On the Propagation and Attenuation of Sound in a Two-phase Dissipative Medium

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Abstract

This work addresses a new theory that concerns the reduction of vibrations of the launch vehicles and satellites during lift-off. The thermal conductivity, mass diffusion and cross-coupling coefficients are extrapolated for a two-phase flow. The obtained results show that the majority of the attenuation effect is induced by the thermal damping; the smallest impact is due to the viscous damping, that when compared to the remaining terms is negligible; and the mass diffusion damping that causes an amplification, reducing the dominant effect of thermal attenuation.

Keywords: Propagation of sound, Attenuation of sound, Dissipative Medium, Two-phase flow, Phenomenological coefficients

1. Introduction

One of the most intense man made sources of noise, is the launch phase of a large rocket. A large rocket motor or cluster can produce noise levels of 150-170 dB affecting the first stage rocket structures. Also of concern are the noise levels of 130-140 dB in the payload shroud at the nose of the rocket that houses the satellite(s). Thus satellite payloads have to be certified against acoustic fatigue at in-flight noise levels. One tried and tested means of reducing lift-off noise levels is to use a water spray: the vaporization of water reduces the high exhaust gas temperature and also absorbs sound. The design of water spray systems for use at rocket launch pads is a costly empirical process of trial and error due to the lack of a suitable theoretical predictive framework. Hence, the main goal of this work is to test the new theory presented here calculating the total acoustic damping based on the thermal conductivity, mass diffusion and cross-coupling coefficients; while establishment the starting point as the six fundamental equations of fluid mechanics where the essential effects of mass and heat fluxes are considered while comparatively less important are omitted, including viscosity, vaporization and chemical reactions.

2. Theoretical Formulation

The equation of continuity states the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \tag{1}$$

where ρ is the mass density and $\vec{v}$ the velocity. The mass diffusion equation:

$$\frac{\partial \xi}{\partial t} + \nabla \cdot (\xi \vec{v}) = -\nabla \cdot \vec{j}, \tag{2}$$

with concentration ξ and diffusive mass flux $\vec{j}$. The heat equation:

$$\rho T \left(\frac{\partial s}{\partial t} + \vec{v} \cdot \nabla s \right) = -\nabla \cdot \vec{q}, \tag{3}$$

involves the temperature T, the entropy s and the heat flux $\vec{q}$. The effect of viscosity was omitted in the heat equation (3), and hence the inviscid momentum equation is used.

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\rho^{-1} \nabla p, \tag{4}$$

where p is the pressure. The medium is assumed to be a perfect gas with equation of state:

$$p = \rho R T, \tag{5}$$

where R is the gas constant. The entropy for an ideal gas, with constant specific heats at constant volume C_V and pressure C_p is given by:

$$s = s_0 + C_V \log p - C_p \log \rho. \tag{6}$$

To close the system of eqs. (1) to (6) the heat $\vec{q}$ and mass $\vec{j}$ fluxes are specified in terms of the gradients of temperature ∇T and concentration $\nabla \xi$ by:

$$\vec{q} = -\chi \nabla T - \beta \nabla \xi, \quad \vec{j} = -D \nabla \xi - \alpha \nabla T, \tag{7a,b}$$

where χ is the thermal conductivity, D the mass diffusion coefficient and α, β the diffusive cross-coupling coefficients.

Substitution of (7a) and (7b) respectively in (3)

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and (2) lead to the heat (8) and mass (9) diffusion equations.

$$\rho T \left(\frac{\partial s}{\partial t} + \vec{v} \cdot \nabla s \right) = \chi \nabla^2 T + \beta \nabla^2 \xi, \qquad (8)$$

$$\frac{\partial \xi}{\partial t} + \nabla \cdot (\xi \vec{v}) = D \nabla^2 \xi + \alpha \nabla^2 T. \qquad (9)$$

For the purpose of elimination among the fundamental equations (1), (4), (6), (8) and (9) it is convenient to put the entropy equation (6) in a differential form through the use of the material derivative and solving it for the pressure leads to eq. (10):

$$\frac{\mathrm{D}p}{\mathrm{D}t} = c^2 \frac{\mathrm{D}\rho}{\mathrm{D}t} + \theta \frac{\mathrm{D}s}{\mathrm{D}t} \qquad (10)$$

where it appears the adiabatic sound speed c^2 and the non-adiabatic coefficient θ. The total state of the fluid is assumed to consist of an uniform mean state with subscript "0" and an unsteady non-uniform perturbation with superscript " ' ".

The linearisation of the material derivative for the mean flow leads to (11a), and the linearisation of the fundamental equations (1), (4), (5) and (8) to (10) leads respectively to eqs. (11b)–(16):

$$\frac{\mathrm{d}}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla : \frac{\mathrm{d}\rho'}{\mathrm{d}t} + \rho_0 (\nabla \cdot \vec{v}') = 0, \quad (11a,b)$$

$$\rho_0 \frac{\mathrm{d}\vec{v}'}{\mathrm{d}t} + \nabla p' = 0, \qquad (12)$$

$$p' = R (\rho_0 T' + T_0 \rho'), \qquad (13)$$

$$\frac{\mathrm{d}p'}{\mathrm{d}t} = c_0^2 \frac{\mathrm{d}\rho'}{\mathrm{d}t} + \theta_0 \frac{\mathrm{d}s'}{\mathrm{d}t}, \qquad (14)$$

$$\rho_0 T_0 \frac{\mathrm{d}s'}{\mathrm{d}t} = \chi \nabla^2 T' + \beta \nabla^2 \xi', \qquad (15)$$

$$\frac{\mathrm{d}\xi'}{\mathrm{d}t} + \xi_0 (\nabla \cdot \vec{v}') = D \nabla^2 \xi' + \alpha \nabla^2 T'. \qquad (16)$$

Applying the linearised material derivative (11a) to (11b) and (16) leads respectively to (17) and (18) after elimination of the velocity perturbation from the momentum equation (12):

$$\frac{\mathrm{d}^2 \rho'}{\mathrm{d}t^2} = \nabla^2 p', \qquad (17)$$

$$\frac{\mathrm{d}^2 \xi'}{\mathrm{d}t^2} - \frac{\xi_0}{\rho_0} \nabla^2 p'$$
$$= D \nabla^2 \left(\frac{\mathrm{d}\xi'}{\mathrm{d}t} \right) + \alpha \nabla^2 \left(\frac{\mathrm{d}T'}{\mathrm{d}t} \right). \quad (18)$$

The entropy perturbation s' appears only in (15), and is eliminated using (14):

$$\chi \nabla^2 T' + \beta \nabla^2 \xi'$$
$$= \frac{C_V}{R} \left(\frac{\mathrm{d}p'}{\mathrm{d}t} - \frac{C_p}{C_V} R T_0 \frac{\mathrm{d}\rho'}{\mathrm{d}t} \right). \quad (19)$$

Applying $\frac{\mathrm{d}}{\mathrm{d}t}$ to eq. (19) leads to:

$$(\gamma - 1) \left[\chi \nabla^2 \left(\frac{\mathrm{d}T'}{\mathrm{d}t} \right) + \beta \nabla^2 \left(\frac{\mathrm{d}\xi'}{\mathrm{d}t} \right) \right]$$
$$= \frac{\mathrm{d}^2 p'}{\mathrm{d}t^2} - c_0^2 \nabla^2 p'. \quad (20)$$

The r.h.s of (20) is the classical wave equation involving the adiabatic sound speed.

The mass density perturbation ρ' appears only in eq. (13) and is eliminated applying $\frac{\mathrm{d}^2}{\mathrm{d}t^2}$ leading to:

$$R \rho_0 \frac{\mathrm{d}^2 T'}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 p'}{\mathrm{d}t^2} - c_1^2 \nabla^2 p'; \qquad (21)$$

the r.h.s of eq. (21) is the classical wave equation involving the isothermal sound speed.

The temperature perturbation T' appears in eq. (18) and after application of $\frac{\mathrm{d}}{\mathrm{d}t}$ is eliminated using eq. (21).

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} \left(\frac{\mathrm{d}\xi'}{\mathrm{d}t} - D \nabla^2 \xi' \right)$$
$$= \nabla^2 \left(\frac{\xi_0}{\rho_0} \frac{\mathrm{d}p'}{\mathrm{d}t} + \frac{\alpha}{\rho_0 R} \frac{\mathrm{d}^2 p'}{\mathrm{d}t^2} - \frac{\alpha T_0}{\rho_0} \nabla^2 p' \right). \quad (22)$$

The term in curved brackets on the l.h.s of eq. (22) is the mass diffusion equation that holds at constant pressure, when the r.h.s vanishes.

Solving eq. (20) for ξ' and applying $\frac{\mathrm{d}}{\mathrm{d}t}$ leads to:

$$\beta \nabla^2 \left(\frac{\mathrm{d}^2 \xi'}{\mathrm{d}t^2} \right) - \frac{1}{\gamma - 1} \left[\frac{\mathrm{d}^3 p'}{\mathrm{d}t^3} - c_0^2 \nabla^2 \left(\frac{\mathrm{d}p'}{\mathrm{d}t} \right) \right]$$
$$= -\frac{\chi}{\rho_0 R} \nabla^2 \left(\frac{\mathrm{d}^2 p'}{\mathrm{d}t^2} \right) + \chi \frac{T_0}{\rho_0} \nabla^4 p'. \quad (23)$$

Applying $\beta \nabla^2$ to eq. (22) and substituting eq. (23) leads to the acoustic wave operator with double, thermal and mass, diffusion:

$$\Box^2 = \frac{1}{\gamma - 1} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - c_0^2 \nabla^2 \right)$$
$$- \left(\frac{D}{\gamma - 1} + \frac{\chi}{\rho_0 R} \right) \nabla^2 \frac{\mathrm{d}^3}{\mathrm{d}t^3}$$
$$+ \left(\frac{c_0^2 D}{\gamma - 1} + \chi \frac{T_0}{\rho_0} - \beta \frac{\xi_0}{\rho_0} \right) \nabla^4 \frac{\mathrm{d}}{\mathrm{d}t}$$
$$+ \frac{\chi D - \alpha \beta}{\rho_0 R} \nabla^4 \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \frac{c_0^2}{\gamma} \nabla^2 \right), \qquad (24)$$

it applies to all scalar wave variables and to the divergence of velocity. In the absence of diffusivities eq. (24) reduces to the adiabatic wave equation in the first brackets; the last brackets in eq. (24) is the isothermal wave equation multiplied by the determinant of the diffusion coefficients in (7a,b).

The solution of the wave equation (24) is sought in the form of plane waves with frequency ω and wave vector $\vec{k}$. The spatial dependence in the wave equation (24) appears only through Laplacians, so only the modulus of the wave vector appears, the

dependence on the wavenumber implies isotropic waves, since there is no preferred direction. The isotropy is in a frame convected with the mean flow velocity $\vec{v}_0$, since the linearised material derivative (11a)≡(25a) leads to the Doppler shifted frequency (25b):

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla \to -i\overline{\omega},$$

$$\overline{\omega} = \omega - \vec{k} \cdot \vec{v}_0. \qquad (25\text{a,b})$$

Substituting the plane wave solution in the doubly-diffusive acoustic wave equation (24) and using eq. (25b) leads to the dispersion relation:

$$0 = \frac{\overline{\omega}^4}{\gamma - 1} + i\overline{\omega}^3 k^2 \left(\frac{D}{\gamma - 1} + \frac{\chi}{\rho_0 R} \right)$$

$$-\overline{\omega}^2 k^2 \left(\frac{c_0^2}{\gamma - 1} + k^2 \frac{\chi D - \alpha\beta}{\rho_0 R} \right) \qquad (26)$$

$$-i\overline{\omega} k^4 \left(\frac{c_0^2 D}{\gamma - 1} + \chi \frac{T_0}{\rho_0} - \beta \frac{\xi_0}{\rho_0} \right) + (\chi D - \alpha\beta) \frac{T_0}{\rho_0} k^6$$

The four modes may be expected to be two damped acoustic waves propagating in opposite directions plus two decaying fields, one thermal and one diffusive.

The diffusivities are usually small, and neglecting their determinant simplifies the dispersion relation (26) from a quartic to a cubic expression. In the absence of dissipation the cubic dispersion relation reduces to (27b) for purely acoustic waves (27c).

$$\chi = D = \beta = 0: \quad \overline{\omega}^2 - c_0^2 k^2 = 0,$$

$$\overline{\omega} = \pm c_0 k. \qquad (27\text{a-c})$$

While in the presence of weak dissipation it may be expected that the cubic dispersion relation leads to three modes:

(i/ii) two sound waves propagating in opposite directions (27c) with weak dampings $\varepsilon_{\pm}$ (28a);

(iii) a purely decaying mode with damping δ (28b).

$$\omega - \vec{k} \cdot \vec{v}_0 = \overline{\omega} = \begin{cases} \pm c_0 k - i\varepsilon_{\pm} = \overline{\omega}_{\pm} \\ -i\delta = \overline{\omega}_* \end{cases} \qquad (28\text{a,b})$$

The dispersion relation with roots (28a,b) must be of the form:

$$0 = (\overline{\omega} - c_0 k + i\varepsilon_+)(\overline{\omega} + c_0 k + i\varepsilon_-)(\overline{\omega} + i\delta). \quad (29)$$

Since the product of diffusivities was neglected in the cubic dispersion relation the product of dampings (30a) is also neglected in eq. (29) leading to eq. (30b):

$$\varepsilon_+\varepsilon_-, \varepsilon_+\delta, \varepsilon_-\delta \ll \overline{\omega}^2: \overline{\omega}^3 + i\overline{\omega}^2 (\varepsilon_+ + \varepsilon_- + \delta)$$

$$-\overline{\omega} c_0 k [c_0 k + i(\varepsilon_- - \varepsilon_+)] - i\delta c_0^2 k^2 = 0. \quad (30\text{a,b})$$

The coincidence of (30b) with the cubic dispersion relation proves that: the "guess" about the modes (28a,b) was correct; the coefficients of $\overline{\omega}$ show that the damping is the same for sound waves propagating in opposite directions (31a) as should be expected; the independent term specifies the decay of the non-acoustic mode (31b); the coefficient of $\overline{\omega}^2$ determines the total damping (32a) leading to the damping for the acoustic modes (32b).

$$\varepsilon_+ = \varepsilon_- \equiv \overline{\varepsilon},$$

$$\delta = k^2 D + \frac{\gamma - 1}{\rho_0} \frac{k^2}{c_0^2} (T_0\chi - \xi_0\beta); \qquad (31\text{a,b})$$

$$2\overline{\varepsilon} + \delta = k^2 \left(D + \frac{\gamma - 1}{\rho_0} \frac{\chi}{R} \right)$$

$$\overline{\varepsilon} = \frac{k^2 (\gamma - 1)^2 \chi}{2\rho_0 R\gamma} + \frac{\gamma - 1}{2\rho_0} \frac{k^2}{c_0^2} \xi_0\beta. \qquad (32\text{a,b})$$

The thermal part of the thermoviscous acoustic dissipation coefficient [1] per unit time is (33):

$$\varepsilon_1 = \frac{k^2\chi (\gamma - 1)^2}{2\rho_0 R\gamma} \qquad (33)$$

which is in agreement with the first term on the r.h.s of (32b). The viscous dissipation coefficient for acoustic waves [1] adds to eq. (33) a term involving the shear η and bulk ζ viscosities (34a):

$$\varepsilon_2 = \frac{\overline{\omega}^2}{2\rho_0 c_0^2} \left(\frac{4}{3}\eta + \zeta \right),$$

$$\varepsilon_3 = \frac{\gamma - 1}{2\rho_0 c_0^2} \frac{\overline{\omega}^2}{c_0^2} \xi_0\beta. \qquad (34\text{a,b})$$

Thus the total dissipation coefficient for acoustic waves, including thermal (33), mass diffusion (34b) and viscous diffusion (34a) is.

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \qquad (35)$$

The damping (35) applies to the acoustic modes propagating in opposite directions (28a). The third mode (28b) is a purely decaying mode.

The energy density per unit volume (36a) in a fluid is:

$$E = \rho \left(\frac{v^2}{2} + U \right),$$

$$\mathrm{d}U = T\,\mathrm{d}s - p\,\mathrm{d}v + \mu_1\,\mathrm{d}N_1 + \mu_2\,\mathrm{d}N_2, \quad (36\text{a,b})$$

where (36b) is the internal energy, v the specific volume, (μ_1, μ_2) the chemical potentials and (N_1, N_2) the mole numbers of the two constituents of the two-phase flow; The two chemical terms last on the r.h.s of eq. (36b) can be rewritten (37):

$$\mu_1\,\mathrm{d}N_1 + \mu_2\,\mathrm{d}N_2 = \mu\,\mathrm{d}\xi, \qquad (37)$$

where μ is the relative chemical potential. Being eq. (38) the equation of energy,

$$\frac{\mathrm{D}E}{\mathrm{D}t} = -\rho \left(\frac{v^2}{2} + U \right)(\nabla \cdot \vec{v})$$

$$- \vec{v} \cdot \nabla p + \rho T \frac{\mathrm{D}s}{\mathrm{D}t} + \frac{p}{\rho} \frac{\mathrm{D}\rho}{\mathrm{D}t} + \mu \frac{\mathrm{D}\xi}{\mathrm{D}t}. \quad (38)$$

The diffusive terms involving the concentration (2) and entropy (4) are separated from the rest in:

$$-\nabla \cdot \vec{q} - \mu \nabla \cdot \vec{j} = \frac{\partial E}{\partial t} + \nabla \cdot [(E + p)\vec{v}]. \quad (39)$$

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The energy equation (39) thus involves eq. (40a), the energy density (36a) and the convective energy flux $\vec{F}$:

$$\frac{\partial E}{\partial t} + \nabla \cdot \vec{F} = -\nabla \cdot \vec{Q},$$

$$\nabla \cdot \vec{Q} = \nabla \cdot \vec{q} + \mu \nabla \cdot \vec{j}, \qquad (40a,b)$$

and the diffusive energy flux is given by (40b). Substitution of eq. (40b) in eq. (3) leads to the equation of entropy.

$$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = -\nabla \cdot \left(\vec{Q} - \mu\vec{j}\right) - \vec{j} \cdot \nabla \mu. \qquad (41)$$

The second principle of thermodynamics leads to eq. (42):

$$\frac{\partial}{\partial t}\left(\rho s\right) = \rho \frac{\mathrm{D}s}{\mathrm{D}t} - \nabla \cdot \left(\rho s \vec{v}\right). \qquad (42)$$

The second term on the r.h.s of eq. (42) is the divergence of the convective entropy flux, and hence unrelated to diffusion leading to eq. (43b):

$$\frac{\partial}{\partial t}\left(\rho s\right) = \rho \frac{\mathrm{D}s}{\mathrm{D}t}$$

$$= -\frac{1}{T}\left[\nabla \cdot \left(\vec{Q} - \mu\vec{j}\right) + \vec{j}\cdot\nabla\mu\right]. \qquad (43a,b)$$

The first term on the r.h.s of (43b) may be rewritten,

$$-\frac{\nabla \cdot \left(\vec{Q} - \mu\vec{j}\right)}{T}$$

$$= -\nabla \cdot \left(\frac{\vec{Q} - \mu\vec{j}}{T}\right) + \left(\vec{Q} - \mu\vec{j}\right) \cdot \nabla\left(\frac{1}{T}\right), \quad (44)$$

only the second term of the r.h.s of eq. (44) contributes to the local entropy production:

$$0 < \dot{S} = -\frac{1}{T^2}\left(\vec{Q} - \mu\vec{j}\right) \cdot \nabla T - \frac{\vec{j}\cdot\nabla\mu}{T}. \quad (45a,b)$$

The entropy production per unit time is a linear function of the fluxes $\dot{x}_n$ whose coefficients are gradients. For small gradients the fluxes are linear functions with kinetic coefficients (46a) that appear also in the entropy production (46b):

$$\dot{x}_n = \sum_l \Gamma_{nl} X_l, \quad \dot{S} = \sum_{n,l} \Gamma_{nl} X_n X_l > 0. \quad (46a,b)$$

Since the entropy production (46b) is a quadratic form, the kinetic coefficients may be taken as symmetric (47a); also since it must be positive-definite, in the case of a 2×2 matrix the Cayley-Hamilton conditions (47b,c) must be satisfied [2]:

$$\Gamma_{nl} = \Gamma_{ln}; \quad \Gamma_{11} > 0,$$

$$\Gamma_{22} > \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{11}} = \frac{\left(\Gamma_{12}\right)^2}{\Gamma_{11}} > 0. \qquad (47a\text{-}c)$$

In the present case (45b) the fluxes are eq. (48a) and hence the gradients eq. (48b):

$$\dot{x}_n = \left\{\vec{Q} - \mu\vec{j}, \vec{j}\right\}, X_n = \left\{-\frac{\nabla T}{T^2}, -\frac{\nabla\mu}{T}\right\}. \quad (48a,b)$$

Thus the diffusion relations (46a) are:

$$\vec{Q} - \mu\vec{j} = -\Gamma_{11}\frac{\nabla T}{T^2} - \Gamma_{12}\frac{\nabla\mu}{T},$$

$$\vec{j} = -\Gamma_{12}\frac{\nabla T}{T^2} - \Gamma_{22}\frac{\nabla\mu}{T}, \qquad (49a,b)$$

where the kinetic coefficients satisfy eq. (47b,c). Solving eq. (49b) for the gradient of the chemical potential $\nabla\mu$ and substituting in eq. (49a):

$$\vec{Q} = -\left[\Gamma_{11} - \frac{\left(\Gamma_{12}\right)^2}{\Gamma_{22}}\right]\frac{\nabla T}{T^2} + \left(\mu + \frac{\Gamma_{12}}{\Gamma_{22}}\right)\vec{j}. \quad (50)$$

In the absence of mass flux (51a) the energy flux coincides with the heat flux (51b) where the thermal conductivity (51c) is positive by (47c),

$$\vec{j} = 0: \quad \vec{Q} = -\overline{\chi}\,\nabla T,$$

$$\overline{\chi} = \frac{1}{T^2}\left[\Gamma_{11} - \frac{\left(\Gamma_{12}\right)^2}{\Gamma_{22}}\right] > 0. \qquad (51a\text{-}c)$$

The energy flux in the presence of mass flux is given by:

$$\vec{Q} = -\overline{\chi}\,\nabla T + \left(\mu + \frac{\Gamma_{12}}{\Gamma_{22}}\right)\vec{j}. \qquad (52)$$

The chemical potential is the derivative of the free enthalpy with regard to the concentration:

$$\mu = \left(\frac{\partial G}{\partial \xi}\right)_{T,p} : \nabla\mu = \left(\frac{\partial\mu}{\partial T}\right)_{p,\xi}\nabla T$$

$$+ \left(\frac{\partial\mu}{\partial p}\right)_{T,\xi}\nabla p + \left(\frac{\partial\mu}{\partial\xi}\right)_{T,p}\nabla\xi, \qquad (53a,b)$$

Substituting eq. (53a,b) specifies the mass (49b) and energy (52) fluxes respectively as:

$$-\vec{j} = D\,\nabla\xi + \alpha\,\nabla T + \varphi\,\nabla p,$$

$$-\vec{Q} = \overline{\overline{\chi}}\,\nabla T + \overline{\beta}\,\nabla\xi + \psi\,\nabla p, \qquad (54a,b)$$

where:

(i) the barodiffusion coefficients φ and ψ can be ommited, because they must be zero;

(ii) the mass flux (49b) is thus given by (55a):

$$\vec{j} = -D\,\nabla\xi - \alpha\,\nabla T: \quad D \equiv \frac{\Gamma_{22}}{T}\left(\frac{\partial\mu}{\partial\xi}\right)_T,$$

$$\alpha \equiv \frac{\Gamma_{12}}{T^2} + \frac{\Gamma_{22}}{T}\left(\frac{\partial\mu}{\partial T}\right)_\xi; \qquad (55a\text{-}c)$$

(iii) using (55a) the energy flux (52) is given by (56a):

$$\vec{Q} = -\overline{\overline{\chi}}\,\nabla T - \overline{\beta}\,\nabla\xi: \quad \overline{\overline{\chi}} = \overline{\chi} + \alpha\left(\mu + \frac{\Gamma_{12}}{\Gamma_{22}}\right),$$

$$\overline{\beta} = \left(\mu + \frac{\Gamma_{12}}{\Gamma_{22}}\right)D; \qquad (56a\text{-}c)$$

(iv) rewriting eq. (40b) leads to (57b):

$$\nabla \cdot \vec{Q} = \nabla \cdot \left(\vec{q} + \mu\vec{j}\right) - \vec{j} \cdot \nabla\mu:$$

$$\vec{Q} = \vec{q} + \mu\vec{j}, \qquad (57a,b)$$

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and substitution of eqs. (55a) and (56a) specifies the heat flux (58a):

$$\vec{q} = -\chi \, \nabla T - \beta \, \nabla \xi : \quad \chi = \overline{\chi} + \alpha \, \frac{\Gamma_{12}}{\Gamma_{22}},$$

$$\beta = D \, \frac{\Gamma_{12}}{\Gamma_{22}}. \qquad \text{(58a-c)}$$

The diffusion relations (7a,b) are valid with coefficients from eqs. (55b,c) and (58b,c) whereas the damping of the decaying mode (31b) and the damping of the acoustic modes (35) are specified in terms of the thermodynamic properties and diffusion coefficients of the binary mixture.

3. Implementation

This section is divided into two parts: the theoretical implementation, where the mathematical relations to obtain the total dissipation coefficient for acoustic waves are calculated; and the computational implementation with the description of a tool created in Matlab which applied the relations leading to the results.

3.1. Theoretical Implementation

The coefficients from eqs. (55b,c) and (58b,c) that appear in the damping of the acoustic modes, depend on the relative chemical potential and kinetic coefficients. Concerning, specifically the viscous dissipation coefficient (34a) it is necessary to perform a mathematical extrapolation for the shear and bulk viscosities.

3.2. Chemical Potential

The chemical potential (??b), as seen previously is strictly necessary to perform a first approach of this new theory. The air has no chemical potential due to the fact that its main components, O_2 and N_2, have a chemical potential of value equal to 0, thus leading to a relative chemical potential dependent only of water (59),

$$\mu = \frac{\mu_1}{m_1} = \frac{\mu_{H_2O}}{m_{H_2O}}. \qquad (59)$$

In order to calculated the water chemical potential it will be used a mass action equation (60) [3, 4],

$$\mu_{H_2O}(T, \xi) = \mu_0 + RT \ln\left(\frac{\xi}{\xi_0}\right) \text{ [kJ/mol]}, \quad (60)$$

determining the difference between a specified state and a reference state calculated under standard conditions, referenced by the subscript 0. It is noteworthy that eq. (60) is only valid for values of $\xi \ll \xi_0$. Substituting eq. (60) in eq. (59) it is obtained the relative chemical potential equation (61).

$$\mu = \left[\mu_0 + RT \ln\left(\frac{\xi}{\xi_0}\right)\right] \frac{1}{m_{H_2O}} \text{ [kJ/kg]}. \quad (61)$$

The chemical potential of water presents a negative value, as a matter of fact, most of the chemical potential values are negative. This represents the stability of a substance, being negative means it will not decompose into their elements, instead it will spontaneously be produced from them [3]. The chemical potential of a substance decreases with increasing temperature and increases with increasing concentration [3]. This means that in warmer environments the substance has less tendency to transform, whereas the more concentrated a substance is, the more is the tendency to decompose into their elements. It is important to note that for higher concentrations the experimental values depart quite noticeable from eq. (61), hence it is recommended to maintain the concentration values between the range of validity, $\xi \leq 100$ mol/m^3 [3].

The mass diffusion coefficient (55b) and the thermal cross-coupling (55c) rely on the derivative of the relative chemical potential presented in eq. (62a,b).

$$\frac{\partial \mu}{\partial \xi} = \frac{RT}{\xi} \frac{1}{m_{H_2O}},$$

$$\frac{\partial \mu}{\partial T} = R \ln\left(\frac{\xi}{\xi_0}\right) \frac{1}{m_{H_2O}}. \qquad \text{(62a,b)}$$

3.3. Kinetic Coefficients

This work involves a thermodynamic system where both heat conduction and mass diffusion coexist and when two or more irreversible transport processes occur simultaneously they may interfere with each other. The phenomenological relations (49a,b) derived by Onsager [2, 5] consist of appropriate flux-force pairs (48a,b) where the kinetic coefficients Γ_{nl} act as a link between them (46a). In an attempt to simplify the visualization of the following equations the subscripts q and j are used, corresponding to heat and mass and the relation between nomenclatures is expressed in eq. (63).

$$\begin{bmatrix} \Gamma_{qq} & \Gamma_{qj} \\ \Gamma_{jq} & \Gamma_{jj} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \qquad (63)$$

What matters the most when analysing the phenomenological coefficients is the difference between variable values and not actually the gradient. Therefore to simplify the calculations it is assumed that the reference position is the beginning of the water jet, referenced as 0 meters, $x_1 = 0$, and the second position is the peak of the jet, considered in this case to have 1 meter, $x_2 = 1$, leading to $\Delta x = 1$ and thus, $\nabla T = \Delta T$ for example. Since the heat conduction and mass diffusion processes, interfere with each other it is necessary to use phenomenological relations (64a,b) [2],

$$\dot{x}_q = -\Gamma_{qq} \frac{\nabla T}{T^2} - \Gamma_{qj} \frac{\nabla \mu}{T},$$

$$\dot{x}_j = -\Gamma_{jq} \frac{\nabla T}{T^2} - \Gamma_{jj} \frac{\nabla \mu}{T}. \qquad \text{(64a,b)}$$

where Γ_{qq} and Γ_{jj} are due to the heat and mass uncoupled case whereas Γ_{qj} and Γ_{jq} result from the coupling between the heat flux and the chemical

potential difference and the mass flux and temperature difference.

The objective is to obtain the values for the phenomenological coefficients, and literature on this subject is scarce, and even scarcer when the subject involves two-phase flows. With this in mind, the chosen approach to perform the calculations is the direct use of the phenomenological equations, where the heat related flow, $\dot{x}_q$, can not be assumed, but the mass flow, $\dot{x}_j$, is going to be treated as the water flow of a deluge system.

When $\Delta T = 0$, eq. (64b) reduces to [6, 7, 8],

$$\dot{x}_j = -\Gamma_{jj} \frac{\nabla \mu}{T}, \qquad (65)$$

where T is the mean temperature.

Similarly, when $\Delta \mu = 0$, from eq. (64b) [6, 7, 8],

$$\dot{x}_j = -\Gamma_{qj} \frac{\nabla T}{T^2}. \qquad (66)$$

Recalling eq. (50), when there is no mass flux (51a), $\vec{j} = 0$, it is obtained [6, 7, 8],

$$\dot{x}_Q = -\left[\Gamma_{qq} - \frac{(\Gamma_{qj})^2}{\Gamma_{jj}} \right] \frac{\nabla T}{T^2}. \qquad (67)$$

Without mass flux the energy flux coincides with the heat flux that is specified by Fourier's law (51b) [6, 9]. It was stated in the beginning of this section that this method is an approximation since it only takes into account the water phase instead of the two-phase flow, therefore, the conductivity here will be different from χ considered in eq. (51c) which combines both phases.

$$\dot{x}_Q = -\kappa \nabla T, \qquad (68)$$

where, κ is the water thermal conductivity in the spray system. Thus, substituting eq. (68) in eq. (67) leads to an equation for Γqq,

$$\kappa T^2 = \Gamma_{qq} - \frac{(\Gamma_{qj})^2}{\Gamma_{jj}}. \qquad (69)$$

Through the Onsager's reciprocal relations it is finally obtained the following eqs. (70) to (72) for the phenomenological coefficients,

$$\Gamma_{jj} = -\dot{x}_j \frac{T}{\nabla \mu} \left[\frac{\mathrm{kg \cdot s \cdot K}}{\mathrm{m}} \right], \qquad (70)$$

$$\Gamma_{qj} = -\dot{x}_j \frac{T^2}{\nabla T} \left[\frac{\mathrm{kg \cdot K \cdot m}}{\mathrm{s}} \right], \qquad (71)$$

$$\Gamma_{qq} = \kappa T^2 + \frac{(\Gamma_{qj})^2}{\Gamma_{jj}} \left[\frac{\mathrm{kg \cdot m^3 \cdot K}}{\mathrm{s^3}} \right], \qquad (72)$$

Γ_{jj} decreases with increasing temperature, increases with water flow and when subject to concentration variation, it increases with it. The Cayley-Hamilton condition in eq. (47c) is satisfied as it was mandatory. Γ_{qj} presents a negative value due to the fact that the exterior temperature is higher than the one from the system. This parameter decreases, in absolute value, with increasing temperature and

increases with the mass flow. In the presence of temperature variation, Γ_{qq} decreases with increasing temperature and the same happens in regard to concentration. The required condition from eq. (47b) is achieved.

3.4. Water Thermal Conductivity

The water thermal conductivity, κ, is dependent of the temperature and since the phenomenological coefficients are defined using a specific temperature, the mean temperature, the thermal conductivity is also defined for this variable to maintain the consistency in the equations. After researching, various models for water thermal conductivity were analysed and the choice for the most fitted model was based on the error between the values obtained by the models and experimental values available in the literature. Equation (73a,b) from [10] is by far the one with better results.

$$\kappa = A + BT + CT^{1.5} + DT^2 + ET^{0.5} \left[\frac{\mathrm{W}}{\mathrm{m \cdot K}} \right];$$

$$\begin{cases} A = 0.5650285, \\ B = 0.0026363895, \\ C = -0.00012516934, \\ D = -1.5154918 \times 10^{-6}, \\ E = -0.0009412945. \end{cases} \qquad (73a,b)$$

3.5. Shear Viscosity

The shear viscosity, η, is crucial for the evaluation of the viscous dissipation coefficient in eq. (34a), it is thus necessary to obtain a formula for two-phase flow and apply it to an air-water system. Once again, as it was done previously for the thermal conductivity, a research was conducted in order to find the most suited model for this work specific case.

It is true that the assessment of the most trustworthy model should rely on errors formulations, however in this case there is no experimental data directly related to the two-phase viscosity. The two-phase viscosity models can be divided into two groups, the first one is suitable for materials in which the thermal conductivity of the continuous phase is higher than the thermal conductivity of the dispersed phase, meaning that the heat flow would avoid the dispersed phase and the dominant phase would be the liquid [11]. Whereas the second group expresses the opposite, the dominant phase is the gas and the heat flow would involve this phase as much as possible [11]. Having this explanation in mind it is possible to relate the first group of viscosities as the one most suitable for the present flow, more specifically the Cicchitti et al. [12] model presented in eq. (74),

$$\eta_m = x\,\eta_l + (1 - x)\,\eta_g \ [\mathrm{Pa \cdot s}]. \qquad (74)$$

where x is the mass-quality and the subscripts l and

g correspond to liquid and gas, respectively. It is important to note that η_m, satisfies an important limiting condition (75) related to the mass quality. [11].

$$\begin{cases} x = 0, \eta_m = \eta_g \\ x = 1, \eta_m = \eta_l \end{cases} \quad (75)$$

3.5.1. Gas viscosity

With respect to air viscosity the commonly used eq. (76a,b) refers to Sutherland's Law [13, 14] and when compared with other theoretical equations and with known viscosity values it presents the lowest percentage of error in all the temperature intervals studied.

$$\eta_g = \eta_{0g} \left(\frac{T}{T_0} \right)^{3/2} \frac{T_0 + S}{T + S};$$

$$\begin{cases} \eta_{0g} = 1.716 \times 10^{-5} \, [\text{Pa} \cdot \text{s}], \\ T_0 = 273 \text{ K}, \\ S = 110.4 \text{ K}, \end{cases} \quad (76\text{a,b})$$

where η_0 is a known viscosity at a known temperature T_0.

3.5.2. Liquid viscosity

The calculation of water viscosity is not so clear from the literature as that from air, hence some formulas were analysed in order to find the one most suitable for our purpose. Similarly to water viscosity, a comparison between the models and experimental values was made and the percentage of error calculated, the formula that proved to be the most appropriate is expressed in eq. (77a,b), which is derived from experimental values, valid within the 273.15 and 643 K temperature range and is given in centipoise[1] [15].

$$\log_{10} \eta_l = A + \frac{B}{T} + CT + DT^2 \, [\text{cP}],$$

$$\begin{cases} A = -10.2158, \\ B = 1.7925 \times 10^3, \\ C = 1.7730 \times 10^{-2}, \\ D = -1.2631 \times 10^{-5}. \end{cases} \quad (77\text{a,b})$$

While the gas viscosity increases with temperature, the liquid viscosity decreases with increasing temperature in an exponential way.

3.6. Bulk Viscosity

In liquids and gases, molecules have translational, rotational and vibrational degrees of freedom. Whilst the shear viscosity is associated with the translational motion, the bulk viscosity ζ concerns to the relaxation of both rotational and vibrational degrees of freedom [16]. It is defined a bulk viscosity formula only for the water phase due to the scarcity of theoretical models and experimental results for a

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[1]1 cP = 1×10^{-3} Pa·s

two-phase flow.

There are three possible experimental techniques to measure the bulk viscosity, the Brillouin spectroscopy, laser transient grating spectroscopy and acoustic spectroscopy, being this last one the most accurate method [16]. Holmes et al [17] made an experiment using acoustic spectroscopy to study the water bulk viscosity temperature dependence. With the experimental results obtained it was developed a model (78a,b) able to fit the data.

$$\zeta = A \exp\left(-BT\right) \, [\text{Pa} \cdot \text{s}],$$

$$\begin{cases} A = 5.091712 \times 10^{-3}, \\ B = 2.545425 \times 10^{-2}. \end{cases} \quad (78\text{a,b})$$

After examining the results obtained it is possible to observe that the bulk viscosity has a bigger value than the dynamic viscosity. In fact it is approximately three times larger than its shear counterpart throughout the experimental temperature range. While the water takes values on the order of 10^{-3}, the air bulk viscosity is on the order of magnitude of 10^{-5} [18], substantiating the initial assumption of considering the water as the predominant phase for viscosity.

3.7. Computational Implementation

After the theoretical implementation it is necessary to implement the chosen formulas and perform the respective calculations, obtaining the necessary results. To do that with the highest degree of confidence a mathematical tool was developed on the MATLAB software. It is important to emphasize that the code implemented is used only as tool, developed with the intent of comparing the results obtained with the existent data in the literature, calculate the error between them and hence validate the formulas implemented and obtain a graphic representation of the various results and equations. Multiple MATLAB functions were developed, each one responsible for a specific area of calculation.

4. Results

4.1. Acoustic Attenuation

The components of the acoustic damping, (33) and (34a,b), were parametrized by the Doppler shifted frequency, $\overline{\omega}^2$, in a way of easing the reading of the graphical results.

$$\vartheta_1 = \frac{\varepsilon_1}{\overline{\omega}^2} = \frac{(\gamma - 1)^2 \chi}{2p_0 \gamma^2 R} \left[\frac{\text{m}^2 \cdot \text{s} \cdot \text{mol}}{\text{kg}} \right], \quad (79)$$

$$\vartheta_2 = \frac{\varepsilon_2}{\overline{\omega}^2} = \frac{1}{2p_0 \gamma} \left(\zeta + \frac{4}{3}\eta \right) \, [\text{s}], \quad (80)$$

$$\vartheta_3 = \frac{\varepsilon_3}{\overline{\omega}^2} = \frac{\gamma - 1}{2p_0 \gamma^2 R T_0} \xi_0 \beta \left[\frac{\text{m}^2 \cdot \text{s} \cdot \text{mol}}{\text{kg}} \right]. \quad (81)$$

$$\vartheta = \vartheta_1 + \vartheta_2 + \vartheta_3 \, [\text{m}^2 \cdot \text{s} \cdot \text{mol}], \quad (82)$$

4.2. Thermal Damping

The thermal damping is proportional to the thermal conductivity (79), thus the results reproduce the same behaviour, with a different, smaller, order of magnitude mainly due to the division by the exterior pressure. In fig. 1 the thermal attenuation decreases with the increasing of temperature, due to the fact that with increasing temperature the energy of thermal motion loosens the molecules decreasing the attraction forces which induces a decreasing in the conductivity and hence in the attenuation. The variation of the thermal attenuation concerning to concentration (fig. 2) is a portrait of the behaviour of chemical potential, since the thermal conductivity also has components derived from mass diffusion, thus decreasing with increasing concentration.
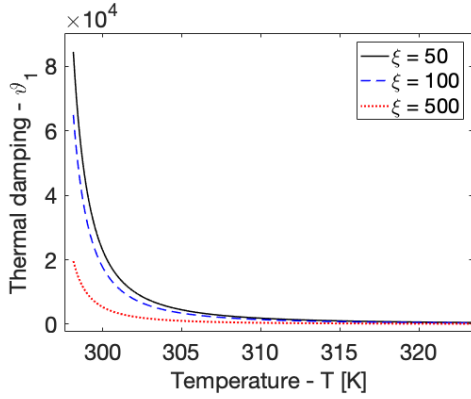
Figure 1: ϑ_1 with temperature variation.
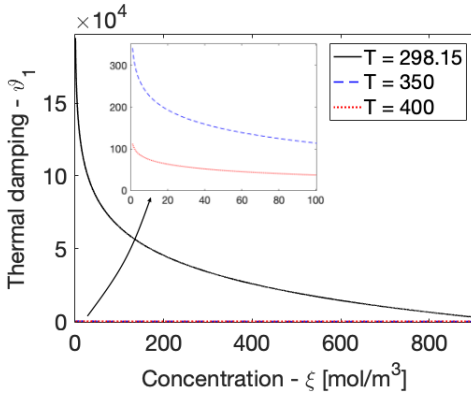
Figure 2: ϑ_1 with concentration variation.

4.3. Viscous Damping

The viscous attenuation (80) depends only on the viscosities and when combining the bulk and shear effects the values obtained, on the order of 10^{-8} (fig. 3) are very small when comparing, for example, with the thermal conductivity (10^{12}) that is the parameter on which the thermal damping depends, hence it is neglected.

There is something to have in mind, while the shear viscosity is calculated for a two-phase flow,

the bulk viscosity was evaluated only for the water component, thus this component is being summed to the entire range of viscosities, meaning it is also being added when it should only exist air. This effect is easily observed if attending to the black filled curve, represented for a mass quality $x = 0$, a pure air phase. While the shear viscosity on gases increases with temperature, this is not observed, instead it reproduces the behaviour of the liquid phase, decreasing with increasing temperature since when there is an increase the mean particles have a greater thermal energy being more easy to overcome the attractive forces that are binding them together, implying a decreasing viscosity and so a decreasing power of attenuation.
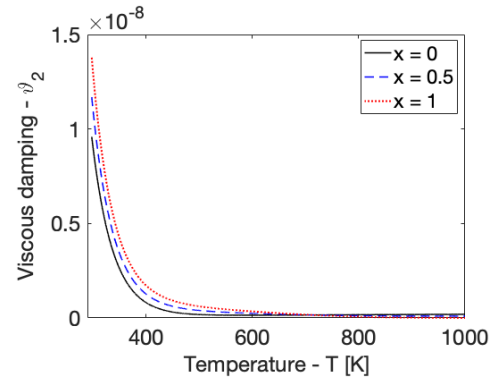

Figure 3: ϑ_2 with temperature variation.

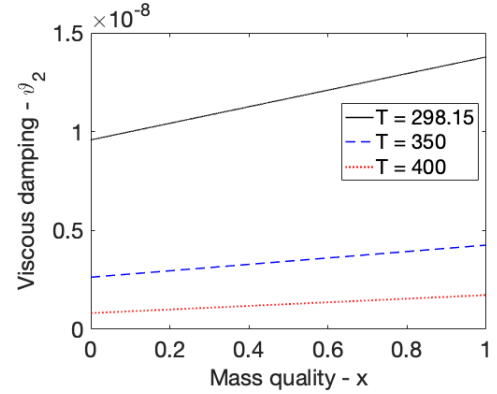Figure 4: ϑ_2 with mass quality variation.

The water presents a higher viscosity value than the air, and so it is expected that when increasing the mass quality, and thus the quantity of water in the system the viscosity also increases leading to an increase of the attenuation (fig. 4).

4.4. Mass Diffusion Damping

The very first result that is important to note in the mass diffusion damping eq. (81) is that it has a negative value, meaning that instead of contributing to the acoustic attenuation it will in fact have an amplification effect, increasing the heat flux. The

second result worth mentioning is that, since it depends directly on the concentration it would be expected to have a variation regarding this parameter, however as can be analysed in fig. 6 this does not happen and the variation in concentration can actually be neglected. This effect is explained by the fact that both terms, ξ and β cancel because of having opposite gradients. The temperature variation effect (fig. 5) reproduces the behaviour of the mass diffusion cross-coefficient, it decreases with increasing temperature in absolute value.
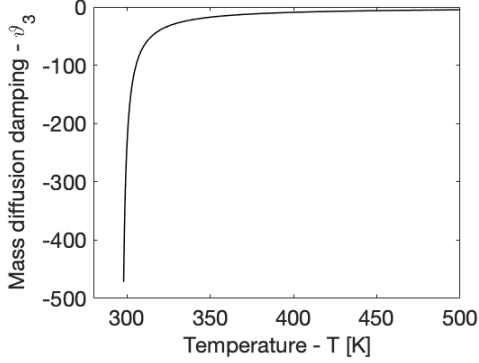
Figure 5: ϑ_3 with temperature variation.
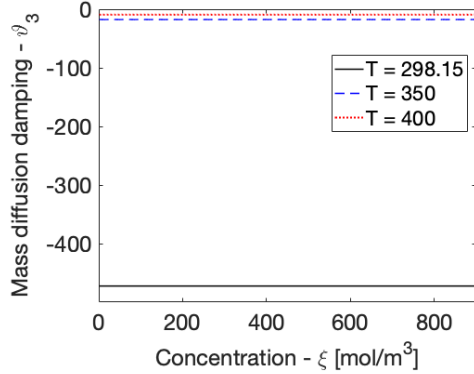
Figure 6: ϑ_3 with concentration variation.

4.5. Total Damping

Te total damping consist of three components: the thermal damping accounts the highest attenuation by a large margin as concerns to the order of magnitude of the three dampings, being expected to dominate the global attenuation. The viscous damping presented the exact opposite, being the smallest attenuation and can be neglected. The mass diffusion damping has the opposite effect to attenuation, meaning that is does not contribute to attenuate but instead to amplification.

When analysing fig. 7 it is clear that the attenuation with temperature variation is a reproduction of the thermal damping, decreasing with increasing temperature due to the thermal conductivity effect, already established before. In fig. 8 it is actually possible to see the reflection of the mass

diffusion amplification effect when attending to the available zoom and comparing it with the one from fig. 2, showing a slightly decrease in value.The mass quality effect (fig. 9) is negligible since the viscous attenuation is the only that depends on this parameter and the magnitude of this damping is infinitesimally small when compared with the thermal damping. The effect of pressure, since it is present in the denominator of the three components it is approximately inversely proportional.
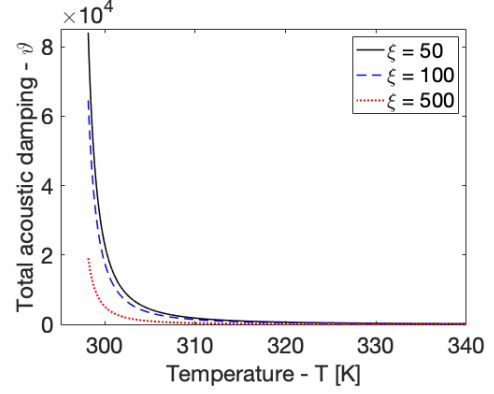

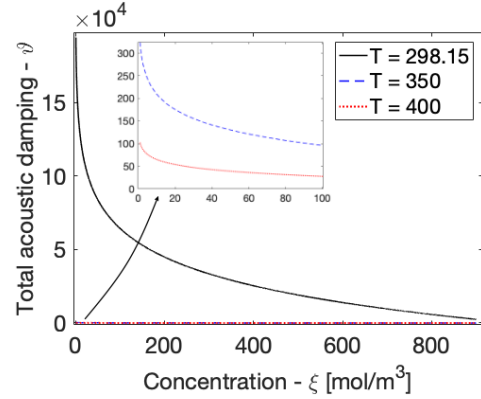
Figure 7: ϑ with temperature variation.

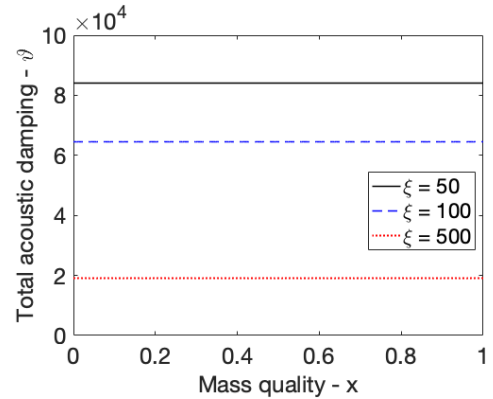Figure 8: ϑ with concentration variation.

Figure 9: ϑ with mass quality variation.

5. Conclusions

Concluding, the acoustic attenuation is dominated by the thermal damping which means the heat conduction irreversible process is the one with the biggest impact, while the mass diffusion damping has the opposite effect increasing the heat flux. In what matters to the viscous part, it was established in this work that the bulk viscosity actually presents a bigger influence in the medium than its shear counterpart. Nonetheless, this effect does not have an influence in the final result due to the very small magnitude when compared with the thermal damping. In a future work both the kinetic coefficients and bulk viscosity must be improved in order to include the effects from the gas phase. The effects from viscosity, vaporization and chemical reactions that are not considered in the six fundamental equations must also be comprised.

Acknowledgements

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