Inverse Method for Identification of Forces and Structural Damage in Steady-State Regime

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Abstract

In this work an inverse method for identification of forces and structural damage is analysed, which stems from the application of the concept of transmissibility for systems with multiple degrees of freedom. The purpose of the proposed method is to evaluate the location of external forces and structural modifications, using the dynamic responses of the structure being analysed. Once the location of the forces or possible structural modifications is identified, one then proceeds with the determination of the applied forces' amplitudes or structural damage quantification. One of the main purposes of this work is to define with clarity an effective methodology concerning the applicability of the proposed method. To this end, the numerical identification of forces and structural modifications is studied upon an Euler-Bernoulli beam and a Kirchhoff-Love plate. The structural modifications analysed throughout this work are: addition of a discrete mass, reduction of stiffness and formation of a single crack. All the cases studied have yielded very successful results, given that the location of forces or structural modifications have been correctly determined and the calculation of the external forces' amplitudes or the structural damage quantification reveal themselves to be nearly exact. Moreover, the numerical application of the inverse method has revealed some of its limitations, which were further examined with the purpose of establishing solutions that make its applicability more viable and efficient.
Keywords: inverse method, transmissibility, identification of forces, identification of structural damage, damage quantification, dynamic responses

1. Introduction

In the structural dynamics field, excitation forces estimation is vital to perform a dynamic structural analysis. Whenever possible, force transducers are used to make direct measurements of the external applied forces [1]. The force sensor needs to be placed in the exact location where it is intended to perform the measurement, in order to make that measurement as precise as possible. However, the location of the sensors on the optimal position is frequently impossible for several reasons: inaccessibility, movement of the region, danger, among others. Therefore, to overcome the problem described above, one uses indirect methods to obtain the amplitude of the external forces applied. Such methods are able to estimate the forces indirectly through structural dynamic response measurements. Several examples of structural dynamic responses are: displacements, accelerations and reaction forces. Thus, the estimation of forces' amplitudes reveals itself to be an inverse problem. Various theories regarding inverse methods have been developed throughout the years. All of them have in common the effects of ill-posed matrices, derived from the nature of the inverse problem. Frequently, these problems are overcome by the use of the pseudo-inverse applied in over-determined systems, applying the singular value decomposition [8] and using Kalman filters [3]. An indirect method for force identification in the frequency domain, proposed by Neves and Maia [6], is presented throughout this dissertation. The method consists in estimating the location and amplitude of external excitation forces that act upon a structure. This estimation is performed through the measurement of the reaction forces or the dynamic displacements of a structure. Concepts of transmissibility in multiple degrees of freedom are used to develop a relationship between the dynamic measurements and the forces applied. The concept of identifying forces using transmissibility can be adapted to several scientific fields. For example, it is possible to extend the inverse method of force identification using transmissibility into a structural damage identification method. Inverse methods of structural damage identification have been the subject of some investigation since the late twenties till today. The great majority of the development made in this field is based on the use of modal data of an undamaged structure as reference data. All the subsequent results derived from structural tests are compared with the reference data and, any deviation in the modal properties is used to estimate possible structural modifications. The structural damage identification method studied throughout this dissertation was first developed by Neves and Maia in [5]. The authors demonstrated that any structural modification in terms of stiffness, mass or damping can be interpreted as additional forces acting upon the original undamaged structure. Therefore, the identification of possible structural modifications is performed by identifying the additional forces, using the inverse force identification method based on transmissibility. The main goal of this dissertation is to define a concise methodology which allows to clarify the inverse method for identification of forces and structural damage. For that purpose, one intends to create some norms to facilitate the interpretation of the method, rendering the analysis of the results more efficient and expedite. Once the methodology is developed, it is intended to apply it to several numerical examples to demonstrate its applicability. The cases studied are: addition of a discrete mass on a beam and on a plate and stiffness reduction and crack formation on a beam.

2. Background

The purpose of this chapter is to briefly summarize all the theoretical foundations which are essential regarding the development of this dissertation.

2.1. Mechanical Vibration

Mechanical Vibration is defined as the measurement of a periodic process of oscillations with respect to an equilibrium point. In general, a vibration system includes a means for storing potential energy (springs or elasticity), a means for storing kinetic energy (mass or inertia) and a means that dissipates energy (damper). The vast majority of systems with practical application are continuous. This kind of systems are characterized by an infinite number of degrees of freedom. However, a continuous system can be modelled as a discrete system with multiple degrees of freedom. In general, the more degrees of freedom are considered, the resultant model generate more accurate results. The dynamic equilibrium equation for a a MDOF system was deduced along this section and is given by,

$$[M]\{\ddot{y}(t)\} + [C]\{\dot{y}(t)\} + [K]\{y(t)\} = \{f(t)\} \quad (1)$$

In this way, a discrete system is characterized by a global mass, stiffness and damping matrices. Assuming that the excitation forces, $\{f(t)\}$, are harmonic and discrete, with a frequency ω, the nodal displacements, $\{y(t)\}$, are harmonic and discrete too and are characterized by the same frequency ω. Using the Fourier transform, one can re-write equation (1) in the frequency domain,

$$\left[-\omega^2[M] + i\omega[C] + [K]\right]\{Y(\omega)\} = \{F(\omega)\} \quad (2)$$

One can define the dynamic stiffness matrix, $[Z(\omega)]$, as,

$$[Z(\omega)] = \left[-\omega^2[M] + i\omega[C] + [K]\right] \quad (3)$$

and equation (2) can be written as,

$$[Z(\omega)]\{Y(\omega)\} = \{F(\omega)\} \quad (4)$$

The matrix $[H(\omega)]$, also called FRF (Frequency Response Function), is simply the inverse of the dynamic stiffness matrix, $[Z(\omega)]$.

2.2. Linear Elasticity Theory

Solid structures will deform when adequate external forces are applied. If the material is elastic and the external forces are removed, the structure will return to its original shape and size. For an elastic body, each displacement has two components. One of them results from deformations of the body itself (strains) and the other component is the rigid body motion, uniform along the elastic body. To describe the stress components one can study an infinitesimal cube. The stress components align with the normal of the three perpendicular planes is designated by normal stress. In turn, the components perpendicular to the normal of each plane are named shear stress. Gathering all this components, one can create the stress tensor. The stress tensor relates linearly with the strain tensor, as stated by the Generalized Hooke's Law.

2.3. Finite Element Method

Engineering problems are, most of the times, expressed in terms of partial differential equations (PDEs). For the vast majority of problems, these PDEs cannot be solved analytically. Instead, an approximation of the equations can be constructed, typically based upon different types of discretizations. These discretization methods approximate the PDEs with numerical model equations, which can be solved using numerical methods. The finite element method is used to compute such approximations. The solution to the numerical model equations are, in turn, an approximation of the real solution to the PDEs.

2.3.1 Euler-Bernoulli Beam

In the Euler-Bernoulli beam theory it is assumed that the plane sections perpendicular to the beam axis remain plane and perpendicular to the same axis after deformation. In this section, one presents the derivation of the elemental mass and stiffness matrices for an Euler-Bernoulli beam element. These matrices are the basis to model a beam in finite elements. Assembling all the elemental mass and stiffness matrices one can create the global mass and stiffness matrices, which characterize the structural behaviour of a beam.

2.3.2 Kirchhoff-Love Plate

A plate is a solid body formed by two parallel planes, in which the lateral dimension is far greater than the thickness. Therefore, it is not necessary to model this structure using 3D elasticity theory. The Kirchhoff-Love theory rely on the following kinematic assumptions: straight lines normal to the mid-surface remain straight after deformation, straight lines normal to the mid-surface remain normal to the mid-surface after deformation and the thickness of the plate does not change during deformation. Likewise the previous section, to model a plate in finite elements one needs to assemble the elemental mass and stiffness matrices. The derivation of the formulation for these matrices is presented in this section.

2.4. Force Transmissibility for MDOF Systems

The concept of force transmissibility relates the dynamic reaction forces with the external applied forces. Force transmissibility for a SDOF system is defined as the ratio between the transmitted force's amplitude and the excitation force's amplitude. to expand this definition to a MDOF system, one needs to define four sets of degrees of freedom:

- *Set K*: degrees of freedom where external forces are applied;
- *Set U*: degrees of freedom matching the location of the reaction forces;
- *Set C*: degrees of freedom without applied forces;
- *Set E*: *Set K*+*Set C*, degrees of freedom without reaction forces.

In order to formulate an equation for the force transmissibility, one adapts equation(4) expanding it to the *sets* defined previously. Assuming that the degrees of freedom of *set U* are fixed, and after some mathematical manipulation, the following equation is derived [4],

$$\{F_U\} = \left([Z_{UE}][Z_{EE}]^{-1}\right)\{F_E\} \tag{5}$$

Having into consideration the definition of transmissibility, one can define the force transmissibility matrix, $\left[T_{UE}^{(f)}\right]$, as follows,

$$\left[T_{UE}^{(f)}\right] = [Z_{UE}][Z_{EE}]^{-1} \tag{6}$$

Using the force transmissibility matrix one can relate the forces acting upon the degrees of freedom of *set K* (*set K* is included in *set E*) and the reaction forces located in the degrees of freedom of the *set U*. One important and useful property of the force transmissibility matrix is that each column refers uniquely to a single applied force. Therefore, one can always use sub-matrices of the force transmissibility matrix. Consequently, it is not necessary to calculate a new transmissibility matrix each time the location of the external forces changes.

2.5. Displacement Transmissibility for MDOF Systems

For a specific set of external harmonic discrete forces applied upon a structure, it is possible to relate the displacements' amplitude between system's degrees of freedom, applying the concept of displacement transmissibility. Taking a SDOF system into consideration, one can define the displacement transmissibility as the ratio between the amplitude of the applied displacements and the amplitude of the transmitted displacements. This definition can be expanded to a MDOF system. For that purpose, one defines four *sets* of degrees of freedom as follows [6]:

- *Set K*: degrees of freedom where the displacement value is known;
- *Set U*: degrees of freedom where the displacement value is unknown;
- *Set A*: degrees of freedom where forces are applied;
- *Set C*: all the remaining degrees of freedom.

Equation (4) can be written in terms of the FRF as follows,

$$\{Y(\omega)\} = [H(\omega)]\{F(\omega)\} \tag{7}$$

Similarly to the previous section, in order to formulate an expression for the displacement transmissibility, one adapts equation (7) expanding it to the *sets* defined above. Assuming the case where the structure is free from supports, all the forces are applied upon the degrees of freedom of *set A*. Taking that assumption into consideration and after some mathematical manipulation, the following equation is derived [2],

$$\{Y_U\} = \left([H_{UA}][H_{KA}]^{+}\right)\{Y_K\} \tag{8}$$

Since that the matrix $[H_{KA}]$ isn't necessarily square, one needs to use the pseudo-inverse to calculate the displacements' amplitude of the *set U*. In order to apply the pseudo-inverse, the number of degrees of freedom of *set K* must be higher or equal than the number of degrees of freedom of *set A*. One can define the displacement transmissibility matrix, $\left[T_{UK}^{A}{}^{(d)}\right]$, as follows,

$$\left[T_{UK}^{A}{}^{(d)}\right] = [H_{UA}][H_{KA}]^{+} \quad (9)$$

Using the displacement transmissibility matrix, for a specific location of the external forces (*set A*), one can relate the known displacements of *set K* and the unknown displacements of *set U*. In contrast to the force transmissibility matrix, the displacement transmissibility matrix has to be recalculated each time the location of the external forces changes.

2.6. Modelling Structural Modifications as Additional Forces

Structural modifications imply changes on the global mass, stiffness and damping matrices, $[\Delta M]$, $[\Delta K]$ and $[\Delta C]$, respectively. Along this section it is shown that structural modifications, also referred to as structural damage, can be interpreted as additional forces acting upon an undamaged structure, as proposed by the authors of the bibliographic reference [5]. In a general form, the dynamic behaviour of a structurally modified discrete system is characterized by the following expression,

$$[M_{\Delta}]\{\ddot{y}(t)\} + [C_{\Delta}]\{\dot{y}(t)\} + [K_{\Delta}]\{y(t)\} = \{f(t)\} \quad (10)$$

where $[M_{\Delta}] = [M + [\Delta M]]$, $[K_{\Delta}] = [K + [\Delta K]]$ and $[C_{\Delta}] = [C + [\Delta C]]$. Equation (10) can be re-written as follows,

$$[M]\{\ddot{y}(t)\}+[C]\{\dot{y}(t)\}+[K]\{y(t)\} = \{f(t)\}+\{f_D(t)\} \quad (11)$$

where $\{f_D(t)\}$ is the additional forces vector, which result from the structural damage, and, omitting the time dependency, is given by,

$$\{f_D\} = -\left([\Delta M]\{\ddot{y_D}\} + [\Delta C]\{\dot{y_D}\} + [\Delta K]\{y_D\}\right) \quad (12)$$

where the index D in the displacements vector $\{y_D\}$) indicates that the displacements apply to the location of the structural modifications. Equation (11) illustrates that structural modifications can be defined as additional forces, $\{f_D(t)\}$, acting upon an undamaged structure, characterized by the matrices $[M]$, $[K]$ and $[C]$. Considering a harmonic excitation in steady-state conditions, the additional forces vector can be written as follows,

$$\{F_D(\omega)\} = -(-\omega^2[\Delta M]+i\omega[\Delta C]+[\Delta K])\{Y_D(\omega)\}) \quad (13)$$

Equation (13) will be used recurrently till the end of this chapter.

2.6.1 Addition of a Discrete Mass

The addition of a discrete mass is modelled based on the assumption that it only influences the mass matrix of the undamaged structure. Consequently, $[\Delta K]$ and $[\Delta C]$ are equal to zero. Based on the assumption described above, equation (13) can be re-written as follows,

$$\{F_D(\omega)\} = \omega^2[\Delta M]\{Y_D(\omega)\} \quad (14)$$

It is also assumed that the mass addition only affects the transverse displacements of the structure. Therefore, the mass rotational inertia is considered negligible.

2.6.2 Localized Reduction of Stiffness

In this section it is considered that the localized reduction of stiffness only influence the original stiffness matrix. Thus, $[\Delta M]$ and $[\Delta C]$ are equal to zero. For the structural damage described above, equation (13) is given by,

$$\{F_D(\omega)\} = -[\Delta K]\{Y_D(\omega)\}) \quad (15)$$

The term "localized" is used because it is assumed that the influence of the reduction of stiffness is confined to the nodes of a structure modelled in finite elements. It is also considered that this structural modification only affects the nodal rotations.

2.6.3 Addition of a Discrete Mass and Localized Reduction of Stiffness

Gathering both the addition of a discrete mass and the localized reduction of stiffness, described in sections 2.6.1 and 2.6.2 respectively, equation (13) can be written as follows,

$$\{F_D(\omega)\} = \left(-[\Delta K] + \omega^2[\Delta M]\right)\{Y_D(\omega)\} \quad (16)$$

2.6.4 Crack in a Beam

J. K. Sinha, M. I. Friswell and S. Edwards presented in [7] a simplified model for open cracks. This model is applicable to Euler-Bernoulli beam elements and it is based on modifications in local flexibility in the vicinity of a crack. In figure 1 one can see a representation of a beam with multiple cracks. Crack formation on a beam implies a reduction in the structure stiffness. Part of the material of the beam surrounding the crack is not under tension and, consequently, the stiffness contribution is limited. The authors of [7] proposed a simplified procedure to model the stiffness variation in the vicinity of a crack, considering that the flexibility varies linearly from a non-cracked section till a cracked section. It is assumed that the variation of flexibility starts from an effective length, l_c, in both

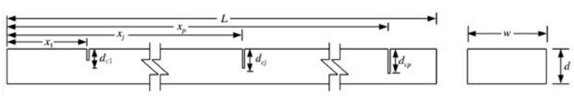

Figure 1: Beam with multiple cracks

sides of the crack. The derivation of an expression for l_c was shown in [7] and it follows,

$$l_c = 1,5d \tag{17}$$

where d is the height of the beam cross-section. The model proposed for the flexibility is called triangular reduction and it is represented in figure 2. The flexibility in the vicinity of a crack is given by,

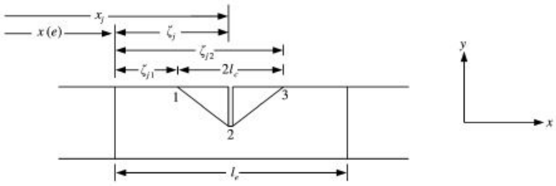

Figure 2: Triangular reduction in an Euler-Bernoulli beam element

$$EI_e(\zeta) = \begin{cases} EI_0 - E(I_0 - I_{cj})\frac{(\zeta-\zeta_{j1})}{(\zeta_j-\zeta_{j1})} \\ EI_0 - E(I_0 - I_{cj})\frac{(\zeta_{j2}-\zeta)}{(\zeta_{j2}-\zeta_j)} \end{cases} \tag{18}$$

where the first equation is applicable when $\zeta_{j1} \leq \zeta \leq \zeta_j$ an the second equation is valid when $\zeta_j \leq \zeta \leq \zeta_{j2}$. In equation (18), ζ_j is the location of the j^{th} crack and ζ_{j1} and ζ_{j2} are the positions, in both sides of the crack, that bound the influence of the crack, as shown in figure 2. Furthermore, E is the longitudinal Young Modulus, I_0 is the second moment of area of the undamaged cross-section and I_{cj} is the second moment of area of the cracked cross-section. I_{cj} is given by,

$$I_{cj} = w\frac{(d-d_{cj})^3}{12} \tag{19}$$

where d_{cj} is the crack depth, as shown in figure 1. The modified elemental stiffness matrix for a cracked beam element can be written as follows,

$$[K_{e,crack}] = [K_e] - [K_{cj}] \tag{20}$$

where $[K_e]$ is the elemental stiffness matrix for an undamaged element and $[K_{cj}]$ is the matrix that characterize the stiffness reduction induced by the crack. Applying equation (18) into the *standard* elemental stiffness matrix integration, one obtains that the stiffness reduction matrix is given by,

$$[K_{cj}] = \begin{bmatrix} k_{11} & k_{12} & -k_{11} & k_{14} \\ k_{12} & k_{22} & -k_{12} & k_{24} \\ -k_{11} & -k_{12} & k_{11} & -k_{14} \\ k_{14} & k_{24} & -k_{14} & k_{44} \end{bmatrix} \tag{21}$$

where the six $k's$ are the stiffness reduction constants, which are functions of ζ_j and d_{cj} only. Applying equation (13) for a single crack,

$$\{F_D(\omega)\} = [K_{cj}^n]\{Y_D(\omega)\} \tag{22}$$

where $[K_{cj}^n]$ is the elemental stiffness reduction matrix for the cracked element n and $\{Y_D(\omega)\}$ is a vector that includes the two transverse displacements and the two rotations of the cracked element n.

3. Methodology

Throughout this chapter one presents a methodology for the force identification method application using force and displacement transmissibility. The same methodology is adapted for the structural damage identification method implementation using this displacement transmissibility. For that purpose, all the structural modifications presented in section 2.6 are taken into account.

3.1. Force Identification Method

Along this chapter, two methodologies for the force identification method are presented. One uses the force transmissibility and the other is based on displacement transmissibility.

3.1.1 Force Transmissibility

The goal of this section is to describe the methodology for the force identification method using measured dynamic reaction forces. For numerical purposes, this measurement is simulated by a numerical calculation using the dynamic equilibrium equation of the structure modelled in finite elements. The location of all reaction forces generates the *set U*, previously presented in section 2.4. The methodology for the force identification method using force transmissibility is based on the following steps:

1 - Determination of all the possible locations of the external forces, also referred to as combinations, using the function *nchoosek(n,k)* available on *MATLAB*'s library of functions, where *n* is the number of locations where the external forces can be applied and *k* refers to the number of external forces taken into account. All this possible locations form a new *set* of degrees of freedom named *set S*;

2 - Calculation of the accumulated error for all the possible combinations of external forces using the following expression [6],

$$\epsilon(\{F_S\}) = \sum_{i=1}^{n_{freq}} \left[\sum_{j=1}^{n_U} \left[\{\tilde{F}_U\}_j - \{F_U\}_j^s \right]^2 \right] \tag{23}$$

where $\{\tilde{F}_U\}$ represents the measured reaction forces and $\{F_U\}^S$ refers to the estimated reaction forces for each combination of the *set S*. n_{freq} indicates the number of excitation frequencies analysed. The estimated reaction forces are calculated using the transmissibility force matrix, as follows,

$$\{F_U\}^s = \left[T_{US}^{(f)}\right]\{F_S\} \tag{24}$$

where $\left[T_{US}^{(f)}\right]$ is a sub-matrix of the force transmissibility matrix, $[T_{UE}^{(f)}]$.

To calculate the accumulated error for each combination one needs to determine the external forces' amplitude, $\{F_S\}$, that minimize equation (23). For that purpose one uses *MATLAB* function *fmincon*. This function has the applied forces' amplitude and the accumulated error as *output* and the function that implements the accumulated error as *input*. The optimal combination is the one that generates the global minimum error. One optimal combination of *k-1* force locations generates a global minimum error when the local minimum error for combinations of *k-1* force locations is lower than the local minimum error for combinations of *k* force locations.

3.1.2 Displacement Transmissibility

To identify the location and amplitude of the external forces using displacement transmissibility it is necessary to measure the displacements for the n_U degrees of freedom of *set U* and for the n_K degrees of freedom of *set K*. This *sets* were previously presented in section 2.5. Once again, for numerical purposes, the displacements measurement is simulated by a numerical calculation using the dynamic equilibrium equation of a structure modelled in finite elements. The methodology for the force identification method using displacement transmissibility follows the following steps:

1 - Determination of all the possible locations of the external forces, in the way presented in the previous section;

2 - Calculation of the accumulated error for each combination of external forces applying the following expression [6],

$$\epsilon^{A_i} = \sum_{i=1}^{n_{freq}} \left[\sum_{j=1}^{n_U} \left[|\tilde{Y}_{Uj}| - |Y_{Uj}| \right]^2 \right] \tag{25}$$

where $\tilde{Y}_U$ represents the measured displacements of *set U* and Y_U refers to the estimated displacements of the same *set*. The estimated displacements are calculated using the displacement transmissibility matrix as follows,

$$\{Y_U\} = \left[T_{UK}^{A_i\,(d)}\right] \{\tilde{Y}_K\} \tag{26}$$

where the vector $\{\tilde{Y}_K\}$ represents the measured displacements of *set K*. The displacement transmissibility matrix, $\left[T_{UK}^{A_i\,(d)}\right]$, is defined for each combination of external forces A_i. In order to determine the combination that includes the correct location of the external forces, one needs to introduce a necessary condition described as follows:

A combination A_i of *k* force locations is optimal if all the combinations that generate a local minimum for *k+1* force locations include combination A_i and, additionally, the amplitude of the force related to the degree of freedom outside combination A_i is nearly null for all the frequencies analysed. One names has "false positives" or "potentially optimal combinations" all the combinations of *k+1* force locations that generate local minimum errors and include the optimal combination of *k* force locations and, additionally, the force related to the degree of freedom outside the optimal combination is nearly null for all the spectrum of frequencies. Therefore, the definition of the necessary condition implies the calculation of the forces' amplitude for all the false positives. This calculation is performed using the following expression,

$$\{F_A\} = [H_{KA}]^+ \{Y_K\} \tag{27}$$

3.2. Identification of Structural Modifications

Throughout section 2.6 it was shown that structural modifications can be interpreted as additional forces acting upon a non-modified structure. To determine the location of the external forces, including the additional forces that result from the structural modifications, one follows the methodology for the force identification method using displacement transmissibility, proposed in section 3.1.2. One needs to apply an external force with known location and amplitude in order to measure the displacements of *set U* and *set K*. Once the location of the additional forces is obtained, one reconstructs the amplitude for all the forces identified, applying equation (27).

3.2.1 Addition of a Discrete Mass

For an addition of a discrete mass, the vector of additional forces, $\{F_D(\omega)\}$, is given by equation (14), presented in section 2.6.1. If only one mass is added to the structure, equation (14) can be written as,

$$F_{mass}(\omega) = \omega^2 m_{mass} Y_{mass}(\omega) \tag{28}$$

where $F_{mass}(\omega)$ is the reconstructed force's amplitude at the location of the additional mass, m_{added} is the amount of mass added and $Y_{mass}(\omega)$ is the transversal displacement at the location of the mass. The value m_{added} is calculated for all the spectrum of frequencies. Therefore, the amount of mass added is obtained performing the mean of all the values obtained.

3.2.2 Localized Stiffness Reduction

Regarding a localized stiffness reduction, the vector of additional forces $\{F_D(\omega)\}$ is given by equation (15), presented in section 2.6.2. It is emphasized that the reduction of stiffness described in section 2.6.2 only influence the rotation of the nodes of a structure modelled in finite elements. With that being said, it is necessary to identify external moments and not only external forces. This feature is not problematic because the method of force identification presented is naturally expanded if one needs to take moments into consideration, without changing the associated methodology. To quantify the variation of stiffness, $[\Delta K]$, one needs to apply equation (15) for all the frequencies of the spectrum of frequencies, calculating *a posteriori* the mean for the values obtained.

3.2.3 Addition of a Discrete Mass and Localized Reduction of Stiffness

The additional forces vector,$\{F_D(\omega)\}$, generated when an addition of a discrete mass and a localized reduction of stiffness occurs simultaneously was presented in section 2.6.3, equation (16). Assuming that the reduction of stiffness only affects the rotation n and that only a single mass was added, one can re-write equation (16) as follows,

$$\{F_D(\omega)\} = -\Delta K_{n\times n} Y_n(\omega) + \omega^2 m_{mass} Y_{mass}(\omega) \tag{29}$$

where $-\Delta K_{n\times n}$ is a scalar that represents the stiffness reduction in the node that has the rotation n and $Y_n(\omega)$ is the rotation's amplitude. To quantify the reduction of stiffness $-\Delta K_{n\times n}$, the following equation is applied for all the spectrum of frequencies,

$$\{F_{DK}(\omega)\} = -\Delta K_{n\times n} Y_n(\omega) \tag{30}$$

where $\{F_{DK}(\omega)\}$ is the line of the additional forces vector, $\{F_D(\omega)\}$, corresponding to the reduction of stiffness. Similarly, to quantify the amount of mass added, one needs to solve the following equation for all the spectrum of frequencies,

$$\{F_{DM}(\omega)\} = \omega^2 m_{mass} Y_{mass}(\omega) \tag{31}$$

where $\{F_{DM}(\omega)\}$ is the line of the additional forces vector, $\{F_D(\omega)\}$, corresponding to the addition of mass.

3.2.4 Crack in an Euler-Bernoulli Beam

Regarding a formation of a single crack on the n^{th} element of an Euler-Bernoulli beam element, the expression for the additional forces vector was presented in section 2.6.4, equation (22). For a beam element n with the degrees of freedom w_1, θ_1, w_2 and θ_2 , one can re-write equation (22) as follows,

$$\begin{bmatrix} k_{11} & k_{12} & -k_{11} & k_{14} \\ k_{12} & k_{22} & -k_{12} & k_{24} \\ -k_{11} & -k_{12} & k_{11} & -k_{14} \\ k_{14} & k_{24} & -k_{14} & k_{44} \end{bmatrix} \begin{Bmatrix} w_1(\omega) \\ \theta_1(\omega) \\ w_2(\omega) \\ \theta_2(\omega) \end{Bmatrix} = \begin{Bmatrix} F_1(\omega) \\ F_2(\omega) \\ F_3(\omega) \\ F_4(\omega) \end{Bmatrix} \tag{32}$$

where $F_1(\omega)$, $F_2(\omega)$, $F_3(\omega)$ and $F_4(\omega)$ are the reconstructed forces and moments corresponding to the element n. Writing equation (32) in the format $[A]\{x\} = \{b\}$, where the vector $\{x\}$ has the six stiffness reduction constants, one obtains an indeterminate system because the matrix $[A]$ has more lines than columns. However, given the frequency dependency of the displacements, rotations, forces and moments, one can adapt the indeterminate system and transform it into an over-determined one, as follows,

$$\begin{bmatrix} [Q(\omega_1)] \\ \vdots \\ [Q(\omega_L)] \end{bmatrix} \begin{Bmatrix} k_{11} \\ k_{12} \\ k_{14} \\ k_{22} \\ k_{24} \\ k_{44} \end{Bmatrix} = \begin{Bmatrix} \{F(\omega_1)\} \\ \vdots \\ \{F(\omega_L)\} \end{Bmatrix} \tag{33}$$

where a matrix $[Q(\omega_n)]$ is defined for each frequency. The matrix presented in equation (33) has $L \times 4$ lines and 6 columns, where L represents the number of frequencies considered. Omitting the dependency of the displacements and rotations in terms of the frequency ω_n, the matrix $[Q(\omega_n)]$ is given by,

$$\begin{bmatrix} w_1 - w_2 & \theta_1 & \theta_2 & 0 & 0 & 0 \\ 0 & w_1 - w_2 & 0 & \theta_1 & \theta_2 & 0 \\ -w_1 + w_2 & -\theta_1 & -\theta_2 & 0 & 0 & 0 \\ 0 & 0 & w_1 - w_2 & 0 & \theta_1 & \theta_2 \end{bmatrix} \tag{34}$$

In order to calculate the value of the six stiffness reduction constants one apply the pseudo-inverse in equation (33). Once the six constants are determined, one can calculate the crack location, ζ_j and the second moment of area in the crack section, I_{cj}. This calculation is done though a system of two equations. Each of the equations correspond to the expression of two of the six stiffness reduction constants determined previously. Given six equations for the six constants, the two unknowns in all of them are precisely ζ_j and I_{cj}. Afterwards, the crack depth d_{cj} is directly calculated applying the definition of I_{cj}, presented in section 2.6.4, equation (19).

4. Results

Throughout this chapter one presents the results for the numerical simulation of the inverse method for identification of forces and structural damage.

4.1. Identification of Forces Applied in a Beam

The results for the numerical simulation of the force identification method in a beam, using force and displacement transmissibility, are presented in this section. The stainless steel beam analysed throughout this section and section 4.2 has a rectangular cross-section of area $A = 1m^2$ and a second moment of inertia $I = 2 \times 10^{-4} m^4$. The finite element model of the beam is shown in figure 3.

4.1.1 Force Transmissibility

To apply the concept of force transmissibility one considers that both ends of the beam are built-in and the external forces of $500N$ are applied at

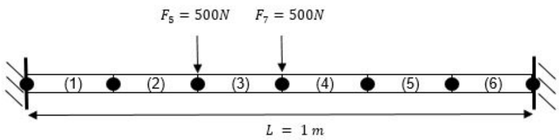

Figure 3: Beam modelled in finite elements

the nodes 3 and 4 of the structure modelled in finite elements, has shown in figure 3. Following the methodology for the force identification using force transmissibility, described in section 3.1.1, one obtained the graph presented in figure 4. Figure 4 displays the accumulated error generated by each combination of external forces. In this par-

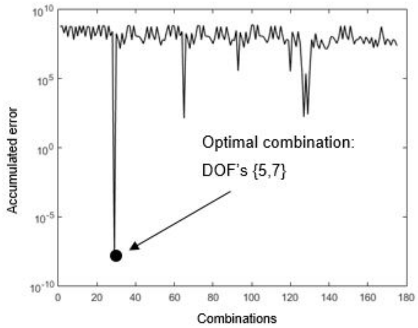

Figure 4: Accumulated error for combinations up to three DOF's

ticular case one calculated the accumulated errors for combinations with one, two and three degrees of freedom, since the optimal solution is a combination of 2 external forces. Analysing figure 4 one can immediately conclude that an optimal combination was found, since there is a combination that minimize globally the accumulated error. This combination includes the degrees of freedom 5 and 7 as shown, which are the transverse displacements for the nodes with applied forces. One obtained a value of $500N$ for both forces' amplitudes.

4.1.2 Displacement Transmissibility

For the application of the concept of displacement transmissibility one follows the methodology described in section 3.1.2. In this section one studies a free-free beam where two external forces of $500N$ are applied at the nodes 3 and 4. In figure 5 one shows the corrected accumulated error calculated for combinations with one, two and three degrees of freedom, for the same reason explained in the previous section. The corrected accumulated error graph is simply the accumulated error graph where a factor is added to all the false positives. In that way, the observation of the optimal combination is more clear. As shown in figure 5, the optimal combination includes the transverse displacements of both nodes with external forces applied. Reconstructing the forces associated with the optimal combination, the exact forces' amplitudes are determined.

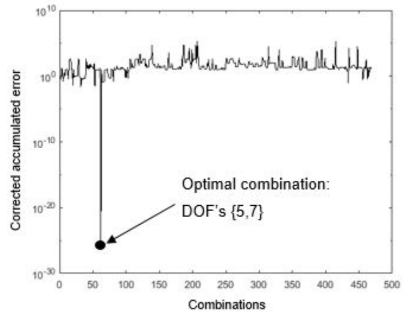

Figure 5: Corrected accumulated error for combinations up to three DOF's

4.2. Structural Modifications Identification in a Beam

In this section one analyses the numerical applicability of the method for structural damage identification in a free-free beam. For each structural modification studied throughout this section, the results obtained for the structural damage quantification are fully or nearly exact.

4.2.1 Addition of a Discrete Mass

In this section one applies the methodology described in section 3.2.1. A discrete mass of $100g$ is added at node 3 and to excite the DOF's of the beam a $500N$ harmonic force is applied at the node 4. Therefore, one needs to determine an optimal combination that includes the DOF 5, transverse displacement of the node where the mass is added and the DOF 7, transverse displacement of the node where the excitation force is applied. The optimal combination is correctly identified, however, the presentation of the corrected accumulated error graph is omitted since it is very similar to the graph shown in figure 5.

4.2.2 Localized Stiffness Reduction

Along this section one applies the methodology described in section 3.2.2. One defined a stiffness reduction of 5% at the node 2 and the beam is excited by a harmonic force of $500N$ applied at the node 3. The optimal combination is identified correctly as shown in figure 6, since it includes the DOF 4, rotation of the node where the reduction of stiffness occurs, and the DOF 5, transverse displacement of the node where the external force is applied.

4.2.3 Addition of a Discrete Mass and Localized Stiffness Reduction simultaneously

In this section one studies the case of a beam with a mass added at the node 4 and, simultaneously, a stiffness reduction of 5% occurs at node 2. An excitation force of $500N$ is applied at the node 3. Therefore, the correct optimal combination

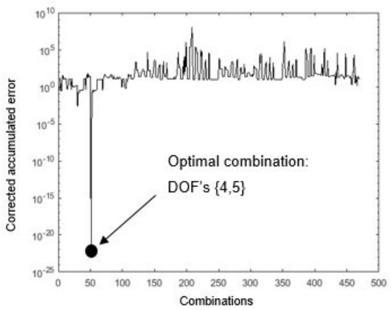

Figure 6: Corrected accumulated error for combinations up to three DOF's

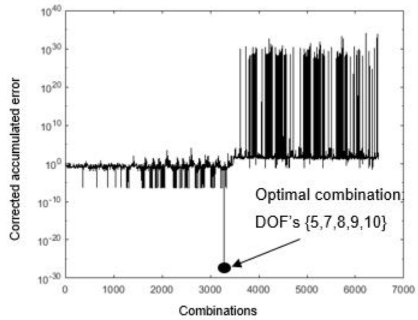

Figure 8: Corrected accumulated error for combinations up to six DOF's

consists of the following DOF's: DOF 4, rotation of node 2, DOF 5, transverse displacement of node 3 and DOF 7, transverse displacement of node 4. As shown in figure 7, the correct combination was identified. In this case, combinations of four degrees of freedom were analysed since one needs to identify the location of two forces and one moment.

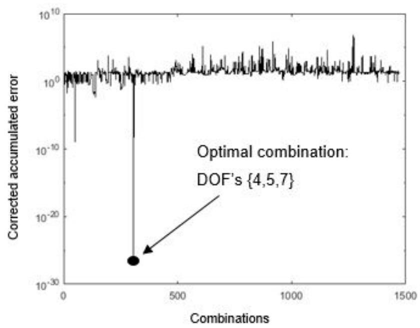

Figure 7: Corrected accumulated error for combinations up to four DOF's

4.2.4 Crack Formation in a Beam

In order to determine the depth and the location of a crack in an Euler-Bernoulli beam one follows the methodology described in section 3.2.4. For the case studied in this section, the crack is located in the center of the element 4 and has a depth of $0,007m$, which corresponds to 70% of the beam height. An excitation harmonic force of $3000N$ is applied at the node 3. Since the crack is located in the element 4, the optimal combination has the following DOF's: DOF 5, transverse displacement of the node where the excitation force is applied and DOF's 7, 8, 9 and 10, displacements and rotations of the element 4. As shown in the figure 8, the correct location of the cracked element is indeed determined. Once it is necessary to identify three forces and two moments, the corrected accumulated error is calculated for combinations up to six DOF's. The calculations of the crack location and depth yields exact results.

4.3. Identification of External Forces Applied in a Plate

In this section one extends the complexity of the force identification method, applying it to the case of a Kirchhoff-Love plate free from supports. The plate is made of steel and has a length and a width of $1m$ with a thickness of $0.02m$. The structure was modelled in 551 finite elements and has a total of 600 nodes.

4.3.1 Displacement Transmissibility

In this section one analyses the applicability of the displacement transmissibility for the identification of external forces applied upon a plate, following the methodology described in section 3.1.2. For that purpose, an external harmonic force of $100N$ is applied at the node 225. The corrected accumulated error calculated for combinations of one and two external forces is presented in figure 9. Analysing figure 9 one can conclude that the force location is successfully determined, since the combination that generates a global minimum error includes only the transverse displacement of node 225. The reconstruction of the force applying equation (27) yields the exact amplitude of the external force.

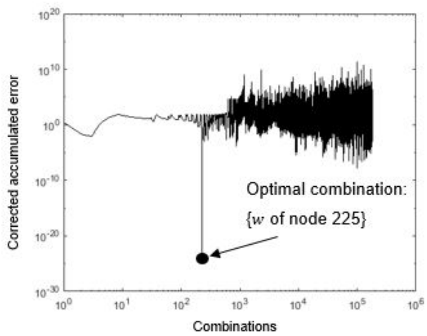

Figure 9: Corrected accumulated error for combinations up to two DOF's

4.4. Addition of a Discrete Mass on a Plate

Throughout this section on studies the applicability of the concept of displacement transmissibility to identify a discrete mass added to a plate, following the methodology presented in section 3.2.1. The plate considered in this section has the physical and dimensional properties of the plate considered on the previous section. In the case of the present section one applied a harmonic force of $100N$ at the node 225 and a mass of $100g$ is added at the node 460. Therefore, the optimal combination must include the transverse displacements of nodes 225 and 460. As shown in figure 10, the optimal combination is correctly identified. The quantification of the amount of mass added is done applying equation (28), presented in section 3.2.1, and yields an exact result.

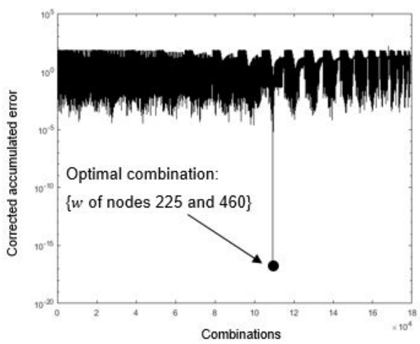

Figure 10: Corrected accumulated error for combinations up to two DOF's

5. Conclusion

Throughout chapter 4 one demonstrated the numerical applicability of the force and structural damage identification method using transmissibility. Despite the successful results obtained, it is necessary to take into account that, typically, an inverse problem is quite sensible to the sensor's noise, which can lead to deviations regarding the results obtained in laboratory. The numerical results obtained in sections 4.3 and 4.4 made clear one of the limitations of the method presented. For a structure modelled in a considerable number of finite elements, the total number of combinations taken into account can be quite high. Therefore, the implementation of the method can lead to a significant simulation time, inhibiting the usage of the method in real time. However, there are several possibilities to overcome this limitation. For instance, one can use parallel computing, where the calculation of the accumulated errors is subdivided into several computers or memories. Another possibility takes into consideration the physics of the problem. If one can predict the structure areas more prone to damage, the possible locations of the external forces can be limited to the vicinity of the critical areas of the structure. One of the main advantages of the inverse method proposed is that the reference data of the structure is defined for its original state. Consequently, once the matrix that characterize the dynamic behaviour of the structure does not change, all the transmissibility matrices required during the application of the method can be calculated and saved in database , prior to the simulation. The experimental application of the structural damage identification method is suggested as future work, since an experimental validation of this method is extremely important to its corroboration and improvement.

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