Aerodynamic Interactions Between Tandem Configuration Rotors

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December 2021

Abstract

The main objective of this work is to explore the performance of tandem configuration rotors (with partial overlap) and compare to isolated configurations, identifying critical parameters for these configurations. Additionally, this work is intended as a source of data for the development of alternative configurations for drones. A summary of Momentum Theory, as a first-approximation reference for isolated rotors, is presented. An adaptation of this theory for tandem-rotor configurations is also explained, as it allows for the computation of the expected experimental results. A testing bench, originally developed by Amado for coaxial configurations, was modified to allow for the study of rotors in tandem. A calibration procedure was carried out and resulting coefficients verified with results supplied by the propellers' manufacturer. The parameters studied were pitch and diameter for both rotors, rotation direction as well as interrotor and interaxial distances. The performance of the downstream rotor and the overall performance were very sensitive to rotation direction (where opposite rotation was observed to be less performant). Interaxial distance was also found to be of high importance. Moderate impact was observed when a smaller downstream diameter was used, or when the pitch of the downstream rotor is greater. A moderate sensitivity to upstream pitch was verified. A low sensitivity to interrotor distance and upstream diameter is also of note.
Keywords: Drones, Tandem-rotor Configurations, Experimental Analysis, Aerial Propulsion Systems, Momentum Theory Analysis

1. Introduction

Unmanned Aerial Vehicles (UAVs), and in particular multi-rotor configurations for these vehicles, have become ubiquitous in countless areas of application. These types of vehicles are of especially high importance in conditions where use of a human worker are either too costly (such as delivery or transportation) or too unsafe (structural integrity assessment or forest-fire warning systems). As payload requirements increase, so does research for rotor configurations that provide more lift for the same overall area, weight or power requirements. The most common configuration for UAVs is a planar, 4-rotor configuration, as it allows for simple implementation of control with reasonable redundancy. Coaxial configurations, with two rotors in the same axis, have been studied and seen some implementation in commercial applications.

To investigate the results of interference between tandem rotors, with particular focus on the downstream rotor, is the main objective of this work. It is also of importance to expand on the influence of several parameters, such as rotation direction, propeller diameter and pitch, as well as the interrotor and interaxial separation between the rotors. Identifying where the resulting interference has a negative effect, where it is mostly irrelevant and where it can have a positive impact on the performance of the downstream rotor is of unequivocally high importance when analysing alternative propulsion systems.

Though there are some empirical and Computational Fluid Dynamics (CFD) simulations done for tandem rotors, an extensive analysis on the effects of varying several parameters independently has not been seen. When tested in hover, parameters such as propeller pitch or diameter have simply not been the main focus of most studies [1, 2]. Theys [3] studied the influence of five different variables on the performance of the propulsion system, comparing to coaxial and planar configurations. Forward-flight CFD analysis has been performed in [4], specifying the changes in advance ratio and both interrotor and interaxial distances.

2. Background

2.1. Theoretical Overview

For isolated rotors, a Momentum Theory Analysis (MTA) is the standard first-order approximation in most applications [5]. Performance parameters for isolated rotors are presented, as well as a modified version suited to tandem rotors.

Power required to hover depends on thrust T, rotor area A and air density ρ, as defined in equation (1):

$$P_{\mathrm{ideal}} = \frac{T^{3/2}}{\sqrt{2\rho A}} \tag{1}$$

The Figure of Merit, FM, is defined as the ratio of power required to hover and power actually used (mechanical power), as in eq. Eq. (2). It is often interpreted as a measure of hover efficiency.

$$FM = \frac{P_{\mathrm{ideal}}}{P_{\mathrm{mech}}} \tag{2}$$

2.2. Modified Momentum Theory for Tandem Rotors

A modified version of MTA can be obtained [5] for two tandem configuration cases, based on inter-rotor distance H: one where the rotors are in very close proximity (where $H \approx 0$, figure 1), and one where the downstream rotor is in the far wake of the upstream one ($H \rightarrow \infty$, fig. 2).

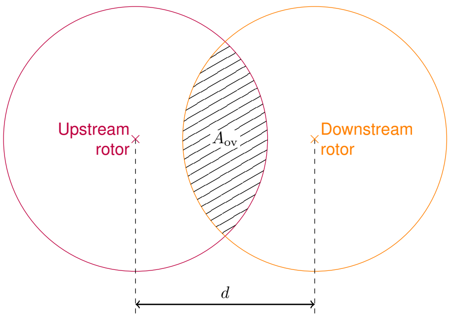

Figure 1: Overlap area definition for rotors in close proximity.

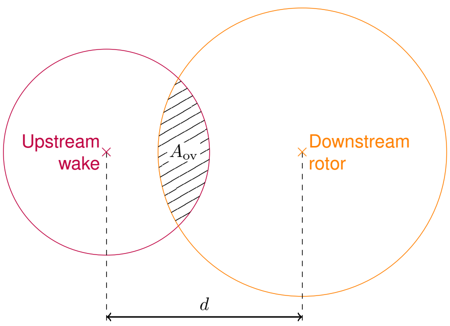

Figure 2: Overlap area definition for when the downstream rotor is in the fully developed wake of the other.

The ratio of area with overlap to total area of one rotor, $m' = A_{\mathrm{ov}}/A$ is used extensively, based on the assumption that power required to hover is proportional to the area of the rotor. In either case, the overlap coefficient κ_{ov} is defined as the ratio of power required to hover with overlap as compared to the power required to hover in a planar configuration. Equation (3) shows this definition:

$$\kappa_{\mathrm{ov}} = \frac{(P_{\mathrm{ind}})_{\mathrm{ov}}}{P_{\mathrm{ind}}} \tag{3}$$

Three cases are shown for an analysis of κ_{ov}. In any case, it is assumed that $T_{\mathrm{u}} = T_{\mathrm{d}}$.

1. If $H/D_{\mathrm{d}} = 0$, i.e. the rotors are very close to one another. Then, the expression for κ_{ov} becomes:

$$\kappa_{\mathrm{ov}} = 1 + \left(\sqrt{2} - 1\right) m' \tag{4}$$

2. If $H/D_{\mathrm{d}} \rightarrow \infty$, i.e. the downstream rotor is in the wake of the upstream one. In this case, m' can be computed using (5) and (6):

$$\begin{cases} \cos\theta = \frac{D_{\mathrm{uw}}^2 - D_{\mathrm{d}}^2 + 4d^2}{4dD_{\mathrm{uw}}} \\ \cos\varphi = \frac{D_{\mathrm{d}}^2 - D_{\mathrm{uw}}^2 + 4d^2}{4dD_{\mathrm{d}}} \end{cases} \tag{5}$$

And the overlap area coefficient can be computed with (6), where D_{uw} is the upstream wake diameter and D_{d} is the downstream diameter.

$$m' = \frac{2}{\pi}\left[\left(\frac{D_{\mathrm{uw}}}{D}\right)^2 \theta + \left(\frac{D_{\mathrm{d}}}{D}\right)^2 \varphi - \frac{d}{D}\frac{D_{\mathrm{uw}}}{D}\sin\theta\right] \tag{6}$$

With the last auxilliary function, $G(m')$:

$$G(m') = \sqrt{5m'^2 + 4m' + 1} - 3m' \tag{7}$$

κ_{ov} can finally be computed with:

$$\kappa_{\mathrm{ov}} = \frac{P_{\mathrm{total}}}{2P_{\mathrm{iso}}} = \frac{(G(m') + 1 + 2m')}{2} \tag{8}$$

3. A correlation by Harris [6], based on $\delta = d/D_{\mathrm{d}}$ which shows good empirical agreement:

$$\kappa_{\mathrm{ov}} \approx \left[\sqrt{2} - \frac{\sqrt{2}}{2}\delta + \left(1 - \frac{\sqrt{2}}{2}\right)\delta^2\right] \tag{9}$$

These results are shown in fig. 3. While there are some differences, the predicted values and correlations do not differ significantly.

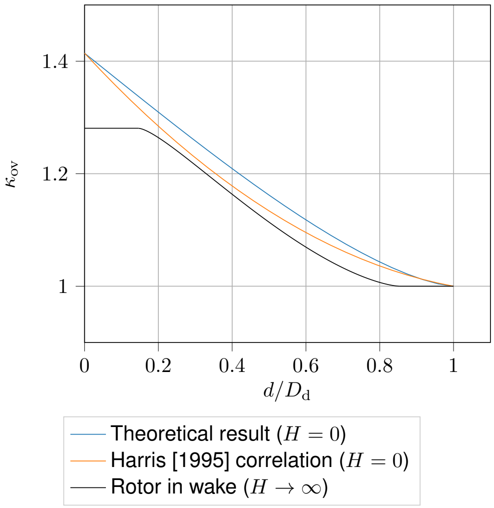

Figure 3: Graphical comparison of κ_{ov}, in three different cases.

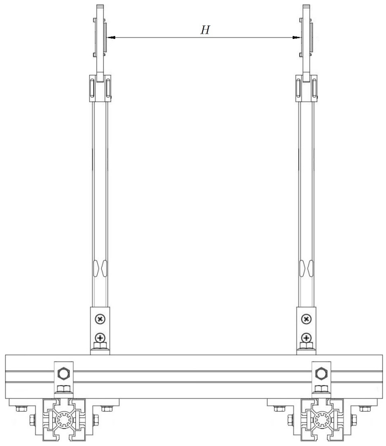

Figure 5: Side view of the setup, with projected modifications, with interaxial distance d indicated.

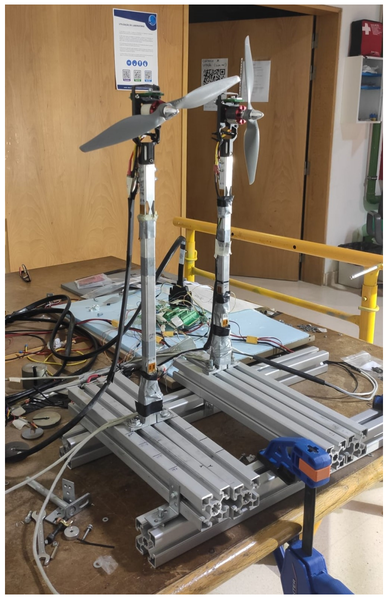

Figure 6: A photograph of the experimental setup at its final configuration.

3. Experimental Measurements

3.1. Test-Bench Design

To do the necessary measurements in tandem configurations, an experimental test-bench designed by Amado [7] was modified. The original test-bench was capable of measuring thrust, torque and RPM, as well as voltage, current and temperature to prevent motor burnout. The modifications were intended to allow movement in both horizontal axes, so that tandem configurations could be tested. The LabView program that was designed with the test-bench was modified, with the addition of a proportional controller for angular velocity. Front and side projected views for the test-bench, with nomenclatures for interrotor and interaxial distances specified, are shown in figures 4 and 5.

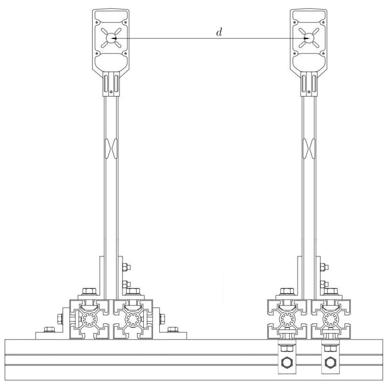

Figure 4: Front view of the setup, with projected modifications, with interrotor distance H indicated.

The modified test-bench is shown in fig. 6

3.2. Experimental Validation

Reference data for propellers in isolated contitions was collected. This data was used to verify and validate the calibration procedure, as well as. Fig. 7 shows the validation of the calibration of the extensometers, with 2σ (95%) confidence intervals.

3.3. Data Collection Methodology

Data was collected at 500 RPM increments, in the range 1500 to 6000 RPM. It was interpolated in 2D to generate a mesh-grid of 200×200 points, of each of the relevant variables. Propellers used were acquired from *APC Propellers*, and are shown in a $D\times P$ E format (i.e. 10×6E). D represents the propeller diameter and P the pitch (both in inches).

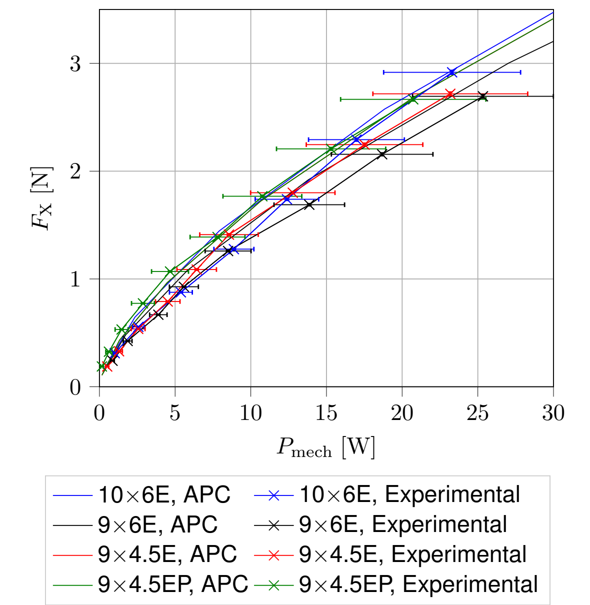

Figure 7: Computed thrust as a function of mechanical power (P_{mech}) for propellers used. 2D confidence intervals provided are 2σ.

The propeller specifications include E or EP, to differentiate between clockwise (CW) and counter-CW (CCW) rotation directions. The following parameters were tested:

1. **Rotation direction sensitivity** was the first test to allow for the reduction of total configurations to be tested. In particular, the downstream propeller was the 10×6E, and the 9×4.5E or 9×4.5EP upstream. Performance of a rotor is proportional to relative velocity (V_{rel} in fig. 8) between the rotor blade (V_{d}) and the incoming air (V_{uw}). In tandem, relative velocity is greater when Equal Rotation (ER) is used. As such, it is expected that ER is superior to an equivalent Opposite Rotation (OR) system.

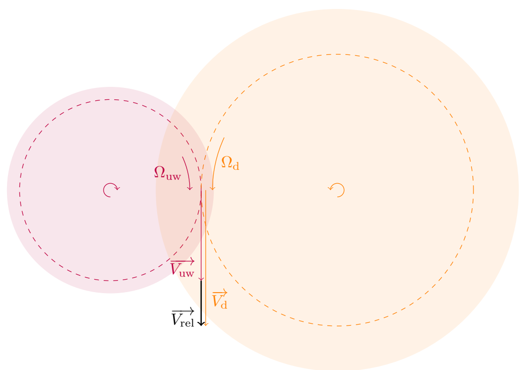

Figure 8: Relative velocity is smaller for OR systems

2. **Upstream propeller pitch**, p_{u}, is expected to be one of the most important parameters, as the wake strength is directly related to pitch: a lower pitch propeller produces a weaker wake, as well as less lift for the same angular velocity, Ω; conversely, a higher pitch means a stronger, more disturbing wake and more lift for the same propeller diameter. Two propellers upstream were tested [9×6E, 9×4.5E]. In these tests the downstream propeller was the 10×6E.

3. **Downstream propeller pitch**, p_{d}, was tested similarly to upstream pitch, using the same propellers and exchanging the upstream and downstream propellers. Qualitatively, a lower pitch propeller will produce less lift, but it will probably experience a smaller influence of the upstream-wake than its higher-pitched counterpart.

4. **Upstream propeller diameter**, D_{u}, was tested using a 9×6E or 10×6E upstream, and a 10×6E downstream. It is expected that this variable is somewhat significant, but not as much as downstream diameter. It is also expected that, for larger H/D_{d}, this parameter loses some importance.

5. **Downstream propeller diameter**, D_{d}, is expected to have a large significance, since m' is proportional to d/D_{d}, and κ_{ov} decreases with d/D_{d}. Increasing D_{d} decreases d/D_{d}, so that κ_{ov} and total power consumption is expected to decrease.

6. **Interrotor distance** H or H/D_{d}, is measured as the distance between the rotor planes, parallel to the axis of the rotors. 4 distances were tested: $[85, 120, 155, 190]$ mm.

7. **Interaxial distance** d or d/D_{d}, (i.e. between the axes of rotation of the propellers) were tested for all available propellers and test cases. 3 different distances were tested: $[136, 186, 230]$ mm.

4. Results

Example plots for each parameter tested are shown, as an illustration of the general case. The closer to vertical the isolines are, the less a given system or rotor is sensitive to this parameter; conversely, the more inclined the isolines are, the more sensitive it is. Some plots are also displayed with one variable fixed (e.g. angular velocity $\Omega_{\mathrm{u}} = 4500$ RPM), while the other is allowed to vary.

4.1. Rotation Direction Sensitivity

The system showed a fairly high sensitivity to rotation direction. Regarding thrust to power, ER performed 10% better relative to OR when $d/D_d = 0.535$ or 0.732. There was close to no difference when $d/D_d = 0.906$. Figure 9 shows thrust isolines as a function of mechanical power supplied to each rotor.

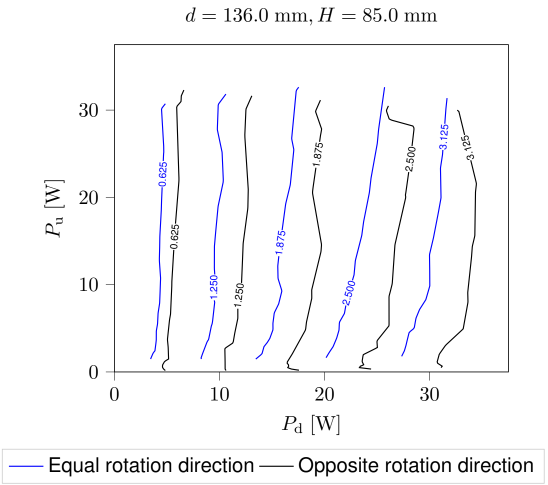

Figure 9: Isolines for downstream thrust as a function of upstream (P_u) an downstream power (P_d), for comparison of rotation direction.

Figure of Merit for a fixed upstream or downstream angular velocity (Ω_u or Ω_d, respectively) are shown in fig. 10. ER is, from the downstream FM point of view, more efficient than OR. It is also of note that, for most distances tested, a steady decrease in efficiency can be observed when Ω_d is fixed and Ω_u increased. For the remaining of the tests done equal rotation direction was used for this reason.

Rotation direction had a clear impact on performance. ER direction performed close to 10% better than OR in close proximity ($d/D_d < 0.732$, $H/D_d = 0.334$ to 0.748). The system as a whole performed slightly better when in OR from the point of view of the overlap coefficient when $d/D_d = 0.732$. In the same conditions, the OR rotor downstream produced more thrust for the same power.

4.2. Upstream Propeller Pitch

A similarly high sensitivity to upstream pitch was verified. A lower pitch had a smaller influence on the downstream rotor at low angular velocities, but as Ω increases both cases behaved similarly.

From the perspective of used power relative to a similar planar configuration (κ_{ov}), it is clear that a larger pitch on the upstream rotor is less effective: the overlap coefficient for the system is significantly

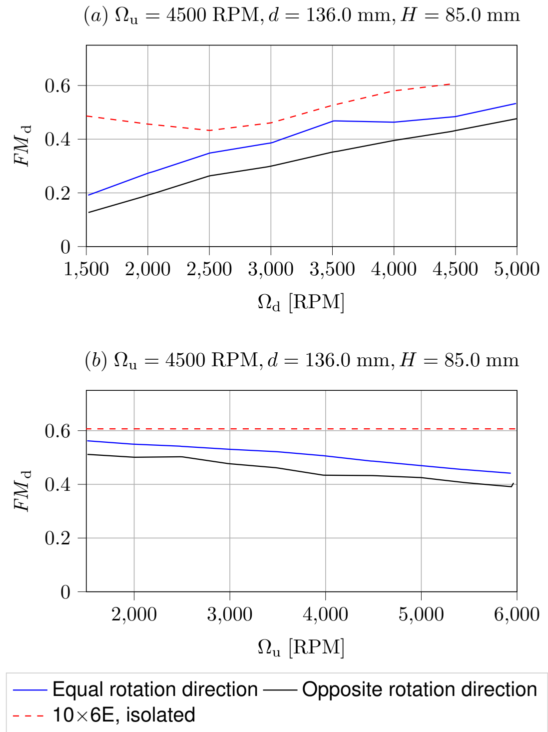

Figure 10: Downstream Figure of Merit as a function of angular velocity, for comparison of rotation directions.

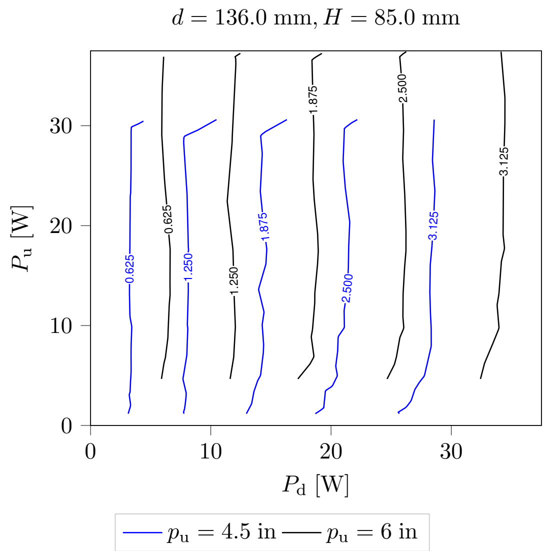

Figure 11: Isolines for downstream thrust as a function of mechanical power on both rotors, for comparison of upstream pitch.

higher. A 6 inches upstream pitch will imply a 30 to 50% increase in power consumption.

Upstream pitch was found to have a moderate effect on performance of the downstream rotor. A larger upstream pitch settled more quickly on some asymptotic behavior, regardless of interrotor

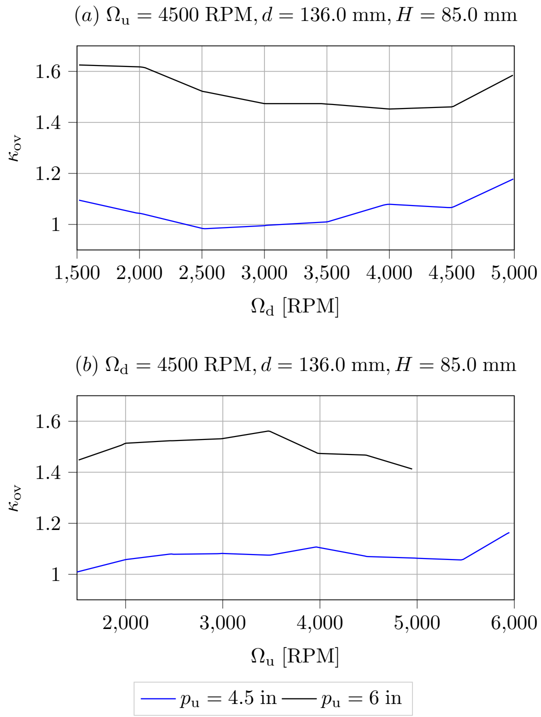

Figure 12: Overlap coefficient κ_{ov}, as a function of Ω. A greater upper pitch provides significantly larger power consumptions overall.

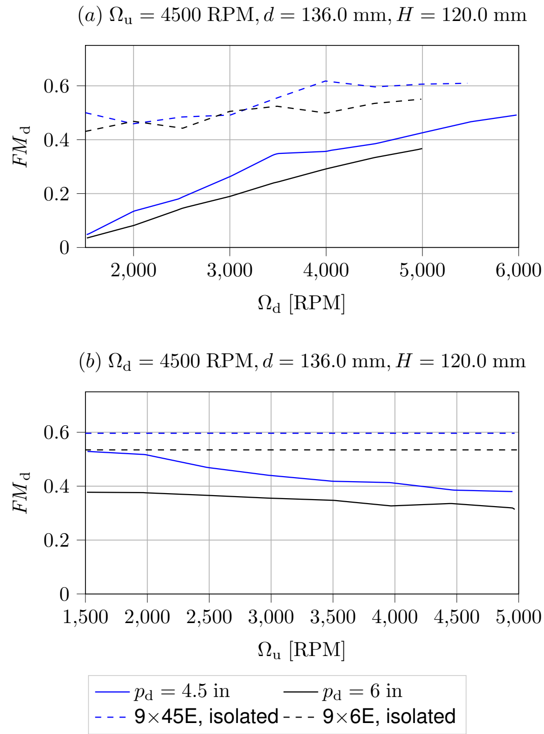

Figure 13: Downstream Figure of Merit for fixed angular velocities for downstream pitch comparison.

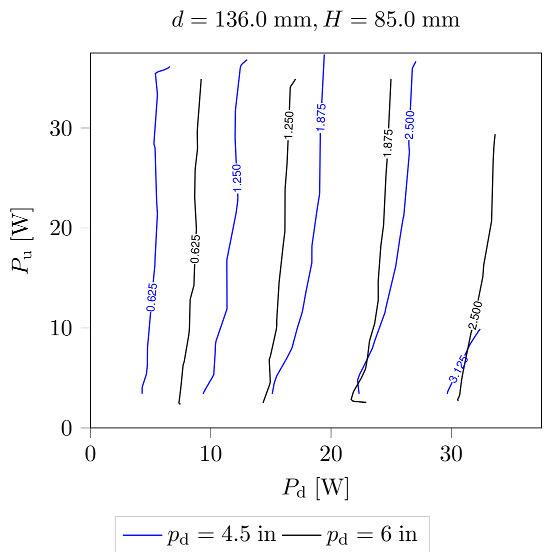

Figure 14: Isolines for downstream thrust, as a function of mechanical power on either rotor, as a measure of downstream pitch sensitivity.

or interaxial distance. The smaller upstream pitch showed a more linear behavior for lift when $d/D_d = 0.535$. At $d/D_d = 0.732$, the smaller upstream pitch showed almost no effect, for $P_u > 10$ W. A very similar effect was observed for FM_d. Total power consumption for the system was significantly higher when $d/D_d = 0.535$, averaging $\kappa_{ov} \approx 1.48$. This suggests that, while the downstream rotor behaves more efficiently, the overall system consumes significantly more power than an equivalent planar system.

4.3. Downstream Propeller Pitch

Downstream pitch showed an asymptotic behavior similar to what was observed with upstream pitch. The differences in the isolated performances were made less relevant by the influence of another propeller upstream, as can be seen in fig. 13(b).

Similar observations can be made from the isolines in fig. 14: a larger pitch is less affected by an increase in upstream power. The smaller-pitched propeller suffers a much more significant impact as P_u is increased, but is still more effective at generating thrust.

4.4. Upstream Propeller Diameter

Upstream diameter had an almost insignificant impact on the lower rotor. Even on the closest proximity tested the inclination of isolines for FM is essentially identical, as can be seen in fig. 15. Some differences can be observed at lower RPM_d and high RPM_u. However, as RPM_d increases, this difference becomes negligible.

In fact, any of the previously mentioned metrics reveal a very small importance of upstream diameter on overall system performance, or even downstream rotor performance. In particular, fig. 16

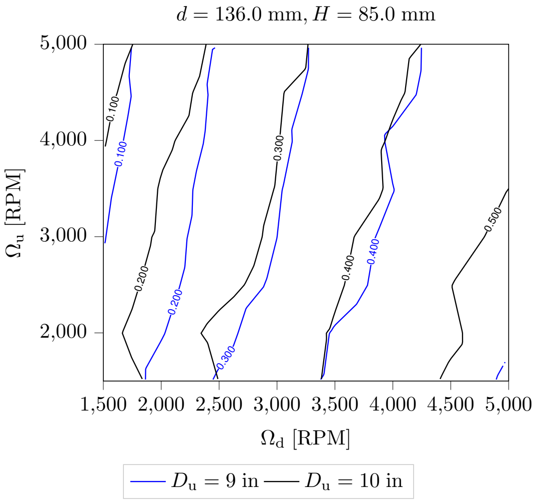

Figure 15: Downstream rotor's Figure of Merit, as a function of angular velocity on either rotor. Sensitivity to upstream diameter was not measurably different.

shows the evolution of κ_{ov} with Ω.

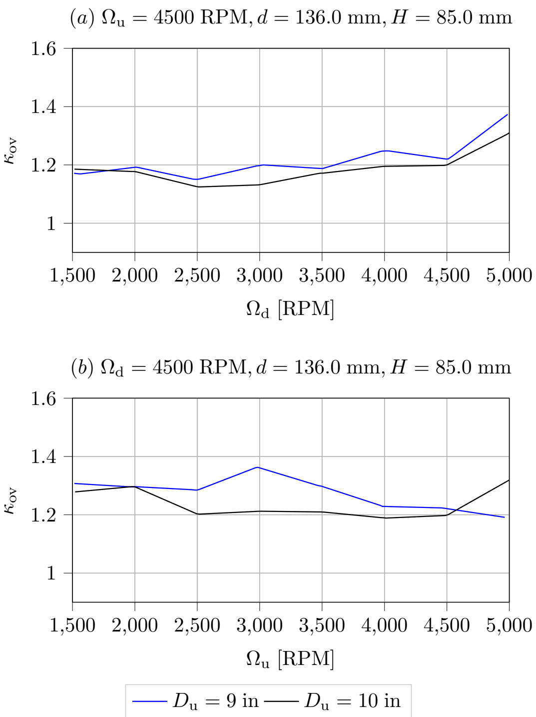

Figure 16: Overall power consumption of the system (relative to a planar configuration) as a function of angular velocity. A smaller upstream diameter showed slightly larger power consumptions in general.

4.5. Downstream Propeller Diameter

Downstream diameter was found to have a larger influence on overal and downstream performance. This is mostly expected: as downstream diameter decreases, so does m' for the same interaxial distance d. This, in turn, implies that a larger influence is seen on the performance of the downstream propeller, as more of its area is under the influence of the upstream propeller, relative to its total area.

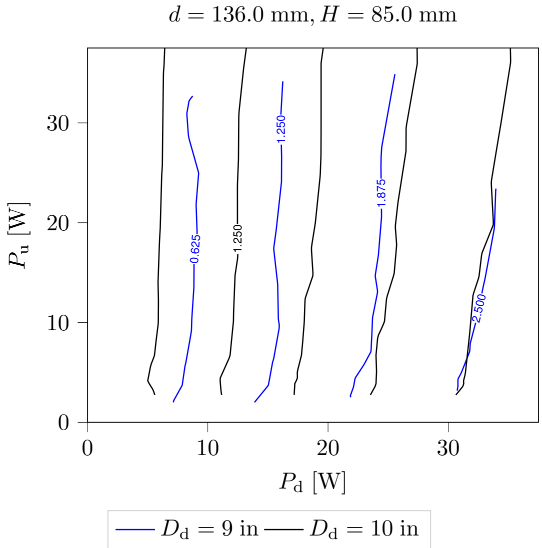

Figure 17: Downstream thrust isolines, as a function of mechanical power. Increasing downstream diameter does not increase sensitivity, as isolines are parallel to one another.

Differences in FM are made more extreme by changing downstream rotor diameter, as is seen in fig. 18. In general, at constant power or Ω on either rotor, the magnitude of the differences between the downstream rotors increased if D_d was varied.

It is also the case that, as a system, a smaller D_d implies more power consumption relative to the planar configuration. For $D_d = 10$ inches, an average κ_{ov} of 1.25 was observed, whereas for $D_d = 9$ inches, $\kappa_{ov} \approx 1.35$.

4.6. Interrotor Distance

Interrotor distance had almost no noticeable effect on performance. This is likely due to fact that the upstream wake is already fully developed, such that changing H has little effect on the downstream rotor. Figure 19 shows the Figure of Merit as a function of either rotor's angular velocity. All isolines are fairly close together and have similar inclinations, suggesting that, indeed, not much sensitivity to H exists in the range $H/D_d = [0.334 - 0.748]$.

In terms of system power consumption, the overall trend confirms theoretical estimations that higher H show lower κ_{ov}, and vice-versa.

Isolines for FM_d for $d/D_d = 0.535$ showed that, for $H/D_d \geq 0.610$ the downstream rotor was more impacted by the upstream rotor, as isolines had

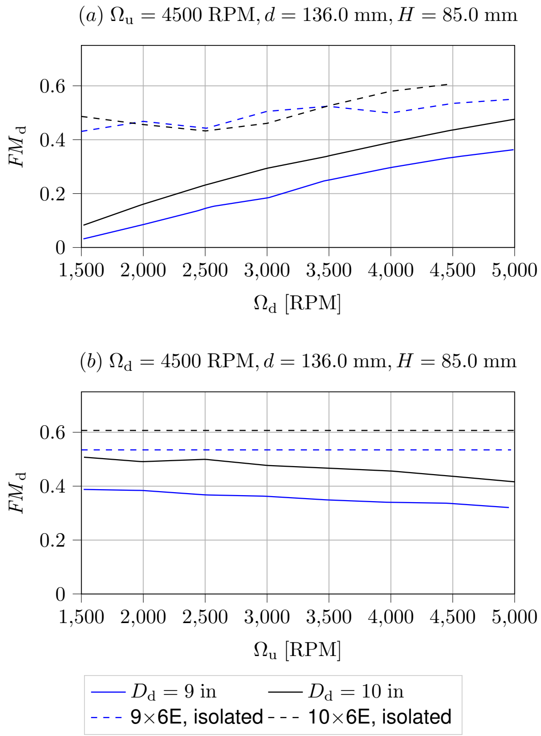

Figure 18: Downstream Figure of Merit as a function of angular velocity. Differences in isolated performance are driven further apart when in the presence of another rotor upstream.

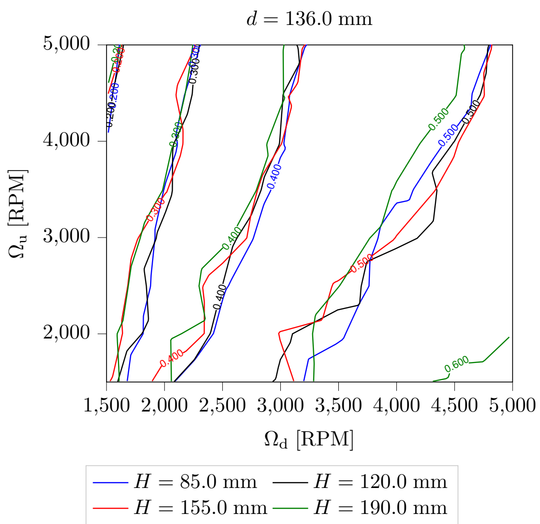

Figure 19: Downstream Figure of Merit isolines, as either rotor's angular velocity is varied. Higher distances appear to provide a small improvement in FM for the same pair of Ω.

more inclination. H/D_d = 0.472 to 0.335 showed no difference in either FM or thrust. No significant impact could be noticed for $d/D_d \geq 0.732$, both in terms of downstream rotor performance and system

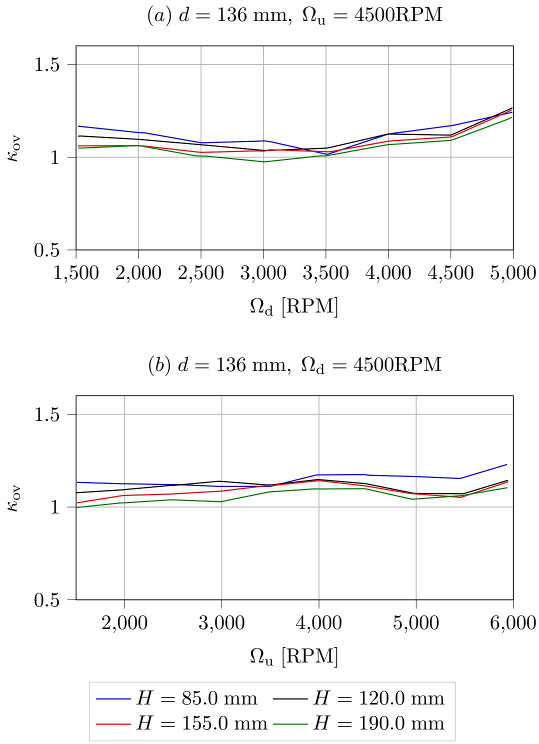

Figure 20: Overlap coefficient κ_{ov}, as a function of Ω. Larger H provide a small improvement in overall power consumption.

power consumption.

4.7. Interaxial Distance

Interaxial distance d showed a much higher impact on the system overall and on the thrust of the rotor downstream. The best performance was achieved at $d/D_d = 0.732$, for the range H/D_d = [0.334 - 0.748]. In figure 21, $H/D_d = 0.748$. It can be seen that the intermediate distance provides more thrust for the same power, being slightly better than the isolated rotor when P_d is in the range [5 - 10] W.

The same observation can be made for FM, seen in fig. 22. When d = 186 mm, the performance of the downstream rotor is significantly better and suffers very little performance losses from the effect of the upstream rotor.

Overall power consumption of the system showed similar results: the largest d showed the greatest κ_{ov}. This was unexpected from a theoretical point of view, where a larger d/D_d would yield a smaller κ_{ov}. For d/D_d = 0.535 or 0.732, κ_{ov} was essentially equal. Fig. 23 shows κ_{ov} as a function of Ω.

An important observation should also be made regarding downstream rotor performance: for small RPM on the upstream rotor ($\Omega_u \leq 2500$ RPM), d/D_d = 0.535 performed better than a larger d/D_d. More research should be conducted to iden-

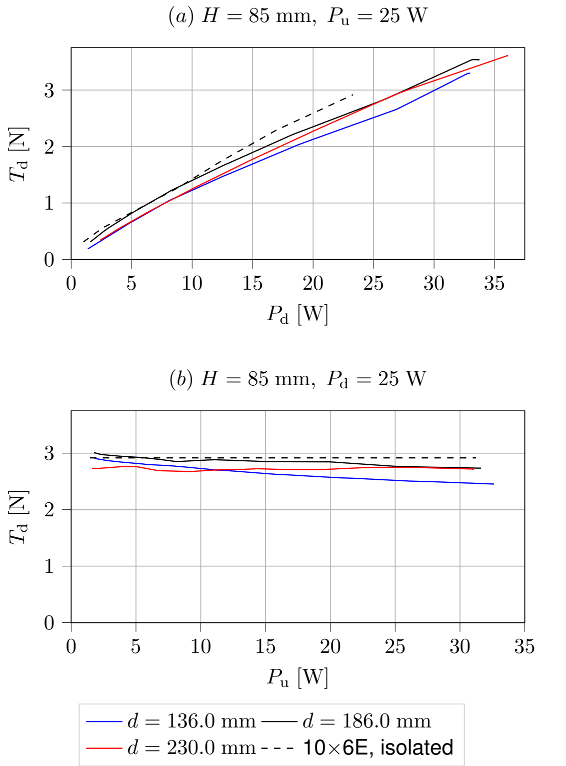

Figure 21: Thrust produced by the downstream rotor, for interaxial distance comparison. Downstream thrust generation was slightly better than an isolated rotor for some intervals of P_d.

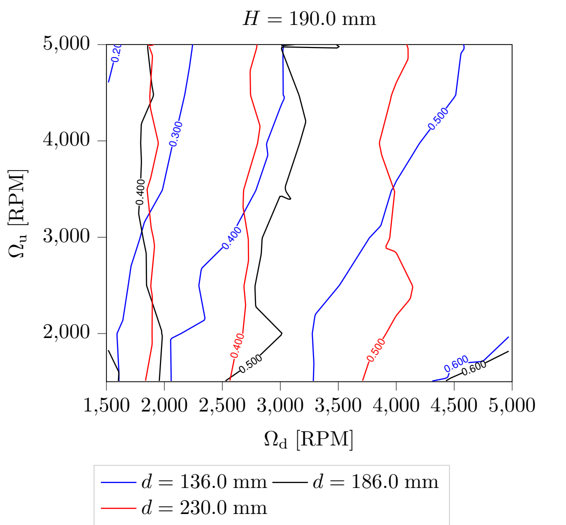

Figure 22: Figure of Merit isolines, for interrotor distance comparison. It is clear that, up to a Ω_u threshold, a smaller d/D_d is preferrable.

tify whether these results can be expanded to include a larger range of operation, and the magnitude of the improvement.

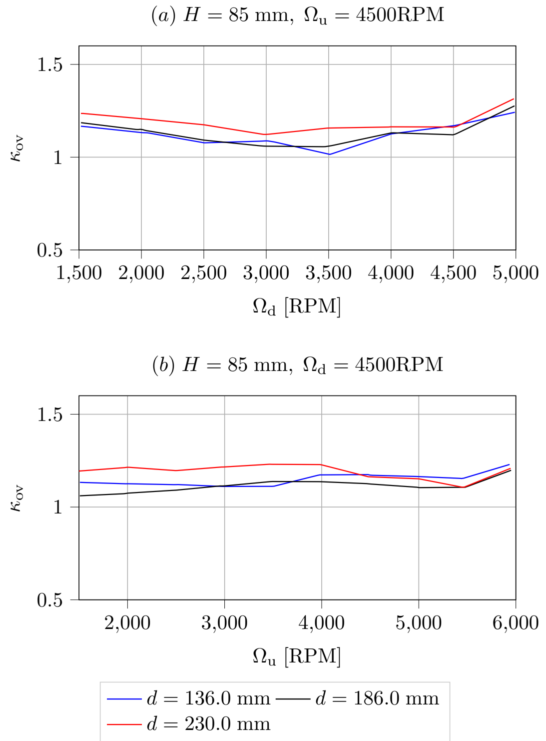

Figure 23: Overlap coefficient, κ_{ov}, as a function of either rotor's Ω. Unexpectedly, the largest d (or d/D_d) showed a slightly greater κ_{ov}, for small RPM.

5. Conclusions

An experimental test-bench originally designed for coaxial configurations was modified for analysing tandem rotors. Measurements were done in a systematic and as consistent as possible way, to identify key parameters to each parameter. Data was collected with flexibility in post processing in mind, using a *python* program to compute the required parameters and generate the plots for further analysis. These plots allow for easier identification of where and in which conditions each configuration is more performant, either relative to a planar configuration or relative to one another.

Some performance improvements over a planar configuration could also be identified. For example, when $H/D_d = 0.334$ and $d/D_d = 0.732$, where a small but noticeable improvement in performance over an equivalent configuration was observed. Expanding this range to include more common use cases is undoubtedly relevant for improving the performance of these propulsion systems.

To further identify the influence of unexamined parameters such as motor constant K_v might be of some importance in the design process of a UAV. The influence of the downstream rotor on the upstream one may be relevant for a further analysis. It was left unexplored in this work, as it is usually assumed to be of small magnitude for most conditions. Constraints of the test-bench did not allow for the lift of both rotors to be measured simultaneously.

To further simulate the behavior of a drone, the introduction of a second upstream rotor could be of interest. In fact, projects were made for this addition, but this was not implemented due to issues with the supply of parts.

Lastly, estimation of parameters not only for hover, but for forward flight and vertical movement is of some importance. It was investigated, in part, in [4], but no empirical studies exist on the topic.

Acknowledgements

The author would like to thank supervisors Filipe Cunha and José Azinheira, for the continued support in the experimental and editorial process. I am also grateful to IST and its faculty, for the facilities and equipment provided. A thank you is also in order to professor André Marta, for lending equipment and expertise, especially regarding safety advice.

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