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Confronting Multi-Higgs Models with Experiment

The Search for Underrated and Understudied Signals

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Abstract. We search for distinctive signals of understudied characteristic of certain Multi-Higgs models. This led to the submission of an article for publication [1].
The main understudied property discussed is the presence of couplings between one Z boson and two charged scalars of different masses present in models with at least two scalar doublets and one charged scalar singlet. We explore this issue in detail, considering $h \to Z\gamma$, $B \to X_s\gamma$, and the decay of a heavy charged scalar into a lighter one and a Z boson. We propose that the latter be actively searched for at the LHC, using the scalar sector of a Zee-type model as a prototype and proposing benchmark points which obey all current experimental data, and could be within reach of the LHC.
We also discuss some topics on models with extra Gauge symmetries. Namely a model with an $SU(2) \times U(1) \times U(1)$ electroweak sector with a scalar doublet and a higher multiplet.

1 Introduction

The Higgs Mechanism [2, 3] was used in the creation of the most successful theory particle physicist proposed so far, the Standard Model (SM)[4, 5]. It consists of a process in which a scalar develops a vacuum, that breaks part of the Electroweak Gauge group, and creates mass terms for the correspondent Gauge bosons. The Goldstone theorem guarantees the existence of Goldstone bosons after the breaking of the symmetries, which get absorbed by the now massive Gauge bosons, solving the renormalizability problem of such theories with massive vector bosons.

The SM has been confirmed to unprecedented precision, culminating with the detection of the Higgs boson [6, 7]. Nonetheless, there are questions the SM leaves unanswered, like the problems of Dark Matter, Neutrino Masses, and Baryon asymmetry. It is then natural to extend the model with additional scalars, since this is the less tested sector so far.

It is known experimentally that the masses of the W and Z bosons bear a relation very close to that predicted in the SM: $M_Z \cos\theta_W / M_W \sim 1$, where θ_W is the Weinberg angle. This holds automatically for models with a scalar sector satisfying what is known as custodial symmetry. The scalar potential satisfies this symmetry if it is only composed of scalar singlet, doublet, septet and specific higher representations of the electroweak gauge group. Thus, we are lead to study theories with any number of scalar doublets and/or singlets; the latter neutral and/or charged.

A case of particular interest is the Zee model [8] with two scalar doublets and one charged singlet, originally proposed to explain naturally small neutrino masses, and later adapted to explain also DM [9, 10]. The Zee model with an extra Z_2 symmetry proposed by Wolfenstein [11] is not consistent with current data from neutrino oscillations [12, 13], but the original proposal is still consistent with all leptonic experimental results [14, 15]. But the scalar sector of the Zee model also has another striking feature which is mostly ignored; it is the minimal model predicting the existence of couplings $ZH_1^\pm H_2^\mp$ between the Z gauge boson and two charged scalars (H_1^+ and H_2^+) of different mass. This is the feature highlighted in this work.

Even before direct detection of the extra charged scalar particles, $ZH_1^\pm H_2^\mp$ couplings could potentially have a virtual effect on current measurements, such as $h_{125} \to Z\gamma$. We discuss this example. In fact, the contribution of the charged scalars to the branching ratio can even vanish, but that is not because the Z couples to two different charged scalars, but rather because there are two charged scalars contributing in the loop. Indeed, this feature is already present for instance in the 3HDM, where there are two charged scalars but the coupling of the Z to them is diagonal. Although there is a modulation of the result with the mixing angle between the two charged Higgs, this is hidden when the sum over all diagrams is performed.

To study this model we took into account all the theoretical and experimental constraints coming from the scalar and quark sectors. In particular, we considered in detail the influence of the bounds coming from BR($B \to X_s\gamma$) [16]. This is especially important because, as there are two charged Higgs, one can evade the 580 GeV limit for the 2HDM [17]. We discuss the implications of this for Zee-type models.

A distinctive signal for this model with its $ZH_1^\pm H_2^\mp$ couplings is the decay of the heavier charged Higgs into the lightest one and one Z. We performed an analysis of the parameter space to look for regions where this decay can be large. This lead us to identify examples of benchmark points where the decay $H_2^+ \to H_1^+ Z$ can be large as well as the decay $H_1^+ \to t\bar{b}$, leading to a clear signature that should be searched for at the LHC.

As a side work, we also study the addition of gauge symmetries to the electroweak sector and how it affects the Higgs mechanism. Specifically, we study models with an extra $U(1)$ gauge symmetry, and present formulae for a theory with a doublet and a higher multiplet.

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2 Models with an arbitrary number of doublets and singlets

We consider models with the content of the SM but a scalar sector composed of the following $SU(2)$ doublets and singlets

$$
\begin{aligned}
\phi_a &= \begin{pmatrix}\varphi_a^+\\ \varphi_a^0\end{pmatrix}, \quad a = 1,2,...,n_d\,,\\
\chi_i^+, &\quad i = 1,2,...,n_c\,,\\
\chi_r^0, &\quad r = 1,2,...,n_n\,,
\end{aligned} \tag{1}
$$

and denote by $\varphi_a^{0\prime}$ and $\chi_r^{0\prime}$ the neutral fields after the removal of the developed vacua. The fields rotate to the physical states through the transformations

$$
\begin{aligned}
\varphi_a^+ &= U_a^\alpha S_\alpha^+\,,\\
\chi_i^+ &= T_i^\alpha S_\alpha^+\,,\\
\varphi_a^{0\prime} &= V_a^\beta S_\beta^0\,,\\
\chi_r^{0\prime} &= R_r^\beta S_\beta^0\,,
\end{aligned} \tag{2}
$$

where the last matrix is real and the others are complex. These matrices are not square, and thus, are not unitary. Only if there is no mixing between the scalar and doublet sectors, these matrices can be brought to a basis where they are presented as unitary matrices plus zeros. This will have an important bearing further in this work.

The scalar potential has the quadratic and quartic terms combining the scalar $\phi_a^\dagger\phi_b$ with the scalar bosons. There is only one more unique term, which is $(\mu_4^{abi}\phi_a i\sigma_2\phi_b\chi_i^- + h.c.)$, where μ_4^{abi} is anti-symmetric in (a,b).

After expanding the vacua, the term above leads to

$$
\begin{aligned}
V \supset \quad & \frac{\mu_4^{abi}}{\sqrt{2}}(\varphi_a^+\varphi_b^{0\prime} - \varphi_a^{0\prime}\varphi_b^+)\chi_i^-\,,\\
& + \frac{\mu_4^{abi*}}{\sqrt{2}}(\varphi_a^-\varphi_b^{0\prime *} - \varphi_a^{0\prime *}\varphi_b^-)\chi_i^+\,,\\
& + \frac{\mu_4^{abi}}{\sqrt{2}}(v_a\varphi_b^+ - v_b\varphi_a^+)\chi_i^-\,,\\
& + \frac{\mu_4^{abi*}}{\sqrt{2}}(v_a^*\varphi_b^- - v_b^*\varphi_a^-)\chi_i^+\,,
\end{aligned} \tag{3}
$$

where we can see that there is only mixing between the doublets and singlets if $\mu_4^{abi} \neq 0$ for some combination of indices. The cubic term in the potencial is then essential for the non-unitary behaviour of the matrix U_a^α, which will soon be of major importance. We also see in this last equation that the coupling $h^0H_1^+H_2^-$ can exist with $\mu_4^{abi} = 0$, but only for H_1^+ and H_2^+ belonging both to the doublet sector or both to the singlet sector, while $\mu_4^{abi} \neq 0$ induces a mixing of the sectors.

Finally, the covariant derivative of the scalars leads to the term

$$
\begin{aligned}
\mathcal{L} \supset -i\frac{g}{2c_W}Z_\mu\ & (2s_W^2\delta^{\alpha\alpha'} - (U^\dagger U)^{\alpha'\alpha})\\
& \times(S_\alpha^+\partial^\mu S_{\alpha'}^- - S_{\alpha'}^-\partial^\mu S_\alpha^+)\,,
\end{aligned} \tag{4}
$$

where we see the appearance of $(U^\dagger U)^{\alpha'\alpha}$. Note that if $\mu_4^{abi} = 0$, then the matrix U_a^α will behave as unitary, meaning that $(U^\dagger U)^{\alpha'\alpha}$ will be diagonal. This means that the coupling $ZH_a^+H_b^-$ for $a \neq b$ does not exist if $\mu_4^{abi} = 0$. The charged scalars will then only have flavour changing neutral currents with the Z boson if $\mu_4^{abi} \neq 0$ for some combination of indices. The exploration of this under-appreciated point is one of the distinguishing features of this work.

3 A Zee-type Model

The minimal models containing the type of coupling highlighted in the last section are Zee-type models, which are composed of two scalar doublets and one singly charged singlet. To study a particular example of these models, we pick a Zee-type model consisting of a type II 2HDM with the singly charged singlet. Our purpose is not to make a global fit to the sectors of the model, but rather highlight those features of such types of model that could be probed at the LHC.

The Higgs potential of the model can be written as

$$
\begin{aligned}
V = {} & m_C^2\chi^+\chi^- + \lambda_C(\chi^+\chi^-)^2 + [\mu_4\ \phi_1 i\sigma_2\phi_2\chi^- + h.c.]\\
& + m_1^2\phi_1^\dagger\phi_1 + m_2^2\phi_2^\dagger\phi_2 - m_{12}^2\left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right)\\
& + \left[k_1\phi_1^\dagger\phi_1 + k_2\phi_2^\dagger\phi_2 - k_{12}\left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right)\right]\chi^+\chi^-\\
& + \frac{\lambda_1}{2}\left(\phi_1^\dagger\phi_1\right)^2 + \frac{\lambda_2}{2}\left(\phi_2^\dagger\phi_2\right)^2 + \lambda_3\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2\\
& + \lambda_4\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1 + \frac{\lambda_5}{2}\left[\left(\phi_1^\dagger\phi_2\right)^2 + \left(\phi_2^\dagger\phi_1\right)^2\right]\,,
\end{aligned} \tag{5}
$$

and we consider all parameters and vacua real.

Denoting the rotation matrices for various angles as

$$
O_\theta = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\,, \tag{6}
$$

we define the angles that rotate to the physical basis as

$$
\begin{pmatrix}S_1^0 \equiv G^0\\ S_4^0 \equiv A\end{pmatrix} = O_\beta\begin{pmatrix}\mathrm{Im}\varphi_1^{0\prime}\\ \mathrm{Im}\varphi_2^{0\prime}\end{pmatrix}\,, \tag{7}
$$

$$
\begin{pmatrix}S_2^0\\ S_3^0\end{pmatrix} = O_\alpha\begin{pmatrix}\mathrm{Re}\varphi_1^{0\prime}\\ \mathrm{Re}\varphi_2^{0\prime}\end{pmatrix}\,, \tag{8}
$$

$$
\begin{pmatrix}S_1^+ \equiv G^+\\ H^+\end{pmatrix} = O_\beta\begin{pmatrix}\varphi_1^+\\ \varphi_2^+\end{pmatrix}\,, \tag{9}
$$

$$
\begin{pmatrix}S_2^+\\ S_3^+\end{pmatrix} = O_\gamma\begin{pmatrix}H^+\\ \chi^+\end{pmatrix}\,, \tag{10}
$$

where β is the angle that brings us to the Higgs basis of the doublets, revealing the Goldstone bosons $G^{0,\pm}$, and the other angles are obtained by diagonalizing the remaining mass matrices.

The U_a^α matrix discussed before becomes, in this theory,

$$
U = \begin{pmatrix}\cos\beta & -\sin\beta\cos\gamma & \sin\beta\sin\gamma\\ \sin\beta & \cos\beta\cos\gamma & -\cos\beta\sin\gamma\end{pmatrix}\,. \tag{11}
$$

And the quantity appearing in the flavour changing neutral current of the charged scalars with the Z is

$$
U^\dagger U = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos^2\gamma & -\sin\gamma\cos\gamma\\ 0 & -\sin\gamma\cos\gamma & \sin^2\gamma\end{pmatrix}\,. \tag{12}
$$

The flavour changing coupling comes exactly from the off diagonal components of this matrix, which are absent in theories like the NHDM. When calculating the masses and angles from the parameters of the potential we are left with the following fourteen independent parameters

$$\begin{aligned} &m_{H_1^0},\, m_{H_2^0},\, m_{A^0},\, m_{H_1^+},\, m_{H_2^+},\, m_{12}^2, \\ &v,\, \alpha,\, \beta,\, \gamma,\, \lambda_C,\, k_1,\, k_2,\, k_{12}\,, \end{aligned} \tag{13}$$

where v is the only one already constrained by experiment thorough the mass of the $W_\mu^\pm$ bosons.

3.1 Theoretical Constraints

To scan the parameter space of the model, we imposed on the generated points some theoretical constraints that the model needs to satisfy.

The bounded from below conditions in the Zee model were studied in Reference [18]. They extend the known conditions for the 2HDM [19, 20], but only find necessary conditions, not sufficient. One of the conditions depending only on α and β cannot be solved analytically. To sidestep this, we took a large sample of those parameters and excluded the ones that do not satisfy the condition.

The analysis of the charged breaking minima was much more complicated than in the 2HDM [20]. Therefore, we took an approach based on Reference [18]. We parameterize the possible minima, and for each neutral minima in the theory, we generate random values of that parameterisation and use the method of gradient descent to obtain the lower minimums.

To ensure perturbative unitarity of the quartic couplings, we implemented the general algorithm presented in Reference [21]. We found the scattering matrices for processes conserving electric charge and hypercharge, which have a total of 19 different eigenvalues. We then exclude points where at least one eigenvalue is bigger than 8π.

Finally, the points have to satisfy the electroweak precision measurements from the oblique parameters S, T and U. We use the fit given in [22] and demand the parameters to be within 2σ of the result.

3.2 Constraints from the LHC

One type of experimental constraints we impose on the points from the LHC, are the constraints on the measured Higgs boson of $125 GeV$. These are forced from the signal streghts for each production mode and final state, which are given in Reference [23].

The rest of the constraints from the LHC are the bounds on other neutral and charged scalars which we implemented using the most recent version of `HiggBounds` 5 [24].

3.3 Constraints from BR($B \to X_s\gamma$)

In models with charged scalar bosons it is well known [16, 17, 25–27] that the experimental limits on the BR($B \to X_s\gamma$) can put important constraints on the parameter space of these models. For instance, in Reference [17] the bound

$$m_{H^+} > 580\,\text{GeV}\,, \tag{14}$$

is derived for the type 2 2HDM at 95% CL (2σ).

We take the approach of considering for the theoretical error a band around the central value of the calculation with an error of 2.5%, and following [27], for the experimental error, we consider 99%CL (3σ), that is,

$$2.78\times10^{-4} < \text{BR}(B \to X_s\gamma) < 3.77\times10^{-4}\,. \tag{15}$$

Our calculation follows closely the original calculation of Reference [16]. The central point in that calculation is that the new contributions from the charged scalar bosons are encoded in the Wilson coefficients. For the input parameters we use those of Reference [16] except for $\alpha_s(M_Z), m_t, M_Z, M_W$ that were updated to the values of the PDG [28].

Before advancing, we checked that, by using the input parameters of [16], we were able to reproduce their results for the SM.

Then, we considered the particular case of a type 2 2HDM, accomplished by setting $\gamma = 0$ in our model, decoupling the singlet completely. In Figure 1 we show the results considering a band corresponding to 2.5% in the calculation and a 3σ band for the experimental result. As can be seen, the limit for the mass of the charged scalar that we get is similar to what was obtained in Reference [17].

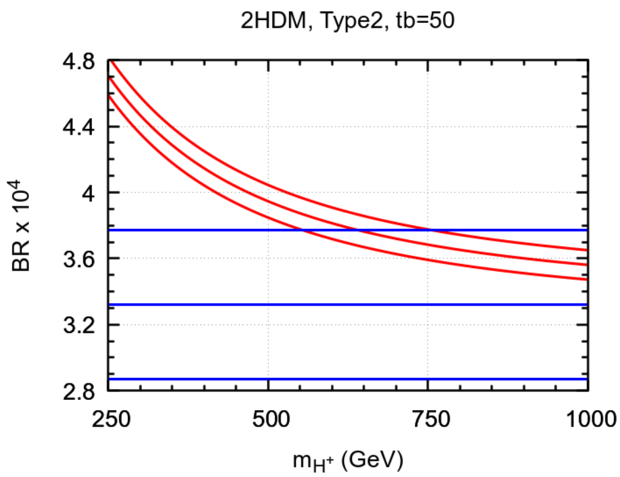

Figure 1. BR($B \to X_s\gamma$) as a function of the charged scalar mass. The lines in blue represent the 3σ experimental limits, and those in red to 2.5% error in the calculation.

Finally, we considered our type 2 Zee model. For now, we do not impose the theoretical and experimental constraints on the model, since our purpose here is just to show how the constraints from BR($B \to X_s\gamma$) can be satisfied. We varied the masses and γ, and obtained the plots in Figure 2. In the upper panel, we see that we cannot have both charged Higgs masses below the approximately $580 GeV$ limit simultaneously. Nonetheless, it is possible that one of the charged Higgs is below that threshold, if the other is above. This depends on the angle γ as can be seen in the lower panel. We see that $m_{H_1^+}$ can be as low as 50 GeV if the mixing angle is close to $\pm\pi/2$ and that we recover the previous result for $\gamma = 0$.

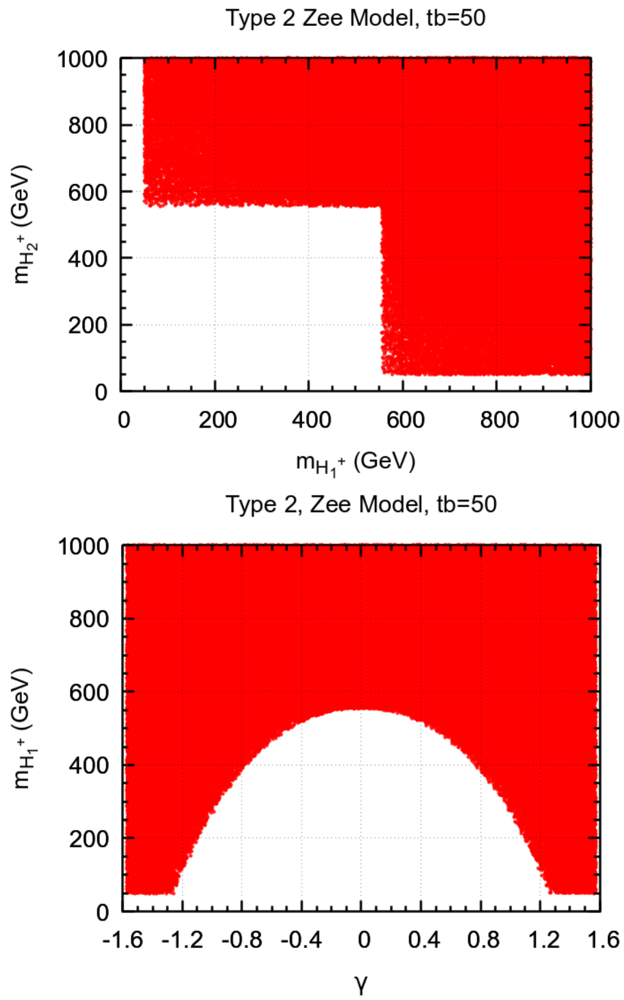

Figure 2. Upper panel: points satisfying Eq. (15) for Zee-type models. Lower panel: mass of the lightest charged scalar boson as a function of the mixing angle γ.

4 The Search for Distinctive Signals of Zee-type Models

4.1 Impact of the charged scalars on the decays $h \to \gamma\gamma$ and $h \to Z\gamma$

The distinctive feature of our implementation of Zee-type models, the appearance of the off-diagonal coupling $ZH_1^{\pm}H_2^{\mp}$, contributes to the loop decay $h \to Z\gamma$. It is then natural to seek for a new feature in this decay, caused by the mentioned coupling. The decay $h \to \gamma\gamma$, on the other hand, will not be influenced by this coupling. The difference between the two processes comes from diagrams where different charged scalars circulate in the loop.

The dependence of the amplitudes μ_4, is intrinsically related to their dependence on the angle γ. The couplings $h_1 H_j^+ H_k^-$ are proportional to $\sin\gamma$ for $j \neq k$, and do not have a strong dependence on γ otherwise. The couplings $ZH_j^+H_k^-$ are proportional to $\sin 2\gamma$ for $j \neq k$ and to $\cos 2\gamma$ otherwise.

We were able to check that the amplitudes of the diagrams depend on γ as the products of the respective couplings would suggest, as expected. However, once we added all the diagrams, we lost the dependence on γ. The dependence on μ_4 is then hidden, and the observed behaviour ends up being the same as in models like the 3HDM.

We conclude that the decay $h \to Z\gamma$ cannot be used to identify the novel coupling $ZH_1^+H_2^-$ appearing in Zee-type models, and move on to search for such a signal in other processes.

4.2 The decay $H_2^+ \to H_1^+ + Z$

Depending on the masses, the following decays of the most massive charged higgs are among the most important,

$$H_2^{\pm} \to H_1^{\pm} + Z\,, \qquad H_2^{+} \to t + \bar{b}\,, \quad H_2^{\pm} \to H_1^{\pm} + h_i\,,$$
$$H_2^{\pm} \to W^{\pm} + h_i\,, \quad H_2^{+} \to \nu_\tau + \tau^+\,.$$

The first decay is unique to this type of models and not present in the NHDM. It has a strong dependence on γ and is only possible for $\gamma \neq 0$. It is then the prefect process to probe the type of models we are interested in. We have checked that it can indeed ocur, and its dependence on γ and the mass of the heavier scalar can be seen in Figure 3, where all the points satisfy the constraints discussed previously.

After the decay into the lighter scalar, this last one can decay, if kinematically available, into the same processes.

There is one chain of processes that has a very clear signature. That chain is

$$H_2^+ \to H_1^+ + Z\,, \quad \text{and} \quad H_1^+ \to t + \bar{b}\,, \tag{16}$$

and it should be search for at the LHC in order to probe models with this characteristic couplings.

Since the model has many independent parameters, it won't help to plot the BR's as a function of the heavier scalar masses, analogous to what was made in [29], since we would get a figure with all points superimposed and no lines. We then turn to benchmark points. This allows for a better visualisation, since we fix most of the parameters and oscillate them slightly, in order to show that the branching ratios of the process in Equation 16 can be important, or even dominant.

In Appendix A we present the dominant branching ratios of H_2^+ and H_1^+ for the parameters in the benchmark regions around each benchmark point.

In the first three benchmark points we see that our signal decay has the largest branching ratio, while H_1^+ decays almost always into $t + \bar{b}$. This should provide clear signatures at the LHC. A detailed analysis, with background studies, should of course be done. The width of the bands comes from the variation of $\tan\beta$ (at the percent level, because the good points have $\alpha \simeq \beta$). All the points pass all the constraints.

For the first benchmark point we chose both masses of the charged scalars to be above the limit shown in Equation 14, while for the second and third benchmark point we chose for the lighter one to be below that limit. This way we can show that there are indeed points passing all

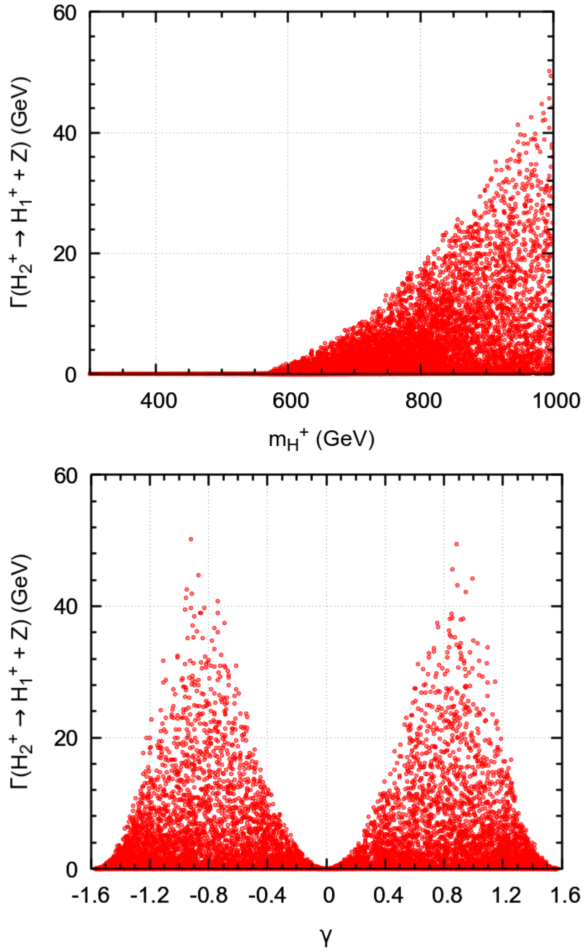

Figure 3. Decay with $H_2^+ \to H_1^+ + Z$. On the upper panel the dependence on the mass of the decaying charged Higgs and on the lower the dependence on γ.

the constraints discussed, including those of Equation 15, while one of the charged scalar lies below the limit of Equation 14.

The forth and last benchmark point was taken from the few points with a large BR($H_1^+ \to W^+ + h_1$). This decay is one of the channels that have not been investigated at the LHC yet [30]. From the figures we see that, in our model, both BR($H_1^+ \to W^+ + h_1$) and BR($H_1^+ \to W^+ + h_2$) can be sizeable. But in this case, the BR($H_2^+ \to H_1^+ + Z$) is very small. However the BR($H_2^+ \to W^+ + h_1$) and BR($H_2^+ \to W^+ + h_2$) can also be large, making this an interesting benchmark point.

5 Models with extra gauge symmetries

As a side work, we explore models with extra Gauge symmetries. Namely theories the electroweak gauge Group $SU(2) \times U(1) \times U(1)$. We also work a model with such an electroweak group and a scalar sector composed of a doublet and a higher multiplet.

5.1 Neutral Gauge Bosons

We consider theories with the following covariant derivative

$$D^\mu = \partial^\mu + i \sum_{r=1}^{m} g_r Y_r X_r^\mu \,, \tag{17}$$

where we consider only the neutral Gauge bosons. We assume the scalars of the theories have the vacua in only one entry, such that

$$Y_r < \phi_i >= y_{ri} < \phi_i > \,, \tag{18}$$

and parameterize that entry as

$$\phi_{iv} = \frac{1}{\sqrt{2}}(v_i + h_i + i a_i) \,. \tag{19}$$

After applying the covariant derivative to the scalars and calculating the kinectic terms of the scalars in the lagrangian, the relevant terms for observing the Higgs mechanism on the nautral gauge bosons are the following

$$\mathcal{L}_{K0} = \frac{1}{2}\bar{a}^T \bar{a} + \bar{a}^T C \bar{X} + \frac{1}{2}\bar{X}^T M^2 \bar{X} \,. \tag{20}$$

Where the components of $\bar{a}$ are $\partial^\mu a_i$, the components of $\bar{X}$ are X_r^μ, C is a matrix whose columns are the vectors

$$\bar{c}_r = \begin{pmatrix} v_1 g_r y_{r1} & v_2 g_r y_{r2} & . & . & v_n g_r y_{rn} \end{pmatrix} , \tag{21}$$

and M^2 is the Graham Matrix of C. By diagonalizing this last matrix with a rotation matrix R, the lagrangian terms are rewritten as

$$\mathcal{L}_{K0} = \frac{1}{2}\bar{a}^T \bar{a} + \bar{a}^T C_D \bar{Y} + \frac{1}{2}\bar{Y}^T M_D^2 \bar{Y} \,. \tag{22}$$

where $\bar{Y}$ are the mass eigenstates, $\bar{Y} = R\bar{X}$, M_D^2 is the diagonal form of M^2, $RM^2R^T = M_D^2$, and $C_D = CR^T$.

We can safely ignore the massless Gauge bosons, since by corresponding to null eigenvalues, the nature of M_D^2 and C_D guarantees us that they do not appear in this term. Besides, the Goldstone theorem guarantees that all massive gauge bosons have a correspondent Goldstone boson to absorb, which means we can safely project the a_i to that space in the C_D term and separate them from the others in the purely kinetic term. We then obtain

$$\mathcal{L}_{K0} = \frac{1}{2}\bar{z}^T \bar{z} + \frac{1}{2}\bar{b}^T \bar{b} + \bar{b}^T C_{DS} \bar{B} + \frac{1}{2}\bar{B}^T M_D^2 \bar{B} \,, \tag{23}$$

where $\bar{B}$ are the massive Gauge bosons, $\bar{b}$ are the Goldstone bosons, and $\bar{z}$ are the rest of the scalars. After a little algebra, this gets written as

$$\mathcal{L}_{K0} = \frac{1}{2}\bar{z}^T \bar{z} + \frac{1}{2}\left(\bar{B}^T + \bar{b}^T (C_D^T)^{-1}\right) M_D^2 \left(\bar{B} + C_D^{-1}\bar{b}\right) . \tag{24}$$

Finally, using the gauge symmetries available, we redefine the Gauge bosons as

$$\bar{Z} = \bar{B} + C_D^{-1}\bar{b} \,, \tag{25}$$

leaving the lagrangian as

$$\mathcal{L}_{K0} = \frac{1}{2}\bar{z}^T \bar{z} + \frac{1}{2}\bar{Z}^T M_D^2 \bar{Z} \,, \tag{26}$$

which illustrates the Higgs mechanism at work in these theories.

5.2 The photon in $SU(2)\times U(1)\times U(1)$ theories

In the case of a theory with an $SU(2)\times U(1)\times U(1)$ electroweak Gauge group, the neutral part of the covariant derivative takes the form

$$D_\mu = \partial_\mu + igT_3W^3_\mu + ig_aY_aX_{a\mu} + ig_bY_bX_{b\mu}\,, \tag{27}$$

and the photon is given by a linear combination of those Gauge bosons as

$$A_\mu = cW^3_\mu + aX_{a\mu} + bX_{b\mu}\,. \tag{28}$$

But, defining B_μ as

$$B_\mu = \frac{1}{\sqrt{a^2+b^2}}(aX_{a\mu} + bX_{b\mu})\,, \tag{29}$$

and X_μ as the combination of $X_{a\mu}$ and $X_{b\mu}$ orthogonal to B_μ, the photon can be written as

$$A_\mu = cW^3_\mu + dB_\mu\,. \tag{30}$$

One can then always decide to start in this basis of the $U(1)\times U(1)$ subgroup, and define the hypercharges of all particles with relation to this basis. This means that the first angle of rotation is nonphysical and one only needs two of the usual three Euler angles to get to the physical basis. These angles can be defined, as an example, as

$$\begin{pmatrix} Z^0_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}\,, \tag{31}$$

$$\begin{pmatrix} Z'_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_X & -\sin\theta_X \\ \sin\theta_X & \cos\theta_X \end{pmatrix}\begin{pmatrix} Z^0_\mu \\ X_\mu \end{pmatrix}\,, \tag{32}$$

where (A_μ, Z_μ, Z'_μ) are the physical basis.

Note that the singlets in a theory do not couple with W^3_μ. If they develop a vacuum, they also cannot couple to B_μ, or they would also couple to A_μ, and break the electromagnetic charge. One must conclude then that, in a theory with singlets, the basis (B_μ, X_μ) can be found by requiring that only the X_μ couples to the singlets with vacua. If such basis is nowhere to be found, the eletromagnetic charge is broken in the theory, and one must exclude it.

5.3 Working Theory: Doublet plus Multiplet

As an example, we consider a theory with an $SU(2)\times U(1)\times U(1)$ electroweak Gauge group and a scalar sector consisting of a doublet and a higher multiplet, parameterized as

$$\varphi = \begin{pmatrix} \cdot \\ \frac{1}{\sqrt{2}}(v + \varphi_R + i\varphi_I) \end{pmatrix}, \chi = \begin{pmatrix} \cdot \\ \cdot \\ \frac{1}{\sqrt{2}}(u + \chi_R + i\chi_I) \\ \cdot \end{pmatrix}, \tag{33}$$

and with hypercharges:

$$\varphi: \quad t_{3V} = -1/2 \quad y_B = 1/2 \quad y_X = y_2\,, \tag{34}$$

$$\chi: \quad t_{3V} = -y_B \quad y_B = y_B \quad y_X = y_X\,. \tag{35}$$

The C matrix is then written as

$$C = \begin{pmatrix} -\frac{1}{2}gv & \frac{1}{2}g_Bv & y_2g_Xv \\ -y_Bgu & y_Bg_Bu & y_Xg_Xu \end{pmatrix}\,, \tag{36}$$

and one can check that a rotation of the first two columns (correspondent to W^3_μ and B_μ respectively) by the angle

$$\cos(\theta_W) = \frac{rg}{\sqrt{r^2g^2 + g'^2}}\,, \tag{37}$$

$$\sin(\theta_W) = \frac{g'}{\sqrt{r^2g^2 + g'^2}}\,, \tag{38}$$

brings us to a basis with a null column, whose entry in the graham matrix of C will be zero, and thus must correspond to the photon. We are then already in the basis discussed in Section 5.2, thanks to our choice of hypercharges.

Removing the photon, the mass matrix becomes

$$M^2 = \begin{pmatrix} g_0^2(\frac{1}{4}v^2 + y_B^2u^2) & \Delta \\ \Delta & g_X^2(y_2^2v^2 + y_X^2u^2) \end{pmatrix}\,, \tag{39}$$

where $\Delta = -g_0g_X(\frac{1}{2}y_2v^2 + y_By_Xu^2)$ and $g_0 = \sqrt{g^2 + g_B^2}$.

Finally, to diagonalize this matrix one uses the angle

$$\cos\theta_X = \sqrt{\frac{1 + \sqrt{1 - 1/(1+K^2/4)}}{2}}\,, \tag{40}$$

$$\sin\theta_X = \frac{sign(K)}{c_X\sqrt{4+K^2}}\,, \tag{41}$$

where:

$$K = \frac{g_0^2(\frac{1}{4}v^2 + y_B^2u^2) - g_X^2(y_2^2v^2 + y_X^2u^2)}{\frac{1}{2}g_0g_X(\frac{1}{2}y_2v^2 + y_By_Xu^2)}\,, \tag{42}$$

and gets for the masses of the physical Gauge bosons

$$M_1^2 = \quad v^2(\tfrac{1}{2}g_0c_X + y_2g_Xs_X)^2 \tag{43}$$

$$+u^2(y_Bg_0c_X + y_Xg_Xs_X)^2\,, \tag{44}$$

$$M_2^2 = \quad v^2(\tfrac{1}{2}g_0s_X - y_2g_Xc_X)^2 \tag{45}$$

$$+u^2(y_Bg_0s_X - y_Xg_Xc_X)^2\,. \tag{46}$$

6 Conclusions

We look for understudied distinctive signals in various Multi-Higgs Models. One such singular feature of models with multiple scalar doublets and charged singlets is the presence of off-diagonal $ZH_1^\pm H_2^\mp$ couplings. We have studied this feature in detail, using the scalar sector of a Zee-type model as an example.

We show that $ZH_1^\pm H_2^\mp$ couplings appear in $h\to Z\gamma$ and $B\to X_s\gamma$, but that there they do not impose features beyond those already present in generic 3HDM (where such off-diagonal couplings are not present).

We stress the importance of looking experimentally for $H_2^+ \to H_1^+Z$ decays and propose interesting benchmark points. We also found in our model interesting values for the decays recently proposed in [30]. We found that there are regions of parameter space consistent with large branching ratios for $H_1^+\to W^+h_{1,2}$ or $H_2^+\to W^+h_{1,2}$. But, in those cases, we found no case where simultaneously $\mathrm{BR}(H_2^+\to H_1^+Z)$ was large. We strongly urge a search for $H_2^+\to H_1^+Z$ decays.

We also work some expressions on the Higgs mechanism for theories with extra Gauge symmetries in the electroweak sector. We give emphasis to $SU(2)\times U(1)\times U(1)$ theories and work the example of theories with a scalar dublet and a higher multiplet.

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A Benchmark points branching ratios

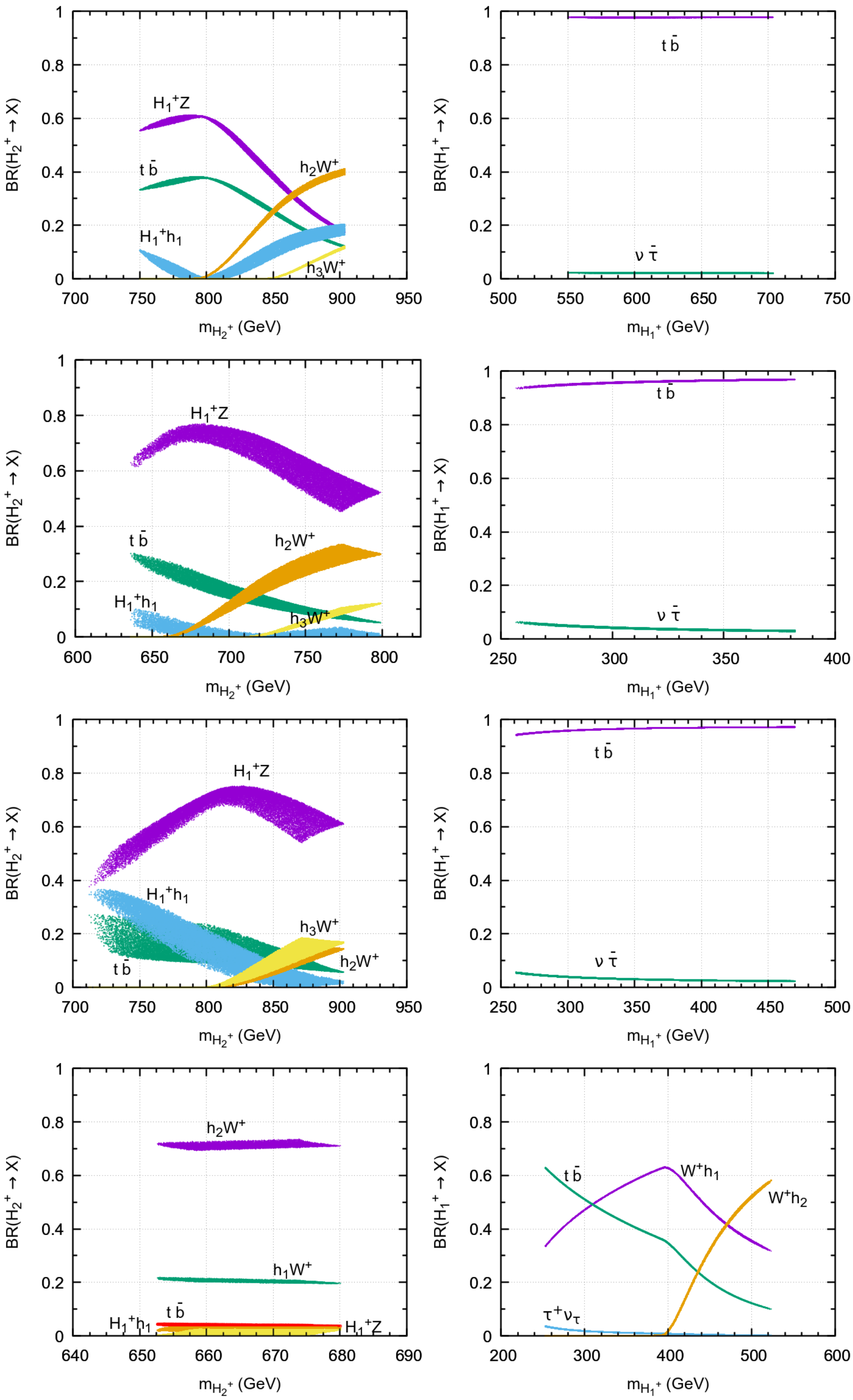

Figure 4. From top to bottom we present the dominant branching ratios of H_2^+ on the left and H_1^+ on the right for each of the benchmark points 1 to 4.