Computational Experiments of a Maintenance Scheduling Problem – CARRIS case study

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In 2018, Rodrigo Arrais Martins developed a Mixed Integer Linear Program model that was implemented in FICO Xpress software, in order to optimize the bus maintenance scheduling of a bus operating company with the goal of reducing its maintenance costs. The results obtained, though improving the company's current schedule, were no great regarding the optimality of the solution and the computational time it took to reach it. The present dissertation searches a way to make improvements in those aspects. A parallel solving multiple model approach based on the Dantzig-Wolfe decomposition was first attempted, resulting in the impossibility to generate results. Then an alteration to the original model by introducing new restrictions, in order to guide the solver to the solution, was implemented. The results regarding computational time showed great enhancement to the original model, but the improvement in terms of optimality was scarce. Lastly, a heuristic approach, in which the problem was solved sequentially for one bus at a time, was developed. This model showed great improvements such in computational time as in optimality. Showing a reduction of 99.7% in computational time and 8.9% in maintenance costs. Both the heuristic approach and the alteration to the original model were validated through an illustrative example.

Keywords: Optimization, Maintenance Scheduling, Computational Experiment, Bus transport, Mixed Integer Linear Programming, Parallel Solving

1. Introduction

1.1. Context

Public Transport has always been seen as a solution for the environmental problems surrounding the cities' increasing pollution and urban congestion. As the Earth's population grows, the urban population grows, at a rate of 70% in four decades according to the United Nations, the urban population along, it is imperative that there are some changes in the way we move within our cities. The preference of the private car as the main mean of transportation poses a big part of the problem, as so, other alternatives should be looked for. The improvement and optimization of the public transports' operations will play a key role in facing the changes caused by the urban concentration growth. Opportunities to use public transport as a lever towards the evolution of the use of energy in mobility are already being explored. The European Commission (EC, 2011) stated that "the objective for the next decade is to create a genuine single European transport area". The focus of this dissertation is bus transportation, whose importance in urban mobility is undisputed. Lisbon as a city faces some challenges, such as a decrease in air quality and an increase in the number of cars in circulation each day. CARRIS bus operating company oversees the bus service in Lisbon and, according to the National Statistics Institute 2017 Inquiry, it is responsible for around 10% of the city's trips. With the purpose of improving its service, CARRIS bus operating company depends on the service provided by its maintenance department. This dissertation focuses exclusively on preventive maintenance.

1.2. Research Objective

The goal of this dissertation is to improve the decision model, able to minimize the cost of maintenance by bus companies, developed by Martins (2018); This is achieved by conducting several computational experiments with various approaches and trying different solving mechanisms, namely by restricting more the problem under analysis. In order to achieve this objective, three different methods were tried: The Dantzig-Wolfe decomposition, a heuristic approach and adding restrictions to the original problem.

1.3. Document Structure

The present dissertation is structured in the following sections. In Section 2, a summary of the most relevant papers to this dissertation is presented. Section 3 gives an insight of the model developed by Martins (2018). In section 4, the experiments made to improve the previous models and its implementation and adaptations are discussed. In Section 5 its results are analysed. Section 6 provides the conclusions, limitations and points out further improvements to the research conducted.

2. Related work – State of the Art

This dissertation follows the research conducted by Rodrigo Arrais Martins in 2018 and represents a continuation of his work. The research conducted in the present document also relies on his state of the art. This section will summarize the contribution of these articles and other relevant ones related to the computational experiments conducted in this dissertation.

2.1 Bus maintenance and scheduling

Haghani and Shafani (2002) focused on finding a way to respond to the problem of scheduling bus maintenance. Based on bus operation schedules, maintenance and inspection needs, their goal is to design the daily supervision for buses that should be inspected, mostly during their idle time, to reduce the number of hours the vehicle is out of service, i.e. reduce unavailability.

Adonyi et al. (2013) developed a solution for the bus maintenance planning problem in public transportation (Adonyi, et al., 2013). In their model, it is ensured that there are enough buses available for the scheduled service and that maintenance and repair tasks can be applied in the bus's downtime during its service day. The model also manages to reduce maintenance costs because buses will only be repaired if required.

Through a real-life crew scheduling problem of public bus transportation, Öztop et al. (2017) studied the ideal number of crewmember drivers to perform a specific set of tasks with minimal cost. The most relevant point of this paper is the presentation of two constraints: i) drivers cannot exceed the maximum limit of total work time and ii) different crew capacities for different types of vehicle.

2.2 Maintenance optimization in transports

Sriram and Haghani (2003) studied how to minimize maintenance costs and how to minimize the cost associated with redistributing aircrafts to flights that were not originally intended. A mathematical formulation is used to solve the aircraft maintenance scheduling problem, as well as a heuristic method since it can obtain feasible solutions in a reasonable computing time. The main point is to analyse the possibility of performing maintenance during flight inactivity.

Bazargan (2015) presented a maintenance optimization at a flight training school. A mixed-integer linear programming (MILP) model was introduced to uncover a strategy that minimizes total maintenance cost during the planning period and increases aircraft availability. A plan with a smaller number of maintenance activities was tested, which, despite having a higher associated cost, obtained better availability indicators, and thus becoming the chosen solution.

Pour et al. (2017) proposed a hybrid framework that uses feasible solutions generated by Constraint Programming, and then uses a mixed-integer programming approach to optimize those solutions. The objective function guarantees the minimization of the number of business days to complete the plan, all tasks are completed within the planning horizon and the minimization of the penalty associated with assigning workers a task on non-consecutive days.

Martins (2018), developed a MILP model that tried to optimize the maintenance costs of a single Lisbon depot from a bus operating company. The model featured restrictions related to the crew availability, bus availability and maintenance line availability. The model also focused in bus availability as a major decision factor. Finally, it provided a bus maintenance schedule that was able to outperform the system already used by that company. The results of this work were used as a comparison basis for the present document, and the model itself was the object of the study here conducted.

2.3 Computational Models

Colombani & Heipcke (2011), describe several examples of sequential and parallel solving of multiple models with FICO Xpress software and Mosel language. The examples showcase concurrent execution of several instances of a model, the (sequential) embedding of a sub-model into a master, and the implementation of decomposition algorithms (Dantzig-Wolfe and Benders decomposition). This article was studied to identify possible approaches to improve the model developed by Martins (2018).

3. The Martins' Model

In this section the model developed by Martins (2018) is described.

3.1 Indexes

b	bus
c	competence
d	day
m	maintenance type
t	time period
v	vehicle type

w worker

3.2 Sets

B set of buses

C set of competences

D set of days

M set of maintenance types

T set of time periods

V set of vehicle types

W set of workers

TO_b set of time periods in which maintenance activities cannot occur for bus b (e.g. time periods for operation)

TM_b set of time periods in which maintenance activities can occur for bus b (e.g. time periods for maintenance)

3.3 Parameters

cc_w competence of worker w

cD_w daily cost of worker type w

g_{bmvc} amount of work that bus b needs to perform maintenance type m, in vehicle type v, with competence c

nw_c number of workers with competence c

v_b vehicle type of bus b

cU bus unavailability cost

nd number of days

nml number of maintenance lines

nt number of time periods

nw number of workers

nsl number of special lines (exclusive to articulated buses)

ntd number of time periods in a day

L large number

3.4 Decision variables

$$x_{bmtw} = \begin{cases} 1 & \text{if the maintenance type m is performed on bus b at time t by the worker w} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{wd} = \begin{cases} 1 & \text{if worker w is assigned at day d} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{bt} = \begin{cases} 1 & \text{if bus unit b in under maintenance at time unit t} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{bd} = \begin{cases} 1 & \text{if bus b in under maintenance at day d} \\ 0 & \text{otherwise} \end{cases}$$

3.5 Objective function

$$Minimize \sum_{w \in W} \sum_{d \in D} cD_w \cdot y_{wd} + \sum_{b \in B} \sum_{d \in D} cU . z_{bd} \quad (1)$$

Subject to:

$$z_{bt} = 0, \quad \forall\, b \in B, t \in TO_b \quad (2)$$

$$x_{bmtw} = 0, \quad \forall\, b \in B, m \in M, t \in TO_b, w \in W \quad (3)$$

$$\sum_{b \in B} \sum_{m \in M} x_{bmtw} \leq 1, \quad \forall\, t \in T, w \in W \quad (4)$$

$$L * \left[1 + \left(x_{bmtw} - x_{bm(t-1)w}\right)\right] \geq \sum_{t_0 \in TM_b : (t_0 > t)} x_{bmt_0w}, \forall\, b \in B, m \in M, t \in T \setminus \{nt\}, w \in W \quad (5)$$

$$\sum_{b_0 \in B} \sum_{m \in M} \sum_{w \in W:\, CC_w = c} x_{b_0 mtw} \leq nw_c, \qquad \forall\, b \in B, c \in C, t \in TM_b \tag{6}$$

$$\sum_{b_0 \in B} \sum_{m \in M} \sum_{w \in W} x_{b_0 mtw} \leq nw, \qquad \forall\, b \in B, t \in TM_b \tag{7}$$

$$\sum_{t \in TM_b} \sum_{w \in W:\, CC_w = c} x_{bmtw} \geq g_{bmvc}, \qquad \forall\, b \in B, c \in C, m \in M, v \in V \tag{8}$$

$$1 - \left(\sum_{w \in W:(CC_w = 3)} x_{bmtw}\right) + \sum_{w \in W:(CC_w \neq 3)} x_{bmtw} \leq L * \left(1 - \sum_{w \in W:(CC_w = 3)} x_{bmtw}\right)$$

$$\forall\, b \in B, t \in TM_b, m = 3 \tag{9.1}$$

$$1 - \left(\sum_{w \in W:(CC_w = 3)} x_{bmtw}\right) + \sum_{w \in W:(CC_w \neq 3)} x_{bmtw} \leq L * \left(1 - \sum_{w \in W:(CC_w = 3)} x_{bmtw}\right),$$

$$\forall\, b \in B, t \in TM_b, m = 4 \tag{9.2}$$

$$\sum_{m \in M} x_{bmtw} \leq z_{bt}, \quad \forall\, b \in B, t \in TM_b, w \in W \tag{10}$$

$$\sum_{b_0 \in B} z_{b_0 t} \leq nml, \quad \forall\, b \in B, t \in TM_b \tag{11}$$

$$\sum_{b_0 \in B} \sum_{m \in M:(VV_{b_0} = 2)} x_{b_0 mtw} \leq nsl, \quad \forall\, b \in B, t \in TM_b, w \in W \tag{12.1}$$

$$\sum_{b_0 \in B} \sum_{m \in M:(VV_{b_0} \neq 2)} x_{b_0 mtw} \leq nml - nsl, \quad \forall\, b \in B, t \in TM_b, w \in W \tag{12.2}$$

$$x_{bmtw} \leq y_{wd}, \forall\, b \in B, m \in M, w \in W, d \in D, \{t \in TM_b :\ ntd.(d-1) + 1 \leq t \leq ntd.d\} \tag{13}$$

$$z_{bt} \leq zd_{bd}, \qquad \forall\, b \in B, d \in D, \{t \in TM_b :\ ntd.(d-1) + 1 \leq t \leq ntd.d\} \tag{14}$$

$$x_{bmtw} = \{0, 1\} \qquad \forall\, b \in B, m \in M, t \in T, w \in W \tag{15}$$

$$y_{wd} = \{0, 1\} \qquad \forall\, w \in W, d \in D \tag{16}$$

$$zd_{bd} = \{0, 1\} \qquad \forall\, b \in B, d \in D \tag{17}$$

$$z_{bt} = \{0, 1\} \qquad \forall\, b \in B, t \in T \tag{18}$$

The objective function (1) is composed of two components: i) the crew maintenance costs, denoted by O_1 in equation 19; and ii) the buses' unavailability costs, denoted by O_2 in equation 20. These two components are daily costs, and thus, by minimizing the days to perform maintenance activities, the minimization of the objective function is achieved. These two components are explained in detail:

$$O_1 = \sum_{w \epsilon W} \sum_{d \epsilon D} cD_w * y_{wd} \tag{19}$$

The parameter cD_w corresponds to the daily cost of each worker w. However, this value can change accordingly to the type of function involved and the years of experience of the worker. Thus, crew maintenance costs E (Equation 19) can be expressed as the sum of all preventive maintenance costs performed by every worker/employee at every day period until the end of the activities. As mentioned before, y_{wd} is a binary decision variable that indicates whether the worker w is assigned on day d (it is equal to one) or not (it is equal to zero).

$$O_2 = \sum_{b \epsilon B} \sum_{d \epsilon D} cU * z_{bd} \quad (20)$$

The cU corresponds to the cost associated with unavailability of buses and it is also an input, but unlike cD_w, it is assumed to be constant for all buses. Note that assuming that cU is constant simplifies the problem as the importance of each bus may change with its type, capacity, service, demand and route satisfaction. In fact, there are also other factors that may influence the value of cU, such as the loss of revenues, impacts on passenger's perceived satisfaction and reliability, regulatory penalties and even opportunity costs. Therefore, these factors make the quantification of term cU (in Equation 20) challenging. Nevertheless, as stated by the maintenance director of the bus operating company, "we prefer to make vehicles available in viable and safety conditions for daily operations", and thus a high value for parameter cU should be assumed, and it should express the sum of all the unavailability costs per bus unit, for each day out of its regular service. As mentioned before, z_{bd} is a binary decision variable that indicates whether the bus, b, is assigned on day d (it is equal to one), or not (it is equal to zero).

In order to facilitate understanding of the constraints, it was decided to divide them into four groups: i) management constraints; ii) crew and competences/skills constraints; iii) maintenance yard constraints; and iv) general constraints. The division is intended to facilitate understanding, though some constraints could be assigned to two or even three groups.

i. Management constraints:

Constraint (2) ensures that no bus is under maintenance during the regular service/operation time. Constraint (3) indicates that no maintenance activity m, no bus b, and no worker w can be scheduled during the regular service/operation time, i.e. there is no maintenance at any time of regular service/operation. Constraint (4) states that all workers at any given time can only perform a task at a time.

ii. Crew and competences constraints

Constraint (5) ensures that when a bus is under maintenance the same worker performs his/her task in consecutive time units, i.e. maintenance tasks cannot be split. Constraint (6) indicates that, for all maintenance times, the number of assigned workers with a specific skill ($CC_w = c$) must be lower or equal than the number of workers with that skill (nw_c). Constraint (7) bounds the number of workers assigned in order to stay lower or equal to the limit number capacity (nw). Constraint (8) guarantees, for any bus b and maintenance m, that the total maintenance time for a type of worker is at least equal to the amount of scheduled maintenance work (g_{bmvw}) for this type of worker. Constraints (9.1 and 9.2) are identical and specific. These restrictions mean that when a bus is carrying out maintenance of type $m = 3$ (9.1) or type $m = 4$ (9.2), workers of type $w = 3$ must labour alone until they finish, i.e. they must work without the presence of any other type of worker.

iii. Constraints related to the maintenance yard

Constraint (10) states that if any maintenance assignment is made, the bus must remain in the maintenance depot for the time t needed to complete the task. Constraint (11) imposes that, for all maintenance times, the number of buses in the depot is lower or equal to the number of maintenance lines (nml). Constraint (12.1) ensures that the number of maintenance activities assigned to buses of type two ($VV_b = 2$) cannot exceed the number of available maintenance lines (nsl) capable of receiving that type of vehicle. For instance, if $nsl = 1$, it means that there is only one line that can be used by the bus of that type, i.e. there can only be one bus of type two ($VV_b = 2$) in maintenance at a time. Constraint (12.2) limits for all the buses that are not of type two ($VV_b \neq 2$), the number of available lines for maintenance activities as $nml - nsl$, i.e. the difference between the total number of maintenance lines and the number of available maintenance lines capable of receiving buses of type two.

iv. General constraints

Constraint (13) states the relation between x_{bmtw} and y_{wd} decision variables, as there is a conversion from hours to days that must be made, with the purpose of determining the schedules of the workers per day, which are needed in the objective function, namely in component O_1. Constraint (14) states the relation between z_{bt} and zd_{bd} decision variables, as there is a conversion from hours to days that must be made, with the purpose of determining the schedules of buses per day, which are needed in the objective function, namely in component O_2. Finally, constraint (15) states that x_{bmtw} is a binary variable for all bus units, maintenance activities, time units and workers; constraint (16) states that y_{wd} is a binary variable for all workers and days units; constraint (17) states that z_{bd} is a binary variable for all bus and days units and constraint (18) states that z_{bt} is a binary variable for all bus and time units.

4. Computational Experiments

Besides the computational experiments analysed in this document, an experiment with the Dantzig-Wolfe decomposition was also conducted. Due to its unfeasibility it is not analysed.

4.1 Heuristic approach

This heuristic approach is highly based in the Martins' model. The great difference is that instead of solving one large ILP, it solves various smaller problems. However, instead of solving each smaller problem in a parallel way, this heuristic approach solves them sequentially. The heuristic. This section will focus on the computational changes from the Martins' model, the model.

This approach resides on solving the problem for one bus at a time, locking the solution of the previous bus, i.e. the problem is solved for bus 1, the time periods occupied the solution for bus 1 are withdrawn from the solution possibilities of the next bus and so on, until the problem is solved for all the buses.

Major computational changes come from: i) the introduction of a new set that dictates the order in which the buses are solved; ii) the introduction of a loop that annexes the buses orderly to the B set, solves and saves the solution to the problem; and iii) a new constraint that prevents a next bus from overwriting or substituting previous buses' allocations.

i) The new set Gb_{NBus} is defined, with values equal to the bus numbers and where i is the index that defines the order, i.e. Gb_1 is the first bus the problem is solved for. It can be defined manually by the user or decision maker, or by using a criteria for choosing the order. The order criteria defined was based on the amount of work needed for that bus.

ii) Using the "repeat" function of the Mosel language a loop where a new variable called $NBus$ is created with the value of one and increased by one at each iteration. This variable is then used to annex a new bus to the set B, being Gb_{NBus} added to that set. The problem formulated by Martins (Martins 2018) is now a procedure and not the only problem. At the end of the procedure the solution found is added to a new set $Prop\_x_{bmtw}$. This procedure happens within the loop, and it stops when the number of iterations ($NBus$) is equal to the number of buses present in the problem.

iii) Finally, a new constraint was added:

$$Prop_{x_{bmtw}} = x_{bmtw}, \forall\ b \in B \backslash \{Gb_{NBus}\},\ m \in M, t \in T, w \in W \qquad (21)$$

This restriction states that every solution already obtained must keep the same value, and thus preventing the new solutions to be allocated to those time slots.

4.2 Introducing New Restrictions Approach

This approach that was tried is even more similar to the original one than the previous one. It is a small variation of Martins' formulation (2018), in which new restrictions are introduced to reduce the number of variables and nodes of the original problem. These new restrictions are based on characteristics that an optimal solution have.

New restrictions:

$$\sum_{b \in B} \sum_{d \in D} zd_{bd} = NB \qquad (22)$$

$$\sum_{m \in M} \sum_{w \in W} x_{1m10w} \geq 1 \qquad (23)$$

$$\sum_{m \in M} \sum_{w \in W} x_{4m34w} \geq 1 \qquad (24)$$

$$\sum_{m \in M} \sum_{w \in W} x_{12m58w} \geq 1 \qquad (25)$$

$$\sum_{d1 \in D: d1 \leq d} zd_{3d1} \geq zd_{6d}, \quad \forall\, d \in D \tag{26}$$

$$\sum_{d1 \in D: d1 \leq d} zd_{9d1} \geq zd_{10d}, \quad \forall\, d \in D \tag{27}$$

$$\sum_{d1 \in D: d1 \leq d} zd_{5d1} \geq zd_{11d}, \quad \forall\, d \in D \tag{28}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmtw} \geq \sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmt-1w}, \quad \forall\, t \in T: 10 > t \leq 17 \tag{29}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmtw} \geq \sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmt-1w}, \quad \forall\, t \in T: 34 > t \leq 41 \tag{30}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmtw} \geq \sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmt-1w}, \quad \forall\, t \in T: 58 > t \leq 65 \tag{31}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmtw} \geq \sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmt-1w}, \quad \forall\, t \in T: 82 > t \leq 89 \tag{32}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmtw} \geq \sum_{b \in B} \sum_{m \in M} \sum_{w \in W} x_{bmt-1w}, \quad \forall\, t \in T: 106 > t \leq 113 \tag{33}$$

$$\sum_{b \in B} \sum_{m \in M} x_{bmtw} \geq L \times \sum_{b \in B} \sum_{m \in M} x_{bmtw-1}, \quad \forall\, t \in T, w \in W: \; CC_w = 1 \cap w > 1 \tag{34}$$

$$y_{w-1} \leq y_w, \quad \forall\, d \in D, w \in W: \; CC_w = 1 \cap w > 1 \tag{35}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} \mathrm{x_{bmtw}} \geq L \times \sum_{b \in B} \sum_{m \in M} \sum_{\mathrm{w} \in \mathrm{W}} \sum_{t \in T: t \leq 24} \mathrm{x_{bmtw}}, \quad t = 34 \tag{36}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} \mathrm{x_{bmtw}} \geq L \times \sum_{b \in B} \sum_{m \in M} \sum_{\mathrm{w} \in \mathrm{W}} \sum_{t \in T: 25 \leq t \leq 48} \mathrm{x_{bmtw}}, \quad t = 58 \tag{37}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} \mathrm{x_{bmtw}} \geq L \times \sum_{b \in B} \sum_{m \in M} \sum_{\mathrm{w} \in \mathrm{W}} \sum_{t \in T: 49 \leq t \leq 72} \mathrm{x_{bmtw}}, \quad t = 82 \tag{38}$$

$$\sum_{b \in B} \sum_{m \in M} \sum_{w \in W} \mathrm{x_{bmtw}} \geq L \times \sum_{b \in B} \sum_{m \in M} \sum_{\mathrm{w} \in \mathrm{W}} \sum_{t \in T: 73 \leq t \leq 96} \mathrm{x_{bmtw}}, \quad t = 106 \tag{39}$$

$$\sum_{b \in B} zd_{\mathrm{bd}-1} \leq L \times \sum_{b \in B} zd_{\mathrm{bd}}, \quad \forall\, d \in D: d > 1 \tag{40}$$

$$\sum_{w \in W} y_{\mathrm{w}d-1} \leq L \times \sum_{w \in W} y_{\mathrm{w}d}, \quad \forall\, d \in D: d > 1 \tag{41}$$

Restriction (22) states that all the buses are assigned to only one day

Restrictions (23) to (25) make sure that the three buses with the greatest need in terms of amount of work are scheduled in different days (which day is irrelevant) i.e. Bus 1 is scheduled for day 1 which starts at $time\ period$ 10.

Restrictions (26) to (28) define the order in which the buses that are due to the same maintenance activities and are the same type of vehicle, and therefore are interchangeable, are scheduled. The goal is to, instead of letting the model test and decide which goes first, decide for the model, avoiding that step, and in this way, reducing enormously the size of potential feasible solutions to test.

Restrictions (29) to (33), one for each day, define that the model should assign the work starting from the beginning of the day.

In restriction (34) the workers with capacity $CC_w = 1$ are being assign in a predefined order, again saving computational time.

In restrictions (35) a second worker w with competence c is only assigned on day d if the previous worker w with competence c is already assigned on that day d;

In restrictions (36) to (39) there are only activities assigned for a determined day if the previous day already has activities

In restrictions (40) and (41), a bus b $(\ldots)$ or a worker w $(\ldots)$ is only assigned to day d, if it was already assigned for the previous day.

5. Results and discussion

In this section an analysis of the results obtained in both the approaches is executed by comparing them to the ones obtained by Martins (2018). A review of the analysis done by Martins (2018) to his results showed that when the model for run with a cU greater than $250\ monetary\ units$, despite the objective value being higher, the optimization of the problem promised a better allocation of the elements. As in the heuristic approach an update of the cU was made to $260\ monetary\ units$ in the solver, although the result exhibited was adapted to $100\ monetary\ units$, it was deemed relevant to provide an analysis cadent of the same adaptation to the Martins model (2018). Therefore, the results from the model ran for cU equal to 260 and $275\ monetary\ units$, the best looking of the ones Martins (2018) tested, were included in this analysis. Those results were examined with the $100\ monetary\ units$' adaptation.

Firstly, an analysis of the final solution, computational time and optimality gap is conducted, followed by an analysis of the weight of the cost components. After that statistics regarding the bus unavailability, the days in which maintenance activities were assign, and the total working days of each type of worker, are presented. Lastly, the money loss in working hours vs paid hours is evaluated.

Table 1 - Objective value, computational time and optimality gap analysis

Model	Objective Value (cU=100, adapted)	Improv to Original	Comp Time (s)	Improv to Original	Optimality gap	Improv to Original
Martins	2520	-	11003.9	-	21.20%	-
Restrictions	2510	0.40%	1287.5	88,30%	16.14%	5.06%
Martins (cU=260)	2465	2.18%	7373.1	33.00%	12.11%	9.09%
Martins (cU=275)	2295	8.93%	6840.6	37.83%	5.16%	16.04%
Heuristic	2295	8.93%	36.7	99.67%	-	-

From *Table 8* it is observed that both the Heuristic approach and the Martins ($cU = 275$) present the best solution to the problem with a value of $2295\ monetary\ units$. The optimality gap from Martins ($cU = 275$) suggests that this solution is close to optimal, and since the Heuristic approach presents the same value it is fair to assume that this applies to it too. This value represents an improvement of 8.93% from the original model and the optimality gap is reduced by 16.04%.

But where the Heuristic approach comes as an isolated champion in the computational time category, an astonishing reduction of 99.67% representing a value of $36.7\ seconds$ makes this approach the most efficient one. It is important to notice that both the approaches developed in this dissertation presented great reductions in computational time even when compared with adapted Martins models. This was one of the main goals of this research.

It is also relevant that both the cU adaptations of the Martins model (2018) present better solutions than the original one.

Table 2 - Cost component weight analysis

Cost Component	Martins		Restrictions		Martins (cU=260)		Martins (cU=275)		Heuristic	
	Value	% total	Value	% total	Value	% total	Value	% total	Value	% total
O_1	1120	44.4%	1210	48.2%	1085	45.5%	995	43.4%	995	43.4%
O_2	1400	55.6%	1300	51.8%	1300	54.5%	1300	56.6%	1300	56.6%

Regarding the cost components, considering the analysis performed by Martins (2018), an increasing of the O_2 component's weigh should evolve with the increase of the solution's optimality. This is not visible in *Table 9* from the Martins model (2018) to the Restrictions approach, this is due to a leap from $14\ assigned\ buses$ (one is repeated) to 13, this is evident in *Table 10*. Although, from the Restrictions approach forward it is possible to identify the evolution described above. It is also interesting to notice that the O_2 component plays a heavier role in all these models.

Table 3 - Bus availability and worker assignment

Aproach	Number of times the buses are unavailable	Number of Days	Number of work days by competence			
			mec (3)	lub	ele	bw
Martins	14	5	12	5	4	5
Restrictions	13	5	13	5	5	5
Martins (cU=260)	13	5	12	5	5	5
Martins (cU=275)	13	4	11	4	4	4
Heuristic	13	4	11	4	4	4

In *Table 10* it is evidenced the leap addressed above. The information provided allows an understanding of effects of the optimization, by the number of days and the work days by competence it is possible to observe that the best solutions present a more compact scheduling. It is also noticeable that if a new restriction to the restriction model, imposing that the solution only had four days, was added, a better solution could be achieved.

After analysing all these results, it is concluded that the heuristic approach is superior to all the other studied models and is by far superior when compared to the original model, presenting great improvements in relation to it, especially regarding computational time.

6. Conclusions and Further Research

6.1 Conclusions

The main objective of the present dissertation was to optimize in terms of computational time, optimality gap and final solution, the model created in 2018 by Martins, on the bus maintenance scheduling and applied to the Carris case study. As stated in before, previous results in Martins (2018), though satisfying and ground breaking, still exhibited a large margin for improvement. The reported optimality gap of 20.15% after a computational time of $13\ hours$ is far from being ideal, and the present research work had the challenge to try to improve the computational time, optimality gap and final solution.

The initial idea was to work with parallel solving mechanisms in order to save computational time. The implementation of this model was a long process and was not possible to validate it for neither the illustrative example nor the real case. After this conclusion, two different approaches were proposed. The first one used a heuristic approach and restructured the Martins' model (2018) in a way that it would reduce the amount of free decision variables and combinations to test. The second one consisted of introducing new constraints to Martins' model (2018) based on characteristics of the optimal solution that could be expected a priori. Both these new were validated for an illustrative example and for the real case study, achieving a better final solution in a shorter computational time than Martins' original model.

6.2 Limitations

There are a few limitations that should be discussed in this work. The three conducted approaches presented limitations of their own. One of them, though could, actually, improve the optimality gap and computational time, is far from ideal. Other has some limitations regarding programming, which made it impossible to be validated. And the last one has no way to prove its optimality.

One important limitation with the approach of introducing more constraints is the fact that it relies completely on the experience of the maintenance planners and knowledge. Nevertheless, the spirit and arguments behind the creation of additional constraints can be adapted to other problems. Another limitation of this approach is that there are options that are disregarded which could represent a better solution than the one found.

The heuristic approach presents a big limitation, which is to know whether the solution found is optimal. Contrary to all the other approaches (except for the restrictions' one) there is no mechanism that guarantees the optimality, or not, of the solution.

6.3 Further Research

There are several ways in which further research on this topic can be pursued.

Regarding the "introducing new restrictions approach" approach, other directions for further research would be adding restrictions to the reformulated model. There are obviously a lot of possibilities here. Improvements to this approach should be pursued if the research is related with this exact same case study. Otherwise, it might still work, but would have to be adapted specifically to the characteristics of the solution to the problem under analysis.

Regarding the heuristic approach, it is imperative that a mechanism to verify the optimality of the solution is implemented. An idea would be to try different types of iterative approaches to optimize the previous result.

Also, the model developed using the Dantzig-Wolfe decomposition can potentially be modified in order to be feasible. The bus operating company operates other depots in the Lisbon area, and an implementation of the parallel solving mechanisms, including the Dantzig-Wolfe, could be more advantageous and less complex in terms of adaptations. An interesting way to decompose the problem would be by depot.

7. References

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