Scalar Mixing in New Physics Models

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Abstract

It is not necessary that the scalar sector of the Standard Model consists of only one doublet. Theoretical physicists try to understand what are the consequences of adding more multiplets to that sector, and whether such extensions would be in agreement with the experimental data. In this work, we study an extension of the Standard Model including scalar singlets, doublets, and triplets. For this class of models, we derive prescriptions to obtain finite results for the oblique parameters S and U, we compute the one-loop corrections to the $Zb\bar{b}$ vertex, we apply these results to the Georgi–Machacek Model, and we compare the results of that model to the experimental data.

Keywords: Scalar triplets, New Physics, Oblique parameters, $Zb\bar{b}$ vertex, Georgi–Machacek Model

1 Introduction

Particle Physics studies the most basic constituents of Nature and their interactions. Our knowledge of this field is encapsulated in the Standard Model (SM) [1–3]. This is an $SU(3)\times SU(2)\times U(1)$ gauge theory that describes all the fundamental particles observed until now (it has, indeed, predicted the existence of some of them before they were experimentally observed) and the way they interact with each other. The SM is one of the most accurate theories in science.

However, there are phenomena that the SM cannot explain. Since the SM is a very accurate theory, particle physicists do not want to replace it by a completely different theory. Rather, they try to complete it by adding to it features that might help explain some of the phenomena that it cannot adequately encompass.

One of the ways to enlarge the SM is by extending its scalar sector, which, originally, contains only one $SU(2)$ doublet. The most well-studied extension is the addition of another $SU(2)$ scalar doublet, obtaining a two-Higgs-doublet model (2HDM).[1] Extensions of the SM with scalar $SU(2)$ singlets are also frequent.

In this work, we consider extensions of the SM including arbitrary numbers of $SU(2)$ scalar singlets, doublets, and triplets. We develop a formalism to address scalar mixing in such models. Using that formalism, we compute both the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex in this class of models. We then apply our results to the special case of the Georgi–Machacek (GM) model, which is a model with $SU(2)$ scalar triplets.

This paper is organized as follows. In section 2 we present the formalism for scalar mixing. In section 3 we discuss the oblique parameters; we explain how we have proceeded to obtain a finite result for the parameters S and U. In section 4 we compute the one-loop corrections to the $Zb\bar{b}$ vertex. In section 5 we make a short description of the GM model and we apply our results from sections 3 and 4 to fit this model to the experimental data.

2 Scalar Mixing Formalism

Consider a $SU(2)\times U(1)$ electroweak model in which the scalar sector includes

- n_d $SU(2)$ doublets with hypercharge $Y=\frac{1}{2}$,

$$\phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix}, \quad k=1,...,n_d; \tag{1}$$

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[1]A review of this type of models can be found in ref. [4]

- n_{t_1} $SU(2)$ triplets with hypercharge $Y=1$,

$$\Xi_p = \begin{pmatrix} \xi_p^{++} \\ \xi_p^{+} \\ \xi_p^{0} \end{pmatrix}, \quad p = 1, ..., n_{t_1}; \tag{2}$$

- n_{t_0} $SU(2)$ real triplets with hypercharge $Y=0$,

$$\Lambda_q = \begin{pmatrix} \lambda_q^{+} \\ \lambda_q^{0} \\ -\lambda_q^{-} \end{pmatrix}, \quad q = 1, ..., n_{t_0}, \tag{3}$$

  where λ_0 is a real scalar field;

- n_{s_1} complex $SU(2)$ singlets with hypercharge $Y=1$,

$$\chi_j^{+}, \quad j = 1, ..., n_{s_1}; \tag{4}$$

- n_{s_0} real $SU(2)$ singlets with hypercharge $Y=0$,

$$\chi_l^{0}, \quad l = 1, ..., n_{s_0}; \tag{5}$$

- n_{s_2} complex $SU(2)$ singlets with hypercharge $Y=2$,

$$\chi_r^{++}, \quad r = 1, ..., n_{s_2}. \tag{6}$$

We have then a total of $n_1 = n_d + n_{t_1} + n_{t_0} + n_{s_1}$ complex scalar fields with electric charge $+1$, $n_0 = 2n_d + 2n_{t_1} + n_{t_0} + n_{s_0}$ real scalar fields with electric charge 0 and $n_2 = n_{t_1} + n_{s_2}$ complex scalar fields with electric charge $+2$.

The neutral fields are allowed to have non-zero vacuum expectation values (VEVs), such that

$$\langle 0|\varphi_k^0|0\rangle = \frac{v_k}{\sqrt{2}}, \qquad \langle 0|\xi_p^0|0\rangle = \frac{w_p}{\sqrt{2}}, \tag{7a}$$

$$\langle 0|\lambda_q^0|0\rangle = x_q, \qquad \langle 0|\chi_l^0|0\rangle = u_l, \tag{7b}$$

where the VEVs v_k and w_p are in general complex and the VEVs x_q and u_l are real. We can then expand the neutral fields around their VEVs as

$$\varphi_k^0 = \frac{1}{\sqrt{2}}(v_k + \varphi_k^{0\prime}), \qquad \xi_p^0 = \frac{1}{\sqrt{2}}(w_p + \xi_p^{0\prime}), \tag{8a}$$

$$\lambda_q^0 = x_q + \lambda_q^{0\prime}, \qquad \chi_l^0 = u_l + \chi_l^{0\prime}. \tag{8b}$$

In this class of models, the masses of the $W^\pm$ and Z bosons are given in terms of the VEVs of the scalar fields as

$$m_Z^2 = \frac{g^2}{c_W^2}\Big(\frac{1}{4}v^2 + w^2\Big), \quad m_W^2 = g^2\Big(\frac{1}{4}v^2 + \frac{1}{2}w^2 + x^2\Big), \tag{9}$$

where c_W is the cosine of the Weinberg angle θ_W and we defined $v = \sqrt{\sum_{k=1}^{n_d}|v_k|^2}$, $w = \sqrt{\sum_{p=1}^{n_{t_1}}|w_p|^2}$ and $x = \sqrt{\sum_{q=1}^{n_{t_0}} x_q^2}$. We note that the relation $m_W = m_Z \cos\theta_W$ is, in general, no longer verified due to the introduction of triplets in the model.

Calling S_c^{++} $(c = 1, ..., n_2)$, S_a^{+} $(a = 1, ..., n_1)$ and S_b^{0} $(b = 1, ..., n_0)$ to the fields with electric charges $+2$, $+1$ and 0, respectively, that are eigenstates of the mass matrices, we can then write

$$\varphi_k^{+} = \sum_{a=1}^{n_1}(U_1)_{ka}S_a^{+}, \qquad \chi_j^{+} = \sum_{a=1}^{n_1}(U_2)_{ja}S_a^{+}, \tag{10a}$$

$$\lambda_q^{+} = \sum_{a=1}^{n_1}(U_3)_{qa}S_a^{+}, \qquad \xi_p^{+} = \sum_{a=1}^{n_1}(U_4)_{pa}S_a^{+}, \tag{10b}$$

$$\varphi_k^{0\prime} = \sum_{b=1}^{n_0}(V_1)_{kb}S_b^{0}, \qquad \xi_p^{0\prime} = \sum_{b=1}^{n_0}(V_2)_{pb}S_b^{0}, \tag{10c}$$

$$\lambda_q^{0\prime} = \sum_{b=1}^{n_0}(R_1)_{qb}S_b^{0}, \qquad \chi_l^{0\prime} = \sum_{b=1}^{n_0}(R_2)_{lb}S_b^{0}, \tag{10d}$$

$$\xi_p^{++} = \sum_{c=1}^{n_2}(T_1)_{pc}S_c^{++}, \quad \chi_r^{++} = \sum_{c=1}^{n_2}(T_2)_{rc}S_c^{++}, \tag{10e}$$

where the matrices U_1, U_2, U_3, U_4, V_1, V_2, R_1, R_2, T_1 and T_2 have dimensions $n_d \times n_1$, $n_{s_1} \times n_1$, $n_{t_0} \times n_1$, $n_{t_1} \times n_1$, $n_d \times n_0$, $n_{t_1} \times n_0$, $n_{t_0} \times n_0$, $n_n \times n_0$, $n_{t_1} \times n_2$ and $n_{s_2} \times n_2$, respectively. The neutral fields S_b^0 are reals fields, which means that the matrices R_1 and R_2 are real, while the others are complex.

Like in the SM, in this theory we will have three Goldstone bosons, $G^\pm$ and G^0, that we will identify with $S_1^\pm$ and S_1^0, such that $S_1^\pm \equiv G^\pm$ and $S_1^0 \equiv G^0$. We will denote the masses of the scalars $S_a^\pm$ by m_a, the masses of the scalars S_b^0 by μ_b and the masses of the scalars S_c^{++} by M_c.

Applying the operators T_3 and $T_\pm$ (which are the generators of the gauge group $SU(2)$) to the vacuum, we get the form of the columns of the mixing matrices relative to the Goldstone bosons. We can thus write

$$(V_1)_{k1} = \frac{i\, v_k}{\sqrt{v^2+4w^2}}, \quad (U_1)_{k1} = \frac{v_k}{\sqrt{v^2+2w^2+4x^2}}, \tag{11a}$$

$$(V_2)_{p1} = \frac{2\, i\, w_p}{\sqrt{v^2+4w^2}}, \quad (U_3)_{q1} = \frac{2x_q}{\sqrt{v^2+2w^2+4x^2}}, \tag{11b}$$

$$(U_4)_{p1} = \frac{\sqrt{2}\, w_p}{\sqrt{v^2+2w^2+4x^2}}. \tag{11c}$$

We get then the Feynman rules for the vertices with scalar and gauge bosons by developing the gauge-kinetic Lagrangian

$$\mathcal{L} = \sum_{k=1}^{n_d}(D^\mu\phi_k)^\dagger(D_\mu\phi_k) + \sum_{p=1}^{n_{t_1}}(D^\mu\Xi_p)^\dagger(D_\mu\Xi_p) \tag{12a}$$

$$+\frac{1}{2}\sum_{q=1}^{n_{t_0}}(D^\mu\Lambda_q)^\dagger(D_\mu\Lambda_q)+\sum_{j=1}^{n_{s_1}}(D^\mu\chi_j^+)^\dagger(D_\mu\chi_j^+) \tag{12b}$$

$$+\frac{1}{2}\sum_{l=1}^{n_{s_0}}(\partial^\mu\chi_l^0)(\partial_\mu\chi_l^0)+\sum_{r=1}^{n_{s_2}}(D^\mu\chi_r^{++})^\dagger(D_\mu\chi_r^{++}). \tag{12c}$$

3 Oblique Parameters

Let $A_{VV'}(q^2)$ be the coefficients of $g^{\mu\nu}$ in the vacuum polarization tensors $\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu}A_{VV'}(q^2) + q^\mu q^\nu B_{VV'}(q^2)$, where VV' may be either AA, AZ, ZZ or WW and q is the four-momentum of the gauge boson. We define $\delta A_{VV'}(q^2)$ as $\delta A_{VV'}(q^2) \equiv A_{VV'}(q^2)|_{NP} - A_{VV'}(q^2)|_{SM}$, being $A_{VV'}(q^2)|_{NP}$ the function $A_{VV'}(q^2)$ computed in a New Physics (NP) model and $A_{VV'}(q^2)|_{SM}$ the function $A_{VV'}(q^2)$ computed in the SM.

An analysis of the "oblique corrections" leads to the identification of six relevant observables that allow us to parametrize the effects of New Physics. Three of those observables are the oblique parameters S, T and U that were defined by Peskin and Takeuchi [5, 6] and are given by

$$T = \frac{1}{\alpha\, m_Z^2}\Big(\frac{1}{c_W^2}\delta A_{WW}(0) - \delta A_{ZZ}(0)\Big), \tag{13a}$$

$$S = \frac{4s_W^2c_W^2}{\alpha}\left(\left.\frac{\partial\,\delta A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=0} - \left.\frac{\partial\,\delta A_{AA}(q^2)}{\partial q^2}\right|_{q^2=0} + \frac{c_W^2-s_W^2}{c_W s_W}\left.\frac{\partial\,\delta A_{AZ}(q^2)}{\partial q^2}\right|_{q^2=0}\right), \tag{13b}$$

$$\begin{aligned} U = \frac{4s_W^2}{\alpha}\Bigg(&\left.\frac{\partial\,\delta A_{WW}(q^2)}{\partial q^2}\right|_{q^2=0} - c_W^2\left.\frac{\partial\,\delta A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=0} - s_W^2\left.\frac{\partial\,\delta A_{AA}(q^2)}{\partial q^2}\right|_{q^2=0} \\ &+2c_W s_W\left.\frac{\partial\,\delta A_{AZ}(q^2)}{\partial q^2}\right|_{q^2=0}\Bigg), \end{aligned} \tag{13c}$$

where α is the fine-structure constant and s_W is the sine of the Weinberg angle θ_W.

The other three observables are the oblique parameters V, W and X that were defined by Maksymyk, Burgess and London [7] and are given by

$$V = \frac{1}{\alpha}\left(\left.\frac{\partial\,\delta A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=m_Z^2} - \frac{\delta A_{ZZ}(m_Z^2)-\delta A_{ZZ}(0)}{m_Z^2}\right), \tag{14a}$$

$$W = \frac{1}{\alpha}\left(\left.\frac{\partial\,\delta A_{WW}(q^2)}{\partial q^2}\right|_{q^2=m_W^2} - \frac{\delta A_{WW}(m_W^2)-\delta A_{WW}(0)}{m_W^2}\right), \tag{14b}$$

$$X = \frac{s_W c_W}{\alpha}\left(\left.\frac{\partial\,\delta A_{AZ}(q^2)}{\partial q^2}\right|_{q^2=0} - \frac{\delta A_{AZ}(m_Z^2)-\delta A_{AZ}(0)}{m_Z^2}\right). \tag{14c}$$

To compute the oblique parameters, we started by identifying the relevant Feynman diagrams that contribute to the vacuum polarization tensors in our NP model and also in the SM. We computed then the vacuum polarization tensor both in our NP model and also in the SM and subtracted both results. This subtraction is not trivial as in the SM the masses of the W and Z gauge bosons obey the relation $m_W = m_Z c_W$ and in a general model with triplets this relation is not verified. Thus, to compute the SM vacuum polarization tensors, we did not use the usual SM Feynman rules (which can be found, for example, in [8] or in [9]). For the triple vertices with only gauge and Goldstone bosons, we used the Feynman rules obtained by requiring gauge invariance without assuming $m_W = m_Z c_W$. Namely, to obtain the Feynman rule for the vertex $ZW^\pm G^\mp$, we required gauge invariance in the process $e^- \to \nu_e\ Z\ W^-$. Knowing this Feynman rule, we can obtain the Feynman rule for vertex ZG^-G^+ by requiring gauge invariance in the process $Z \to e^-\ \bar{\nu}_e\ \mu^+\ \nu_\mu$ and the Feynman rule for the vertex $G^0W^\pm W^\mp$ by requiring gauge invariance in the process $W^- \to e^-\ \bar{\nu}_e\ \nu_\mu\ \bar{\nu_\mu}$. These Feynman rules obtained by gauge invariance can be found in figure 1. Using the usual SM Feynman rules for the other SM vertices, we got a divergent and gauge dependent result for parameter T (as expected, because T should be divergent for models that violate custodial symmetry [10,11]), we got a divergent and gauge independent result for parameters S and U and we got finite and gauge independent results for parameters V, W and X.

To get a finite result for the oblique parameters S and U, we must multiply the SM Feynman rules for the vertices ZG^0H and ZZH by $\sqrt{1-\left(\frac{c_W^2 m_Z^2}{m_W^2}-1\right)^2}$ (which is equal to one for $m_W = m_Z c_W$) and the SM Feynman rules for the vertices $W^\pm G^\mp H$ and $W^\pm W^\mp H$ by $\sqrt{4-3\frac{c_W^2 m_Z^2}{m_W^2}}$ (which is also equal to one for $m_W = m_Z c_W$). The SM Feynman rules multiplied by these factors (*i.e.*, in the form they were used to compute the oblique parameters) can be found in figures 2 and 3. Doing these multiplications we get finite results for parameters S and U without compromising their gauge independence. By multiplying the SM Feynman rules by these factors, we get a finite result for these two oblique parameters in a model with any scalar content, as can be shown by generalizing the formalism so that it allows us to work with models with scalar multiplets of any dimension and using this generalized formalism to compute the divergent diagrams that contribute to each of the oblique parameters.

Using all the diagrams that contribute to $\Pi^{\mu\nu}_{AA}$ at one-loop level, we can show that, for our class of NP models, we have $A_{AA}(q^2=0)=0$ as required by the Ward-Takahashi identities [12,13].

4 One-loop corrections to the $Zb\bar{b}$ vertex

Another way to indirectly detect heavy scalars can be through radiative corrections to the $Zb\bar{b}$ vertex. At tree-level, the left- and right-handed b-quark couplings are given by $g^0_{Lb} = s_W^2/3 - 1/2$ and $g^0_{Rb} = s_W^2/3$. These couplings will have one-loop contributions that will be different for models with different scalar content. As such, we should be able to probe for New Physics through these one-loop corrections. The two observables which are influenced by these corrections due to New Physics are the hadronic branching ratio of Z to b quarks:

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}, \tag{15}$$

and the b quark asymmetry (measured in the process $e^-e^+ \to b\bar{b}$),

$$A_b = \frac{\sigma(e^-_L \to b_F) - \sigma(e^-_L \to b_B) + \sigma(e^-_R \to b_B) - \sigma(e^-_R \to b_F)}{\sigma(e^-_L \to b_F) + \sigma(e^-_L \to b_B) + \sigma(e^-_R \to b_B) + \sigma(e^-_R \to b_F)}, \tag{16}$$

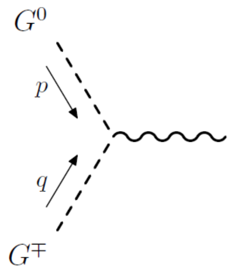
where $e^-_{L,R}$ are left and right handed initial-state electrons and $b_{F,B}$ are final-state b-quarks moving in the forward and backward directions with respect to the direction of the initial-state electrons [14].

In our calculations, we will use the approximation where the CKM matrix element $V_{tb}=1$ and we will neglect the mass of the bottom quark m_b.

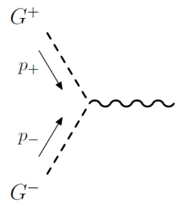
We follow the on-shell renormalization scheme from Hollik [15,16]. We are looking for terms that change the tree-level couplings, which, after renormalization may be written as

$$i\Gamma^{Zbb}_\mu = i\gamma_\mu \frac{g}{c_W}\left((g^0_{Lb}+\Delta g_L)P_L + (g^0_{Rb}+\Delta g_R)P_R\right), \tag{17}$$

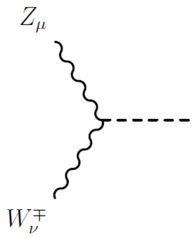
where $\Delta g_\aleph$ ($\aleph = L, R$) are the one-loop corrections to the couplings $g_{\aleph b}$ after renormalization, including the corrections that are also present in the Standard Model. Thus, we are not interested in terms proportional to p_i^μ, being p_i, with $i=1,2,3$, the momenta



$$W^{\pm}_{\mu} = \frac{g}{2}\frac{m_Z c_W}{m_W}(q-p)_{\mu}$$



$$Z_{\mu} = -i\, g\, c_W \left(1 - \frac{m_Z^2}{2m_W^2}\right)(p_- - p_+)_{\mu}$$



$$G^{\pm} = -i g m_W c_W g_{\mu\nu}\left(\frac{m_Z^2}{m_W^2} - 1\right)$$

Figure 1: Feynman rules for the triple vertices with gauge and Goldstone bosons that are obtained by requiring gauge invariance and without assuming $m_W = m_Z c_W$.

of each of the external particles in the vertex.

According to Hollik's renormalization scheme [15, 16], $\Delta g_{\aleph}$ ($\aleph = L, R$) have a contribution from, not only the one-loop diagrams of the $Zb\bar{b}$ vertex, but also from the one-loop diagrams of the b quark self-energy. Consider the part of the b quark self-energy proportional to $\not{p}$, which we may write as $\Sigma(p) = \not{p}\left(\Omega_L(p^2)P_L + \Omega_R(p^2)P_R\right)$. Then, we will have a contribution from the self-energy of the b quark to the one loop corrections to the couplings given by $\Delta g_{Lb} = -g^0_{Lb}\Omega_L(p^2 = m_b^2)$ and $\Delta g_{Rb} = -g^0_{Rb}\Omega_R(p^2 = m_b^2)$.

We can then compute the contributions to the one loop corrections to the couplings from the diagrams with charged scalars and from the diagrams with neutral scalars separately. Both of those give, separately, a finite result for models with triplets.

To obtain a gauge independent result, we should change from the couplings $g_{\aleph b}$ ($\aleph = L, R$) parametrized as $g_{\aleph b} = g^0_{\aleph b} + \Delta g_{\aleph b}$, being $g^0_{\aleph b}$ the tree-level coupling and $\Delta g_{\aleph b}$ the one-loop contribution; to a parametrization $g_{\aleph b} = g^{SM}_{\aleph b} + \delta g_{\aleph b}$, where $g^{SM}_{\aleph b}$ is the SM part and $\delta g_{\aleph b}$ is the NP part. To do that, we must subtract the SM one-loop contribution to the couplings from $\Delta g_{\aleph b}$. In the limit $m_b \to 0$, the SM results for Δg^c_{Rb}, Δg^n_{Lb} and Δg^n_{Rb} (where the superscripts c and n refer to the contributions from the diagrams with charged and neutral scalars, respectively) are equal to 0, because these SM results are proportional to m_b^2. Thus, we get $\delta g^c_{Rb} = \Delta g^c_{Rb}$, $\delta g^n_{Lb} = \Delta g^n_{Lb}$ and $\delta g^n_{Rb} = \Delta g^n_{Rb}$ and these three quantities are finite and gauge independent. In the case of g^c_{Lb}, by subtracting the SM result (computed using the Feynman rules for the SM triple vertices with gauge and Goldstone bosons obtained by requiring gauge invariance), we get a gauge independent but divergent result for δg^c_{Lb} for models in which $m_W \neq m_Z c_W$. This happens because, although the result for Δg^c_{Lb} is finite, its gauge dependent terms are divergent for $m_W \neq m_Z c_W$. Furthermore, when we use the Feynman rules required by gauge invariance to compute the SM result, we cancel the gauge dependent terms of the result in our NP model by subtracting the SM result. Thus, we are subtracting a divergent quantity from a finite one, such that we get a divergent result for δg^c_{Lb}. If, otherwise, we had computed the SM result using the usual SM Feynman rules, we would get a finite result for δg^c_{Lb} but it would be gauge dependent. This would happen because this way we would be subtracting two finite quantities but now their gauge dependent terms would be different for $m_W \neq m_Z c_W$.

5 The Georgi-Machacek Model

Having computed the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex to a general model with scalar singlets, doublets and triplets, next we apply these results to the concrete case of the

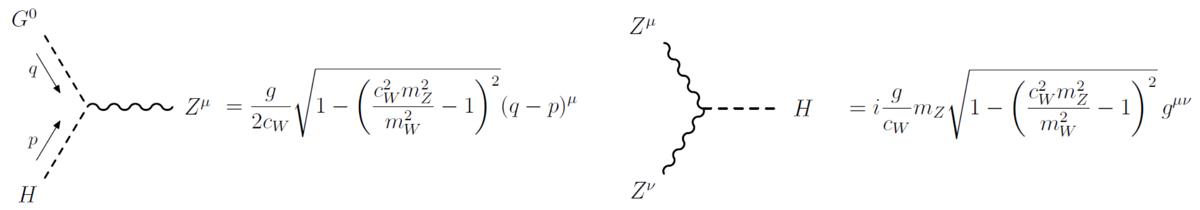


Figure 2: SM Feynman rules for the vertices ZG^0H and ZZH multiplied by $\sqrt{1-\left(\frac{c_W^2 m_Z^2}{m_W^2}-1\right)^2}$. This quantity is equal to 1 when we have $m_W = m_Z c_W$ and by performing this multiplication, we get a finite result for the oblique parameter S in models in which $m_W \neq m_Z c_W$.

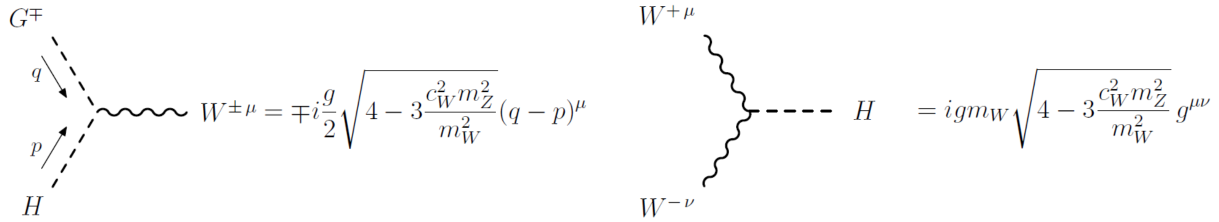


Figure 3: SM Feynman rules for the vertices $W^\pm G^\mp H$ and $W^\pm W^\mp H$ by $\sqrt{4-3\frac{c_W^2 m_Z^2}{m_W^2}}$. This quantity is equal to 1 when we have $m_W = m_Z c_W$ and by performing this multiplication, together with the multiplication performed to obtain the Feynman rules in figure 2, we get a finite result for the oblique parameter U in models in which $m_W \neq m_Z c_W$.

Georgi-Machacek model [17] with an additional $\mathbb{Z}_2$ symmetry which will eliminate the cubic terms, making the model simpler without changing significantly the physics [10].

This model contains:

- one complex doublet with hypercharge $Y = \frac{1}{2}$,

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}; \tag{18}$$

- one real triplet with hypercharge $Y = 0$,

$$\Lambda = \begin{pmatrix} \lambda^+ \\ \lambda^0 \\ -\lambda^- \end{pmatrix}; \tag{19}$$

- one complex triplet with hypercharge $Y = 1$,

$$\Xi = \begin{pmatrix} \xi^{++} \\ \xi^+ \\ \xi^0 \end{pmatrix}. \tag{20}$$

These fields can be written in the matrix form

$$\Phi = \begin{pmatrix} \varphi^{0*} & \varphi^+ \\ -\varphi^- & \varphi^0 \end{pmatrix}, \quad \Psi = \begin{pmatrix} \xi^{0*} & \lambda^+ & \xi^{++} \\ -\xi^- & \lambda^0 & \xi^+ \\ \xi^{--} & -\lambda^- & \xi^0 \end{pmatrix}. \tag{21}$$

These matrices transform under a global $SU(2)_L \times SU(2)_R$ symmetry as $\Phi \to U_{L2}\Phi U_{R2}^\dagger$ and $\Psi \to U_{L3}\Psi U_{R3}^\dagger$, where $U_{R,L\,2} = \exp\left(it^a\theta^a_{L,R}\right)$ and $U_{R,L\,3} = \exp\left(iT^a\theta^a_{L,R}\right)$, being t^a and T^a the two and three dimensional generators of $SU(2)$, respectively.

The most general potential which is invariant under the global $SU(2)_L \times SU(2)_R$ symmetry and also under a $\mathbb{Z}_2$ symmetry $\Psi \to -\Psi$ is [18]

$$\begin{aligned} V =& \frac{\alpha_2^2}{2}\operatorname{Tr}\left(\Phi^\dagger\Phi\right) + \frac{\alpha_3^2}{2}\operatorname{Tr}\left(\Psi^\dagger\Psi\right) + \beta_1\left(\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\right)^2 + \beta_2\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\operatorname{Tr}\left(\Psi^\dagger\Psi\right) \\ &+ \beta_3\operatorname{Tr}\left(\Psi^\dagger\Psi\Psi^\dagger\Psi\right) + \beta_4\left(\operatorname{Tr}\left(\Psi^\dagger\Psi\right)\right)^2 - \beta_5\operatorname{Tr}\left(\Phi^\dagger t^a\Phi t^b\right)\operatorname{Tr}\left(\Psi^\dagger T^a\Psi T^b\right), \end{aligned} \tag{22}$$

where α_i for $i \in \{2,3\}$ and β_j for $j \in \{1,...,5\}$ are all real parameters because each trace term is also real.

This potential admits a vacuum structure such that $\langle 0|\varphi^0|0\rangle = a/\sqrt{2}$, $\langle 0|\lambda^0|0\rangle = b$ and $\langle 0|\xi^0|0\rangle = b$, where $a, b \in \mathbb{R}$ and are related to the parameters couplings of the model by

$$\frac{\alpha_2^2}{2} + 2a^2\beta_1 + 3b^2\beta_2 - \frac{3}{2}b^2\beta_5 = 0, \tag{23a}$$

$$\alpha_3^2 + 2a^2\beta_2 + 4b^2\beta_3 + 12b^2\beta_4 - a^2\beta_5 = 0. \tag{23b}$$

This means that, in our notation, we have $n_d = 1$, $n_{t_1} = 1$, $n_{t_0} = 1$, $n_{s_1} = 0$, $n_{s_0} = 0$, $n_{s_2} = 0$, $n_0 = 5$, $n_1 = 3$, $n_2 = 1$. We also have $v = v_1 = a$, $x = x_1 = b$, $w = w_1 = \sqrt{2}b$. We can then write the masses of the W and Z bosons as $m_W^2 = \frac{g^2}{4}(a^2 + 8b^2)$ and $m_Z^2 = \frac{g^2}{4c_W^2}(a^2 + 8b^2)$, such that we have $m_W = m_Z c_W$.

From the potential in equation 22, we get the form of the matrices U_i $(i = 1,3,4)$, V_j $(j = 1,2)$ and R_1 in the GM model. The matrix T_1 is equal to 1 as there is only one doubly charged scalar, which means that it cannot mix. The matrices U_2, R_2 and T_2 are not defined in this model because it does not contain scalar singlets.

The masses of the scalars are given in terms of the VEVs of the neutral fields and the parameters of the potential by $m_2^2 = \frac{1}{2}\beta_5(a^2+8b^2)$, $m_3^2 = 8b^2\beta_3 + \frac{3}{2}a^2\beta_5$, $\mu_2^2 = \frac{1}{2}\beta_5(a^2 + 8b^2)$, $\mu_3^2 = 8b^2\beta_3 + \frac{3}{2}a^2\beta_5$,

$$\mu_4^2 = 4a^2\beta_1 + 4b^2(\beta_3 + 3\beta_4) - 2\sqrt{(2a^2\beta_1 - 2b^2(\beta_3 + 3\beta_4))^2 + 3a^2b^2(\beta_5 - 2\beta_2)^2}, \tag{24a}$$

$$\mu_5^2 = 4a^2\beta_1 + 4b^2(\beta_3 + 3\beta_4) + 2\sqrt{(2a^2\beta_1 - 2b^2(\beta_3 + 3\beta_4))^2 + 3a^2b^2(\beta_5 - 2\beta_2)^2} \tag{24b}$$

and $M_1^2 = 8\beta_3 b^2 + \frac{3}{2}\beta_5 a^2$.

Computing the oblique parameters for the GM model, we get a divergent and gauge dependent result (as expected, once again) and we get finite and gauge independent results for the oblique parameters S, U, V, W and X. In this model, the subtraction of the SM results for the vacuum polarization tensors from the NP ones is trivial as the relation $m_W = m_Z c_W$ is verified. This means that we would get a finite results for the oblique parameters S and U in this model even if we had not multiplied the SM Feynman rules by the factors mentioned in section 3.

In this model, as we have only one doublet, then, similarly to what happens in the SM, we will have $\delta g_{Rb}^c = \Delta g_{Rb}^c = 0$, $\delta g_{Lb}^n = \Delta g_{Lb}^n = 0$ and $\delta g_{Rb}^n = \Delta g_{Rb}^n = 0$ in the limit $m_b \to 0$, because these quantities are proportional to m_b^2. Furthermore, as the relation $m_W = m_Z c_W$ is verified, the we get a finite and gauge independent result for δg_{Lb}^c. Our result for δg_{Lb}^c depends only on the mass m_2, such that it is independent of the masses of the other scalars.

To compare our results for the GM model with experiment we must relate the couplings $g_{\aleph b}$ ($\aleph = L, R$) with the observables A_b and R_b. We can relate these quantities by [19]

$$\frac{g_{Lb}}{g_{Rb}} \equiv \varrho = \frac{\sqrt{1-4\mu_b}\left(1 \pm \sqrt{1-(1+2\mu_b)A_b^2}\right) + (1+2\mu_b)A_b}{\sqrt{1-4\mu_b}\left(1 \pm \sqrt{1-(1+2\mu_b)A_b^2}\right) - (1+2\mu_b)A_b}, \tag{25a}$$

$$g_{Rb}^2 = \frac{s_c + s_u + s_s + s_d}{c^{\text{QCD}}c^{\text{QED}}\left((2-6\mu_b)(1+\varrho^2) + 12\mu_b\varrho\right)}\frac{R_b}{1-R_b}, \tag{25b}$$

where $\mu_b = \left(m_b(m_Z^2)\right)^2/m_Z^2$, $s_c+s_u+s_s+s_d = 1.3184$ and $c^{\text{QCD}} = 0.9953$ and $c^{\text{QED}} = 0.99975$ are QCD and QED corrections, respectively.

The Standard Model predictions for the couplings $g_{L,Rb}$ are $g_L^{SM} = -0.420875$ and $g_R^{SM} = 0.077362$ [20], such that we get $A_b^{SM} = 0.9347$ and $R_b^{SM} = 0.21581$.

An overall fit of various electroweak observables gives [21] $R_b^{\rm fit} = 0.21629 \pm 0.00066$, $A_b^{\rm fit} = 0.923 \pm 0.020$. However, making the average of two direct measurements of A_b done at LEP1 and SLAC in two different ways, we get [22] $A_b^{\rm average} = 0.901 \pm 0.013$. We have then that $R_b^{\rm fit}$ deviates from its SM value by 0.7σ and $A_b^{\rm fit}$ deviates from its SM value by 0.6σ, while $A_b^{\rm average}$ deviates from its SM value by 2.6σ.

According to equation 25a, we get two solutions for ϱ. According to equation 25b, g_{Rb} can either be positive or negative. Thus, we get four solutions for g_{Lb} and g_{Rb} for the two pairs of experimental values $(R_b^{\rm fit}, A_b^{\rm fit})$ and $(R_b^{\rm fit}, A_b^{\rm average})$. Using the central values for the experimental results of A_b and R_b and equations 25a and 25b, we get the values displayed in table 1 for g_{Lb}, g_{Rb}, $\delta g_{Lb} = g_{Lb} + 0.420875$ and $\delta g_{Rb} = g_{Rb} - 0.077362$.

solution	g_{Lb}	g_{Rb}	δg_{Lb}	δg_{Rb}
$1^{\rm fit}$	-0.420206	0.084172	0.000669	0.006810
$2^{\rm fit}$	-0.419934	-0.082806	0.000941	-0.160168
$3^{\rm fit}$	0.420206	-0.084172	0.841081	-0.161534
$4^{\rm fit}$	0.419934	0.082806	0.840809	0.005444
$1^{\rm average}$	-0.417814	0.095496	0.003061	0.018134
$2^{\rm average}$	-0.417504	-0.094139	0.003371	-0.171501
$3^{\rm average}$	0.417518	-0.095496	0.838688	-0.172858
$4^{\rm average}$	0.417504	0.094139	0.838379	0.016777

Table 1: Results for g_{Lb} and g_{Rb} computed from the experimental values for A_b and R_b. Solutions labelled by "fit" were computed using $A_b^{\rm fit}$, while solutions labelled by "average" were computed using $A_b^{\rm average}$.

We can see that in solutions 3 and 4 the value of δg_{Lb} is too large, which indicates that solutions 1 and 2 might be preferred over solutions 3 and 4.

To make the numerical fit to the experimental data, we make a further simplification: we put $\beta_5 = 2\beta_2$ on the scalar potential. In this case, we get alignment. We identify then S_5^0 with the SM Higgs boson, such that $\mu_5^2 = 8a^2\beta_1 \equiv m_h^2 = (125.09\ \text{GeV})^2$.

The strategy used to fit the experimental data was to scan the allowed regions for the potential parameters by the BFB conditions and the unitarity conditions and select the ones for which the deviation of the oblique parameters S and U from their experimental values were less than 1σ. For each of those points, we computed δg_L (δg_R is equal to 0). We fitted only solution number 1 from table 1 as in the GM model we have $\delta g_{Rb} = 0$, which means that we will not be able to get a good fit to the other solutions. We have used `LoopTools` [23, 24] to perform the numerical integration of the Passarino-Veltman functions.

We do not get a better agreement with solution 1 than in the SM. In fact, we cannot even reach the 2σ interval of $A_b^{\rm average}$. This happens because in this model, as in any model with only one scalar doublet (and possibly other additional $SU(2)$ multiplets of higher dimension), in the limit $m_b \to 0$, we will have $\delta g_{Rb} = 0$, such that only g_{Lb} will be changed by the additional scalar content of the GM model. However, the result for g_{Lb} in the GM model is always bigger than the SM one (*i.e.*, δg_{Lb} is always positive), such that the GM fit is always worse than the SM one.

6 Conclusion

In this paper we have presented a formalism that allows to study a general class of models with $SU(2)$ scalar singlets, doublets, and triplets. For this class of models we have computed the oblique parameters. We have found that a special prescription is needed for the parameters S and U to be finite one needs to multiply some Feynman rules of the SM by extra factors, that become equal to 1 when $m_W = m_Z \cos\theta_w$. On the other hand, for the parameter T we got a divergent and gauge-dependent result, because our models do not contain any custodial symmetry that preserves $m_W = m_Z \cos\theta_w$. For the parameters V, W, and X our results are finite and gauge-independent. Still using the same formalism, we have computed the one-loop corrections to the $Zb\bar{b}$ vertex. We obtained a gauge-independent but divergent result for the contribution to g_{Lb} from the diagrams with charged scalars; we have been unable to eliminate this problem. On the other hand, both the contribution to g_{Lb} from the diagrams with neutral scalars and all the contributions to g_{Rb} are finite and gauge-independent.

We have applied our results to the concrete case of the Georgi–Machacek model; this is a model with scalar triplets in which the relation $m_W = m_Z \cos\theta_w$ holds because of an *ad hoc* custodial symmetry. We have obtained a divergent and gauge-dependent result for the oblique parameter T, because the custodial symmetry is not a genuine symmetry of the model; the other oblique parameters are both finite and gauge-independent. The one-loop corrections to the $Zb\bar{b}$ vertex in this model are all finite because the relation $m_W = m_Z \cos\theta_w$ holds at the tree level. We have made a fit to the experimental data by using the oblique parameters S and U and the observables A_b and R_b. This fit turned out to be no better than the SM one.

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