Planning hospital networks:
a case study of the hospital network in Portugal

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Abstract

One of the most important goals in NHS-based countries is to ensure the efficient provision of healthcare services to its population while balancing costs and access. Thus, planning an optimized hospital network is crucial for providing good quality healthcare, since decisions related with the location of the hospital, demand allocation and installed capacity directly impact the daily activities of the hospitals and, consequently, the service level of the healthcare. This thesis aims to develop and implement an optimization approach to plan a hospital network, within the scope of a National Health Service, considering relevant aspects of hospital networks and apply it to a real case study in the Portuguese health system. In order to do this, a bi-objective mixed-integer linear programming model is presented in which two objective functions are minimized. The first one minimizes expected travel time to reach hospitals weighted by demand, which relates to improvement in access to healthcare. The second one minimizes expected operational and investment hospital costs, which relates to efficiency. Uncertainty in the demand for service was also incorporated. The model was applied to the national continental network of Cardiology inpatient service and to the Regional Health Administration of Alentejo's network of Internal Medicine inpatient service. The results demonstrated that decentralizing care can improve geographical access and reinforced the need to make a compromise between equity in access to healthcare and costs.
Keywords: Hospital Referral Network, Location-allocation, Multi-Objective Programming, Uncertainty Modelling, Operational Research in Healthcare

1. Introduction

Planning a hospital network is an extremely important task in healthcare. Every person needs healthcare services and that care is best provided when hospitals are placed in optimal locations and have sufficient resources to serve the demand. Decisions like hospital location and demand allocation are often involved in the strategic planning of a network of hospitals and they directly impact the life of the patients. Questions like "How long should a person take to get to a hospital?", "How large should a hospital be?", "Should a transfer be necessary, to which hospital should a patient be transferred?" and "How many hospitals should a certain region/city/country have?" are key to this type of planning.

Over the last few years, the organization of hospitals in Portugal has gone through some changes. With the goal of improving Portuguese healthcare, the focus has been on building a connected network of hospitals that provides healthcare in a coherent manner and is based on principles of rationality, complementarity and efficiency [15]. Due to the recent organizational modifications, there is a lack of updated investigation that depicts the present state of healthcare services in Portugal. According to the research done for this thesis, and until the time of completion and delivery of this work, there is no model for the planning of hospital networks that considers hospitals as multi-level structures according to medical specialties, adapted to the Portuguese case.

This being said, the main goals of this work, in the context of supplying hospital healthcare services in a country with an National Health Service (NHS), are to develop a mathematical model to support decisions concerning planning hospital networks. And, in this way, help optimize hospital services in order to improve access, while balancing costs and efficiency.

2. Background and related work

Despite the legal and political commitments to social rights, health inequalities caused by some social determinants are still a large concern in Portugal's NHS [19]. One of these determinants is geography. Due to the insufficient supply of healthcare services in the interior, more rural, regions of Portugal, people from these localities experience more difficulties in accessing these services when compared with people who live closer to cities. This represents a considerable gap in the provision of care to elderly populations since these regions have a larger percentage of older populations [7]. To be able to bridge this gap and walk towards a more equitable society, careful and intelligent planning is fundamental. Healthcare planners in countries with an NHS have to make several decisions in terms of hospital location, organization and resource allocation in order to reach certain policy objectives (e.g. geographic equity of access, quality and efficiency while minimizing costs) [12]. A task that is usually complex because some of these goals can be conflicting. Improving geographical access may require building smaller hospital facilities closer to the populations, which can lead to higher inefficiencies and costs [13]. The next section presents some of the most relevant work done in health care facility location modeling worldwide and in Portugal.

Location models in the healthcare sector

In a real world scenario, every logistic operation will have budget constraints. Some authors tackled this problem together with the problem of accessing health facilities in rural populations in a province in Iran [5]. The model they designed took into account the costs of opening the facilities and building network links. However, this model assumed facilities to be uncapacitated which can be unrealistic. On this topic, there was an attempt to design a reliable healthcare network whose facilities have a limited capacity [16]. In their work, these authors recognized the risk of deterioration of patients' health conditions due to limits in the capacity and investigated a queue system that was designed, in advance, to address this situation. Another paper presented a general mathematical formulation that tried to maximize access to public services and then apply it to a realistic case based on the Toronto hospital network [1]. The authors also addressed aggregate capacity decisions on top of determining the configuration of the network. Other researchers also tried to improve local accessibility, equity and efficiency in health by developing a location model in a multi-objective framework [11]. The object of study, in this case, were community based organizations. In the context of the Portuguese health system, the literature about locating hospitals is not extensive. Nonetheless, some work has been done regarding hospital network planning. In specific, work that considers hospitals as diverse multi-service structures inserted in a complex network [17, 18, 12, 13, 2, 3].

3. Mathematical Model

The problem at hand consists of locating hospitals and allocating demand to those hospitals. Each hospital is viewed as a multi-service facility with n medical specialties. Each hospital has a certain level l according to each medical specialty. Consequently, it is part of a hierarchical structure, where hospitals refer to each other when needed, creating a hospital referral network. Transfers can occur between lower level hospitals to hospitals in the same level or higher. This network must take into account the improvement of access and the minimization of costs.

The problem described is now mathematically formulated in multi-objective Mixed-Integer Linear Programming. The model presented here is based on Model 1 of "Location-allocation approaches for hospital network planning under uncertainty" [13], which considers location a first-stage decision and allocation a scenario dependent decision. The indices, sets, parameters, weights and decision variables used are described in sections 3.1, 3.2 and 3.3. The objective functions and constraints are presented in 3.4.

3.1. Indexes and sets

$t, \tau \in T$: Set of time periods in which the planning horizon is divided
$i \in I$: Set of demand points
$j, j', k, k' \in J$: Potential locations for a hospital
$J_o \subset J$: Set of hospitals initially existing
$J_c \subset J$: Set of hospitals that are not opened at the initial moment
$J_m \subset J$: Set of hospitals that are initially existing and must be kept open
$s \in S$: Set of possible scenarios for demand
$n \in N$: Set of medical specialties that can exist in a hospital
$l, p, p' \in L$: Set of levels for a certain medical specialty

3.2. Parameters and weights

d^1_{ij} : Average travel time from demand point i to hospital j
d^2_{jk} : Average travel time from hospital j to hospital k
α : Weight to differentiate a first entry in the system and a transfer
P_s : Probability of scenario s
D^{SNLt}_{inls} : Demand for medical specialty n in level l in location i scenario s and time t
$capmin^t_j$: Minimum capacity required in hospital j in time t
$capmax^t_j$: Maximum capacity required in hospital j in time t
d^{max} : Maximum travel time allowed for a population to access hospital care
N^t : Maximum number of hospitals operating in time t
pop^t_j : Population in demand point j in time t
pop^{min} : Minimum population required to open a hospital in location j
$year^t$: Number of years of time period t
OC^t_j : Unit cost for providing care in hospital j in time t
IC^t_j : Fixed investment cost in a new hospital ($j \in J_c$) providing care in time t
CC^t_j : Fixed cost of closing an existing hospital ($j \in J_o$) providing care in time t
n^{spec} : Total number of existing medical specialties
n^{min} : Minimum number of medical specialties in a hospital
M : Large coefficient that is chosen to be larger than any reasonable value that a variable may take (used in big-M constraints)

3.3. Decision Variables

X^t_j : =1 if a hospital is located at site j in time t; 0 otherwise
Z^t_{jnl} : =1 if a hospital located at site j has medical specialty n in level l in time t; 0 otherwise
Y^t_{ijnls} : Flow from demand point i to hospital j in medical specialty n in level l in time t and scenario s (flow from demand points to hospitals)
Y^{Tt}_{jknlps} : Flow from medical specialty n in level l in hospital j to medical specialty n in level p in hospital k (flow related to intra- and inter-hospitals transfers)
Y^{Ct}_{jnls} : Patients that stay (receive care) in hospital j in medical specialty n in level l and scenario s
cap^t_j : Capacity in hospital j in time t
$Scap^t_{js}$: Expected utilization of hospital j in time t and scenario s

3.4. Objective functions and constraints

$$\text{Min} \sum_{s\in S} P_s \Big(\sum_{i\in I}\sum_{j\in J}\sum_{n\in N}\sum_{l\in L}\sum_{t\in T} d^1_{ij}\, Y^t_{ijnls} + \sum_{j\in J}\sum_{k\in J}\sum_{n\in N}\sum_{l\in L}\sum_{p\in L}\sum_{t\in T} \alpha\, d^2_{jk}\, Y^{Tt}_{jknlps} \tag{1}$$

$$\text{Min} \sum_{s\in S}\sum_{t\in T} P_s \Big(\sum_{j\in J} Scap^t_{js}\, OC^t_j\, year^t \Big) + \sum_{t\in T\setminus\{1\}}\sum_{j\in J_c} \big(X^t_j - X^{t-1}_j\big) IC^t_j + \sum_{t\in T\setminus\{1\}}\sum_{j\in J_o} \big(X^{t-1}_j - X^t_j\big) CC^t_j \tag{2}$$

Subject to:

$$\sum_{j\in J} Y^t_{ijnls} = D^{SNLt}_{inls} \quad \forall_{t\in T\ i\in I\ n\in N\ l\in L\ s\in S} \tag{3}$$

$$\sum_{i\in I} Y^t_{ijnls} + \sum_{k'\in J}\sum_{p'\in L} Y^{Tt}_{k'jnp'ls} - \sum_{k\in J}\sum_{p\in L} Y^{Tt}_{jknlps} = Y^{Ct}_{jnls} \tag{4}$$

$$\forall_{t\in T\ j\in J\ n\in N\ l\in L\ s\in S}$$

$$Scap^t_{js} = \sum_{n\in N}\sum_{l\in L} Y^{Ct}_{tjnls} \quad \forall_{t\in T\ j\in J\ s\in S} \tag{5}$$

$$capmin^t_j X^t_j \leqslant Scap^t_{js} \leqslant capmax^t_j X^t_j \quad \forall_{t\in T\ j\in J\ s\in S} \tag{6}$$

$$cap^t_j \geqslant Scap^t_{js} \quad \forall_{t\in T\ j\in J\ s\in S} \tag{7}$$

$$Y^t_{ijnls} + D^{SNLt}_{inls} \sum_{l'\in L} Z^t_{j'nl'} \leqslant D^{SNLt}_{inls} \quad \forall_{t\in T\ i\in I\ j\in J\ j'\in\{j'|d^1_{ij'}<d^1_{ij}\}\ n\in N\ l\in L\ s\in S} \tag{8}$$

$$Y^t_{ijnls} = 0 \quad \forall_{t\in T\ i\in I\ j\in\{j|d^1_{ij}>d^{max}\}\ n\in N\ l\in L\ s\in S} \tag{9}$$

$$Y^t_{ijnls} \leqslant D^{SNLt}_{inls} \times X^t_j \quad \forall_{t\in T\ i\in I\ j\in J\ n\in N\ l\in L\ s\in S} \tag{10}$$

$$\sum_{j\in J} X^t_j \leqslant N^t \quad \forall_{t\in T} \tag{11}$$

$$X^t_j = 0 \quad \forall_{t\in T\ j\in\{j|pop^t_j<pop^{min}\}} \tag{12}$$

$$X^{t-1}_j \leqslant X^t_j \quad \forall_{t\in T\setminus\{1\}\ j\in J_c} \tag{13}$$

$$X^{t-1}_j \geqslant X^t_j \quad \forall_{t\in T\setminus\{1\}\ j\in J_o} \tag{14}$$

$$X^\tau_j - X^{\tau-1}_j = 0 \quad \forall_{j\in J_c} \tag{15}$$

$$X^{\tau-1}_j - X^\tau_j = 0 \quad \forall_{j\in J_o} \tag{16}$$

$$\sum_{l\in L} Z^t_{jnl} \leqslant 1 \quad \forall_{t\in T\ j\in J\ n\in N} \tag{17}$$

$$\sum_{n\in N}\sum_{l\in L} Z^t_{jnl} \leqslant n^{spec} \times X^t_j \quad \forall_{t\in T\ j\in J} \tag{18}$$

$$\sum_{n\in N}\sum_{l\in L} Z^t_{jnl} \geqslant n^{min} \times X^t_j \quad \forall_{t\in T\ j\in J} \tag{19}$$

$$\sum_{k\in J}\sum_{p\in L} Y^{Tt}_{kjnpls} \leqslant M \times Z^t_{jnl} \quad \forall_{t\in T\ j\in J\ n\in N\ l\in L\ s\in S} \tag{20}$$

$$Y^{Ct}_{jnls} \leqslant M \times Z^t_{jnl} \quad \forall_{t\in T\ j\in J\ n\in N\ l\in L\ s\in S} \tag{21}$$

$$Y^{Tt}_{jknlps} = 0 \quad \forall_{t\in T\ j\in J\ k\in J\ n\in N\ l\in L\ p\in\{p|p<l\}\ s\in S} \tag{22}$$

$$Y^{Tt}_{jjnlls} = 0 \quad \forall_{t\in T\ j\in J\ n\in N\ l\in L\ s\in S} \tag{23}$$

$$Z^{t-1}_{jnl} \leqslant Z^t_{jnl} \quad \forall_{t\in T\setminus\{1\}\ j\in J\ n\in N\ l\in L} \tag{24}$$

$$X^t_j\ Z^t_{jnl} \in \{0,1\} \quad Y^t_{ijnls} Y^{Tt}_{jknlps} \geqslant 0 \quad \forall_{t\in T\ i\in I\ j\in J\ k\in K\ n\in N\ l\in L\ p\in L\ s\in S} \tag{25}$$

The model considers two objective functions representing the access and costs objectives. Equation 1 minimizes the expected travel time to reach hospital care weighted by demand. Equation 2 minimizes expected costs, both operational and investment costs. Equations 3 and 4 refer to demand satisfaction and flow conservation constraints. Equations 5, 6 and 7 refer to hospital capacity. Equations 8, 9 and 10 are closest assignment constraints. Equations 11-16 are related to opening, closing and locating hospitals. Equations 17-24 are constraints about medical specialties. Finally, equation 25 represents standard integrality and non negativity constraints.

4. Results and Discussion

In this section, the model presented in the last section is implemented, solved for two real cases described in subsection 4.1 and its results are presented and discussed in subsection 4.2.

4.1. Characterization of the cases

As previously mentioned, the model was tested for two real cases, each case with one medical specialty and one hospital service at a time. The choice of the medical specialties and services was made according to the information available and how recent this information was. Consequently, the two cases are: the inpatient service in Cardiology in continental Portugal (case 1) and the inpatient service in Internal Medicine in the Regional Health Administration of Alentejo (case 2). This decision meant to demonstrate the full potential of applicability of the model, since a bigger scale was explored in the first case and a smaller scale, with uncertainty in demand incorporated, was explored in the second case. All the sets and subsets are described in table 1 and the parameters and weights (except d^1_{ij}, d^2_{jk}, D^{SNLt}_{inls} and pop^t_j) are described in table 2.

Table 1: Definition of sets and subsets.

Sets	Case 1	Case 2
T :	{1, 2, 3}	{1, 2, 3}
I :	{1, 2, 3, ..., 18}	{1, 2, 3, ..., 11}
J :	{1, 2, 3, ..., 39}	{1, 2, 3, ..., 9}
J_o	$= J$	$\{\emptyset\}$
J_c	$= \{\emptyset\}$	J
J_m	$= J_o$	$\{\emptyset\}$
S :	{1}	{3}
N :	{1} = {Cardiology}	{1} = {Int. Medicine}
L :	{1, 2, 3}	{1, 2, 3}

Table 2: Definition of parameters and weights

Parameters	Case 1	Case 2
α :	0.5	0.5
P_s :	1	1/3, 1/3, 1/3
$capmin^t_j$:	50, 500, 1000	0
$capmax^t_j$:	1400, 1900, 6000	$(3500, 5500, 6000) \times 5$
d^{max} :	70	90
N^t :	39	9
pop^{min} :	10 000	10 000
$year^t$:	1	5
OC^t :	-	347×7.4, 491×8
IC^t :	-	200000, 224500
CC^t :	-	200000, 224500
n^{spec} :	50	50
n^{min} :	1 = Cardiology	1 = Int. Medicine
M :	1 000 000	1 000 000

Note: Values of capacities correspond to values of capacities for hospitals in levels I, II and III, respectively. The first value of the costs corresponds to costs in hospitals in level I and the second value of the costs corresponds to costs in hospitals in level II and III.

4.1.1 Case 1

The document that was central as a source of information for this referral network was the *"Rede de Referenciação de Cardiologia - Proposta de Atualização"* (Cardiology Referral Network - Proposal for an update) [14].

The planning horizon (T set) is divided into three periods. The demand points (I set) being considered are the districts of Portugal, which makes a total of 18 demand points. The J set represents the potential locations for siting hospitals. For the Cardiology test situation, the J set comprises of exact 39 locations where there were already hospitals with a Cardiology service in 2013 [14]. Since this test situation was one primarily for allocation of demand, the subset J_o, which represents the hospitals initially existing, has all the locations of J. In other words, it is equal to J. As a result, J_c is an empty set since it represents all the closed facilities at the beginning of the planning horizon. Also for the reason stated, J_m is equal to J_o, as all the hospital that are initially existing must be kept open. Furthermore, there is only one scenario s for demand and one medical specialty (Cardiology) n as previously said. The number of levels l each hospital can have, according to this specialty, is three.

The value for the α parameter acts as a weight in the objective function that minimizes travel times. It serves to distinguish first entries in the system and transfers between hospitals. Here, it is defined as 0.5 because it was considered more crucial for a person to reach a hospital quickly as a direct entry than as transfer from another hospital, since the latter may already have received preliminary hospital care. The probability of scenario s, P_s, is 1 since there is only one scenario. In terms of minimum and maximum capacity for the hospitals, the values are the same for each time period, and were based on the minimum and maximum number of people admitted in the Cardiology service per level in the year 2013, respectively. The d^{max} parameter represents the maximum amount of time that a person has to travel to reach a hospital (as a direct entry) and it is defined as 70 minutes. The maximum number of open hospitals at a given time period (N^t) is set at 39. The minimum population (pop^{min}) for a hospital to open is set at 10 000 people (parameter not used in this case since all the hospitals are open). The parameter $year^t$ is set at one so each time period has the duration of one year. The maximum number of medical specialties (n^{spec}) a hospital can have is 50 and the minimum (n^{min}) is 1. The M (big m) parameter is defined as 1 000 000. To calculate the travel time between every demand point i and every hospital candidate site j, d^1_{ij}, and the travel time between every pair of hospitals j and k, d^2_{jk}, the Haversine distance between the pair of points mentioned above was calculated and converted into time. For distances equal or less than 50 km, the converting velocity used was 50 km/h. For distances greater than 50 km, the velocity used was 100 km/h. The demand for healthcare (D^{SNLt}_{inls}) to be calculated was the number of people per district in need of care in the Cardiology specialty per level in a year. Given that each time period was only a year, it was safe to assume the demand did not change significantly, so the demand in every time period was the same. The values for D^{SNLt}_{inls} were calculated using information from the *Instituto Nacional de Estatística* and from the Ministry of Health [9, 14]. Finally, the population in demand in the area of each hospital (pop^t_j) was defined as the population that existed in the location of every hospital with Cardiology service in 2013 [6].

4.1.2 Case 2

In this case, the document that provided the information needed for the Internal Medicine referral network was the *"Rede de Referenciação de Medicina Interna"* (Internal Medicine Referral Network) [4].

The planning horizon (T set) is divided in three periods. The demand points (I set) considered are 11 in total. Each sub-region of Alentejo's RHA is divided in 3 demand points, except one (Alentejo Litoral) that is divided in 2. In terms of hospital locations (J set), 9 were considered. All locations, similarly to case 1, were locations where a hospital already exists or where there is a possibility for building one. Two of the locations are in Lisbon but, due to the lack of higher level hospitals in the region being studied, the hospitals at those locations are part of the Internal Medicine referral network. No hospitals were considered to be opened in the first time period (J_c is equal to J and J_o is empty) and no hospitals were force to be kept open (J_m is also empty). In this case, three scenarios for demand were considered. One representing low demand (s=1), another high demand (s=2) and another representing a baseline level of demand (s=3). Finally, the number of levels each hospital can have, according to this specialty, is four. However, no hospital is ranked at level III, so level III and IV are merged into one. Subsequently, the L set has three levels.

The value for α is the same as in the first case. Regarding the probability of the three scenarios, each scenario is considered to be equally likely so P_s is 1/3 for every s. In terms of minimum and maximum capacity for the hospitals, the values are the same for each time period. The minimum is zero and the maximum is based on the number of people admitted in the Internal Medicine service per level in the year 2016. The d^{max} parameter is 90 minutes. Since this value is very high for a travel time, some other lower travel times were explored further in the solutions. The maximum number of open hospitals at a given time period (N^t) is set at 9. The minimum population (pop^{min}) for a hospital to open is set at 10 000 people and every region for hospital candidate sites fulfilled this condition. This means pop^{min} parameter will not limit the opening of a hospital in this case. The parameter $year^t$ is equal to 5 years, so each time period has that duration. Thus, the planning horizon lasts 15 years. Regarding hospital costs, the information was based on the paper mentioned before [13]. The maximum number of medical specialties (n^{spec}) a hospital can have is 50 and the minimum (n^{min}) is 1. The M (big m) parameter is defined as 1 000 000. The values for d^1_{ij} and d^2_{jk} were calculated similarly as in case 1. The velocities of travel considered were 80 km/h for distances equal/below 60 km and 100 km/h for the remainder. According to a study performed by the National Institute of Statistics [8], the population of Alentejo will decrease in the next decades. In order to have reliable results and because demand is considered a great source of uncertainty, three possible scenarios were created. In all scenarios, the population never grows. The difference in each scenario is the rate of population decrease. In a pessimistic scenario (s1), the population decreases at a faster rate than the present one. In a baseline scenario (s3), it continues to decrease in a rate equal to the present rate. Lastly, in an optimistic (s2) scenario, it decreases at a slower rate than the present rate. The prediction for the resident population of Alentejo in these three scenarios are represented in figure 1.

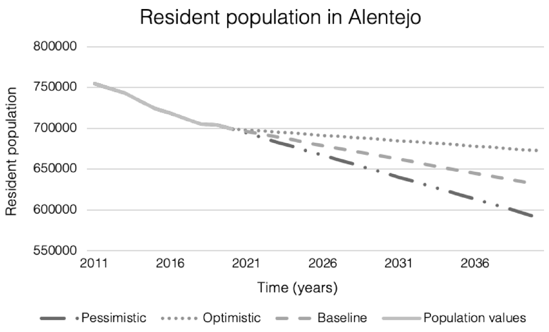

Figure 1: Projections for population growth in Alentejo (data taken from [8]).

The resident population in each demand point varied, according to the different projection scenarios and across the different time periods. In consequence, the total number of expected people in demand for Internal Medicine services also varied, in the different scenarios and time periods. The demand per level was then calculated similarly to case 1 and the final values of D_{inls}^{SNLt} were obtained, for all elements of S and T. It is relevant to note that the distribution of population through the different sub-regions did not change. In other words, the total population of Alentejo and the population of each sub-region changed but the size of the population in each sub-region, in relation to each other, was assumed to be always the same. The projections for the population were calculated using data from [8]. The pop_j^t values were defined according to the population that existed in the location of every hospital with Internal Medicine service in 2016. It was assumed to be the same for every time period of the planning horizon [10].

4.2. Computational results

Before testing with real instances, the model was first tested with fictional data in order to be validated. The model was implemented and solved in Python™, using the *docplex - IBM Decision Optimization CPLEX* library. Every test was performed in a dual-core Intel® Core™ i5-5250U CPU @ 1.60GHz and 4GB 1600MHz DDR3 memory computer with the macOS Big Sur (Version 11.6) operating system.

4.2.1 Case 1

As explained before, for this case, only objective function 1 (relative to improvement of access) was optimized since there would be no costs of opening/closing hospitals. The flows from demand points to hospitals, from hospitals to other hospitals and the number of people served at each hospitals are represented, in figure 2 (in respect to time $t = 1$).

The location of the hospitals was not decided by the model. However, the level, the number of people each hospital serves and the flows between hospitals were. Through the analysis of figure 2, the first thing that can be verified is that the demand is allocated to the nearest hospital for every demand point except one. The demand from Porto is allocated to a hospital in another district (hospital 31 in Aveiro) instead of being allocated to any of the hospitals in Porto that may be, technically, at a closer distance (e.g. 4, 5, 6, 30 or 36). According to the travel times d_{ij}^1 calculated by the model, $d_{1,30}^1 < d_{1,31}^1 < d_{1,6}^1 < d_{1,36}^1 < d_{1,5}^1 < d_{1,4}^1$ (where Aveiro is demand point $i = 1$ and the j's are the hospital locations). This means that the demand from Aveiro should have been assigned to hospital 30, since it is the closest hospital. However, hospital 30, being a level III hospital, is at full capacity already. So, the model assigned the demand from Aveiro to the next closest hospital, which is hospital 31. In reality, hospital 31 is not the closest hospital but, since the travel times were calculated through an approximation, in the model it is. An issue like this could be solved by adjusting the values used for velocity, for the distance turning point or by attributing the actual travel times to each demand point-location pair.

Besides this, it can verified that each hospital is only treating the patients that need care at the level the hospital is or at a level below (and transfers the rest), as it was intended. Another observation that can be made regarding transfers flows is the fact that some hospitals, due to overcrowding, are transferring patients to multiple other hospitals. That is, the hospitals are transferring patients, not because they do not have the "level required", but because they are at full capacity. It is the case of hospital 38 for example. This hospital is located in Lisbon, where the demand for hospital care is high. Because this hospital is the nearest to the demand point that represents the Lisbon district, it has to receive all patients from that district plus some level III patients from other hospitals in other districts, since hospital 38 is also the closest level III hospital in the Lisbon vicinities. Level III patients are a priority for hospital 38 because they can only be treated in level III hospitals. Thus, other patients that can be treated in other hospitals (level I patients for example) are being transferred to other hospitals (23, 32, 33, 39). Even though this does not represent exactly the situation in real life, some conclusions can still be drawn if the demand from these hospitals is seen as aggregated. It is clear that these hospitals are serving the demand from Lisbon plus the higher level patients from other districts, which is in fact what happens in real life. If needed, the demand point from Lisbon could be divided into several demand points and the demand would be distributed more uniformly by other hospitals and would not be allocated to one single hospital. Still on this topic, some hospitals (19, 21, 20...) do not receive any direct entries. Thereby, a conclusion that can be drawn from these results is that the choice of demand points for these types of models is crucial. Differences in demand point size and location can influence the model tremendously and, due to that fact, should be chosen carefully.

In terms of classifying each hospital in a level, there are some differences between the results of the model

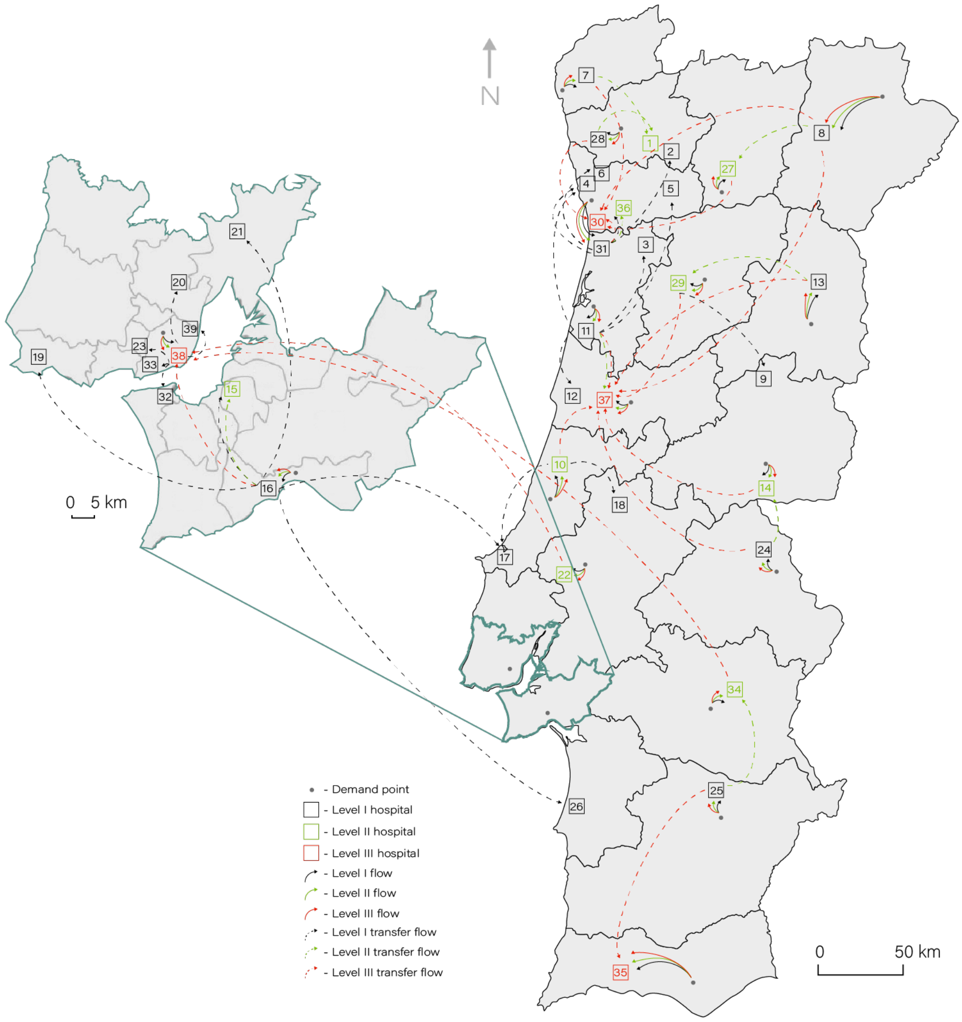

Figure 2: Model results for demand allocation in case 1.

and reality. There were no costs included in this version of the model, which means, without any constraints in this aspect, the model would have classified the majority of hospitals as level III because these are the ones that can treat the most types of patients. However, level III hospitals are more expensive to build and to maintain than level II or level I hospitals, so some restrictions had to be imposed. In 2013, the number of hospitals/hospital centers classified at level I was 29, level II was 9 and level III was 4 [14]. In the model, the number of hospitals allowed to be classified at each level was kept the same but the model was allowed to choose which of the hospitals were classified at each level. The results show that the model assigned level III to hospitals 30, 35, 37 and 38 instead of hospitals 36, 37, 38 and 39. Two of the classifications (37 and 38) coincide but two of them do not. The choice of classifying hospital 30 as a level III hospital instead of level 36 is not too odd because they are both in localized in Porto. The more interesting choice was the decision to classify as level III a hospital in Faro (hospital 35) instead of hospital 39 in Lisbon. Not considering costs, this choice seems to make more sense than locating another level III hospital in Lisbon since in the south of Portugal there are few hospitals and the ones that exist are not that specialized. If it were considering costs, the model might make a different decision since it may not be worth to maintain a level III hospital for the demand that exists in the south. In addition to changes in the classification of level III hospitals, there were also changes in the other levels. In general, the model also improved access to level II care since it classified hospitals, not previously classified as level II, as level II that are located further away from large cities (e.g. 10, 14). Again, this was expected since the model did not include costs. Nonetheless, considering only accessibility, these classifications would be the best choices for improving access to health care services and, consequently, to increase equity in access to those services.

4.2.2 Case 2

In case 2, the model was solved for a smaller geographic area. In this way, it was possible to make several iterations by varying the values of the parameters and incorporating uncertainty. In this case, the two objective functions (eq. 1 and eq. 2) were considered, as well as the three demand scenarios and the three time periods. In the beginning of the planning horizon, no hospitals were considered to be open. Some additional constraints related to levels were added to the model to increase its realistic aspect. The hospitals in Lisbon were forced to be classified as level III in the medical specialty in question and the number of hospitals at that level was limited to two. This means no other hospital, except those in Lisbon, could have that classification. Also, the number of hospitals classified with level II was not allowed to be more than 2. These restrictions served as additional baseline budget constraints since an operating hospital has different expenses depending on its level.

Analysis of trade-offs between costs and travel times

The solutions obtained for the model are represented as points in figure 3 and information about the solutions is described in table 3.

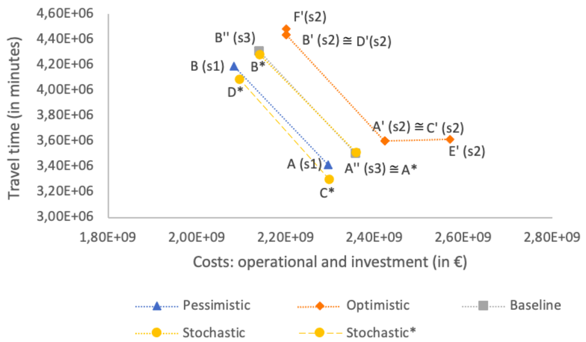

Figure 3: Solutions obtained for case 2 with deterministic and stochastic results.

Table 3: Results obtained with different parameters values and scenarios.

Point	Costs (in €)	Travel time(in min)
A (s1)	$2,30 \times 10^9$	$3,41 \times 10^6$
B (s1)	$2,08 \times 10^9$	$4,19 \times 10^6$
A' (s2)	$2,42 \times 10^9$	$3,60 \times 10^6$
B' (s2)	$2,20 \times 10^9$	$4,44 \times 10^6$
A" (s3)	$2,36 \times 10^9$	$3,50 \times 10^6$
B" (s3)	$2,14 \times 10^9$	$4,31 \times 10^6$
C' (s2)	$2,42 \times 10^9$	$3,60 \times 10^6$
D'(s2)	$2,20 \times 10^9$	$4,44 \times 10^6$
E' (s2)	$2,57 \times 10^9$	$3,61 \times 10^6$
F'(s2)	$2,20 \times 10^9$	$4,49 \times 10^6$
A*	$2,36 \times 10^9$	$3,51 \times 10^6$
B*	$2,14 \times 10^9$	$4,28 \times 10^6$
C*	$2,30 \times 10^9$	$3,30 \times 10^6$
D*	$2,10 \times 10^9$	$4,08 \times 10^6$

Each point represents a value of minimized costs in euros (€), which can be operational or related to opening/closing hospitals, and a value of minimized travel times to reach hospital services weighted by demand in minutes. The results for the deterministic model in the three scenarios are represented by points A, A', A", B, B' and B". The location of each point represents different configurations of the hospital network, which implies trade-offs between costs and time travelled to access hospitals services. Each of these points was calculated by minimizing each objective function separately. Points A, A' and A" were obtained by first minimizing the time/distance travelled to reach hospitals services, fixing that objective function on that minimum value and then minimizing the objective function about costs. Points B, B' and B" were calculated similarly but the objective functions switched places. First, the costs objective function was minimized and fixed on the minimum value discovered, then the travel time objective function was minimized. This was done for every demand scenario. Therefore, it can be said that points A, A' and A" represent the improved access solution, while points B, B' and B" represent the minimum cost solution, respectively for low (s1), high (s2) and intermediate (s3) demand. Points A* and B* refer to the combination of all scenarios in one solution. Thus, they represent the stochastic results, where each scenario was considered and had the same probability of happening.

Table 4 introduces the calculated trade-off values of costs and travel times going from one improved access solution to an improved costs solution, as well as the calculated trade-off values of costs and travel times going from one improved costs solution to an improved access solution. Table 5 introduces the calculated trade-off values of costs and travel times going from one improved access solution to a different improved access solution, as well as the calculated trade-off values of costs and travel times going from one improved costs solution to a different improved costs solution.

Table 4: Trade-offs between costs and travel times (improved access/costs solutions and improved costs/access solution).

Difference between going from an improved access solution to an improved costs solution			Difference between going from an improved costs to an improved access solution		
Solutions	Costs	Travel time	Solutions	Costs	Travel time
A - B	-9,2%	22,8%	**B - A**	10,1%	-18,6%
A' - B'	-9,2%	23,2%	**B' - A'**	10,1%	-18,8%
A" - B"	-9,2%	23,0%	**B" - A"**	10,2%	-18,7%
C' - D'	-9,2%	23,2%	**D' - C'**	10,1%	-18,8%
E' - F'	-9,2%	24,1%	**F' - E'**	16,7%	-19,4%
A* - B*	-9,2%	22,1%	**B* - A***	10,1%	-18,1%
C* - D*	-8,8%	23,7%	**D* - C***	9,6%	-19,2%

Table 5: Trade-offs between costs and travel times (improved access/access solutions and improved costs/costs solution).

Difference between going from one improved access solution to another			Difference between going from one improved costs solution to another		
Solutions	Costs	Travel time	Solutions	Costs	Travel time
A - A"	2,7%	2,7%	**B - B"**	2,7%	2,8%
A" - A'	2,8%	2,8%	**B" - B'**	2,8%	2,9%
A' - C'	0,0%	0,0%	**B' - D'**	0,0%	0,0%
A' - E'	6,0%	0,3%	**B' - F'**	0,0%	1,1%
A* - C*	-2,6%	-5,9%	**B* - D***	-2,1%	-4,6%
A" - A*	0,0%	0,1%	**B" - B***	0,0%	-0,7%

Table 6 relates to the network configurations. The rows symbolize hospital locations and the columns symbolize solutions (or hospital configurations). The last column corresponds to the real configuration of the network [4]. Each square corresponds to the state of each hospital in each solution. The possible states for each hospital (in the medical specialty in question) are: open and in level I (I); open and in level II (II); open and in level III (III); or closed (-). It should be noted that in the R column, hospital F is not closed but does not belong to the NHS so it is considered to be closed.

Table 6: Configuration of the hospital network for the different solutions.

	A	B	A'	B'	A"	B"	C'	D'	E'	F'	A*	B*	C*	D*	R
A	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
B	II	II	II	II	II	II	II	II	I	II	II	II	II	II	I
C	I	I	I	I	I	I	I	I	II	I	I	I	I	I	II
D	I	I	I	I	I	I	I	I	I	I	I	I	I	I	-
E	II	I	II	I	II	I	II	I	I	I	II	I	III	I	I
F	I	I	I	I	I	I	I	I	I	I	I	I	I	III	-
G	I	II	I	II	I	II	I	II	II	II	I	II	II	II	I
H	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
I	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III

From a preliminary analysis of the deterministic results, it can be verified that optimistic scenarios generate the most expensive solutions, pessimistic scenarios generate the most low-cost solutions and baseline scenarios generate an intermediate cost solution ($cost_A < cost_{A''} < cost_{A'}$ and $cost_B < cost_{B''} < cost_{B'}$). From the observation of table 6, it is possible to see that the configurations in A, A" and A' are the same; as well as the configurations in B, B" and B'. This indicates that the change in costs and travel times is not related to network configurations or hospital classifications. It is related to the size of the demand being served. Optimistic scenarios correspond to a prediction in which the values for demand are highest, followed by the values from the baseline scenario and the values from the pessimistic scenario. Higher values for demand equates to more hospital utilization and that can lead to higher costs. From table 5, it can be verified that the difference in costs, of going from a solution in a pessimistic scenario to a solution in a baseline scenario, is of 2,7%. This is true for comparisons between both improved access solutions and improved costs solutions (A-A" and B-B"). The difference in costs, of going from a solution in a baseline scenario to a solution in an optimistic scenario, is 2,8%, which is slightly higher. This is verified for passing from one improved access solution to another (A"-A') and for passing from one improved costs solution to another (B"-B').

In can also be verified that the trend stays the same for time travelled. The travel time increases with the increasing of the demand ($time_A < time_{A''} < time_{A'}$ and $time_B < time_{B''} < time_{B'}$). This can be explained by the need to cater to more people. More people in need of reaching hospitals services, which means more people for which the model has to minimize travel time, can lead to more travel time for everyone. Through the analysis of table 5, it can be verified that the difference in travel times, of going from an improved access solution in a pessimistic scenario to an improved access solution in a baseline scenario, is of 2,7% (A-A"). However, the difference in travel times, of going from an improved costs solution in a pessimistic scenario to another improved costs solution in a baseline scenario, is of 2,8% (B-B"). Furthermore, the difference in travel times, of going from an improved access solution in a baseline scenario to another improved access solution in an optimistic scenario, is 2,8% (A"-A'). And the difference in travel times, of going from an improved costs solution in a baseline scenario to another improved costs solution in an optimistic scenario, is 2,9% (B"-B').

Regarding the comparison (in table 4) between going from one improved access solution to an improved costs solution, it is possible to see that the costs decrease by 9.2% in every scenario (A-B, A'-B' and A"-B"). However, the increase in travel times is not equal for all those cases. For the same decrease in costs, going from an improved access solution to an improved costs solution in a pessimistic scenario (A-B), corresponds to the lowest increase in travel times (followed by A"-B" and then A'-B'). When going from an improved costs solution to an improved access solution, the difference in costs is the same for the pessimistic and optimistic scenario (B-A and B'-A') and is equal to 10,1%. For the baseline scenario, the value is 10,2%. Similar to the previous situation, the decrease in travel times is not equal for all those cases but is very close. The largest decrease (18,8%) in travel times going from an improved costs solution to an improved access solution happens in the optimistic scenario (B-A).

Looking at the stochastic results (points A* and B*), it is possible to affirm that the values of costs and travel times are quite similar to the ones obtained for the deterministic solution in the intermediate scenario (A* ≅ A" and B* ≅ B"). The differences between solutions, both in costs and travel times, is less than 1% and there are no differences in the configurations of the network. In terms of comparing the price of going from an improved access solution to an improved costs solution, it is clear that for a decrease in costs of 9,2%, the increase in travel times is only 22,1%. In the case of going from an improved costs solution to an improved access solution, it is can be seen that for a decrease of 18,1% in travel times, the costs increase 10,1%.

Points C', D', E' and F' represent other relevant deterministic solutions obtained by changing the values of some of the parameters. These variations were all performed using the optimistic scenario since this was the scenario that predicted the highest value for demand. Points C' and D' are the solution for when the maximum travel time allowed - d^{max} - was lowered to 50 minutes (it was 90 minutes before). Points E' and F' are the solution for when the maximum capacity - $capmax_j^t$ - for all hospitals was reduced to 80% of what it was before. Points C* and D* are the stochastic results for a case in which the model was allowed to place an extra level III hospital in Alentejo, which represented a possibility to stop transferring patients to hospitals outside this region. By observation of these new results, it is clear that the solution found for points C' and D' (where the maximum travel time was reduced) is very similar, both in time and in costs, to the deterministic solution found with the original value of d^{max} (C' ≅ A' and D' ≅ B'). It is also apparent that the values of the improved cost solution corresponding to a lowered $capmax_j^t$ are very close to the corresponding improved cost solutions in the optimistic scenario (F' ≅ B' ≅ D'). However, the improved access solution, even though it has the same travel time value ($time_{E'} \cong time_{A'} \cong time_{A}*$), it differs significantly on the costs value. The different of lowering the maximum capacity correspond to a increase in costs of 6,0% and an increase of travel times (0,3%). This difference can be justified by some differences in the configurations of the hospitals and by a bigger percentage of patients being treated in higher level hospitals.

Regarding the points C* and D*, related to a solution where an additional level III hospital was allowed to be placed, it is possible to see that the improved access solution had a lower value of travel time when compared to the other stochastic improved access solution ($time_{C*} < time_{A*}$), corresponding to a 5,9% difference. This verifies that, when there is a higher level hospital inside the Alentejo region, the patients do not need to be transferred to hospitals in Lisbon and the total travel time is lower. The same happens with the improved costs solutions ($costs_{D*} < costs_{B*}$), which seems counter-intuitive because a solution that has 3 hospitals in level III (D*) is less expensive than another with 2 hospitals in level III (B*), for the same number of hospitals in level I and II. This, however, can be explained by the operational costs considered for these locations and by some transfers the hospitals are doing. In the future, it would be of great interest to explore different decisions regarding parameter settings to compare how similar the solutions would be.

Analysis of changes in network configurations

In addition to analysing travel time and cost values, it is important to look at the actual solutions found for the configuration of the network of hospitals. To aid in this discussion, table 6 must be analysed. From observing the table, it can be confirmed that all hospitals were opened in every solution. It can also be verified that there are some changes in the classification of the hospitals. The solution that is closest to the real configuration is the the improved costs solution for when maximum capacity is at 80% (E'). It is also clear that the model, when given the possibility of placing two level II hospitals, always chose to do it, even when costs were minimized first. In part, this validates the government's decision to build another higher level hospital to provide better care at those levels in this region. Moreover, when given the possibility of placing an extra level III hospital, the model chose to do it in both solutions (C*, D*). Additionally, in all solutions but one, the model decided to locate a level II hospital in location B. However, the second location for the level II hospital varied according to which objective function was minimized first. In improved access solutions, that hospital was placed in location E, while in improved costs, it was placed in location G. These decisions are to be expected since location E is more central and closer to more demand points than location G. The same can be said for the decision on where to locate the III level hospital in C* and D*.

Other conclusions, related to the reliability of the model, can be inferred. When comparing solutions with different values for maximum travel time allowed d^{max}, it can be stated that the configurations are the same for $d^{max} = 90$ (A' and B') and $d^{max} = 50$ (C' and D'). When comparing solutions with different values for maximum capacity $capmax_j^t$, it can be concluded that the configuration is the same for the full capacity improved cost solution (B') and the capacity at 80% improved cost solution(F'). For the improved access solutions (A' and E'), the configuration are very similar but the hospital in locations B and C are "switched".

Comparing the solutions' configurations to the real one, it is possible to observe that in the latter the location of the only level II hospital is very central and localized near the most populated demand point ($i = 6$). While in the results obtained, where it was possible to place at least two level II hospitals, these hospitals were placed in opposites sides of the region. Location B is near the top of the geographic area under evaluation and locations E and G are near the bottom. To see if these results may be in part due to the differences in operational costs explained earlier and if the model would behave differently if the costs (and maximum capacities) were the same for every location, a quick version of the model, was designed and solved. The results show that the solution would not be very different. Therefore, the model seems to suggest that the optimal solution may involve the decentralization of higher level care, instead of building all specialized hospitals in the more populated areas.

In order to better visualize the solutions proposed by the model, an example of a configuration based off of solution A* was mapped in figure 4.

From a first glance, it can be verified that all demand from demand points seems to be being assigned to the closest hospitals, as expected. Another anticipated conclusion is the transfer of all level II patients from level I hospitals to the closest level II hospitals (B and E). Also an expected decision is the transfer of all level III patients from all hospitals to the closest level III hospitals (H and I). In addition, it is possible to see that some hospitals are being more used than others. For example, hospital E is receiving the level II transfers of all, but one, level I hospitals and hospital I is receiving the level III transfers of all, but two, hospitals. One other observation that can be made is that people from one sub-region are being assigned to a hospital outside their sub-region. It is the case of demand point 5. The demand from this demand point, which belongs to Alentejo Central, is being assigned to a hospital in Alto Alentejo. This may not be a very critical issue since all sub-regions are still a part of the Alentejo's Regional Health Administration, which is the most important unit in health issues.

5. Conclusions and Future Work

In summary, there are still many questions to answer in this field. Nonetheless, some things can be concluded. The importance of multi-objective models, in order to obtain realistic solutions, is highlighted. The implemented model also suggests that, in general, the solutions that most improve access to services are the most expensive ones. So, a compromise must be made between equity in access and costs to obtain an optimal feasible solution. The results also suggest that, as it was expected, decentralizing care, or building more hospitals in non-central areas, can improve geographical access.

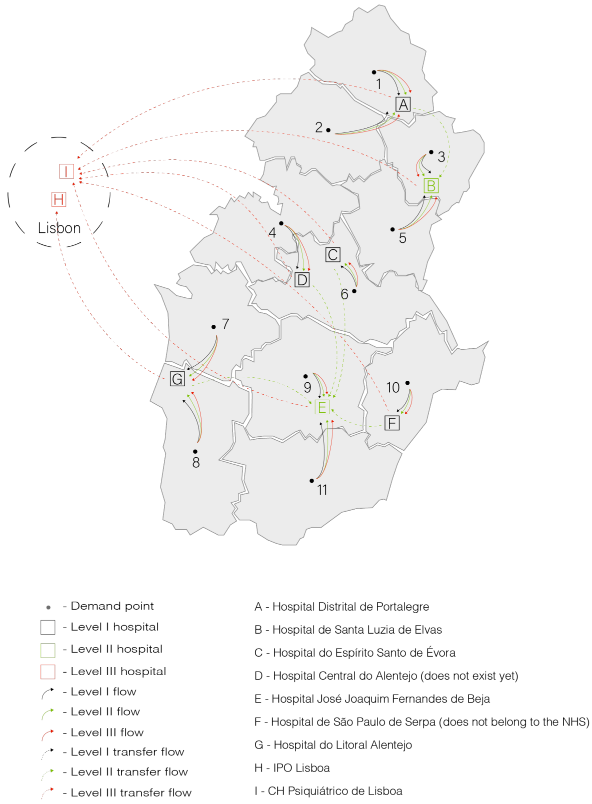

Figure 4: Model results for situation A* in case 2 - stochastic results with minimization of objective function 1 (minimization of travel times). Demand points 1, 2 and 3 correspond to Alto Alentejo; 4, 5, 6 correspond to Alentejo Central; 7 and 8 correspond to Alentejo Litoral; and 9, 10 and 11 correspond to Baixo Alentejo

Especially, hospitals providing higher level and specialized care. Another conclusion is the fact that the choice of the size and location of demand points can influence greatly the solution. In terms of robustness and reliability, the model was tested for different numbers of demand and values for parameters. The solutions did not differ too much from each other so it is possible to say the model is quite robust and reliable. Although, the model must be tested with more variations of inputs in order to reach that conclusion with a higher degree of certainty.

In the future, it would be interesting to solve the model for a superior number of N. It should be noted that for these cases it may be necessary to use heuristics or metaheuristics, since including more specialties (or more referral networks) would increase the dimension of the problem. Furthermore, since the model presented does not allow transfers between medical specialties, it would be interesting to incorporate that in the model. It would also be of interest to work alongside decision makers in this field to get their inputs about their preferences and the values for some weights (e.g. α weight, or objective function weights if the bi-objective model is solved through Weighted Sum method (WSM)).

Finally, it can be concluded that location-allocation modeling is a broad topic that has multiple promising and yet unexplored questions. More specifically, location-allocation models in the health sector can be an extremely useful tool to aid in the government's decision-making and to help increase equity in health care.

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