Planning hospital networks

A case study of the hospital network in Portugal

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Dedicated to my parents.

Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Preface

The work presented in this thesis was performed at Instituto Superior Técnico (Lisbon, Portugal), during the period March-October 2021, under the supervision of Prof. Ana Póvoa and Prof. Daniel Santos.

Acknowledgments

Firstly, I would like to thank my supervisors for all their help. Thank you to Prof. Ana Póvoa for the helpful advice and guidance in every meeting. Thank you to Prof. Daniel Santos for all the patience and tireless help with every question and problem I had. Secondly, I want to thank Maria Lopes who read through my thesis, multiple times, and gave very valuable insights every time. Thirdly, I would like to thank my friends who helped me, not only in these past months but the last five years. And last, but not least, I want to thank my parents, my siblings and my family for their constant love and support.

Abstract

One of the most important goals in NHS-based countries is to ensure the efficient provision of healthcare services to its population while balancing costs and access. Thus, planning an optimized hospital network is crucial for providing good quality healthcare, since decisions related with the location of the hospital, demand allocation and installed capacity directly impact the daily activities of the hospitals and, consequently, the service level of the healthcare. This thesis aims to develop and implement an optimization approach to plan a hospital network, within the scope of a National Health Service, considering relevant aspects of hospital networks and apply it to a real case study in the Portuguese health system. In order to do this, a bi-objective mixed-integer linear programming model is presented in which two objective functions are minimized. The first one minimizes expected travel time to reach hospitals weighted by demand, which relates to improvement in access to healthcare. The second one minimizes expected operational and investment hospital costs, which relates to efficiency. Uncertainty in the demand for service was also incorporated. The model was applied to the national continental network of Cardiology inpatient service and to the Regional Health Administration of Alentejo's network of Internal Medicine inpatient service. The results demonstrated that decentralizing care can improve geographical access and reinforced the need to make a compromise between equity in access to healthcare and costs.

Keywords

Hospital Referral Network, Hospital Location, Demand Allocation, Multi-Objective Programming, Uncertainty Modelling, Operational Research in Healthcare

Resumo

Um dos objetivos mais importantes num país com um sistema de saúde baseado num Serviço Nacional de Saúde é garantir a prestação eficiente de serviços hospitalares à população, equilibrando custos e acesso. Assim, o planeamento de uma rede hospitalar otimizada é crucial para prestar serviços de saúde de boa qualidade, visto que decisões relacionadas com a localização do hospital, a alocação da procura e a capacidade instalada de um hospital impactam diretamente as atividades diárias dos hospitais e, consequentemente, o nível de serviço dos cuidados hospitalares. Esta dissertação pretende desenvolver e implementar uma abordagem de otimização ao planeamento de uma rede hospitalar, no âmbito de um Serviço Nacional de Saúde, considerando aspetos relevantes característicos das redes hospitalares e aplicá-la a um caso de estudo real dentro do sistema de saúde português. Desta forma, foi criado um modelo de programação linear inteira mista bi-objetivo que minimiza duas funções objetivo. A primeira minimiza o tempo de viagem para obter serviços hospitalares ponderado pela procura, relacionando-se assim com a melhoria do acesso a cuidados de saúde. A segunda minimiza os custos dos hospitais, relacionando-se portanto com a eficiência. A incerteza relativa à procura foi também incorporada. O modelo foi aplicado ao serviço de internamento na especialidade de Cardiologia em todo o território continental português e ao serviço de internamento na especialidade de Medicina Interna na Administração Regional de Saúde do Alentejo. Os resultados obtidos demonstraram que a descentralização dos serviços de saúde pode melhorar o acesso geográfico a esses serviços e evidenciaram a importância de haver um compromisso entre equidade de acesso à saúde e custos.

Palavras Chave

Rede de Referenciação Hospitalar, Localização de Hospitais, Alocação de Procura, Programação Multi-Objetivo, Modelação de Incerteza, Investigação Operacional aplicada à Saúde

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Acronyms

ACS	Alto Comissariado da Saúde
ACSS	Administração Central do Sistema de Saúde
DGH	Directorate-General for Health
DGS	Direção-Geral da Saúde
EPE	Entidade Pública Empresarial
ERS	Entidade Reguladora da Saúde
EU	European Union
GDP	Gross Domestic Product
HFLP	Hierarchical Facility Location Problems
HIV	Human Immunodeficiency Virus
LP	Linear Programming
LHU	Local Health Units
LTV	Lisbon and Tagus Valley
LTC	Long-term Care
MILP	Mixed-Integer Linear Programming
NHP	National Health Plan
NHS	National Health Service
NUTS	Nomenclatura das Unidades Territoriais para Fins Estatísticos
OR	Operational Research
RHA	Regional Health Administrations

Chapter 1

Introduction

This chapter gives an introduction to the problem of hospital network planning. In section 1.1, the problem of hospital network planning is contextualized and the motivation for this thesis is presented. Section 1.2 describes the main objectives of the dissertation. Section 1.3 explains the methodology with the specific steps taken to reach the objectives. Finally, section 1.4 details the outline and structure of the document.

1.1 Contextualization and motivation

Health is one of the fundamental rights of every human being and the health of all people is crucial to the attainment of peace and security (WHO, 1946). To improve and maintain the health of the individual and the population, access to healthcare is one of the most important factors. Good healthcare needs human and material resources and intelligent planning to evaluate how to best use those resources. The need for better organization and management is ever growing because the need for healthcare is also growing (Watts and Crimmins, 2008). Older populations are increasing worldwide and older people have higher per capita healthcare needs than young people for most types of healthcare (Watts and Crimmins, 2008). For these reasons, the demand for tools to help make better, and more informed, decisions is high.

Planning a hospital network is an extremely important task in healthcare. Every person needs healthcare services and that care is best provided when hospitals are placed in optimal locations and have sufficient resources to serve the demand. Decisions like hospital location and demand allocation are often involved in the strategic planning of a network of hospitals and they directly impact the life of the patients. Questions like ”How long should a person take to get to a hospital?”, ”How large should a hospital be?”, ”Should a transfer be necessary, to which hospital should a patient be transferred?” and ”How many hospitals should a certain region/city/country have?” are key to this type of planning. Finding the balance between budget constraints and equity in access is often the main problem location theory has to deal with. Therefore, location decisions have a significant impact in the community around them. In healthcare, poor facility location decisions can have serious consequences that might not be immediate or obvious but are nonetheless there. For example, increased mobility and mortality rates have been proven to be associated to difficult access to healthcare facilities (Ahmadi-Javid et al., 2017).

Over the last few years, the organization of hospitals in Portugal has gone through some changes. With the goal of improving Portuguese healthcare, the focus has been on building a connected network of hospitals that provides healthcare in a coherent manner and is based on principles of rationality, complementarity and efficiency (Ministério da Saúde, 2021a). Due to recent modifications (explained in the next chapter), there is a lack of updated investigation that depicts the present state of healthcare services in Portugal. According to the research done for this thesis, and until the time of completion and delivery of this work, there is no model for the planning of hospital networks that considers hospitals as multi-level structures according to medical specialties, adapted to the Portuguese case. This being said, the present dissertation aims to do that, following the objectives presented in the next section.

1.2 Objectives

The main goals of this study, in the context of supplying hospital healthcare services in a country with a National Health Service (NHS), are to:

- Develop a mathematical model to support decisions concerning planning hospital networks;
- Optimize hospital services to improve access to those services, taking into account efficiency and cost issues.

In order to do this, a mathematical model that captures the characteristics of the hospital network in the Portuguese NHS is designed. Although the model is based on the Portuguese case, it can be generalized to any hospital network, which adds to the value of the work developed. A solution approach is also explored to solve the model.

1.3 Methodology

In order to reach the goals stated in the last section, some steps have to be taken. This section presents those steps, represented graphically in figure 1.1.

1. Describe the case study in the Portuguese hospital network, highlighting the nature of hospitals as multi-service suppliers, organized in a hierarchical structure with ascending (and descending) flows of patients and with limits on installed capacity;

2. Review the most relevant literature about this problem, analysing the contributions of Operational Research (OR) to facility location using Mixed-Integer Linear Programming (MILP), detailing models and solution approaches;

3. Propose a mathematical model in Mixed-Integer Linear Programming that represents the case study presented, but that is also general enough to be applied in other contexts (e.g. other NHS), and use it, in order to obtain suggested location sites for hospitals and respective capacities, services, medical specialties and flows;

4. Implement the model with data from real situations that prove its applicability to the problem at hand, analyzing the obtained results and discussing its limitations.

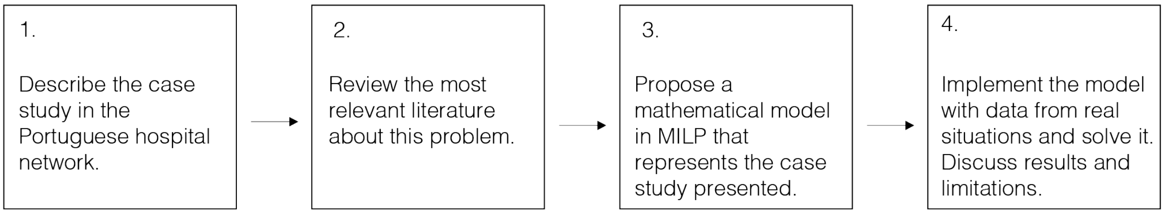

Figure 1.1: Thesis methodology.

With the objectives of the thesis clear, the next section introduces its organization and structure.

1.4 Thesis outline

The present thesis has the following structure:

- **Introduction** The present chapter aims to introduce, contextualize and motivate the reader, explaining also the objectives of the work;
- **Case Study** The second chapter characterizes healthcare in Portugal, what it was in the past and what it is now, and the existing network of hospitals;
- **Literature Review** The third chapter presents a review of the most relevant scientific articles about the problem to be studied;
- **Mathematical Model** The fourth chapter details the notation and formulation of the mixed-integer linear programming model designed;
- **Results and Discussion** The fifth chapter describes the application of the model to the case study and the computational results are obtained and discussed;

- **Conclusions and Future Work** The sixth, and final, chapter summarizes the most important conclusions and gives some suggestions regarding future work on the topic at hand.

Chapter 2

Case Study

This chapter aims to characterize the problem at hand by representing the existing health system in Portugal and detailing how it evolved along the years to what it is today. Thus, providing context to this thesis. Section 2.1 presents general geographic and demographic information about Portugal. It also highlights a few key points in the history of health in the country and describes the current organization of the Portuguese Health System. Section 2.2 discusses objectives in the health industry and how they might differ from objectives in other industries. It also briefly reviews the role of equity and accessibility in health. Finally, section 2.3 presents the conclusions of the chapter and connects the two previous sections.

2.1 Healthcare in Portugal

The present section aims to introduce an overview of the health care in Portugal, including some notions about Portuguese geography and demography in section 2.1.1, the context and current organization of the health system in section 2.1.2, the hospitals referral networks in section 2.1.3 and a brief characterization of the Regional Health Administration of Alentejo in section 2.1.4.

2.1.1 Geography and demography

Currently, the Portuguese territory, excluding the autonomous regions, is divided into 18 districts. The district unit is the most socially relevant subdivision of the country but there are others that are often used in the health literature. The European Statistical Office proposes a Nomenclature of Territorial Units for Statistics - *Nomenclatura das Unidades Territoriais para Fins Estatísticos (NUTS)* - for statistical purposes that divides the country into different regions (Ministério do Planeamento e da Administração do Território, 1989). This nomenclature comprises three levels of aggregation of the Portuguese territory

(NUTS I, II and III). The broadest subdivision is between continental Portugal and the two autonomous regions of the Azores and Madeira (NUTS I). These can be further subdivided into 7 regions (NUTS II), 5 in mainland Portugal and 2 that constitute the archipelagos. This five region division of mainland Portugal (North, Centre, Lisbon Metropolitan Area, Alentejo and Algarve) is frequently used in statistics to aggregate data about the population (Ministério do Planeamento e da Administração do Território, 1989). In 2019, the estimated number for the resident population of Portugal was 10 295 909 people according to Instituto Nacional de Estatística (2020a). The North region is the most populated area with 3 575 338 people, followed by the Lisbon and Tagus Valley (LTV) region (2 863 272 people) and Centre region (2 217 285 people). The Alentejo (704 558 people) and Algarve (438 406 people) regions are the less populated regions.

2.1.2 Portuguese Health System: Historical context and current organization

Until 1974, the Portuguese Health System was not a system but a combination of several scattered social institutions, public and private health services and state hospitals. Throughout the 1960s, Portugal had some of the worst values of health indicators compared with the other European Union (EU) countries (Baganha et al., 2002). It performed poorly regarding population coverage, it had higher child mortality rates and it spent less of its Gross Domestic Product (GDP) on health than the EU average (Baganha et al., 2002). In the next decade, the Portuguese society went through some significant shifts including a very important revolution in 1974, followed by profound social changes that also affected the health sector. It was in this environment that the National Health Service (NHS) was born. It was officially established in 1979 and the goal was to provide a universal, general and free health service (República Portuguesa, 2005). Years later, in 1990, the ”Health Basic Law” (”Lei de Bases de Saúde”) introduced the establishment of the Regional Health Administrations (RHA) (Ministério da Saúde, 1990), which will be explained in a later section. After 1990, the Portuguese health system could be characterised by three clearly individualized and articulated segments: the NHS which encapsulated all institutions and official health care service providers conditioned by the Ministry of Health; every public entity that developed activities of promotion, prevention and treatment in health; and all private entities that also provided health care services (Baganha et al., 2002).

On the subject of health systems, the Beveridge and Bismarck models come easily to mind. The health systems based on the Beveridge model are usually NHS-based, funded by general taxation and exist in many European countries (e.g. Spain, Italy, UK, Denmark, Sweden, Greece). Some characteristics of this type of health system are the provision of universal coverage and the free or nearly free access to health services at the point of use. On the opposite side, lie the social insurance based systems or Bismarckian-based models, which are funded by pay-roll contributions (e.g. Benelux countries, France, Germany, Austria) (Freeman, 1998). Each system obviously has its advantages and disadvantages. NHS-based systems are reported to allow a greater degree of financial equity among users. However, social insurance systems are usually the ones associated with higher user satisfaction. This can be explained due to the possibility of choice of provider and the generation of higher levels of services of high quality encouraged by the fee-for-service payments (Elola, 1996). In fact, most of the problems identified in countries with an NHS are related to the quality of care: long waiting lists, difficult access to specialists due to gatekeeping, inefficient management and limitations in terms of the choice of the provider (Freeman, 1998).

In the context of the NHS framework, decentralization is an important concept. It has been defined ”in public planning, management and decision-making as the transfer of authority and power from higher to lower levels of government or from national to subnational levels” (World Health Organization, 2007). Many European countries regard it as an effective way to improve service delivery, to bring the community into health decision-making, to improve allocation of resources according to needs and to reduce health inequalities (Simões and Hernández-Quevedo, 2017). In Portugal, this is accomplished by the RHA. There are five RHA in total (North, Centre, Lisbon and Tagus Valley, Alentejo and Algarve) and they are responsible for the management of the NHS at a regional level (seen in figure 2.1). They coordinate everything regarding health care provision within the scope of their respective territorial circumscription. They are responsible for executing national health policies, helping the preparation of the National Health Plan (NHP) and monitor its implementation, ensure the regional planning of resources (human, financial and material), establishing and reviewing contracts within the scope of public-private partnerships, etc Ministério da Saúde (2012).

Despite the legal and political commitments to social rights, health inequalities caused by some social determinants are still a large concern in Portugal's NHS (Simões and Hernández-Quevedo, 2017). One of these determinants is income. Low income populations face more challenges when paying for medicine and when accessing health care not covered by the NHS. Health literacy is another factor. Older individuals and/or individuals with a low education level may experience more barriers in accessing the internet and, as a consequence, information regarding health available online. Besides income and health literacy, Simões and Hernández-Quevedo (2017) argues that geography is another factor contributing to the health inequities verified. Due to the insufficient supply of healthcare services in the interior, more rural, regions of Portugal, people from these localities experience more difficulties in accessing these services when compared with people who live closer to cities. This represents a considerable gap in the provision of care to elderly populations since these regions have a larger percentage of older populations (Instituto Nacional de Estatística, 2020a). Therefore, solutions to this problem are a frequently studied topic. In a recent study about disparities in geographical access to hospitals in Portugal, it was shown that municipalities with a higher percentage of older people, isolated communities and municipalities close to the Spanish border are the ones with worst health access (Costa

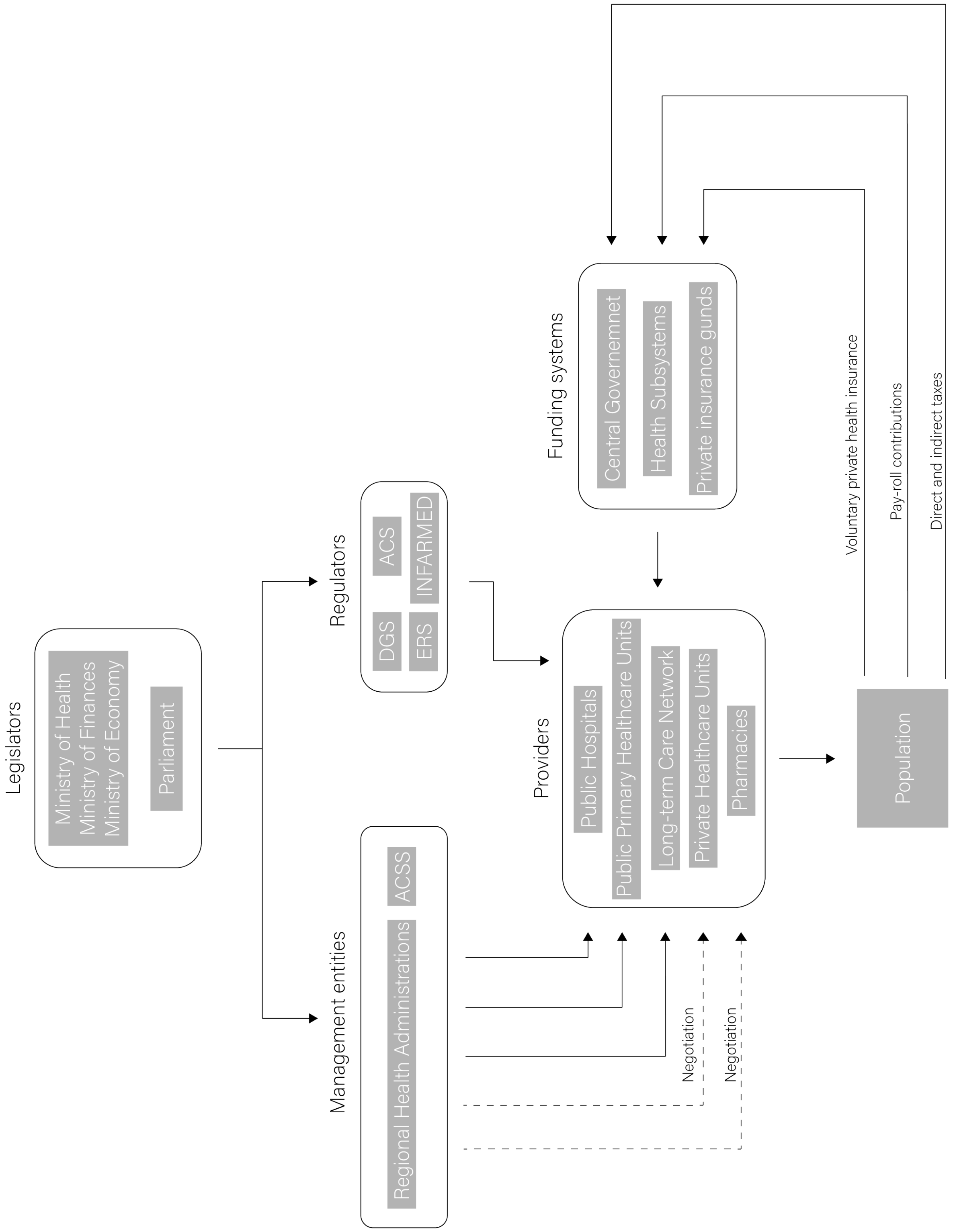

Figure 2.1: General overview of the Portuguese health system (Administração Central do Sistema de Saúde (ACSS), Direção-Geral da Saúde (DGS), Alto Comissariado da Saúde (ACS) and Entidade Reguladora da Saúde (ERS) correspond to Portuguese initials/acronyms).

Source: Author's own compilation based on "Fig. 2.1 - Overview chart of the health system" in Simões and Hernández-Quevedo (2017) and on "Figura 8 - Principais stakeholders no sector da saúde em Portugal" in Deloitte (2011).

et al., 2020). Despite these disparities, the authors verify that there has been some improvement in the last 20 years in relation to the average time it takes to get from a municipality to a hospital (25 minutes, 3 minutes less than in 1991).

To be able to bridge this gap and walk towards a more equitable society, careful and intelligent planning and regulation are fundamental. Healthcare planners in countries with an NHS have to make several decisions in terms of hospital location, organization and resource allocation to reach certain policy objectives (e.g. geographic equity of access, quality and efficiency while minimizing costs) (Mestre et al., 2012). A task that is usually complex because some of these goals can be conflicting. Improving geographical access may require building smaller hospital facilities closer to the populations, which can lead to higher inefficiencies and costs (Mestre et al., 2015). In addition to the hospital location problem, there are several other that need to be thought of. Some examples would be optimal structure for a hospital network, the best way to define a hospital's catchment area[1] and the limits in the costs required to improve access (Mestre et al., 2012).

Planning under a budget implies making trade-offs between generally two or more relevant aspects. In the Portuguese case, the Ministry of Health allocates its budget to the different institutions of the NHS. This allocation of funds is done based on a mix of historical expenditure and capitation (Simões and Hernández-Quevedo, 2017). This is why collecting data and understanding the current world tendencies are extremely important to make choices that could save resources and, consequently, decrease costs. According to World Health Organization (2000), the long-term care sector has been growing across European countries. The longer life expectancy, the rise in chronic illness' prevalence and the increasing participation of women in the workforce (which leaves them no longer available to give care) have contributed to this growth (World Health Organization, 2000). Currently, the supply is not enough to meet the demand, which ultimately leads to inefficiencies and higher costs since patients who actually require Long-term Care (LTC) are consuming health resources from the acute care sector (Cardoso et al., 2016).

2.1.3 Hospital referral networks

In 2019, there were 238 hospitals in Portugal. Of these 238 hospitals, 127 were private hospitals (45.4%), 108 were public hospitals (53.4%) and 3 were public-private partnerships. Of the 108 public hospitals, 103 were universal access hospitals and 5 were military or prison hospitals (Instituto Nacional de Estatística, 2019a). This translates into approximately 1 universal access hospital per 100 000 inhabitants. These hospitals can belong to a group of hospitals (hospital centers) or can function independently. Other entities also responsible for providing healthcare to the population are the Local

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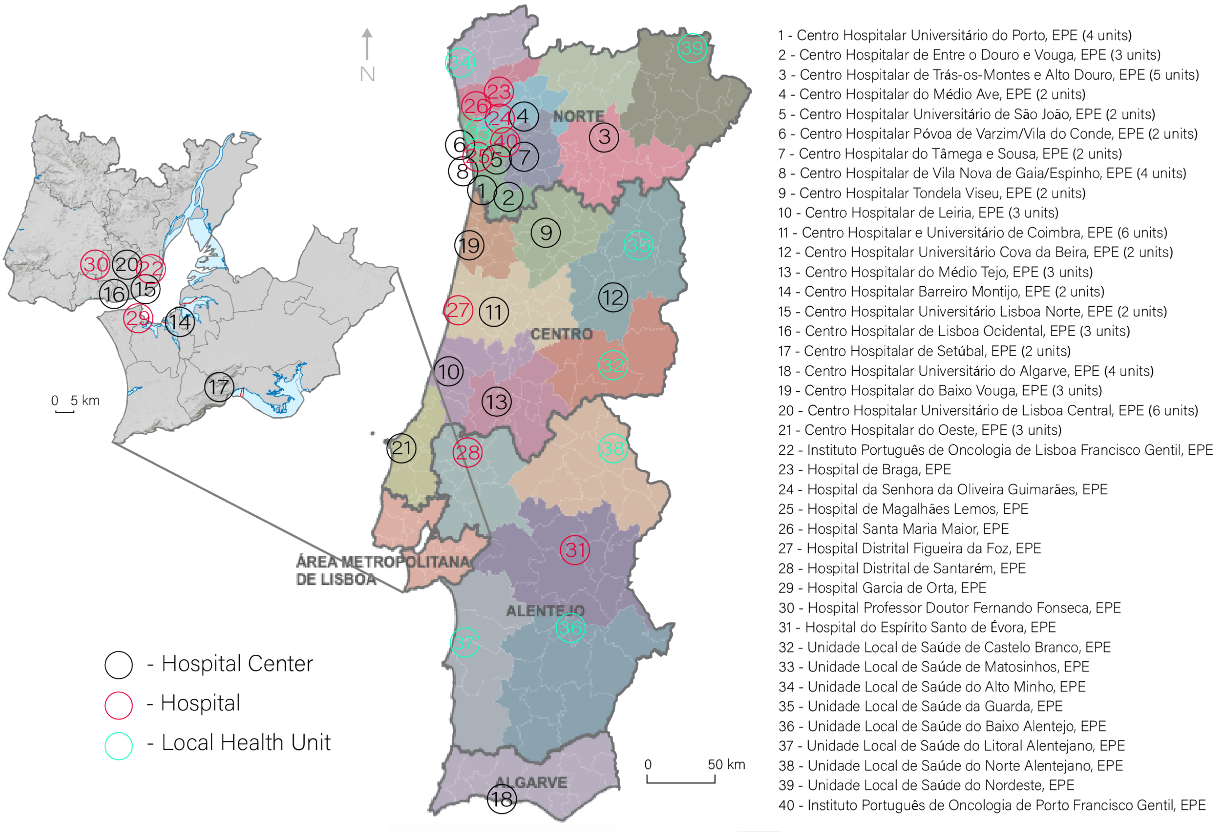
[1] Hospital catchment areas are the areas whose population is served by that hospital. The definition of these areas can be done using different methods (e.g. K-means clustering)(Gilmour, 2010).

Health Units (LHU). The goal of the LHU is to assemble in one health unit the care provision for a given population, improving the connection between primary and hospital care provision (and eventually long-term care). There are 40 entities (hospitals, hospital centers and LHU) in the public business sector *Entidade Pública Empresarial (EPE)*. The names and locations of these entities are depicted in figure 2.2 (Ministério da Saúde, 2021b). As it was previously mentioned, there is higher density of healthcare providers in larger cities (Lisbon and Porto) and along the coast. Consequently, the regions of Algarve, Alentejo and Norte (east side) have a very short supply of healthcare services.



1 - Centro Hospitalar Universitário do Porto, EPE (4 units)
2 - Centro Hospitalar de Entre o Douro e Vouga, EPE (3 units)
3 - Centro Hospitalar de Trás-os-Montes e Alto Douro, EPE (5 units)
4 - Centro Hospitalar do Médio Ave, EPE (2 units)
5 - Centro Hospitalar Universitário de São João, EPE (2 units)
6 - Centro Hospitalar Póvoa de Varzim/Vila do Conde, EPE (2 units)
7 - Centro Hospitalar do Tâmega e Sousa, EPE (2 units)
8 - Centro Hospitalar de Vila Nova de Gaia/Espinho, EPE (4 units)
9 - Centro Hospitalar Tondela Viseu, EPE (2 units)
10 - Centro Hospitalar de Leiria, EPE (3 units)
11 - Centro Hospitalar e Universitário de Coimbra, EPE (6 units)
12 - Centro Hospitalar Universitário Cova da Beira, EPE (2 units)
13 - Centro Hospitalar do Médio Tejo, EPE (3 units)
14 - Centro Hospitalar Barreiro Montijo, EPE (2 units)
15 - Centro Hospitalar Universitário Lisboa Norte, EPE (2 units)
16 - Centro Hospitalar de Lisboa Ocidental, EPE (3 units)
17 - Centro Hospitalar de Setúbal, EPE (2 units)
18 - Centro Hospitalar Universitário do Algarve, EPE (4 units)
19 - Centro Hospitalar do Baixo Vouga, EPE (3 units)
20 - Centro Hospitalar Universitário de Lisboa Central, EPE (6 units)
21 - Centro Hospitalar do Oeste, EPE (3 units)
22 - Instituto Português de Oncologia de Lisboa Francisco Gentil, EPE
23 - Hospital de Braga, EPE
24 - Hospital da Senhora da Oliveira Guimarães, EPE
25 - Hospital de Magalhães Lemos, EPE
26 - Hospital Santa Maria Maior, EPE
27 - Hospital Distrital Figueira da Foz, EPE
28 - Hospital Distrital de Santarém, EPE
29 - Hospital Garcia de Orta, EPE
30 - Hospital Professor Doutor Fernando Fonseca, EPE
31 - Hospital do Espírito Santo de Évora, EPE
32 - Unidade Local de Saúde de Castelo Branco, EPE
33 - Unidade Local de Saúde de Matosinhos, EPE
34 - Unidade Local de Saúde do Alto Minho, EPE
35 - Unidade Local de Saúde da Guarda, EPE
36 - Unidade Local de Saúde do Baixo Alentejo, EPE
37 - Unidade Local de Saúde do Litoral Alentejano, EPE
38 - Unidade Local de Saúde do Norte Alentejano, EPE
39 - Unidade Local de Saúde do Nordeste, EPE
40 - Instituto Português de Oncologia de Porto Francisco Gentil, EPE

Figure 2.2: Map of continental Portugal with the location of hospital centers, hospitals and local health units. Zoom in on the Lisbon Metropolitan Area.

Source: Author's own compilation based on existent maps of Portugal and of the Lisbon Metropolitan Area.

In terms of complexity, hospitals are not all the same. They differ in terms of amount of resources, technological complexity, size of catchment population and type of care (from basic to specialized). With respect to supply, hospitals can be viewed as having a multi-service structure, as depicted in figure 2.3, since they offer three main types of services: inpatient care (patients whose condition requires admission to a hospital for a period longer than 24 hours), emergency care and outpatient care. Outpatient or ambulatory care can be divided into external consultations and surgical interventions (where the patient is discharged no more than 24 hours after the procedure). According to the annual report about access to healthcare in NHS's establishments (Serviço Nacional de Saude, 2019), the number of outpatient surgical interventions has been growing for several years. In 2019, the percentage of these type of surgeries was 66.1%, while in 2010 was 49.5% and 2000 was 10%. A bigger percentage of ambulatory interventions equals to a smaller percentage of hospital beds occupied by inpatients, which ultimately leads to lesser costs and better use of hospital resources. Still with respect to supply, hospitals can be classified as multi-service facilities since they offer care in the different medical fields, or as they are more commonly known, in medical specialties (not represented in 2.3). Each service in a hospital can have physicians from the various medical specialties available to treat the patients that arrive at a hospital.

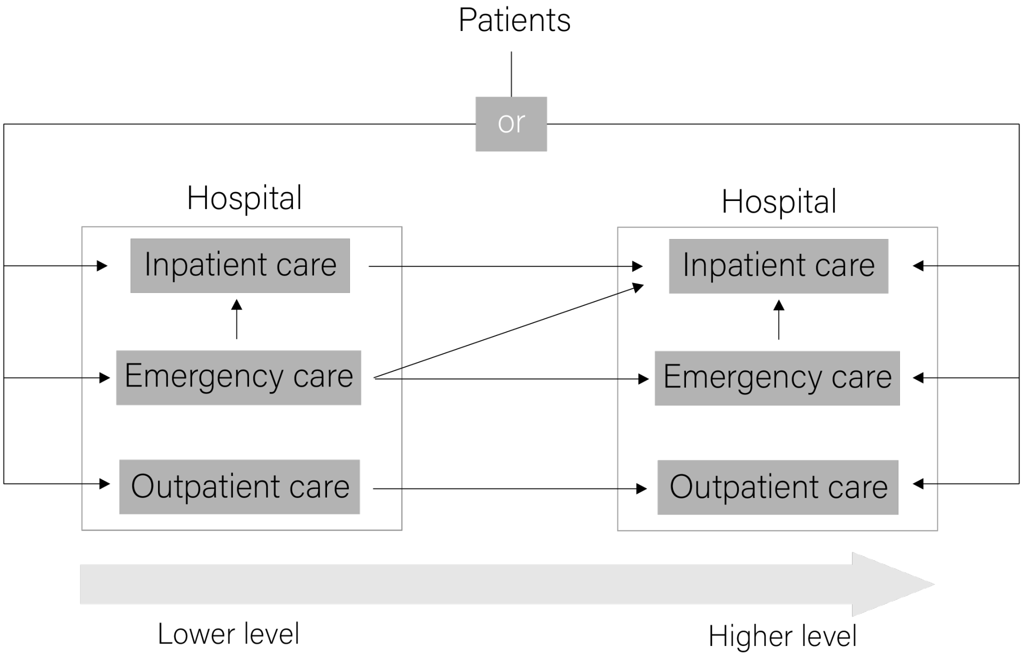


Figure 2.3: Representation of hospitals as multi-service facilities in a hierarchical structure. The diagram represents the flow between services (intra- and inter-hospital) regarding only one medical specialty.

The point of entry into the Portuguese NHS is either through a primary care consultation or through emergency care (in the case of an emergency episode), meaning Portuguese healthcare is subject to a gatekeeping referral process (Simões and Hernández-Quevedo, 2017). Ideally, every NHS user should be included in a general practitioner's list, preferably one in their geographical area of residence. From a primary care consultation, the general practitioner can refer the patient to another more specialized practitioner.

Even before the establishment of the NHS in 1979, there were already attempts at classifying hospitals based on a hierarchical structure and according to geographical and hospital size criteria. But it was only after that year that hospital organization truly evolved. In 1986, according to the minister of health at the time, some guidelines were defined in a document named *Carta Hospitalar Portuguesa*. This document was never fully implemented but the concepts developed there were applied in the *Estatuto do SNS* in 1993, which determined that facilities and NHS integrated services should be classified in accordance with the nature of their responsibilities and their valence chart. In 2008, the emergency services that would be nodes in the emergency referral network (*Rede de Referenciação de Urgência/Emergência*) were defined and classified, creating an articulated and hierarchical network of emergency services.

Until 2014, the public hospitals could be classified as central, district, county or specialized. However, on that year, the classification system changed. The new system required the distribution of hospitals into four groups (Group I, Group II, Group III and Group IV), according to the complexity of the services supplied to their population (guaranteeing proximity and hierarchy of care). Two years later, this classification system changed again since it presented some flaws in its conceptual elaboration and, consequently, in its applicability. Thus, in 2016, a new updated model based on the hospital referral networks emerged. This model attempted to be more efficient and sustainable and it is still being used today. Now, hospitals, hospital centres and local health units are classified in groups according to the respective medical specialties developed, population covered, training ability, human resources differentiation, financing model, emergency services classification and complexity of hospital production. The characteristics of each medical specialty are detailed in a document called ”National Network of Hospital Specialty and Referral”, which is specific for each specialty. This way, a hospital can be on a different level/group depending on the medical specialty in question and patients referral between hospital facilities of the NHS follows the rules of each hospital referral network. The criteria that distinguishes the different levels for a medical specialty are several and are dependent on the medical specialty itself. As mentioned above, some factors that place a medical specialty in a hospital on a certain level are population coverage, ability to diagnose and treat certain pathologies, inpatient capacity, number of specialized professionals, presence of other medical specialties in the hospital, special equipment and/or rooms, ability to perform ambulatory surgery, ability to keep emergency services open every day. In conclusion, a certain hospital can have level *X* in medical specialty *A*, level *Y* in medical specialty *B* and so on so forth for every medical specialty in that hospital. It is common that a larger hospital in a city like Lisbon or Porto ranks at higher levels in most medical specialties since they have more capacity and resources. In the present day there are 33 approved hospital referral networks. Apart from those, 3 are being revised and 1 is being created (Ministério da Saúde, 2021a).

One important note about the notation that will be used throughout the thesis, especially in chapter 4, is the distinction between the terms medical specialty and referral network. A medical specialty is a branch of medicine that focuses on a defined group of patients or diseases. A referral network is an integrated system of connected hospitals, health facilities and health professionals that guarantees the best care for the patient in a certain medical specialty. In other words, the referral network is what dictates the fate of the patient when he needs care (to which hospital he needs to go or to which hospital he is transferred) from a certain medical specialty. Consequently, there is not a direct correspondence between the two concepts. Each medical specialty can have a referral network (not every specialty has one but it is a possibility). But each referral network does not need to correspond to a medical specialty. It is the case of the referral network for Human Immunodeficiency Virus (HIV) infection. In Portugal, HIV infection on its own does not correspond to a medical specialty, it is integrated in another specialty. Even though the two expressions mean different things, sometimes they are used interchangeably throughout the thesis.

2.1.4 Characterization of the Regional Health Administration of Alentejo

The model defined in chapter 4 was solved for two cases in the Portuguese health system. The first involved all the national continental territory and the second zoomed in on the Regional Health Administration of Alentejo. To provide some context for the latter, this sub-section will give a brief overview of this RHA.

The RHA of Alentejo is one of the five RHA. The geographical area of this region, represented in figure 2.4, is divided into four NUTS: Alto Alentejo, Alentejo Central, Alentejo Litoral and Baixo Alentejo. This corresponds to an area of about 27 330 square kilometres and to an estimated resident population of 478 110 people (about 5% of the total Portuguese continental population) (Administração Regional de Saúde do Alentejo, 2019).

In terms of providing hospital care, this region is covered by different LHU. The sub-region of Alto Alentejo is covered by the *Unidade Local de Saúde do Norte Alentejano*; the sub-region of Alentejo Central is served by the *Hospital do Espírito Santo de Évora*, the sub-region of Baixo Alentejo is covered by *Unidade Local de Saúde do Baixo Alentejo*; and the Alentejo Litoral is served by the *Unidade Local de Saúde do Litoral Alentejano*. Alentejo is one of the regions with less public hospitals per geographic location, second only to Algarve (Instituto Nacional de Estatística, 2019b). This can be explained by the lower population density in this region, as mentioned earlier. According to a study done by the Portuguese National Institute of Statistics, in 2080, the number of old people (65+ years) will go from 2.2 million to 3 million and the population in Portugal will have decreased by more than 2 million people (Instituto Nacional de Estatística, 2020b). Some regions, like the Lisbon Metropolitan Area and Algarve, are predicted to grow in population size and other regions, like Alentejo, are predicted to decrease in population size.

According to the 2014 definition, all but one of the five, public owned, hospitals located in this health region, are in level I, as can be seen in figure A.2. There is only one hospital that is ranked at level II:

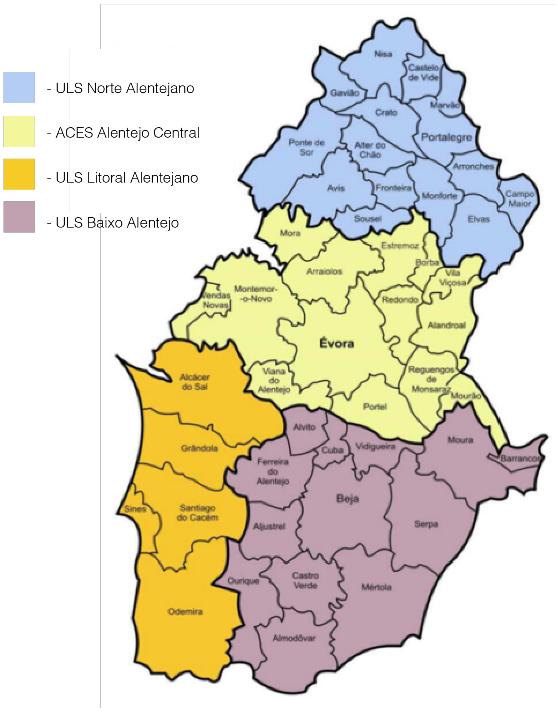


Figure 2.4: Geographic area of Alentejo's Regional Health Administration.

Source: Author's own compilation based on map in Administração Regional de Saúde do Alentejo (2019).

Hospital do Espírito Santo de Évora. This means this is the only hospital, in close proximity, that can provide higher level care to Alentejo's population. Even so, the patients with the most severe conditions have to be transferred to hospitals in Lisbon. Due to this reason, in the past few years, the quality of service and, consequently, the level of satisfaction of the patients has decreased (Administração Regional de Saúde do Alentejo, 2019). Therefore, the need for a new hospital arose and a proposal was submitted. The new hospital - *Hospital Central do Alentejo* - to be built in Évora, is supposed to complement the older hospital, improving access to highly differentiated health care for the population of Alentejo (Administração Regional de Saúde do Alentejo, 2019). The predicted date of of opening is in 2023. Lastly, it is important to note there is one hospital in this region - *Hospital de São Paulo* - that was once publicly managed but, currently, does not belong to the NHS. Some political parties want to reverse this decision and integrated it again in the National Health Service (Grupo Parlamentar, 2021).

With the presentation of an overview of health in Portugal, the next section explores some goals that are important in healthcare.

2.2 Goals in healthcare

This section presents important objectives in healthcare in section 2.2.1 and some considerations about accessibility and equity in section 2.2.2.

2.2.1 Objectives in the Health Industry

Objectives in the healthcare industry differ from objectives in other industries. Profit maximization or costs minimization cannot be the main concern in the public health industry. Improving health care quality and access are some of the most common goals when drafting health policies and guidelines. In the strategic plan for the years 2020-2022 (*"Plano Estratégico da DGS 2020-2022"*) (Direção-Geral de Saúde, 2020), the Directorate-General for Health (DGH) defined "Ensuring an Integrated Approach to Health Planning and Intervention" as one of the six strategic objectives. Another objective particularly relevant in the current pandemic context is the "Public Health Emergency Preparation and Response". Other objectives combine improving aspects of quality, safety, humanization of healthcare, together with strengthening health monitoring and promoting health literacy.

Besides this, there is the NHP, also designed and implemented by the DGH, which sets the main guidelines, strategies and goals for the country, for a given period of time (Simões and Hernández-Quevedo, 2017). This suggests to the Ministry of Health a minimum set of health system activities to be put into effect. The NHP has four main principles: health citizenship, equity and adequate access to health care, healthy policies and health quality. In alignment with those values, the plan in place until last year (Direção Geral de Saúde, 2015) established some goals including annual reduction of premature (before 70 years of age) mortality and the increase in healthy life-years by at least 30% for both men and women. Objectives that can be achieved by health promotion and prevention as well as better access to hospital care.

2.2.2 Accessibility and Equity

Until 1970, considerations on equity were not addressed in facility location literature, efficiency measures being, by far, the most studied (Mcallister, 1976). In the years after, equity started to be incorporated in location models (Mcallister, 1976; Savas, 1978) and, since then, it has become a key word in every health policy maker's discourse. Nowadays, even though it is heavily discussed, there is still no single, universal definition of equity and what it means to have an equitable distribution of health care resources among a certain population (Culyer, 2001). With the aim of presenting policy makers with a useful and operational concept of equity, Oliver and Mossialos (2004) outlined three principles of equity: equal access to healthcare for those in equal need of healthcare, equal utilization of healthcare for those in equal need of healthcare and equal (or equitable) health outcomes. The first principle being the one most appropriate to pursue by policy makers, according to the International Forum on Common Access to Health Care Services (Oliver and Mossialos, 2004). Last year, a study was published synthesizing the effects of barriers and enablers of the implementation of clinical commissioning policy on the success of the English NHS efforts to reduce health inequalities (Regmi and Mudyarabikwa, 2020).

In terms of measuring accessibility, there is no consensus on a unique way to measure how accessible a service is. Some authors find it helpful to break the concept of accessibility down into several elements. For example, Joseph and Phillips (1984) distinguish between two aspects of accessibility: physical and socio-economic. Location theory is concerned with the former. A service is considered geographically or physically accessible, if there is, in fact, a supply of said service in a certain area and the means to reach it are available. The other, no less important, type is socio-economic accessibility. It measures the affordability of the service, whether people regard it as appropriate and whether they are permitted to use it. There has been some articles reviewing some of the most relevant works on this subject. For example, Marsh et al. (1995) compiled some of the earliest works and Wang (2012) reviewed published works focused on measurement, optimization and the impact of healthcare accessibility.

Studying accessibility is crucial in a country like Portugal that relies a great deal on the presence of a NHS. Not only because allocating resources among the population in a an equitable manner is challenging, specially when working with the pressure of a budget, but because the demand is constantly changing Neutens (2015). The average person in need of equal access to health today does not look the same as the average person in need of equal access to health a hundred years ago. Western societies (including the Portuguese one) have undergone significant demographic changes, over the last few years. As can be observed in figure 2.5, there has been a shift in the population distribution towards older ages. This is due to a decline in fertility rate and an increase in longevity (Neutens, 2015). Therefore, there is a need for new and innovative academic efforts in order to maintain adequate care for a growing, more demanding, less mobile, older society (Neutens, 2015). The investment in planning is an example of a way to increase efficiency and minimize costs (Cardoso-Grilo et al., 2020).

The next section presents the most important conclusions of this chapter, making the connection with the literature review.

2.3 Chapter conclusions

As can be seen in this chapter, health in Portugal has evolved considerably along the years. It became better and more accessible. This is due in part to the effort to build a network of integrated care by strengthening the partnership and the complementary relations between the different structures of the NHS and by improving coordination and articulation with other agents and levels of care. Officially, the

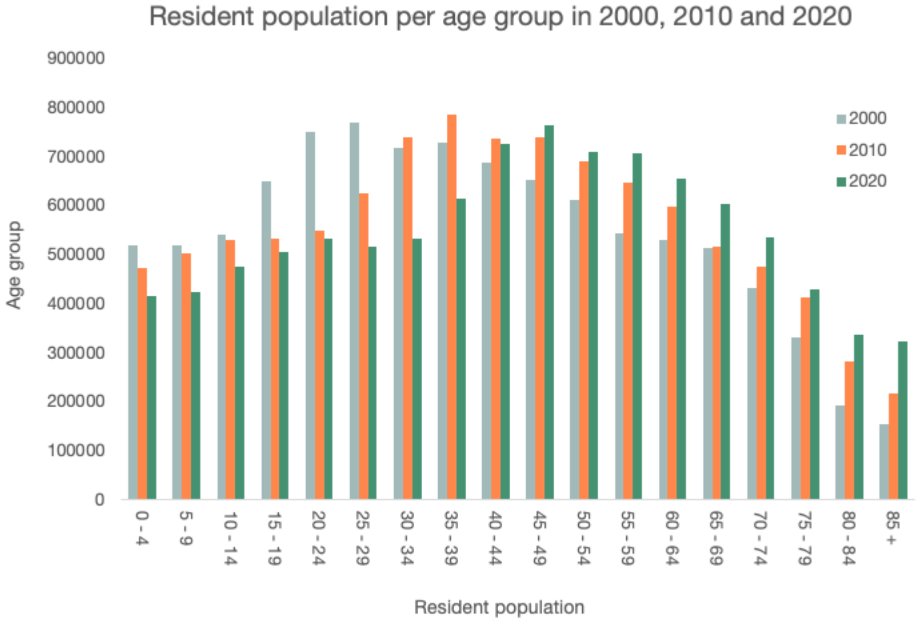


Figure 2.5: Age pyramid in the years 2000, 2010 and 2020 (Instituto Nacional de Estatística, 2021c).

idea of a network of hospitals is relatively new. However, hospitals have been working with each other for many years, transferring patients when necessary to provide better care. Formalizing and giving structure to this network is a step to improve healthcare and access to healthcare. The present thesis is an attempt to study how such a network could work, given all its characteristics and participants.

Therefore, one approach to solving the problem of the hospital network will be presented later. In the next chapter, the most recent and relevant literature on the subject will be reviewed, presenting the solutions approaches developed for similar problems.

Chapter 3

Literature Review

This chapter gathers some of the most relevant literature about hospital network design and facility location and allocation models, obtained through the search for scientific articles in the databases *Web of Science* and *ScienceDirect*, between the months of March of 2021 and October of 2021. Some key words or key expressions that were used during this search were *"Health System"*, *"Facility Location"*, *"Hospital Network Design"*, *"Multi-service facilities"*, *"Equity"* and *"Accessibility"*. Other similar expressions and combinations of those expressions were also used. This research included articles written in English (for the most part) and articles written in Portuguese. Official documents from the Portuguese government and information from the website of the Portuguese National Health Service (NHS) were also consulted. Priority was given to articles that were recent, peer reviewed and whose proposed models were implemented in real instances. Through the analysis of some citations in these articles, a deeper investigation was performed to build a literature review as complete as possible. Some books on these topics like *Network and Discrete Location* by Daskin (1995) and *Location Science* by Laporte et al. (2015b) were also considered.

This chapter adopts the following order: section 3.1 introduces some facility location modeling literature. It gives an overview of the types of problems that can be found and the models that can be applied to those problems. It also explores the array of solutions approaches available to solve the models mentioned. In the last subsection of this section, there is a brief discussion on the topic of uncertainty in locating a facility. Section 3.2 presents an analysis on the state-of-the-art of location modeling. It analyses what has been done in hierarchical location modeling and, more specifically, in location modeling in the healthcare sector. Table 3.1 presents an overview of the articles studied about planning hospital location. This section also includes a study of the work performed in location modeling in healthcare in Portugal. Finally, section 3.3 presents some relevant conclusions of the chapter and acts as a starting point and motivation for the chapter that comes next.

3.1 Classification of modeling frameworks: models and solutions

The field that uses models to help decision-making in complex implementation problems is called Operational Research (OR). These models can be qualitative or quantitative and achieve optimal or near-optimal solutions. There are countless areas in our lives that would benefit from some modeling to help find good solutions to the problems that arise. As will be clear later on, each problem can be modeled in different ways and, in turn, each model can also be solved differently. In this section, some models (subsection 3.1.1) and solution approaches (subsection 3.1.2) are discussed, along with their advantages and disadvantages. Uncertainty in location modeling is also discussed in subsection 3.1.3.

3.1.1 Problem types and modeling approaches

Modeling approaches are as varied and as numerous as the problems that exist and can be modeled. Table B.1 gathers some modeling approaches and their respective abbreviation. Linear Programming (LP) is one of the most common ways to solve small and simple problems but also slightly more complex problems. LP or linear optimization is, like the name suggests, a method to achieve the best outcome or solution in a mathematical model that is constrained by linear relationships (Daskin, 1995). However, there are problems that cannot be modeled linearly and so nonlinear programming also exists. Within LP, there are different categories that differ on the nature of the variables used. For example, integer linear programming and mixed-integer programming (Daskin, 1995). Besides LP, there is also fuzzy programming and robust optimization, when one is dealing with uncertainty; dynamic programming, when one is trying to solve a problem of higher complexity; multiple-criteria decision making, where one explicitly evaluates multiple conflicting criteria in decision making; and many more.

In terms of classifying location models there are also many ways to do it and several criteria that can be utilized. One common criterion is whether facilities offer single or multiple products/services. For example, hospitals are often seen as multi-services facilities since they offer different types of care (Mestre et al., 2012). Another way to classify models is according to whether they consider one or more objectives. Models can bi- (Davari et al., 2015; Mousazadeh et al., 2018b), multi-objective (Zhou et al., 2018; Beheshtifar and Alimoahmmadi, 2015) or have only one objective. The case of locating healthcare facilities is frequently viewed as having multiple objectives. This is due to the fact that the decision maker is usually interested in minimizing resources/costs, while also wanting to minimize travel time (or distance) or maximize coverage (Shishebori and Jabalameli, 2013; Davari et al., 2015). Capacity is another criterion. Facilities can have limited (Mohammadi et al., 2014) or unlimited (Ghaderi and Jabalameli, 2013) capacity. Another way to classify models is between models that attempt to locate desirable facilities (e.g. hospitals, schools) and models that try to locate undesirable facilities (e.g. landfills and prisons). The nature of inputs is also relevant for classification processes: whether they are static (constant over time) (Zhou et al., 2018) or dynamic (change over time) (Mousazadeh et al., 2018b); or whether they are deterministic/certain (Davari et al., 2015) or probabilistic/uncertain (Aboolian et al., 2015) . Furthermore, problems can exist in the private sector or in the public sector (e.g. site hospitals in a NHS); demand can be elastic (dependent on the level of service provided) or inelastic (independent on the level of service provided); and models can be hierarchical (Khodaparasti et al., 2017) versus single-level models. Finally, classifying models based on topography is also common. Problems can be planar (in which demand and facilities can be located anywhere on a plane) or be constrained to a network (in which demands and travel between demand sites and facilities can only occur on a graph/network made of nodes and links).

One classification system was proposed by Şahin and Süral (2007) to classify hierarchical facility location models. The hierarchical system of facilities is characterized as a network whose nodes represent facilities and demand sites. This network can be categorized according to flow pattern, service varieties, spatial configuration and objective (Şahin and Süral, 2007). As detailed below:

- **Flow pattern** describes the way customers and/or goods flow through the levels of hierarchical systems and can be either *single-flow* or *multi-flow*. Single-flow starts from the lowest level and ends at the highest level (or it starts from the highest level and ends at the lowest), passing through all the levels. Multi-flow can start in any level and can got to any other level. Both flow patterns can be identified in *referral* or *non-referral* systems.

- **Service varieties** distinguishes systems between *nested* and *non-nested*, according to the service availability at the levels of hierarchy. In a *nested* hierarchy, a higher-level facility provides all the services that its lower-level facilities provide and at least one additional service. In a *non-nested* hierarchy, facilities at each level offer services that are unique to that level. This categorization is similar to Narula (1984)'s categorization of *successively inclusive* and *exclusive* systems, where *successively inclusive* corresponds to a *nested* relationship between facilities.

- **Spatial configuration** divides networks between *coherent* and *non-coherent*. In a *coherent* network, a lower-level facility must receive or send services from or to one and only one higher-level facility. While in a *non-coherent* network there are less constrictions on the spatial configurations of levels.

- **Objective** differentiates models according to the different types of goals when locating facilities. The most common are *median*, *covering*, *center* and *fixed charged* objectives. The *median* or *p-median* problem (ReVelle and Swain, 1970) is characterised by the minimization of the total weighted travel distance between all the demands and facilities (where average distance and average time are used interchangeable, representing only the ”cost” of travelling from one point to another). For some facilities, minimizing the average distance may not be appropriate since the critical nature of the demand for service imposes a maximum ”acceptable” travel distance or time (e.g. locating fire stations or ambulances) (Owen and Daskin, 1998). In these situations a covering model might be more useful. In *covering* models, a demand is considered *covered* if it can be served in a specified time. The covering objectives can be *set covering* and *maximum coverage*. In a *set covering* problem (Daskin, 1995), the aim is to minimize the number of facilities needed to cover all demands. Which means the decision maker establishes a desired level of coverage and the model tries to minimize the cost of facility location. The issue of applying such models to real life situations is that, often times, the decision makers do not have sufficient resources to build the facilities dictated by the model, for the established level of coverage. A more suitable goal in these cases would be a *maximum coverage* goal (Revelle, 1975), which tries to maximize the number of customers with the desired level of coverage, within the limits of the available resources (Owen and Daskin, 1998). Another class of problems that avoids the potential unachievable demands of the *set covering* approach are the *P-center problems* (Daskin, 1995). Here, the objective is to give coverage to all demands by placing a number of facilities in a way that minimizes coverage distance. In other words, the model calculates what the minimum coverage distance is if we want to locate P facilities. Since, the model tries to minimize the maximum distance between any demand and its nearest facility, it is also called the minimax problem. Finally, *fixed charge* location models (Daskin, 1995) aim to minimize total facility construction and transportation costs. It should be noted that there can be made an additional differentiation between problems with single and multiple objectives (as mentioned previously).

3.1.2 Solution methods

A decision problem can be approached in a number of ways, which results in different models. In turn, these models can be solved by several solution methods. Depending on the complexity of the location model, the search for a solution can consume more or less computational resources (and more or less time). Every solution method can usually be placed into one of two main classes, summarized in table B.2. The first one is composed by *accurate methods*, which result in optimal/exact solutions or near-optimal bounded-error solutions. These methods are commonly used in small-size linear problems since they are efficient in this environment (Afshari and Peng, 2014). If the complexity of the problem increases (due to change in size or in any other aspect of the problem), exact methods cease to be efficient and other methods should be applied. A significant amount of models in the existing literature are solved by general-purpose optimization software packages like CPLEX (Cocking et al., 2012; Shishebori and Jabalameli, 2013; Aboolian et al., 2015), GAMS (Shishebori and Jabalameli, 2013; Rahmaniani et al., 2014; Mestre et al., 2015), LINGO (Chu, 2000) or Xpress (Mitropoulos et al., 2006) which use solving algorithms like simplex, branch and cut... Other methods also used are Lagrangian relaxation (Galvão et al., 2002; Syam, 2008; Kim and Kim, 2013), branch and bound (Verter and Lapierre, 2002) and cutting plane (Vidyarthi and Kuzgunkaya, 2015).

The second class is composed by non-exact or *inaccurate methods* that provide solutions that are not necessarily optimal and are not error-bound. This class comprises of all the heuristic solutions. In cases where the search for the solution is very time-consuming, it becomes advantageous applying heuristic methods. Another application of heuristics is when the exact methods, used in complex models, fail to find an global optimum solution. In Zhang et al. (2009), the mathematical formulation that emerged was nonlinear so the authors compared four heuristics and applied the most efficient one in a real life problem related to a breast screening center network in Montreal. Besides heuristic methods there are metaheuristic methods. A metaheuristic ”is a high-level problem-independent algorithmic framework that provides a set of guidelines or strategies to develop heuristic optimization algorithms” (Sörensen and Glover, 2013). Popular methods in this category are tabu search (Blais et al., 2003; Stummer et al., 2004), genetic algorithm (Beheshtifar and Alimoahmmadi, 2015) and simulated annealing (Ghaderi and Jabalameli, 2013). It is not uncommon that authors combine exact methods with heuristics to solve their model. For example, Ghaderi and Jabalameli (2013) use a greedy heuristic and a fix-and-optimize heuristic based on simulated annealing, branch and bound and cutting methods and Marianov and Taborga (2001) use heuristics and branch and bound solutions.

3.1.3 Uncertainty in location models

One of the aspects that contributes the most to the complexity of location analysis is uncertainty. Location problems are deeply connected to uncertain events and conditions. In the design of each location model, the question of how much uncertainty should be represented is always present. Using uncertainty as a criterion, Daskin and Dean (2006) consider that there are three major categories of healthcare location literature: *accessibility*, *adaptability* and *availability*.

The authors define *accessibility* by ”the ability of patients or clients to reach the health care facility or, in the case of emergency services, the ability of the health care providers to reach patients”. Papers in the *accessibility* category seek to find facility locations that perform well with respect on static inputs, ignoring the needs of the system to evolve in response to changing conditions. The models in these papers take a snapshot of reality and plan for those circumstances only. This is why they are called *static* (and deterministic) or single-period models (Aboolian et al., 2015). They are the most straightforward models where all inputs (such as demands, distances and travel times) are considered to be known quantities and outputs are a one-time decision values (Owen and Daskin, 1998). In these cases, to address the problem of uncertainty and determine the robustness of these solutions, a sensitivity analysis is usually performed. Such investigation tries to quantify the effect of a change in the values of parameters on the optimal solution obtained (Owen and Daskin, 1998).

However, these models are often not enough to represent in a realistic manner the uncertainties inherent to real-live situations. It is not guaranteed that the future will resemble the past or even the present. As Daskin (1995) put it, even though ”most location *models* are static, most *problems* are *dynamic*” because the inputs (and the outputs) depend on time. Examples of such inputs may be costs, available preexisting candidate facility locations or, the most common, demand. To address the problems of static models, dynamic or multi-period models were created. Ballou (1968) was the first to acknowledge the limitations of static and deterministic models in 1968. In his article, he used a series of static deterministic optimal solutions to solve the dynamic problem of trying to locate a single warehouse so as to maximize profits over a finite planning horizon (Ballou, 1968). By considering multiple periods of time, there is the possibility of capturing differences in the demand for services on weekdays vs weekends or to account for increases in demand or costs over the years (Daskin, 1995). In the framework suggested by Daskin and Dean (2006), the dynamic models correspond to the ones in the *adaptability* and *availability* categories. *Adaptability* models address long-term uncertainty about the conditions under which a system will operate and attempt to find solutions that perform adequately in a range of possible scenarios. Examples of authors that explore this type of uncertainty are Ghaderi and Jabalameli (2013) and Mousazadeh et al. (2018a,b). The process to do this often involves defining multiple future conditions and find a good compromise solution. The ’best’ compromise solution might not be optimal in any particular scenario but must do well across all scenarios (Daskin and Dean, 2006). Hence, it is possible to define performance measures for these solutions using the notion of ”regret”.

Availability models reflect short-term changes that derive from facilities being busy. They try to take into account the balance between the demand for services that is always change and the supply of those services. They can be deterministic, queuing-based and probabilistic models. Such models are most often applied to emergency service systems (ambulances), since one vehicle providing services to one demand and be requested to serve another one at the same time. In Motallebi Nasrabadi et al. (2020), the authors take into account long and short term uncertainty.

After this overview regarding model classification, the next section will present the state-of-the-art of location modelling.

3.2 Location models: state-of-the-art

The problem of locating a hospital is not a new problem nor an easy one. In fact, the problem of locating a facility in general is quite old. The formal study of location theory began in 1909, when Alfred Weber had to decide where to build a warehouse so as to minimize the distance between the several customers and this new facility (Owen and Daskin, 1998; Weber, 1929). Throughout the years, this field slowly grew, but it was in the mid-1960’s that it truly flourished (Owen and Daskin, 1998). Since then, a lot of work on this topic has been published due to the crucial nature of location decisions in strategical planning. This type of decisions have an impact on the environment around the facility and on other logistical and operational decisions. Additionally, the high costs of property acquisition and construction make location and relocation projects long-term investments (Owen and Daskin, 1998). Due to the expensive nature of these decisions, decision makers should select sites that will perform well in the time the facility is built but also during the facility's lifetime. This means that the locations chosen need to withstand environmental changes, populations shifts and keep up with the evolution of market trends (Owen and Daskin, 1998). This section reviews hierarchical location modeling in 3.2.1, location modeling in the healthcare sector in 3.2.2 and location modeling in the Portuguese healthcare sector in 3.2.3.

3.2.1 Hierarchical location models

The first survey that compiled the research on the topic of hierarchical location-allocation problems on a network was Narula (1986). A few decades later, Şahin and Süral (2007) published a review that consolidated the main results in the literature dated from 1986 until 2004. In this article, the authors introduced the classification scheme for Hierarchical Facility Location Problems (HFLP) presented in section 3.1. However, horizontal interactions and capacity limits on facilities were not contemplated in this scheme (Şahin and Süral, 2007). Seven years later, Farahani et al. (2014) published an article reviewing over 40 years of HFLP models, adding to the research done by Şahin and Süral (2007). In 2018, Ortiz-Astorquiza et al. (2018) conducted a comprehensive review covering multi-level facility location problems, which can be viewed as a subgroup within HFLP.

The study of hierarchical problems in the context of healthcare systems is extremely useful since healthcare facilities are usually naturally organised as a hierarchical structure. In 1973, Calvo and Marks (1973) formulated the hierarchical health facility location-allocation problem and, years later, Tien et al. (1983) improved upon this formulation, contributing, at the time, to the limited research on this topic. Some reviews have been written regarding the subject. It is the case of Rahman and Smith (2000), who reviewed the literature about location-allocation models in health systems in developing nations (including hierarchical problems) and Laporte et al. (2015a) who wrote about healthcare facility location focusing on the different objective functions and gave an an overview of the different models. Daskin and Dean (2006) reviewed some discrete facility location models like the set covering model, maximal covering model and the P-median model, since these are the ones most used in location planning in healthcare. In the last few years, several hierarchical models were explored.

3.2.2 Location models in the healthcare sector

In the last two decades, healthcare facility location has become an increasingly important object of study of the OR community. This issue is relevant in the organization of resources for healthcare systems but also in disaster management. The term healthcare facility is a broad one and encompasses a few different types of healthcare providers. The first division that can be made is between non-emergency and emergency facilities (based on the proposed framework by Ahmadi-Javid et al. (2017)). Non-emergency facilities include primary care facilities (e.g. hospitals, clinics, off-site public access devices, etc), blood banks, specialized service facilities (e.g. organ transplant centers, detection and prevention centers), medical laboratories, mobile healthcare units, home care centers, rehabilitation centers, doctors' offices, drugstores and long-term nursing care centers (Ahmadi-Javid et al., 2017). Emergency facilities can be further divided into permanent (e.g. emergency centers, trauma centers, ambulance centers) and temporary (Ahmadi-Javid et al., 2017). The group of facilities that is of relevance to this thesis is the non-emergency facilities group, more specifically hospitals.

With the goal to understand the work already done in this area, table 3.1 was created compiling and classifying several healthcare facility models, focusing on primary care providers. Due to the current situation of the SARS-CoV-2 pandemic, it also seemed relevant to include a section reviewing articles exploring disaster management models. The analysis of the articles contemplates the existence of considerations of uncertainty or not, if the period setting of the model was single-period (static) or multi-period (dynamic), the nature of the decision variables (i.e. location, allocation of demand...), the problem type and/or the model used, the solution approach, the application and the country of the case study.

In a real world scenario, every logistic operation will have budget constraints. Ghaderi and Jabalameli (2013) tackled the problem of access to health facilities for rural populations in a province in Iran. The model they designed took into account the costs of opening the facilities and building network links. However, this model assumed facilities to be uncapacitated which can be unrealistic. On this topic, Mohammadi et al. (2014) attempted to design a reliable healthcare network whose facilities have a limited capacity. In their work, they recognized the risk of deterioration of patients' health conditions due to limits in the capacity and investigated a queue system that was designed, in advance, to address this situation. Another paper by Aboolian et al. (2015) presented a general mathematical formulation that tried to maximize access to public services and then apply it to a realistic case based on the Toronto hospital network. The authors also addressed aggregate capacity decisions on top of determining the configuration of the network. Khodaparasti et al. (2017) also tried to improve local accessibility, equity and efficiency in health by developing a location model in a multi-objective framework. The object of study, in this case, were community based organizations, which are groups that have been present in Iran for centuries and are becoming increasingly important in terms of promoting community health and education. Still on the topic of trying to increase equity and effectiveness, Núñez Ares et al. (2016)

proposed a model for locating facilities, more specifically road side clinics in Africa. Other articles also written on the topic of health network design are Mousazadeh et al. (2018b), Evangelista et al. (2019) and Motallebi Nasrabadi et al. (2020).

The majority of the articles presented describe models that are designed to perform well under ”normal” circumstances but are not prepared for when something extreme and out of the ordinary happens. Disruptive events like natural disasters or terrorist attacks are not common events but when they happen they are highly destructive and the demand for health care services increases immensely in a short period of time. Articles like Shishebori and Yousefi Babadi (2015), Zarrinpoor et al. (2017) and Acar and Kaya (2019) propose reliable models where the facilities are subject to the risk of disruptions.

Table 3.1: Overview of models regarding the planning of non-emergency healthcare facilities (primary care facilities like hospitals and clinics) and of models considering disruptions (disaster relief). It should be noted that acronyms were used to improve the organization. All the acronyms used were first mentioned in tables B.1 and B.2.

	Authors (year)	Uncertainty	Multi-period setting	Decision variable	Problem type/model	Solution approach	Application	Country
Non-emergency facilities (primary care facilities)	Blais et al. (2003)	N	-	-	Multi-criteria optimization	Tabu search heuristic	Home-care districting	Canada
	Cocking et al. (2012)	N	Single	Location and allocation of demand	MILP	Exact methods (CPLEX)	Health care facilities	Burkina Faso
	Kim and Kim (2013)	N	Single	Location and allocation of demand	ILP	Heuristic based on Lagrangian relaxation	Public healthcare facilities	Korea
	Ghaderi and Jabalameli (2013)	N	Dynamic (long term)	Location and allocation of demand	MINLP	Greedy heuristic and metaheuristic (simulated annealing)	Healthcare facilities in rural population	Iran
	Shishebori and Jabalameli (2013)	N	-	Location and allocation of demand	Multi-objective MINLP	Exact methods	Medical system centers and link roads	Iran
	Rahmaniani et al. (2014)	Y	Single	Location and allocation of demand	INLP, 2-SSP, MCDM	Gams, meta-heuristics	Hospitals	Iran
	Mohammadi et al. (2014)	Y	-	Location	Multi-objective optimization, QT	Meta-heuristics	Healthcare network	Iran
	Beheshtifar and Alimoahmmadi (2015)	N	Single	Location and allocation of demand	MILP, MCDM, INLP, 2-SSP	Meta-heuristics (genetic algorithm)	Clinics	Iran
	Davari et al. (2015)	N	-	-	Multi-objective optimization with budget constraints	Fuzzy goal programming and fuzzy chance constrained optimization	Preventive healthcare	Turkey
	Mestre et al. (2015)	Y (Demand)	Dynamic (long term)	Location and allocation of demand	MILP, 2-SSP, MCDM	Exact methods (Gams)	Hospital network	Portugal
	Núñez Ares et al. (2016)	N	Single	Location	MILP	Column generation algorithm	Roadside clinics	Africa
	Aboolian et al. (2015)	Y	Single	Location and allocation of demand	MINLP, QT	Exact methods (CPLEX)	Hospital network	Canada
	Khodaparasti et al. (2017)	N	Single	Location-allocation	Multi-objective hierarchical model	Fuzzy goal programming	Community Based Organizations	Iran

Table 3.1: Overview of models regarding the planning of non-emergency healthcare facilities (primary care facilities like hospitals and clinics) and of models considering disruptions (disaster relief). It should be noted that acronyms were used to improve the organization. All the acronyms used were first mentioned in tables B.1 and B.2.

	Authors (year)	Uncertainty	Multi-period setting	Decision variable	Problem type/model	Solution approach	Application	Country
	Zhou et al. (2018)	N	Single	Capacity allocation	Multi-objective stochastic programming	ε-constraint algorithm and a multi-objective genetic algorithm combined with a neighborhood search algorithm	Hospital wards capacity allocation	China
	Mousazadeh et al. (2018b)	Y	Dynamic (long term)	Location and allocation of demand	Multi-objective MINLP	Robust mixed possibilistic-flexible programming	Three-level health service network	Iran
	Mousazadeh et al. (2018a)	Y	Dynamic (long term)	Location and allocation of demand	Bi-objective MINLP	Hybrid robust possibilistic programming	Three-level health service network	Iran
	Motallebi Nasrabadi et al. (2020)	Y	Dynamic (short and long term)	Location and allocation	QT, RO	Evolutionary algorithm	Healthcare Networks	Iran
Disaster relief (disruptions)	Shishebori and Yousefi Babadi (2015)	Y	Single	Location and allocation of demand	MILP, RO	Exact methods	Medical services network design	Iran
	Acar and Kaya (2019)	Y	Dynamic (long term)	Location, allocation and relocation	2-SSP	Exact methods	Mobile hospitals	Turkey
	Zarrinpoor et al. (2017)	Y (demand and service)	Dynamic	Location, allocation	2-stage RO, QT	Benders decomposition	Healthcare network	Iran

3.2.3 Location models in the Portuguese healthcare

In the context of the Portuguese health system, the literature about locating hospitals is not extensive. Nonetheless, important work that is very relevant and that serves as a starting point for this thesis has been done regarding hospital network planning. In specific, work that considers hospitals as diverse multi-service structures inserted in a complex network.

In 2005, Monteiro and Pascoal (2005) proposed a linear integer multi-objective model in order to find the optimal location of hierarchical facilities in a network system, contributing to the planning of a primary health care delivery system in Portugal. On the topic of hospital supply redistribution, Oliveira and Bevan (2006) used a multi-modelling approach with two location-allocation models whose objective functions reflected different concepts of equity of access. Until Mestre et al. (2012) published their article, hospitals had been modelled as single-service facilities with only ascendant flows in the hierarchy's structure. In this study, the authors proposed a new and more accurate model that accounted for the multi-service and hierarchical nature of healthcare delivery. They also included a two-directional referral system of patients between facilities in different levels. Furthermore, an article handling uncertainty in strategical planning of hospital networks written by Mestre et al. (2015) was published in 2015. It focused primarily on uncertainty related with the demand for hospital services. Also in this year, Cardoso et al. (2015) studied an approach for the planning of a long-term care network with considerations of uncertainty, strategic policy and equity. More recently, Cardoso-Grilo et al. (2020) explored further the topic of long-term care networks and tackled the modelling of key health policies with a stochastic multi-objective model.

Given the review of the literature presented in this chapter, the next section summarizes the most relevant conclusions.

3.3 Chapter conclusions

The problem of locating a healthcare facility and allocating demand to it is a very relevant topic. Due to its dimension and the different ways it can be looked at, this particular problem can be the subject of extensive investigation in OR.

The type of healthcare facility (hospital, primary care center...), the type of flows between facilities (ascendant, descendant ...), the type of services (urgent care, specialized care...) that can be provided by each facility and the uncertainty (on costs, demand...) that can be considered are examples of the differentiating characteristics of a problem similar to the one at hand. The majority of articles explores the task of locating a general healthcare facility in countries with a network of healthcare in its beginning stages, where a high level of detail might not be as important as guaranteeing that every person is provided with basic healthcare. In addition to this, there is a lack of research that explores hospitals in relation to each other and the relations between the various departments intra-hospital. Both of which are crucial to represent in a more truthful manner the reality we live in.

In order to solve the problem presented in the case study, it is necessary to choose and develop a mathematical programming model that suits the particularities of the problem. These particularities must be captured in restrictions and objective functions based on, more general, models of facility location. It is possible to conclude that, occasionally, the problems are too complex and the model becomes ”too heavy” to solve. To deal with this situation, one should try to simplify the model or use more powerful methods for solving.

All this information was taken into account when designing the model in the next chapter. The model was an attempt at exploring some answers to the questions highlighted in this chapter.

Chapter 4

Mathematical Model

The problem introduced in Chapter 2 is mathematically formulated in this chapter in multi-objective Mixed-Integer Linear Programming (MILP). The model presented here is based on Model 1 of ”Location-allocation approaches for hospital network planning under uncertainty” (Mestre et al., 2015), which considers location a first-stage decision and allocation a scenario dependent decision.

The aim of the model in the article was to explore how the hospital network could be (re)organized when the two primary concerns of the decision maker were to improve geographical access and minimize costs. It makes sense to use a bi-objective model to ensure balance between these two aspects, since most NHS-based countries wish to provide equal access to hospital services but are limited by budget constraints. The same aim was defined for the model presented in this thesis. However, considering several years have passed since the publication of the mentioned article, some changes were necessary to build a model that represented, as accurately as possible, the hospital network today. Similar to Mestre et al. (2015)’s model, hospitals are viewed as multi-service facilities, serving several demand points, operating in a hierarchical structure. The main difference is in the nature of the services considered. In the 2015’s paper, hospitals are considered to be multi-service facilities because they offer inpatient, outpatient and emergency services. In the present work, hospitals are also considered to be multi-service but the focus is on the medical specialties which they provide. The inclusion of medical specialties, hospital referral networks and the mechanisms of referral from hospital to hospital gives hospitals a multi-level structure. The designed model attempts to generalize the location-allocation problem, so that it is possible to apply it in a case study, not only to the Portuguese hospital network, but to any country with a comparable network system.

The characteristics of the problem are briefly introduced in section 4.1 based on the case study presented. Section 4.2 describes the formulation of the model, including notation used, indexes, sets, parameters, weights, constraints and objective functions. Lastly, section 4.3 summarizes the chapter conclusions.

4.1 Problem description

The problem at hand consists of locating hospitals and allocating demand to those hospitals, in a certain time period. These are two different tasks since the question of whether a hospital should be placed in a given location does not depend on the demand scenario, while allocation does depend on it. This means it is possible to have, for the same location results, varying allocation decisions. In regards to hospital placement, not all location sites are good candidates. On the one hand, the demand for health services in the hospital's catchment area should be great enough to justify a hospital. On the other hand, there should be a maximum acceptable travel distance/time for a patient to reach a hospital facility. As previously stated, a compromise between improving access (e.g. opening a hospital in every street) and minimizing costs (e.g. opening only one hospital for the whole country) is crucial.

Each hospital can supply health care services across a number of medical specialties (e.g. cardiology, oncology...). The model presented later on in this chapter captures this characteristic and can be solved for one service (e.g. inpatient) or *n* services, depending on the input given to the model. As stated in chapter 2, hospitals have no longer a single classification or belong to a single group/type/level. They have multiple classifications that depend on the referral network or medical specialty in question. The distinction between these two concepts has also been explained in the mentioned chapter and in the present chapter, for simplicity, model notation will only include and will only ever refer to the term "medical specialty". Having said that, it is clear that each medical specialty at each hospital can only have one level. Not every hospital has to have every medical specialty. However, there is a minimum number of medical specialties that must exist in one hospital.

The number of medical specialties offered by each hospital is limited by the capacity of each hospital. The capacity of a hospital is the sum of the capacities of all medical specialties in that hospital. It is also necessary to have in mind that a hospital too large will operate under diseconomies of scale and a hospital too small may not be able to offer proper hospital care to the demand.

In terms of demand allocation, the central aspect to bear in mind is the patient's proximity to the facility. Every person in need of hospital care goes to the nearest hospital. If the hospital does not have the required conditions to serve that patient, he/she should be transferred to the closest hospital with the right conditions. These conditions could be human resources, specialized equipment, space, etc... Transfers between hospitals are done according to the respective referral network.

Figure 4.1 depicts a fraction of a network of hospitals. It portrays three hospitals (H_1, H_2 and H_3). Each hospital has the possibility to offer healthcare services in four medical specialties (MS_1, MS_2, MS_3 and MS_4). Each medical specialty has one level in each hospital (in this case the levels that are possible are I, II, III or IV). Hospital H_1 has three medical specialties: MS_1 (level I), MS_2 (level II) and MS_4 (level III). Hospital H_2 has four medical specialties: MS_1 (level II), MS_2 (level I),MS_3 (level I) and MS_4 (level II). Hospital H_3 also has three medical specialties: MS_1 (level III), MS_2 (level IV) and MS_3

(level I).

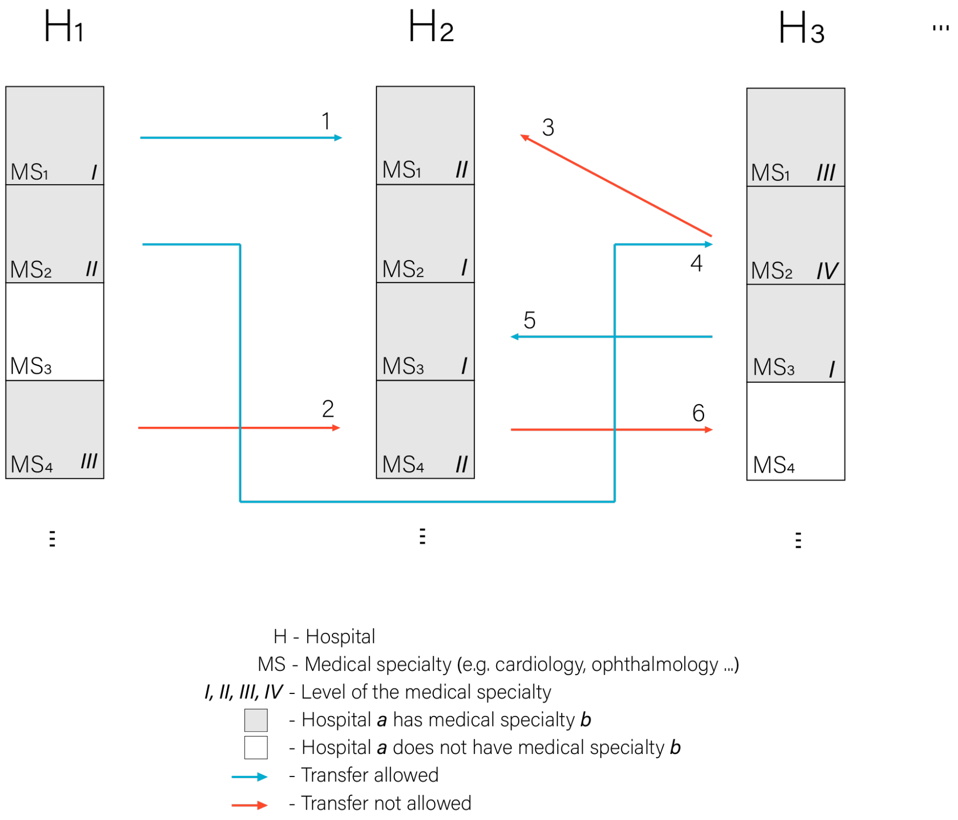


Figure 4.1: Diagram with examples of flows that are allowed (blue) and not allowed (red). The example shows only interactions between 3 hospitals and 4 medical specialties but it is possible to have more/less hospitals and medical specialties.

Source: Author's own compilation.

Additionally, figure 4.1 displays six transfers (three are permitted and three are not). For example, transition 2 is forbidden because the patient is being transferred from a higher level (III) to a lower level (II). Transition 3 is also forbidden since it represents a transfer between two different medical specialties (from MS_2 to MS_1). Finally, transition 6 is not allowed since it represents a transfer where the patient is being transferred to a hospital that does not have the medical specialty that the patient needs. Transfers 1, 4 and 5 are allowed since they are all transfers inside the same medical specialty, from one level to another equal or higher level.

Following the description of the problem, the next section introduces its mathematical formulation.

4.2 Mixed-Integer Linear Programming formulation of the problem

This section introduces the notation used throughout the modeling process (subsections 4.2.1, 4.2.2 and 4.2.3), as well as the objective functions (subsection 4.2.4) and the constraints (subsection 4.2.5). The model is considered to be linear since all equations (objective functions and constraints) contain only linear relations between variables. It is also a mixed-integer model since some of the variables can only assume integer values (it is the case of binary variables, which can only be 0 or 1). Finally, the model is multi-objective since it has two objective functions.

4.2.1 Indexes and sets

$t, \tau \in T$: Set of time periods in which the planning horizon is divided

$i \in I$: Set of demand points

$j, j', k, k' \in J$: Potential locations for a hospital

$J_o \subset J$: Set of hospitals initially existing

$J_c \subset J$: Set of hospitals that are not opened at the initial moment

$J_m \subset J$: Set of hospitals that are initially existing and must be kept open

$s \in S$: Set of possible scenarios for demand

$n \in N$: Set of medical specialties that can exist in a hospital

$l, p, p' \in L$: Set of levels for a certain medical specialty

4.2.2 Parameters and weights

d^1_{ij} : Average travel time from demand point i to hospital j

d^2_{jk} : Average travel time from hospital j to hospital k

α : Weight to differentiate a first entry in the system and a transfer

P_s : Probability of scenario s

D^{SNLt}_{inls} : Demand for medical specialty n in level l in location i scenario s and time t

$capmin^t_j$: Minimum capacity required in hospital j in time t

$capmax^t_j$: Maximum capacity required in hospital j in time t

d^{max} : Maximum travel time allowed for a population to access hospital care

N^t : Maximum number of hospitals operating in time t

pop^t_j : Population in demand point j in time t

pop^{min} : Minimum population required to open a hospital in location j

$year^t$: Number of years of time period t

OC^t_j : Unit cost for providing care in hospital j in time t

IC^t_j : Fixed investment cost in a new hospital $(j \in J_c)$ providing care in time t

CC^t_j : Fixed cost of closing an existing hospital $(j \in J_o)$ providing care in time t

n^{spec} : Total number of existing medical specialties

n^{min} : Minimum number of medical specialties in a hospital

M : Large coefficient that is chosen to be larger than any reasonable value that a variable may take (used in big-M constraints)

4.2.3 Decision variables

X_j^t : =1 if a hospital is located at site j in time t; 0 otherwise

Z_{jnl}^t : =1 if a hospital located at site j has medical specialty n in level l in time t; 0 otherwise

Y_{ijnls}^t : Flow from demand point i to hospital j in medical specialty n in level l in time t and scenario s (flow from demand points to hospitals)

Y_{jknlps}^{Tt} : Flow from medical specialty n in level l in hospital j to medical specialty n in level p in hospital k (flow related to intra- and inter-hospitals transfers)

Y_{jnls}^{Ct} : Patients that stay (receive care) in hospital j in medical specialty n in level l and scenario s

cap_j^t : Capacity in hospital j in time t

$Scap_{js}^t$: Expected utilization of hospital j in time t and scenario s

4.2.4 Objective functions

The purpose of objective function 4.1 is to optimize the geographical access to hospitals. In order to do this, it minimizes the expected travel time to reach hospital care weighted by demand. The first term of the sum concerns the time travelled between demand points and hospitals. The second term is related to the time travelled between hospitals (patient transfers) and has a weight coefficient - α - to differentiate transfers from first entries, if needed.

$$\begin{aligned} Min \sum_{s \in S} P_s \Bigg(& \sum_{i \in I} \sum_{j \in J} \sum_{n \in N} \sum_{l \in L} \sum_{t \in T} d_{ij}^1 \, Y_{ijnls}^t \\ & + \sum_{j \in J} \sum_{k \in J} \sum_{n \in N} \sum_{l \in L} \sum_{p \in L} \sum_{t \in T} \alpha \, d_{jk}^2 \, Y_{jknlps}^{Tt} \Bigg) \end{aligned} \tag{4.1}$$

Expression 4.2, which represents the other objective function, minimizes expected costs. The first term relates to the operating costs of supplying hospital services (daily activities that consume resources). The second and third term represent the costs of opening a new facility and closing existing ones, respectively. The only element influenced by the possible demand scenarios is the first one since it is the only one that contains a variable concerning expected utilization of services.

$$
\begin{aligned}
Min \sum_{s \in S} \sum_{t \in T} P_s \Big(\sum_{j \in J} Scap_{js}^t \, OC_j^t \, year^t \Big) \\
+ \sum_{t \in T \setminus \{1\}} \sum_{j \in J_c} \Big(X_j^t - X_j^{t-1} \Big) IC_j^t \\
+ \sum_{t \in T \setminus \{1\}} \sum_{j \in J_o} \Big(X_j^{t-1} - X_j^t \Big) CC_j^t
\end{aligned}
\tag{4.2}
$$

4.2.5 Constraints

4.2.5.A Demand satisfaction and flow conservation

Equation 4.3 ensures that the demand from demand point i for medical specialty n in level l in scenario s in time period t should be considered and satisfied. In other words, all demand, in each scenario, should be satisfied by a hospital.

$$
\sum_{j \in J} Y_{ijnls}^t = D_{inls}^{SNLt} \quad \forall_{t \in T \; i \in I \; n \in N \; l \in L \; s \in S} \tag{4.3}
$$

Flow conservation constraints determine, for each scenario, the balance between different hospitals in the system. Equation 4.4 states that, for a certain time period t and scenario s, every patient that arrives at a hospital must either be treated there or be transferred to another hospital. In other words, the sum of patients that goes into a hospital minus the sum of patients that goes out (those that are transferred) equals the sum of patients that stay and are treated there. The first two terms of the left side of the equation represent the flow of demand coming to a hospital in location j in medical specialty n in level l, either from a demand point i (Y_{ijnls}^t) or from another hospital in location k' in medical specialty n in level p' ($Y_{k'jnp'ls}^{Tt}$). The third term of the left side of the equation refers to the flow of patients being transferred to another hospital in location k in medical specialty n in level p (Y_{jknlps}^{Tt}). The right side of the equation represents the patients that stay and receive care in hospital j, medical specialty n, level l (Y_{jnls}^{Ct}).

$$
\sum_{i \in I} Y_{ijnls}^t + \sum_{k' \in J} \sum_{p' \in L} Y_{k'jnp'ls}^{Tt} - \sum_{k \in J} \sum_{p \in L} Y_{jknlps}^{Tt} = Y_{jnls}^{Ct} \quad \forall_{t \in T \; j \in J \; n \in N \; l \in L \; s \in S} \tag{4.4}
$$

4.2.5.B Hospital capacity

Equations 4.5, 4.6 and 4.7 refer to the hospital capacity. Equation 4.5 is an auxiliary constraint that estimates the expected utilization in each hospital j in scenario s and time period t. It is calculated through the sum of patients that are treated in a hospital (see equation 4.4) for every medical specialty n and level l.

$$Scap^t_{js} = \sum_{n \in N} \sum_{l \in L} Y^{Ct}_{tjnls} \quad \forall_{t \in T \; j \in J \; s \in S} \tag{4.5}$$

Equation 4.6 establishes that, in each scenario, the expected utilization ($Scap^t_{js}$) is constrained by hospital capacity, which means both minimum ($capmin^t_j$) and maximum capacity limits ($capmax^t_j$). This makes it impossible for a hospital to be too small or too large.

$$capmin^t_j X^t_j \leqslant Scap^t_{js} \leqslant capmax^t_j X^t_j \quad \forall_{t \in T \; j \in J \; s \in S} \tag{4.6}$$

In addition to imposing extreme limits for capacity, it is also important that each hospital is able to serve every patient that is expected to use that hospital. For that, equation 4.7 ensures that the value of the capacity to install (cap^t_j) cannot be less than the expected utilization ($Scap^t_{js}$) in each scenario.

$$cap^t_j \geqslant Scap^t_{js} \quad \forall_{t \in T \; j \in J \; s \in S} \tag{4.7}$$

4.2.5.C Closest assignment constraints

The assignment of demand can be done in a few ways but the most common way, and the way it is done here, is by proximity (closest assignment constraints). Every patient should be allocated to the nearest open hospital, in each scenario. Equation 4.8 assures that if a hospital j' is open, than the assignment cannot be done to a more distant hospital j.

$$Y^t_{ijnls} + D^{SNLt}_{inls} \sum_{l' \in L} Z^t_{j'nl'} \leqslant D^{SNLt}_{inls} \quad \forall_{t \in T \; i \in I \; j \in J \; j' \in \left\{ j' | d^1_{ij'} < d^1_{ij} \right\} \; n \in N \; l \in L \; s \in S} \tag{4.8}$$

Equation 4.9 states that, in each scenario, demand should not take more than a maximum acceptable travel time to access hospital services in a hospital (maximum catchment). For a given demand point i, if the average travel time d^1_{ij} to a hospital in location j is greater than a maximum acceptable travel time d^{max}, the demand flow for care in medical specialty n in level l, from i to j, is zero.

$$Y^t_{ijnls} = 0 \quad \forall_{t \in T \; i \in I \; j \in \left\{ j | d^1_{ij} > d^{max} \right\} \; n \in N \; l \in L \; s \in S} \tag{4.9}$$

Equation 4.10 guarantees that a hospital that is closed cannot receive any patients. If $X^t_j = 0$, then

$Y^t_{ijnls} \leqslant 0$ which becomes $Y^t_{ijnls} = 0$ due to 4.25. If $X^t_j = 1$, then the upper limit of Y^t_{ijnls} is a big enough value, which is all the demand from demand point i ($Y^t_{ijnls} \leqslant D^{SNLt}_{inls}$).

$$Y^t_{ijnls} \leqslant D^{SNLt}_{inls} \times X^t_j \;\; \forall_{t\in T\; i\in I\; j\in J\; n\in N\; l\in L\; s\in S} \tag{4.10}$$

4.2.5.D Opening, closing and locating hospitals

Equation 4.11 limits, in each time period, the number of hospitals opened, which allows for indirect modelling budget restrictions. The sum of the number of open hospitals in location $j \in J$ must be less than or equal to a maximum number N^t, for every time period t.

$$\sum_{j\in J} X^t_j \leqslant N^t \;\; \forall_{t\in T} \tag{4.11}$$

Equation 4.12 assists the choice of potential locations for hospitals sites and establishes that a facility may only open if the population is above a minimum value. If the population in demand in location j, pop^t_j, is less than the minimum required value to open a hospital, pop^{min}, a hospital cannot be opened in location j, for every time period t.

$$X^t_j = 0 \;\; \forall_{t\in T\; j\in \left\{j | pop^t_j < pop^{min}\right\}} \tag{4.12}$$

In a real scenario, hospitals or other facilities are considerable investments, which means once they are open/closed they stay that way for a very long period of time, longer than the planning horizon considered in the model. So constraints like 4.13 and 4.14 are necessary.

Equation 4.13 states that new hospitals ($j \in J_c$) may open, yet once opened, they must remain opened until the end of the planning horizon. In time period $t-1$, if a hospital is opened in location j ($X^{t-1}_j = 1$), then in the next time period t that same hospital has to be open ($X^t_j \geqslant 1$ which means $X^t_j = 1$ because of 4.25).

$$X^{t-1}_j \leqslant X^t_j \;\; \forall_{t\in T\setminus\{1\}\; j\in J_c} \tag{4.13}$$

Equation 4.14 establishes that initial existing hospitals ($j \in J_o$) may close but, once closed, they cannot be reopened. In time period $t-1$, if a hospital is closed in location j ($X^{t-1}_j = 0$), then in the next time period t that same hospital has to be closed ($X^t_j \leqslant 1$ which means $X^t_j = 0$ because of 4.25).

$$X^{t-1}_j \geqslant X^t_j \;\; \forall_{t\in T\setminus\{1\}\; j\in J_o} \tag{4.14}$$

The last period ($t = \tau$) marks the end of the planning horizon and the opening and closure of facilities is not allowed. Equation 4.15 reflects this constraint. If, in penultimate time period ($t = \tau - 1$), the hospital in location j is closed, $X_j^{\tau-1} = 0$, then in the last period that same hospital has to be still be closed, $X_j^{\tau} = 0$.

$$X_j^{\tau} - X_j^{\tau-1} = 0 \;\; \forall_{j \in J_c} \tag{4.15}$$

Equation 4.16 says that if, in penultimate time period ($t = \tau - 1$), a hospital in location j is open , $X_j^{\tau-1} = 1$, then in the last period that same hospital has to be still be open, $X_j^{\tau} = 1$.

$$X_j^{\tau-1} - X_j^{\tau} = 0 \;\; \forall_{j \in J_o} \tag{4.16}$$

4.2.5.E Medical specialties

In real life, every hospital is divided into medical specialties and some patients need treatment from a multi-disciplinary team of doctors from different specialties. However, in this model, that is not possible. It is considered that every patient that constitutes the demand needs care from only one medical specialty and can only be transferred inside that medical specialty, and in consequence, to hospitals that have that specialty.

If a medical specialty exists in a hospital, it can have one, and only one, level. Equation 4.17 says that the sum of levels ($l \in L$) for a given hospital j medical specialty n and time period t, is at most 1. It can be 0, if the medical specialty does not exist in that hospital.

$$\sum_{l \in L} Z_{jnl}^{t} \leqslant 1 \;\; \forall_{t \in T \; j \in J \; n \in N} \tag{4.17}$$

If a hospital is closed (there is no hospital) then it cannot have any medical specialties. Equation 4.18 declares that if a hospital is closed ($X_j^t = 0$) then the sum of medical specialties has to be less than or equal to 0 ($Z_{jnl}^t \leqslant 0$), which means $Z_{jnl}^t = 0$ due to 4.25. If the hospital is open ($X_j^t = 1$), then the number of specialties has an upper limit of equal to the total number of existing specialties ($Z_{jnl}^t \leqslant n^{spec}$).

$$\sum_{n \in N} \sum_{l \in L} Z_{jnl}^{t} \leqslant n^{spec} \times X_j^t \;\; \forall_{t \in T \; j \in J} \tag{4.18}$$

If there is a hospital at site j, then there is a minimum number of medical specialties in that hospital. Equation 4.19 ensure that the sum of specialties (Z_{jnl}^t) is greater than or equal to a minimum number of specialities (n^{min}) if the hospital is open ($X_j^t = 1$).

$$\sum_{n \in N} \sum_{l \in L} Z_{jnl}^{t} \geqslant n^{min} \times X_{j}^{t} \ \forall_{t \in T\, j \in J} \tag{4.19}$$

A patient can only be transferred to another hospital if the receiving hospital has the medical specialty that the patient needs. Equation 4.20 states that if a hospital in location site j does not have a medical specialty n in level l ($Z_{jnl}^{t} = 0$), then the flow concerning hospital transfers in that medical specialty and level is less than or equal to zero in that hospital ($Y_{kjnpls}^{Tt} \leqslant 0$), which means is $Y_{kjnpls}^{Tt} = 0$ because of the constraints in 4.25. If a hospital in location site j has a medical specialty n in level l ($Z_{jnl}^{t} = 1$), then the flow concerning hospital transfers in that medical specialty and level has an upper limit of M (which is a sufficiently large value).

$$\sum_{k \in J} \sum_{p \in L} Y_{kjnpls}^{Tt} \leqslant M \times Z_{jnl}^{t} \ \forall_{t \in T\, j \in J\, n \in N\, l \in L\, s \in S} \tag{4.20}$$

Equation 4.21 states that if there is no medical specialty n in a certain level l in hospital j ($Z_{jnl}^{t} = 0$), the patient has to be "transferred" to the level of that hospital or actually be transferred to another hospital ($Y_{jnls}^{Ct} = 0$).

$$Y_{jnls}^{Ct} \leqslant M \times Z_{jnl}^{t} \, \forall_{t \in T\, j \in J\, n \in N\, l \in L\, s \in S} \tag{4.21}$$

Not all transfers between hospitals are possible. If the hospital the patient is allocated to has all conditions necessary for treatment, then the patient must be treated there if there is enough capacity. Transfers between hospitals from one level to another level below, in a certain specialty, are not allowed. This can sometimes happen in real case scenarios, if the patient is treated in a higher level hospital and then is referred to a lower level hospital for recovery (higher level care is not required anymore). However, for simplicity, the model described here does not allow that. To sum up, only ascendant (from a lower level to a level that is higher) and horizontal (from a lower level to a the same level) flows are possible. Equation 4.22 ensures that a patient being treated by medical specialty n in level l can only be transferred to another hospital if the receiving hospital has that medical specialty in the same level or above.

$$Y_{jknlps}^{Tt} = 0 \ \forall_{t \in T\, j \in J\, k \in J\, n \in N\, l \in L\, p \in \left\{p | p < l\right\}\, s \in S} \tag{4.22}$$

Equation 4.23 says that transfers from one level to the same level in the same hospital are not allowed.

$$Y_{jjnlls}^{Tt} = 0 \ \forall_{t \in T \ j \in J \ n \in N \ l \in L \ s \in S} \tag{4.23}$$

Equation 4.24 guarantees that if in a time period $t-1$ there is a medical specialty n in a certain level l in hospital j ($Z_{jnl}^{t-1} = 1$), then in the next time period t that same medical specialty has to exist in that hospital and it has to have the same level ($Z_{jnl}^{t} = 1$).

$$Z_{jnl}^{t-1} \leqslant Z_{jnl}^{t} \ \ \forall_{t \in T \setminus \{1\} \ j \in J \ n \in N \ l \in L} \tag{4.24}$$

4.2.5.F Standard integrality and non negativity constraints

Equation 4.25 states that variables X_j^t and Z_{jnl}^t are binary and can only take the value of zero or one. It also establishes that all the other variables cannot be negative.

$$X_j^t \ Z_{jnl}^t \in \{0,1\} \ \ Y_{ijnls}^t \ Y_{jknlps}^{Tt} \ Y_{jnls}^{Ct} \ cap_j^t \ Scap_{js}^t \geqslant 0 \ \forall_{t \in T \ i \in I \ j \in J \ k \in K \ n \in N \ l \in L \ p \in L \ s \in S} \tag{4.25}$$

After describing the mathematical model, the next section briefly states the conclusions of the present chapter.

4.3 Chapter conclusions

The present chapter described the model created and its characteristics. The bi-objective model is constrained by equations about demand satisfaction, flow conservation, hospital capacity, demand assignment, hospital location, hospital specialties and standard integrality and negativity. The model was designed to be general enough to be able applied to the hospital network in Portugal or in another country. Other constraints can be added to make the model more specific. The next chapter addresses all aspects of the implementation of the model, including the data used regarding the case study at hand. Still in the next chapter, the results obtained and discussion of said results are presented.

Chapter 5

Results and Discussion

In this chapter, the model presented in the last chapter is implemented, solved with real instances of the case study described in Chapter 2 and its results are presented and discussed. Section 5.1 presents the characteristics of the cases to which the model was applied. Section 5.2 presents the computational results obtained through the resolution of the model and an analysis of the obtained solution. Lastly, section 5.3 summarizes the most important conclusions of the chapter.

5.1 Characteristics of the cases

The model developed in Chapter 4 is tested here for one medical specialty and one hospital service at a time. The choice of the medical specialties and services was made according to the information available and how recent this information was. Consequently, it was ultimately decided to solve the model for two cases: the inpatient service in Cardiology in continental Portugal and the inpatient service in Internal Medicine in the Regional Health Administration of Alentejo. These two cases were chosen to demonstrate the full potential of applicability of the model. In the first case (subsection 5.1.1), a bigger scale was explored since the model was solved for the whole country. In the second case (subsection 5.1.2), the model was solved for a smaller region and uncertainty in demand was incorporated.

5.1.1 Case 1: Cardiology network in Portugal

The document that was central as a source of information for this referral network was the *”Rede de Referenciação de Cardiologia - Proposta de Atualização”* (Cardiology Referral Network - Proposal for an update) (Ministério da Saúde, 2015). It is important to note that Cardiology is a vast branch of the medical field that encompasses various diseases and procedures, each with a need for a different set of skills and resources. Therefore, in Portugal, Cardiology has two sub-specialties: Interventional

Cardiology and Cardiac Eletrophysiology. These two sub-specialties were not included in the case because the network would be too complex and their particular characteristics justify a separate network.

Even though there was an attempt to search for current information, the most recent data found was from 2013 and 2015, which is before the re-organizational changes that occurred in 2016 (mentioned in section 2.1). Therefore, some assumptions had to be made in order to test the model. In the document referenced, all the hospitals that offer services in Cardiology are listed and classified based on the old classification system. This system places each hospital in only one of four groups, according to a service offer that goes from less complex (Group I) to more complex (Group III). Group IV corresponds to the specialized hospitals in a set of medical specialties, which does not include Cardiology. Accordingly, the hospitals listed in the document and used in this case are only in the first three groups. The sets, the parameters and the weights presented in section 4.2 were defined to suit the case.

5.1.1.A Definition of sets

All the sets and subsets used in this case are described in table 5.1.

Table 5.1: Definition of sets and subsets (case 1).

Sets and subsets	
$T:$	$\{1, 2, 3\}$
$I:$	$\{1, 2, 3, ..., 18\}$
$J:$	$\{1, 2, 3, ..., 39\}$
J_o	$= J$
J_c	$= \{\emptyset\}$
J_m	$= J_o$
$S:$	$\{1\}$
$N:$	$\{1\}$ = {Cardiology}
$L:$	$\{1, 2, 3\}$

The planning horizon (T set) is divided into three periods. The demand points (I set) being considered are the districts of Portugal (represented in figure A.1), which makes a total of 18 demand points. The J set represents the potential locations for siting hospitals. For the Cardiology test situation, the J set comprises of exact 39 locations where there were already hospitals with a Cardiology service in 2013. Since this test situation was one primarily for allocation of demand, the subset J_o, which represents the hospitals initially existing, has all the locations of J. In other words, it is equal to J. As a result, J_c is an empty set since it represents all the closed facilities at the beginning of the planning horizon. Also for the reason stated, J_m is equal to J_o, as all the hospital that are initially existing must be kept open. Furthermore, there is only one scenario s for demand and one medical specialty (Cardiology) n as previously said. The number of levels l each hospital can have, according to this specialty, is three.

5.1.1.B Definition of parameters

All the parameters, except d^1_{ij}, d^2_{jk}, D^{SNLt}_{inls} and pop^t_j are defined in table 5.2. The exceptions are explained outside the table, since they require some further explanation.

Table 5.2: Definition of parameters and weights (case 1).

Parameters and weights	
α :	0.5
P_s :	{1}
$capmin^t_j$:	50($l = 1$), 500($l = 2$), 1000($l = 3$)
$capmax^t_j$:	1400($l = 1$), 1900($l = 2$), 6000($l = 3$)
d^{max} :	70
N^t :	39
pop^{min} :	10 000
$year^t$:	1
n^{spec} :	50
n^{min} :	1 = {Cardiology}
M :	1 000 000

The value for the α parameter acts as a weight in the objective function that minimizes travel times. It serves to distinguish first entries in the system and transfers between hospitals. Here, it is defined as 0.5 because it was considered more crucial for a person to reach a hospital quickly as a direct entry than as transfer from another hospital, since the latter may already have received preliminary hospital care. The probability of scenario s, P_s, is 1 since there is only one scenario. In terms of minimum and maximum capacity for the hospitals, the values are the same for each time period, and were based on the minimum and maximum number of people admitted in the Cardiology service per level in the year 2013, respectively. The d^{max} parameter represents the maximum amount of time that a person has to travel to reach a hospital (as a direct entry) and it is defined as 70 minutes. The maximum number of open hospitals at a given time period (N^t) is set at 39. The minimum population (pop^{min}) for a hospital to open is set at 10 000 people (parameter not used in this case since all the hospitals are open). The parameter $year^t$ is set at one so each time period has the duration of one year. The maximum number of medical specialties (n^{spec}) a hospital can have is 50 and the minimum (n^{min}) is 1. The M (big m) parameter is defined as 1 000 000.

Travel time

One common objective of location-allocation problems is the minimization of travel time or distance travelled from demand points to facilities and from facilities to other facilities. In this model, what was minimized was the average travel time weighted by demand. Therefore, it was necessary to calculate the travel time between every demand point i and every hospital candidate site j, d^1_{ij}, and the travel time between every pair of hospitals j and k, d^2_{jk}. In order to do this, the coordinates of every demand point and every candidate location was determined. Then, the Haversine distance between the pair of points mentioned above was calculated. Finally, the travel time was computed using specific values of velocity for converting distance into time. For distances equal or less than 50 kilometres, the velocity used was 50 kilometers per hour. For distances greater than 50 kilometres, the velocity used was 100 kilometers per hour. The different velocity values are meant to take into account the slower traffic that can exist when travelling smaller distances, for example inside cities and localities, and the faster traffic in larger distances due to highways where the velocity limit is higher.

Demand

Another input the model requires is the demand for healthcare (D^{SNLt}_{inls}). This demand needs to be broken down by time period t, demand point i, specialty n, level l and scenario s. In this particular case, it was assumed that the demand did not change over the different time periods. Given that each time period was only a year, it is safe to assume the demand does not change significantly. More specifically, because there is only one medical specialty and one scenario, the demand to be calculated was the number of people per district in need of care in the Cardiology specialty per level in a year. In order to do this, the percentage of resident population in each district was calculated (Instituto Nacional de Estatística, 2021b). Secondly, the total number of patients in need of care in the Cardiology specialty per level needed to be computed. However, there was no information about demand discriminated by level in this specialty (or any specialty). Therefore, to obtain this type of data some assumptions had to be made. The number of people admitted to hospitals that were ranked at level I was known (Ministério da Saúde, 2015). With this knowledge, the percentage of people treated in the service of Cardiology in those hospitals was calculated. Because level I patients can be treated at hospitals ranked in any level, assuming that percentage is the percentage of patients in need of care in level I is a huge underestimation. In order to get a more realistic number, the value was doubled. Thus, the percentage of patients in need of care in level I was assumed to be 60% of all patients in need of Cardiology care. The percentage of patients in need of care in level II was computed by taking 60% of the patients left. Finally, these percentages were converted to number of people by multiplying them by the total number of patients in need of Cardiology care. The number of patients per level per district was then obtained through the multiplication of these numbers by the percentage of resident population in each district (calculated earlier). The demand was assumed to be the same in all the time periods and it is described in table C.3.

The population in demand in the area of each hospital (pop_j^t) was defined as the population that existed in the location of every hospital with Cardiology service in 2013. It was also assumed to be the same in every t since the planning horizon was of three years in total (Instituto Nacional de Estatística, 2013).

5.1.2 Case 2: Internal Medicine in the Alentejo's Regional Health Administration

In this case, the document that provided the information needed for the Internal Medicine referral network was the *"Rede de Referenciação de Medicina Interna"* (Internal Medicine Referral Network) (Carvalho et al., 2016). As it was said for the first case, there was an effort to procure recent and reliable information. However, also for this case, some assumption had to be made, mainly regarding demand.

5.1.2.A Definition of sets

All the sets and subsets used in this case are described in table 5.3.

Table 5.3: Definition of sets and subsets (case 2).

Sets and subsets	
T :	$\{1, 2, 3\}$
I :	$\{1, 2, 3, ..., 11\}$
J :	$\{1, 2, 3, ..., 9\}$
J_o	$= \{\emptyset\}$
J_c	$= J$
J_m	$= \{\emptyset\}$
S :	$\{3\}$
N :	$\{1\}$ = {Internal Medicine}
L :	$\{1, 2, 3\}$

The planning horizon (T set) is divided in three periods. The demand points (I set) considered are 11 in total. Each sub-region of Alentejo's Regional Health Administrations (RHA) is divided in 3 demand points, except one (Alentejo Litoral) that is divided in 2. In terms of hospital locations (J set), 9 were considered. All locations, similarly to case 1, were locations where a hospital already exists or where there is a possibility for building one. Two of the locations are in Lisbon but, due to the lack of higher level hospitals in the region being studied, the hospitals at those locations are part of the Internal Medicine referral network. No hospitals were considered to be opened in the first time period (J_c is equal to J and J_o is empty) and no hospitals were forced to be kept open (J_m is also empty). In this case, three scenarios for demand were considered. One representing low demand (s=1), another high demand (s=2) and another representing a baseline level of demand (s=3). Finally, the number of levels each hospital can have, according to this specialty, is four. However, no hospital is ranked at level III, so level III and IV are merged into one. Subsequently, the L set has three levels.

5.1.2.B Definition of parameters

All the parameters, except d^1_{ij}, d^2_{jk}, D^{SNLt}_{inls} and pop^t_j are defined in table 5.4. The exceptions are explained outside the table, since they require some further explanation.

Table 5.4: Definition of parameters and weights (case 2).

Parameters and weights	
α :	0.5
P_s :	{1/3, 1/3, 1/3}
$capmin^t_j$:	0
$capmax^t_j$:	3500 × 5 ($l = 1$), 5500× 5 ($l = 2$), 6000× 5 ($l = 3$)
d^{max} :	90
N^t :	9
pop^{min} :	10 000
$year^t$:	5
OC^t_j :	347 × 7.4 ($l = 1$), 491 × 8 ($l = 2, 3$)
IC^t_j :	200000 ($l = 1$), 224500 ($l = 2, 3$)
CC^t_j :	200000 ($l = 1$), 224500 ($l = 2, 3$)
n^{spec} :	50
n^{min} :	1 = {Internal Medicine}
M :	1 000 000

The value for α is the same as in the first case. Regarding the probability of the three scenarios, each scenario is considered to be equally likely so P_s is 1/3 for every s. In terms of minimum and maximum capacity for the hospitals, the values are the same for each time period. The minimum is zero and the maximum is based on the number of people admitted in the Internal Medicine service per level in the year 2016. The d^{max} parameter is 90 minutes. Since this value is very high for a travel time, some other lower travel times were explored further in the solutions. The maximum number of open hospitals at a given time period (N^t) is set at 9. The minimum population (pop^{min}) for a hospital to open is set at 10 000 people and every region for hospital candidate sites fulfilled this condition. This means pop^{min} parameter will not limit the opening of a hospital in this case. The parameter $year^t$ is equal to 5 years, so each time period has that duration. Thus, the planning horizon lasts 15 years. Regarding hospital costs, the information was based on the Mestre et al. (2015)'s paper. The maximum number of medical specialties (n^{spec}) a hospital can have is 50 and the minimum (n^{min}) is 1. The M (big m) parameter is defined as 1 000 000.

Travel time

The travel time between every demand point i and every hospital candidate site j, d^1_{ij}, and the travel time between every pair of hospitals j and k, d^2_{jk} were calculated similarly as in case 1. The velocities of travel considered were 80 kilometres per hour for distances equal/below 60 kilometres and 100 kilometres per hour for the remainder.

Demand

According to a study performed by the National Institute of Statistics (Instituto Nacional de Estatística, 2020b), the population of Alentejo will decrease in the next decades. In order to have reliable results and because demand is considered a great source of uncertainty, three possible scenarios were created. In all scenarios, the population never grows. The difference in each scenario is the rate of population decrease. In a pessimistic scenario, the population decreases at a faster rate than the present one. In a baseline scenario, it continues to decrease in a rate equal to the present rate. Lastly, in an optimistic scenario, it decreases at a slower rate than the present rate. The prediction for the resident population of Alentejo in these three scenarios are represented in figure 5.1.

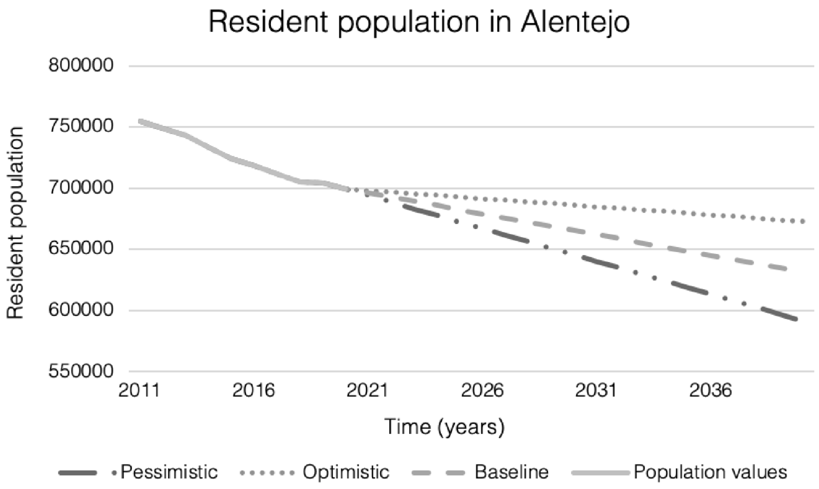


Figure 5.1: Projections for population growth in Alentejo (data taken from Instituto Nacional de Estatística (2020b)).

The resident population in each demand point varied, according to the different projection scenarios and across the different time periods. In consequence, the total number of expected people in demand for Internal Medicine services also varied, in the different scenarios and time periods. The demand per level was then calculated similarly to case 1 and the final values of D_{inls}^{SNLt} were obtained, for all elements of S and T. The demand values are described in tables C.4, C.5 and C.6. It is relevant to note that the distribution of population through the different sub-regions did not change. In other words, the total population of Alentejo and the population of each sub-region changed but the size of the population in each sub-region, in relation to each other, was assumed to be always the same. The projections for the population were calculated using data from Instituto Nacional de Estatística (2020b) and the relevant data can be seen in tables C.1 and C.2.

The population in demand in the area of each hospital (pop_j^t) was defined as the population that existed in the location of every hospital with Internal Medicine service in 2016. It was assumed to be the same for every time period of the planning horizon (Instituto Nacional de Estatística, 2021a).

In the next section, the results obtained, through the model solution using the cases characterized in this section, will be presented.

5.2 Computational results

The present section introduces steps taken towards the validation of the model (subsection 5.2.1) and the computational results of the application of the model to each of the described cases (subsection 5.2.2). The model was implemented and solved in Python™, using the *docplex - IBM Decision Optimization CPLEX* library. Every test was performed in a dual-core Intel® Core™ i5-5250U CPU @ 1.60GHz and 4GB 1600MHz DDR3 memory computer with the macOS Big Sur (Version 11.6) operating system.

5.2.1 Model validation

Before testing for the described cases, the model was first tested with simple fictional data, to find the potential errors in the writing of the constraints or when implementing the model itself in Python. This allowed to validate the model. These test instances included only a few hospitals and demand points in order to be easier to figure out the incongruities. In each of the tests, the constraints defined in section 4.2.5 were sequentially included. Different values for the sets and parameters were also used, to test the model for several configurations of locations, number of medical specialties and levels, capacities, number of hospitals, demand, etc. In this way, it was guaranteed that each of the restrictions made sense and they produced coherent and expected results. These validation test instances were numerous but will not be discussed further since they do not have any applicability to real life scenarios.

5.2.2 Discussion of the results obtained

5.2.2.A Case 1: Demand allocation in the Cardiology network in Portugal

In case 1, the goal was to primarily test the model regarding allocation of demand using all the national continental territory. The model was solved for only one medical specialty and the model was forced to open all the hospitals. As explained before, for this case, only objective function 4.1 (relative to improvement of access) was optimized since there would be no costs of opening/closing hospitals. The model was solved in 36.72 seconds and the value of the minimized objective function was 897245.335 minutes.

The flows from demand points to hospitals, from hospitals to other hospitals and the number of people served at each hospitals are described, in detail, in figure C.1 in the appendix C. In figure 5.2, these flows, in respect to period time $t = 1$, are represented in a map of Portugal.

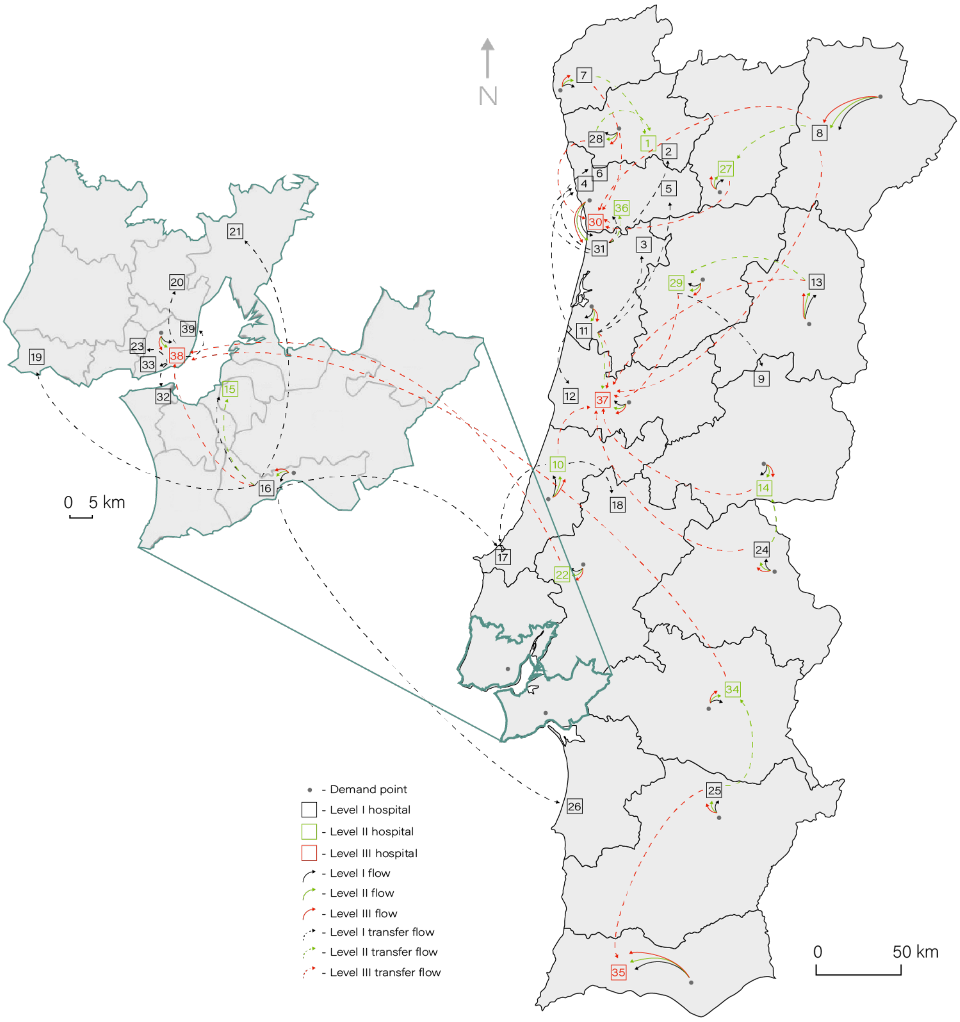


Figure 5.2: Model results for demand allocation in case 1.

The hospitals were represented by squares with numbers (correspondence of hospital's numbers to the respective names is presented in table A.1). The location of the hospitals was not decided by the model. However, the level, the number of people each hospital serves and the flows between hospitals were. Hospitals in level I were represented by black squares, level II were represented by green squares and level III were represented by red squares. The demand points were represented by small grey circles. There were 18 demand points that represented the 18 districts. For simplicity, it was assumed that everyone from a certain district "lives" in the same point. Flows from demand points to hospitals (direct entries in the system) were represented by black, green and red arrows (corresponding to patients in need of level I, level II and level III care, respectively). Flows from hospitals to other hospitals (transfers) were represented by black, green and red dashed arrows (corresponding to patients in need of level I, level II and level III care, respectively).

Through the analysis of figure 5.2, the first thing that can be verified is that the demand is allocated to the nearest hospital for every demand point except one. The demand from Porto is allocated to a hospital in another district (hospital 31 in Aveiro) instead of being allocated to any of the hospitals in Porto that may be, technically, at a closer distance (e.g. 4, 5, 6, 30 or 36). According to the travel times d^1_{ij} calculated by the model, $d^1_{1,30} < d^1_{1,31} < d^1_{1,6} < d^1_{1,36} < d^1_{1,5} < d^1_{1,4}$ (where Aveiro is demand point $i = 1$ and the j's are the hospital locations). This means that the demand from Aveiro should have been assigned to hospital 30, since it is the closest hospital. However, hospital 30, being a level III hospital, is at full capacity already (as seen in figure C.1). So, the model assigned the demand from Aveiro to the next closest hospital, which is hospital 31. In reality, hospital 31 is not the closest hospital but, since the travel times were calculated through an approximation, in the model it is. An issue like this could be solved by adjusting the values used for velocity, for the distance turning point or by attributing the actual travel times to each demand point-location pair.

Besides this, it can verified that each hospital is only treating the patients that need care at the level the hospital is or at a level below (and transfers the rest), as it was intended. An example of this is hospital 24 which is a level I hospital, that receives patients from every level, and transfers level II patients to hospital 14 and level III patients to hospital 37. Another observation that can be made regarding transfers flows is the fact that some hospitals, due to overcrowding, are transferring patients to multiple other hospitals. That is, the hospitals are transferring patients, not because they do not have the "level required", but because they are at full capacity. It is the case of hospital 38 for example. This hospital is located in Lisbon, where the demand for hospital care is high. Because this hospital is the nearest to the demand point that represents the Lisbon district, it has to receive all patients from that district plus some level III patients from other hospitals in other districts, since hospital 38 is also the closest level III hospital in the Lisbon vicinities. Level III patients are a priority for hospital 38 because they can only be treated in level III hospitals. Thus, other patients that can be treated in other hospitals (level I patients for example) are being transferred to other hospitals (23, 32, 33, 39). Even though this does not represent exactly the situation in real life, some conclusions can still be drawn if the demand from these hospitals is seen as aggregated. It is clear that these hospitals are serving the demand from Lisbon plus the higher level patients from other districts, which is in fact what happens in real life. If needed, the demand point from Lisbon could be divided into several demand points and the demand would be distributed more uniformly by other hospitals and would not be allocated to one single hospital. Still on this topic, some hospitals (19, 21, 20...) do not receive any direct entries. Thereby, a conclusion that can be drawn from these results is that the choice of demand points for these types of models is crucial. Differences in demand point size and location can influence the model tremendously and, due to that fact, should be chosen carefully.

In terms of classifying each hospital in a level, there are some differences between the results of the model and reality. There were no costs included in this version of the model, which means, without any constraints in this aspect, the model would have classified the majority of hospitals as level III because these are the ones that can treat the most types of patients. However, level III hospitals are more expensive to build and to maintain than level II or level I hospitals, so some restrictions had to be imposed. In 2013, the number of hospitals/hospital centers classified at level I was 29, level II was 9 and level III was 4 as can be seen in table A.1. In the model, the number of hospitals allowed to be classified at each level was kept the same but the model was allowed to choose which of the hospitals were classified at each level. The results show that the model assigned level III to hospitals 30, 35, 37 and 38 instead of hospitals 36, 37, 38 and 39. Two of the classifications (37 and 38) coincide but two of them do not. The choice of classifying hospital 30 as a level III hospital instead of level 36 is not too odd because they are both in localized in Porto. The more interesting choice was the decision to classify as level III a hospital in Faro (hospital 35) instead of hospital 39 in Lisbon. Not considering costs, this choice seems to make more sense than locating another level III hospital in Lisbon since in the south of Portugal there are few hospitals and the ones that exist are not that specialized. If it were considering costs, the model might make a different decision since it may not be worth to maintain a level III hospital for the demand that exists in the south. In addition to changes in the classification of level III hospitals, there were also changes in the other levels. In general, the model also improved access to level II care since it classified hospitals, not previously classified as level II, as level II that are located further away from large cities (e.g. 10, 14). Again, this was expected since the model did not include costs. Nonetheless, considering only accessibility, these classifications would be the best choices for improving access to health care services and, consequently, to increase equity in access to those services.

It should be noted that each square is said to be representing a hospital, when in reality it can be representing a hospital, a hospital center or a local health unit. This means that some of those squares are representing multiple facilities and a single location had to be chosen for all of the facilities in that group of health care facilities. The multiplication of the single locations into several locations would also contribute to a more realistic model, as it was said for demand points. Furthermore, including uncertainty in demand would also be an interesting approach to see how similar the solutions would be.

In conclusion, it can be said the model behaved as expected, correctly assigning demand to the closest facilities and transferring patients when needed. It also provided very useful insights in terms of improving geographic accessibility countrywide. Regarding possible work to be done in the future, this case could benefit from the inclusion of more demand points per district, more hospital location points per hospital group, from the consideration of hospital costs and from the consideration of uncertainty in demand (total and per level). As can be predicted, the more complex the case is, the more realistic and accurate it is. However, it also takes more time to prepare the data and for the model run. Some of the mentioned aspects for model improvement were not included due to lack of reliable and recent data. Nonetheless, they would be interesting topics of explore in future work.

5.2.2.B Case 2: Internal Medicine in the Alentejo's Regional Health Administration

In case 2, the model was solved for a smaller geographic area. In this way, it was possible to make several iterations by varying the values of the parameters and incorporating uncertainty. The model was solved for the inpatient service of the Internal Medicine medical specialty in the health region of Alentejo. In this case, the two objective functions (eq. 4.1 and eq. 4.2) were considered, as well as the three demand scenarios and the three time periods. In the beginning of the planning horizon, no hospitals were considered to be open. Some additional constraints related to levels were added to the model to increase its realistic aspect. The hospitals in Lisbon were forced to be classified as level III in the medical specialty in question and the number of hospitals at that level was limited to two. This means no other hospital, except those in Lisbon, could have that classification. Also, the number of hospitals classified with level II was not allowed to be more than 2. These restrictions served as additional baseline budget constraints since an operating hospital has different expenses depending on its level.

Two important notes must be mentioned before the analysis of the results, considering they have a significant impact on the results. The first one relates to the $capmin_j^t$ values. The minimum capacity allowed in a hospital was set to 0 in all locations. Since all capacities were in number of people treated per time period t, it did not make sense, at the time, to impose a minimum number of inpatients in a specialty in a certain period of time. Hospitals have several services and specialties and, in some extents of times, the number of people treated in some of them is low or even null. Adding to this, there was the initial cost of opening a hospital that should discourage the model to open a hospital where no patients are treated. However, through the analysis of the results, it is possible to conclude that this initial ”penalty” is not sufficiently deterrent.

The second one relates to both the values of maximum capacity allowed in a hospital ($capmax_j^t$) and the values for the operational costs (OC_j^t). These two groups of values differ for the different hospital locations j, according to the level classification that was given to the hospital that was in that location at the time of the data acquisition. Every location j corresponds to a real hospital location where there is a hospital (with the exception of location $j = 4$ which will be the location of a hospital in the near future). These hospitals are all fully operating hospitals with a level assigned to them as described in figure A.2 and table A.2. Therefore, this information was used to inform the decision of maximum capacity and costs attributed to each location. Locations that, in real life, corresponded to a higher level hospital location were given higher values for maximum capacity and operational costs. Likewise, locations that corresponded to a lower level hospital location were given lower values for maximum capacity and operational costs. Consequently, this made the model biased towards choosing certain solutions. Even though the model was able to choose at which locations to open hospitals and what levels to attribute to those hospitals, some locations had an ”advantage” (since they had more capacity or lower daily activity costs).

Analysis of trade-offs between costs and travel times

The solutions obtained for the model are represented as points in figure 5.3 and information about the solutions is described in table 5.5. Each point represents a value of minimized costs in euros (€), which can be operational or related to opening/closing hospitals, and a value of minimized travel times to reach hospital services weighted by demand in minutes. The results for the deterministic model in the three scenarios are represented by points A, A', A", B, B' and B". The location of each point represents different configurations of the hospital network, which implies trade-offs between costs and time travelled to access hospitals services. Each of these points was calculated by minimizing each objective function separately. Points A, A' and A" were obtained by first minimizing the time/distance travelled to reach hospitals services, fixing that objective function on that minimum value and then minimizing the objective function about costs. Points B, B' and B" were calculated similarly but the objective functions switched places. First, the costs objective function was minimized and fixed on the minimum value discovered, then the travel time objective function was minimized. This was done for every demand scenario. Therefore, it can be said that points A, A' and A" represent the improved access solution, while points B, B' and B" represent the minimum cost solution, respectively for low (s1), high (s2) and intermediate (s3) demand. Points A* and B* refer to the combination of all scenarios in one solution. Thus, they represent the stochastic results, where each scenario was considered and had the same probability of happening.

Table 5.6 introduces the calculated trade-off values, of costs and travel times, going from one improved access solution to a different improved access solution, as well as the calculated trade-off values, of costs and travel times, going from one improved costs solution to a different improved costs solution. Table 5.7 introduces the calculated trade-off values, of costs and travel times, going from one improved access solution to an improved costs solution, as well as the calculated trade-off values, of costs and travel times, going from one improved costs solution to an improved access solution.

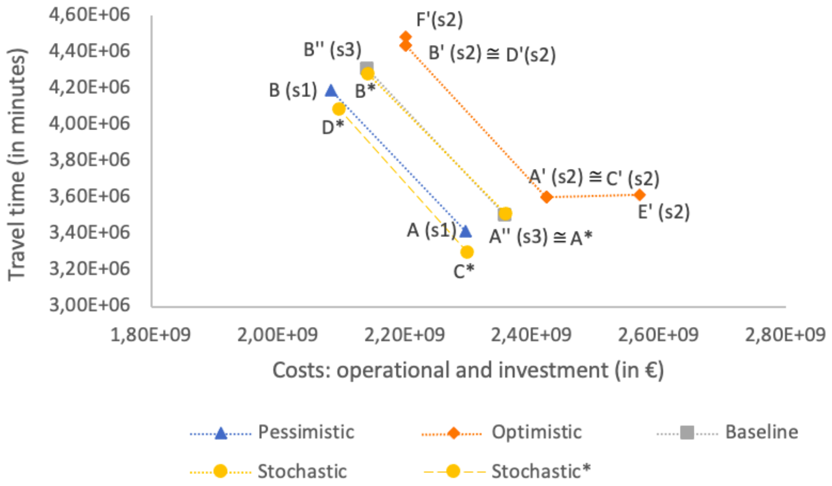


Figure 5.3: Solutions obtained for case 2 with deterministic and stochastic results.

Table 5.5: Results obtained with different parameters values and scenarios.

Point	Costs (in €)	Travel time(in min)	First objective function	Scenario and parameters values	Type of model
A (s1)	$2,30 \times 10^9$	$3,41 \times 10^6$	Minimize times	Pessimistic	Deterministic
B (s1)	$2,08 \times 10^9$	$4,19 \times 10^6$	Minimize costs		
A' (s2)	$2,42 \times 10^9$	$3,60 \times 10^6$	Minimize times	Optimistic	
B' (s2)	$2,20 \times 10^9$	$4,44 \times 10^6$	Minimize costs		
A" (s3)	$2,36 \times 10^9$	$3,50 \times 10^6$	Minimize times	Baseline	
B" (s3)	$2,14 \times 10^9$	$4,31 \times 10^6$	Minimize costs		
C' (s2)	$2,42 \times 10^9$	$3,60 \times 10^6$	Minimize times	Optimistic (max travel time: 50 min)	Other deterministic results
D'(s2)	$2,20 \times 10^9$	$4,44 \times 10^6$	Minimize costs		
E' (s2)	$2,57 \times 10^9$	$3,61 \times 10^6$	Minimize times	Optimistic (max capacity: 80 per cent)	
F'(s2)	$2,20 \times 10^9$	$4,49 \times 10^6$	Minimize costs		
A*	$2,36 \times 10^9$	$3,51 \times 10^6$	Minimize times	All	Stochastic
B*	$2,14 \times 10^9$	$4,28 \times 10^6$	Minimize costs		
C*	$2,30 \times 10^9$	$3,30 \times 10^6$	Minimize times	All	Stochastic*
D*	$2,10 \times 10^9$	$4,08 \times 10^6$	Minimize costs		

Stochastic* - refers to results obtained by making changes in the sets values (explained in the text).

Table 5.8 relates to the network configurations. The rows symbolize hospital locations and the columns symbolize solutions (or hospital configurations). The last column corresponds to the real configuration of the network, as described in Chapter 2. Each square corresponds to the state of each hospital in each solution. The possible states for each hospital (in the medical specialty in question) are: open and in level I (I); open and in level II (II); open and in level III (III); or closed (-). It should be

Table 5.6: Trade-offs between costs and travel times (improved access/access solutions and improved costs/costs solution).

Difference between going from one improved access solution to another			Difference between going from one improved costs solution to another		
Solutions	Costs	Travel time	Solutions	Costs	Travel time
A - A”	2,7%	2,7%	**B - B”**	2,7%	2,8%
A” - A’	2,8%	2,8%	**B” - B’**	2,8%	2,9%
A’ - C’	0,0%	0,0%	**B’ - D’**	0,0%	0,0%
A’ - E’	6,0%	0,3%	**B’ - F’**	0,0%	1,1%
A* - C*	-2,6%	-5,9%	**B* - D***	-2,1%	-4,6%
A” - A*	0,0%	0,1%	**B” - B***	0,0%	-0,7%

noted that in the R column, as mentioned previously, hospital F is not closed but does not belong to the National Health Service (NHS) so it is considered to be closed.

From a preliminary analysis of the deterministic results, it can be verified that optimistic scenarios generate the most expensive solutions, pessimistic scenarios generate the most low-cost solutions and baseline scenarios generate an intermediate cost solution ($cost_A < cost_{A''} < cost_{A'}$ and $cost_B < cost_{B''} < cost_{B'}$). From the observation of table 5.8, it is possible to see that the configurations in A, A” and A’ are the same; as well as the configurations in B, B” and B’. This indicates that the change in costs (and travel times) is not related to network configurations or hospital classifications. It is related to the size of the demand being served. Optimistic scenarios correspond to a prediction in which the values for demand are highest, followed by the values from the baseline scenario and the values from the pessimistic scenario. Higher values for demand equates to more hospital utilization and that can lead to higher costs. From table 5.6, it can be verified that the difference in costs, of going from a solution in a pessimistic scenario to a solution in a baseline scenario, is of 2,7%. This is true for comparisons between both improved access solutions and improved costs solutions (A-A” and B-B”). The difference in costs, of going from a solution in a baseline scenario to a solution in an optimistic scenario, is 2,8%, which is slightly higher. This is verified for passing from one improved access solution to another (A”-A’) and for passing from one improved costs solution to another (B”-B’).

In can also be verified that the trend stays the same for time travelled. The travel time increases with the increasing of the demand ($time_A < time_{A''} < time_{A'}$ and $time_B < time_{B''} < time_{B'}$). This can be explained by the need to cater to more people. More people in need of reaching hospitals services, which means more people for which the model has to minimize travel time, can lead to more travel time for everyone. Through the analysis of table 5.6, it can be verified that the difference in travel times, of going from an improved access solution in a pessimistic scenario to an improved access solution in a baseline scenario, is of 2,7% (A-A”). However, the difference in travel times, of going from an improved costs solution in a pessimistic scenario to another improved costs solution in a baseline scenario, is of 2,8% (B-B”). Furthermore, the difference in travel times, of going from an improved access solution in a baseline scenario to another improved access solution in an optimistic scenario, is 2,8% (A”-A’). And the difference in travel times, of going from an improved costs solution in a baseline scenario to another improved costs solution in an optimistic scenario, is 2,9% (B”-B’).

Table 5.7: Trade-offs between costs and travel times (improved access/costs solutions and improved costs/access solution).

Difference between going from an improved access solution to an improved costs solution			Difference between going from an improved costs to an improved access solution		
Solutions	Costs	Travel time	Solutions	Costs	Travel time
A - B (s1)	-9,2%	22,8%	**B - A (s1)**	10,1%	-18,6%
A’ - B’ (s2)	-9,2%	23,2%	**B’ - A’ (s2)**	10,1%	-18,8%
A” - B” (s3)	-9,2%	23,0%	**B” - A” (s3)**	10,2%	-18,7%
C’ - D’ (s2)	-9,2%	23,2%	**D’ - C’ (s2)**	10,1%	-18,8%
E’ - F’ (s2)	-9,2%	24,1%	**F’ - E’ (s2)**	16,7%	-19,4%
A* - B*	-9,2%	22,1%	**B* - A***	10,1%	-18,1%
C* - D*	-8,8%	23,7%	**D* - C***	9,6%	-19,2%

Regarding the comparison (in table 5.7) between going from one improved access solution to an improved costs solution, it is possible to see that the costs decrease by 9.2% in every scenario (A-B, A’-B’ and A”-B”). However, the increase in travel times is not equal for all those cases. For the same decrease in costs, going from an improved access solution to an improved costs solution in a pessimistic scenario (A-B), corresponds to the lowest increase in travel times (followed by A”-B” and then A’-B’). When going from an improved costs solution to an improved access solution, the difference in costs is the same for the pessimistic and optimistic scenario (B-A and B’-A’) and is equal to 10,1%. For the baseline scenario, the value is 10,2%. Similar to the previous situation, the decrease in travel times is not equal for all those cases but is very close. The largest decrease (18,8%) in travel times going from an improved costs solution to an improved access solution happens in the optimistic scenario (B’-A’). Changes in configurations between all these solutions will be discussed later.

Looking at the stochastic results (points A* and B*), it is possible to affirm that the values of costs and travel times are quite similar to the ones obtained for the deterministic solution in the intermediate scenario (A* ≅ A” and B* ≅ B”). The differences between solutions, both in costs and travel times, is less than 1% and there are no differences in the configurations of the network. In terms of comparing the price of going from an improved access solution to an improved costs solution, it is clear that for a decrease in costs of 9,2%, the increase in travel times is only 22,1%. In the case of going from an improved costs solution to an improved access solution, it can be seen that for a decrease of 18,1% in travel times, the costs increase 10,1%. The network configurations of these solutions will also be discussed later.

Points C', D', E' and F' represent other relevant deterministic solutions obtained by changing the values of some of the parameters. These variations were all performed using the optimistic scenario since this was the scenario that predicted the highest value for demand. Points C' and D' are the solution for when the maximum travel time allowed - d^{max} - was lowered to 50 minutes (it was 90 minutes before). Points E' and F' are the solution for when the maximum capacity - $capmax_j^t$ - for all hospitals was reduced to 80% of what it was before. Points C* and D* are the stochastic results for a case in which the model was allowed to place an extra level III hospital in Alentejo, which represented a possibility to stop transferring patients to hospitals outside this region. By observation of these new results, it is clear that the solution found for points C' and D' (where the maximum travel time was reduced) is very similar, both in time and in costs, to the deterministic solution found with the original value of d^{max} (C' $\cong$ A' and D' $\cong$ B'). It is also apparent that the values of the improved cost solution corresponding to a lowered $capmax_j^t$ are very close to the corresponding improved cost solutions in the optimistic scenario (F' $\cong$ B' $\cong$ D'). However, the improved access solution, even though it has the same travel time value ($time_{E'} \cong time_{A'} \cong time_A*$), it differs significantly on the costs value. The difference of lowering the maximum capacity corresponds to a increase in costs of 6,0% and an increase of travel times (0,3%). This difference can be justified by some differences in the configurations of the hospitals and by a bigger percentage of patients being treated in higher level hospitals (seen in figure C.3).

Regarding the points C* and D*, related to a solution where an additional level III hospital was allowed to be placed, it is possible to see that the improved access solution had a lower value of travel time when compared to the other stochastic improved access solution ($time_{C*} < time_{A*}$), corresponding to a 5,9% difference. This verifies that, when there is a higher level hospital inside the Alentejo region, the patients do not need to be transferred to hospitals in Lisbon and the total travel time is lower. The same happens with the improved costs solutions ($costs_{D*} < costs_{B*}$), which may seem counter-intuitive because a solution that has 3 hospitals in level III (D*) is less expensive than another with 2 hospitals in level III (B*), for the same number of hospitals in level I and II. Here, it is important to remember that the costs can be divided into two types: operational costs and investment costs. The former relates to hospital daily activities and varies with the expected utilization of the hospital and with the location of each hospital (as mentioned before). The latter relates to the opening/closing of hospitals and only depends on that. In the solutions found for all points of figure 5.3, the model decides to open all hospitals in every time period (figure C.2), which means that all solutions will have a baseline cost that is the same for every solution. Thus, the difference in costs in B* and D* (decrease of 2,1%) comes down to differences in operational costs. Through the analysis of the number of people being treated in each hospital (figure C.4), it can be verified that in the D* solution no patients are being treated in hospitals C($j = 3$), D($j = 4$), H($j = 8$) and I($j = 9$), even though they are open. This means that the model is opening hospitals and assigning demand, from demand points close by, to those hospitals and then transferring it all to other hospitals that have a lower rate for operational costs. This way the model is able to ”compensate” the initial cost of opening hospitals that are not used in two ways. The first is by minimizing the total travel times. The travel time between demand points and hospitals is being minimized because the patients are being assigned to hospitals near by, they are just not being treated there. The second is by minimizing operational costs. In this solution, location F ($j = 6$) was the location chosen to place a level III hospital, additionally to the ones in Lisbon. This location was associated with lower operational costs since it was originally the location of a level I hospital. Therefore, all level III patients are being treated inside the region of Alentejo and at a lower cost. So, even though the results could be more realistic, they make sense given the parameters and the constraints of the model. In the future, it would be of great interest to explore different decisions regarding parameter settings in order to obtain an even more realistic solution.

Finally, still looking at figure 5.3 and at tables 5.6 and 5.7, it is possible to conclude that the solutions obtained do not vary that much in relation to each other, in terms of the values evaluated. The calculated differences between improved access solutions, both in costs and travel times, is never higher than 6%; and the calculated differences between improved costs solutions, both in costs and travel times, is never higher than 5%. In general, the points are all relatively close together. This is probably due to the somewhat strict constraints that were imposed to the model, regarding level classification and number of hospitals. The changes in the hospital network configurations are going to be discussed in the next section.

Analysis of changes in network configurations

In addition to analysing travel time and cost values, it is important to look at the actual solutions found for the configuration of the network of hospitals. To aid in this discussion, table 5.8 and figure 5.4 must be analysed.

From observing the table, it can be confirmed that all hospitals were opened in every solution. It can also be verified that there are some changes in the classification of the hospitals. The solution that is closest to the real configuration is the the improved costs solution for when maximum capacity is at 80% (E’). It is also clear that the model, when given the possibility of placing two level II hospitals, always chose to do it, even when costs were minimized first. In part, this validates the government’s decision to

Table 5.8: Configuration of the hospital network for the different solutions. Classification of levels of each hospital in black, green and red for levels I, II and III, respectively. The *R* column corresponds to the real configuration of the hospitals in the NHS.

		Points														
		A	B	A’	B’	A”	B”	C’	D’	E’	F’	A*	B*	C*	D*	R
Hospitals	A	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
	B	II	II	II	II	II	II	II	II	I	II	II	II	II	II	I
	C	I	I	I	I	I	I	I	I	II	I	I	I	I	I	II
	D	I	I	I	I	I	I	I	I	I	I	I	I	I	I	-
	E	II	I	II	I	II	I	II	I	I	I	II	I	III	I	I
	F	I	I	I	I	I	I	I	I	I	I	I	I	I	III	-
	G	I	II	I	II	I	II	I	II	II	II	I	II	II	II	I
	H	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
	I	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III

build another higher level hospital to provide better care at those levels in this region. Moreover, when given the possibility of placing an extra level III hospital, the model chose to do it in both solutions (C*, D*). Additionally, in all solutions but one, the model decided to locate a level II hospital in location B. However, the second location for the level II hospital varied according to which objective function was minimized first. In improved access solutions, that hospital was placed in location E, while in improved costs, it was placed in location G. These decisions are to be expected since location E is more central and closer to more demand points than location G. The same can be said for the decision on where to locate the III level hospital in C* and D*.

Other conclusions, related to the reliability of the model, can be inferred. When comparing solutions with different values for maximum travel time allowed d^{max}, it can be stated that the configurations are the same for $d^{max} = 90$ (A’ and B’) and $d^{max} = 50$ (C’ and D’). When comparing solutions with different values for maximum capacity $capmax_j^t$, it can be concluded that the configuration is the same for the full capacity improved cost solution (B’) and the capacity at 80% improved cost solution(F’). For the improved access solutions (A’ and E’), the configuration are very similar but the hospital in locations B and C are ”switched”.

Comparing the solutions’ configurations to the real one, it is possible to observe that in the latter the location of the only level II hospital is very central and localized near the most populated demand point $(i = 6)$. While in the results obtained, where it was possible to place at least two level II hospitals, these hospitals were placed in opposites sides of the region. Location B is near the top of the geographic area under evaluation and locations E and G are near the bottom. To see if these results may be in part due to the differences in operational costs explained earlier and if the model would behave differently if the costs (and maximum capacities) were the same for every location, a quick version of the model,

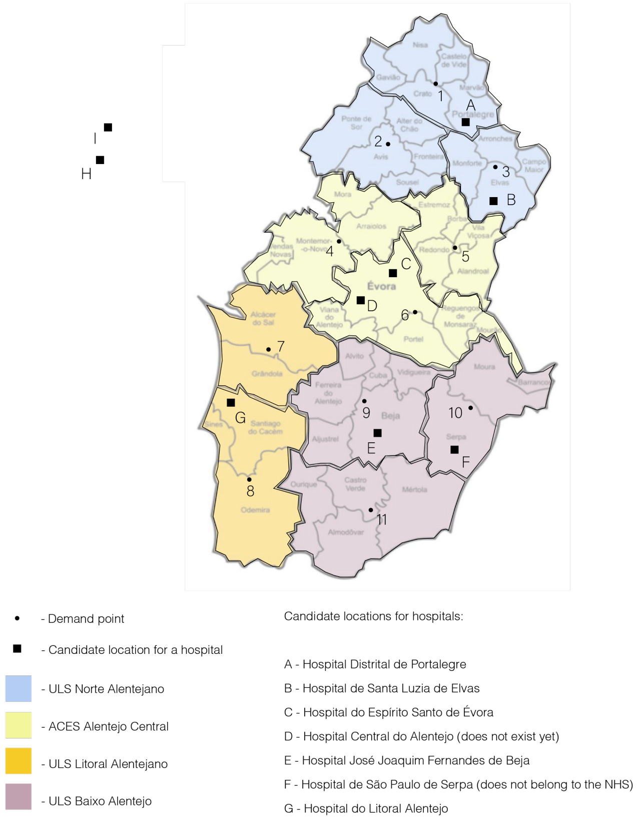


Figure 5.4: Regional Health Administration of Alentejo divided by sub-regions and demand points. Candidate locations for hospitals.

was designed and solved. The results (presented in figure C.5 included in appendix C) show that the solution would not be very different. Therefore, the model seems to suggest that the optimal solution may involve the decentralization of higher level care, instead of building all specialized hospitals in the more populated areas.

In order to better visualize the solutions proposed by the model, an example of a configuration based off of solution A* was mapped in figure 5.5. The flows are described in figure C.6. The icons and legend are equal to the one used in figure 5.2 and a detailed explanation about them is written in the section of that figure.

From a first glance, it can be verified that all demand from demand points seems to be being assigned to the closest hospitals, as expected. Another anticipated conclusion is the transfer of all level II patients from level I hospitals to the closest level II hospitals (B and E). Also an expected decision is the transfer of all level III patients from all hospitals to the closest level III hospitals (H and I). In addition, it is possible to see that some hospitals are being more used than others. For example, hospital E is receiving the level II transfers of all, but one, level I hospitals and hospital I is receiving the level III transfers of all, but two, hospitals. One other observation that can be made is that people from one sub-region are being assigned to a hospital outside their sub-region. It is the case of demand point 5. The demand from this demand point, which belongs to Alentejo Central, is being assigned to a hospital in Alto Alentejo. This may not be a very critical issue since all sub-regions are still a part of the Alentejo's Regional Health Administration, which is the most important unit in health issues.

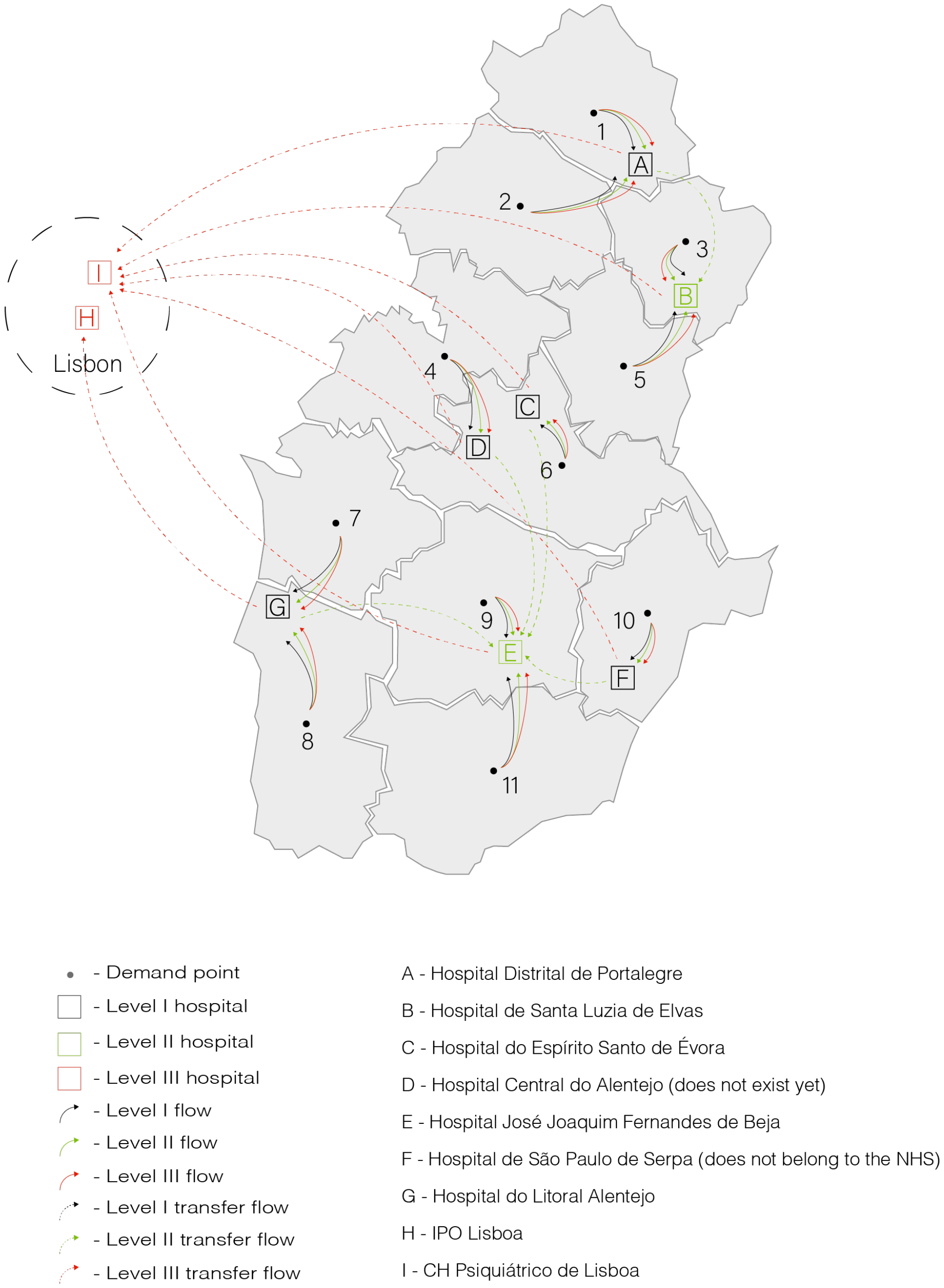
In conclusion, even though there are some differences in the solution presented, the changes are minor. Nonetheless, an analysis with more relaxed constraints would be interesting to gauge how different the solutions would be.

The next section will summarize and present the conclusions drawn from the discussion of the results.

5.3 Chapter conclusions

In this chapter, the model was implemented and results were obtained, presented, analysed and discussed. Throughout the discussion some conclusions were already drawn, but this section includes a summary of the most important ones.

From the solving the model for the first case, the importance of including multiple objectives, in order to obtain a more realistic solution, is highlighted. Only taking into account maximization of access can lead to solutions that, often times, may be difficult to implement in real life. And the same can be said for only taking into account minimization of costs. However, some valuable information about improving accessibility was gathered from solving the model for this case. It was also possible to conclude that the choice of demand points, both in terms of size and location, influences substantially the decisions of the model and, consequently, can lead to different solutions. The same can be stated for choices regarding representation of facilities. The choice to represent a group of hospitals in a single point also has consequences in demand assignment. The results in the second case prove that the model is quite robust and reliable. However, they also highlight the importance of a careful choice of parameters since different values for parameters can lead to different solutions if the model is not robust enough. From both cases' results, it is possible to say that decentralizing specialized and high level care leads to improved and more equal access to healthcare and more solutions, having this aspect in mind, should be explored. Another conclusion is the importance of good quality and recent data. If the information given as input to the model is not good, then more uncertainty must be incorporated to obtain reliable results. Consequently, more time and resources are spent solving the model. Finally, one of the most important conclusions is that location modeling involves a lot of difficult compromises and decisions.



● - Demand point
☐ - Level I hospital
☐ - Level II hospital
☐ - Level III hospital
- Level I flow
- Level II flow
- Level III flow
- Level I transfer flow
- Level II transfer flow
- Level III transfer flow

A - Hospital Distrital de Portalegre
B - Hospital de Santa Luzia de Elvas
C - Hospital do Espírito Santo de Évora
D - Hospital Central do Alentejo (does not exist yet)
E - Hospital José Joaquim Fernandes de Beja
F - Hospital de São Paulo de Serpa (does not belong to the NHS)
G - Hospital do Litoral Alentejo
H - IPO Lisboa
I - CH Psiquiátrico de Lisboa

Figure 5.5: Model results for situation A* in case 2 - stochastic results with first place minimization of objective function 4.1 (minimization of travel times).

Chapter 6

Conclusions and Future Work

The present chapter intends to describe all the conclusions from this dissertation and to suggest possible related topics that may be interesting to explore in future work.

From the research done about health in Portugal, it can be concluded that health provision has evolved a lot in the last decades along with the National Health Service (NHS). Healthcare became more accessible to all and increased in quality. Some of these improvements are related to an effort in growing a network of care, where all hospitals work with each other. An important step towards a more articulated and coordinated NHS was the creation of the Hospital Referral Networks. The creation of these networks formalized and gave structure to the existing partnerships between hospitals and reinforced the complementary nature of the relationships in public healthcare. Nonetheless, there are still some improvements to be made. Most of the documents describing the referral networks would benefit from an update and the use of a universal homogeneous format. Taking into account the interdisciplinary care (with multiple medical specialties) that occurs in hospitals would also contribute to a more complete vision of healthcare. As well as contemplating other health institutions beyond the NHS (Ministério da Saúde, 2015).

Following the review of the literature in location modeling, it was concluded that there is a need for research about hospital networks in countries with already established networks, where most hospitals are already built. The main goal to be explored in these networks is the optimization of services, which can be obtained by changes in hospital capacities, resources and the occasional opening/closing of a hospital. Intra-hospital modeling, that takes into account the relations between services and medical specialties, is also something that would be interesting to explore.

Taking into account all of the above, the model, in this thesis, was developed. It meant to explore a new aspect of a relatively well studied topic: location modeling where hospitals are viewed as multi-service facilities in a hierarchical structure. In the past, hospitals were seen as having only one classification and one place in the hierarchy. This thesis introduced the notion of plurality in hospital hierarchy, through presenting a model in which there was the possibility for the same hospital to have a different classification, and consequently, a different place in the hierarchy of hospitals, according to the referral network in question. The cases for which the model was solved included only one specialty at a time. In the future, it would be interesting to solve the model for a superior number of N. It should be noted that for these cases it may be necessary to use heuristics or metaheuristics, since including more specialties (or more referral networks) would increase the dimension of the problem. Furthermore, since the model presented does not allow transfers between medical specialties, it would be interesting to incorporate that in the model. Similarly, it would also contribute to the generalization of the model to add the possibility of distinguishing the medical specialties into different services (e.g. emergency, inpatient...). Still in the same line of thought, other types of health facilities that operate alongside hospitals to provide other types of health care, like primary care centres or long-term care facilities, could be included. The incorporation of the long-term care network (*Rede Nacional de Cuidados Continuados Integrados*) would also contribute to a more realistic model, since that could lead to a more accurate number of inpatients.

Most of the data used throughout the thesis is not up to date; this means it was necessary to make some assumptions and predictions to obtain the data needed. Since the results are only as good as the inputs the model receives, it is very important to work with updated information. This is true in relation to the values for both the sets and the parameters used. In this sense, it would be useful to get access to more accurate numbers for the demand per medical specialty and per level or for hospitals costs. Getting an approximated number for operational costs for a hospital to work in each level in each medical specialty would improve the quality of the results. It would also be of interest to work alongside decision makers in this field to get their inputs about their preferences and the values for some weights (e.g. α weight, or objective function weights if the bi-objective model is solved through Weighted Sum method (WSM)). Since the model is multi-objective, it could be solved through other methods like lexicographic method or WSM in order to compare differences in the results obtained.

As previously stated, it would be interesting to explore some different values for minimum and maximum capacity. The unit used for capacity was the number of patients treated per time period. This unit may not be the best measure for capacity because it allows, for example, for all the patients that would normally be treated in the span of a year be treated in one day. This is impossible since hospitals have a limited number of beds. Thus, in the future, it would be of interest to convert hospitals capacity into beds, using the average length of stay (ALOS).

Two different instances of the case study were used to solve the model. In future work, it could be useful to apply the model to each of the Regional Health Administrations. These units have already a degree of independence. However, the most high level care can only still be found in Lisbon and Porto. Options to increase each administration's independence could be explored, since it could improve access for more people. In terms of uncertainty, the type that was included was uncertainty in the demand for services. Each scenario used predicted the decline in growth of the population of Alentejo. However, due to the COVID-19 pandemic, there was an increase of the working-from-home culture which may create lasting impacts on the way work is viewed in the future. People may not need to live full time in larger cities where their offices and places of work are located. This said, it could be beneficial to study a scenario where less populated areas, like Alentejo, become more populated. In this sense, some other locations for hospitals could be explored. Also, some other types of uncertainty could be incorporated (e.g. travel times).

Summarizing, there are still many questions to answer in this field. Nonetheless, some things can be concluded. The importance of multi-objective models, in order to obtain realistic solutions, is highlighted. The implemented model also suggests that, in general, the solutions that most improve access to services are the most expensive ones. So, a compromise must be made, between equity in access and costs, to obtain an optimal feasible solution. The results also suggest that, as it was expected, decentralizing care, or building more hospitals in non-central areas, can improve geographical access. Especially, hospitals providing higher level and specialized care. Another conclusion is the fact that the choice of the size and location of demand points can influence greatly the solution. In terms of robustness and reliability, the model was tested for different numbers of demand and values for parameters. The solutions did not differ too much from each other so it is possible to say the model is quite robust and reliable. Although, the model must be tested with more variations of inputs in order to reach that conclusion with a higher degree of certainty. Finally, it can be concluded that location-allocation modeling is a broad topic that has multiple promising and yet unexplored questions. More specifically, location-allocation models in the health sector can be an extremely useful tool to aid in the government's decision-making and to help increase equity in health care.

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Appendix A

Case Study

Figure A.1: Districts of Portugal.

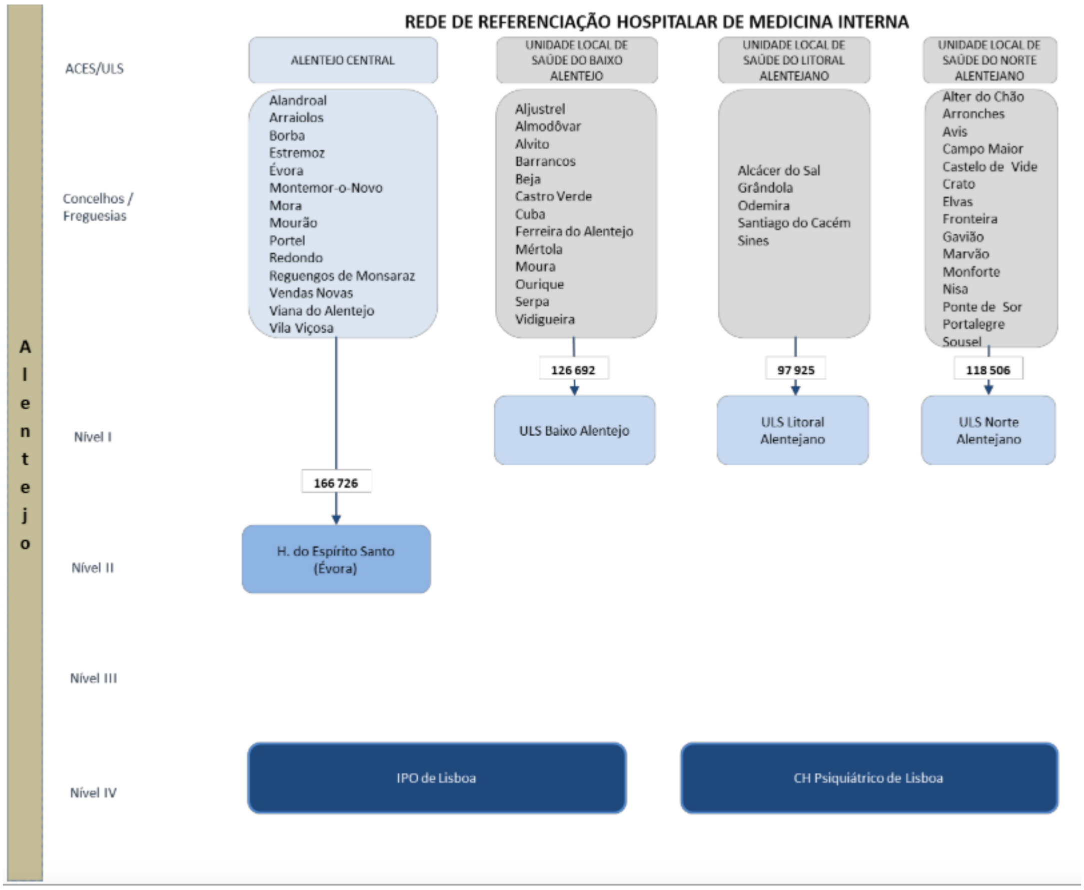


Figure A.2: Hospital referral network for Internal Medicine in 2015.

Source: Image from Carvalho et al. ((2016)).

Table A.1: Hospitals with the Cardiology medical specialty in 2013, with respective number in the model, region and level.

height Number in the model	Facility	Region	Level
1	Centro Hospitalar do Alto Ave, EPE (Guimarães)	North	I
2	Centro Hospitalar do Médio Ave, EPE (Santo Tirso/ Famalicão)		
3	Centro Hospitalar Entre Douro e Vouga, EPE (Santa Maria da Feira)		
4	Centro Hospitalar Póvoa de Varzim/Vila do Conde, EPE		
5	Centro Hospitalar Tâmega e Sousa, EPE (Penafiel)		
6	Unidade Local de Saúde de Matosinhos, EPE		
7	Unidade Local de Saúde do Alto Minho, EPE (Viana do Castelo)		
8	Unidade Local de Saúde do Nordeste, EPE (Bragança / Mirandela)		
9	Centro Hospitalar Cova da Beira, EPE (Covilhã)	Center	
10	Centro Hospitalar de Leiria, EPE		
11	Centro Hospitalar do Baixo Vouga, EPE (Aveiro)		
12	Hospital Distrital da Figueira da Foz, EPE		
13	Unidade Local de Saúde da Guarda, EPE		
14	Unidade Local de Saúde de Castelo Branco, EPE		
15	Centro Hospitalar Barreiro/Montijo, EPE	LTV	
16	Centro Hospitalar de Setúbal, EPE		
17	Centro Hospitalar do Oeste (Torres Vedras / Caldas da Rainha)		
18	Centro Hospitalar Médio Tejo, EPE (Torres Novas)		
19	Hospital de Cascais, PPP		
20	Hospital de Loures, PPP		
21	Hospital de Vila Franca de Xira, PPP		
22	Hospital Distrital de Santarém, EPE		
23	Hospital Fernando da Fonseca, EPE (Amadora / Sintra)		
24	Unidade Local de Saúde Norte Alentejo, EPE (Portalegre)	Alentejo and Algarve	
25	Unidade Local de Saúde do Baixo Alentejo, EPE (Beja)		
26	Unidade Local de Saúde do Litoral Alentejano, EPE (Santiago do Cacém)		
27	Centro Hospitalar Trás-os-Montes e Alto Douro, EPE (Vila Real)	North	II
28	Hospital de Braga, PPP		
29	Centro Hospitalar Tondela-Viseu, EPE		
30	Centro Hospitalar de Vila Nova de Gaia/Espinho, EPE - Hospital Eduardo Santos Silva		
31	Centro Hospitalar do Porto, EPE		
32	Hospital Garcia de Orta, EPE (Almada)	LTV	
33	Centro Hospitalar de Lisboa Ocidental, EPE		
34	Hospital do Espírito Santo de Évora, EPE	Alentejo	
35	Centro Hospitalar do Algarve, EPE	Algarve	
36	Centro Hospitalar de São João, EPE	North	III
37	Centro Hospitalar e Universitário de Coimbra, EPE	Center	
38	Centro Hospitalar de Lisboa Central, EPE	LVT	
39	Centro Hospitalar Lisboa Norte, EPE		

Source: Data from Ministério da Saúde ((2015)).

Table A.2: Hospitals with the Internal Medicine medical specialty in 2015, with respective label in the model, region, sub-region and level.

Label in the model	Facility	Region	Sub-region	Level
A	Hospital Distrital de Portalegre	Alentejo	ULS Norte Alentejano	I
B	Hospital Santa Luzia de Elvas	Alentejo	ULS Norte Alentejano	I
C	Hospital do Espírito Santo de Évora	Alentejo	ACES Alentejo Central	II
D	Hospital Central do Alentejo (Évora)	Alentejo	ACES Alentejo Central	-
E	Hospital José Joaquim Fernandes de Beja	Alentejo	ULS Baixo Alentejo	I
F	Hospital de São Paulo (Serpa)	Alentejo	ULS Baixo Alentejo	I
G	Hospital do Litoral Alentejano (Santiago do Cacém)	Alentejo	ULS Litoral Alentejano	I
H	Instituto Português de Oncologia de Lisboa, Francisco Gentil,	Lisbon	-	IV-a
I	Hospital Júlio de Matos (Centro Hospitalar Psiquiátrico de Lisboa)	Lisbon	-	IV-c

Source: Data from Carvalho et al. ((2016)).

Appendix B

Literature Review

Table B.1: Modeling approaches with the respective code (used in table 3.1).

Modeling approach		Code
Integer linear programming		ILP
Integer nonlinear programming		INLP
Mixed-integer linear programming		MILP
Mixed-integer nonlinear programming		MINLP
Goal programming		GP
Nonlinear programming		NLP
Fuzzy programming		FP
Dynamic programming		DP
Stochastic dynamic programming		SDP
Robust optimization		RO
Multi-level programming		MLP
Constraint programming		CP
Multi-criteria decision making		MCDM
Multi-person decision making (Game Theory)		MPDM
Queuing theory		QT
Stochastic programming	Probabilistic programming	PSP
	Single-stage stochastic programming	1-SSP
	Two-stage stochastic programming	2-SSP
	Multi-stage stochastic programming	M-SSP
Other		O

Source: Author's own compilation based on "Table 4 - The survey descriptive dimensions from computational perspective: modeling approach, solution method, and case study inclusion." in Ahmadi-Javid et al. ((2017)).

Table B.2: Solution methods divided into two classes: accurate and inaccurate methods (used in table 3.1).

Solution method		
Accurate methods (exact or bounded-error methods)	General-purpose optimization software (Lingo, CPLEX, Xpress, Gams, ...)	
	Branch and bound	
	Branch and cut	
	Branch and price	
	Branch and cut and price	
	Cutting plane	
	Lagrangian relaxation	
	Benders decomposition	
	Dynamic programming	
Inaccurate methods (without any error analysis)	Heuristic	
	Metaheuristic	Tabu search
		Genetic algorithm
		Simulated Annealing
		Ant Colony
	Approximate stochastic optimization	Simulation-based optimization
		Stochastic approximation
		Sample average approximation
		Scenario optimization

Source: Author's own compilation based on "Table 4 - The survey descriptive dimensions from computational perspective: modeling approach, solution method, and case study inclusion." in Ahmadi-Javid et al. ((2017)).

Appendix C

Results and Discussion

Table C.1: Projections of total population of Alentejo for the year of 2080, according to the different scenarios. Data taken from Instituto Nacional de Estatística ((2020b)).

Region	Projection scenarios	Total popualtion	
		2018	2080
Alentejo	Low demand (s1)	705 478	375 970
	Baseline demand (s3)		495 189
	High demand (s2)		619 745
	Without migration		431 980

Table C.2: Resident population in Alentejo (2015-2021) and projection of resident population in Alentejo (2022-2040), according to the different scenarios. Data taken form Instituto Nacional de Estatística ((2020b)) and projections calculated, by linear regression, with data taken from table C.1.

Years	Scenarios			
	Pessimistic (s1)	Optimistic (s2)	Baseline (s3)	
2015	724391	724391	724391	
2016	718087	718087	718087	
2017	711950	711950	711950	
2018	705478	705478	705478	
2019	704558	704558	704558	
2020	699420	699420	699420	
2021	694029	698092	696016	
2022	688638	696764	692612	t = 1
2023	683248	695436	689208	
2024	677857	694108	685805	
2025	672466	692780	682401	
2026	667075	691453	678997	
2027	661684	690125	675593	t = 2
2028	656293	688797	672189	
2029	650903	687469	668785	
2030	645512	686141	665382	
2031	640121	684813	661978	
2032	634730	683485	658574	t = 3
2033	629339	682157	655170	
2034	623948	680829	651766	
2035	618558	679501	648362	
2036	613167	678173	644958	
2037	607776	676845	641555	
2038	602385	675518	638151	
2039	596994	674190	634747	
2040	591603	672862	631343	
2037	607776	676845	641555	
2038	602385	675518	638151	
2039	596994	674190	634747	
2040	591603	672862	631343	

Table C.3: Demand D from all time periods t, scenario s, medical specialty n, level l, demand point i (case 1).

t	s	n	l	i	D	t	s	n	l	i	D	t	s	n	l	i	D
1	1	1	1	1	1589	1	1	1	2	1	636	1	1	1	3	1	424
1	1	1	1	2	320	1	1	1	2	2	128	1	1	1	3	2	85
1	1	1	1	3	1881	1	1	1	2	3	753	1	1	1	3	3	502
1	1	1	1	4	281	1	1	1	2	4	112	1	1	1	3	4	75
1	1	1	1	5	404	1	1	1	2	5	162	1	1	1	3	5	108
1	1	1	1	6	923	1	1	1	2	6	369	1	1	1	3	6	246
1	1	1	1	7	345	1	1	1	2	7	138	1	1	1	3	7	92
1	1	1	1	8	997	1	1	1	2	8	399	1	1	1	3	8	266
1	1	1	1	9	325	1	1	1	2	9	130	1	1	1	3	9	87
1	1	1	1	10	1039	1	1	1	2	10	416	1	1	1	3	10	277
1	1	1	1	11	5217	1	1	1	2	11	2087	1	1	1	3	11	1391
1	1	1	1	12	236	1	1	1	2	12	94	1	1	1	3	12	63
1	1	1	1	13	4054	1	1	1	2	13	1621	1	1	1	3	13	1081
1	1	1	1	14	977	1	1	1	2	14	391	1	1	1	3	14	261
1	1	1	1	15	1940	1	1	1	2	15	776	1	1	1	3	15	517
1	1	1	1	16	522	1	1	1	2	16	209	1	1	1	3	16	139
1	1	1	1	17	434	1	1	1	2	17	174	1	1	1	3	17	116
1	1	1	1	18	804	1	1	1	2	18	322	1	1	1	3	18	215

Table C.4: Demand D from time period $t = 1$, scenario s, medical specialty n, level l, demand point i (case 2).

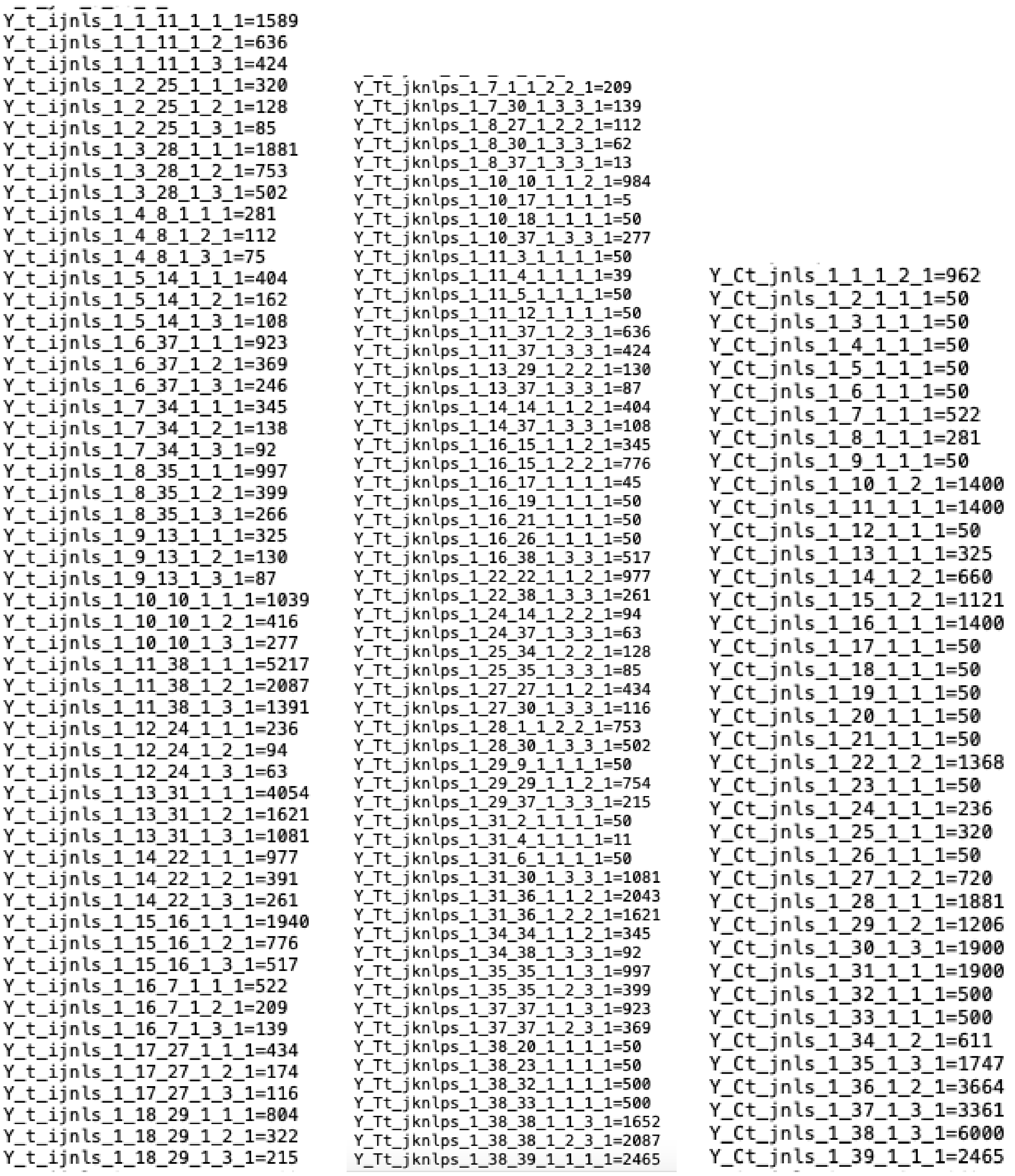
t	s	n	l	i	D	t	s	n	l	i	D	t	s	n	l	i	D
1	1	1	1	1	3870	1	2	1	1	1	3960	1	3	1	1	1	3915
1	1	1	1	2	2760	1	2	1	1	2	2825	1	3	1	1	2	2795
1	1	1	1	3	3255	1	2	1	1	3	3335	1	3	1	1	3	3295
1	1	1	1	4	3555	1	2	1	1	4	3640	1	3	1	1	4	3600
1	1	1	1	5	3560	1	2	1	1	5	3645	1	3	1	1	5	3600
1	1	1	1	6	7235	1	2	1	1	6	7410	1	3	1	1	6	7320
1	1	1	1	7	2350	1	2	1	1	7	2405	1	3	1	1	7	2375
1	1	1	1	8	6735	1	2	1	1	8	6895	1	3	1	1	8	6815
1	1	1	1	9	5815	1	2	1	1	9	5955	1	3	1	1	9	5885
1	1	1	1	10	2680	1	2	1	1	10	2745	1	3	1	1	10	2710
1	1	1	1	11	2320	1	2	1	1	11	2375	1	3	1	1	11	2345
1	1	1	2	1	775	1	2	1	2	1	790	1	3	1	2	1	780
1	1	1	2	2	550	1	2	1	2	2	565	1	3	1	2	2	560
1	1	1	2	3	650	1	2	1	2	3	665	1	3	1	2	3	660
1	1	1	2	4	710	1	2	1	2	4	730	1	3	1	2	4	720
1	1	1	2	5	710	1	2	1	2	5	730	1	3	1	2	5	720
1	1	1	2	6	1445	1	2	1	2	6	1480	1	3	1	2	6	1465
1	1	1	2	7	470	1	2	1	2	7	480	1	3	1	2	7	475
1	1	1	2	8	1345	1	2	1	2	8	1380	1	3	1	2	8	1360
1	1	1	2	9	1165	1	2	1	2	9	1190	1	3	1	2	9	1175
1	1	1	2	10	535	1	2	1	2	10	550	1	3	1	2	10	540
1	1	1	2	11	465	1	2	1	2	11	475	1	3	1	2	11	470
1	1	1	3	1	195	1	2	1	3	1	200	1	3	1	3	1	195
1	1	1	3	2	140	1	2	1	3	2	140	1	3	1	3	2	140
1	1	1	3	3	165	1	2	1	3	3	165	1	3	1	3	3	165
1	1	1	3	4	180	1	2	1	3	4	180	1	3	1	3	4	180
1	1	1	3	5	180	1	2	1	3	5	180	1	3	1	3	5	180
1	1	1	3	6	360	1	2	1	3	6	370	1	3	1	3	6	365
1	1	1	3	7	120	1	2	1	3	7	120	1	3	1	3	7	120
1	1	1	3	8	335	1	2	1	3	8	345	1	3	1	3	8	340
1	1	1	3	9	290	1	2	1	3	9	300	1	3	1	3	9	295
1	1	1	3	10	135	1	2	1	3	10	135	1	3	1	3	10	135
1	1	1	3	11	115	1	2	1	3	11	120	1	3	1	3	11	115

Table C.5: Demand D from time period $t = 2$, scenario s, medical specialty n, level l, demand point i (case 2).

t	s	n	l	i	D	t	s	n	l	i	D	t	s	n	l	i	D
2	1	1	1	1	3715	2	2	1	1	1	3925	2	3	1	1	1	3815
2	1	1	1	2	2650	2	2	1	1	2	2800	2	3	1	1	2	2725
2	1	1	1	3	3125	2	2	1	1	3	3300	2	3	1	1	3	3210
2	1	1	1	4	3415	2	2	1	1	4	3605	2	3	1	1	4	3510
2	1	1	1	5	3415	2	2	1	1	5	3610	2	3	1	1	5	3510
2	1	1	1	6	6950	2	2	1	1	6	7340	2	3	1	1	6	7140
2	1	1	1	7	2255	2	2	1	1	7	2380	2	3	1	1	7	2320
2	1	1	1	8	6465	2	2	1	1	8	6830	2	3	1	1	8	6645
2	1	1	1	9	5585	2	2	1	1	9	5900	2	3	1	1	9	5740
2	1	1	1	10	2575	2	2	1	1	10	2720	2	3	1	1	10	2645
2	1	1	1	11	2225	2	2	1	1	11	2350	2	3	1	1	11	2290
2	1	1	2	1	745	2	2	1	2	1	785	2	3	1	2	1	765
2	1	1	2	2	530	2	2	1	2	2	560	2	3	1	2	2	545
2	1	1	2	3	625	2	2	1	2	3	660	2	3	1	2	3	640
2	1	1	2	4	685	2	2	1	2	4	720	2	3	1	2	4	700
2	1	1	2	5	685	2	2	1	2	5	720	2	3	1	2	5	700
2	1	1	2	6	1390	2	2	1	2	6	1465	2	3	1	2	6	1425
2	1	1	2	7	450	2	2	1	2	7	475	2	3	1	2	7	465
2	1	1	2	8	1295	2	2	1	2	8	1365	2	3	1	2	8	1330
2	1	1	2	9	1115	2	2	1	2	9	1180	2	3	1	2	9	1145
2	1	1	2	10	515	2	2	1	2	10	545	2	3	1	2	10	530
2	1	1	2	11	445	2	2	1	2	11	470	2	3	1	2	11	455
2	1	1	3	1	185	2	2	1	3	1	195	2	3	1	3	1	190
2	1	1	3	2	135	2	2	1	3	2	140	2	3	1	3	2	135
2	1	1	3	3	155	2	2	1	3	3	165	2	3	1	3	3	160
2	1	1	3	4	170	2	2	1	3	4	180	2	3	1	3	4	175
2	1	1	3	5	170	2	2	1	3	5	180	2	3	1	3	5	175
2	1	1	3	6	350	2	2	1	3	6	365	2	3	1	3	6	355
2	1	1	3	7	115	2	2	1	3	7	120	2	3	1	3	7	115
2	1	1	3	8	325	2	2	1	3	8	340	2	3	1	3	8	330
2	1	1	3	9	280	2	2	1	3	9	295	2	3	1	3	9	285
2	1	1	3	10	130	2	2	1	3	10	135	2	3	1	3	10	130
2	1	1	3	11	110	2	2	1	3	11	120	2	3	1	3	11	115

Table C.6: Demand D from time period $t = 3$, scenario s, medical specialty n, level l, demand point i (case 2).

t	s	n	l	i	D	t	s	n	l	i	D	t	s	n	l	i	D
3	1	1	1	1	3560	3	2	1	1	1	3885	3	3	1	1	1	3720
3	1	1	1	2	2540	3	2	1	1	2	2775	3	3	1	1	2	2655
3	1	1	1	3	2995	3	2	1	1	3	3270	3	3	1	1	3	3130
3	1	1	1	4	3275	3	2	1	1	4	3570	3	3	1	1	4	3420
3	1	1	1	5	3275	3	2	1	1	5	3575	3	3	1	1	5	3420
3	1	1	1	6	6660	3	2	1	1	6	7270	3	3	1	1	6	6960
3	1	1	1	7	2160	3	2	1	1	7	2360	3	3	1	1	7	2260
3	1	1	1	8	6200	3	2	1	1	8	6765	3	3	1	1	8	6475
3	1	1	1	9	5355	3	2	1	1	9	5840	3	3	1	1	9	5595
3	1	1	1	10	2465	3	2	1	1	10	2690	3	3	1	1	10	2575
3	1	1	1	11	2135	3	2	1	1	11	2330	3	3	1	1	11	2230
3	1	1	2	1	710	3	2	1	2	1	775	3	3	1	2	1	745
3	1	1	2	2	510	3	2	1	2	2	555	3	3	1	2	2	530
3	1	1	2	3	600	3	2	1	2	3	655	3	3	1	2	3	625
3	1	1	2	4	655	3	2	1	2	4	715	3	3	1	2	4	685
3	1	1	2	5	655	3	2	1	2	5	715	3	3	1	2	5	685
3	1	1	2	6	1330	3	2	1	2	6	1455	3	3	1	2	6	1390
3	1	1	2	7	430	3	2	1	2	7	470	3	3	1	2	7	450
3	1	1	2	8	1240	3	2	1	2	8	1350	3	3	1	2	8	1295
3	1	1	2	9	1070	3	2	1	2	9	1170	3	3	1	2	9	1120
3	1	1	2	10	495	3	2	1	2	10	540	3	3	1	2	10	515
3	1	1	2	11	425	3	2	1	2	11	465	3	3	1	2	11	445
3	1	1	3	1	180	3	2	1	3	1	195	3	3	1	3	1	185
3	1	1	3	2	125	3	2	1	3	2	140	3	3	1	3	2	135
3	1	1	3	3	150	3	2	1	3	3	165	3	3	1	3	3	155
3	1	1	3	4	165	3	2	1	3	4	180	3	3	1	3	4	170
3	1	1	3	5	165	3	2	1	3	5	180	3	3	1	3	5	170
3	1	1	3	6	335	3	2	1	3	6	365	3	3	1	3	6	350
3	1	1	3	7	110	3	2	1	3	7	120	3	3	1	3	7	115
3	1	1	3	8	310	3	2	1	3	8	340	3	3	1	3	8	325
3	1	1	3	9	270	3	2	1	3	9	290	3	3	1	3	9	280
3	1	1	3	10	125	3	2	1	3	10	135	3	3	1	3	10	130
3	1	1	3	11	105	3	2	1	3	11	115	3	3	1	3	11	110



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Y_t_ijnls_1_1_11_1_1_1=1589
Y_t_ijnls_1_1_11_1_2_1=636
Y_t_ijnls_1_1_11_1_3_1=424
Y_t_ijnls_1_2_25_1_1_1=320
Y_t_ijnls_1_2_25_1_2_1=128
Y_t_ijnls_1_2_25_1_3_1=85
Y_t_ijnls_1_3_28_1_1_1=1881
Y_t_ijnls_1_3_28_1_2_1=753
Y_t_ijnls_1_3_28_1_3_1=502
Y_t_ijnls_1_4_8_1_1_1=281
Y_t_ijnls_1_4_8_1_2_1=112
Y_t_ijnls_1_4_8_1_3_1=75
Y_t_ijnls_1_5_14_1_1_1=404
Y_t_ijnls_1_5_14_1_2_1=162
Y_t_ijnls_1_5_14_1_3_1=108
Y_t_ijnls_1_6_37_1_1_1=923
Y_t_ijnls_1_6_37_1_2_1=369
Y_t_ijnls_1_6_37_1_3_1=246
Y_t_ijnls_1_7_34_1_1_1=345
Y_t_ijnls_1_7_34_1_2_1=138
Y_t_ijnls_1_7_34_1_3_1=92
Y_t_ijnls_1_8_35_1_1_1=997
Y_t_ijnls_1_8_35_1_2_1=399
Y_t_ijnls_1_8_35_1_3_1=266
Y_t_ijnls_1_9_13_1_1_1=325
Y_t_ijnls_1_9_13_1_2_1=130
Y_t_ijnls_1_9_13_1_3_1=87
Y_t_ijnls_1_10_10_1_1_1=1039
Y_t_ijnls_1_10_10_1_2_1=416
Y_t_ijnls_1_10_10_1_3_1=277
Y_t_ijnls_1_11_38_1_1_1=5217
Y_t_ijnls_1_11_38_1_2_1=2087
Y_t_ijnls_1_11_38_1_3_1=1391
Y_t_ijnls_1_12_24_1_1_1=236
Y_t_ijnls_1_12_24_1_2_1=94
Y_t_ijnls_1_12_24_1_3_1=63
Y_t_ijnls_1_13_31_1_1_1=4054
Y_t_ijnls_1_13_31_1_2_1=1621
Y_t_ijnls_1_13_31_1_3_1=1081
Y_t_ijnls_1_14_22_1_1_1=977
Y_t_ijnls_1_14_22_1_2_1=391
Y_t_ijnls_1_14_22_1_3_1=261
Y_t_ijnls_1_15_16_1_1_1=1940
Y_t_ijnls_1_15_16_1_2_1=776
Y_t_ijnls_1_15_16_1_3_1=517
Y_t_ijnls_1_16_7_1_1_1=522
Y_t_ijnls_1_16_7_1_2_1=209
Y_t_ijnls_1_16_7_1_3_1=139
Y_t_ijnls_1_17_27_1_1_1=434
Y_t_ijnls_1_17_27_1_2_1=174
Y_t_ijnls_1_17_27_1_3_1=116
Y_t_ijnls_1_18_29_1_1_1=804
Y_t_ijnls_1_18_29_1_2_1=322
Y_t_ijnls_1_18_29_1_3_1=215
```

(a) Flow from demand points to hospitals.

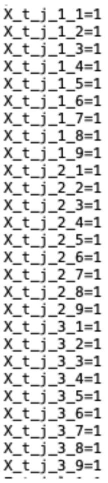
```
Y_Tt_jknlps_1_7_1_1_2_2_1=209
Y_Tt_jknlps_1_7_30_1_3_3_1=139
Y_Tt_jknlps_1_8_27_1_2_2_1=112
Y_Tt_jknlps_1_8_30_1_3_3_1=62
Y_Tt_jknlps_1_8_37_1_3_3_1=13
Y_Tt_jknlps_1_10_10_1_1_2_1=984
Y_Tt_jknlps_1_10_17_1_1_1_1=5
Y_Tt_jknlps_1_10_18_1_1_1_1=50
Y_Tt_jknlps_1_10_37_1_3_3_1=277
Y_Tt_jknlps_1_11_3_1_1_1_1=50
Y_Tt_jknlps_1_11_4_1_1_1_1=39
Y_Tt_jknlps_1_11_5_1_1_1_1=50
Y_Tt_jknlps_1_11_12_1_1_1_1=50
Y_Tt_jknlps_1_11_37_1_2_3_1=636
Y_Tt_jknlps_1_11_37_1_3_3_1=424
Y_Tt_jknlps_1_13_29_1_2_2_1=130
Y_Tt_jknlps_1_13_37_1_3_3_1=87
Y_Tt_jknlps_1_14_14_1_1_2_1=404
Y_Tt_jknlps_1_14_37_1_3_3_1=108
Y_Tt_jknlps_1_16_15_1_1_2_1=345
Y_Tt_jknlps_1_16_15_1_2_2_1=776
Y_Tt_jknlps_1_16_17_1_1_1_1=45
Y_Tt_jknlps_1_16_19_1_1_1_1=50
Y_Tt_jknlps_1_16_21_1_1_1_1=50
Y_Tt_jknlps_1_16_26_1_1_1_1=50
Y_Tt_jknlps_1_16_38_1_3_3_1=517
Y_Tt_jknlps_1_22_22_1_1_2_1=977
Y_Tt_jknlps_1_22_38_1_3_3_1=261
Y_Tt_jknlps_1_24_14_1_2_2_1=94
Y_Tt_jknlps_1_24_37_1_3_3_1=63
Y_Tt_jknlps_1_25_34_1_2_2_1=128
Y_Tt_jknlps_1_25_35_1_3_3_1=85
Y_Tt_jknlps_1_27_27_1_1_2_1=434
Y_Tt_jknlps_1_27_30_1_3_3_1=116
Y_Tt_jknlps_1_28_1_1_2_2_1=753
Y_Tt_jknlps_1_28_30_1_3_3_1=502
Y_Tt_jknlps_1_29_9_1_1_1_1=50
Y_Tt_jknlps_1_29_29_1_1_2_1=754
Y_Tt_jknlps_1_29_37_1_3_3_1=215
Y_Tt_jknlps_1_31_2_1_1_1_1=50
Y_Tt_jknlps_1_31_4_1_1_1_1=11
Y_Tt_jknlps_1_31_6_1_1_1_1=50
Y_Tt_jknlps_1_31_30_1_3_3_1=1081
Y_Tt_jknlps_1_31_36_1_1_2_1=2043
Y_Tt_jknlps_1_31_36_1_2_2_1=1621
Y_Tt_jknlps_1_34_34_1_1_2_1=345
Y_Tt_jknlps_1_34_38_1_3_3_1=92
Y_Tt_jknlps_1_35_35_1_1_3_1=997
Y_Tt_jknlps_1_35_35_1_2_3_1=399
Y_Tt_jknlps_1_37_37_1_1_3_1=923
Y_Tt_jknlps_1_37_37_1_2_3_1=369
Y_Tt_jknlps_1_38_20_1_1_1_1=50
Y_Tt_jknlps_1_38_23_1_1_1_1=50
Y_Tt_jknlps_1_38_32_1_1_1_1=500
Y_Tt_jknlps_1_38_33_1_1_1_1=500
Y_Tt_jknlps_1_38_38_1_1_3_1=1652
Y_Tt_jknlps_1_38_38_1_2_3_1=2087
Y_Tt_jknlps_1_38_39_1_1_1_1=2465
```

(b) Transfer flow between hospitals.

```
Y_Ct_jnls_1_1_1_2_1=962
Y_Ct_jnls_1_2_1_1_1=50
Y_Ct_jnls_1_3_1_1_1=50
Y_Ct_jnls_1_4_1_1_1=50
Y_Ct_jnls_1_5_1_1_1=50
Y_Ct_jnls_1_6_1_1_1=50
Y_Ct_jnls_1_7_1_1_1=522
Y_Ct_jnls_1_8_1_1_1=281
Y_Ct_jnls_1_9_1_1_1=50
Y_Ct_jnls_1_10_1_2_1=1400
Y_Ct_jnls_1_11_1_1_1=1400
Y_Ct_jnls_1_12_1_1_1=50
Y_Ct_jnls_1_13_1_1_1=325
Y_Ct_jnls_1_14_1_2_1=660
Y_Ct_jnls_1_15_1_2_1=1121
Y_Ct_jnls_1_16_1_1_1=1400
Y_Ct_jnls_1_17_1_1_1=50
Y_Ct_jnls_1_18_1_1_1=50
Y_Ct_jnls_1_19_1_1_1=50
Y_Ct_jnls_1_20_1_1_1=50
Y_Ct_jnls_1_21_1_1_1=50
Y_Ct_jnls_1_22_1_2_1=1368
Y_Ct_jnls_1_23_1_1_1=50
Y_Ct_jnls_1_24_1_1_1=236
Y_Ct_jnls_1_25_1_1_1=320
Y_Ct_jnls_1_26_1_1_1=50
Y_Ct_jnls_1_27_1_2_1=720
Y_Ct_jnls_1_28_1_1_1=1881
Y_Ct_jnls_1_29_1_2_1=1206
Y_Ct_jnls_1_30_1_3_1=1900
Y_Ct_jnls_1_31_1_1_1=1900
Y_Ct_jnls_1_32_1_1_1=500
Y_Ct_jnls_1_33_1_1_1=500
Y_Ct_jnls_1_34_1_2_1=611
Y_Ct_jnls_1_35_1_3_1=1747
Y_Ct_jnls_1_36_1_2_1=3664
Y_Ct_jnls_1_37_1_3_1=3361
Y_Ct_jnls_1_38_1_3_1=6000
Y_Ct_jnls_1_39_1_1_1=2465
```

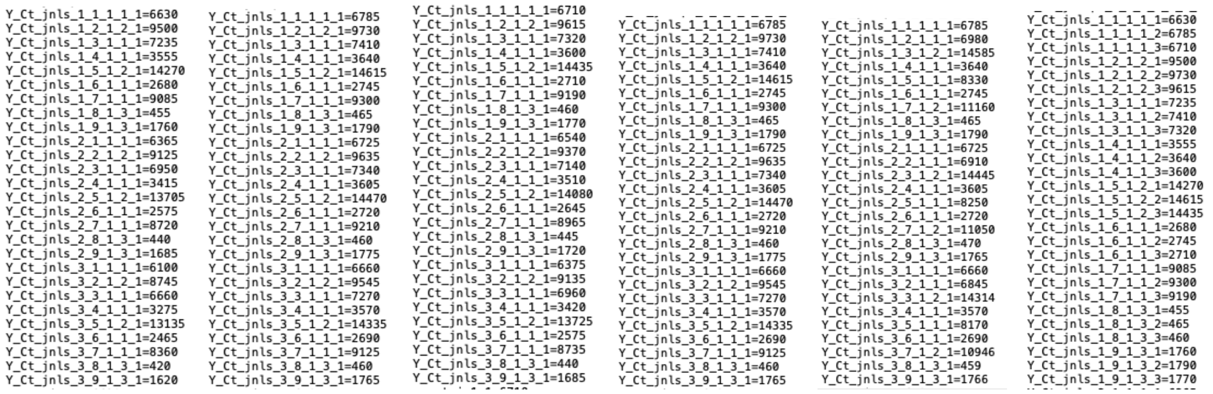
(c) Number of people treated in each hospital.

Figure C.1: Output of solving the model for case 1 (time period considered was $t = 1$).



```
X_t_j_1_1=1
X_t_j_1_2=1
X_t_j_1_3=1
X_t_j_1_4=1
X_t_j_1_5=1
X_t_j_1_6=1
X_t_j_1_7=1
X_t_j_1_8=1
X_t_j_1_9=1
X_t_j_2_1=1
X_t_j_2_2=1
X_t_j_2_3=1
X_t_j_2_4=1
X_t_j_2_5=1
X_t_j_2_6=1
X_t_j_2_7=1
X_t_j_2_8=1
X_t_j_2_9=1
X_t_j_3_1=1
X_t_j_3_2=1
X_t_j_3_3=1
X_t_j_3_4=1
X_t_j_3_5=1
X_t_j_3_6=1
X_t_j_3_7=1
X_t_j_3_8=1
X_t_j_3_9=1
```

Figure C.2: Hospital locations for all solutions in case 2.



```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_2_1_2_1=9500
Y_Ct_jnls_1_3_1_1_1=7235
Y_Ct_jnls_1_4_1_1_1=3555
Y_Ct_jnls_1_5_1_2_1=14270
Y_Ct_jnls_1_6_1_1_1=2680
Y_Ct_jnls_1_7_1_1_1=9085
Y_Ct_jnls_1_8_1_3_1=455
Y_Ct_jnls_1_9_1_3_1=1760
Y_Ct_jnls_2_1_1_1_1=6365
Y_Ct_jnls_2_2_1_2_1=9125
Y_Ct_jnls_2_3_1_1_1=6950
Y_Ct_jnls_2_4_1_1_1=3415
Y_Ct_jnls_2_5_1_2_1=13705
Y_Ct_jnls_2_6_1_1_1=2575
Y_Ct_jnls_2_7_1_1_1=8720
Y_Ct_jnls_2_8_1_3_1=440
Y_Ct_jnls_2_9_1_3_1=1685
Y_Ct_jnls_3_1_1_1_1=6100
Y_Ct_jnls_3_2_1_2_1=8745
Y_Ct_jnls_3_3_1_1_1=6660
Y_Ct_jnls_3_4_1_1_1=3275
Y_Ct_jnls_3_5_1_2_1=13135
Y_Ct_jnls_3_6_1_1_1=2465
Y_Ct_jnls_3_7_1_1_1=8360
Y_Ct_jnls_3_8_1_3_1=420
Y_Ct_jnls_3_9_1_3_1=1620
```

(a) A.

```
Y_Ct_jnls_1_1_1_1_1=6785
Y_Ct_jnls_1_2_1_2_1=9730
Y_Ct_jnls_1_3_1_1_1=7410
Y_Ct_jnls_1_4_1_1_1=3640
Y_Ct_jnls_1_5_1_2_1=14615
Y_Ct_jnls_1_6_1_1_1=2745
Y_Ct_jnls_1_7_1_1_1=9300
Y_Ct_jnls_1_8_1_3_1=465
Y_Ct_jnls_1_9_1_3_1=1790
Y_Ct_jnls_2_1_1_1_1=6725
Y_Ct_jnls_2_2_1_2_1=9635
Y_Ct_jnls_2_3_1_1_1=7340
Y_Ct_jnls_2_4_1_1_1=3605
Y_Ct_jnls_2_5_1_2_1=14470
Y_Ct_jnls_2_6_1_1_1=2720
Y_Ct_jnls_2_7_1_1_1=9210
Y_Ct_jnls_2_8_1_3_1=460
Y_Ct_jnls_2_9_1_3_1=1775
Y_Ct_jnls_3_1_1_1_1=6660
Y_Ct_jnls_3_2_1_2_1=9545
Y_Ct_jnls_3_3_1_1_1=7270
Y_Ct_jnls_3_4_1_1_1=3570
Y_Ct_jnls_3_5_1_2_1=14335
Y_Ct_jnls_3_6_1_1_1=2690
Y_Ct_jnls_3_7_1_1_1=9125
Y_Ct_jnls_3_8_1_3_1=460
Y_Ct_jnls_3_9_1_3_1=1765
```

(b) A’.

```
Y_Ct_jnls_1_1_1_1_1=6710
Y_Ct_jnls_1_2_1_2_1=9615
Y_Ct_jnls_1_3_1_1_1=7320
Y_Ct_jnls_1_4_1_1_1=3600
Y_Ct_jnls_1_5_1_2_1=14435
Y_Ct_jnls_1_6_1_1_1=2710
Y_Ct_jnls_1_7_1_1_1=9190
Y_Ct_jnls_1_8_1_3_1=460
Y_Ct_jnls_1_9_1_3_1=1770
Y_Ct_jnls_2_1_1_1_1=6540
Y_Ct_jnls_2_2_1_2_1=9370
Y_Ct_jnls_2_3_1_1_1=7140
Y_Ct_jnls_2_4_1_1_1=3510
Y_Ct_jnls_2_5_1_2_1=14080
Y_Ct_jnls_2_6_1_1_1=2645
Y_Ct_jnls_2_7_1_1_1=8965
Y_Ct_jnls_2_8_1_3_1=445
Y_Ct_jnls_2_9_1_3_1=1720
Y_Ct_jnls_3_1_1_1_1=6375
Y_Ct_jnls_3_2_1_2_1=9135
Y_Ct_jnls_3_3_1_1_1=6960
Y_Ct_jnls_3_4_1_1_1=3420
Y_Ct_jnls_3_5_1_2_1=13725
Y_Ct_jnls_3_6_1_1_1=2575
Y_Ct_jnls_3_7_1_1_1=8735
Y_Ct_jnls_3_8_1_3_1=440
Y_Ct_jnls_3_9_1_3_1=1685
```

(c) A”.

```
Y_Ct_jnls_1_1_1_1_1=6785
Y_Ct_jnls_1_2_1_2_1=9730
Y_Ct_jnls_1_3_1_1_1=7410
Y_Ct_jnls_1_4_1_1_1=3640
Y_Ct_jnls_1_5_1_2_1=14615
Y_Ct_jnls_1_6_1_1_1=2745
Y_Ct_jnls_1_7_1_1_1=9300
Y_Ct_jnls_1_8_1_3_1=465
Y_Ct_jnls_1_9_1_3_1=1790
Y_Ct_jnls_2_1_1_1_1=6725
Y_Ct_jnls_2_2_1_2_1=9635
Y_Ct_jnls_2_3_1_1_1=7340
Y_Ct_jnls_2_4_1_1_1=3605
Y_Ct_jnls_2_5_1_2_1=14470
Y_Ct_jnls_2_6_1_1_1=2720
Y_Ct_jnls_2_7_1_1_1=9210
Y_Ct_jnls_2_8_1_3_1=460
Y_Ct_jnls_2_9_1_3_1=1775
Y_Ct_jnls_3_1_1_1_1=6660
Y_Ct_jnls_3_2_1_2_1=9545
Y_Ct_jnls_3_3_1_1_1=7270
Y_Ct_jnls_3_4_1_1_1=3570
Y_Ct_jnls_3_5_1_2_1=14335
Y_Ct_jnls_3_6_1_1_1=2690
Y_Ct_jnls_3_7_1_1_1=9125
Y_Ct_jnls_3_8_1_3_1=460
Y_Ct_jnls_3_9_1_3_1=1765
```

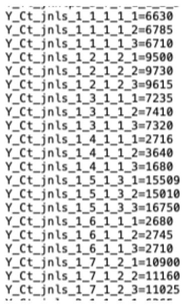
(d) C’.

```
Y_Ct_jnls_1_1_1_1_1=6785
Y_Ct_jnls_1_2_1_1_1=6980
Y_Ct_jnls_1_3_1_2_1=14585
Y_Ct_jnls_1_4_1_1_1=3640
Y_Ct_jnls_1_5_1_1_1=8330
Y_Ct_jnls_1_6_1_1_1=2745
Y_Ct_jnls_1_7_1_2_1=11160
Y_Ct_jnls_1_8_1_3_1=465
Y_Ct_jnls_1_9_1_3_1=1790
Y_Ct_jnls_2_1_1_1_1=6725
Y_Ct_jnls_2_2_1_1_1=6910
Y_Ct_jnls_2_3_1_2_1=14445
Y_Ct_jnls_2_4_1_1_1=3605
Y_Ct_jnls_2_5_1_1_1=8250
Y_Ct_jnls_2_6_1_1_1=2720
Y_Ct_jnls_2_7_1_2_1=11050
Y_Ct_jnls_2_8_1_3_1=470
Y_Ct_jnls_2_9_1_3_1=1765
Y_Ct_jnls_3_1_1_1_1=6660
Y_Ct_jnls_3_2_1_1_1=6845
Y_Ct_jnls_3_3_1_2_1=14314
Y_Ct_jnls_3_4_1_1_1=3570
Y_Ct_jnls_3_5_1_1_1=8170
Y_Ct_jnls_3_6_1_1_1=2690
Y_Ct_jnls_3_7_1_2_1=10946
Y_Ct_jnls_3_8_1_3_1=459
Y_Ct_jnls_3_9_1_3_1=1766
```

(e) E’.

```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_1_1_1_2=6785
Y_Ct_jnls_1_1_1_1_3=6710
Y_Ct_jnls_1_2_1_2_1=9500
Y_Ct_jnls_1_2_1_2_2=9730
Y_Ct_jnls_1_2_1_2_3=9615
Y_Ct_jnls_1_3_1_1_1=7235
Y_Ct_jnls_1_3_1_1_2=7410
Y_Ct_jnls_1_3_1_1_3=7320
Y_Ct_jnls_1_4_1_1_1=3555
Y_Ct_jnls_1_4_1_1_2=3640
Y_Ct_jnls_1_4_1_1_3=3600
Y_Ct_jnls_1_5_1_2_1=14270
Y_Ct_jnls_1_5_1_2_2=14615
Y_Ct_jnls_1_5_1_2_3=14435
Y_Ct_jnls_1_6_1_1_1=2680
Y_Ct_jnls_1_6_1_1_2=2745
Y_Ct_jnls_1_6_1_1_3=2710
Y_Ct_jnls_1_7_1_1_1=9085
Y_Ct_jnls_1_7_1_1_2=9300
Y_Ct_jnls_1_7_1_1_3=9190
Y_Ct_jnls_1_8_1_3_1=455
Y_Ct_jnls_1_8_1_3_2=465
Y_Ct_jnls_1_8_1_3_3=460
Y_Ct_jnls_1_9_1_3_1=1760
Y_Ct_jnls_1_9_1_3_2=1790
Y_Ct_jnls_1_9_1_3_3=1770
```

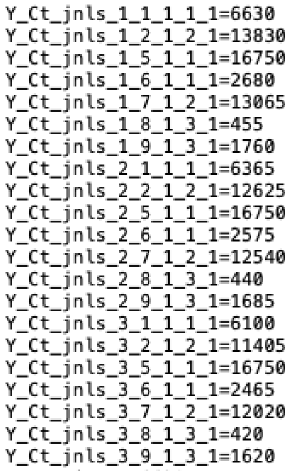
(f) A* $(t = 1)$.



```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_1_1_1_2=6785
Y_Ct_jnls_1_1_1_1_3=6710
Y_Ct_jnls_1_2_1_2_1=9500
Y_Ct_jnls_1_2_1_2_2=9730
Y_Ct_jnls_1_2_1_2_3=9615
Y_Ct_jnls_1_3_1_1_1=7235
Y_Ct_jnls_1_3_1_1_2=7410
Y_Ct_jnls_1_3_1_1_3=7320
Y_Ct_jnls_1_4_1_1_1=2716
Y_Ct_jnls_1_4_1_1_2=3640
Y_Ct_jnls_1_4_1_1_3=1680
Y_Ct_jnls_1_5_1_3_1=15509
Y_Ct_jnls_1_5_1_3_2=15010
Y_Ct_jnls_1_5_1_3_3=16750
Y_Ct_jnls_1_6_1_1_1=2680
Y_Ct_jnls_1_6_1_1_2=2745
Y_Ct_jnls_1_6_1_1_3=2710
Y_Ct_jnls_1_7_1_2_1=10900
Y_Ct_jnls_1_7_1_2_2=11160
Y_Ct_jnls_1_7_1_2_3=11025
```

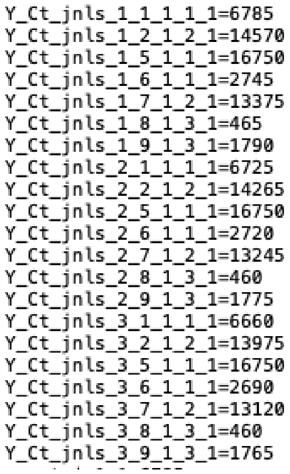
(g) C*$(t = 1)$.

Figure C.3: Comparison of number of people treated in each hospital in all improved access solutions (case 2).



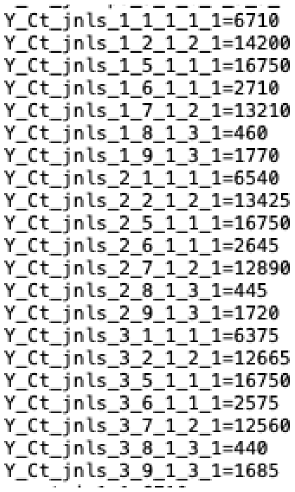
```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_2_1_2_1=13830
Y_Ct_jnls_1_5_1_1_1=16750
Y_Ct_jnls_1_6_1_1_1=2680
Y_Ct_jnls_1_7_1_2_1=13065
Y_Ct_jnls_1_8_1_3_1=455
Y_Ct_jnls_1_9_1_3_1=1760
Y_Ct_jnls_2_1_1_1_1=6365
Y_Ct_jnls_2_2_1_2_1=12625
Y_Ct_jnls_2_5_1_1_1=16750
Y_Ct_jnls_2_6_1_1_1=2575
Y_Ct_jnls_2_7_1_2_1=12540
Y_Ct_jnls_2_8_1_3_1=440
Y_Ct_jnls_2_9_1_3_1=1685
Y_Ct_jnls_3_1_1_1_1=6100
Y_Ct_jnls_3_2_1_2_1=11405
Y_Ct_jnls_3_5_1_1_1=16750
Y_Ct_jnls_3_6_1_1_1=2465
Y_Ct_jnls_3_7_1_2_1=12020
Y_Ct_jnls_3_8_1_3_1=420
Y_Ct_jnls_3_9_1_3_1=1620
```

(a) B.



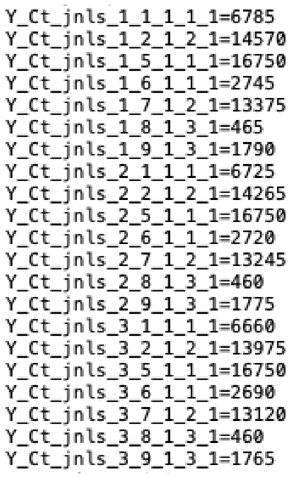
```
Y_Ct_jnls_1_1_1_1_1=6785
Y_Ct_jnls_1_2_1_2_1=14570
Y_Ct_jnls_1_5_1_1_1=16750
Y_Ct_jnls_1_6_1_1_1=2745
Y_Ct_jnls_1_7_1_2_1=13375
Y_Ct_jnls_1_8_1_3_1=465
Y_Ct_jnls_1_9_1_3_1=1790
Y_Ct_jnls_2_1_1_1_1=6725
Y_Ct_jnls_2_2_1_2_1=14265
Y_Ct_jnls_2_5_1_1_1=16750
Y_Ct_jnls_2_6_1_1_1=2720
Y_Ct_jnls_2_7_1_2_1=13245
Y_Ct_jnls_2_8_1_3_1=460
Y_Ct_jnls_2_9_1_3_1=1775
Y_Ct_jnls_3_1_1_1_1=6660
Y_Ct_jnls_3_2_1_2_1=13975
Y_Ct_jnls_3_5_1_1_1=16750
Y_Ct_jnls_3_6_1_1_1=2690
Y_Ct_jnls_3_7_1_2_1=13120
Y_Ct_jnls_3_8_1_3_1=460
Y_Ct_jnls_3_9_1_3_1=1765
```

(b) B’.



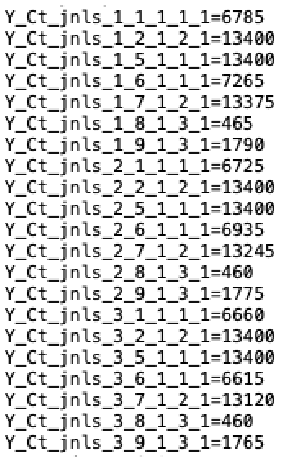
```
Y_Ct_jnls_1_1_1_1_1=6710
Y_Ct_jnls_1_2_1_2_1=14200
Y_Ct_jnls_1_5_1_1_1=16750
Y_Ct_jnls_1_6_1_1_1=2710
Y_Ct_jnls_1_7_1_2_1=13210
Y_Ct_jnls_1_8_1_3_1=460
Y_Ct_jnls_1_9_1_3_1=1770
Y_Ct_jnls_2_1_1_1_1=6540
Y_Ct_jnls_2_2_1_2_1=13425
Y_Ct_jnls_2_5_1_1_1=16750
Y_Ct_jnls_2_6_1_1_1=2645
Y_Ct_jnls_2_7_1_2_1=12890
Y_Ct_jnls_2_8_1_3_1=445
Y_Ct_jnls_2_9_1_3_1=1720
Y_Ct_jnls_3_1_1_1_1=6375
Y_Ct_jnls_3_2_1_2_1=12665
Y_Ct_jnls_3_5_1_1_1=16750
Y_Ct_jnls_3_6_1_1_1=2575
Y_Ct_jnls_3_7_1_2_1=12560
Y_Ct_jnls_3_8_1_3_1=440
Y_Ct_jnls_3_9_1_3_1=1685
```

(c) B”.



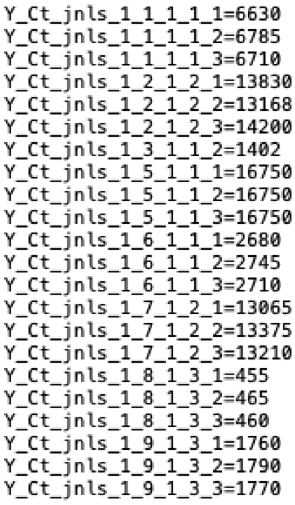
```
Y_Ct_jnls_1_1_1_1_1=6785
Y_Ct_jnls_1_2_1_2_1=14570
Y_Ct_jnls_1_5_1_1_1=16750
Y_Ct_jnls_1_6_1_1_1=2745
Y_Ct_jnls_1_7_1_2_1=13375
Y_Ct_jnls_1_8_1_3_1=465
Y_Ct_jnls_1_9_1_3_1=1790
Y_Ct_jnls_2_1_1_1_1=6725
Y_Ct_jnls_2_2_1_2_1=14265
Y_Ct_jnls_2_5_1_1_1=16750
Y_Ct_jnls_2_6_1_1_1=2720
Y_Ct_jnls_2_7_1_2_1=13245
Y_Ct_jnls_2_8_1_3_1=460
Y_Ct_jnls_2_9_1_3_1=1775
Y_Ct_jnls_3_1_1_1_1=6660
Y_Ct_jnls_3_2_1_2_1=13975
Y_Ct_jnls_3_5_1_1_1=16750
Y_Ct_jnls_3_6_1_1_1=2690
Y_Ct_jnls_3_7_1_2_1=13120
Y_Ct_jnls_3_8_1_3_1=460
Y_Ct_jnls_3_9_1_3_1=1765
```

(d) D’.



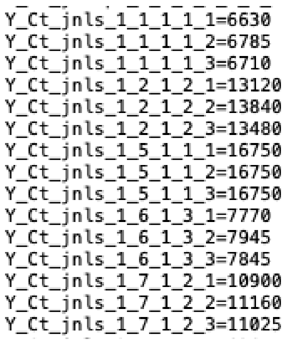
```
Y_Ct_jnls_1_1_1_1_1=6785
Y_Ct_jnls_1_2_1_2_1=13400
Y_Ct_jnls_1_5_1_1_1=13400
Y_Ct_jnls_1_6_1_1_1=7265
Y_Ct_jnls_1_7_1_2_1=13375
Y_Ct_jnls_1_8_1_3_1=465
Y_Ct_jnls_1_9_1_3_1=1790
Y_Ct_jnls_2_1_1_1_1=6725
Y_Ct_jnls_2_2_1_2_1=13400
Y_Ct_jnls_2_5_1_1_1=13400
Y_Ct_jnls_2_6_1_1_1=6935
Y_Ct_jnls_2_7_1_2_1=13245
Y_Ct_jnls_2_8_1_3_1=460
Y_Ct_jnls_2_9_1_3_1=1775
Y_Ct_jnls_3_1_1_1_1=6660
Y_Ct_jnls_3_2_1_2_1=13400
Y_Ct_jnls_3_5_1_1_1=13400
Y_Ct_jnls_3_6_1_1_1=6615
Y_Ct_jnls_3_7_1_2_1=13120
Y_Ct_jnls_3_8_1_3_1=460
Y_Ct_jnls_3_9_1_3_1=1765
```

(e) F’.



```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_1_1_1_2=6785
Y_Ct_jnls_1_1_1_1_3=6710
Y_Ct_jnls_1_2_1_2_1=13830
Y_Ct_jnls_1_2_1_2_2=13168
Y_Ct_jnls_1_2_1_2_3=14200
Y_Ct_jnls_1_3_1_1_2=1402
Y_Ct_jnls_1_5_1_1_1=16750
Y_Ct_jnls_1_5_1_1_2=16750
Y_Ct_jnls_1_5_1_1_3=16750
Y_Ct_jnls_1_6_1_1_1=2680
Y_Ct_jnls_1_6_1_1_2=2745
Y_Ct_jnls_1_6_1_1_3=2710
Y_Ct_jnls_1_7_1_2_1=13065
Y_Ct_jnls_1_7_1_2_2=13375
Y_Ct_jnls_1_7_1_2_3=13210
Y_Ct_jnls_1_8_1_3_1=455
Y_Ct_jnls_1_8_1_3_2=465
Y_Ct_jnls_1_8_1_3_3=460
Y_Ct_jnls_1_9_1_3_1=1760
Y_Ct_jnls_1_9_1_3_2=1790
Y_Ct_jnls_1_9_1_3_3=1770
```

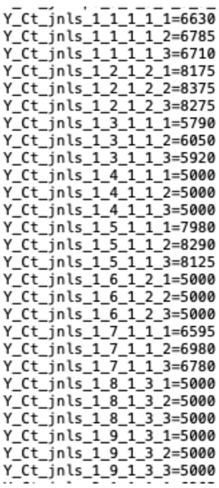
(f) B*($t = 1$).



```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_1_1_1_2=6785
Y_Ct_jnls_1_1_1_1_3=6710
Y_Ct_jnls_1_2_1_2_1=13120
Y_Ct_jnls_1_2_1_2_2=13840
Y_Ct_jnls_1_2_1_2_3=13480
Y_Ct_jnls_1_5_1_1_1=16750
Y_Ct_jnls_1_5_1_1_2=16750
Y_Ct_jnls_1_5_1_1_3=16750
Y_Ct_jnls_1_6_1_3_1=7770
Y_Ct_jnls_1_6_1_3_2=7945
Y_Ct_jnls_1_6_1_3_3=7845
Y_Ct_jnls_1_7_1_2_1=10900
Y_Ct_jnls_1_7_1_2_2=11160
Y_Ct_jnls_1_7_1_2_3=11025
```

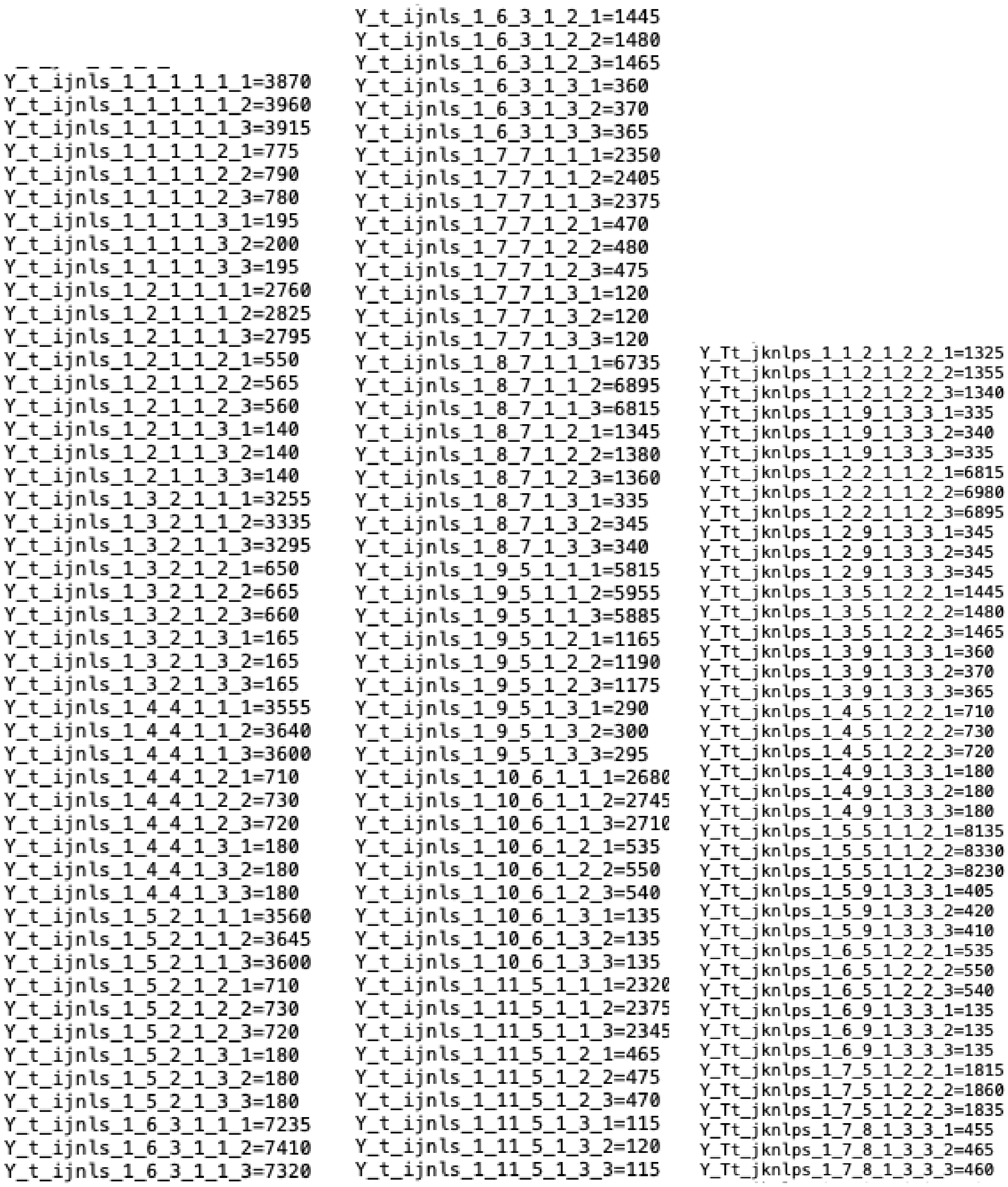
(g) D*($t = 1$).

Figure C.4: Comparison of number of people treated in each hospital in all improved cost solutions (case 2).



```
Y_Ct_jnls_1_1_1_1_1=6630
Y_Ct_jnls_1_1_1_1_2=6785
Y_Ct_jnls_1_1_1_1_3=6710
Y_Ct_jnls_1_2_1_2_1=8175
Y_Ct_jnls_1_2_1_2_2=8375
Y_Ct_jnls_1_2_1_2_3=8275
Y_Ct_jnls_1_3_1_1_1=5790
Y_Ct_jnls_1_3_1_1_2=6050
Y_Ct_jnls_1_3_1_1_3=5920
Y_Ct_jnls_1_4_1_1_1=5000
Y_Ct_jnls_1_4_1_1_2=5000
Y_Ct_jnls_1_4_1_1_3=5000
Y_Ct_jnls_1_5_1_1_1=7980
Y_Ct_jnls_1_5_1_1_2=8290
Y_Ct_jnls_1_5_1_1_3=8125
Y_Ct_jnls_1_6_1_2_1=5000
Y_Ct_jnls_1_6_1_2_2=5000
Y_Ct_jnls_1_6_1_2_3=5000
Y_Ct_jnls_1_7_1_1_1=6595
Y_Ct_jnls_1_7_1_1_2=6980
Y_Ct_jnls_1_7_1_1_3=6780
Y_Ct_jnls_1_8_1_3_1=5000
Y_Ct_jnls_1_8_1_3_2=5000
Y_Ct_jnls_1_8_1_3_3=5000
Y_Ct_jnls_1_9_1_3_1=5000
Y_Ct_jnls_1_9_1_3_2=5000
Y_Ct_jnls_1_9_1_3_3=5000
```

Figure C.5: Number of people treated in each hospital in a improved access solution from a different version of model ($capmin_j^t = 1000 * 5$, $capmax_j^t = 6000 * 5$, $OC_j^t = 347 * 7.4$, $IC_j^t = CC_j^t = 200000$).



```
Y_t_ijnls_1_1_1_1_1_1=3870
Y_t_ijnls_1_1_1_1_1_2=3960
Y_t_ijnls_1_1_1_1_1_3=3915
Y_t_ijnls_1_1_1_1_2_1=775
Y_t_ijnls_1_1_1_1_2_2=790
Y_t_ijnls_1_1_1_1_2_3=780
Y_t_ijnls_1_1_1_1_3_1=195
Y_t_ijnls_1_1_1_1_3_2=200
Y_t_ijnls_1_1_1_1_3_3=195
Y_t_ijnls_1_2_1_1_1_1=2760
Y_t_ijnls_1_2_1_1_1_2=2825
Y_t_ijnls_1_2_1_1_1_3=2795
Y_t_ijnls_1_2_1_1_2_1=550
Y_t_ijnls_1_2_1_1_2_2=565
Y_t_ijnls_1_2_1_1_2_3=560
Y_t_ijnls_1_2_1_1_3_1=140
Y_t_ijnls_1_2_1_1_3_2=140
Y_t_ijnls_1_2_1_1_3_3=140
Y_t_ijnls_1_3_2_1_1_1=3255
Y_t_ijnls_1_3_2_1_1_2=3335
Y_t_ijnls_1_3_2_1_1_3=3295
Y_t_ijnls_1_3_2_1_2_1=650
Y_t_ijnls_1_3_2_1_2_2=665
Y_t_ijnls_1_3_2_1_2_3=660
Y_t_ijnls_1_3_2_1_3_1=165
Y_t_ijnls_1_3_2_1_3_2=165
Y_t_ijnls_1_3_2_1_3_3=165
Y_t_ijnls_1_4_4_1_1_1=3555
Y_t_ijnls_1_4_4_1_1_2=3640
Y_t_ijnls_1_4_4_1_1_3=3600
Y_t_ijnls_1_4_4_1_2_1=710
Y_t_ijnls_1_4_4_1_2_2=730
Y_t_ijnls_1_4_4_1_2_3=720
Y_t_ijnls_1_4_4_1_3_1=180
Y_t_ijnls_1_4_4_1_3_2=180
Y_t_ijnls_1_4_4_1_3_3=180
Y_t_ijnls_1_5_2_1_1_1=3560
Y_t_ijnls_1_5_2_1_1_2=3645
Y_t_ijnls_1_5_2_1_1_3=3600
Y_t_ijnls_1_5_2_1_2_1=710
Y_t_ijnls_1_5_2_1_2_2=730
Y_t_ijnls_1_5_2_1_2_3=720
Y_t_ijnls_1_5_2_1_3_1=180
Y_t_ijnls_1_5_2_1_3_2=180
Y_t_ijnls_1_5_2_1_3_3=180
Y_t_ijnls_1_6_3_1_1_1=7235
Y_t_ijnls_1_6_3_1_1_2=7410
Y_t_ijnls_1_6_3_1_1_3=7320
```

(a) Flow from demand points to hospitals.

```
Y_t_ijnls_1_6_3_1_2_1=1445
Y_t_ijnls_1_6_3_1_2_2=1480
Y_t_ijnls_1_6_3_1_2_3=1465
Y_t_ijnls_1_6_3_1_3_1=360
Y_t_ijnls_1_6_3_1_3_2=370
Y_t_ijnls_1_6_3_1_3_3=365
Y_t_ijnls_1_7_7_1_1_1=2350
Y_t_ijnls_1_7_7_1_1_2=2405
Y_t_ijnls_1_7_7_1_1_3=2375
Y_t_ijnls_1_7_7_1_2_1=470
Y_t_ijnls_1_7_7_1_2_2=480
Y_t_ijnls_1_7_7_1_2_3=475
Y_t_ijnls_1_7_7_1_3_1=120
Y_t_ijnls_1_7_7_1_3_2=120
Y_t_ijnls_1_7_7_1_3_3=120
Y_t_ijnls_1_8_7_1_1_1=6735
Y_t_ijnls_1_8_7_1_1_2=6895
Y_t_ijnls_1_8_7_1_1_3=6815
Y_t_ijnls_1_8_7_1_2_1=1345
Y_t_ijnls_1_8_7_1_2_2=1380
Y_t_ijnls_1_8_7_1_2_3=1360
Y_t_ijnls_1_8_7_1_3_1=335
Y_t_ijnls_1_8_7_1_3_2=345
Y_t_ijnls_1_8_7_1_3_3=340
Y_t_ijnls_1_9_5_1_1_1=5815
Y_t_ijnls_1_9_5_1_1_2=5955
Y_t_ijnls_1_9_5_1_1_3=5885
Y_t_ijnls_1_9_5_1_2_1=1165
Y_t_ijnls_1_9_5_1_2_2=1190
Y_t_ijnls_1_9_5_1_2_3=1175
Y_t_ijnls_1_9_5_1_3_1=290
Y_t_ijnls_1_9_5_1_3_2=300
Y_t_ijnls_1_9_5_1_3_3=295
Y_t_ijnls_1_10_6_1_1_1=2680
Y_t_ijnls_1_10_6_1_1_2=2745
Y_t_ijnls_1_10_6_1_1_3=2710
Y_t_ijnls_1_10_6_1_2_1=535
Y_t_ijnls_1_10_6_1_2_2=550
Y_t_ijnls_1_10_6_1_2_3=540
Y_t_ijnls_1_10_6_1_3_1=135
Y_t_ijnls_1_10_6_1_3_2=135
Y_t_ijnls_1_10_6_1_3_3=135
Y_t_ijnls_1_11_5_1_1_1=2320
Y_t_ijnls_1_11_5_1_1_2=2375
Y_t_ijnls_1_11_5_1_1_3=2345
Y_t_ijnls_1_11_5_1_2_1=465
Y_t_ijnls_1_11_5_1_2_2=475
Y_t_ijnls_1_11_5_1_2_3=470
Y_t_ijnls_1_11_5_1_3_1=115
Y_t_ijnls_1_11_5_1_3_2=120
Y_t_ijnls_1_11_5_1_3_3=115
```

(b) Flow from demand points to hospitals (continuation).

```
Y_Tt_jknlps_1_1_2_1_2_2_1=1325
Y_Tt_jknlps_1_1_2_1_2_2_2=1355
Y_Tt_jknlps_1_1_2_1_2_2_3=1340
Y_Tt_jknlps_1_1_9_1_3_3_1=335
Y_Tt_jknlps_1_1_9_1_3_3_2=340
Y_Tt_jknlps_1_1_9_1_3_3_3=335
Y_Tt_jknlps_1_2_2_1_1_2_1=6815
Y_Tt_jknlps_1_2_2_1_1_2_2=6980
Y_Tt_jknlps_1_2_2_1_1_2_3=6895
Y_Tt_jknlps_1_2_9_1_3_3_1=345
Y_Tt_jknlps_1_2_9_1_3_3_2=345
Y_Tt_jknlps_1_2_9_1_3_3_3=345
Y_Tt_jknlps_1_3_5_1_2_2_1=1445
Y_Tt_jknlps_1_3_5_1_2_2_2=1480
Y_Tt_jknlps_1_3_5_1_2_2_3=1465
Y_Tt_jknlps_1_3_9_1_3_3_1=360
Y_Tt_jknlps_1_3_9_1_3_3_2=370
Y_Tt_jknlps_1_3_9_1_3_3_3=365
Y_Tt_jknlps_1_4_5_1_2_2_1=710
Y_Tt_jknlps_1_4_5_1_2_2_2=730
Y_Tt_jknlps_1_4_5_1_2_2_3=720
Y_Tt_jknlps_1_4_9_1_3_3_1=180
Y_Tt_jknlps_1_4_9_1_3_3_2=180
Y_Tt_jknlps_1_4_9_1_3_3_3=180
Y_Tt_jknlps_1_5_5_1_1_2_1=8135
Y_Tt_jknlps_1_5_5_1_1_2_2=8330
Y_Tt_jknlps_1_5_5_1_1_2_3=8230
Y_Tt_jknlps_1_5_9_1_3_3_1=405
Y_Tt_jknlps_1_5_9_1_3_3_2=420
Y_Tt_jknlps_1_5_9_1_3_3_3=410
Y_Tt_jknlps_1_6_5_1_2_2_1=535
Y_Tt_jknlps_1_6_5_1_2_2_2=550
Y_Tt_jknlps_1_6_5_1_2_2_3=540
Y_Tt_jknlps_1_6_9_1_3_3_1=135
Y_Tt_jknlps_1_6_9_1_3_3_2=135
Y_Tt_jknlps_1_6_9_1_3_3_3=135
Y_Tt_jknlps_1_7_5_1_2_2_1=1815
Y_Tt_jknlps_1_7_5_1_2_2_2=1860
Y_Tt_jknlps_1_7_5_1_2_2_3=1835
Y_Tt_jknlps_1_7_8_1_3_3_1=455
Y_Tt_jknlps_1_7_8_1_3_3_2=465
Y_Tt_jknlps_1_7_8_1_3_3_3=460
```

(c) Transfer flow between hospitals.

Figure C.6: Output of solving the model for solution A* case 2 (time period considered was $t = 1$).