Staff rescheduling with minimum disruption at Emergency Medical Services

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“Deus quer, o homem sonha, a obra nasce”

in *A Mensagem*, Fernando Pessoa

Declaration

I declare that this document is an original work of my own authorship and that it fulfils all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Declaração

Declaro que o presente documento é um trabalho original da minha autoria e que cumpre todos os requisites do Código de Conduta e Boas Práticas da Universidade de Lisboa.

Abstract

Emergency Medical Services (EMS) play a critical role in pre-hospital care and directly affect the medical outcome of emergency patients. Given the increase of emergency requests and the restricted resources that EMS systems have at their disposal, there is an urgency to operate at the highest efficiency. In order to find fields to improve, planning problems at EMS were studied and it was found that rescheduling, which is the process of finding a new feasible schedule after a disruption, constitutes an interesting area to develop further analysis.

Since the Portuguese Emergency Medical Institute – INEM – still performs this complex task manually and, being an almost daily activity, there is the opportunity to improve its efficiency by developing more sophisticated methods capable of providing strong insights to decision-makers. Therefore, a mathematical model was developed in order to assist EMS systems on their rescheduling activity, providing accurate solutions in a short amount of time.

To assess its effectiveness, the model was tested for different INEM scenarios, including the most extreme absenteeism cases in the EMS sector and delivered optimal solutions always in less than 11 minutes. Additionally, it was also possible to compare the differences between starting the month with a cyclic and a non-cyclic schedule. Results show that the non-cyclic outperforms the cyclic in both under and oversupply situations due to its flexibility to adapt to different contexts. Therefore, as INEM currently builds a cyclic initial schedule, it could be beneficial to consider a shift to non-cyclic.

Keywords: Emergency Medical Services, Staff Rescheduling, Optimization, Mathematical Programming, Health Services, Human Resources Planning

Resumo

Os EMS desempenham um papel crítico nos cuidados pré-hospitalares e sua capacidade de resposta afeta diretamente o resultado médico dos pacientes em casos de emergência. Dado o aumento dos pedidos e os escassos recursos que os sistemas de EMS têm à sua disposição, existe a necessidade de atuar com a máxima eficiência. Assim, com intuito de descobrir áreas por melhorar, os problemas de planeamento dos EMS foram estudados e verificou-se que o reescalonamento, processo de reconstrução do horário após uma disrupção, seria uma área de interesse para desenvolver uma análise aprofundada.

Uma vez que o INEM ainda executa esta complexa tarefa manualmente e, sendo uma atividade quase diária, existe a oportunidade de melhorar a sua eficiência através do desenvolvimento de métodos mais sofisticados capazes de gerar informações relevantes para os decisores. Por conseguinte, foi desenvolvido um modelo matemático para ajudar os sistemas EMS a resolver este problema, que pretende fornecer soluções precisas em pouco tempo, para responder diariamente.

Para comprovar a sua eficácia, o modelo foi testado para diferentes cenários do INEM, incluindo os casos mais extremos de absentismo no sector do EMS, tendo encontrado soluções ótimas sempre em menos de 11 minutos. Adicionalmente, foi também possível comparar a resposta usando um calendário cíclico e um não cíclico. Os resultados evidenciam que, neste caso, o não cíclico supera o cíclico devido à sua flexibilidade para se adaptar a contextos diferentes. Por conseguinte, como o INEM atualmente constrói um calendário inicial cíclico, poderia ser benéfico considerar uma mudança para não cíclico.

Palavras-chave: Serviços de Emergência Médica, Reescalonamento de Staff, Otimização, Programação Matemática, Serviços de Saúde, Planeamento de Recursos Humanos

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When I first entered Técnico's building in 2016, I never expected to have such a thrilling experience. After 5 years, this dissertation marks the end, for now, of my academic path. There are so many stories to tell and many people to thank. Let us stay with the people.

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List of Acronyms

A – Afternoon

AEM – Ambulância de Emergência Médica (Medical Emergency Ambulance)

ALS – Advance Life Support

BLS – Basic Life Support

CIAV - Centro de Informação Antivenenos (Antipoisoning Information Centre)

CODU - Centro de Orientação de Doentes Urgentes (Urgent Patients Orientation Centre)

COVID19 – Corona Virus Disease 2019

DGS – Direção Geral de Saúde (General Health Office)

EMS – Emergency Medical Services

FM – Financial Management (Department)

FMSE – Fuzzy Multi-criteria Simulated Evolution

GDP – Gross Domestic Product

HRM – Human Resources Management (Department)

HU – Health Unit

iCARE – Integrated Clinical Ambulance Record

ILP – Integer Linear Programming

ILS – Immediate Life Support

INEM – Instituto Nacional de Emergência Médica (Medical Emergency National Institute)

IM – Installations Management (Department)

LO – Logistics and Operations (Department)

LP – Linear Programming

M – Morning

ME – Medical Emergency (Department)

MEM – Medical Emergency Motorcycle

MET – Medical Emergency Training (Department)

MIP – Mixed Integer Programming

N – Night

NAP – Nurse Addition Problem

NHS – National Health Service

NINEM – Non-INEM Ambulances

O – Day Off

CDOS – Centro Distrital de Operações de Socorro (Aid Operations District Centre)

CAPIC – Centro de Apoio Psicológico e Intervenção em Crise (Psychological Support and Crisis Intervention Centre)

PEM – Ambulâncias em Postos de Emergência Médica (Medical Emergency Stations)

PMPH – Purchase Management and Public Hiring (Department)

PSP – Polícia Segurança Pública (Public Security Police)

RD – Regional Delegations

RES – Ambulâncias em Estações de Reserva (Reserve Station Ambulances)

SA – Simulated Annealing

SE – Simulated Evolution

SHEM – Serviço de Helicópetro de Emergência Médica (Medical Emergency Helicopter Service)

SIADEM – Sistema Integrado de Atendimento e Despacho de Emergência Médica (Integrated System of Medical Emergency Answering and Dispatching)

SIEM – Sistema Integrado de Emergência Médica (Integrated Medical Emergency System)

SIT – Systems and Information Technologies (Department)

SIV – Suporte Imediato de Vida (Immediate Life Support Vehicle)

SNS – Serviço Nacional de Saúde (National Health Service)

SNS24 – Serviço Nacional de Saúde 24

SUMC – Serviço de Urgência Médico-Cirúrgico (Urgency Medical-Surgical Services)

SUP – Serviços de Urgência Polivalente (Urgency Polyvalent Services)

TEPH – Técnicos de Emergência Pré-Hospitalar (Pre-Hospital Emergency Techincians)

TETRICOSY – TElephonic TRIage and COunseling SYstem

TIP – Ambulâncias de Transporte Inter-Hospitalar Pediátrico (Inter-Hospital Pediatric Transport Ambulances)

UMIPE – Unidade Móvel de Intervenção Psicológica de Emergência (Mobile Unit of Phycological Emergency Intervention)

VMER – Viatura Médica de Emergência e Reanimação (Vehicle of Medical Emergency and Reanimation)

VND – Variable Neighbourhood Descendent

1. Introduction

Technological improvement and easier access to health care services, alongside a better distribution of goods and better living conditions, have increased the medium life expectancy worldwide. In Portugal, in the last 20 years, it increased more than five years to 81 years (Simões *et al.* 2017). This factor has contributed to an increase in the proportion of elder people in society. In Portugal, in the last 20 years, the ratio of an elder person per youngster increased from 1 to 1.61 (PORDATA, 2020). Consequently, the need for proper health care services has been increasing over the past years.

Effectively, in 2018, Portugal spent 18,300 million euros on health care, which means an expense of 1,784 euros per capita and 5.1% more than in the previous year, which represents 9.1% of the national Gross Domestic Product – GDP (INE, 2019). Within the health care service, there are emergency medical services, which are designated to save lives and play a key role in pre-hospital medical care, directly impacting the medical outcome of emergency patients (World Health Organization, 2005). Although, in 2018, Portugal had the same percentage of expenses allocated to emergency medical services as the average of the European Union, the percentage of the GDP allocated to health care services was almost 10% lower when compared to the average of 9.9% from the European Union (Eurostat, 2021). These factors may indicate that, in Portugal, the Emergency Medical National Institute – INEM – operates with scarce resources.

Thus, in this context, arises the need to deliver the most effective and efficient service to patients using restricted available resources. Operations Research techniques have been applied to a variety of problems in health care environments and have largely contributed to providing better solutions for these problems (Brailsford & Vissers, 2011). Personnel scheduling is one of the most extensively studied topics as it can directly influence the performance of the health organization and staff costs have the highest proportion on the overall costs of a health organization (Clark *et al*. 2015).

However, staff rescheduling, which is the task of rebuilding a schedule that suffered a disruption by an absent employee, has not received a lot of attention. Although it has been considered as a complex and time-consuming activity, accounting for 10-20% of managers' day-to-day activities, few academic research has been made on this topic (Clark *et al*., 2015).

In most organizations, rescheduling is still a task performed manually and few models have been implemented due to lack of staff training, high complexity of solutions or lack of financial resources to implement them. INEM is no exception and still performs this activity manually. In fact, INEM planners must often rely on intuition and experience to make challenging planning decisions in the face of uncertainty, limited by budget restrictions and balancing various stakeholders' objectives. Therefore, there is room for improvement, to achieve a higher efficiency which will lead to a higher service level.

This context motivates the present study, which addresses the staff rescheduling problem in the EMS field, exploiting particularly the case of INEM. The use of more sophisticated techniques to support decision-making could potentially contribute to more effective and efficient solutions, which may enhance the overall performance of an organization.

1.1 Dissertation Goals

The ultimate goal of the dissertation is to, in the context of EMS, automatize the staff rescheduling process, improving, not only the overall quality of the final schedule but also the sense of fairness and satisfaction in the workforce. The proposed model should be flexible to allow different goals and to be able to cope with different EMS systems. Additionally, the model results must be analysed in order to provide key insights to INEM that, at the same time, may be relevant and adaptable to all EMS systems.

Besides these main goals, there are also secondary objectives, such as (i) characterizing the Portuguese EMS, focusing on the most important aspects of its operation, highlighting their complex nature, (ii) assessing what previous and current practices are being used by EMS practitioners in order to identify potential gaps and proposing alternative approaches and (iii) contribute to the existing literature and possibly to increase the efficiency of INEM's operations.

1.2 Methodology

To achieve the aforementioned objectives, a research methodology with eleven steps is proposed, as can be observed in figure 1.

The first step is (1) Problem Definition, which includes the characterization of the Portuguese Emergency Medical Service, focusing on the most important aspects of its operation, highlighting their complex nature, to then be able to propose and define the problem that will be addressed. This step enables a clear overview of the context. The second is (2) Literature Review, which assesses previous literature on both the emergency medical services field and the rescheduling process, in order to identify research gaps and fundaments that support the development of the model.

Following, the third step is (3) First Mathematical Problem Definition. This is done taking key aspects from the literature studied, namely from the most relevant studies from Maenhout & Vanhoucke (2011), Maenhout & Vanhoucke (2018), Wickert *et al.* (2019) and Wolbeck *et al*. (2020). This includes, for example, the fair penalization scheme proposed by Wolbeck *et al*. (2020). Then, in step (4), a first draft of the problem and its characteristics are presented to INEM's TEPHs, in order to get a clear overview of what are the most important factors to take into account and if those being considered are relevant, e.g., it is crucial to understand if the different stakeholders prefer fairness and employees' satisfaction over service quality or overall costs for the organization. The data collected from these interactions is treated in step (5), extracting the main conclusions that will be then presented to TEPH's responsible, following a review in step (6), having in mind its validation.

In case of validation, it is possible to construct the (7) Final Mathematical Problem Definition, determining then sets, subsets, parameters, decision variables, the objective function and constraints. After, in step (8), it will be possible to implement the model with CPLEX. In step (9), the model will be solved, considering progressively more complex instances from INEM's dataset. If these solutions are validated, then a strong analysis will be performed in step (10). This analysis intends to understand which is the best initial schedule type, cyclic and non-cyclic and how it can affect the results. Then, in (11) this analysis will be presented, reviewed and subject to validation from the TEPH's responsible. Additionally, recommendations to INEM are given.

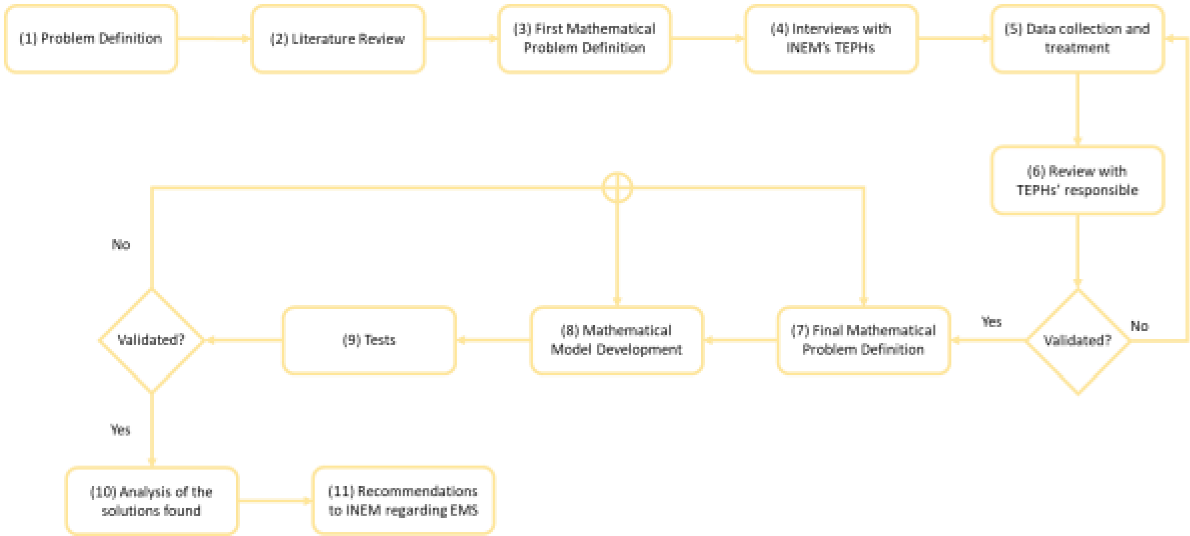

Figure 1: Proposed methodology for the future dissertation

1.3 Thesis Outline

The dissertation is structured into seven chapters:

- **Chapter 1 – Introduction**

The dissertation starts with the current chapter, an introduction comprehending the motivations of this study and what the goals pretended to be achieved through a research methodology. Furthermore, it specifies the structure of the document.

- **Chapter 2 – Problem Definition**

The second chapter introduces INEM and explains in detail SIEM's activities, from emergency request to patient treatment. Following, the problem at stake is defined and its importance is explained in the context of INEM.

- **Chapter 3 – Literature Review**

The third chapter reviews the literature on emergency medical planning problems, concluding that rescheduling is worth being deeply studied. It also reviews the approaches researchers have been proposing over the last years for rescheduling problems in different fields, which is crucial for the development of the model.

- **Chapter 4 – Model Formulation**

This chapter introduces a refined problem statement and, supported by the methodologies explored in the literature review and INEM's case study, an optimization model to solve EMS staff rescheduling problems is presented, detailing the sets, subsets, parameters, variables and the constraints required to minimize the objective function that seeks to generate as little disruption as possible for workers, maintaining a good service level.

- **Chapter 5 – INEM Case Study and Instance Generation**

  In chapter 5, the data required to apply the model above defined to the real case of INEM is introduced, explaining how data was collected. It also provides a more detailed exposition of the problem context started in chapter 2 and, finally, the instance structured is presented as well as the tests done in order to assess the model accuracy to find solutions for different scenarios.

- **Chapter 6 – Results and Discussion**

  Chapter 6 exhibits the results obtained for the different experiments, takes key insights from them and makes make recommendations both specific for INEM and generally applicable to all EMS systems.

- **Chapter 7 – Conclusion and Future Work**

  The last chapter summarises the most relevant insights and conclusions of this study, highlighting opportunities for future work.

2. Case Study Description

This chapter aims to describe the case study which is addressed in this Master dissertation. It starts by providing an overview of the Portuguese entity responsible for the Emergency Medical Services – INEM – to have a wide view of the context. Then, this analysis will be narrowed and focused on the rescheduling at INEM.

Section 2.1 introduces INEM to better perceive its mission in the Portuguese society and define its organizational structure, focusing on the operational related areas. Next, it explains in detail the whole set of processes that constitute pre-hospital care. Section 2.2 discusses rescheduling at INEM for a specific class of workers, TEPHs (Pre-Hospital Emergency Technicians). Due to increasing demand and lack of resources, efficiency improvement can be a pillar to fulfil the population's requirements. Considering the aforesaid analysis, Section 2.3 presents the research problem definition. At last, section 2.4 presents the chapter conclusions and main considerations.

2.1 Portuguese Emergency Medical Service

The Medical Emergency National Institute – INEM – was established, in 1981, as the entity of the Portuguese Health Ministry responsible for running the Integrated Medical Emergency System – SIEM –, ensuring a prompt and correct pre-hospital health care provision in mainland Portugal (INEM, 2020). In order to achieve its mission, INEM must define, organize, coordinate and assess SIEM operation. These activities can be divided into two groups: (i) in-scene emergency and respective support activities and (ii) administrative and planning activities. The first group includes the following:

- Emergency request reception, triage and counselling;
- Proper medical emergency vehicles dispatch and pre-hospital medical care provision at the scene;
- Transportation of patients to the hospital;
- Correct coordination between the adequate Health Units (HU) (e.g., hospitals and Portuguese Red Cross) and other entities (e.g., police officers, firefighters).

The second group encompasses activities such as:

- Planning and coordination of medical emergency training for staff and National Health Service – SNS – entities, as well as competencies' certification;
- Continuous designing of emergency civil planning policies (e.g., Covid-19 measures), partnering with the General Health Office – DGS;
- Collaboration in emergency and catastrophe plan development with Health Region Administrations, the General Health Office and the Civil Protection National Authority (e.g., fires and natural disasters);
- Development of prevention, awareness and information actions, concerning SIEM and its activities;
- Maintenance and extension of a wide emergency telecommunications network.

2.1.1 Organizational Structure

There are three Regional Delegations (RD) responsible for the operational management of INEM's activities, working as Decentralized Services. These RDs are established in the North, Centre and South of Portugal, and work in collaboration with the Centralized Services, which cover three main areas: (i) Operational, (ii) Logistics and Operations Support and (iii) Management Support (INEM, 2019).

Considering the Operational area, it includes two departments: Medical Emergency (ME) and Medical Emergency Training (MET). The ME Department coordinates SIEM, on both normative and technical sides, and assesses periodically its performance. In 2019, 42 people worked directly in this department, involving Pre-Hospital Emergency Technicians (TEPH), Nurses, Doctors and Leaders. The MET Department, besides defining and planning the medical emergency training, also guides and executes the training for the different SIEM entities. In 2019, 17 people were shared by both departments.

Concerning the Logistics and Operations Support area, it includes seven departments: Human Resources Management (HRM), Financial Management (FM), Logistics and Operations (LO), Systems and Information Technologies (SIT), Purchase Management and Public Hiring (PMPH), Juridic and Installations Management (IM). For this study, we are going to focus only on the LO Department and on the SIT Department.

The main function of the LO Department is to manage INEM's fleet, assuring its successful operation as well as defining which patients should be transported and in which vehicle. In 2019, 50 people worked directly in this department, mainly TEPH and Technical Assistants. The SIT Department is responsible for implementing and maintaining information systems in INEM and defining interfaces with other internal and external information systems. The department aims to guarantee the homogenization and optimization of procedures.

Regarding the Management Support area, it is composed of three departments: Quality, Planning and Control Management (PCM), and, finally, Marketing and Communication. Lately, the PCM Department is gaining more importance in INEM's activities, measuring the accurate KPI's, assuring that the organization meets its goals (INEM, 2019).

In figure 2, it is possible to observe a graphical summary of the information explained above, presenting the organizational structure of INEM.

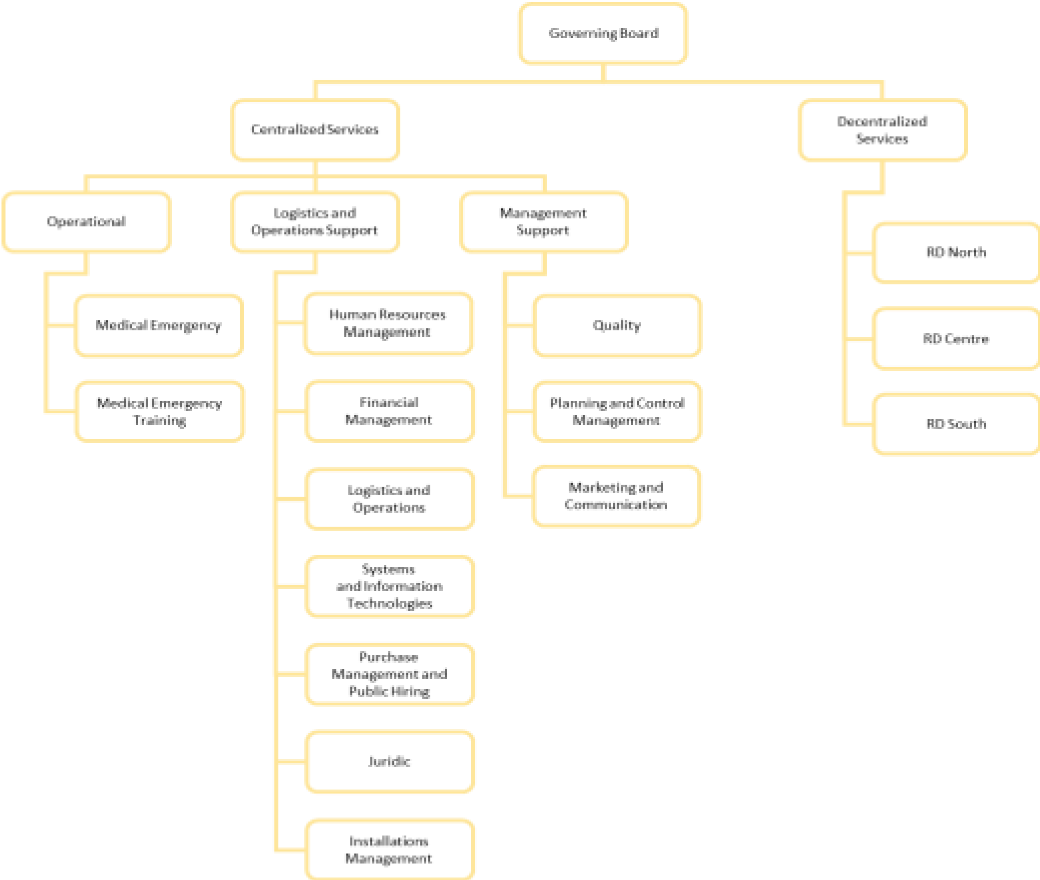

Figure 2: INEM's organizational structure (INEM, 2020)

2.1.2 From emergency request to patient treatment

In this section, the patient's emergency care pathway, in terms of pre-hospital care, is going to be presented and SIEM's most relevant activities for this analysis will be studied afterwards. SIEM is a set of coordinated activities executed by different entities within the emergency medical care delivery structure, including INEM, firefighters, police officers, the Portuguese Red Cross, hospitals and health units.

Firstly, an incident that requires emergency assistance is detected, usually by civilians near the event, that must call the emergency number – 112 –, which, in case of a medical emergency, is directed to a Centre of Orientation of Urgent Patients – CODU. After receiving the information and prioritizing the case, an emergency vehicle with a crew may be dispatched (normally, at least one TEPH is present). While it is reaching the incident, if needed, the caller may be receiving instructions on Basic Life Support by the TEPH at the CODU. Once the emergency vehicle arrives, medical care to stabilize the victim is provided. When necessary, while the victim is transported to a health unit, care continues to be provided in transit (Ferreira dos Santos, 2019). Finally, the victim is received at the closest health unit to continue the required treatment. At this moment, the vehicle and the crew are released and become available for the next operation (INEM, 2020).

2.1.2.1 Call reception and Triage

Emergency calls to 112 are handled in two Operational Centres managed by the Public Security Police - PSP. The North Operational Centre answers calls from districts north of Leiria and Castelo Branco, while the South Operational Centre answers the remaining calls. Police officers evaluate the situation and direct the call to a specific SIEM stakeholder who is able to better respond and solve the incident. In case of a health-related emergency, the call is sent to a CODU (Ferreira dos Santos, 2019).

The four CODUs operate on a 24/7 basis in mainland Portugal and their activity is managed by TEPHs, backed by doctors and psychologists. In these centres, TEPHs are required to screen and prioritize each situation, advise in-scene actors and dispatch the adequate emergency vehicle and crew, in order to ensure an accurate response. The first National CODU was founded as a result of the regional differences in emergency call answering in 2011, improving the service level significantly by reducing the answer time, i.e., from the moment a police officer transfers the call to CODU until a TEPH is in line, from 13 (2010) to 7 seconds (2011). Besides 112 calls, CODU also answers to other types of requests: (i) inter-hospital emergency transportation requests, which are treated as 112 calls and (ii) calls transferred from SNS 24, a 24-hour service that provides medical advice through the phone. In 2020, CODU predicts to have answered 1.443.155 calls, around 3.943 per day with a lead time, i.e., since a person calls to the moment a CODU TEPH is in line, of 18s.

Table 1: Evolution of Emergency Calls (INEM, 2020)

Emergency Calls	**2017**	**2018**	**2019**	**P 2020**
Answered Calls (Total Per Year)	1,368,141	1,393,594	1,414,858	1,443,155
Answered Calls (Average Per Day)	3,748	3,818	3,876	3,943
Answering Time (Average in seconds)	36	26	24	18

Since 2012, CODUs began a partnership with SNS 24, where all non-urgent calls are transferred from CODU to SNS 24, and all the urgent calls go the opposite way. This measure was key to improve efficiency, allowing INEM to allocate its resources to situations that are, in fact, urgent. In 2020, CODU transferred 95.601 calls to SNS 24, around 262 per day, representing a 34% increase from 2019. The number of calls transferred from SNS 24 to CODU was 76.012, around 208 per day, representing a 24% increase from 2019 (INEM, 2021a). This growth may be explained by the COVID 19 pandemic, as SNS 24 was the entity responsible for tracking COVID 19 infected. Figure 3 shows the growth of each type of calls since 2015, which leads to an efficiency increase, as the population receives more accurate assistance to their needs.

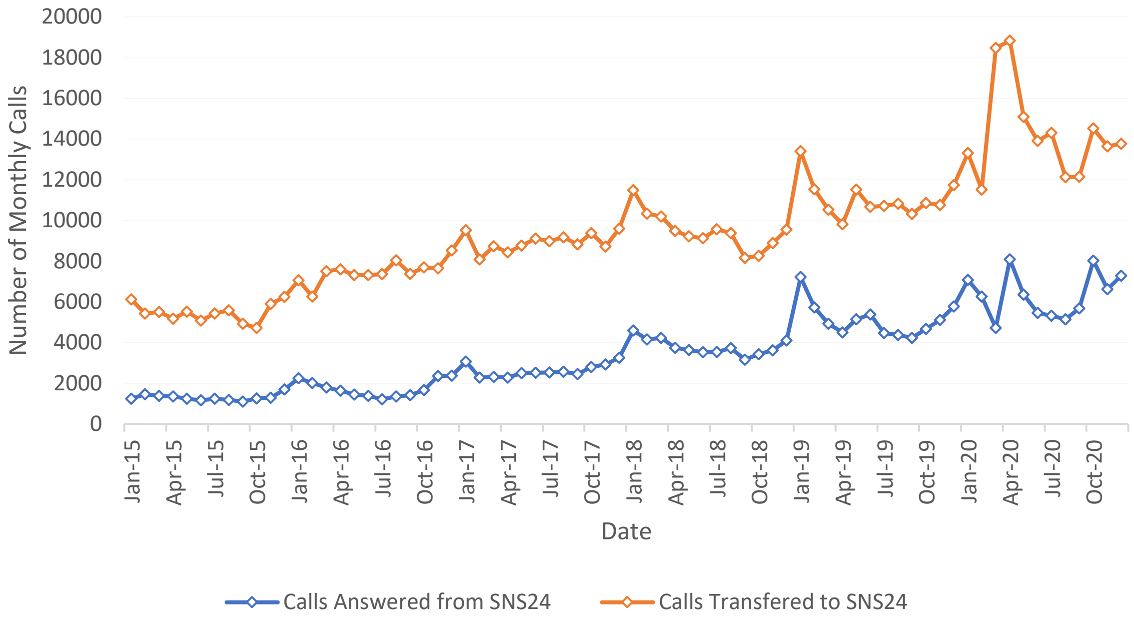

Figure 3: Evolution of the monthly number of medical emergency calls answered from and transferred to SNS 24, from 2015 to 2021 (INEM, 2021a)

Emergency calls are answered by the TEPH who has been idle for the longest time, regardless of the caller's location or the TEPH's CODU (Ferreira dos Santos, 2019). Then, the triage stage starts, in which the TEPH asks the caller specific questions provided by the medical triage algorithm TETRICOSY (TElephonic TRIage and COunseling SYstem). As the TEPH records the answers in the software, priority levels change and new questions are suggested. Finally, TETRICOSY Response Plan will determine if and which emergency vehicle and crew should be assigned. This model is based on a set of decision support algorithms that ease TEPHs' job, allowing higher objectivity, while reducing possible errors. This triage is based on the following classification:

Table 2: Priority Classification (INEM, 2019)

P1 – Emergent Situations	Life-threatening situation, where the victim requires immediate intervention
P3 – Urgent Situations	Not immediate life-threatening situation but the victim requires assistance in a short period of time
P5 – Non-Urgent Situations	No vehicles or crew are required at the scene and the call may be transferred to SNS24
Other Priorities – CIAV, P4 Authority, P4 CDOS, P6 CAPIC	No vehicles or crew are required at the scene and different assistance may be given (e.g., psychological and anti-poisoning assistance)

In figure 4, the evolution of the number of monthly emergency calls by priority is shown and it is possible to conclude that there is a general growth since 2015, except for 2020. In addition, most of SIEM's activities are driven by occurrences of Priority 3. Moreover, Priority 3 accounts for 67% – 78% of the total emergency calls answered per month, being the minimum percentage observed during the first lockdown in Portugal, from March to April of 2020 when more people were at home, contributing to a smaller number of emergency requests. The second most frequent is Priority 1, accounting for 10% – 14% of the emergency calls, which was also reduced drastically during the first lockdown. Then, Priority 5 accounts for 6% – 16%, having reached the maximum value at the beginning of the COVID 19 pandemic. Since SNS 24 is the entity responsible for tracking COVID 19 infected, the number of calls transferred from CODU to SNS 24 increased significantly. The remaining priorities represent 3% – 8% of the monthly emergency calls answered by CODU.

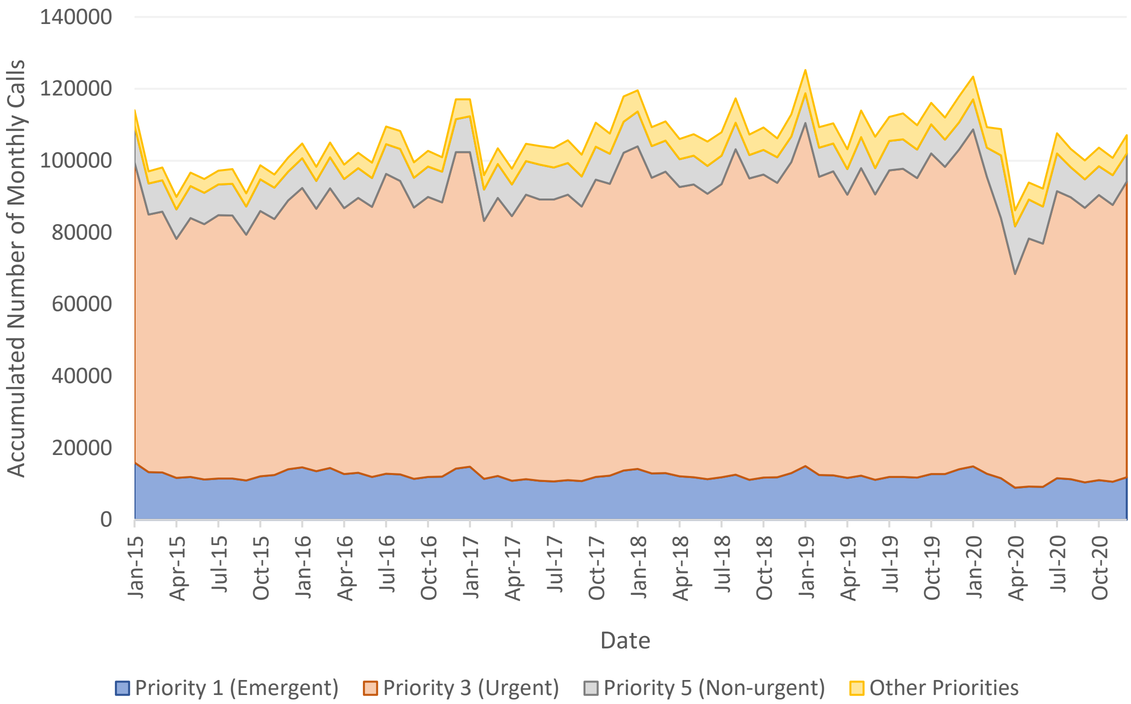

Figure 4: Evolution of the monthly number of emergency calls of each priority level answered by CODU, from 2015 to 2021 (INEM, 2021b)

This study will continue to focus on situations that require emergency vehicles with a crew to assist at the occurrence location. In the next section, emergency vehicles will be presented.

2.1.2.2 Emergency vehicles

In order to provide an efficient and high-quality medical service to the population, INEM and its SIEM's partners, namely the Portuguese Red Cross, firefighters and health units, own different emergency vehicles. These are specialized mobile resources, that transport patients and victims from the occurrence location to the nearest hospital, and allow the transporting crew to both provide medical care at the scene and during the conveyance. In 2017, INEM reached a historical mark of having, at least, one ambulance in each of Portugal's mainland councils (278+).

In June of 2020, INEM owned 671 operational emergency vehicles distributed by the Portuguese inland, with the possibility to require more vehicles – not owned by the entity – according to seasonality. The location of these vehicles is crucial to assure fast and flexible assistance (INEM, 2020). Most of them are located in emergency departments of SNS hospitals, but may also be based in regions where Urgency Polyvalent Services – SUP – and Urgency Medical-Surgical Services – SUMC – hospitals exist, which can be hospitals, health centres, firefighters or police stations, according to the type of vehicle (Ferreira dos Santos, 2019). Table 3 shows INEM's fleet composition, the number of vehicles in 2020, the staff required for each and their function. Emergency vehicles may be equipped in order to provide Basic Life Support – BLS – or Advanced Life Support – ALS – also known as ILS – Immediate Life Support. Additionally, in the Appendix, it is possible to find an image of each vehicle.

Table 3: INEM's fleet (Ferreira dos Santos, 2019; INEM, 2013; Namorado Rosa, 2017)

Emergency Vehicle	Number of Vehicles in 2020	Staff Requirement	Function
Medical Emergency Ambulance (AEM) (BLS)	56	2 TEPHs	Transport TEPHs to the scene, stabilize and provide medical care while transporting victims to a HU
Medical Emergency Motorcycle (MEM) (BLS)	9	1 TEPH	Promptly transport a TEPH to the scene to stabilize the victim and prepare for transportation
Inter-Hospital Pediatric Transport Ambulances (TIP) (BLS)	4	1 TEPH + 1 Nurse + 1 Physician	Transport the crew and stabilize or transport a minor (0-18 years old)
Immediate Life Support Vehicle (SIV) (ALS)	40	2 TEPHs + 1 Nurse	Transport the crew and provide a differentiated medical care (e.g., resuscitation, defibrillation, medication)
Vehicle of Medical Emergency and Reanimation (VMER) (ALS)	44	1 Nurse + 1 Physician	Promptly transport the crew to the scene to stabilize the victim and prepare for transportation
Medical Emergency Helicopter Service (SHEM) (ALS)	4	2 Pilots + 1 Nurse + 1 Physician	Transport risk patients between HUs or from the scene to a HU
Mobile Unit of Phycological Emergency Intervention (UMIPE) (BLS)	4	1 TEPH + 1 Psychologist	Intervene in potentially traumatic situations, where victim's emotional stress requires special care
Medical Emergency Stations Ambulances (PEM) (BLS)	364	2 SIEM's Partner Members	Transport the crew, stabilize and provide medical care while transporting victims to a HU
Reserve Station Ambulances (RES) (BLS)	123	2 SIEM Partner's Members	Transport the crew, stabilize and provide medical care while transporting victims to a HU
Non-INEM Ambulances (NINEM) (BLS)	22	Partners' Members	Used when others are unavailable to stabilize and provide medical care while transporting victims to a HU

As the dissertation thesis will focus on the work done by TEPHs, it is going to be displayed in order to have a better overview of INEM's operations.

2.1.2.3 TEPHs

TEPHs are health professionals that work full-time at INEM and their activities are exclusively dedicated to assisting in a pre-hospital context, being crucial to the survival of victims of sudden illness or accidents. TEPHs can do two types of activities: (i) manage CODUs, being responsible to answer calls, triage the situation and allocate the correct vehicle and crew to the location and (ii) operate a specific ambulance and assist victims at the scene, according to vehicle's requirements (Table 3). Additionally, they may also have certain tasks related to exceptional situations or international missions. TEPHs are an essential part of INEM's workforce, representing more than 70% of the total (INEM, 2018a). However, this number needs to be reinforced, therefore, new TEPHs must be hired every year.

Table 4 displays the evolution of the number of TEPHs since 2017. For each year, there are two columns. The first with a P, which corresponds to a Prediction according to the estimated needs for that year, i.e., the required number of TEPHs to perform tasks. The second with an O, which corresponds to the actual number of TEPHs that Occupied each function at the end of the year (except for 2020, in which these values are only accounted until June). It is noticeable that the effective number of TEPHs has been roughly increasing. Nonetheless, the percentage of required TEPHs that is fulfilled, i.e., $\frac{Total\ Occupied_i}{Total\ Predicted_i}, i = 2017, 2018, 2019, 2020$, has been varying between 72% and 79%. This suggests that current TEPHs are overworking to meet population needs.

Table 4: Evolution of the number of TEPHs per year, between 2017 and June 2020 (INEM, 2017, 2018b, 2019, 2020)

Function/Year	**2017 P**	**2017 O**	**2018 P**	**2018 O**	**2019 P**	**2019 O**	**2020 P**	**2020 O**
General Coordinator TEPH	11	12	4	4	4	7	4	4
Operational Coordinator TEPH	-	-	8	8	23	18	23	7
TEPH with CODU functions	1264	178	295	212	279	229	279	224
TEPH with EV functions	-	721	920	737	1018	717	1018	815
TEPH Backoffice	-	28	24	32	27	5	21	5
Total	1275	939	1251	993	1351	976	1345	1055
% of required TEPHs	73.7%		79.4%		72.2%		78.4%	

To ensure that the highest standards of quality and safety are met, TEPHs must complete a rigorous training plan credited by the Health Minister and Medical Doctors' Order. This formation requires internships in ambulances and hospitals, representing a pioneer model in Portugal but with excellent results shown internationally in countries such as Spain or the United States of America (INEM, 2018a).

Table 5 shows, in the first column, the competencies required by TEPHs in order to perform their tasks – Driving, CODU responsible and operator, Glucagon administrator, Defibrillator and Driving MEM. From these tasks, which include CODU and emergency vehicles related, some obligatorily require certain competencies. The first column displays the percentage of tasks that require each competence. It is important to note that some tasks may require more than one specific competence. The second column demonstrates the percentage of tasks for which additional competencies may be important but not critical. Therefore, the third column results from the sum of the first two columns. The last column provides the percentage of TEPHs allocated to Lisbon CODU that possess these competencies.

Comparing the two last columns, one can remark that the percentage of TEPHs with a particular competence is higher than the necessity of that competence, with an exception for "CODU responsible". For this case, it may indicate two possible outcomes: (i) TEPHs that have CODU responsible competence must overwork to meet tasks requirements leading to extreme fatigue or (ii) tasks that require mandatorily a TEPH with CODU responsible competence may be performed by other TEPHs without this competence, decreasing the quality level of the task executed. Both possibilities may impair CODU's proper functioning and harm INEM's pre-hospital processes.

This analysis was done for the Lisbon CODU due to being the focus of the dissertation.

Table 5: Competences, % of tasks requiring mandatory or desired competence and TEPHs with those competences for Lisbon CODU

Competences	% of tasks with mandatory competence	% of tasks with desired competence	Total tasks	TEPHs with competence
Driving	53.1%	31.3%	84.4%	87.2%
CODU responsible	7.8%	-	7.8%	3.8%
CODU operator	10.9%	-	10.9%	48.9%
Glucagon	35.9%	34.4%	70.3%	96.6%
Defibrillator	37.5%	60.9%	98.4%	100.0%
Driving MEM	4.7%	-	4.7%	5.3%

2.1.2.4 Dispatching and routing

According to the priority and local accessibility of the emergency, the most appropriate vehicle must be selected to be dispatched. This decision is made by the responsible CODU TEPH and validated by a regulating doctor.

Emergency situations require both BLS and ALS or ILS emergency vehicles. On the other hand, urgent situations only require a BLS vehicle. In some cases, MEMs are dispatched to a P3 occurrence in order to quickly reach the scene to stabilize the victim as soon as possible or to confirm that it is not a false alarm. Some situations demand a psychologist or a paediatrician, therefore an UMIPE or a TIP may be dispatched. Table 6 compiles vehicles requirements for the different priorities.

Table 6: Vehicle Types Requirements According to Priority Level (INEM, 2019)

Priority Level	Vehicle Types	Possible Combination of Vehicles
P1 - Emergency Situations	BLS + ALS/ILS	AEM/PEM/RES/NINEM + VMER/SIV VMER + SIV SHEM
P3 - Urgent Situations	BLS	AEM/PEM/RES/NINEM/SIV MEM
P5 - Non-Urgent Situations	No vehicle	-
Other Priorities	UMIPE, TIP or others	Variable

Figure 5 presents the average number of dispatches per day for all types of vehicles. One can observe that nearly all vehicles dispatches decreased in 2020. This happened mostly due to the confinement that Portugal lived in the first semester of that year, reducing the number of emergency occurrences. Additionally, it seems evident that PEM is the most dispatched emergency vehicle, mainly because it represents more than 50% of INEM's fleet. It is followed by AEM and RES, which are identical. Then, VMER is more used than NINEM, SIV and MEMs. Finally, UMIPE, SHEM and TIP are barely dispatched.

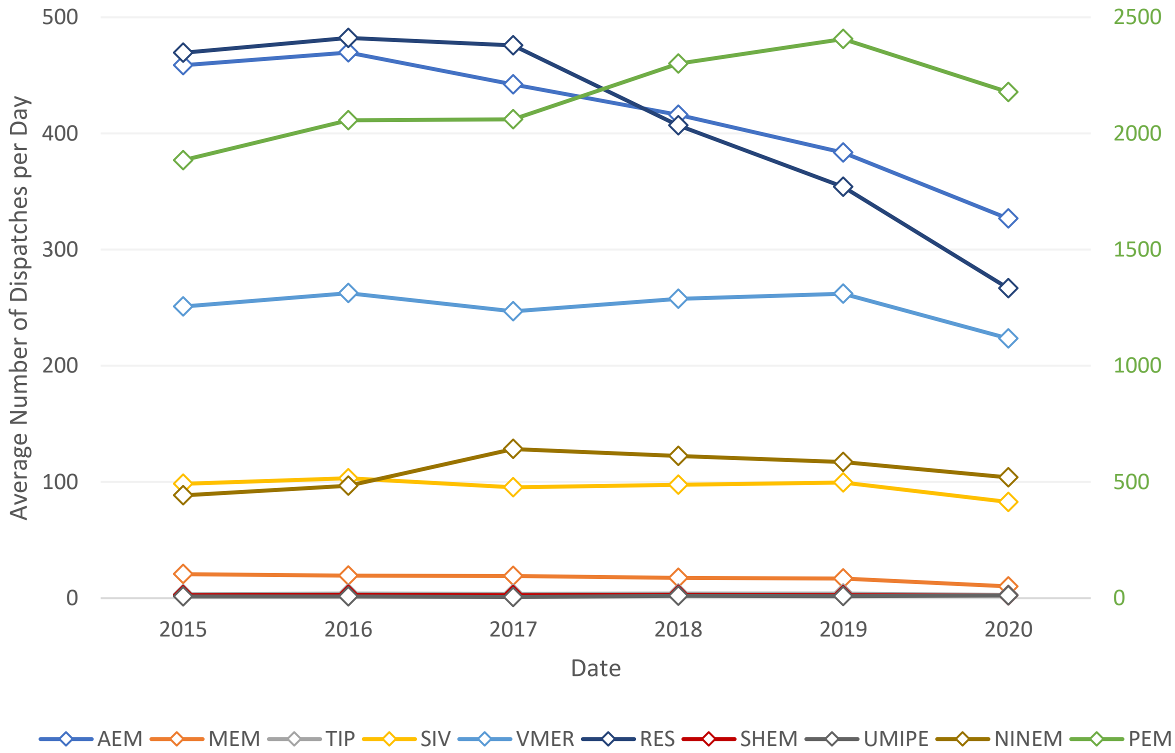

Figure 5: Average number of vehicle dispatches per day, from 2015 to 2020 (INEM, 2021a)

After a vehicle is selected, it must be dispatched based on location and availability. To support TEPHs' decision, INEM uses SIADEM – Integrated System of Medical Emergency Answering and Dispatching – that gives updated information on the vehicles' exact location and their availability, whether parked in a base on in-transit. The system displays a list with the vehicles ordered by the proximity to the occurrence and following a priority rule established by INEM. On any occasion that is possible to send an INEM owned vehicle (AEM), the TEPH must dispatch it first. If there are no INEM vehicles available, a partner vehicle (PEM or RES from the Portuguese Red Cross or Firefighters Associations) must be dispatched. If none of these options is possible, the TEPH should dispatch a NINEM, which represent ambulances for which INEM has not an established protocol, for which the entity must pay a prize each time it is operated (INEM, 2019). Normally, the TEPH chooses the closest available vehicle, such that vehicles and crews operate within their usual region of activity, which may be a county or a parish.

Regarding TEPHs, Figure 6 indicates an average of how many times a team of TEPHs is dispatched in each shift of 8 hours, assuming that each crew is assigned to one emergency vehicle for that day's shift. Once more, AEM is the most dispatched vehicle, therefore, TEPHs allocated to that transport will handle occurrences more frequently.

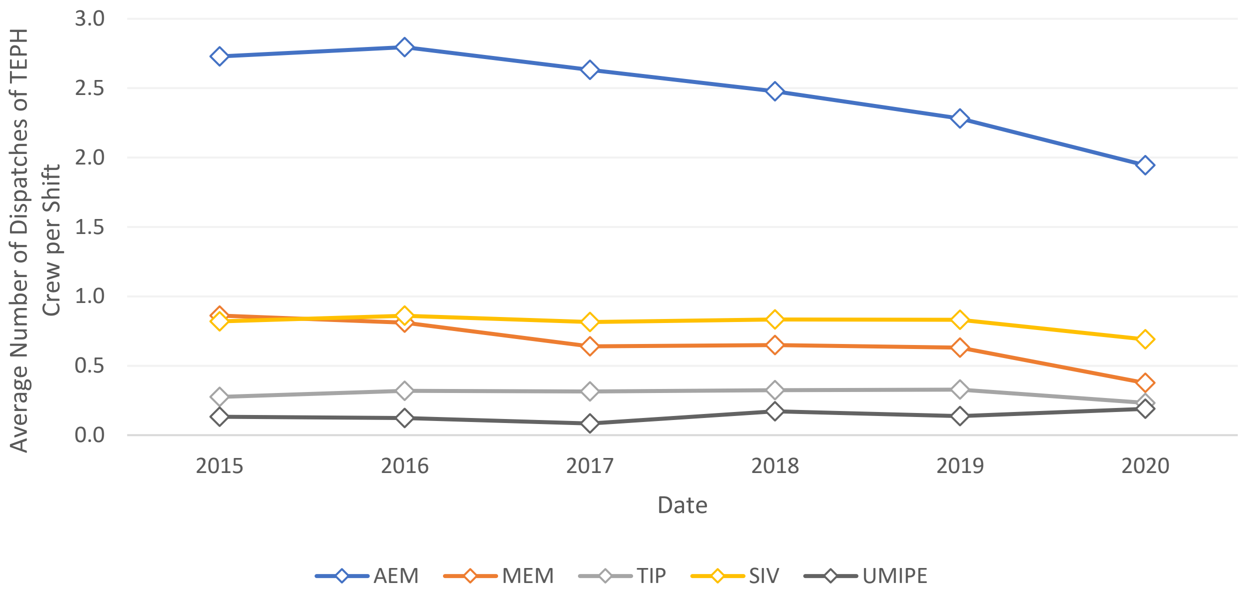

Figure 6: Average number of dispatches of a TEPH's crew per shift from 2015 to 2020 (INEM, 2021a)

Finally, to assess the quality of its dispatching process, INEM uses two Key Performance Indicators: (i) % of occurrences in rural zones where response time is less than 30 minutes and (ii) % of occurrences in urban areas where response time is less than 15 minutes. INEM reached 90% for the first and 73% for the second in 2018 (INEM, 2019), being these results in line with what is achieved in countries with high-performance medical emergency services, such as Spain and the United Kingdom.

2.1.2.5 Articulation and Coordination

To guarantee that all involved actors are in line, there is, for each emergency, a CODU TEPH that has two intermediating functions: (i) counsel people that are with the victim to provide any possible care, according to the situation, (ii) contact the closest Health Unit to confirm that is possible to receive that person. Additionally, TEPHs in the emergency vehicle must use iCare – Integrated Clinical Ambulance REcord – that allows sharing real-time information about the victim so that the CODU team can inform the Health Unit in order to prepare for the treatment, accelerating this process, which is especially important in serious emergent cases (INEM, 2019).

In Figure 7, it is possible to view the whole SIEM's process, summarizing previous sections. Activities in yellow represent those studied in section 2.1.2.1 and 2.1.2.2, comprehending the steps since an emergency request is made until a vehicle is dispatched, in green those analysed in section 2.1.2.4, regarding the vehicle's dispatching and routing, and, finally, in dotted blue those in this last section.

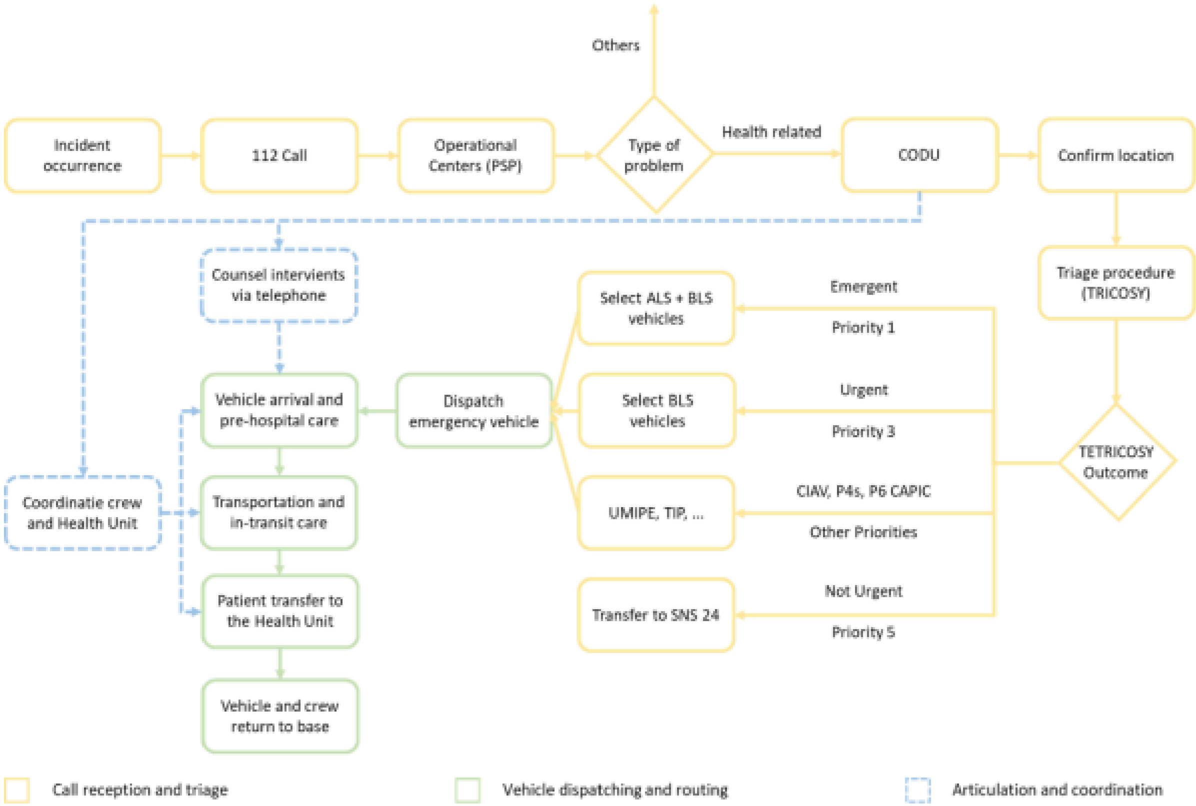

Figure 7: SIEM's Process

2.2 Staff rescheduling at INEM

From this analysis, one can understand that INEM is a continuous operating entity, in which work is structured into shifts. Managing staff properly is of crucial importance as these represent a vast percentage of INEM's operational costs. Periodically, a staff schedule is presented by the responsible who assigns each individual to a specific duty over the planning horizon. Nevertheless, as this operating system is embedded in a dynamic and uncertain environment where unpredictable events may happen, disruptions may affect the initial schedule, creating the necessity of rescheduling. This phenomenon urges when individuals are unable to perform a specific task to which they were assigned and therefore it is mandatory to reconstruct the schedule, fulfilling demand, while trying to minimize shift changes (Wolbeck *et al.*, 2020). Since TEPHs represent the greatest share of the staff, this study will focus on this group of workers allocated to the CODU of Lisbon.

In this section, a brief overview of INEM's current scheduling process is going to be presented so that the context can be understood. Then, the rescheduling process is going to be introduced in detail, as well as the problems that may arise from the current *modus operandi* and performance indicators that must be improved.

2.2.1 Scheduling at INEM

Until recently, TEPHs' schedules were made based on their preferences. TEPHs started by determining and managing their own agenda in a platform called *Gestão de Horários* to allow a certain degree of flexibility. Then, in a second step, both coordinators of CODU and EVs would construct a first schedule for each activity based on the data provided. Rearrangements due to infeasibilities would then be done until forecasted demand was met. Finally, the last step was the coordination between the heads of CODU and EVs, reaching a final schedule (Namorado Rosa, 2017). Effectively, this is a simple process for TEPHs but highly complex to the coordinators, as the rearrangements' processes can have several iterations.

Nowadays, TEPHs operate under a cyclical schedule where they give their available shifts to the responsible, who will later build the schedule based on the following 10-days cycle: *M – M; A – A; O; N – N; O – O – O*. These letters represent the shift that is done in a specific day (N – Night (00:00 a.m. – 8 a.m.), M – Morning (08:00 a.m. – 04:00 p.m.), A – Afternoon (04:00 p.m. – 12:00 p.m.) and O – Day Off). Table 7 displays an example of a schedule for 10 TEPHs for a planning horizon of 10-days. It is possible to remark that all TEPHs account for 6 shifts during this period, assuring equality of worked hours.

Table 7: Example of a cyclical schedule

TEPH/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
TEPH 1	M	M	A	A	O	N	N	O	O	O
TEPH 2	O	M	M	A	A	O	N	N	O	O
TEPH 3	O	O	M	M	A	A	O	N	N	O
TEPH 4	O	O	O	M	M	A	A	O	N	N
TEPH 5	N	O	O	O	M	M	A	A	O	N
TEPH 6	N	N	O	O	O	M	M	A	A	O
TEPH 7	O	N	N	O	O	O	M	M	A	A
TEPH 8	A	O	N	N	O	O	O	M	M	A
TEPH 9	A	A	O	N	N	O	O	O	M	M
TEPH 10	M	A	A	O	N	N	O	O	O	M

2.2.2 Rescheduling at INEM

Rescheduling is triggered by a schedule disruption such as the absence of an assigned TEPH, leading to understaffing. Since the beginning of the COVID 19 pandemic, these disruptions have been more frequent, as a result of new factors that can lead to an absence, such as a contact with an infected – requiring quarantine – or a TEPHs' child being in contact with an infected – which may require that the parent stays at home.

When in need to reconstruct a new schedule, similar problems to the scheduling process arise. It is mandatory to guarantee that tasks that ask for specific competencies are done by qualified TEPHs, while meeting forecasted demand and not incurring excessive costs. It is critical to comply with work and legal regulation, while reducing the occurrence of both over and understaffing. It is crucial to maintain labour equality between staff, while contributing to improve their satisfaction. However, the rescheduling process must consider three additional aspects.

First, disruptions are based on usually unforeseen events that happen close to their date. Therefore, it is not possible to manage them in advance. Moreover, these require that the reconstructing process is done in a short period – days or weeks – being present in the short term operational phase. With daily disruptions, the responsible may become overwhelmed as solving these disruptions every day is not always desired, since it generally leads to higher costs. Secondly, it is not just necessary to build a schedule but rather do it with the fewer shift changes possible, because doing it every time that there is a disruption would lead to overwork and it could harm TEPHs' satisfaction due to constant changes. Finally, TEPHs adapt their personal lives according to the initial schedule and when there is a disruption that affects their agenda, their willingness to accept changes can decrease, being this a vital factor, as their refusal or resistance may make the process very difficult or even impossible to advance.

Nowadays, both INEM's scheduling and rescheduling for Lisbon's CODU area are performed by three local TEPHs, two for emergency vehicles and one for the CODU. Looking to rescheduling, it is an informal process, where the three rebuild the schedule by calling other TEPHs to determine whether or not they are available for the disrupted shift. Typically, they contact people that are in one of the three consecutive days off (Table 6) and try to maintain equity factors – extra hours, weekends availability, legal constraints and others – immutable. Often, TEPHs communicate between themselves to assure that another worker replaces the absent.

Table 8: Cyclical schedule with disruptions

TEPH/Day	**Day 1**	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEPH 1	M	M	A	A	O	N	N	O	O	O
TEPH 2	O	M	M	A	A	O	N	N	O	O
TEPH 3	O	O	M	M	A	A	O	N	N	O
TEPH 4	O	O	O	M	M	A	A	O	N	N
TEPH 5	N	O	O	O	M	M	A	A	O	N
TEPH 6	N	N	O	O	O	M	M	A	A	O
TEPH 7	O	N	N	O	O	O	M	M	A	A
TEPH 8	A	O	N	N	O	O	O	M	M	A
TEPH 9	A	A	O	N	N	O	O	O	M	M
TEPH 10	M	A	A	O	N	N	O	O	O	M

To evaluate the process performance, INEM intends to achieve neither overtime nor undertime. On the one hand, overtime must happen only when strictly necessary, as the entity is incurring higher costs. On the other hand, undertime might not fulfil demand, causing understaffing, which is also undesirable. Additionally, some issues related to TEPHs' wages arise when working hours are not constant. However, another interesting performance indicator could be the time spent to rebuild a schedule or the number of changes from the previous schedule to the updated version. Considering the example of Table 7, assume now that there are five disruptions to the initial situation, highlighted in red in Table 8. These would create the necessity to reconstruct the schedule.

Table 9 displays the last schedule, where red cells represent TEPHs that had *Day Off* and now must work in a shift and green cells represent those TEPHs who will have a *Day Off* in the disrupted shift. In this case, although there is undertime or overtime for 8 TEPHs, one may believe that for a longer period this could be balanced and INEM's goals would be me. Nevertheless, what this method demands in terms of labour and time spent, when compared to an automatic solution, is considerably high. Especially, it is possible to think of the increased difficulties that arise when scaling these 10 TEPHs to the actual number and when staff's capabilities are introduced, knowing that certain tasks require specific competencies.

Table 9: Reconstructed cyclical schedule

TEPH/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
TEPH 1	M	M	A	A	O	N	N	N	O	O
TEPH 2	O	M	M	A	A	O	N	O	O	O
TEPH 3	O	O	M	O	A	A	O	N	N	O
TEPH 4	O	O	O	M	M	A	A	O	N	N
TEPH 5	N	O	N	O	M	M	O	A	O	N
TEPH 6	N	N	O	M	O	M	M	O	A	O
TEPH 7	O	N	N	O	O	O	M	M	A	A
TEPH 8	A	O	O	N	O	O	O	M	M	A
TEPH 9	A	A	O	N	N	O	A	O	M	M
TEPH 10	M	A	A	O	N	N	O	A	O	M

2.3 Problem Definition

Currently, INEM's managers reconstruct disrupted schedules manually, which makes it a time-consuming task. As it is a frequent task and it happens almost every day, the time spent can be higher than what would be desired. Moreover, the rescheduling process has more factors needed to be taken into account than the scheduling, which increases its complexity. Thus, being an informal process, some errors may occur. These are, certainly, not desired, since when equity is not accomplished, employees' satisfaction may decrease, which will ultimately affect the quality of the service provided.

Therefore, this dissertation aims to develop and apply a mathematical model that can act as support to the rescheduling process at INEM. The main goal of the model is to provide a tool for this process to perform this activity in a short amount of time and causing little as less disorder as possible for TEPHs. Ultimately, it aims as well to improve the quality of current solutions. It should automate the process of constructing a new updated schedule after a disruption caused by a worker. The model must consider current restrictions, such as demand fulfilment, legal regulation and labour equity maintenance without incurring over or understaffing. It will be applied to the Lisbon CODU area that is composed of 266 TEPHs.

2.4 Chapter Considerations

The goal of this chapter is to present the context and define the case study that is addressed in the dissertation. After this analysis, it is possible to perceive that INEM plays a key role in providing pre-hospital medical care in Portugal, as it is the entity responsible for running a set of activities and coordinating a group of entities in a complex EMS system.

Demand for this service has been increasing in the past years, which, naturally, hinders its mission. In spite of 2020's figures, it is expected that this tendency will continue after recovering from the damages caused by the pandemic. Lately, INEM has been training and hiring more TEPHs to follow this growth. Nevertheless, another possibility to go along with this movement might come from increasing processes' efficiency, reducing the necessity of employing more workers and following the restricted budget constraints. Having in mind that rescheduling is a time-consuming and exhaustive process that is still performed manually in INEM, it is possible to conclude that there is room to improve.

Therefore, the goal of this dissertation is to create a mathematical model to assist EMS and, especially, INEM in the rescheduling process. The model aims to produce optimal solutions and in a short amount of time. In the next chapter, a literature review on rescheduling is presented, in order to understand that this is a problem that must be deepened in the context of the EMS and also to study the various approaches that researchers have been introducing to cope with the rescheduling problem in other fields. These are crucial to present an accurate model for EMS.

3. Literature Review

Although it has not received a lot of attention in the literature when compared to scheduling, the staff rescheduling is a common problem for most organizations where each worker operates a shift. This chapter shows why it should be more studied, especially in the EMS context. Section 3.1 presents the different planning problems that have been arising in EMS. After considering that rescheduling is a problem to EMS, section 3.2 shows how poor rescheduling decisions can harm the performance of a health organization. Then, section 3.3 reviews the methodologies that authors follow when approaching the rescheduling problem. In section 3.4, the main solution techniques are outlined, distinguishing between exact and heuristic approaches. Finally, section 3.5 presents the main conclusions of this chapter and highlights the gap in the literature.

3.1 Planning Problems in EMS

Emergency Medical Service (EMS) is one of the most essential health care entities as it largely contributes to saving people's lives and lower death and morbidity rates. Operations research scientists, EMS planners and health care professionals, who investigated many problems arising in the EMS systems' management since the 1960s, have recognized the significance and sensitivity of decision making in this field (Aringhieri, Bruni, Khodaparasti, & van Essen, 2017).

Frequently, decision making is related to planning, as these represent the most important managerial functions. Planning can be perceived as deciding what to do in advance, having two main components, goals and action statements. Decision making is the process of identifying several different alternatives, resulting in the choice of a course of action to detriment of the others. By planning, a team discovers these alternatives and, by testing and evaluating their effectiveness, they choose a path to follow.

Additionally, planning in EMS has the goal to measure the performance and costs of the system, as well as to assess its process quality, i.e., response time. Moreover, planning also aims to compute the expected service level based on the current state and future changes to be implemented (Reuter-Oppermann *et al.* 2017). Reuter-Oppermann *et al*., (2017) divided EMS planning into three main groups: (i) the general design, (ii) the logistics and (iii) the analytics.

The general design is mainly emergency hotline dispatch, emergency rescue and patient transport, considering existing laws and regulations, which may differ according to the country or the region. The logistics group must ensure that the designed services can be provided, encompassing two main aspects, ambulance planning and workforce planning. Finally, the analytics includes two parts: forecasts for the logistical planning problems (number of ambulances, location bases, hiring crew and others) and data analysis of historical data to control the provided service levels and to confirm that laws and regulations were met.

This thesis will focus on the logistics field, concerning a specific planning problem: workforce planning. Reuter-Oppermann *et al*., (2017) split EMS staff into two groups: (i) staff working in the ambulances and (ii) staff working in dispatching centres. In most countries, the two groups are completely disconnected so the corresponding planning problems can be studied independently. However, both in Portugal and Germany, dispatchers also work in ambulances and vice-versa. Furthermore, staff scheduling has been widely discussed in the literature, which is motivated by economic factors, since labour cost is the major direct cost component in many cases (Van Den Bergh *et al*. 2013).

Despite shift scheduling being an extensively studied subject, rescheduling has not received the same consideration in the literature (Maenhout & Vanhoucke, 2011). The importance of staff rescheduling as an every-day complex and time consuming task has not been widely recognized, although some authors have noted that the lack of a specific methodological base that considers its complexity increases the likelihood to prejudice patients and stress working staff and managers (Clark *et al*., 2015). Therefore, this thesis will focus on this gap, studying the staff rescheduling in EMS.

3.2 Staff Rescheduling Problems

In any organization, staff scheduling is a balancing act between service needs and the available workforce resources. Poor scheduling decisions carry risks that may negatively impact the staff and, ultimately, the performance of the service provided. Rescheduling is the act of rediscovering this balance in a short period. Thus, poor rescheduling decisions may also prejudice the effectiveness of the service (Clark *et al*., 2015). Clark *et al*. (2015) concluded that, as well as nurse scheduling, data suggests that rescheduling can impact patient care, staff morale and costs.

Regarding the deterioration of patient care, it can be originated by three consequences of poor rescheduling decisions: (i) poor skill mix, (ii) under-staffing and (iii) long hours. McGillis Hall *et al*. (2004) found that, in the hospital context, inappropriate staff skill mix results in higher medication errors and wound infections, especially concerning more complex problems. Aiken *et al*. (2002) and Taylor *et al*. (1999) identified that when there are high patient-to-nurse ratios, there is also a higher mortality risk and failure-to-rescue rate. It can be transposed to the EMS, as when there are fewer technicians, response time – both on the dispatching centre and for the ambulance – may be higher, causing a delayed service, which may lead to mortality risk increase. Finally, Rogers *et al*. (2004) detected that when shift durations exceeded 12.5 hours per day, nurses were more fatigued, leading to an increase in errors occurrence and that risk of errors started increasing when shifts exceeded 8.5 hours per day.

Considering staff morale decline, it can be caused by three consequences of poor rescheduling: (i) long hours (ii) inequitable shifts and (iii) excessive changes on the initial schedule. As observed in Rogers *et al.* (2004), when working long hour shifts, staff will be more fatigued, which according to Hegney *et al*. (2006), can lead to a decrease in nurse morale. According to Wilson, (2002), several of characteristics contribute to the acceptability of shift work, including equitable shift rotas. Also, these rotas must be as regular as possible, having few changes so that employees can find a routine for themselves. Finally, low staff morale can lead to high staff turnover and high absenteeism (Silvestro & Silvestro, 2008).

Concerning the loss of costs effectiveness, it can be generated by two consequences of poor staff morale already mentioned, high staff turnover and high absenteeism, and two consequences of poor rescheduling decisions, over-staffing and low efficiency in the rescheduling process. Bland Jones (2008) conducted a study where the most obvious insight was that by building an environment that increases nurse retention and mitigates its absence, the health organization would save money from turnover costs and other indirect gains, such as improved staff and patient satisfaction. Hayes & Bonnet, (2010) also concluded that it is possible to decrease costs by retaining valued staff. In addition, overstaffing can directly increase costs (Newbold, 2008). Finally, Clark *et al.* (2015) described rescheduling as time-consuming, as some senior managers were spending 10-20% of most working days rescheduling, repairing absenteeism and changing required staff.

When poorly done, the rescheduling process in health organization can, indirectly, hinder patient care, staff morale and costs increase. Thus, it can affect the whole performance of the health organization, being a topic demanding urgent attention. Figure 8 shows a graphical summary of the analysis above to have a clearer view of the causality relation between these events originated by poor rescheduling decisions. The green highlighted boxes show the long term effects poor rescheduling may have on the organization, while in yellow are highlighted the local consequences that lead to those long terms effects.

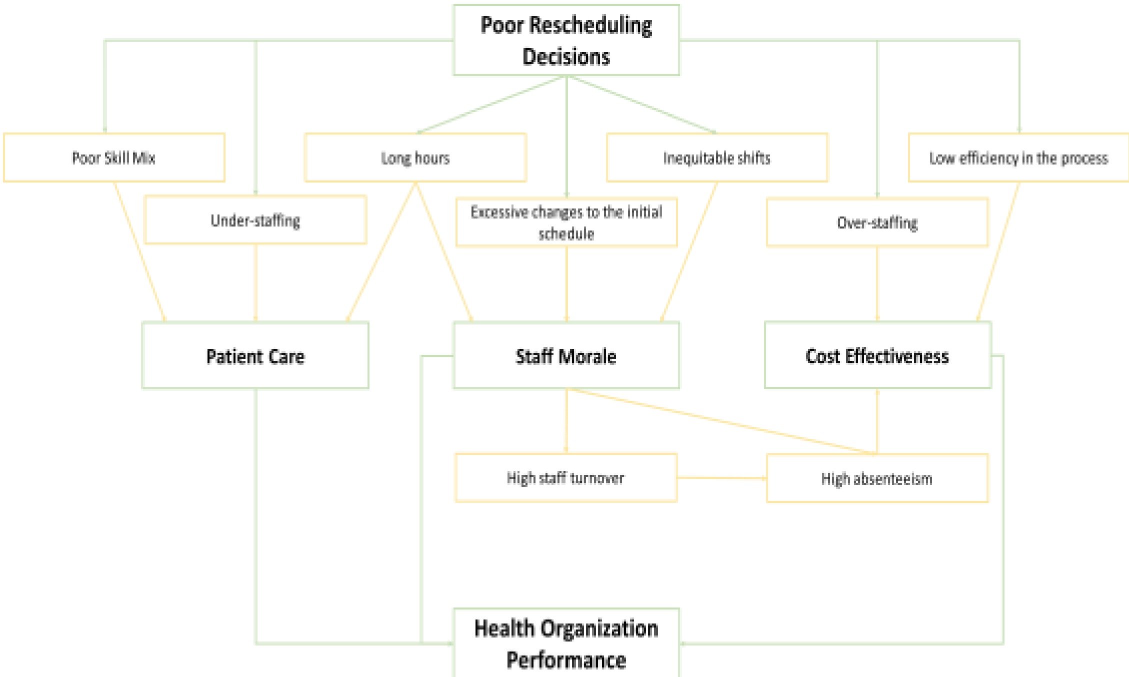

Figure 8: Effects of poor rescheduling decisions

3.3 Methodologies

In the rescheduling problem, some characteristics are translated into a mathematical formulation through an accurate modelling. This section introduces the most common methodologies used over the years. First, it introduces the most important decision criteria. These give an insight into the quality of the reschedule solution. Then, the most common constraints to which rescheduling decisions are subjected will be discussed.

Finally, it presents some different modelling strategies regarding the horizon and the number of workers considered, and their impact on the quality of the final solution.

3.3.1 Common Decision Criteria

The reconstructed schedule quality is measured by multiple goals with different priority levels which can be grouped in health organisation's and staff related objectives, as it is demonstrated in table 10 (Maenhout & Vanhoucke, 2011).

Table 10: Rescheduling objectives (Maenhout & Vanhoucke, 2011, 2013; Mutingi & Mbohwa, 2015)

Health Organization's Objectives	**Staff Related Objectives**
i) Maximize or maintain the quality of service as was intended in the original roster before disruptions	iii) Maintain or maximize or the satisfaction of individual nurse preferences
ii) Maintain or minimize labour cost	iv) Maintain or maximize schedule fairness
-	v) Maintain or minimize workload variation
-	vi) Maintain or minimize schedule changes as much as possible

The objectives related to the health organization normally consist of assuring the level of service is maintained while not incurring extra costs. To that end, the aim is to guarantee the number and skills of workers needed in each shift, while avoiding extra costs (Maenhout & Vanhoucke, 2013). In the literature review of Mutingi & Mbohwa, (2015), these objectives are designated as i) Maximize or maintain the quality of service as was intended in the original roster before disruptions and ii) Maintain or minimize labour cost. Usually, these are achieved by penalizations in the objective function. In Maenhout & Vanhoucke, (2011) they use a penalty cost for scheduling a deficient number of nurses for a specific shift and one for when there is a surplus of nurses. Another way can be to penalize if a proper skill mix is not reached and to penalize extraordinary hours worked.

The staff related objectives concern individuals' preferences and satisfaction which are determined by fair and even scheduling practices, personal contract stipulations, collective union agreement requirements, labour laws and others. In Mutingi & Mbohwa, (2015) these objectives are known as iii) Maintain or maximize or the satisfaction of individual nurse preferences, iv) Maintain or maximize schedule fairness and v) Maintain or minimize workload variation. Again, these objectives are usually obtained by penalization in the objective function. Wolbeck *et al.* (2020)**,** to maximize fairness, considers a penalization depending on (i) the type of shift change (e.g., if the change is from a working day to a day off or to another shift type), on (ii) the time of a shift change by taking into account the lead time between the announcement of the disruption and the current shift execution and on (iii) the distribution of shift changes by looking to past accumulated penalties.

Moreover, related to rescheduling, there is another objective that must be achieved. Workers organize their personal lives according to the expected duties. Hence, any change can create inconveniences. It is crucial, then, that the new schedule created retain as much the current individual shifts as possible (Maenhout and Vanhoucke, 2013), which is designated in Mutingi & Mbohwa, (2015) as vi) Maintain or minimize schedule changes as much as possible. This the most common objective in rescheduling as models are intended to minimize a function that counts the differences between the previous and the current schedule.

3.3.2 Common Constraints

Rescheduling models share many constraints with scheduling models. Regarding scheduling, following the work done by Cheang *et al*. (2003) and Namorado Rosa, (2017), it is possible to divide these constraints into four groups: (i) coverage requirements, (ii) time-related constraints, (iii) work regulation constraints and (iv) internal ward constraints. Maenhout and Vanhoucke (2011) added one more group concerning rescheduling (v) disruption constraints, as it is displayed in table 11.

Table 11: Common constraints in rescheduling (Cheang *et al*., 2003; Maenhout & Vanhoucke, 2011; Namorado Rosa, 2017)

Coverage requirements	**Time-related constraints**	**Work regulation constraints**	**Internal ward constraints**	**Disruption constraints**
Requirements of different types of workers for any shift	Staff workload	Preferences or requirements	Shift patterns	The set of disruptions must be performed by other workers
Constraints among shifts	Consecutive working shifts/days	Rest time between working shifts	Historical record	Some nurses cannot be assigned to particular shifts
Shift type assignment (requirements for each shift type)	Consecutive same working shifts/days	Working weekend	-	-
-	-	Holidays and vacations	-	-
-	-	Staff free days	-	-

Coverage requirements refer to the necessity of providing proper care with the right number of skilled workers. Then, time-related constraints concern staff's periodic work. Restrictions that deal with legislative and preferences' issues are presented in work regulation constraints. Additionally, some specific policies practised by wards are considered in the internal ward constraints. Finally, the last column considers restrictions regarding the post-disruption moment. The latter are specific constraints only found in rescheduling models, being comprised of two sets of constraints. The first guarantees that the disruptor worker that was firstly assigned to a shift no longer has to operate that shift. The second ensures that, despite the need to rebuild the schedule, some workers cannot perform certain shifts, e.g., when a worker is absent on a night shift, the one that will substitute him/her cannot have made two night shifts in the past two days if there is a restriction of a maximum of two consecutive night shifts. However, the substitute can still operate a different shift.

The constraints here presented are usually divided into two categories: hard and soft. Hard constraints are those that must be satisfied in any case, e.g., a worker can be assigned to at most one shift per day. These are normally related to minimum service levels and regulatory or contractual constraints, i.e., guaranteeing that there is a minimum number of workers not absent, with required skills for the shift and that do not work consecutive shifts or more than one shift per day (Wickert *et al*., 2019). Soft constraints are those that can, to some point, be violated but, when this violation occurs, there is a penalty in the objective function (Paias *et al*. 2021). These include all the remaining constraints.

Moreover, Wickert *et al.* (2019) studied the impact on the model of relaxing soft constraints. They observed that a model that considers soft constraints is 30 times slower than without them. Hence, although they considered that these are often important for generating good quality solutions, in critical cases when it is required to be as fast as possible, it may be useful to ignore them, as performing these urgent tasks should be prioritized over staff preferences or month workload.

3.3.3 Reschedule Strategies

When constructing a schedule, it is logical that the whole time horizon must be considered. However, when rebuilding an affected schedule, it may not be necessary to reschedule for the whole horizon. Some authors investigated the outcome for different scheduling horizons in order to understand what the impact was.

Maenhout and Vanhoucke (2013) concluded that it is not required to rebuild for the complete horizon. In truth, to effectively perform this task, the planning period must start two days before the first schedule disruption. Starting sooner than these two days can lead to a deterioration of the solution's quality. This is particularly important because the responsible manager is frequently noticed of the absence less than two days before its occurrence. Concerning the period after the last disruption, these authors also found that it is not necessary to go until the end of the time horizon, but rather two days after the last disruption. This will ensure an effective rescheduling and increasing the after-period is useless, as the quality will not improve. Additionally, Wickert *et al. (2019)* generated near-optimum results confirming that if the planning period had already started, it may only be necessary to consider from the first day of absence.

Moreover, Maenhout & Vanhoucke, (2013) also considered the variation of the number of workers on to reroster. In traditional rescheduling, the whole staff size is considered. They hypothesized that with fewer workers, solutions could still achieve a good quality in less time. However, this theory fell apart, as they identified that a higher number of workers would lead to lower variation, which would, in turn, guarantee higher quality.

3.4 Solutions

Several solution techniques have been applied to solve the personnel rescheduling problem. Based on the different classifications made, a broader classification is proposed (Clark *et al*., 2015; Mutingi & Mbohwa, 2015; Namorado Rosa, 2017). It divides these techniques into (i) manual methods (ii) exact algorithms and (iii) heuristic algorithms.

3.4.1 Manual Methods

Manual methods are the simplest. When a worker responsible for a shift cannot operate it, s/he can bargain with other colleagues to do their shift instead or s/he can contact the schedule responsible. Then, the responsible must contact the available staff to ensure that all shifts are operated and that working and legal regulations are met. It is extremely time consuming, difficult and more susceptible to errors occurrence (Mutingi & Mbohwa, 2015). It is interesting to note that even with a simple Excel solution, the number of ward managers tasks for covering absences, which includes phone calls, conversations to substitute staff and others, decreased significantly (Tuominen *et al.* 2016). However, it still is the most common method used by managers.

Clark *et al.* (2015) conducted a survey on four different health organizations to understand if automation scheduling processes were used and what were the reasons for its non-utilization. In the first, staff did not perceive the advantage of using a scheduling software, thus the paper-based system kept being used. The second had a software introduced. Nevertheless, due to lack of training, its adoption was poor and most areas still used manual methods. The third health organization could not find financial resources to implement them. Only the fourth had introduced the software across all areas. Although manual methods are very complex, only a few alternative models have been implemented, namely the one proposed by Bard & Purnomo (2006) in a hospital in the United States of America. These authors approached the problem in a different view, where organizations could hire part-timers, agency nurses on a daily basis – Nurse Addition Problem (NAP).

3.4.2 Exact Algorithms

Exact methods used in rescheduling comprise Linear Programming (LP) – to solve problems with continuous decision variables –, Integer Linear Programming (ILP) – to solve problems with discrete decision variables – techniques and Mixed Integer Programming (MIP) that combines both discrete and continuous decision variables (Namorado Rosa, 2017).

The first researchers to approach the rescheduling problem were Moz & Pato (2003), who conducted the tests for the ILP formulation of the integer multicommodity flow model, where they concluded that it is possible to obtain optimal schedules in reasonable computational times from the ILP formulation of the multicommodity flow problem. Moz & Pato (2004) presented two new integer multicommodity flow models. The first aims to optimize an integer multicommodity flow in a multi-level network with additional constraints. The second model aggregates the nodes corresponding to the same shift tasks in a single node. The latter achieved much better results than both the former and the formulation of Moz & Pato (2003), where 22 test instances were solved in less than five hours against 36 test instances solved in one minute or less.

Later, Do *et al*. (2017) proposed a method based on MIP to provide a new schedule with minimum changes, while satisfying all constraints as in the original situation, for the Lai Chau hydropower station. This model is capable of providing very good solutions in less than 180 seconds. However, they noticed that, as time passes, it is more difficult to find a feasible solution. Additionally, if a worker asks for an absence, while all his/her day-offs have been satisfied, the solution tends to be infeasible.

Taking into account that schedules are often affected by uncertain events that require its reconstruction, Wolbeck *et al*. (2018) used a dynamic approach where a monthly schedule is created and then uncertain events such as illness are generated stochastically. In addition, the authors tested different optimization rescheduling strategies, random and ratio approaches where no differences were found in terms of employee satisfaction and extra hours. Moreover, they tested with and without constraint relaxation and concluded that allowing constraint violation may decrease satisfaction, while extra hours increase.

Wolbeck *et al.* (2020) developed a MIP model where instead of penalizing each shift change equally, they evaluated them according to a Fair Shift Change Penalization Scheme. In this scheme, penalty costs are determined dynamically depending on (i) the type of shift change (e.g., if the change is from a working day to a day off or to another shift type), on (ii) the time of a shift change by taking into account the lead time between the announcement of the disruption and the current shift execution and on (iii) the distribution of shift changes by looking to past accumulated penalties. The higher the penalty the less satisfied the employee is. This model provides a fair distribution of shift changes among all workers. Also, Wolbeck *et al.* (2020) considered refusals (when the manager calls a nurse that refuses) as additional absences in the model, which is an important characteristic to consider in the implementation in a health organization.

Finally, Paias *et al.* (2021) approached the bus driver rerostering problem with a MIP multicommodity flow assignment model, enabling the reconstruction based on calling standby drivers, good candidates to replace absents, depot drivers, those who have no duties in that day, and the postponement of already assigned days off. The concept of standby workers is also addressed by Wolbeck *et al.* (2020) as a way of considering rescheduling when performing the scheduling process, increasing the robustness of the original schedule.

3.4.3 Heuristic Algorithms

Heuristics are a powerful family of algorithms for numerical optimization. It is a method designed for finding a solution that is reasonably good in an acceptable time frame. It may not be the optimal solution but it is still helpful as it is found in a short amount of time.

Moz & Vaz Pato (2007) were the first to propose heuristic methods for the rescheduling problem. They developed two versions of a constructive heuristic, in which the first, orders the nurses in a fixed way and assigns tasks accordingly, whilst, in the second, tasks and nurses were randomly ordered in the initialization step. The randomly ordered outperformed the fixed ordered model. Additionally, they also developed various versions of a Genetic Algorithm (GA), depending on the encoding of permutations and on the genetic operators used for each encoding, and also developed hybridized versions. Despite longer computational times, GA had almost always better solution quality than constructive heuristics. Pato & Moz (2008) developed an utopic Pareto genetic heuristic to deal with a bi-objective nurse rerostering problem, where the main differences, compared to the previous model, were the building of the initial population and the inclusion of an utopic individual to the population to improve quality and diversification.

Kitada & Morizawa (2010) proposed a heuristic model incorporating a tree-search algorithm into the recursive search algorithm to achieve an optimal roster while minimizing reassigned tasks. Following this model, in Kitada & Morizawa (2013), the authors proposed to solve the problem of staff absence for several consecutive days by separating several days of absence into a set of one day absences and then solving each subproblem. Finally, they followed the approach as in their previous study.

In Maenhout & Vanhoucke (2011), the authors developed an evolutionary meta-heuristic model. They also tested different optimization strategies and compared their results with the existing literature, concluding that their procedure outperforms the best in class from Pato & Moz, (2008). Later, Maenhout & Vanhoucke (2018) proposed a method that thrives on a perturbation mechanism to diversify the search and a variable neighbourhood search to intensify the search in the region of a solution point. The benchmark revealed that this model is capable of obtaining near-optimal results for different scenarios for short computational times.

Bäumelt *et al*. (2013) dealt with the rerostering problem by performing a parallel algorithm (constructive heuristic) on a Graphics Process Unit (GPU) to shorten the required computational time, which is a non-trivial task, bringing many issues, such as the utilization of the memory and the minimization of the communication overhead between the PC and the GPU. When comparing to the results of Moz & Pato (2004), they found that their solutions were within 18% (4%) from the optimal for a dataset of 19 (32) nurses, while the average speed was up to 1.9 (2.5) times faster. While the results were still far from optimal, the speed was significantly improved. Later, Bäumelt *et al*. (2016) improved the model and achieved a better quality, at the level of Moz & Vaz Pato (2007), but with a speed of 13 (18) times faster for a dataset of 19 (32) nurses. GPU is an important tool to accelerate complex problems and it is clear that it will play a key role in the future of computational science (Brodtkorb *et al*. 2013).

Chiaramonte & Caswell (2016) developed a modified agent-based nurse rostering system that provides solutions for both scheduling and rescheduling and also considers the negative impact on nurses preferences. Despite producing good results, as it solved over 90% of the disruptions, it has still some limitations. The model only allows one-to-one shift trade, i.e., if a worker is absent for two days, it will only consider four shifts, two from the disrupter and two from the worker that will fill those shifts. However, this model presents a promising methodology, having the potential to be as good as others.

Mutingi & Mbohwa (2016) created a Fuzzy Multi-criteria Simulated Evolution (FMSE) approach, an iterative algorithm generated from the general Simulated Evolution (SE), where some of the original SE operators are fuzzified. It allows decision-makers to use expert opinions based on information from patients and staff to make adjustments to the solution process based on the relative contribution of each element. Uhmn *et al*. (2017) introduced a deterministic algorithm, iterative deepening depth-first search. They compared with a

Simulated Annealing (SA) and remarked that the model outperformed the SA in terms of the number of instances solved, the average time to find a solution and its quality. Simić, Corchado, Simić, Đorđević, & Simić (2019) presented a hybrid strategy for the rescheduling problem with a methodology based on efficient cooperation between fuzzy logic ordered weighted averaging and variable neighbourhood descent search.

Finally, Wickert *et al*. (2019) presented a Variable Neighbourhood Descent (VND) heuristic, a simple algorithm but able to integrate several neighbourhood structures and generate good results quickly. They approached different strategies, such as relaxing constraints, reschedule only a part of the time horizon and, when comparing the results with a MIP solver, they always found errors lower than 2%, which shows the robustness of the model and provides an alternative to commercial solvers.

3.5 Chapter Considerations

This chapter aimed to understand rescheduling as a problem in the context of the EMS and to learn which approaches have been developed to deal with it. In fact, staff scheduling in EMS is known as a problem due to the necessity of making efficient use of scarce resources – both personal and financial – and has been studied for many years.

In this context, rescheduling arises as a frequent and time-consuming task required to repair schedules. It carries several risks and, when poorly performed, it has implications for patient care, staff morale and cost effectiveness, which will ultimately affect the performance of the health organization. However, there is currently little computational support for rescheduling and even less implementation, being, in most cases, an informal task that occupies a serious fraction of managers' time.

Although exact algorithms guarantee an optimal solution, the larger the problem, the more complex the solution space is, thus the algorithm will be slower. Despite the significant advances in computer hardware and commercial solvers over the past few decades, it remains difficult to guarantee the optimality of staff scheduling solutions. Additionally, as rescheduling problems have been becoming more complex with the introduction of more constraints, researchers have been dealing with it by relaxing soft constraints *(Wickert et al.* 2019). Furthermore, researchers have been designing different heuristic algorithms, which solve the problem in a faster and more efficient way than the latter, even if they compromise optimality, accuracy or precision.

Several heuristic approaches, such as evolutionary algorithms and constructive heuristic, were introduced and it is possible to conclude that these have been delivering good solutions in a considerably lower time than exact algorithms. As rescheduling happens almost every day and, in specific cases, needs to be as fast as possible, the time is certainly an aspect to take into account. Thus, these methods may become crucial for the evolution of staff rescheduling literature and model implementation.

To solve this problem, most of the approaches are exact or heuristic algorithms and provide optimal or near-optimal results in a short period. The next chapter proposes a mathematical model to solve the staff rescheduling problem at EMS.

4. Model Formulation

This chapter presents an optimization model to solve EMS staff rescheduling problems, supported by the methodologies explored in the literature review and INEM's case study. Therefore, section 4.1 presents the statement of the problem that will be addressed. Section 4.2 describes the mathematical formulation, where the parameters, sets and subsets are primarily established, followed by the objective function and constraints. Final remarks and the chapter's conclusions are made in section 4.3.

4.1 Problem Statement

The rescheduling problem occurs when one or more workers cannot be present in a day and shift to which they were assigned before in the initial schedule due to unforeseen events. These events may harm the quality of the service provided and thus it is crucial to recreate a schedule in a short period. For this, workers that were on day-off may be requested to work on that day which may oblige workers to perform it out of their team but only if they have the required skills to do so.

Each day is divided into three shifts, as presented in table 12, and each shift is composed of several tasks to be done with fixed starting times and lengths in order to avoid the possibility of overlapping. Most duties start at the same time and have the same duration of 8 hours.

Table 12: The three shift types and their corresponding starting and ending time

Shift Type	Starting Time	Ending Time
Night (1)	00:00 a.m.	08:00 a.m.
Morning (2)	08:00 a.m.	04:00 p.m.
Afternoon (3)	04:00 p.m.	00:00 a.m.

The reconstructed schedule reallocates, when needed, workers to different tasks to respond quickly to the disruption and guarantees along the planning horizon that both work and regulation factors are respected.

Additionally, the following elements are also key to rebuild a schedule: the definition of the planning horizon, namely the number of days of the schedule, and the total number of weekends during the planning horizon. Furthermore, all the public holidays, scheduled holidays and non-working days during that specific planning horizon must be known, once they decrease the number of working hours.

To simplify the problem under consideration, all personnel is assumed similar and exceptional situations are not considered in this model. As a result, seniority rights, personal limitations, and breaks during working shifts are not considered. To summarize, the problem can be stated as:

<u>Given:</u>

1. The number of contractual working hours;

2. The maximum number of consecutive working night shifts;
3. The minimum resting time between two consecutive shifts;
4. The minimum number of Sundays off;
5. The maximum number of consecutive days off;
6. The maximum number of consecutive working days;
7. The number of public and scheduled holidays on the planning horizon (/per worker);
8. The composition of each team;
9. The set of skills needed to perform each task;
10. The set of desired skills associated to each task;
11. The allocation of tasks to teams and correspondent service;
12. The shifts starting and ending time;
13. The staff requirements for each task on each shift of each day of the planning horizon;
14. The days in which a worker prefers not to work;
15. The days in which a worker is absent.

Determine:

1. The reallocation of tasks to workers on each shift of each day of the planning horizon starting from the day in which the schedule is disrupted.

So as to minimize:

1. Changes between initial and rebuilt schedule;
2. Number of tasks done by people which do not belong to the team to which the task is allocated;
3. Under and overstaffed tasks;
4. Allocation of tasks to workers who do not have the desired skills;
5. Allocation of tasks to workers who do not have preference for that shift;
6. Deviation of the number of working hours from those established in the contract (excess and deficit);
7. The number of incomplete weekends off among the workforce.

4.2 Mathematical Formulation

This section introduces the mathematical model taking into account the problem statement. It presents a detailed explanation of the sets, subsets, parameters, variables and the objective function. Finally, the constraints to which the model is subject are defined.

4.2.1 Sets and Subsets

The following are the sets and indices used in the mathematical model:

– $\boldsymbol{i} \in \boldsymbol{I}$ is the set of individuals
– $\boldsymbol{t} \in \boldsymbol{T}$ is the set of tasks

- $\boldsymbol{d} \in \boldsymbol{D}$ is the set of days in the planning horizon
- $\boldsymbol{s} \in \boldsymbol{S}$ is the set of working shifts, (e.g., S = {1(Night), 2(Morning), 3(Afternoon)})
- $\boldsymbol{g} \in \boldsymbol{G}$ is the set of teams
- $\boldsymbol{w} \in \boldsymbol{W}$ is the set of Sundays on the planning horizon

The following are the subsets used in the mathematical model:

- $\boldsymbol{I_g^G}$ is the set of workers that belong to team g
- $\boldsymbol{I_t^T}$ is the set of workers that have the required skills to perform task t
- $\boldsymbol{T_i^I}$ is the set of tasks that can be performed by worker i
- $\boldsymbol{I_t^{Td}}$ is the set of workers that have the desired skills to perform task t
- $\boldsymbol{T_g^G}$ is the set of tasks that are assigned to team g
- $\boldsymbol{H_i^I}$ is the set of holidays scheduled by worker i
- $\boldsymbol{P_i^I}$ is the set of days a worker i prefer not to work
- $\boldsymbol{C_t^T}$ is the set of tasks that belong to the dispatch centre

4.2.2 Parameters

The parameters used in the mathematical model are defined as:

- $\boldsymbol{\theta_{days}^{max}}$ as the maximum number of consecutive working days
- $\boldsymbol{\theta_{nights}^{max}}$ as the maximum number of consecutive working nights
- $\boldsymbol{\theta_{d-off}^{max}}$ as the maximum number of consecutive days off
- $\boldsymbol{\theta_i^{min}}$ as the minimum number of working hours for worker i
- $\boldsymbol{\theta_{s-off}^{min}}$ as the minimum number of Sundays off that worker must have during the planning horizon
- $\boldsymbol{\theta_s^{min-s}}$ as the minimum number of shifts that must be performed for each shift s
- $\boldsymbol{L_t}$ as the length of the task t
- η as the number of public holidays on the planning horizon
- ξ as the number of hours to discount from the contract hours
- $\boldsymbol{A_i}$ as the percentage of work that must be allocated to the dispatch centre for worker i
- $\boldsymbol{R_{d,s,t}}$ as the staff requirements for day d, shift s and task t
- $\boldsymbol{z_{i,d,s,t}^0}$ as the initial schedule for worker i
- $\boldsymbol{z_{i,d}^D}$ as the disruptions caused by worker i and day d
- $\boldsymbol{Dist_{i,t}}$ as the distance between worker i location and the place where the task t must be performed
- $|\boldsymbol{D}|$ as the number of days during the planning horizon
- $|\boldsymbol{W}|$ as the number of Sundays during the planning horizon

The objective function weights are now defined as:

- $\boldsymbol{w^{WF-}}$ the weight of the penalty variable for understaffed tasks for a certain day and shift

- $\boldsymbol{w^{WF+}}$ the weight of the penalty variable for overstaffed tasks for a certain day and shift
- $\boldsymbol{w^{OT-}}$ the weight of the penalty variable for shortage of hours worked
- $\boldsymbol{w^{OT+}}$ the weight of the penalty variable for excess of hours worked
- $\boldsymbol{w^{Sun}}$ the weight of the penalty for worker i being assigned to work on Sunday w but not on Saturday $w-1$
- $\boldsymbol{w^{Sat}}$ the weight of the penalty for worker i being assigned to work on Saturday $w-1$ but not on Sunday w
- $\boldsymbol{w^{SP}}$ the weight of the penalty for not satisfying staff preferences
- $\boldsymbol{w^{des}_{skill}}$ the weight of the penalty for worker i not meeting desired skills for task t
- $\boldsymbol{w^{C-}}$ the weight of the penalty variable for the difference between required working hours in the dispatch centre and the hours worked (undertime)
- $\boldsymbol{w^{C+}}$ the weight of the penalty variable for the difference between required working hours in the dispatch centre and the hours worked (overtime)
- $\boldsymbol{w^{G}}$ the weight of the penalty for team swaps
- $\boldsymbol{w^{Dist}}$ the weight of the penalty for tasks performed outside of workers' location
- $\boldsymbol{w^{R-}}$ the weight of the penalty variable for decrease in working hours by worker i from previous schedule
- $\boldsymbol{w^{R+}}$ the weight of the penalty variable for increase in working hours by worker i from previous schedule

4.2.3 Variables

This section presents decision and auxiliary variables along with their respective domain. Regarding decision variables, this problem presents only a set of decision variables, concerning the assignment of a tasks on a day and shift to a worker:

$\boldsymbol{x_{i,d,s,t}} \in \{0,1\}$ is a decision variable that determines if worker i is assigned on day d and shift s to task t (1) or not (0)

There are various auxiliary/penalty variables, which account for the quality of the reschedule solution, as these are present in the objective function. In this model, the auxiliary variables are the following:

- $\boldsymbol{Y^{OT-}_{i}} \in \mathbb{N}_{\boldsymbol{0}}$ is the auxiliary/penalty variable for the number of deficit working hours by worker i
- $\boldsymbol{Y^{OT+}_{i}} \in \mathbb{N}_{\boldsymbol{0}}$ is the auxiliary/penalty variable for the number of excess working hours by worker i
- $\boldsymbol{Y^{WF-}_{d,s,t}} \in \mathbb{N}_{\boldsymbol{0}}$ is the auxiliary/penalty variable measuring the lack of workers on day d, shift s to perform task t
- $\boldsymbol{Y^{WF+}_{d,s,t}} \in \mathbb{N}_{\boldsymbol{0}}$ is the auxiliary/penalty variable measuring the excess of workers on day d, shift s to perform task t
- $\boldsymbol{Y^{Sun}_{i,w}} \in \mathbb{N}_{\boldsymbol{0}}$ is the auxiliary/penalty variable defining if worker i is assigned to work on Sunday w but not on Saturday $w-1$

- $\boldsymbol{Y_{i,w}^{Sat}} \in \mathbb{N}_0$ is the auxiliary/penalty variable defining if worker i is assigned to work on Saturday $w-1$ but not on Sunday w
- $\boldsymbol{Y_{i,d}^{SP}} \in \mathbb{N}_0$ is the auxiliary/penalty variable that evaluates if the preference of a day-off of worker i on day d is satisfied
- $\boldsymbol{Y_{i,t}^{des}} \in \mathbb{N}_0$ is the auxiliary/penalty variable for assigning a task t to a worker i that does not have the desired skills to perform t
- $\boldsymbol{Y_{i}^{C+}} \in \mathbb{N}_0$ is the auxiliary/penalty variable measuring the difference between required working hours in the dispatch centre for worker i
- $\boldsymbol{Y_{i}^{C-}} \in \mathbb{N}_0$ is the auxiliary/penalty variable measuring the difference between required working hours in the dispatch centre and the hours worked for worker i
- $\boldsymbol{Y_{g}^{G}} \in \mathbb{N}_0$ is the auxiliary/penalty variable for allocating tasks to team g to workers that do not belong to team g
- $\boldsymbol{Y_{i,d,s}^{R+}} \in \mathbb{N}_0$ is the auxiliary/penalty variable measuring the increase in working hours by worker i from previous schedule
- $\boldsymbol{Y_{i,d,s}^{R-}} \in \mathbb{N}_0$ is the auxiliary/penalty variable measuring the decrease in working hours by worker i from previous schedule

4.2.4 Objective Function

The objective function presented in equation (4.1) aims to minimize the weighted sum of the penalty variables, as stated in problem statement on section 4.1.

$$
\begin{aligned}
Minimize \quad & \left(\sum_{d\in D}\sum_{s\in S}\sum_{t\in T} \boldsymbol{w^{WF-}} \times \boldsymbol{Y_{d,s,t}^{WF-}} + \boldsymbol{w^{WF+}} \times \boldsymbol{Y_{d,s,t}^{WF+}}\right) \\
& + \left(\sum_{i\in I} \boldsymbol{w^{OT-}} \times \boldsymbol{Y_{i}^{OT-}} + \boldsymbol{w^{OT+}} \times \boldsymbol{Y_{i}^{OT+}}\right) + \left(\sum_{i\in I}\sum_{w\in W} \boldsymbol{w^{Sat}} \times \boldsymbol{Y_{i,w}^{Sat}} + \boldsymbol{w^{Sun}} \times \boldsymbol{Y_{i,w}^{Sun}}\right) \\
& + \left(\sum_{i\in I}\sum_{d\in D} \boldsymbol{w^{SP}} \times \boldsymbol{Y_{i,d}^{SP}}\right) + \left(\sum_{i\in I}\sum_{d\in D}\sum_{s\in S}\sum_{t\in T} \boldsymbol{w_{skill}^{des}} \times \boldsymbol{Y_{i,d,s,t}^{des}}\right) \\
& + \left(\sum_{i\in I} \boldsymbol{w^{C-}} \times \boldsymbol{Y_{i}^{C-}} + \boldsymbol{w^{C+}} \times \boldsymbol{Y_{i}^{C+}}\right) + \left(\sum_{g\in G} \boldsymbol{w^{G}} \times \boldsymbol{Y_{g}^{G}}\right) \\
& + \left(\sum_{i\in I}\sum_{t\in T} \boldsymbol{w^{Dist}} \times \left(\sum_{d\in D}\sum_{s\in S} \boldsymbol{x_{i,d,s,t}}\right) \times \boldsymbol{Dist_{i,t}}\right) \\
& + \left(\sum_{i\in I}\sum_{d\in D}\sum_{s\in S} \boldsymbol{w^{R-}} \times \boldsymbol{Y_{i,d,s}^{R-}} + \boldsymbol{w^{R+}} \times \boldsymbol{Y_{i,d,s,t}^{R+}}\right)
\end{aligned}
\tag{4.1}
$$

The first four lines concern aspects regarding the scheduling and the last line is related to the rescheduling. The first line includes the most important factor of the scheduling: (i) under ($\boldsymbol{Y^{WF-}}$) and overstaffed ($\boldsymbol{Y^{WF+}}$) tasks on particular days and schedules. It represents the coverage maximization, which has a direct impact on any organization, especially EMS, as the lack of workers may compromise the service quality and thus jeopardize patient care. Effectively, the objective function will be more penalized in case of understaffed tasks.

Then, terms (ii) to (viii) seek to reduce factors that do not contribute to the schedule quality, such as (ii) shortage ($\boldsymbol{Y_i^{OT-}}$) and excess ($\boldsymbol{Y_i^{OT+}}$) of hours worked, (iii) incomplete weekends off, which means having only the Saturday off ($\boldsymbol{Y_{i,w}^{Sat}}$) or only the Sunday off ($\boldsymbol{Y_{i,w}^{Sun}}$), (iv) completing tasks on days in which individuals preferred not to work ($\boldsymbol{Y_{i,d}^{SP}}$), and (v) had not the desired skills to perform that task ($\boldsymbol{Y_{i,t}^{des}}$). Additionally, it also considers (vi) the shortage ($\boldsymbol{Y_i^{C-}}$) and excess ($\boldsymbol{Y_i^{C+}}$) of hours that should be allocated to the dispatch centre and (vii) the tasks done outside of the corresponding team ($\boldsymbol{Y_g^{G}}$). Finally, the objective function penalizes (viii) the distance that each worker must do to perform a specific task. Each term corresponds to a penalty that impacts the general satisfaction for the schedule, weighed correspondingly.

Lastly, the objective function also takes into account (ix) the differences between the current and previous schedule, by penalizing for each worker both the decrease ($\boldsymbol{Y}_{i,d,s}^{R-}$) and the increase ($\boldsymbol{Y}_{i,d,s}^{R+}$) of shifts to perform. As it was explained before, it is of critical importance that the rebuilt schedule creates as little disorder as possible from the previous.

4.2.5 Constraints

Finally, this section presents the model's constraints. These were developed based on previous works done by various authors (Carmo, 2021; A. Clark & Walker, 2011; Maenhout & Vanhoucke, 2018a; Namorado Rosa, 2017; Wickert *et al*., 2019; Wolbeck *et al*., 2020). Constraints can be divided into two categories, hard and soft constraints, which will be exposed in the next sections.

4.2.5.1 Hard Constraints

This category is composed of all the constraints that must not be violated. Thus, the model aims to find a rebuilt schedule that respects all the hard constraints. It is important to note that there are constraints that concern the building of any schedule, so an initial schedule can be built applying this set of constraints. Moreover, there are also constraints related to rebuilding a new schedule from a previous one. Therefore, with the same set of constraints, it is possible to build scheduling and rescheduling models (Wickert *et al*., 2019).

Scheduling Constraints

Skills Requirement

Certain tasks demand precise skills and constraint (4.2) guarantees that there are no workers assigned to tasks for which they are not capable of, i.e., tasks can only be done by the group of workers that have the skills to perform it. This is critical to assure that patients have the best health care with the right staff.

$$\boldsymbol{x_{i,d,s,t} = 0}\ , \forall\ \boldsymbol{i} \notin \boldsymbol{I_t^T}\ , \forall\ \boldsymbol{d} \in \boldsymbol{D}\ , \forall\ \boldsymbol{s} \in \boldsymbol{S}\ , \forall\ \boldsymbol{t} \in \boldsymbol{T} \qquad (4.2)$$

Minimum resting time between two consecutive shifts

Workers at EMS need a minimum resting time between two consecutive shifts of twelve hours. As each shift has eight hours, it is considered that there must be two shifts off between the last finished and the following. Therefore, constraints (4.3), (4.4) and (4.5) assure this. Constraint (4.3) states that each worker can only carry out at most one task (that belongs to the group of tasks that this worker is capable of performing) per day, while constraint (4.4) expresses that a worker can only operate at most one task within the morning and the afternoon of one day and the night of the following. Constraint (4.5) considers the remaining alternative where the individual can only perform at most one task within the afternoon of one day and the night and the morning of the following. Figure 9 explain the three shifts sequences that are avoided with these equations.

$$\sum_{t \in T_i^I} \boldsymbol{x_{i,d,1,t}} + \sum_{t \in T_i^I} \boldsymbol{x_{i,d,2,t}} + \sum_{t \in T_i^I} \boldsymbol{x_{i,d,3,t}} \leq \mathbf{1}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{d} \in \boldsymbol{D} \tag{4.3}$$

$$\sum_{t \in T_i^I} \boldsymbol{x_{i,d+1,1,t}} + \sum_{t \in T_i^I} \boldsymbol{x_{i,d,2,t}} + \sum_{t \in T_i^I} \boldsymbol{x_{i,d,3,t}} \leq \mathbf{1}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{d} \in \boldsymbol{D} \setminus \{|\boldsymbol{D}|\} \tag{4.4}$$

$$\sum_{t \in T_i^I} \boldsymbol{x_{i,d+1,1,t}} + \sum_{t \in T_i^I} \boldsymbol{x_{i,d+1,2,t}} + \sum_{t \in T_i^I} \boldsymbol{x_{i,d,3,t}} \leq \mathbf{1}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{d} \in \boldsymbol{D} \setminus \{|\boldsymbol{D}|\} \tag{4.5}$$

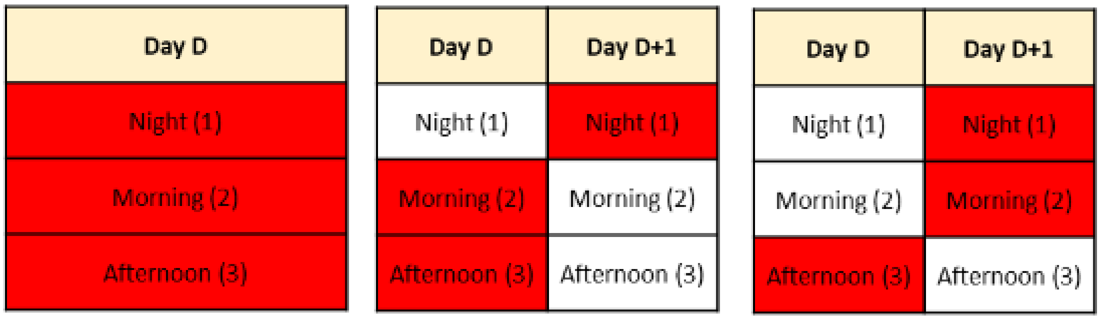

Figure 9: Shift sequences to be avoided

Maximum consecutive working days per worker

Each worker must not be assigned to tasks in more than θ_{days}^{max} consecutive days. Thus, constraint (4.6) guarantees that, for each set of θ_{days}^{max} + 1 consecutive days in the planning horizon, each individual works a maximum of θ_{days}^{max}, having at least one day off in the set.

$$\sum_{r \in \{d, d+1, \dots, d+\theta_{days}^{max}\}} \sum_{s \in S} \sum_{t \in T_i^I} \boldsymbol{x_{i,r,s,t}} \leq \boldsymbol{\theta_{days}^{max}}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{d} \in \boldsymbol{D} \setminus \{|\boldsymbol{D}|, |\boldsymbol{D}| - \mathbf{1}, \dots, |\boldsymbol{D}| - \boldsymbol{\theta_{days}^{max}}\} \tag{4.6}$$

Maximum consecutive working nights per month per worker

Constraint (4.7) functions as the previous but regarding nights (shift = 1). Hence, for each set of θ_{nights}^{max} + 1 days during the planning horizon, each individual works a maximum of θ_{night}^{max}, having at least one night off in the set.

$$\sum_{r\in\{d,d+1,\dots,d+\theta_{days}^{max}\}}\sum_{s\in S}\sum_{t\in T_i^l} x_{i,r,1,t} \leq \theta_{nights}^{max}, \forall\, i \in I, \forall\, d \in D\ \{|D|, |D|-1, \dots, |D|-\theta_{nights}^{max}\} \quad (4.7)$$

Maximum consecutive days-off per month per worker

Each worker has also a maximum limit of θ_{d-off}^{max} consecutive days-off. In this case, constraint (4.8) assures that, for each set of θ_{d-off}^{max} + 1 consecutive days during the planning horizon, each individual must work at least one day. It is important to note that, when implementing the model, holidays must be taken into account so there is no conflict between this constraint and scheduled holidays.

$$\sum_{r\in\{d,d+1,\dots,d+\theta_{d-off}^{max}+1\}}\sum_{s\in S}\sum_{t\in T_i^l} x_{i,r,s,t} \geq 1, \forall\, i \in I, \forall\, d \in D \setminus \{|D|, |D|-1, \dots, |D|-\theta_{d-off}^{max}-1\} \quad (4.8)$$

Minimum number of Sundays off per month per worker

Additionally, when building the schedule, each worker must be entitled to have a minimum of θ_{s-off}^{min} Sundays off. Constraint (4.9) allows individuals to only be assigned to Sundays' tasks the number of Sundays in the planning horizon $|W|$ minus the number of minimum Sundays off, θ_{s-off}^{min}.

$$\sum_{r\in W}\sum_{s\in S}\sum_{t\in T_i^l} x_{i,r,s,t} \leq |W| - \theta_{s-off}^{min}, \forall\, i \in I \quad (4.9)$$

Minimum number of shifts per shift type

In order to avoid that workers only operate in a specific shift, constraint (4.10) requires that each one has at least θ_s^{min-s} tasks in each shift. It allows to balance the type of shifts performed by the workers, while this equilibrium may also contribute to increase staff satisfaction.

$$\sum_{d\in D}\sum_{t\in T_i^l} x_{i,d,s,t} \geq \theta_s^{min-s}, \forall\, i \in I, s \in S \quad (4.10)$$

Scheduled holidays

Finally, scheduled holidays must be considered. Thus, constraint (4.11) guarantees that workers are not allocated to tasks on days that belong to the group of days in which they have holidays, as they are absent.

$$\sum_{d\in H_i^l}\sum_{s\in S}\sum_{t\in T_i^l} x_{i,d,s,t} = 0, \forall\, i \in I \quad (4.11)$$

Rescheduling Constraints

Absent workers do not work

When there is a disruption, rescheduling constraints are used, as workers that will be absent in a day of the schedule may not perform tasks to be done in that day. Therefore, constraint (4.12) set that, for a worker

i on a specific day d, if s/he is absent, i.e., $\boldsymbol{z}_{i,d}^{D} = 1$, then s/he cannot be allocated to tasks on that day, i.e., $\sum_{s \in S} \sum_{t \in T_i^I} \boldsymbol{x}_{i,d,s,t} = 0$. If s/he is not absent on that, i.e., $\boldsymbol{z}_{i,d}^{D} = 0$, then s/he can operate on that day, i.e., $\sum_{s \in S} \sum_{t \in T_i^I} \boldsymbol{x}_{i,d,s,t} = 1$. The sum of these two terms is equal or lower than 1, because there can be the case where the worker is not absent, $\boldsymbol{z}_{i,d}^{D} = 0$, and was not allocated to a task on that day, $\sum_{s \in S} \sum_{t \in T_i^I} \boldsymbol{x}_{i,d,s,t} = 0$.

$$\sum_{s \in S} \sum_{t \in T_i^I} \boldsymbol{x}_{i,d,s,t} + \boldsymbol{z}_{i,d}^{D} \leq \mathbf{1} \text{ , } \forall\, \boldsymbol{i} \in \boldsymbol{I} \text{ , } \forall\, \boldsymbol{d} \in \boldsymbol{D} \qquad (4.12)$$

4.2.5.2 Soft Constraints

In contrast to hard, soft constraints can be violated but these violations are penalized in the objective function. The global objective is to minimize the weighted sum of all the penalty variables, minimizing the real impact of violating soft constraints and obtaining a better quality for the solution.

Scheduling Constraints

Overtime and undertime of workers

Each worker performs certain tasks in specific days and shifts ($\boldsymbol{x}_{i,d,s,t}$), which multiplied by the length of each task $\boldsymbol{L}_t$ returns the number of hours worked in each month. This number must be in agreement with the number of hours present in the contract $\boldsymbol{\theta}_i^{min}$, subtracted by the number of scheduled H_i^I and public holidays η multiplied by the number of hours to be subtracted ξ. However, to meet the demand, sometimes there are differences between the number of hours worked and those established in the contract, being counted as overtime $\boldsymbol{Y}_i^{OT+}$ or undertime $\boldsymbol{Y}_i^{OT-}$. Effectively, reducing both over and undertime are of the greatest interest to the organization, thus these factors are penalized in the objective function.

$$\sum_{d \in D} \sum_{s \in S} \sum_{t \in T_i^I} \boldsymbol{L}_t \times \boldsymbol{x}_{i,d,s,t} = \boldsymbol{\theta}_i^{min} - (\eta + H_i^I) \times \xi + \boldsymbol{Y}_i^{OT+} - \boldsymbol{Y}_i^{OT-} \text{ , } \forall\, \boldsymbol{i} \in \boldsymbol{I} \qquad (4.13)$$

Coverage requirements

Constraint (4.14) intends to avoid understaffed tasks that may harm the quality of the service provided and overstaffed tasks that lead the organization to incur extra costs. Therefore, for each task on a day and shift, the number of workers allocated must be equal to the demand $\boldsymbol{R}_{d,s,t}$. Nevertheless, current workforce may not be able to answer to the demand, which will take to understaffed tasks $\boldsymbol{Y}_{d,s,t}^{WF-}$ or it may exceed the demand, leading to overstaffed tasks $\boldsymbol{Y}_{d,s,t}^{WF+}$. As explained above, reducing both is crucial to the organization.

$$\sum_{i \in I_t^T} \boldsymbol{x}_{i,d,s,t} = \boldsymbol{R}_{d,s,t} - \boldsymbol{Y}_{d,s,t}^{WF-} + \boldsymbol{Y}_{d,s,t}^{WF+} \text{ , } \forall\, \boldsymbol{d} \in \boldsymbol{D} \text{ , } \forall\, \boldsymbol{s} \in \boldsymbol{S} \text{ , } \forall\, \boldsymbol{t} \in \boldsymbol{T} \qquad (4.14)$$

Incomplete weekends off per month per worker

Usually, organizations are legally bounded to offer their workers a particular number of Sundays off enforced by the hard constraint (4.9). Still, organizations should, whenever it is possible, give the whole weekend off to increase staff's satisfaction. Constraint (4.15) takes this aspect into consideration, penalizing the objective function if a person works on a Sunday but not on Saturday ($\boldsymbol{Y_{i,w}^{Sun}}$) and if a person works on Saturday but not on a Sunday ($\boldsymbol{Y_{i,w}^{Sat}}$).

$$\sum_{s \in S} \sum_{t \in T_i^I} (\boldsymbol{x_{i,w,s,t}} - \boldsymbol{x_{i,w-1,s,t}}) - \boldsymbol{Y_{i,w}^{Sun}} + \boldsymbol{Y_{i,w}^{Sat}} = \boldsymbol{0}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{w} \in \boldsymbol{W} \tag{4.15}$$

The following soft constraints related to scheduling were not used in the present model. However, they will be introduced, since they can be interesting to the development of the literature and future works.

Staff preferences

In order to increase staff satisfaction, it is important to meet their preferences. Therefore, constraint (4.16) penalizes when a worker must perform a task on day and shift that s/he did not give preference to.

$$\sum_{t \in T_i^I} \boldsymbol{x_{i,d,s,t}} = \boldsymbol{Y_{i,d,s}^{SP}}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{d} \in \boldsymbol{P_i^I}, \forall\, \boldsymbol{s} \in \boldsymbol{S} \tag{4.16}$$

Satisfy desired skill requirements

Besides the skills required to execute a task, some organizations also consider those skills that are relevant and an added value but not mandatory to operate. Even if a worker has not these skills, s/he may be allocated to that task. Thus, constraint (4.17) penalizes when a person is allocated to a task and does not belong to the set of workers that have the desired skills to perform it $\boldsymbol{I_t^{Td}}$.

$$\sum_{d \in D} \sum_{s \in S} \boldsymbol{x_{i,d,s,t}} = \boldsymbol{Y_{i,t}^{des}}, \forall\, \boldsymbol{i} \notin \boldsymbol{I_t^{Td}}, \forall\, \boldsymbol{t} \in \boldsymbol{T_i^I} \tag{4.17}$$

Hours allocated to each class of task

Some organizations demand that workers have a mix of tasks from different departments. Hence, there is a percentage $\boldsymbol{A_i}$ of hours worked in the dispatch centre that the staff must respect. However, this is not considered an obligation but rather a recommendation to keep operating in emergency vehicles and thus it is penalized with lack $\boldsymbol{Y_i^{C-}}$ and excess $\boldsymbol{Y_i^{C+}}$of hours in the dispatch centre.

$$\sum_{d \in D} \sum_{s \in S} \sum_{t \in C_t^T} (\boldsymbol{x_{i,d,s,t}} \times \boldsymbol{L_t}) = \boldsymbol{A_i} \times \sum_{d \in D} \sum_{s \in S} \sum_{t \in T} (\boldsymbol{x_{i,d,s,t}} \times \boldsymbol{L_t}) - \boldsymbol{Y_i^{C-}} + \boldsymbol{Y_i^{C+}}, \forall\, \boldsymbol{i} \in \boldsymbol{I} \tag{4.18}$$

Performing tasks within the respective team

To facilitate the organization of operations, organizations group their workers according to the tasks and skills requirement, in such way that each person is part of a team. Therefore, it is crucial to have tasks that belong to a team being allocated to workers that also belong to that team, being the contrary penalized in the objective function with the counting variable $\boldsymbol{Y_g^G}$.

$$\sum_{d \in D} \sum_{s \in S} \sum_{t \in T_g^G} \sum_{i \in I_t^T \setminus I_g^G} \boldsymbol{x_{i,d,s,t}} - \boldsymbol{Y_g^G} = \boldsymbol{0}, \forall\, \boldsymbol{g} \in \boldsymbol{G} \tag{4.19}$$

Rescheduling Constraints

Changes from original schedule

When rebuilding a schedule after a person's absence, it is critical to reduce the disruption caused to the remaining staff. Therefore, constraint (4.20) assures that the new schedule $\boldsymbol{x_{i,d,s,t}}$ is equal to the previous one $\boldsymbol{z^0_{i,d,s,t}}$, except for specific cases when it is required to change workers' schedule in order to respect hard constraints or to achieve a better solution quality. $\boldsymbol{Y^{R+}_{i,d,s}}$ reflects the increase of shifts worked that an individual has from the previous schedule and $\boldsymbol{Y^{R-}_{i,d,s}}$ the decrease.

$$\sum_{t \in T_i^I} (\boldsymbol{x_{i,d,s,t}} - \boldsymbol{z^0_{i,d,s,t}}) + \boldsymbol{Y^{R+}_{i,d,s}} - \boldsymbol{Y^{R-}_{i,d,s}} = \boldsymbol{0}, \forall\, \boldsymbol{i} \in \boldsymbol{I}, \forall\, \boldsymbol{d} \in \boldsymbol{D}, \forall\, \boldsymbol{s} \in \boldsymbol{S} \tag{4.20}$$

4.3 Chapter Considerations

This chapter presented the mathematical model to solve the EMS rescheduling problem. First, the problem statement was presented to have a clear overview of the issue being addressed. The model has the goal of reallocating tasks to workers, while minimizing eight important factors abovementioned.

Then, the mathematical formulation was introduced showing sets and subsets, parameters, variables, objective function and constraints. The developed model intends to be general for all EMS and flexible to integrate multiple goals. As seen in section 4.2.5, it is capable of producing both scheduling and rescheduling, being the constraints grouped in those used to build a first schedule and those that must be taken into account when rebuilding a schedule. In the next chapter, the INEM case will be presented and a real data set will be used in order to validate the present model.

5. INEM Case Study and Instance Generation

This chapter introduces the data required to apply the model above defined to the real case of INEM. Section 5.1 explains the three data collection procedures used in this work. Following, section 5.2 presents the problem context, detailing the demand area, the tasks needed to be done, the workforce and respective team allocation to both services, CODU and EVs. Section 5.3 describes the instance structure. Then, section 5.4 presents the three groups of instances generated to verify the consistency of the model, evaluating the difference between starting from a cyclic and a non-cyclic schedule, assessing the feasibility of the model for the INEM universe and then varying the absenteeism rate and disruptions generation. Finally, section 5.5 briefly discuss the main considerations taken from this chapter.

5.1 Data Collection Procedures

To apply the proposed model to the case study of INEM, appropriate data must be collected and processed in order to create model inputs. To accomplish this, three data collection methods were used. First, an analysis of the data treated in previous INEM's works was done. In the last years, Namorado Rosa, (2017), Ferreira dos Santos, (2019) and Carmo, (2021) studied issues at INEM, including staff scheduling and then developed a complete database alongside INEM's technicians. This dataset is composed of, for example, workers, their skills and teams' composition. However, when the data required was not present in the database, it was important to discuss with other stakeholders, including experts that had previous contacts INEM's technicians. This was important to add relevant data, such as parameters settings and also the value of the weights. Finally, when it was not possible to access demanded data via the two abovementioned methods, it was important to resort to the existing literature. To illustrate, INEM did not keep records of the absences that happen during a month. Therefore, the absence rate was estimated based on the work done by (Wolbeck *et al*., 2018) and Wickert *et al*., (2019). Moreover, it would be interesting that INEM could keep these records and also the reconstructed schedule that arose from a disruption in order to compare with the results obtained from the model present in section 6.

5.2 Problem Context

INEM operates every day, being the workers possibly allocated to one of three shifts: night, morning and afternoon. INEM's initial schedule is cyclic, where TEPHs work in 10-days cycles (M-M-A-A-O-N-N-O-O-O) as it was explained in section 2.2.1.

This section presents the demand area which will be addressed in this case, then the tasks that must be done and, finally, the workforce composition, as well as the composition of the teams.

5.2.1 Demand Area

This study focuses on rescheduling for both INEM's services, CODU and EVs in the areas that respond to the Lisbon CODU displayed in figure 10. Surely, it encompasses CODU's location (in red) but it also includes several areas of the Lisbon Metropolitan Area (below CODU in the left column). Moreover, there are also other cities, scattered through the south to the centre region of Portugal that are ordered from the northernmost to the southernmost in the right column of the figure.

Figure 10: Geographic distribution of INEM services that respond to the Lisbon CODU

5.2.2 Tasks

For each of the two services, there are specific tasks. These also have an associated duration, almost all last for eight hours, except for MEM drive which lasts for twelve hours. Additionally, each task also demands certain skills so that they can be performed by TEPHs. These skills are displayed in the right column of table 13.

TEPHs can execute two jobs at CODU: the CODU shift responsible and the CODU operator. The CODU shift responsible is responsible for supervising and coordinating activities at CODU throughout each shift, monitoring the various operations and making crucial decisions as necessary. Concerning the high level of responsibility that this work implies, it necessitates a particular skill - CODU responsible -, which comprises more expertise with CODU service and leadership abilities. Once a TEPH has this skill, he also has the CODU operator skill and can perform both jobs. On the other hand, the CODU operator is responsible for answering incoming calls, routing and dispatching vehicles, and gathering emergency data, and the CODU operator competence is required to accomplish this work.

Considering EVs service, each task is called for the kind of medical vehicle to which it is linked. To illustrate, SIV task is related to the SIV vehicle and the TIP task is related to the TIP vehicle. There is just one exception for the AEM category, because this vehicle demands two distinct tasks, requiring then two TEPHs. These TEPHs have separate roles: one is in charge of driving, as an AEM driver, while the other is in charge of the shift, as an AEM team responsible. Each of these activities needs the acquisition of specialized abilities in accordance with the work requirements of each TEPH. For example, driving and MEM driving skills are equivalent to having a vehicle and a motorbike license, respectively and both glucagon administration and defibrillator skills call for specialized training.

Table 13: INEM tasks, duration and required skills

Tasks		Duration (in hours)	Required Skills
CODU	CODU shift responsible	8	CODU responsible
	CODU operator	8	CODU operator
EVs	AEM driver	8	Driving
	AEM team responsible	8	Glucagon, Defibrillator
	SIV driver	8	Driving
	TIP driver	8	Driving, CODU operator
	UMIPE driver	8	Driving, CODU operator, Defibrillator
	MEM driver	12	MEM driving, Defibrillator, Glucagon

5.2.3 Workforce and Team Allocation

The workforce considered in this case study is composed of 266 Pre-Hospital Emergency Technicians (TEPHs). At INEM, every TEPH holds a full-time contract, working 35 hours per week, which corresponds to 7 hours per day, 5 days a week. As the time horizon in this case study is 31 days, if there are no public or scheduled holidays, TEPHs must work 156 hours given by the following equation:

$$\theta_i^{min} = \text{int}\left((35 \times 4) \times (|D|/28) + 0.5\right) - (\eta + H_i^I) \times \xi \qquad (5.1)$$

However, due to the need of satisfying the demand, TEPHs may have under or overtime, despite not being recommended. To have an idea, given the horizon of 31 days and the cyclic schedule that was implemented by INEM, workers perform 18 or 19 shifts within a month (depending on which day of the 10-days cycle the month starts) without extra hours. Assuming 19, it means that there are 5,054 operated shifts by the 266 TEPHs. Nevertheless, for this area, INEM's demand for the whole month corresponds to 6,080 shifts, which shows two main aspects. On the one hand, there is a huge lack of TEPHs for the Lisbon CODU area. In order to not have extra hours and meet all the demand, 54 new TEPHs should be hired, assuming that each one would execute 19 shifts. On the other hand, there is also a great need for efficiency in INEM's operations to assure that these scarce human resources are used in the best possible way and rescheduling is definitely one aspect to take into account.

As seen before, INEM has two services: CODU and EVs. TEPHs can be allocated to one or more teams and each team is responsible for certain tasks that belong to one or the other service. Nonetheless, TEPHs may perform tasks outside of their team, but this should be avoided. The number of TEPHs assigned to each working team is determined by the following factors: required skills and the location of the tasks. The efficient and well-planned distribution of TEPHs between teams and tasks is critical to ensure the organization of INEM's activities. The allocation of each TEPH to the correspondent team was provided by INEM.

5.2.3.1 CODU Service

At CODU, there are 5 teams and each one is formed by 23 TEPHs, having a total of 115 technicians that can perform CODU's tasks. It is important to note that these may also belong to other EVs' teams. Each one of the 5 teams is responsible for the same two tasks: CODU shift responsible and CODU operator, as is shown in table 14.

Table 14: Distribution of the number of workers, task number and task type per working team for CODU

Team	Number of workers	Task number	Task
Team 1	23	Task 1 Task 2	CODU shift responsible CODU operator
Team 2	23	Task 3 Task 4	CODU shift responsible CODU operator
Team 3	23	Task 5 Task 6	CODU shift responsible CODU operator
Team 4	23	Task 7 Task 8	CODU shift responsible CODU operator
Team 5	23	Task 9 Task 10	CODU shift responsible CODU operator

In this service, the daily demand for each shift is defined a priori based on past events. Each CODU shift responsible task requires one TEPH and each CODU operator task requires 20 TEPHs with the required capabilities, which means there is a need of 63 TEPHs for CODU's activities per day.

5.2.3.1 EV Service

In the EVs case, there are 194 TEPHs capable of performing tasks that belong to EVs. In this case, each team is responsible for a set of activities related to a set of vehicles. There are teams without associated TEPHs, however, due to workers' cross training, the corresponding tasks are still performed. The information regarding teams, allocated workers and the tasks that they are responsible for is summarized in Table 15.

Furthermore, associated with each vehicle there is a correspondent task and a required number of workers. For AEM, there must be two TEPHs, one responsible for driving and another as the team responsible. For the other vehicles, SIV, TIP, UMIPE and MEM, there is only one TEPH required to perform the task associated with the vehicle.

Moreover, not all vehicles operate continuously, 24 hours per day. Most of AEMs work every day all three shifts but some only function two shifts a day (morning and afternoon). Then, SIVs, UMIPE and TIP operate constantly. Finally, MEMs only operate on morning shifts on weekdays.

Table 15: Distribution of the number of workers, task number and task type per working team for EVs

Team	Number of workers	Task number	Task
Team 6	28	Task 11 to Task 17	AEM 1, AEM 10, AEM 15, SIV Lisboa
Team 7	24	Task 18 to Task 23	AEM 2, AEM 9, AEM 13
Team 8	25	Task 24 to Task 29	AEM 3, AEM 11, AEM 12
Team 9	27	Task 30 to Task 37	AEM 4, AEM 7, AEM 14, AEM Sacavém
Team 10	21	Task 38 to Task 43	AEM 6, AEM 5, AEM Amadora
Team 11	18	Task 44 to Task 47	AEM Setúbal 1, AEM Setúbal 2
Team 12	17	Task 48 to Task 51	AEM Almada, AEM Seixal
Team 13	4	Task 52	SIV Estremoz
Team 14	4	Task 53	SIV Torres Novas
Team 15	6	Task 54	SIV Tomar
Team 16	5	Task 55	SIV Elvas
Team 17	3	Task 56	SIV Ponte de Sor
Team 18	0	Task 57	TIP Lisboa
Team 19	0	Task 58	UMIPE Lisboa
Team 20	4	Task 59	MEM Lisboa
Team 21	3	Task 60	MEM Cascais
Team 22	6	Task 61	MEM Setubal
Team 23	4	Task 62	SIV Odemira
Team 24	5	Task 63	SIV Moura
Team 25	4	Task 64	SIV Castro Verde

The next section explains what inputs are given to the model to obtain accurate solutions regarding the case study.

5.3 Instance structure

Instances are composed of tables, parameters and weights. Instances are composed of 8 tables, 10 parameters and 12 weights. These will be explained in the following sections.

5.3.1 TEPH vs Team

The first table allocates each TEPH to one or more teams within which s/he is expected to perform most of her/his tasks. Each line corresponds to a TEPH identification and each column to a team identification. If TEPH i belongs to team g, the entry (i, g) of the table 16 is equal to 1 and 0 otherwise.

Table 16: Table TEPH vs Team

		Team																								
TEPH		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	162	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	163	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	164	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	165	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	166	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

5.3.2 TEPH vs Task

The second table indicates if a TEPH has the skills that are considered to be required to perform a specific task. Each line corresponds to a TEPH identification and each column to a task identification. In case a TEPH i has the required skills to perform task t, the entry (i, t) of the table 17 is equal to 1. In case it does not happen, the entry (i, t) is equal to 0.

Table 17: Table TEPH vs Task

		Task																								
TEPH		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**…**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	162	0	0	0	0	0	0	0	0	0	0	1	1	…	1	1	1	1	1	1	1	1	1	1	1	1
	163	0	1	0	1	0	1	0	1	0	1	1	1	…	1	1	1	1	1	1	0	0	0	1	1	1
	164	0	0	0	0	0	0	0	0	0	0	1	1	…	1	1	1	1	1	1	0	0	0	1	1	1
	165	0	1	0	1	0	1	0	1	0	1	1	1	…	1	1	1	1	1	1	0	0	0	1	1	1
	166	0	1	0	1	0	1	0	1	0	1	1	1	…	1	1	1	1	1	1	0	0	0	1	1	1
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

5.3.3 Demand during the planning horizon

The following table displays the demand defined a priori for each day and each shift of the planning horizon. Each line corresponds to a task identification and each column to a shift of the day above, for example the top left 0 is related to the demand for task 9 on the day 1 and shift 1. The entry $(t(d, s))$ corresponds to the number of TEPHs required for that task in that day and shift.

Table 18: Demand during the planning horizon

	Horizon																									
Task	**Day**	**1**			**2**			**3**			**4**			**...**	**28**			**29**			**30**			**31**		
	Shift	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**	**...**	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	9	0	0	0	0	0	0	0	1	0	0	1	0	...	1	0	0	1	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	20	0	0	20	0	...	20	0	0	20	0	0	0	0	0	0	0	0
	11	1	1	1	1	1	1	1	1	1	1	1	1	...	1	1	1	1	1	1	1	1	1	1	1	1
	12	1	1	1	1	1	1	1	1	1	1	1	1	...	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	...	1	1	1	1	1	1	1	1	1	1	1	1
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

5.3.4 Duration of each task

Table 19 presents the length in hours of each task t. As it was abovementioned all tasks take 8 hours to be completed with exception to MEM tasks, 59, 60 and 61, which take 12 hours to be carried out.

Table 19: Duration of each task

	Task																								
Duration	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**...**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**
	8	8	8	8	8	8	8	8	8	8	8	8	...	8	8	8	8	8	8	12	12	12	8	8	8

5.3.5 Task vs Team

The next table allocates each task to a team. Each line corresponds to a task identification and each column to a team identification. If task t belongs to team g, the entry (t, g) of the table 19 is equal to 1 and 0 otherwise.

Table 20: Task vs Team

	Team																									
Task		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	52	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	53	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

5.3.6 Scheduled holidays

The following table identifies the days in which TEPHs have scheduled vacations. Effectively, there are months where more people have holidays than others, which corresponds to lower demand months and summer months. Since it was not possible to access INEM's holidays record, for this case study, it was decided with the decision maker to have a 0.5% of vacation days for all the 266 TEPHs, accounting for around 40 days. It was assumed that each TEPH takes 5 days of holidays at a time, so there were 8 TEPHs chosen randomly to have holidays. Then, the starting day of the beginning of the holidays was also randomly chosen.

Table 21: Scheduled holidays

		Day																							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**…**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
TEPH	**…**	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	162	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	163	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	164	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	165	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	166	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

5.3.7 Initial Schedule

The following tables represent the initial schedule that suffers a disruption, which are joined together. However, for the better understanding of what is at stake, they are presented here separately. For both tables, each line corresponds to a TEPH identification and each column to a day. The difference between the tables is what is in the entry (i, d) of each. In the first, table 22, in the entry (i, d), there is the shift that TEPH i is going to execute on day d. If the entry is 0, TEPH i does not work on day d. In the second, table 23, the entry (i, d) corresponds to the task that TEPH i must perform on day d. Therefore, in the end, there will be a tuple (i, d, s, t) with the task t that a TEPH i must carry out on day d and shift s. To illustrate, there is the following example of (162, 1, 3, 30).

Table 22: Initial Schedule – Task

		Day																							
	Shift	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**…**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
TEPH	**…**	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	162	3	3	0	1	1	0	0	0	2	2	3	3	…	2	3	3	0	1	1	0	0	0	2	2	3
	163	0	2	2	3	3	0	1	1	0	0	0	2	…	0	0	2	2	3	3	0	1	1	0	0	0
	164	2	2	3	3	0	1	1	0	0	0	2	2	…	0	2	2	3	3	0	1	1	0	0	0	2
	165	2	2	3	3	0	1	1	0	0	0	2	2	…	0	2	2	3	3	0	1	1	0	0	0	2
	166	1	0	0	0	2	2	3	3	0	1	1	0	…	1	1	0	0	0	2	2	3	3	0	1	1
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

Table 23: Initial Schedule – Task

		Day																								
	Task	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**…**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	162	30	32	0	30	37	0	0	0	30	30	30	30	…	31	30	30	0	30	30	0	0	0	36	30	33
	163	0	34	2	2	2	0	2	30	0	0	0	35	…	0	0	2	2	2	2	0	2	2	0	0	0
TEPH	**164**	4	35	33	30	0	31	30	0	0	0	31	4	…	0	30	31	4	4	0	4	30	0	0	0	30
	165	30	31	32	35	0	30	31	0	0	0	32	37	…	0	35	35	30	30	0	31	30	0	0	0	31
	166	6	0	0	0	37	33	6	6	0	31	31	0	…	30	6	0	0	0	36	6	6	6	0	30	31
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

5.3.8 Disruptions

The following tables present the disruptions generated for the initial schedule, i.e., the days in which TEPHs will be absent. For both tables, each line corresponds to a TEPH identification and each column to a day. If the entry (i, d) of both tables is 1, it means that TEPH i is absent on day d, thus will not execute any task. If it is 0, there is no problem and TEPH i can work on day d.

Since it was not possible to access the records of disruptions and changes on the initial schedule at INEM for a month, disruption generation was based on the works done by Ingels & Maenhout, (2015), Wolbeck *et al*., (2018) and Wickert *et al*., (2019) and interactions with the decision maker, deciding for an absenteeism rate of 1.6%.

The difference between the two tables lies in the way disruptions were generated. For table 24, which refers to type I, disruptions were generated with a Binomial distribution with binary scenarios (success – 1 or unsuccess – 0) with a probability of success of 1.6%. The number of trials was the number of TEPHs multiplied by the number of days in the planning horizon (266 x 31). Disruptions could happen every day for any TEPH.

Table 24: Disruption Type I

		Day																								
	Task	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**…**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	162	0	0	0	0	0	0	0	1	0	0	0	0	…	0	0	0	0	1	0	0	0	0	0	0	0
	163	0	0	0	0	0	0	0	0	1	0	0	0	…	0	0	0	0	0	0	0	0	1	0	0	0
TEPH	**164**	0	0	0	0	0	0	0	0	0	0	0	0	…	0	0	1	0	0	0	0	0	0	0	0	0
	165	0	0	0	0	0	0	0	0	0	0	1	0	…	0	1	0	0	0	0	0	0	0	0	0	0
	166	0	0	0	0	0	0	1	0	0	0	0	0	…	0	0	0	1	0	0	0	0	0	0	0	0
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

Disruption of type II, not uniformly distributed through the planning horizon, was generated since the study made by Vahtera et. al (2001) showed that there are week days where the probability to be absent is higher. According to the work developed, people tend to be absent more in the beginning – Monday – or in the end of the week – Friday. They found that the absenteeism rates on these days is up to 1.9 times higher than on others. Therefore, for each day, it was generated a trial using a Binomial distribution where the probability of being a day with a disruption was 80% for Mondays and Fridays and 40% for the rest of the days. This resulted in 14 days with disruptions, 6 Fridays or Mondays and 8 from the other weekdays, which accounts for around 45% of the days in the planning horizon, as presented in figure 11.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S

Figure 11: 14 disruption days for disruptions of Type II

Then, to equal the probability of 1.6% of the previous type, disruptions were generated again for each worker on the days with disruptions using a Binomial distribution where the probability of each person being absent was 3.6% (3.6% x 45% = 1.62%). Table 25 displays the disruptions of type II.

Table 25: Disruption Type II

	Day																								
	Task	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**...**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	162	0	1	0	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0	0	0
	163	0	1	0	0	1	0	0	0	0	0	0	1	...	0	0	0	1	0	0	0	0	0	0	1	0
TEPH	**164**	0	0	1	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0	0	0
	165	0	0	0	0	1	0	0	0	0	0	0	0	...	0	0	0	0	0	1	0	0	0	1	1	0
	166	0	0	0	0	1	0	0	0	0	0	0	1	...	0	0	0	0	0	0	0	0	0	1	0	0
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

5.3.9 Parameters and Weights

Regarding INEM case study, the parameters used are those displayed in table 26. All of them were obtained via the collaboration with INEM's technicians.

The objective function attempts to minimize the weighted sum of the penalties associated with the soft constraints. Nonetheless, the factors are not all equally important and should be weighted accordingly. The relative and then the quantitative relevance of the criteria was discussed and adjusted with INEM's decision-makers, aiming to achieve accurate weights in order to adequately assess the models' performance. Table 27 summarizes the weights used for the tests, which are kept for each instance tested.

As explained throughout the thesis, the main goal of the model is to produce as little disruption as possible in the following days of the schedule, whilst minimizing uncovered demand. Therefore, changes in the rebuilt schedule from the previous schedule are the most penalized factors. It is important to note that for CODU's instances, changes from previous schedules weight had both a value of 20, after some adjustments made with the decision-maker.

Moreover, understaffed tasks are considered less desirable to happen than overstaffed tasks and these understaffed tasks are much more harmful for the organization if they happen in EVs rather than in CODU.

Having less workers in CODU may represent an increase in the time that patients wait for a vehicle. However, having less workers in EVs may lead to non-operating vehicles and, consequently, the patients may not receive any kind of assistance. Finally, as EVs teams are located in different sites, assigning workers to tasks outside of their teams is less wanted than for CODUs.

Table 26: Parameters for INEM case study

Parameter	Value
$\boldsymbol{\theta_{days}^{max}}$: Maximum number of consecutive working days	6
$\boldsymbol{\theta_{nights}^{max}}$: Maximum number of consecutive working nights	3
$\boldsymbol{\theta_{d-off}^{max}}$: Maximum number of consecutive days off	3
$\boldsymbol{\theta_{i}^{min}}$: Minimum number of working hours for worker i	156
$\boldsymbol{\theta_{s-off}^{min}}$: Minimum number of Sundays off that worker must have during the planning horizon	1
$\boldsymbol{\theta_{s}^{min-s}}$: Minimum number of shifts that must be performed for each shift	3
$\lvert\boldsymbol{D}\rvert$: Number of days during the planning horizon	31
$\lvert\boldsymbol{W}\rvert$: Number of Sundays during the planning horizon	4
η : Number of public holidays on the planning horizon	0
ξ : Number of hours to discount from the contract hours	8

Table 27: Weights for INEM case study

Penalty	Weight	Value
Understaffed tasks for EVs	w^{EV_WF-}	1000
Overstaffed tasks for EVs	w^{EV_WF+}	10
Understaffed tasks for CODU	w^{CODU_WF-}	100
Overstaffed tasks for CODU	w^{CODU_WF+}	10
Shortage of hours worked	w^{OT-}	1
Excess of hours worked	w^{OT+}	50
Incomplete weekend off (Sunday)	w^{Sun}	10
Incomplete weekend off (Saturday)	w^{Sat}	10
Team swaps for Evs	w^{EV_G}	5
Team swaps for CODU	w^{CODU_G}	1
Changes from previous schedule (-)	w^{R-}	7000
Changes from previous schedule (+)	w^{R+}	7000

5.4 Test instances

The model was tested on real data provided by INEM and the tests made can be divided into four groups.

The first group concerns a comparison between rescheduling starting from a cyclic schedule and from a non-cyclic schedule with the same disruptions scenario. The cyclic initial schedule was generated from previous works at INEM and the non-cyclic schedule was generated from the model presented in this thesis. As it was demonstrated in section 4.2.5, some constraints are related to scheduling and others that deal with rescheduling, thus this model is able to perform both schedule and reschedule.

Additionally, these tests were made considering only CODU service, where there are 115 workers that can perform the 10 tasks in 31 days and 3 shifts. Also, the two types of disruptions were considered, generating 4 instances, as can be observed in table 28.

Table 28: First group instances

Instance	TEPHs	Tasks	Days	Variables	Rate
CODU Cyclic Schedule – Type I	115	10	31	106,950	1.6%
CODU Cyclic Schedule – Type II	115	10	31	106,950	1.6%
CODU Non-Cyclic Schedule – Type I	115	10	31	106,950	1.6%
CODU Non-Cyclic Schedule – Type II	115	10	31	106,950	1.6%

It is important to remark that, similarly to the following instances, a month was simulated and the model only run in the days where there was a disruption, day r. In this case, the past schedule was blocked from day 1 until day (r – 3) and, from day (r – 2) until day 31, variables were free to change from the previous schedule. It means that TEPHs must notice the manager until two days before their absence, which seems reasonable. Therefore, during the month the number of variables for this instance is expressed by the following equation:

$$\#\, variables = 115 \times 10 \times 3 \times (31 - (\mathrm{r} - 2)) \tag{5.2}$$

Equation (5.2) applies to the other instances only changing the number of TEPHs and the number of tasks.

The second group concerns tests for an initial cyclic schedule for the universe of EVs, composed of 194 workers that can perform the 54 tasks in 31 days and 3 shifts and for the whole INEM, CODU and EVs services, of 266 workers that can perform 64 tasks in the same planning horizon. This group also includes the two types of disruptions generated with a 1.6% absenteeism rate. Table 29 provides a summary of the second group instances.

Table 29: Second group instances

Instance	TEPHs	Tasks	Days	Variables	Rate
EVs Cyclic Schedule – Type I	194	54	31	974,268	1.6%
EVs Cyclic Schedule – Type II	194	54	31	974,268	1.6%
INEM Cyclic Schedule – Type I	266	64	31	1,538,232	1.6%
INEM Cyclic Schedule – Type II	266	64	31	1,538,232	1.6%

The third group involves tests for an initial schedule for the whole INEM, CODU and EVs services, with three variants of the first type of generated disruptions. The first variant starts from the disruptions used in the second group (with an absenteeism rate of 1.6%) but the next day of the schedule for each TEPH is also a disruption day. This results in two consecutive absent days, which accounts for an absenteeism rate of approximately 3.2%. The second variant is very similar but the two next days of a disruption also have disruptions. Hence, three consecutive absent days are generated, which accounts for an absenteeism rate of approximately 4.8%. The third variant is equal to type I, disruptions are also generated randomly using Binomial distribution but different values of probability will be used, 5% and 10%, because these are the absenteeism rate reported for emergency services and health care, due to their stress working conditions and high rates of overtime (Wickert *et al*., 2019). Third group cases have been intensely studied over the years in the rescheduling literature, hence it is critical to assure that the model is capable of finding a solution for the most severe cases (Kitada & Morizawa, 2013; Maenhout & Vanhoucke, 2013, 2018b; Wickert *et al*., 2019; Wolbeck et al., 2020, 2018). Table 30 presents a summary of third group instances.

Table 30: Third group instances

Instance	TEPHs	Tasks	Days	Variables	Rate
INEM Cyclic Schedule – Type I Variant I	266	64	31	1,538,232	3.2%
INEM Cyclic Schedule – Type I Variant II	266	64	31	1,538,232	4.8%
INEM Cyclic Schedule – Type I	266	64	31	1,538,232	5%
INEM Cyclic Schedule – Type I	266	64	31	1,538,232	10%

The fourth group includes tests for an initial schedule for the whole INEM, CODU and EVs services, with an absenteeism rate of 5%, the same disruptions used for the third group, starting from a cyclic and a non-cyclic schedule. In this case, it is intended to study the effect on the other variables of the objective function after varying the weight of changing shifts from the previous schedule w^{R-} and w^{R+}. These will have a value of 1000, equal to the weight of understaffing, and 500, half of the weight of understaffing so that in the objective function two changes (remove a shift from an absent person and add one to another person) have the same value, in theory, of an understaffed task. It is also intended to go deeper in the study of the differences between starting with a cyclic and a non-cyclic schedule.

Table 31: Fourth group instances

Instance	TEPHs	Tasks	Days	Variables	Rate	w^{R-}/w^{R+}
INEM Cyclic Schedule – Type I	266	64	31	1,538,232	5%	1000
INEM Cyclic Schedule – Type I	266	64	31	1,538,232	5%	500
INEM Non-Cyclic Schedule – Type I	266	64	31	1,538,232	5%	1000
INEM Non-Cyclic Schedule – Type I	266	64	31	1,538,232	5%	500

5.5 Chapter Considerations

This chapter presents mainly the data needed to be inputted in the model in order to validate its accuracy in the case of INEM. First, it was explained how data was collected via three procedures, using mainly data from previous works at INEM and from the literature. Then, the problem context was explained to deepen even more the knowledge on INEM activities, its workforce and how it is divided into teams that belong to one of the two services. Following, the main structure for all instances was exposed, ending with the different tests that were done in order to assess the consistency of the model for different scenarios.

6. Results and Discussion

This chapter presents the computational results of the model application to the different test instances introduced in chapter 5. Section 6.1 presents the results of group 1 regarding the comparison between starting from a cyclic and a non-cyclic initial schedule. Section 6.2 demonstrates the results for the second group, which includes both EVs and INEM with two types of disruptions and an absenteeism rate of 1.6%. Section 6.3 presents the results that assure that the model is capable of performing for the most severe cases of the emergency services sector with absenteeism rates of 5% and 10% and considering 2 and 3 consecutive absent days. Section 6.4 presents the results considering the effects of varying the weight of changing shifts from previous to the new schedule starting from both cyclic and non-cyclic schedules. Section 6.5 analyses the results obtained for the different scenarios with a 5% absenteeism rate and some recommendations are made along the way. Finally, section 6.6 draws the main considerations of the results provided by the model for the different tests.

The model is implemented in Python with the combination of the library docplex - IBM Decision Optimization CPLEX and solving through IBM ILOG CPLEX Optimizing Studio. All tests were executed on a PC with an Intel Core i7- 1165G7 processor of 2.4 GHz and 16 GB of RAM running under the Windows 10 operating system.

The results below were obtained after simulating a month with disruptions. On each day there was a disruption, the simulation would stop and run the model in order to solve the disruptions. Therefore, there is a reconstructed schedule for each day that there was a disruption. Then, the column *Time (s)* is the time that the model took to run in each iteration and *Changes (+)* is the number of sets (i, d, s) that did not exist in a previous schedule but exist in the following one, i.e., the number of additional shifts that each person must perform compared to a previous schedule. *Changes (–)* is the number of sets (i, d, s) that existed in a previous schedule but do not exist in the following, i.e., the number of shifts that each has not to perform anymore from a previous schedule. Finally, *OFV* is the Objective Function Value that allows to compare the quality between each solution. Most commonly the averages of these values are going to be considered in the following analysis. Furthermore, CPLEX was capable of finding an optimal solution for all models.

6.1 Results – Group 1

This section presents the results that provide key insights on the differences between starting the month with a cyclic and a non-cyclic initial schedule.

The two following tables, 32 and 33 display the results for the first type of generated disruptions with 1.6% of absenteeism rate, where the first of them concerns the cyclic and the second the non-cyclic initial schedule. For this group of instances, it is important to have in mind that it is expected to have some overstaffing, because the number of TEPHs that can perform CODU's tasks is 115 and, if we consider the assumption that in section 5.2.3 where each TEPH performs 19 shifts per month, there is a total of 2,185 carried out shifts, which is higher than the demand of 1,953 shifts. In fact, both initial schedules have overstaffing. Compared to the demand, in the cyclic there are more 178 shifts done than what is required and in the non-cyclic 150.

Table 32: Results for CODU Cyclic Schedule – Type I Disruption – 1.6%

CODU Cyclic Schedule – Type I				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	19.14	0	1	3,515
Schedule 2	20.33	0	1	3,513
Schedule 3	15.16	1	2	3,551
Schedule 4	20.00	1	2	3,539
Schedule 5	14.75	1	2	3,537
Schedule 6	12.8	0	1	3,495
Schedule 7	12.53	0	1	3,493
Schedule 8	10.63	2	4	3,699
Schedule 9	9.28	2	3	3,587
Schedule 10	8.48	1	1	3,537
Schedule 11	5.38	1	2	3,545
Schedule 12	4.52	1	1	3,525
Schedule 13	2.67	0	1	3,503
Schedule 14	2.39	1	2	3,541
Schedule 15	2.58	0	2	3,517
Schedule 16	2.19	0	1	3,505
Schedule 17	1.72	0	1	3,485
Schedule 18	1.31	0	1	3,493
Schedule 19	1.21	0	2	3,501
Schedule 20	1.03	0	1	3,499
Schedule 21	0.64	0	1	3,479
Schedule 22	0.66	0	1	3,497
Schedule 23	0.63	1	1	3,477
Schedule 24	0.42	0	1	3,495
Schedule 25	0.34	0	1	3,475
Schedule 26	0.33	0	2	3,511
Mean	6.58	0.46	1.50	3,519.77
SD	6.72	0.63	0.75	44.30

After running the two monthly simulations, some findings may be worth considering. Firstly, it takes more time, on average, to solve the same disruptions for the cyclic schedule than for the non-cyclic. However, since for this case the running time is a matter of seconds, it is not concerning. Nevertheless, it may become a concern on larger services, such as to the whole INEM service. Then, to solve the same disruption scenario, it is needed to make less changes for cyclic schedule than for non-cyclic. These changes include both to add people to tasks that they were not allocated in the previous schedule – Changes (+) – and to remove people from tasks that they were supposed to – Changes (-) – due to disruptions or to increase solution's quality. The number of Changes (+) is effectively lower when starting with the cyclic due to the fact that there is higher overtime from the initial schedule, so it is not always required to add more TEPHs since the demand is being met. In a more restricted instance, where there can be more understaffing shifts, this phenomenon may not happen.

Finally, it is key to remark that the Objective Function Value – OFV – when starting from the cyclic schedule is higher than when starting from the non-cyclic schedule. It means that the quality of the solutions found is lower by an average difference of approximately 17.5%. Although the first fact that comes to mind is the difference in the overstaffing of the initial schedules, this aspect is dissipated throughout the month. To illustrate, the last created schedule that started from the cyclic has more 151 shifts done that what is required and in the non-cyclic 149 but the difference in the OFV remains of 532. Therefore, there must be another reason for this difference. Since the sum of the shortage and excess of hours worked is similar, this factor is no reason either. For example, for the last schedule, it is 959 when for the cyclic and 1,019 for the non-cyclic.

Table 33: Results for CODU Non-Cyclic Schedule – Type I Disruption – 1.6%

CODU Non-Cyclic Schedule – Type I				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	10.28	1	1	2,721
Schedule 2	10.39	1	2	2,727
Schedule 3	9.91	2	2	2,821
Schedule 4	8.08	3	5	2,937
Schedule 5	7.88	3	2	2,879
Schedule 6	4.78	1	1	2,819
Schedule 7	6.17	1	1	2,819
Schedule 8	5.67	5	5	3,099
Schedule 9	4.41	3	3	3,069
Schedule 10	2.33	3	1	2,913
Schedule 11	1.89	2	2	2,903
Schedule 12	1.72	2	1	2,885
Schedule 13	1.63	1	1	2,865
Schedule 14	1.95	1	1	2,865
Schedule 15	1.44	3	4	2,971
Schedule 16	1.45	1	1	2,911
Schedule 17	1.34	1	2	2,871
Schedule 18	0.72	1	1	2,911
Schedule 19	0.78	1	1	2,931
Schedule 20	0.67	1	2	2,959
Schedule 21	0.64	1	2	2,899
Schedule 22	0.55	1	1	2,939
Schedule 23	0.52	3	2	2,899
Schedule 24	0.61	1	1	2,939
Schedule 25	1.11	2	1	2,899
Schedule 26	0.78	2	2	2,979
Mean	3.37	1.81	1.85	2,901.15
SD	3.31	1.04	1.17	81.80

Additionally, the number of tasks performed out of each's team is small, in average 4 for all schedules. Hence, the difference between the OFV lays in the number of incomplete weekends off. To illustrate, for the last schedule, there are 185 incomplete weekends off for the cyclic schedule and 116 for the non-cyclic. Regarding that the weight associated to this penalty is 10, the difference is equal to 690 in the OFV.

In the two following tables, 34 and 35 simulations of a month with disruptions of type II are displayed. The insights taken from these are similar to those of the previous simulations. The time required to rebuild a schedule is now lower for cyclic but there are less Changes (+) again for the cyclic schedule and relatively the same for Changes (-) for both schedules. In this case, the OFV is again lower for the non-cyclic and the difference of incomplete weekends off for the last schedule is 65 (196 for the cyclic and 131 for the non-cyclic), which accounts for a difference in the OFV of 650. Since the difference between the averages is 710.14, it is reasonable to assume that the main aspect that affects the differences in OFVs is the incomplete weekends off.

Table 34: Results for CODU Cyclic Schedule – Disruptions Type II – 1.6%

CODU Cyclic Schedule – Type II				
Schedule	**Time**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	17.28	3	5	3,763
Schedule 2	18.31	1	4	3,857
Schedule 3	6.25	2	4	3,893
Schedule 4	5.02	0	4	3,855
Schedule 5	6.28	1	1	3,825
Schedule 6	3.75	0	1	3,803
Schedule 7	4.39	2	2	3,863
Schedule 8	3.02	0	1	3,801
Schedule 9	3.05	0	1	3,809
Schedule 10	2.66	3	5	4,205
Schedule 11	0.92	1	1	4,085
Schedule 12	0.86	2	2	4,125
Schedule 13	0.77	1	3	4,032
Schedule 14	0.56	1	1	4,014
Mean	5.22	1.21	2.50	3,923.57
SD	5.46	1.01	1.55	135.67

In fact, these two scenarios of table 34 and 35 were simulated again but, in this case, the weight associated with incomplete weekends off was fixed at 0 and the results confirmed the hypothesis presented before, since the average of OFVs for the scenario starting with the cyclic schedule was 1,988.14 and for the scenario starting with the non-cyclic was 1,928.43, which represents a difference of 3%. Although rescheduling benefits from starting from the non-cyclic schedule, the difference compared to the cyclic schedule is significantly lower than when considering the scenarios that penalized incomplete weekends off (table 34 and 35). Therefore, it is possible to state that starting with a cyclic schedule jeopardizes schedules' quality, as the OFV is much higher than when starting with a non-cyclic schedule mainly due to the penalization of incomplete weekends off – IWO.

Table 35: Results for CODU Non-Cyclic Schedule – Disruptions Type II – 1.6%

CODU Non-Cyclic Schedule – Type II				
Schedule	**Time**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	24.25	6	6	3,051
Schedule 2	19.61	4	4	3,171
Schedule 3	8.86	5	4	3,213
Schedule 4	12.30	2	4	3,169
Schedule 5	7.89	2	2	3,169
Schedule 6	7.77	1	1	3,129
Schedule 7	4.98	4	2	3,213
Schedule 8	5.02	1	1	3,143
Schedule 9	3.89	1	1	3,163
Schedule 10	2.25	3	2	3,345
Schedule 11	1.47	1	2	3,245
Schedule 12	0.94	1	1	3,197
Schedule 13	0.59	4	2	3,379
Schedule 14	0.56	2	1	3,401
Mean	7.17	2.64	2.36	3,213.43
SD	6.97	1.63	1.49	95.59

Regarding CODU, the 5 different teams start the month on different days of the 10-days cycle. To illustrate, for team 1, the beginning of the cycle matches the first day of the month, so their first ten days shifts are: M-M-A-A-O-N-N-O-O-O. However, for team 2, the first day of the month matches the third day of the cycle, so their first ten days shifts are: A-A-O-N-O-O-O-M-M. This implies that, every weekend, there are always two teams with incomplete weekend off. For the non-cyclic, this aspect is more flexible, being able to have fewer incomplete weekends off. Although it may be more practical for TEPHs, as it is easier to know their schedules, it impacts the quality of the recreated schedules, which may harm the service quality.

It is important to note that these tests, since they are for a CODU instance, function as a first idea to the cyclic vs non-cyclic schedule problem but it is crucial to study this problem for the whole INEM service. Finally, table 36 presents a summary of the averages of the results discussed in this section.

Table 36: Summary of the solutions found for the first group tests

Instance	**Rate**	**Changes (+)**	**Changes (-)**	**OFV**	**IWO**
CODU Cyclic Schedule – Type I	1.6%	0.46	1.50	3,519.77	183.92
CODU Non-Cyclic Schedule – Type I	1.6%	1.81	1.85	2,901.15	126.42
CODU Cyclic Schedule – Type II	1.6%	1.21	2.50	3,923.57	191.25
CODU Non-Cyclic Schedule – Type II	1.6%	2.64	2.36	3,213.43	126.92
CODU Cyclic Schedule – Type II	1.6%	1.21	2.50	1,988.14	0
CODU Non-Cyclic Schedule – Type II	1.6%	2.64	2.36	1,928.43	0

6.2 Results – Group 2

Before proceeding with additional studies, it is critical to validate the suggested model to confirm its accuracy in describing the real system, as well as to ensure that any simplifying assumptions and accompanying mathematical formulations created throughout the modelling stages conform to reality (Williams, 2013). In order to validate the proposed model, four instances were created, two for the EVs service (tables 37 and 38) and two for the whole INEM service (tables 39 and 40), being both services tested for two types of generated disruptions.

Table 37: Results for EVs Cyclic Schedule – Disruptions Type I – 1.6%

EVs Cyclic Schedule – Type I				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	420.38	0	27	193,900
Schedule 2	372.2	0	8	59,908
Schedule 3	358.86	2	2	29,906
Schedule 4	325.69	1	2	22,916
Schedule 5	240.09	1	12	98,024
Schedule 6	224.41	2	12	105,922
Schedule 7	225.38	3	3	46,946
Schedule 8	206.05	0	12	96,962
Schedule 9	188.09	7	1	60,932
Schedule 10	180.63	0	8	62,950
Schedule 11	170.52	1	5	48,958
Schedule 12	164.97	2	2	32,986
Schedule 13	157.80	2	2	32,966
Schedule 14	152.79	0	7	52,966
Schedule 15	149.53	1	2	25,982
Schedule 16	141.98	0	7	52,004
Schedule 17	181.81	3	1	33,016
Schedule 18	123.08	0	2	19,032
Schedule 19	122.91	2	3	40,050
Schedule 20	112.61	0	1	12,038
Schedule 21	129.03	0	5	39,084
Schedule 22	181.81	1	1	19,108
Schedule 23	162.89	0	4	33,140
Schedule 24	129.73	1	1	10,140
Schedule 25	109.13	0	2	19,156
Schedule 26	93.36	4	2	40,140
Schedule 27	59.52	2	1	22,172
Schedule 28	48.17	0	1	7,156
Schedule 29	41.81	2	2	32,980
Mean	178.45	1.28	4.76	46,601.38
SD	91.00	1.55	5.44	37,322.48

Table 38: Results for EVs Cyclic Schedule – Disruptions Type II – 1.6%

EVs Cyclic Schedule – Type II				
Schedule	**Time**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	451.27	2	28	214,892
Schedule 2	219.84	2	4	46,928
Schedule 3	201.83	2	5	53,942
Schedule 4	177.98	1	4	39,966
Schedule 5	170.63	1	3	33,012
Schedule 6	151.75	3	9	89,044
Schedule 7	151.05	1	12	96,042
Schedule 8	136.33	3	4	54,058
Schedule 9	106.94	2	6	62,197
Schedule 10	104.55	3	7	75,182
Schedule 11	90.56	5	15	157,220
Schedule 12	116.95	2	3	61,219
Schedule 13	80.44	1	1	33,268
Schedule 14	74.58	1	1	15,003
Mean	159.62	2.07	7.29	73,712.36
SD	91.61	1.10	6.91	51,618.22

Regarding the EVs service, the simulation of the month starts from a cyclic schedule. For the first case, disruptions were generated using a Binomial distribution with a probability of 1.6%, creating disruptions in 29 days. For the second case, disruptions were generated using type II, as it was explained in section 5.3.8. For both cases, it was possible to find reasonable solutions in a short time, being the average of time spent 178.45 seconds and 159.62 seconds, respectively. Therefore, one of the objectives of the model is guaranteed, to find solutions quickly. This makes it possible for managers to run the model on a daily basis to solve disruption problems or even to adjust the current schedule over the month to better meet workers' and organizations' goals, present in the objective function. It is interesting to note that, throughout the month, the time required to find optimal solutions reduces due to the decrease in the number of decision variables, since the past schedule is blocked.

Looking at the changes made in relation to previous schedules, both models present acceptable results, changing in average only 0.16% and 0.26% of the previous schedule. Hence, it is reasonable to declare that the model provides accurate results for the EVs service.

Concerning the whole INEM service, the simulation also started from a cyclic schedule and the disruptions of the first type generated absences in all of the 31 days. For both instances, the model found again optimal solutions in fairly short times, with an average of 212.08 seconds for the first and 214.71 seconds for the second, being the maximum less than 11 minutes, which is remarkably a reasonable time to spend in one day to create a new schedule.

Table 39: Results for INEM Cyclic Schedule – Disruptions Type I – 1.6%

INEM Cyclic Schedule – Type I				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	349.28	0	7	13,430
Schedule 2	621.67	1	10	48,462
Schedule 3	478.86	1	5	27,498
Schedule 4	442.2	2	13	34,536
Schedule 5	361.73	7	14	90,590
Schedule 6	338.97	5	14	83,630
Schedule 7	326.5	3	10	27,620
Schedule 8	295.01	6	11	62,646
Schedule 9	394.09	8	9	97,660
Schedule 10	237.69	3	10	48,706
Schedule 11	220.38	3	7	48,732
Schedule 12	201.44	3	11	55,738
Schedule 13	190.72	8	2	76,736
Schedule 14	178.59	0	1	8,032
Schedule 15	180.84	0	4	13,744
Schedule 16	162.98	2	9	48,768
Schedule 17	152.05	3	2	41,807
Schedule 18	162.36	0	2	20,850
Schedule 19	146.69	1	2	27,831
Schedule 20	129.29	0	1	13,849
Schedule 21	116.31	0	3	27,883
Schedule 22	109.81	0	2	20,926
Schedule 23	97.57	0	4	34,931
Schedule 24	93.18	0	4	34,963
Schedule 25	145.63	2	3	41,971
Schedule 26	112.8	1	5	49,003
Schedule 27	103.45	3	4	56,011
Schedule 28	70.98	0	3	28,035
Schedule 29	57.31	1	5	49,094
Schedule 30	48.86	3	6	70,118
Schedule 31	47.23	0	3	28,115
Mean	212.08	2.13	6.00	42,965.00
SD	138.65	2.39	3.93	22,762.11

Looking at the changes from past schedules, both models present acceptable results, changing on average only 0.17% and 0.21% of the previous schedule. Thus, it is plausible to declare that the model provides accurate results for the whole INEM service under the circumstances discussed and adjusted with the decision-maker.

Table 40: Results for INEM Cyclic Schedule – Disruptions Type II – 1.6%

INEM Cyclic Schedule – Type II				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	562.13	3	14	69,413
Schedule 2	498.59	1	8	34,437
Schedule 3	379.3	2	17	62,469
Schedule 4	195.3	5	14	69,480
Schedule 5	167.69	6	16	104,518
Schedule 6	149.13	4	7	83,606
Schedule 7	136.28	0	9	41,646
Schedule 8	174.19	3	4	55,642
Schedule 9	141.05	1	4	41,686
Schedule 10	117.3	1	2	69,712
Schedule 11	166.56	4	5	41,706
Schedule 12	128.59	1	4	27,738
Schedule 13	105.92	0	1	13,714
Schedule 14	83.97	1	2	20,760
Mean	214.71	2.29	7.64	52,609.07
SD	145.56	1.83	5.31	24,575.88

6.3 Results – Group 3

This section aims to demonstrate the model accuracy to deal with the strictest scenarios that can happen in the emergency services sector regarding disruptions. These include managing situations with sudden absences for consecutive days and also situations with one of the highest absenteeism rates among the different sectors, ranging from 5% to 10%. These have been deeply examined over the years in the rescheduling literature (Kitada & Morizawa, 2013; Maenhout & Vanhoucke, 2013, 2018b; Wickert *et al*., 2019; Wolbeck *et al*., 2020, 2018).

Table 41 shows the results for a scenario with two consecutive absent days and an absenteeism rate of 3,2%. It shows that the model is capable of finding optimal solutions in a short amount of time, with an average of 234.58 seconds per disruption day. Regarding the changes it is interesting to remark that, throughout the month, it is uncommon to have Changes (+) higher than Changes (-), i.e., there are more shifts being eliminated than those being added. This means that, sometimes, it is better for the organization to have understaffed tasks than to cause perturbation for their workers. However, it can be adjusted in the weight associated with penalizing the changes in the rebuilt schedule, as will be explained afterwards.

Table 41: Results for INEM Cyclic Schedule – Disruptions Type I Variant I – 3.2%

INEM Cyclic Schedule – Type I Variant I 3.2%				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	580.75	0	1	13,430
Schedule 2	634.53	1	5	55,437
Schedule 3	475.16	3	6	69,570
Schedule 4	444.27	3	5	62,539
Schedule 5	416.06	3	7	76,712
Schedule 6	374.14	5	9	111,758
Schedule 7	315.91	6	6	97,731
Schedule 8	301.98	5	4	69,779
Schedule 9	424.53	6	6	97,828
Schedule 10	425.73	4	7	90,890
Schedule 11	221.86	4	6	76,904
Schedule 12	211.44	3	8	83,978
Schedule 13	210.14	6	9	111,994
Schedule 14	179.78	2	2	34,984
Schedule 15	160.31	2	3	35,024
Schedule 16	172.88	1	6	70,058
Schedule 17	160.53	3	7	98,042
Schedule 18	138.14	5	4	35,054
Schedule 19	153.34	0	4	49,104
Schedule 20	126.84	2	2	21,120
Schedule 21	118.64	0	5	42,150
Schedule 22	114.5	2	6	63,192
Schedule 23	160.95	2	6	63,224
Schedule 24	149.56	3	9	91,272
Schedule 25	149.27	4	7	84,296
Schedule 26	111.08	1	6	56,336
Schedule 27	97.27	4	9	105,368
Schedule 28	77.73	1	6	56,408
Schedule 29	64.33	4	8	91,440
Schedule 30	48.2	4	11	112,496
Schedule 31	52.06	0	8	63,560
Mean	234.58	2.87	6.06	70,699.29
SD	157.40	1.81	2.26	26,618.69

Table 42 exhibits the results for a scenario with three consecutive absent days and an absenteeism rate of 4.8%. This represents situations of TEPHs' illness, which makes it important to study, especially within the COVID-19 pandemic context. In this case, the ratio Changes (+) per Changes (-) is higher due to the fact that the maximum of absent days that TEPHs can have according to the model is three. Thus, in the day right after coming from the three days absent period it was needed to add a shift if there was none in order not to violate that constraint.

Table 42: Results for INEM Cyclic Schedule – Disruptions Type I Variant I – 4.8%

INEM Cyclic Schedule – Type I Variant I 4.8%				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	520.83	1	2	27,397
Schedule 2	579.08	3	6	69,460
Schedule 3	487.17	6	8	104,558
Schedule 4	421.31	6	10	118,558
Schedule 5	370.44	8	8	118,638
Schedule 6	320.11	7	11	139,692
Schedule 7	310.05	8	7	118,704
Schedule 8	338.78	6	8	104,726
Schedule 9	265.64	6	9	111,792
Schedule 10	280.48	7	9	118,902
Schedule 11	389.72	4	10	104,938
Schedule 12	283.78	4	10	105,016
Schedule 13	340.41	8	13	154,058
Schedule 14	310.59	8	8	119,038
Schedule 15	225.17	1	4	42,062
Schedule 16	310.19	5	8	98,104
Schedule 17	195.63	7	9	119,120
Schedule 18	191.22	3	8	84,110
Schedule 19	210.45	7	8	112,172
Schedule 20	157.78	1	4	42,196
Schedule 21	216.31	0	5	42,206
Schedule 22	138.59	5	8	98,248
Schedule 23	130.27	5	10	112,288
Schedule 24	207.42	8	12	147,344
Schedule 25	162.01	6	10	119,376
Schedule 26	167.63	4	11	112,432
Schedule 27	99.94	8	12	147,464
Schedule 28	88.78	2	8	77,512
Schedule 29	111.89	5	10	112,552
Schedule 30	58.67	6	15	154,624
Schedule 31	96.22	1	13	105,720
Mean	257.63	5.03	8.84	104,613.13
SD	129.86	2.44	2.76	31,978.38

Table 43 displays the results for an instance with an absenteeism rate of 5%. It shows that the model can find optimal solutions in a short amount of time, with an average of 192.91 seconds per disruption day. It is interesting to observe that the average of the OFV for this case is 15% lower when compared to the previous instance despite having a higher absenteeism rate. This occurrence is related to the higher likelihood, in the previous scenario, of adding a shift, as previously explained. Additionally, the previous scenarios were 17% and

25% more time-consuming than this one, which may reflect a higher severity and, consequently, higher difficulty on dealing with absences on consecutive days.

Table 43: Results for INEM Cyclic Schedule – Disruptions Type I – 5%

INEM Cyclic Schedule – Type I 5%				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	496.25	1	10	83,494
Schedule 2	581.63	1	7	69,550
Schedule 3	457.86	4	12	118,690
Schedule 4	408.56	4	9	97,776
Schedule 5	364.58	4	10	111,880
Schedule 6	359.63	10	8	139,892
Schedule 7	315.97	8	6	111,876
Schedule 8	185.89	9	10	139,940
Schedule 9	160.44	4	6	76,958
Schedule 10	154.14	2	9	84,048
Schedule 11	141.79	8	7	112,102
Schedule 12	136.31	2	9	84,168
Schedule 13	126.27	5	13	133,224
Schedule 14	162.95	2	3	42,240
Schedule 15	151.23	5	12	126,296
Schedule 16	148.23	6	11	126,336
Schedule 17	144.36	4	6	77,400
Schedule 18	272.73	4	5	70,432
Schedule 19	127.47	2	8	77,520
Schedule 20	117.98	1	7	63,558
Schedule 21	105.39	2	5	56,602
Schedule 22	99.92	4	12	119,666
Schedule 23	144.23	0	4	35,762
Schedule 24	130.36	2	5	56,730
Schedule 25	119.33	4	9	98,706
Schedule 26	103.56	3	11	105,826
Schedule 27	66.72	4	6	77,842
Schedule 28	71.23	3	11	105,906
Schedule 29	52.09	2	5	56,930
Schedule 30	36.97	0	7	56,986
Schedule 31	36.11	1	3	36,002
Mean	192.91	3.58	7.94	88,849.61
SD	139.56	2.49	2.77	30,067.57

Table 44 shows the results obtained for a 10% absenteeism rate. Once more, the time required to recreate each schedule is perfectly reasonable to be performed on a daily basis if needed. Effectively, in this case, the average OFV is two times higher than for the 5% absenteeism rate. Finally, the number of changes and the OFV decreasing over the month shows that there are less absent workers at the end of the month compared to the beginning of the month, which in this case is a random event.

Table 44: Results for INEM Cyclic Schedule – Disruptions Type I – 10%

INEM Cyclic Schedule – Type I 10%				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	500.28	4	25	209,582
Schedule 2	309.53	5	27	237,760
Schedule 3	391.03	4	22	196,022
Schedule 4	309.71	10	19	210,268
Schedule 5	203.42	15	36	357,630
Schedule 6	200.41	15	22	266,744
Schedule 7	202.97	9	17	196,784
Schedule 8	179.19	15	22	266,890
Schedule 9	163.69	12	18	217,950
Schedule 10	160.39	16	24	288,146
Schedule 11	150.83	14	29	309,390
Schedule 12	181.09	8	14	162,502
Schedule 13	122.59	7	15	162,566
Schedule 14	113.13	3	11	106,618
Schedule 15	104.94	5	11	120,666
Schedule 16	117.05	7	17	176,766
Schedule 17	120.19	6	10	120,738
Schedule 18	106.86	5	17	162,884
Schedule 19	98.11	2	17	142,024
Schedule 20	88.75	5	21	191,202
Schedule 21	84.84	5	20	184,372
Schedule 22	88.56	2	11	100,458
Schedule 23	76.47	4	11	114,514
Schedule 24	108.13	5	9	107,546
Schedule 25	97.91	3	8	79,578
Schedule 26	85.77	2	8	79,626
Schedule 27	71.61	2	5	58,650
Schedule 28	50.84	4	12	121,714
Schedule 29	47.81	0	7	65,778
Schedule 30	38.69	3	15	135,874
Schedule 31	42.38	2	10	93,938
Mean	148.94	6.42	16.45	169,199.35
SD	103.00	4.52	7.11	74,057.44

6.4 Results – Group 4

This section aims to determine the effects of varying the weight of making changes from previous to new schedules for both cyclic and non-cyclic starting schedules. It presents the results obtained for the main aspects that have been studied in the past sections, time to create the new optimal schedule, the changes that were made from the past and the objective function value. In section 6.5 there will be a more in-depth analysis.

Table 45 displays results for an initial cyclic schedule for INEM with a 5% absenteeism rate and a penalization of changes of 1000, which is very similar (except for OFV) to those in table 43, where the penalization was 7000. At a first glance, it seems that, for this case, when this penalization is bigger or equal to the penalization associated with understaffing, the model will return the same schedule for the same scenario.

Table 45: Results for INEM Cyclic Schedule – Disruptions Type I – 5% – Changes Weight = 1000

INEM Cyclic Schedule – Type I 5% - 1000				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	548.73	1	10	17,494
Schedule 2	596.97	1	7	15,550
Schedule 3	473.63	4	12	22,690
Schedule 4	420.31	4	9	19,776
Schedule 5	369.69	4	10	21,880
Schedule 6	352.53	10	8	25,892
Schedule 7	326.27	8	6	21,876
Schedule 8	187.59	9	10	25,940
Schedule 9	166.41	4	6	16,958
Schedule 10	164.73	6	9	22,066
Schedule 11	146.14	4	7	18,120
Schedule 12	140.31	2	9	19,204
Schedule 13	128.88	8	13	28,278
Schedule 14	165.02	1	3	11,284
Schedule 15	161.84	5	12	24,340
Schedule 16	206.47	7	11	25,380
Schedule 17	147.13	3	6	16,434
Schedule 18	154.28	2	5	14,468
Schedule 19	131.09	2	8	17,556
Schedule 20	121.89	1	7	15,594
Schedule 21	107.67	2	5	14,638
Schedule 22	103.61	4	12	23,702
Schedule 23	146.33	4	9	20,742
Schedule 24	142.14	2	5	14,766
Schedule 25	128.48	0	4	11,798
Schedule 26	110.83	3	11	21,862
Schedule 27	98.73	4	6	17,878
Schedule 28	61.83	3	11	21,942
Schedule 29	52.53	2	5	14,966
Schedule 30	44.59	0	7	15,022
Schedule 31	36.89	1	3	12,038
Mean	198.18	3.58	7.94	19,036.58
SD	143.91	2.57	2.77	4,483.84

Table 46 presents the results for the same scenario but where the weight associated with changes in the schedule is 500. This was made because, as it was observed in section 6.3 for table 41, the solutions found, normally, deal with the absences by accepting understaffing rather than replacing the missing people with other TEPHs that can perform the task. To no surprise, the number of Changes (+) is on average higher than in the past table and OFV is lower. Once more, the model is able to generate a new schedule in a short time.

Table 46: Results for INEM Cyclic Schedule – Disruptions Type I – 5% – Changes Weight = 500

INEM Cyclic Schedule – Type I Variant I 5% - 500				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	499.75	3	10	11,994
Schedule 2	582.33	3	7	11,050
Schedule 3	523.36	4	12	14,690
Schedule 4	430.63	3	9	13,276
Schedule 5	483.31	7	10	15,380
Schedule 6	362.06	11	8	15,892
Schedule 7	322.95	9	6	14,376
Schedule 8	214.58	7	10	15,420
Schedule 9	176.22	2	6	10,976
Schedule 10	171.34	7	9	15,062
Schedule 11	146.91	6	7	13,606
Schedule 12	150.13	3	8	11,672
Schedule 13	150.05	8	13	17,728
Schedule 14	166.47	1	3	9,234
Schedule 15	163.75	5	12	15,790
Schedule 16	159.05	6	11	15,830
Schedule 17	162.69	3	6	11,906
Schedule 18	216.92	2	5	10,940
Schedule 19	133.91	3	8	12,528
Schedule 20	121.64	3	7	11,566
Schedule 21	107.31	4	6	12,110
Schedule 22	103.41	5	12	15,674
Schedule 23	155.66	2	10	15,214
Schedule 24	135.03	2	5	11,238
Schedule 25	127.95	2	4	9,770
Schedule 26	109.23	3	10	13,834
Schedule 27	98.25	3	5	11,850
Schedule 28	62.75	3	11	14,914
Schedule 29	53.83	2	5	11,438
Schedule 30	39.02	2	7	11,494
Schedule 31	35.55	1	3	10,010
Mean	205.36	4.03	7.90	13,111.68
SD	148.25	2.43	2.75	2,165.55

In table 47, it is possible to observe the results obtained from running a simulation starting with a non-cyclic initial schedule conceived by part of the model, the same procedure explained in section 6.1 for CODU instances. In this case, the model runs for 7 hours and achieved a relative gap of 4.05%. Being a case where the demand is higher than the general supply, as it was observed in section 5.2.3, this schedule has less understaffed tasks and TEPHs has less undertime than the cyclic, which is caused by the higher flexibility to respond to the needs.

Table 47: Results for INEM Non-Cyclic Schedule – Disruptions Type I – 5% – Changes Weight = 1000

INEM Non-Cyclic Schedule – Type I 5% - 1000				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	339.36	0	24	28,034
Schedule 2	263.97	1	9	18,001
Schedule 3	267.48	2	10	20,045
Schedule 4	264.21	1	7	16,081
Schedule 5	248.84	1	9	17,153
Schedule 6	258.75	3	11	22,215
Schedule 7	242.95	2	7	18,281
Schedule 8	216.39	9	12	29,060
Schedule 9	266.63	5	9	24,068
Schedule 10	258.64	4	11	23,162
Schedule 11	250.97	4	4	16,206
Schedule 12	155.69	1	3	12,232
Schedule 13	138.79	2	10	20,296
Schedule 14	145.75	1	9	17,368
Schedule 15	122.83	4	13	25,164
Schedule 16	115.27	1	9	18,249
Schedule 17	119.31	1	6	14,289
Schedule 18	116.11	3	6	17,333
Schedule 19	100.64	1	7	16,391
Schedule 20	106.75	3	6	17,415
Schedule 21	97.52	3	8	19,439
Schedule 22	89.84	0	7	15,219
Schedule 23	81.25	2	7	17,280
Schedule 24	121.17	1	4	12,312
Schedule 25	103.77	1	4	12,344
Schedule 26	89.73	1	9	18,408
Schedule 27	78.27	2	8	19,848
Schedule 28	55.71	2	16	21,034
Schedule 29	48.17	1	11	20,664
Schedule 30	42.67	1	14	22,776
Schedule 31	44.59	1	5	13,816
Mean	156.52	2.06	8.87	18,844.61
SD	83.83	1.76	4.07	4,159.49

It is interesting to note that the average time required to conceive each new schedule is significantly lower, around 21% lower than for the same situation but with a cyclic starting schedule, table 45. A possible explanation can be related to the fact that this initial schedule was generated by a variance of the model applied for rescheduling. To illustrate, the results of table 48 were obtained, on average, in 35% of the time of the same situation, exhibited in table 46.

Table 48: Results for INEM Non-Cyclic Schedule – Disruptions Type I – 5% – Changes Weight = 500

INEM Non-Cyclic Schedule – Type I Variant I 5% - 500				
Schedule	**Time (s)**	**Changes (+)**	**Changes (-)**	**OFV**
Schedule 1	98.33	5	24	14,075
Schedule 2	89.94	2	9	9,340
Schedule 3	101.01	3	10	9,983
Schedule 4	96.67	3	7	8,530
Schedule 5	101.56	3	9	9,202
Schedule 6	98.44	5	11	10,473
Schedule 7	89.99	3	7	8,722
Schedule 8	91.08	4	12	11,460
Schedule 9	155.34	2	9	10,017
Schedule 10	84.98	5	11	10,659
Schedule 11	82.78	2	4	8,303
Schedule 12	83.94	2	3	8,029
Schedule 13	73.23	2	10	10,492
Schedule 14	71.63	3	9	9,664
Schedule 15	67.55	4	13	12,132
Schedule 16	69.49	4	10	10,396
Schedule 17	65.38	3	6	9,215
Schedule 18	60.69	2	6	9,557
Schedule 19	131.61	2	7	9,616
Schedule 20	57.12	3	6	9,939
Schedule 21	54.34	3	8	10,552
Schedule 22	51.92	3	7	9,402
Schedule 23	49.01	2	7	10,042
Schedule 24	48.03	2	4	8,574
Schedule 25	50.34	3	4	8,607
Schedule 26	42.67	2	8	10,171
Schedule 27	38.44	4	8	9,995
Schedule 28	36.23	6	16	13,047
Schedule 29	34.55	3	10	10,718
Schedule 30	34.16	4	14	12,030
Schedule 31	34.01	3	5	9,370
Mean	72.40	3.13	8.84	10,074.58
SD	28.90	1.07	4.06	1,342.37

Additionally, Changes (-) is higher than in table 45, which shows that the likelihood of a person that is considered to be absent on a certain day was supposed to work is higher for the same disruption scenario, demonstrating that cyclic provides a wider response to the demand.

Table 48 shows the results obtained for the same scenario, starting from a non-cyclic schedule, but with the weight associated with changes being 500, instead of 1000. Compared to table 46, the same observations made to the past table maintain. However, in this case, the average OFV is relatively lower, which is caused by the lower understaffing. Compared to table 47, as was expected the average number of Changes (-) is higher.

6.5 Results Analysis

In the following section, there will be made a more extensive analysis of the results featured in the past section. Given the equal or relatively close absenteeism rate, scenarios from tables 42, 43, 45, 46, 47 and 48 will be examined. Since the changes from previous schedules were already discussed before, in this section the remaining most relevant elements of the objective function are going to be taken into account, which are understaffing, overstaffing, incomplete weekends off and undertime. Finally, the objective function will also be compared between scenarios.

6.5.1 Understaffing

Regarding understaffing, as was above remarked, there is a tendency of avoiding replacing an absent TEPH with another and instead accepting understaffing. Three insights can be retrieved from figure 12.

First, for the cyclic initial schedule, when the weight associated is 1000 or 7000, the level of understaffing is essentially the same, as it was for the changes from previous schedules, presented in section 6.4. It shows that when the weight associated with changes is equal or higher than the one associated with understaffing only obligatory changes to respect the hard constraints are made.

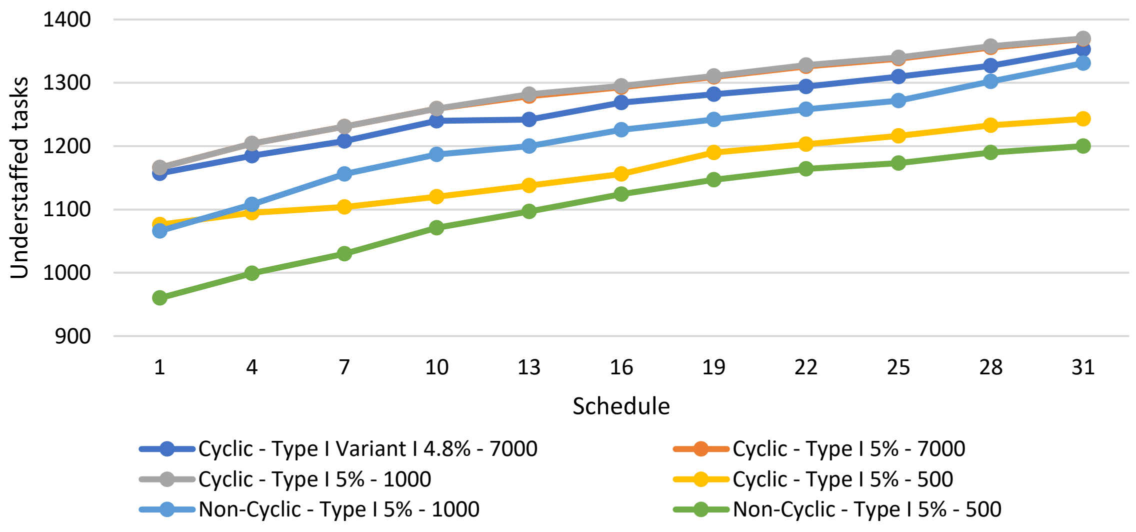

Figure 12: Tasks not performed compared to the demand for the different schedules created throughout the month

Secondly, for the same scenario, the level of understaffing is always lower when the weight associated with changes is equal to 500 compared to 1000. Fixing the weight at 500 is a good alternative if INEM wishes to have more replaced shifts and more perturbation for TEPHs but also less understaffing, which can be crucial to a provide a proper service.

Finally, for the first schedule created, both scenarios that start with non-cyclic have the lowest number of understaffed tasks. This aspect comes from the fact that the non-cyclic initial schedule has less understaffed and it is more flexible, which, for a very stressed set of human resources as INEM has, can be more beneficial. Additionally, even for both scenarios where the weight associated with changes is equal to 500, schedules that start with a non-cyclic respond better on what understaffing is concerned, having less understaffed tasks than the scenario that starts with a cyclic schedule.

6.5.2 Overstaffing

Considering overstaffing – tasks that are done in excess compared to what is established in the a priori demand – cyclic starting schedules, in general, achieve better results than non-cyclic starting schedules, which can be due to the fact that non-cyclic schedule starts with more performed tasks and there will be effectively more redundant tasks. However, there is still the case of the cyclic schedule with three consecutive days absences. Since the model requires to after being absent for three days, having one working day some allocated tasks may again be redundant. Table 49 displays the average overstaffed tasks per instance.

Table 49: Average overstaffed tasks per instance

Instance	Overstaffing Average
Cyclic - Type I Variant I 4.8% - 7000	10.09
Cyclic - Type I 5% - 7000	7.73
Cyclic - Type I 5% - 1000	7.09
Cyclic - Type I 5% - 500	5.36
Non-Cyclic - Type I 5% - 1000	9.18
Non-Cyclic - Type I 5% - 500	9.36

6.5.3 Incomplete weekends off

In respect to incomplete weekends off, scenarios that start with non-cyclic schedules seem to perform worse regarding scenarios starting with cyclic schedules. In contrast to what was discussed in section 6.1, this event occurs, since in this instance the supply is not capable of meeting the demand, thus the flexible non-cyclic schedule aims to respond with a higher effort from workers, which results in the increase of only one weekend day with a shift to perform. In figure 13, it is possible to view three groups of scenarios with similar incomplete weekends off for all the TEPHs throughout the month: the upper group, where there are the non-cyclic starting schedules, in the middle, there are scenarios with a 5% absenteeism rate, despite the value of the weight associated to changes and the bottom with only the instance where there are three consecutive absent days. Therefore, scenarios with the same disruption context and initial schedule tend to be affected in the same way on what incomplete weekends off is concerned.

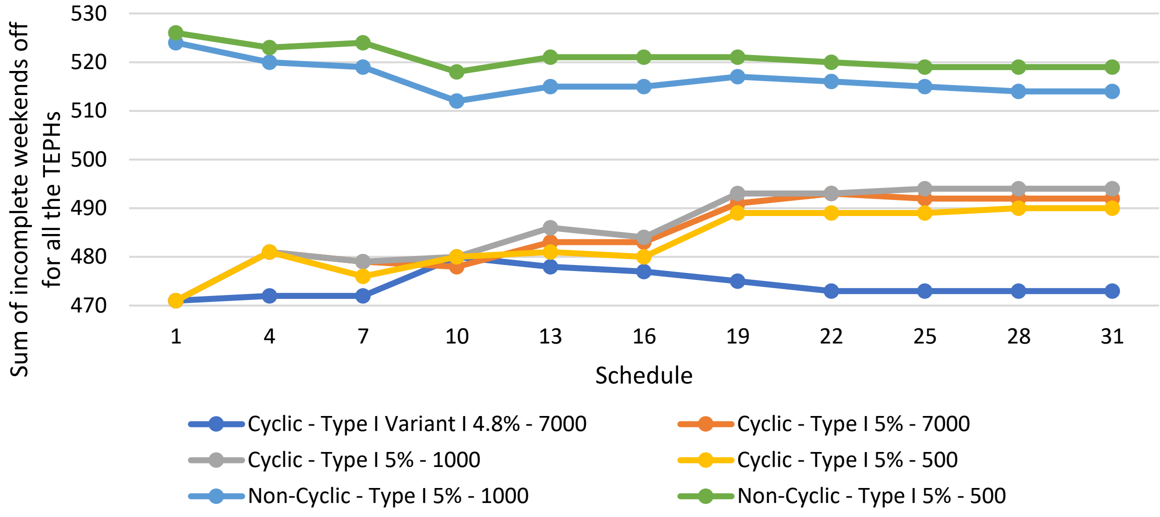

Figure 13: Incomplete weekends off for the different schedules created throughout the month

6.5.4 Undertime

Regarding the hours that TEPHs do not work compared to what is established in the celebrated contract, scenarios that start with cyclic schedules tend to perform worse than scenarios starting with non-cyclic schedules, as it is presented in figure 14.

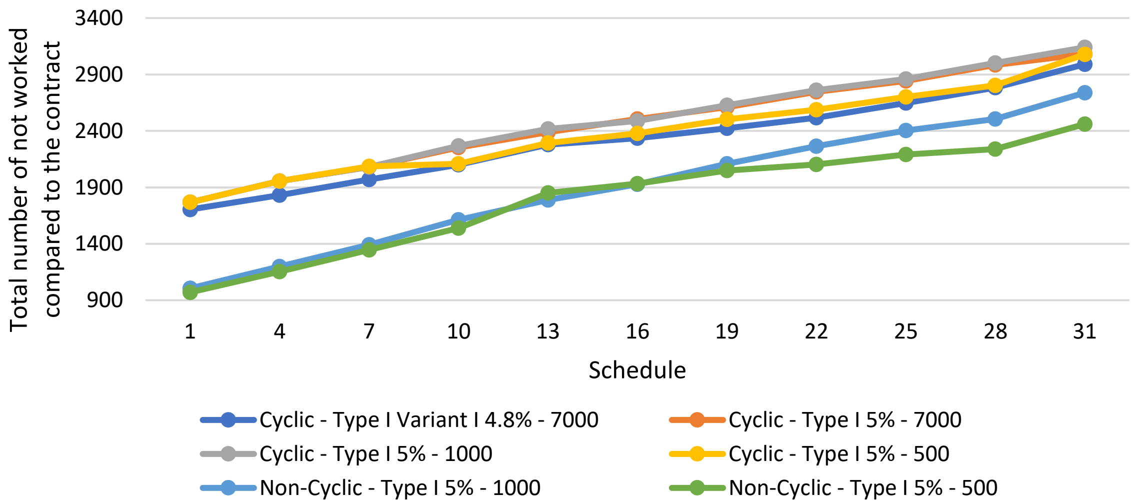

Figure 14: Hours not worked compared to the contract for the different schedules created throughout the month

For the cyclic starting schedule TEPHs have limited days that they can work constrained by the 10-days cycle, which for 31 days is, at maximum, 19. If on average they perform shifts of 8 hours, they work 152 hours, which is less than what is established in the contract for this case, 156 hours. Hence, for the first schedule, TEPHs have at least 4 hours of undertime. For the non-cyclic starting schedule, since there are less schedule constraints, TEPHs can carry out more than 19 shifts in 31 days, having less undertime. ´

Finally, the increasing tendency for each scenario verified in figure 14 refers to the fact already mentioned that, for certain cases, it seems to be preferable to have one more understaffed task rather than replace the absent worker with another that was not performing any task in that day and shift.

6.5.5 Objective Function Value

After analysing how these elements react in the different scenarios, it is thus possible to perceive the reasons behind each OFV. The following three figures demonstrate the evolution of the OFVs over the month.

Figure 15 displays two scenarios starting with a cyclic initial schedule, one with three consecutive absent days and one with an absenteeism rate of 5%. For this case, although the second (in yellow) has higher understaffing, higher incomplete weekends off and higher undertime, the first (in blue) has higher OFV throughout the month mainly due to great need of making changes, especially adding shifts that were not being done in the previous schedule. Being the weight associated with schedule changes the highest by far, the objective function is much more penalized.

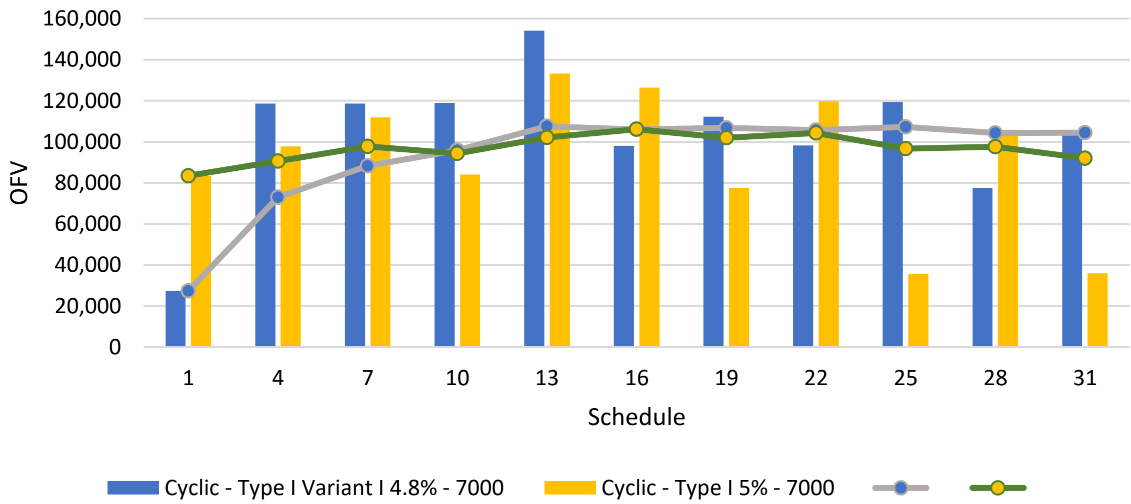

Figure 15: OFV for the different schedules created throughout the month (1)

In figure 16, the two scenarios presented are a simulation starting with a cyclic schedule and one with a non-cyclic schedule, both with the weight associated with changes equal to 1000. Additionally, figure 17 exhibits the same scenarios but with the weight associated with changes equal to 500. In both cases, the non-cyclic starting schedule performs better than the cyclic, overall. However, both cyclic schedules start the month more prepared with lower understaffing and lower undertime, which can even compensate for the fact that they have, in general, more incomplete weekends off and overstaffing. For the second case, the non-cyclic initial schedule simulation makes even more changes on average to the schedules and still has lower OFV.

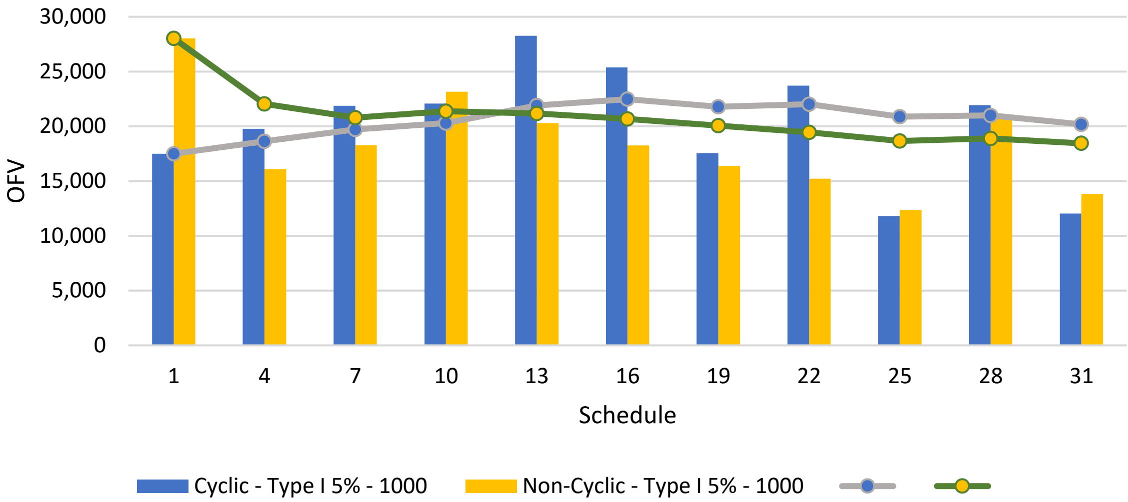

Figure 16: OFV for the different schedules created throughout the month (2)

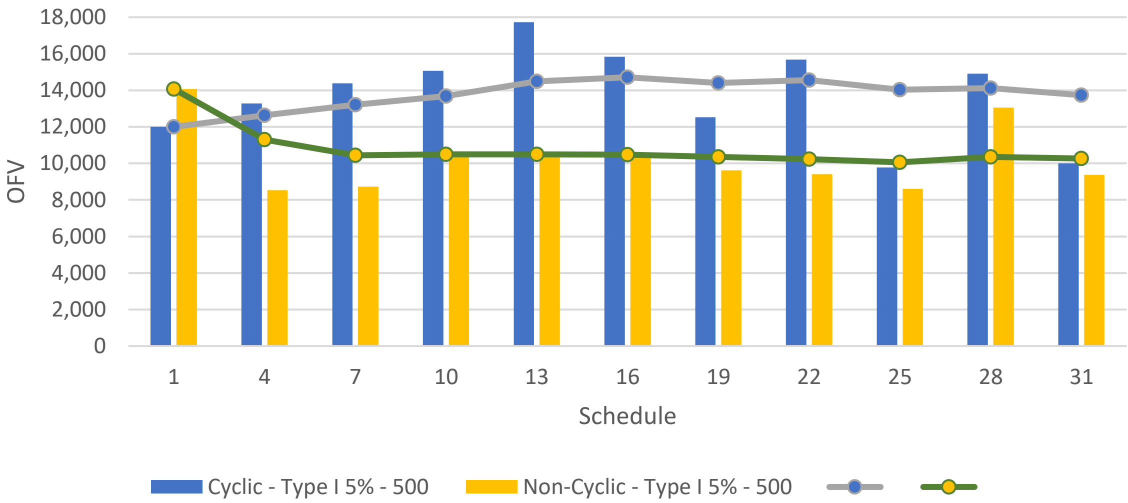

Figure 17: OFV for the different schedules created throughout the month (3)

6.6 Chapter Considerations

This chapter describes the results obtained from simulating the different month scenarios, extracting from those key insights. The model is capable of producing both scheduling and rescheduling effectively for the whole INEM service. Additionally, it is able to achieve optimal solutions for rescheduling in a short time (maximum of 11 minutes) for the worst-case scenarios, according to the literature, regarding two and three consecutive absent days and also absenteeism rates ranging from 5% to 10%.

Moreover, scenarios starting with non-cyclic initial schedules provide better results for cases with under and over supply compared to the demand. The non-cyclic initial schedule is more flexible, being able to adapt better to different cases, for example in CODU decreasing incomplete weekends off and in INEM decreasing the understaffing. Being less strict than the cyclic initial schedule, the non-cycle initial schedule appears as a good alternative for INEM current way of operating. For the same disruption scenario, starting the month with the latter provides better solutions than with the cyclic.

7. Conclusions and Future Work

EMS are complex and uncertain systems that are critical to save patients' lives and provide high-quality health care. Moreover, over the past years, tendencies show that requests for EMS systems are growing. Although countries are increasing the total expenses in health care and also expenses relative to their GDP, Portugal is still behind the European average. Operating with tight budget constraints and, thus, limited resources create the necessity to improve efficiency so that no activities are spending unnecessary crucial resources. SIEM's process is complex and, despite the great effort to modernize INEM's activities, accurate performance indicators show that there is still room for improvement, especially in the rescheduling process, which is a reconstruction of a schedule after a disruption caused by an absent employee. It is performed manually by schedule managers and it is, usually, a daily and time-consuming task.

Some authors found that both scheduling and rescheduling constitute two of the EMS planning problems regarding workforce planning. Effectively, the scheduling problem in EMS has been extensively studied and researchers have proposed several solutions. However, the rescheduling process has not received the same attention. Some interesting studies approached this problem through mainly exact and heuristic algorithms. Albeit there are relevant contributions to other contexts, such as hospitals and bus driving, there are no applications to EMS systems. Hence, the main contribution of this dissertation is a model to assist the rescheduling process for EMS systems in a short time and causing little perturbation to the workers. This model should be flexible to integrate multiple goals, while providing a realistic representation of the most important components of the rescheduling at INEM.

To assess its effectiveness, the model was tested for progressively more complex INEM scenarios. Results show the capacity to manage well INEM's situation, presenting the model as a good alternative to the current manual way of rebuilding schedules at the organization. Additionally, the two main objectives were achieved, as the model was able to provide optimal solutions in less than 11 minutes per day. Moreover, the most severe situations studied in the literature were also considered, being the model able to show its consistency for cases with two and three consecutive absent days and also for the highest absenteeism rates observed in the emergency services sector ranging from 5% to 10%.

Furthermore, since INEM currently operates under a cyclic initial schedule, the differences between starting the month with a cyclic and a non-cyclic schedule were studied. For scenarios with oversupply and undersupply compared to the demand, the non-cyclic initial schedule month simulation outperformed the cyclic, mainly due to the lack of flexibility shown by the latter. On the one side, since all workers must follow a 10-days cycle, it presents inefficiencies related to overstaffing and it is not capable to deal with specific days penalizations, such as having incomplete weekends off. There is always the same number of workers with incomplete weekends off, where for the non-cyclic it can adjust both incomplete weekends off and balance overstaffing with undertime in order to provide better quality solutions. On the other side, regarding undersupply scenarios, the cyclic initial schedule is not able to respond to the existing understaffing, since workers must follow the 10-days cycle. However, the non-cyclic initial schedule is not so strict and is capable of countering, in some way, this situation balancing the reduction of understaffing with the increase of overtime, finding an optimal balance and creating schedules with higher quality.

Hence, the recommendation to INEM holds with rethinking the way its initial schedule is being generated. Although it is more practical for TEPHs, as it is easier for them to follow a cycle every month, knowing which shifts they must perform, it is not the solution that offers the highest quality and as explained in Chapter 3, bad rescheduling techniques may ultimately jeopardize the quality of the provided health service.

7.1 Future Work

Despite the advancements made in this area, there are still numerous ways to develop this subject. The results obtained in this dissertation aimed to create new schedules in a matter of minutes and with little perturbation for the workers, only considering three groups of elements in the objective function. In severe cases, where a quick response is required, it may be fair but performing it every day may neglect important aspects, such as staff preferences and hours allocated to each service. Thus, it would be interesting to solve rescheduling also concerning those key factors – present in chapter 5 – even if periodically during the month, to adjust the latter, assuring that staff satisfaction is being considered. Probably, it would take a longer time to find optimal solutions and, if needed, a heuristics approach should be developed in order to find good results quickly.

Additionally, it would be also important to consider disruptions for shifts instead of only considering for days, which is a more severe scenario, because when is not related to illness, workers' absences may only happen for a short period of the day.

Regarding INEM, it is essential to study TEPHs absenteeism more in-depth in order to better understand the reasons behind them and, possibly, introduce factors taking that into consideration in the initial schedule to reduce disruptions. It would also be interesting to have the records of one month of the created schedules after disruptions to be able to precisely compare the solutions provided by the model and those obtained by performing this task manually. It would be key to have this other type of records so that future work could be even more relatable to INEM's operations and to clearly conclude whether or not the solutions presented represent effective gains for INEM's efficiency and ultimately for its patients.

Furthermore, INEM's planners must often rely on intuition and experience to make challenging planning decisions and, according to the literature, schedule managers, often spend 10-20% of their days performing this task. Hence, it would be a great advantage if this process could be automatized. For its implementation, there are two tools that outstand to be critical for its acceptance. The first is a Graphical User Interface, which must be user-friendly and intuitive to ease managers job. The second is a communication tool concerned with the difficulty for the manager to communicate with TEPHs and get people to perform tasks that were going to be done by an absent worker.

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Appendix

Emergency Vehicle	Image of the Vehicle
Medical Emergency Ambulance (AEM) (BLS)	
Medical Emergency Motorcycle (MEM) (BLS)	
Inter-Hospital Pediatric Transport Ambulances (TIP) (BLS)	
Immediate Life Support Vehicle (SIV) (ALS)	
Vehicle of Medical Emergency and Reanimation (VMER) (ALS)	
Medical Emergency Helicopter Service (SHEM) (ALS)	
Mobile Unit of Phycological Emergency Intervention (UMIPE) (BLS)	
Medical Emergency Stations Ambulances (PEM) (BLS)	
Reserve Station Ambulances (RES) (BLS)	
Non-INEM Ambulances (NINEM) (BLS)