Cooperation between Humans and Robots Underwater

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Abstract—In underwater environments, human divers face enormous challenges commonly related to poor visibility, lack of orientation, heavy equipment, limited breathing time and pressure-related issues. This does not only hinder the diver’s work performance but also increases the probabilities of occurring accidents. With this in mind, the present work addresses the problem of enabling cooperative strategies between the diver and autonomous underwater vehicle (AUV) aiming to increase task safety and efficiency levels. From a practical perspective, our work contributes by proposing a cooperation architecture between a diver and the AUV guidance, navigation and control systems. Under this cooperation framework, we present state of the art solutions to localize and track the diver via algorithms and onboard sensors of the AUV and follow up by designing its control system. From a theoretical perspective, the derivation of the controllers exploit nonlinear Lyapunov based techniques and geometric control analysis tools, achieving robust properties and stable equilibria that are proved formally. Simulations results are presented and discussed, in the presence of measurement noise, constant ocean current disturbances and uncertainty in the model parameters of our vehicle, illustrating the performance achieved with the proposed control system in a realistic cooperative scenario with a diver.

Index Terms—Autonomous underwater vehicles, Diver-robot interaction, Geometric control, Path following, Stability of nonlinear systems, Target tracking.

I. INTRODUCTION

Diving has come a long way since humans started exploring underwater environments. These activities were and still are motivated by the exploitation of marine resources and exploring unknown areas, which still cover a large percentage of our aquatic planet. During the maturing process along these years, diving experience has shown that, in contrast with other land-based activities, is not easily performed, and ultimately requires extreme caution and standardised procedures to minimise risk faced by divers [1]. For this reason, there has recently been an interest in developing new strategies and work ethics in carrying out these diving operations, which raises the question of whether the use of cooperative mechanisms between divers and robots can increase efficiency and reduce risk levels. Moreover, the marine environment raises several challenges and opportunities for new solutions embedded in navigation and control literature [2]. Applications of these mechanisms will surely result in solutions to problems in the field of robotics and lead to new questions, producing cyclical research and development work, boosting innovation and contributing towards science.

Cooperation strategies can be enacted in various different solutions, but the present thesis focuses on the guidance, navigation and control solutions applied to autonomous underwater vehicles (AUVs), partially motivated with state of the art solutions regarding the problem of target tracking [3], [4], localization [5]–[7] and nonlinear control [8]–[12]. Together with the nonexistence of tether cables and human element, always present in remotely operated vehicles (ROVs), this paves the ground to robust and efficient autonomous architectures. Previous works [13]–[17] were carried out under many objectives but with the common goal of developing and researching cooperation frameworks between autonomous marine vehicles (AMRs) and divers, and serve as a basis of guidance and support to this work. In summary, this paper addresses the problem of designing a 6 Degrees-of-Freedom (DOF) dynamic and kinematic control system for a single AUV within a cooperation framework system that will enhance the divers’ capabilities and/or reduce associated risks with diving operations.

The paper is organized as follows. The problem addressed in the paper is expanded in Section II. Section III refers to possible cooperation frameworks in various diving operations. Section IV expands state of the art solutions towards locating and tracking a diver, wrapping up with an architecture proposal towards designing the AUV control system, presented in Section V. Simulations results are presented in Section VI and Section VII summarizes the main conclusions of the paper.

A. Mathematical Notation

Throughout the paper we used bold symbols and letter to denote vectors or matrices and normal script for scalar. Vectors and matrices are represented in lower and upper case, respectively. The symbol \(\mathbf{x}\) denotes an \(n \times n\) identity matrix. The mapping \(\mathbf{S}(\cdot) : \mathbb{R}^3 \mapsto \mathbb{S}\) denotes the skew-symmetric operator, i.e. \(\mathbf{S}(\mathbf{a})\mathbf{b} = \mathbf{a} \times \mathbf{b}\), where \(\mathbf{a}, \mathbf{b} \in \mathbb{R}^3\) and \(\times\) is the vector cross product. The asymptotic and supremum norm are denoted by \(\lim_{t \to \infty} \| \mathbf{x}(t) \|\) and \(\| \mathbf{x} \|_\infty = \sup_{t \geq 0} \| \mathbf{x}(t) \|\), respectively. For simplicity of notation, except when explicitly stated, \(\| \cdot \|\) denotes the Euclidean 2-norm. For an arbitrary \(n \times m\) matrix \(\mathbf{A}\), if the p-norm for vectors is used, the corresponding (induced) norm is defined as \(\| \mathbf{A} \|_p = \sup_{\mathbf{x} \neq 0} \frac{\| \mathbf{A}\mathbf{x} \|_p}{\| \mathbf{x} \|_p}, \forall \mathbf{x} \in \mathbb{R}^n\).

II. PROBLEM STATEMENT

The present article tackles the problem of enabling cooperating behaviours between a diver and an AUV through the
design of an autonomous control and navigation system, both from a theoretical and practical point of view.

A. Diver-AUV Cooperation

The problem of cooperation between an AUV and a diver can be formulated in various ways, and there is no optimal approach as there exists different types of diving operations, each with respective risks and tasks. It is imperative to first identify the mission at hand and then proceed to formulate the possible cooperation strategies.

B. Diver Tracking

Gathering information about the diver whereabouts in underwater environments is beneficial for vehicle guidance, control and navigation systems. This can be formulated in a two-fold problem approach: i) target localization; ii) target tracking. The former concerns the estimation of a target position, e.g., diver, object, robot, dock, animal, via measurements accessed by an agent from a suite of sensors. The latter, through solving this localization problem and obtaining estimates of the target position, addresses the problem of obtaining additional information about other kinematic components of the target state, e.g., velocity, acceleration, etc. This paper focuses on solving the localization (and tracking) problem of estimating linear motion quantities of a target, that is, the target is assumed to have no intrinsic attitude and, consequently, no associated angular velocity. We instead interpret the target attitude/angular velocity to be associated with the mission requirements, which is then provided by a guidance system.

C. AUV Motion Control

Assuming we have a functional vehicle navigation system, with the means to locate and track a diver and determine the AUV pose as well as velocity, we address the control problem of designing a 6 DOF dynamic and kinematic controller for a single fully-actuated AUV under the proposed cooperation architecture.

III. DIVER-AUV UNDERWATER COOPERATION

While the concept of cooperation is not new in the world of robotics, only recent development in the areas of underwater communication, navigation and electronics [18] have allowed for such strategies to be carried out in the challenging marine environment. Specific to cooperation is the concept of diver-AUV cooperation which concerns the different ways an AUV can cooperate with the diver to help enhance the tasks' feasibility and/or reduce the associated risks faced by the diver. Possible cooperation strategies were identified, such as: (i) follow the diver within a safety radius, allowing different tasks to be performed such as, pointing towards the desired location underwater to aid the diver navigation, or carrying certain tools/equipment; (ii) observe the diver in the surrounding area, in a station-keeping manoeuvre to provide light or other improvements to the working area; (iii) Monitor the diver and check their vital signs, reducing potential diving associated risks; (iv) Dynamically plan the robot mission on-line via gesture or other communication interfaces with the diver. Common risks associated with diving operations include low visibility, decompression sickness, nitrogen narcosis, currents and a general lack of orientation. Other risks may arise, but it depends on the diving operation at hand, so an initial brief review and discussion are held to contextualize our problem, highlighting the specific risks/challenges and benefits towards implementing such cooperation strategies.

Commercial/Industrial Diving: These concern the engineering work performed in underwater environments such as building, repair, examination, or maintenance. In commercial diving, common risks are associated with the surrounding hazardous working environments and the use of specialized equipment. By implementing some cooperation strategies between a diver and an AUV, the diver fatigue can be reduced during diving, the diver could have a safer and faster approach towards the worksite, or have a helping hand from the robot in hard-to-operate tools, e.g., valves, levers, with recent work done towards this latter approach [19].

Scientific Diving: These diving operations have their purpose built on the pursuit of knowledge and research development related to science, with tasks performed underwater being of a scientific nature [20]. Common challenges in these missions are associated with not knowing the location of potential objects or areas of interest (e.g., underwater archaeology) or ruining data (e.g., marine biology). These cooperation strategies could benefit the mission development, allowing a quicker and safer delivery of material of interest to the surface, a better site inspection or finding new areas of interest.

Media Diving: Media diving operations specialize in underwater cinematography and photography, related to oceanography, engineering, cinema and television industry. Divers often use specialized equipment (e.g., video cameras, underwater lighting). There is almost no need to navigate accurately and know the whereabouts of the environment as they usually accompany other divers/objects. Relative to cooperation strategies, automation behaviours have been proven to benefit filming operations [21], with steadier recording, self-recording and transportation of certain apparatus.

Military Diving: Military diving operations concern the tasks performed by military personnel in underwater environments. These can involve more risks due to the military nature of these operations, lack of time to prepare and plan. But, contrary to popular belief, accidents are less likely to happen during these types of diving operations compared to recreational diving, as pointed out by the Poland Military Institute of Medicine [22]. AUVs can support the diver towards search-and-rescue missions via acoustic or visual technology, and safely dispose of found hazardous material.

A. Proposed Cooperation Architecture

Given this discussion, we propose a general-purpose architecture, presented in Fig. 1, promoting some cooperation strategies mentioned above and serving as a basis of development and motivation for the control system derived afterwards. Under this architecture, we then desire the AUV to: (R1) follow and accompany the diver during diving operations, maintaining a safe distance; (R2) Position itself, in any given desired location according to mission requirements; (R3) Orient itself independently, depending on position, according to mission requirements.
motion quantities, i.e. position and velocity, defined as
\[ x_{\Phi} \] where \( k, m, n \) designate the measurement matrix that yields the measurement \( z \in \mathbb{R}^n \) taken at time \( m \), with \( m, n \) real valued matrices, and \( H \in \mathbb{M}(m, n) \) denotes the measurement matrix that yields the measurement \( z \in \mathbb{R}^m \) taken at time \( k \) with the target at a given state \( x_k \), where \( k, m, n \in \mathbb{N} \). Our state vector assumes the target linear motion quantities, i.e. position and velocity, defined as \( x = [p^T, v^T]^T \in \mathbb{R}^6 \), where \( p_k \in \mathbb{R}^3 \) and \( v_k \in \mathbb{R}^3 \) denote the target’s inertial position and body velocity. Relative to the structures of the transition matrix \( \Phi \), tracking Kalman based filter structures used for underwater target tracking is usually similar, with some having slight differences [3]. There are three frequently used target motion models: the constant velocity (CV) model, the constant acceleration (CA) model and the turning model. The most popular and common approach [23] is to consider the target dynamics with having a constant velocity (CV) between each sampling time. Empirically, this is compatible with the movement that a diver performs underwater whose movements are smooth, with almost non-existent acceleration, and do not change direction rapidly. With this, assuming a constant velocity (CV) target model the transition matrix structure takes the following form

\[
\Phi = \begin{bmatrix}
1 & 0 & 0 & T_s & 0 & 0 \\
0 & 1 & 0 & 0 & T_s & 0 \\
0 & 0 & 1 & 0 & 0 & T_s \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},
\]

where \( T_s \in \mathbb{R} \) denotes the sampling time interval.

The structure of the measurement model \( H \), since we are dealing with a second-order system, we consider a measurement model structure regarding position-only (POM). We do not discuss a position-velocity-measurement (PVM) approach as it is not common for underwater sensors to measure a target velocity. It is fair to assume that we only have measures of a target position, therefore our measurement matrix takes the following structure

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}.
\]

**B. A Kalman Filter Solution**

The Kalman filter algorithm based on these models recursively estimates the state vectors in a mean-square sense via the Kalman filter equations of prediction and estimation, respectively given by

\[
\hat{x}_k = \Phi \hat{x}_{k-1},
\]

\[
\hat{x}_k = \hat{x}_k + K_k (z_k - H \hat{x}_k),
\]

where the parameter \( K_k \) denotes the Kalman gain that minimizes the steady-state expected value of the error covariance, given by \( \lim_{n \to \infty} E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \), following the standard Kalman practice of solving the Ricatti equation.

**C. Filter Parameter Design**

The next step is to design the parameters of the filter that relate to the noise effect on the system state and sensor measurements. It is not known a general approach to this problem [24] but in conventional tracking systems, a common model of process noise design is the random acceleration (RA) model [24], with the covariance matrix taking the following structure
The baseline can either be beacons (transducers) are fixed in some known location and of triangulation and clock synchronization. These acoustic localization of targets is the use of fixed beacons via principles of their position.

Having defined the solution towards the tracking problem, we now move towards solving the challenging problem of localizing the diver via measurements of their position.

**LBL/SBL Systems**: A customary approach towards the localization of targets is the use of fixed beacons via principles of triangulation and clock synchronization. These acoustic beacons (transducers) are fixed in some known location and can either be long baseline (LBL) and short baseline (SBL) acoustic positioning systems. The difference lies in distances (baselines) between these beacons, the former usually fixated in the seabed or buoys separated by long distances between each transducer, and the latter on a vessel or another platform of choice, with only meters separating each transducer. LBL systems are well known and robust solutions towards locating a target providing an alternative solution to GNSS, or inertial based navigation systems. However, they can be difficult to implement due to the need for physical beacons with the known position being built on-site - sometimes unfeasible for divers to fixate such beacons. On the other hand, SBL systems surge as a mobile alternative, being easier to deploy and operate, favoured for research operations centred around a host vessel. The accuracy of the estimate is largely dependent on the length of the baseline and may require the agents to operate in the close vicinity of the ship or platform of choice, translating in a lesser range of operation compared to LBL systems. However, both LBL and SBL suffer the problem of complex infrastructure and require extra agents at play to estimate the target position.

**Visual Based Localization**: Another strategy used towards localization of targets is via the use of optical sensors or acoustic imaging sensors. These can provide additional information about the surrounding environment that are not accessible using traditional acoustic sensors. This field is relatively new in the Marine Robotics literature due to the recent increase in onboard computing power and advances in image processing techniques. Common approaches using optical sensors are based on detecting a target based on visual data, processing the data and proceeding to maintain a visual reference. These systems do not scale well with distance, can have a limited field of views (depending on the optical sensor) and in poor-visibility conditions, the performance is drastically reduced. Acoustic imaging sensors provide a robust alternative as acoustic propagation is more suited in marine environments. Standard procedure passes through obtaining an image from an acoustic sensor and implementing image processing or computer vision techniques towards interpreting the data, obtaining a location of the diver in the visual frame.
and then tracking it [29]. But these acoustic sensors can be more complex, costly and power-hungry compared to optical sensors, do not possess a high resolution and can suffer from scattering issues due to multi-path propagation of acoustic waves in underwater scenarios [18].

**Single Range Measurements:** The concept of single range localization, roughly speaking, considers an agent that has access to single range measurements to a target, from a set of onboard sensors and aims to estimate the position of the target. This strategy has been the subject of recent interest as a cost-efficient solution with relatively easy implementation, with current available underwater acoustics technology. The principle behinds this concept, as shown in Fig. 3, considers that, via certain excitation conditions (e.g., agent motion), it is possible to design an observable filter that estimates the position of the target, satisfying certain stability criteria for the estimation error dynamics [6]. These solutions can also be extended to multiple agents, relaxing the necessary constraints to observe the target, allowing for different motions to be performed by different agents (e.g., straight lines, cycloid-type, etc) [30]. However, these suffer from a huge drawback of constraining the vehicle motion to be specific otherwise the target becomes "unobservable".

![Fig. 3](image)

**V. AUV Nonlinear Controller Design**

This section proposes a nonlinear control law under the proposed architecture to regulate the motion of an AUV to follow a prescribed path (R1), position itself at any point in this path (R2) and stabilize its attitude at an arbitrary reference position (R3), in the presence of a constant unknown ocean current disturbance. The kinematic and dynamic model of an underwater vehicle moving in a three-dimensional space is first derived and then proceed to derive the control law at a kinematic and dynamic level. It is assumed that the AUV inertial position \( p \in \mathbb{R}^3 \), attitude \( R_0 \in SO(3) \) and velocity \( \nu = (v, \omega)^T \in \mathbb{R}^6 \) is known, where \( v \in \mathbb{R}^3 \) and \( \omega \in \mathbb{R}^3 \) denotes the AUV linear and angular velocity, respectively, provided by a navigation system. It is also assumed that the diver inertial position \( p_t \in \mathbb{R}^3 \) and linear velocity \( v_t \in \mathbb{R}^3 \) is known, provided by a target tracking filter. The diver attitude \( R_t \in SO(3) \) and angular velocity \( \omega_t \in \mathbb{R}^3 \) is provided by a guidance system, as these components are not measured but instead defined depending on the mission requirements.

**A. Vehicle Modeling**

Following standard practise and notation in marine craft literature [31], the general kinematic and dynamic equations of motion of the vehicle in three dimensions can be developed using an inertial coordinate frame \( \{N\} \) and a body-fixed coordinate frame \( \{B\} \) attached to the centre of mass of the AUV. Considering an irrotational ocean current with inertial velocity \( V_c = [V_c, 0]^T \in \mathbb{R}^6 \) the 6 DOF kinematic equations of motion of an underwater vehicle for linear and angular motion can be expressed as

\[
\dot{p} = R_0 v_r + V_{c},
\]

(9a)

\[
\dot{R}_0 = S(\omega)R_0,
\]

(9b)

where \( v_r = v - V_{c} \) are the body axis components of the vehicle’s linear velocity with respect to the water.

Considering that an underwater vehicle motion through water is subject to external forces and possesses actuation capabilities, the 6 DOF equations of motion take the vector form

\[
M \nu_r + C(\nu_r) \nu_r + D(\nu_r) \nu_r = \tau,
\]

(10)

where \( \nu_r = [v_r, \omega]^T \) represents the vehicle linear and angular velocity w.r.t to the water, \( M \in \mathbb{M}(6,6) \) denotes the matrix of rigid-body inertia and added mass terms, \( C(\nu_r) \in \mathbb{M}(6,6) \) is the matrix of rigid-body and added mass Coriolis and centripetal terms, \( D(\nu_r) \in \mathbb{M}(6,6) \) represents the matrix of linear and nonlinear hydrodynamic damping terms, and \( \tau \in \mathbb{R}^6 \) is a vector of forces and moments generated from the vehicle actuators.

**B. Position and Attitude Control**

Central to the inner-outer loop control design, the outer loop acts on a kinematic level of the vehicle motion. Considering that our vehicle is fully-actuated, at a kinematic level the control objective consists of recruiting the linear and angular relative velocities \( v_r \) and \( \omega_r \) respectively, to solve a two-fold problem: 1) regulate the vehicle position to follow a path and meet the desired position in this path; 2) regulate the vehicle attitude to achieve an arbitrary reference. At this stage, it is assumed that the ocean current linear velocity \( V_{c} \) is known.

For the former problem of controlling the vehicle position, illustrated in Fig. 4, we consider an a priori specified path expressed to a moving target, that is a parametric closed \( C^2 \) continuous curve. This path is parameterized by continuous variable \( \gamma \in \mathbb{R} \), whose derivative \( \dot{\gamma} \) is an extra input control parameter. Let the virtual point \( p_d(\gamma) \in \mathbb{R}^3 \) denote the position of the virtual reference point for the vehicle to follow, expressed in a target frame \( \{T\} \) whose origin is attached to the target position \( p_t \). To obtain its inertial position and velocity, we can express its virtual position for a given \( \gamma \), as follows

\[
p_d(\gamma) = p_t + R_t p_d(\gamma),
\]

(11)

\[
\dot{p}_d(\gamma) = v_t + R_t \left( \frac{\partial p_d(\gamma)}{\partial \gamma} \dot{\gamma} + S(\omega_t)p_d(\gamma) \right).
\]

(12)

With this, we can formulate the following moving path-following problem, as follows:
Problem 1. Consider an AUV with kinematics equation given by (9a) and (9b). Let \( p_d^i(\gamma) \) be sufficiently smooth and its derivatives with respect to \( \gamma \) are bounded. Derive a control law for \( v_r \) and \( \gamma \) such that i) the position of the vehicle \( p \) converges to \( p_d \), i.e., the positioning error \( p - p_d \) has a globally asymptotically stable (GAS) equilibrium point at the origin and ii) the parameterization variable \( \gamma \) converges to \( \gamma_d \), i.e., the virtual particle position error \( \gamma - \gamma_d \) has a globally asymptotically stable (GAS) equilibrium point at the origin.

Proposition 1. The following control law is proposed to solve Problem 1, where our vehicle is represented in blue, and an arbitrary target is represented in red, e.g., a diver, other vehicle, etc. (adapted from [10]).

The error associated with the vehicle and the desired position is re-defined in the body-fixed frame as follows, with respective dynamics

\[
e_p = R_v^T (p - p_d),
\]

\[
\dot{e}_p = -S(\omega_r)e_p + v_r + R_v^T \left( V_{c_v} - v_t - R_t \left( \frac{\partial p_d^i(\gamma)}{\partial \gamma} \gamma + S(\omega_t)p_d^i(\gamma) \right) \right).
\]

The path position error and respective dynamics are, respectively, given by

\[
\check{\gamma} = \gamma - \gamma_d,
\]

\[
\check{\tilde{\gamma}} = \tilde{\gamma} - \gamma_d.
\]

The following control law is proposed to solve Problem 1, as follows:

We define a saturation function according to the following definition:

**Proposition 1.** Consider the system described in (9a) in closed-loop with the control laws

\[
v_d = -\lambda_p \sigma \left( k_p \frac{e_p}{\lambda_p} \right) - R_v^T \left( V_{c_v} - v_t - R_t \left( \frac{\partial p_d^i(\gamma)}{\partial \gamma} \gamma + S(\omega_t)p_d^i(\gamma) \right) \right),
\]

\[
\dot{\gamma} = \gamma_d - k_\gamma \left( \rho_\gamma \sigma(k_\gamma \tilde{\gamma}) - e_p^T R_v^T R_t \frac{\partial p_d^i(\gamma)}{\partial \gamma} \right),
\]

where \( \sigma \) represents a saturation function [32, Appendix C.1], and it is assumed that \( v_d = v_r \) at a kinematic level. Additionally, \( \rho_\gamma \) and \( k_\gamma \) are positive parameter gains, and \( k_\gamma \) is a positive controller gain. Then, the origin of positioning error system \( e_p = 0 \) and path parameterization position error system \( \tilde{\gamma} = 0 \) are GAS.

**Proof.** Define the following Lyapunov candidate

\[
V_p(\tilde{\gamma}, e_p) = \frac{1}{2} e_p^T e_p + \frac{\rho_\gamma}{k_\gamma} \int_0^\gamma \sigma(k_\gamma s) ds > 0,
\]

for every \( e_p \in \mathbb{R}^3 \setminus \{0\} \) and \( \tilde{\gamma} \in \mathbb{R} \setminus \{0\} \). The time derivative of (18) and using the error dynamics (14) and (15b) is given by

\[
\dot{V}_p = e_p^T \left( v_r + R_v^T \left( V_{c_v} - v_t - R_t \left( \frac{\partial p_d^i(\gamma)}{\partial \gamma} \gamma + \tilde{\gamma} + S(\omega_t)p_d^i(\gamma) \right) \right) - \tilde{\gamma} \left( \rho_\gamma \sigma(k_\gamma \tilde{\gamma}) - e_p^T R_v^T R_t \frac{\partial p_d^i(\gamma)}{\partial \gamma} \right) \right).
\]

Substituting the control law (16)-(17) in (19) yields

\[
\dot{V}_p = -e_p^T \lambda_p \sigma \left( k_p \frac{e_p}{\lambda_p} \right) - k_\gamma \left( \rho_\gamma \sigma(k_\gamma \tilde{\gamma}) - e_p^T R_v^T R_t \frac{\partial p_d^i(\gamma)}{\partial \gamma} \right)^2,
\]

making \( \dot{V}_p \) negative definite. Since \( V_p(0,0) = 0 \) and \( V_p(\tilde{\gamma}, e_p) \to \infty \) when \( \|e_p\|, \|\tilde{\gamma}\| \to \infty \) is GAS.

Having solved the former position control problem, we now focus on the latter problem of regulating the vehicle attitude to meet an arbitrary reference. In broad terms, the problem at hand consists of controlling the vehicle body frame \( \{B\} \) with an associated rotation \( R_a \) and ensuring convergence to the desired frame \( \{D\} \) with respective rotation matrix \( R_d \), where both frame with respect to the inertial frame \( \{N\} \).

Additionally, suppose there exist a matrix \( Q \) such that it satisfies the following assumption:

**Assumption 1.** The matrix \( Q \in M(3, m) \) with \( m > 0 \) is such that the singular values are all distinct.

As pointed out by the author [33], this can be interpreted as an observability condition. Then, we can formulate the following attitude set-point regulation problem, as follows:

**Problem 2.** Consider an AUV with rotational kinematics equation given by 9b. Let \( R_d(\Theta_d) \) be a target rotation matrix parameterized by a fixed reference, defined a priori, via \( \Theta_d = [\phi_d, \theta_d, \psi_d]^T \). Additionally suppose that there exists a matrix \( Q \) that satisfies Assumption 1. Derive a feedback control for \( \omega_r \) such that the body frame rotation matrix \( R_v \) converges to \( R_d \), i.e., the rotational error \( R_v^3 R_d^3 \) has an almost globally asymptotically stable (AGAS) equilibrium point at \( I_3 \), where \( I_3 \) is the identity matrix in three dimensions.
For this purpose, define the error rotation matrix, with the respective dynamics, as follows
\[
\begin{align*}
    R_e &= R_o^\tau R_d, \quad (21) \\
    \dot{R}_e &= -S(\omega_r)R_e. \quad (22)
\end{align*}
\]

Following the strategy of the authors in [11], it is convenient to express the error as a function on $SO(3)$, as follows
\[
\varepsilon_\Theta(R_e) = \text{Tr} \left( (I_3 - R_e) QQ^T \right), \quad (23)
\]
where $\text{Tr}(\cdot)$ denotes the trace of a square matrix, defined as the sum of all elements in the matrix diagonal. If the assumption made regarding matrix $Q$ holds, the error function $\varepsilon_\Theta$ is a positive definite Morse function i.e., it is a function with nondegenerate isolated critical points [34] with a global minimum at $R_e = I_3$, a maximum and two saddle points. Computing the time derivative, we have
\[
\dot{\varepsilon}_\Theta(R_e) = -S^{-1}(R_e QQ^T - QQ^T R_e^T)\omega_r, \quad (24)
\]
where $S^{-1} : SO(3) \to \mathbb{R}^3$ corresponds to the inverse mapping of the cross-product operator.

With this, the following control law is proposed to solve Problem 2, as follows:

**Proposition 2.** Consider the system given by (9b) and the following control law in closed-loop
\[
\omega_d = K_s S^{-1}(R_e QQ^T - QQ^T R_e^T),
\]
where $K_s$ is a positive definite gain matrix, and it is assumed that $\omega_d = \omega_r$ at a kinematic level. Then, the equilibrium point of the error rotation matrix $R_e = I_3$ is AGAS. Moreover, there exists a neighborhood of $R_e = I_3$, such that all solutions starting inside it converge exponentially fast to $R_e = I_3$.

The proof will follow a similar approach as done by the authors in [11], disregarding the vehicle translation kinematics and dynamics. Define a Lyapunov candidate for the vehicle error rotation matrix, as follows
\[
V_\Theta(R_e) = \varepsilon_\Theta(R_e) = \text{Tr} \left( (I_3 - R_e) QQ^T \right). \quad (26)
\]

Taking the time derivative of (26) and using the attitude error dynamics (24) in closed-loop with the control law (25), we have
\[
\dot{V}_\Theta = -S^{-1}(R_e QQ^T - QQ^T R_e^T)K_s S^{-1}(R_e QQ^T - QQ^T R_e^T). \quad (27)
\]

It follows immediately that $\dot{V}_\Theta$ is negative semi-definite. We have that the largest invariant set $\mathcal{M} = \{R_e \in SO(3) \mid V_\Theta = 0\}$ is the set of points $R_e$ that are critical points of $V_\Theta$, which are the global minimum $R_e = I_3$, a maximum and two saddle points. Applying LaSalle’s invariance principle, we conclude that the closed-loop trajectories of the error system (24) converge to $\mathcal{M}$ as $t \to \infty$. Additionally, the authors in [11] prove that except for the point $R_e = I_3$ all equilibrium points $R_e = R_e \in \mathcal{M}$ have an unstable manifold. In loose terms, this prevent trajectories remaining in this manifold, allowing them to asymptotically converge to the desired equilibrium. However, it may happen that the convergence to the equilibrium can be affected near this thin set [11]. With this, the equilibrium point $R_e = I_3$ is proven to be AGAS.

C. AUV Dynamics Control

Now, having defined the position and attitude control laws for the outer loop controller, we proceed to derive an inner loop whose task is to meet the desired velocity assignments by inverting the plant dynamics, in this case, our AUV nonlinear dynamics. With this, the following control problem is formulated:

**Problem 3.** Consider the 6 DOF dynamical model of the vehicle given by (10). Let $\nu_d = [\nu_d, \omega_d]^T \in \mathbb{R}^6$ be a desired speed requirement from the outer loop, and suppose that $\nu_d$ is sufficiently smooth and its derivative is bounded. Derive a feedback control law $\tau$ such that the relative velocity error $\nu_r - \nu_d$ has a globally exponentially stable (GES) equilibrium point at the origin.

For this purpose, we first define the error $e_d = \nu_r - \nu_d \in \mathbb{R}^3$. With this, we can rewrite the AUV equations of motion in error form, as follows
\[
M \dot{e}_d = \tau - M \dot{\nu}_d - C(\nu_r) \nu_r - D(\nu_r) \nu_d - D(\nu_r) e_d. \quad (28)
\]

Then, the following inner loop control law is proposed:

**Proposition 3.** Consider the system described by (28) and the following control law in closed-loop
\[
\tau = -K_d e_d + M \dot{\nu}_d + D(\nu_r) \nu_d + C(\nu_r) \nu_r, \quad (29)
\]
where $K_d$ is a positive definite gain matrix. Then, the origin of the velocity error system $e_d = 0$ has a GES equilibrium point.

**Proof.** Consider the following Lyapunov candidate
\[
V_d(e_r) = \frac{1}{2} \varepsilon_r^T M \varepsilon_r. \quad (30)
\]

Taking the time derivative of $V_d(e_d)$ and using the error dynamics (28) coupled with the control law (29), we have that
\[
\dot{V}_d = e_d^T M \dot{e}_d = -e_d^T (K_d + D(\nu_r)) e_d. \quad (31)
\]

Knowing that $D(\nu_r)$ is a positive-definite matrix of damping forces, the following inequality is satisfied
\[
\dot{V}_d \leq -\lambda_{\text{min}}(K_d + D(\nu_r)) \|e_d\|^2, \quad \forall e_d \in \mathbb{R}^3,
\]
where $A = K_d + D(\nu_r)$ and $\lambda_{\text{min}}(A)$ denote the smallest eigenvalue of matrix $A$. Since $\frac{1}{2} \lambda_{\text{min}}(M) \|e_d\|^2 \leq V_d(e_d) \leq \frac{1}{2} \lambda_{\text{max}}(M) \|e_d\|^2$, $\forall e_d \in \mathbb{R}^3$, we have that $e_d = 0$ is GES.
in closed-loop with the following control law

\[
\tau = -K_d e_d + D(\nu_r)\nu_d + C(\nu_r)\nu_r,
\]

\[
\omega_d = K_e S^{-1}(R_e Q Q^T - Q Q^T R_e^T),
\]

where \(K_d\) and \(K_p\) are positive definite gain matrices, and \(k_p\), \(\rho_1\), \(\rho_2\) and \(\lambda_p\) are positive parameter gains. Let \(\frac{\partial p^d(\gamma)}{\partial \gamma}\) and \(V_{v_t}\) be bounded signals. Then, there are sufficiently large gains \(K_d\) such that the closed-loop system is finite-gain \(\mathcal{L}\) stable, with restriction on \(e_v(0)\) and \(v_t(0)\) initial conditions.

\[
\nu_d = -\lambda_p \sigma \left(\frac{k_p}{\lambda_p} e_p\right) - R_v^T \left(\dot{V}_{v_t} - \dot{v}_t - R_s \left(\frac{\partial p^d(\gamma)}{\partial \gamma} \gamma_d + S(\omega_t) p^d(\gamma)\right)\right),
\]

\[
\gamma = \gamma_d - k_r + V_{v_t} - k_r \left(\rho_2 \sigma (k_r \gamma) - e_p^T R_v R_v^T \frac{\partial p^d(\gamma)}{\partial \gamma}\right),
\]

where \(K_d\) and \(K_p\) are positive definite gain matrices, and \(k_p\), \(\rho_1\), \(\rho_2\) and \(\lambda_p\) are positive parameter gains. Let \(\frac{\partial p^d(\gamma)}{\partial \gamma}\) and \(V_{v_t}\) be bounded signals. Then, there are sufficiently large gains \(K_d\) such that the closed-loop system is finite-gain \(\mathcal{L}\) stable, with restriction on \(e_v(0)\) and \(v_t(0)\) initial conditions.

\[
\nu_d = -\lambda_p \sigma \left(\frac{k_p}{\lambda_p} e_p\right) - R_v^T \left(\dot{V}_{v_t} - \dot{v}_t - R_s \left(\frac{\partial p^d(\gamma)}{\partial \gamma} \gamma_d + S(\omega_t) p^d(\gamma)\right)\right),
\]

\[
\gamma = \gamma_d - k_r + V_{v_t} - k_r \left(\rho_2 \sigma (k_r \gamma) - e_p^T R_v R_v^T \frac{\partial p^d(\gamma)}{\partial \gamma}\right),
\]

where \(K_d\) and \(K_p\) are positive definite gain matrices, and \(k_p\), \(\rho_1\), \(\rho_2\) and \(\lambda_p\) are positive parameter gains. Let \(\frac{\partial p^d(\gamma)}{\partial \gamma}\) and \(V_{v_t}\) be bounded signals. Then, there are sufficiently large gains \(K_d\) such that the closed-loop system is finite-gain \(\mathcal{L}\) stable, with restriction on \(e_v(0)\) and \(v_t(0)\) initial conditions.
that are evidenced in the range plot, in the upper-left graph, are due to these model parameter uncertainty and noisy data, which translates into oscillations on the desired position for the AUV to follow, as expected. The attitude controller is also able to reduce the relative bearing between the AUV attitude and mission site. Moreover, due to the model uncertainty, our ocean current estimation error norm $\|e_2\|$ is bounded and converges to a neighbourhood of the origin.

VII. Conclusions

This thesis addressed several modelling, navigation and control problems to promote cooperation synergies between a diver and an AUV. We highlighted the importance of establishing crucial mechanisms for AUV to observe and locate the diver. We presented classical and novel solutions to solve this problem of diver localization and discussed their different implementations and propose a cooperation architecture, highlighting the different systems that promote a specific cooperation strategy, and the requirements for the design of a control system. This controller was derived based on Lyapunov techniques on non-linear systems, translated into an inner-outer loop design problem of providing the desired velocities for the vehicle to achieve, to stabilize its kinematic components of attitude and position independently. Regarding the position control, a moving path-following controller was proposed to solve the problem of stabilizing the vehicle position along a desired time-independent path, with the equilibrium points satisfying GAS. The attitude control towards stabilizing at an arbitrary desired attitude was formulated as a set-point regulation problem, where the control design regarded the attitude error dynamics as a system evolving on $SO(3)$, with equilibria satisfying AGAS. The inner-outer loop interconnection was also analyzed and given arbitrary initial conditions, the system is always stable for a certain appropriate choice of gains. Simulation results were presented in the presence of realistic measurement noise of the diver position, unknown ocean currents and parametric uncertainty, illustrating the performance and robustness achieved with the proposed controller under the presented cooperation architecture. Future work will extend the derived controller to be more robust against parametric uncertainty in an adaptive approach such as [8] or take into account under-actuated vehicles [41], [42], develop different controller solutions to the various cooperation strategies highlighted in this work, expand the diver tracking filter taking into account velocity measurements in addition (PVM filters), a more thorough stability analysis for systems evolving in rotation manifolds by considering tangent spaces in these groups [43].

References

Fig. 7. Evolution in time of the different estimation error variables and position/attitude errors.


