network by an average user. Although this solution represents a significant reduction in the time spent in the network, it did in fact include the three alternatives given for the L96 route, which would not be practical in a real-life situation, as identified by the AMB experts, given the redundancy it implies. Though, one specific case was aroused by the AMB experts after these results, where this redundancy obtained could be used but only in two of the three alternatives. It consisted in considering routes C and D, but at different frequencies, so that a direct alternative could be given to the population, and a split option would be available too. The results are presented in Annex B, represented by case 4.2.6, and it can be seen that the values obtained were worse than previous alternatives, therefore not being considered in the rest of the analysis.

In order to overcome the problem of redundancy, a constraint was added to the model, where only one route could be chosen between the current route and its alternatives (case 4.2.2). This solution included the routes L80, L82, L86, L94, L96, L97, BRT and the alternative proposed for the L85. The frequencies of each route that optimize the problem are shown in Table 4.6. A reduction of 8.4 minutes for each average user is achieved, a lower value than the case 4.2.1, as expected.

Table 4.6: Results of the third scenario: current routes + BRT + Expert Based routes.

Case	Description	Routes	Frequency [bus/min]	Objective function value	Computational Time [s]	Optimal gap [%]
4.2.0 Current routes + BRT + Expert based routes	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT	1/10	1945.02	342495.9 (≈ 3 days and 23 hours)	14.04%
		L81	1/45			
		L82	1/10			
		L86	1/10			
		L94	1/20			
		L96	1/15			
		L97	1/20			
		L85 new	1/12			
		Route C	1/30			
		Route D - 1	1/45			
		Route D - 2	1/10			
4.2.1 Current routes + BRT + Expert based routes	No imposed frequencies (only for BRT) Bus capacity constraint applied Exclusivity constraint in the routes with various alternatives	BRT	1/10	1959.13	372713.5 (≈ 4 days and 8 hours)	< 0.01%
		L80	1/15			
		L82	1/10			
		L86	1/10			
		L94	1/20			
		L96	1/10			
		L97	1/15			
		L85 new	1/15			

Table 4.7 presents the main results obtained for each case. It is worth noting that a reduction of 15% in the time an average user spends in the network is achieved for the best (and realistic) solution. On the operator's side, it would imply the elimination of route L81, the addition of the BRT route, as imposed, and also the modification of route L85, which will be shortened and will guarantee the connection between the end of the BRT and the city of L'Hospitalet de Llobregat. The graphical representation of this optimal solution can be found in Figure 4.7.

Table 4.7: Main results of the three scenarios presented.

Case	Objective function value	Time [min]	Expected Reduction [%]
4.0 - Current routes	2306.51	55.8	-
4.0.1 - Current routes	2133.7	51.6	-7.5%
4.1.1 - Current routes + BRT	1977.45	47.8	-14.3%
4.2.0 - Current routes + BRT + Expert based routes	1945.02	47.0	-15.7%
4.2.1 - Current routes + BRT + Expert based routes	1959.13	47.4	-15.0%

Figure 4.7: Representation of the new network, including the BRT and the new L85 routes.

More cases regarding this last scenario were studied, and the results are present in Appendix B. These cases aimed to study the influence of alternative routes in the network. Thus, the difference between them includes the pool of routes that was given as an input to the model, since it only contained one of the alternatives of each route in each case. However, none of them presented better results than the ones explained in this section. More information on this analysis can be found in Appendix B.

Chapter 5

Conclusion

In this chapter, the main conclusions of the thesis are presented, along with the limitations faced throughout this work. Several topics for future research are also discussed.

5.1 Main Conclusions

This work studied the redesign of an existing bus network due to the implementation of a BRT route. An adaptation of the mathematical formulation of [37] was made, and then applied to the case study of the Barcelona Metropolitan Area.

A third term was added to the objective function to penalize the paths between nodes in which the buses travel empty, as this may be perceived as poor planning in the user's view. Furthermore, two constraints were added to the base formulation to account for the typology of vehicles used in the network, namely the number of available vehicles (fleet size) and their capacity. Since the buses operating the BRT route are different from the ones operating the rest of the network, two different types of buses were included in the analysis, for the same network.

Moreover, a step-by-step method to simplify a complex network into a graph that can be used as an input for the model is presented. In the present case, this method relied on expert judgment, to simplify hundreds of bus stops into twenty-nine nodes, which allowed for a faster, but yet accurate analysis.

The model was run for three different scenarios, differentiated by the set of routes that were analyzed at each time. As a result, it was found that the routes of the current bus network were not optimized, since the time spent in the network by the average user in this configuration is approximately 55.78 minutes, contrasting with the 51.6 minutes obtained for the same routes, but with different operational frequencies. With the introduction of the BRT route, this time decreases even more to a value of 47.2 minutes. The last case studied, the one that presented the more realistic results, involving the routes of the current network, the BRT route, and the expert-based routes, achieved a value of 47.03 minutes.

This study led to the conclusion that, by changing the operational frequencies or the trajectory of part of the network, it is possible to optimize the network. The needed modifications in the network include the elimination of one route (L81), the shortening of route L85, in order to connect the end of the

BRT route with the city of L'Hospitalet de Llobregat, and the introduction of the BRT route. Also, some frequency changes should be applied to the remaining routes. Thus, one can conclude that, with simple modifications, an optimization of the network can be achieved, while saving on average 8.4 minutes for each user, representing a 15% reduction in the travel time. This is of extreme importance in the dynamic of a city since people can then apply this time more productively, both in leisure or working time.

The model developed and discussed in this thesis can be applied to the design of any kind of transit network, namely in the airline network design.

5.2 Limitations

During the development of this work, certain limitations were identified.

Regarding the OD demand matrix, it is important to notice that the method used for its estimation made too many assumptions. Since the focus of this thesis was the optimization problem regarding the bus network, this prediction problem was not totally explored and should be addressed. This method did not model the user's behavior or the changes that may happen with the introduction of the BRT. As the network will be modified, with new routes and different frequencies, it is expected to provide a better service and hopefully attract more users to the network. With this in mind, an assessment of the predicted demand for this new network (especially the induced demand) should be done, in order to have better inputs to the model and to have more accurate results.

Besides that, a multimodal analysis was not considered. Although the existence of other transportation modes was examined when simplifying the OD matrix, with the help of an expert from AMB, the interaction between them should be further explored, both regarding the demand estimation for each one and the design of the network itself, to have fewer overlapping paths.

Additionally, it is important to understand that this work applies only to a restricted number of routes from Barcelona's network, as these are the most affected by the introduction of the BRT route. Thus, it is reasonable to consider that the work may have been constrained by this simplification and, in future applications, the remaining routes could be integrated into the model as well.

5.3 Further Research

On top of the research developed, several topics can be added and further explored.

As said before, the existence of other public transport modes in the area of study should be considered and its inclusion in the analysis could be beneficial for the design of a multimodal network.

Concerning the number of transfers that are allowed by the model, the possibility of including more than 1 transfer in the analysis would be an interesting topic of research. As noted before, the BRT route will overlap part of the existing routes and it connects the main cities of the Barcelona Metropolitan Area, but it does not connect the city centers. Then, having at least two transfers might allow for more flexibility in the use of the BRT route. Though, it is important to keep in mind that allowing a higher number of transfers may influence the user's experience in the network, and at a certain point, other modes of transport may become more attractive than the bus network.

As it was explained in Chapter 2, this thesis addresses only two of the five steps of a transit network design. With that in mind, it is left as a recommendation that the next steps of this problem can be explored, namely the development of timetables and crew and vehicle scheduling.

Regarding the model applied, different topics can be further detailed. The introduction of a term that takes into account the interest of the operators (i.e. the minimization of costs, for example, by minimizing the fleet size needed) in the objective function can lead to a better compromise between the satisfaction of the users and the costs of the operator. In this work, this was not considered, since it was not a priority in the case study.

Moreover, the value of the objective function considers the average time spent by the user in each OD pair. An interesting analysis that can be done in a post-processing stage is the evaluation of the effect of the network's modifications in the travel time of each OD pair, since it may affect the acceptability of the modifications by the community if this time increases too much in some of the OD pairs. By doing this, one could understand which ones were the most affected by the proposed changes, i.e., in which OD pairs the time spent in the network increases (or decreases) and by how much. In this case, there was no constraint regarding this aspect, as it was not a limitation on the work. Nevertheless, by using a MILP model, it is possible to account for this issue, by adding constraints that can limit the maximum increase or decrease that any OD pair can support.

Furthermore, other aspects can be considered in the objective function, regarding the area in consideration. For example, and as explored in the case of the aviation sector, it could be necessary to assure some routes due to social reasons, and so a term concerning this should be included in the objective function or as constraints.

Besides that, it is also suggested that a more robust network design is explored, to consider different scenarios of demand, since the introduction of the BRT route may induce demand that is not predicted in this work, but that should be taken into account.

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Appendix A

Origin-Destination demand matrix determination

The estimation of the OD demand matrix is not the focus of this thesis. In this regard, a draft estimation was obtained by a generation and attraction method, together with real data provided by AMB. Table A.1 presents scores used for this classification, as well as the weights for each combination of scores.

Table A.2 presents the classification attributed to each node, regarding its generation/attraction capability.

Table A.1: Weighted matrix used for the determination of OD demand matrix.

		Attraction		
		1	**2**	**3**
Generation	**1**	0.1	0.2	0.4
	2	0.2	0.6	0.8
	3	0.4	0.8	1.0

Table A.2: Classification of each node according to its generation/attraction.

Node	**Generation**	**Attraction**
1	2	1
2	3	3
3	3	3
4	3	2
5	2	1
6	2	1
7	2	3
8	3	3
9	2	2
10	1	1
11	3	2
12	3	2
13	2	3
14	3	2
15	3	2
16	3	3
17	1	1
18	2	3
19	2	2
20	2	2
21	3	3
22	2	2
23	2	2
24	2	3
25	3	3
26	2	3
27	1	2
28	1	1
29	2	2

Appendix B

Results of extra cases

In this Appendix, the results of every case considered during this work are presented. One can find in table B.1 the cases, a brief description of some of the constraints applied, the set of routes and frequencies that optimize the objective function in each case, along with the value obtained for the objective function and the time that an average user spends in the network for the solution presented.

Some notes regarding the interpretation of the table should be addressed. In cases 4.2.2 to 4.2.6, only one of the alternative routes for L82, L85, and L96 are considered at each time. This is described in the first column. For example, in case 4.2.2, the routes given as an input to the model were the BRT, L80, L81, L86, L94, L97, route A (instead of route L82), route L85 new (instead of route L85), and route C (instead of route L96). The same process was repeated for the other cases. In case 4.2.6, the pool of routes given was: BRT, L80, L81, L82, L86, L94, L97, route L85 new (instead of route L85), route C, and route D (instead of route L96). As explained in Chapter 4, this case appeared from an exchange of ideas with experts from AMB, when the first results were presented. Since the optimal solution for case 4.2.0 included routes L96, C, and D, and this does not make sense in real life due to the redundancy of routes, the experts suggested having only routes C and D (instead of the three) with specific frequencies (1/30 and 1/15 respectively). This case was analyzed, but, as it can be seen, it does not present the most optimal results.

The last column (Time) refers to the time spent by an average user inside the network and is given in minutes.

Table B.1: Results of every case considered.

Case	Description	Routes	Frequency [bus/min]	Objective function value	Time [min]
4.0 Current routes	Imposed frequencies for each route Bus capacity constraint applied: 63 people	L80 L81 L82 L85 L86 L94 L96 L97	1/45 1/15 1/20 1/20 1/15 1/30 1/12 1/15	2306.51	55.8
4.0.1 Current routes	No imposed frequencies for each route Bus capacity constraint applied: 63 people	L80 L81 L82 L85 L86 L94 L96 L97	1/20 1/45 1/12 1/15 1/12 1/20 1/12 1/20	2133.7	51.6
4.1.0 Current routes + BRT	Imposed frequencies for each route Bus capacity constraint applied: 63 people	BRT L80 L81 L82 L85 L86 L94 L96 L97	1/10 1/45 1/15 1/20 1/20 1/15 1/30 1/12 1/15	2166.57	52.4
4.1.1 Current routes + BRT	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT L80 L82 L85 L86 L94 L96 L97	1/10 1/15 1/12 1/15 1/12 1/20 1/10 1/20	1977.45	47.8

4.2.0 Current routes + BRT + Expert based routes	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT L81 L82 L86 L94 L96 L97 L85 new Route C Route D - 1 Route D - 2	1/10 1/45 1/10 1/10 1/20 1/15 1/20 1/12 1/30 1/45 1/10	1945.02	47.0
4.2.1 Current routes + BRT + Expert based routes	No imposed frequencies (only for BRT) Bus capacity constraint applied Exclusivity constraint in the routes with various alternatives	BRT L80 L82 L86 L94 L96 L97 L85 new	1/10 1/15 1/10 1/10 1/20 1/10 1/15 1/15	1959.13	47.4
4.2.2 Current routes + BRT + L85 new + A + C	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT L81 L86 L94 L97 Route A L85 new Route C	1/10 1/20 1/10 1/20 1/15 1/12 1/10 1/10	2031.71	49.1
4.2.3 Current routes + BRT + L85 new + B + C	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT L81 L86 L94 L97 Route B L85 new Route C	1/10 1/12 1/45 1/20 1/12 1/10 1/12 1/10	2012.62	48.7

4.2.4 Current routes + BRT + L85 new + A + D	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT L80 L86 L94 L97 Route A L85 new Route D - 1 Route D - 2	1/10 1/15 1/10 1/20 1/12 1/10 1/10 1/15 1/10	2002.46	48.4
4.2.5 Current routes + BRT + L85 new + B + D	No imposed frequencies (only for BRT) Bus capacity constraint applied	BRT L80 L81 L86 L94 L97 Route B L85 new Route D - 1 Route D - 2	1/10 1/30 1/15 1/30 1/15 1/10 1/10 1/12 1/20 1/10	1990.78	48.1
4.2.6 Current routes + BRT + L82 + L85 new + C + D	Imposed frequency for routes BRT, C and D Bus capacity constraint applied	BRT L80 L82 L86 L94 L97 L85 new Route C Route D - 1 Route D - 2	1/10 1/45 1/10 1/10 1/15 1/12 1/10 1/30 1/15 1/15	1991.19	48.1