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Structures & Foundations Supporting Vibrating Machines

Case study: Pile cap supporting a reciprocating machine

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Declaration

I hereby declare that this document is an original work of my own and is in compliance with all requirements of the Code of Conduct and Good Practice of the University of Lisbon.

ABSTRACT

Foundations and structures supporting vibrating machines require a specific design in order to minimize the negative effects of vibrations. The performance (as well as safety and stability) of machines depend largely on their design, manufacturing and ultimately on the interaction with the environment. Therefore, machine foundations shall be designed in such a way that the dynamic forces caused by the machines are transmitted to the soil through the foundation avoiding all kinds of harmful effects. Excessive vibrations adversely affect persons, buildings and equipment. Risk mitigation of excessive vibrations foresees the control of the frequency and amplitude of the machine vibrations. The source of excitation, in this case mechanical vibration, should be properly characterized, regarding operating speed, magnitude of the dynamic forces and nature of the excitation. It's up to the design engineer to evaluate, in line with the machine design parameters and the geotechnical context at the location of the machine foundation, the best design to avoid excessive vibration, beyond the limits of acceptance. Such measures include either the stiffening (over-tuning) of the foundation or the opposite, namely under-tuning of the foundation. Design methodologies, such as finite element analysis (FEA), predict results close to reality that can be compared with the imposed boundaries. Limits of acceptance are properly defined, either by national standards, suppliers of equipment and ultimately, good practice. Such guidelines impose boundaries to avoid any damage of the machine and any of its components (for long term operation) as well as people discomfort (mainly, plant operators).

Keywords: Vibrating machines, Dynamic analysis, Reciprocating compressors, Deep foundations, Unbalanced forces, Time history analysis, Dynamic response, Dynamic tuning.

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NOTATIONS

α_A	Axial displacement interaction factor for a typical reference pile in the group
α_L	Lateral displacement interaction factor for a typical reference pile in the group
ξ	Damping ratio
ϕ	Phase angle
γ_P	Total unit weight of the pile material
γ	Total unit weight of the soil
γ_S	Total unit weight of the backfill soil
μ; μ_S	Poisson's ration of the soil
μ_P	Poisson's ration of the pile material
η	Isolation efficiency
ρ	Soil mass density
ρ_P	Density of the pile material
σ'_M	Mean effective normal stress
$\left\{\psi_j\right\}$	Vector of the j-th mode at frequency p_j
ω	Frequency of excitation

a_0	Dimensionless frequency factor
A	Cross-sectional area of the pile
c	Damping constant
c_c	Critical damping
$[C]$	Damping matrix
c_x^1	Damping constant for horizontal motion for a single pile
c_x^f	Damping constant due to embedment of the pile cap subject to horizontal motion
c_x^{G}	Pile group geometric damping constant subject to horizontal motion
c_z^1	geometric damping constant for vertical motion for a single end-bearing pile
c_z^f	Damping constant due to embedment of the pile cap subject to vertical motion
c_z^{G}	Pile group geometric damping constant subject to vertical motion
c_φ^1	Damping constant for uncoupled rocking motion
c_φ^f	Damping constant due to embedment of the pile cap subject to uncoupled rocking motion
D	Material or hysteric damping
D_m	Material damping ratio
D_{min}	Minimum material damping
D_f	Depth of the embedment of the pile cap
e	Eccentricity of the rotating mass
e_v	Void ratio
E_P	Young modulus of the pile material
f	Operating speed (rpm)
$f_{j1}, f_{j,2}$	Dimensionless pile stiffness and damping functions for the *j*-th direction
F_0	Amplitude of a forcing function (frequency independent)
F	Force of excitation
$F_T(t)$	Dynamic excitation force
$F_E(t)$	Transmitted force
G_{max}	Dynamic Shear Modulus
G_s	Shear Modulus of the backfill soil
I_p ; I	Inertia of the pile
J	Torsional constant of the pile
k	Spring stiffness
$[K]$	Stiffness matrix
K_x^1	Spring constant for horizontal motion for a single pile
K_x^f	Stiffness constant due to embedment of the pile cap subject to horizontal motion
K_x^G	Pile group stiffness subject to horizontal motion
K_z^1	Spring constant for vertical motion for a single end-bearing pile
K_z^f	Stiffness constant due to embedment of the pile cap subject to vertical motion
K_z^G	Pile group stiffness subject to vertical motion

K_{φ}^{1}	Spring constant for uncoupled rocking motion
K_{φ}^{f}	Spring constant due to embedment of the pile cap subject to uncoupled rocking motion
L_p	Length of pile
L	length of connecting rod
M	Magnification factor
m	Mass
$[M]$	Mass matrix
m_r	rotating mass
m_{rot}	rotating mass in a reciprocating machine
m_{rec}	reciprocating mass in a reciprocating machine
N	SPT blow count
NT	normal torque
p	Natural frequency
p_d	Damped natural frequency
P_a	Atmospheric pressure
P_S	power being transmitted by the shaft at the connection
q_c	CPT point resistance
Q	Normal balance criteria according ASA/ANSI S2.19
r	Frequency ratio or length of crank
r_o	Pile radius or equivalent radius
S	Pile spacing
S_f	service factor, used to account for increasing unbalance during the design service life of the machine
t	Time variable
T_r	Transmissibility ratio
u	Generic representation of Displacement
$\dot{u}$	Generic representation of Velocity
$\ddot{u}$	Generic representation of Acceleration
u_{st}	Static displacement
U_0	Amplitude of the steady-state response
V_C	Compressive velocity of a pile
V_S	Shear wave velocity of the soil
W_D	Energy dissipated in one cycle of loading
W_S	Maximum strain energy stored during the cycle
y_c	crank pin displacement in local y-axis
$\ddot{y}_g$	Ground acceleration
z_P	Piston displacement
z_C	Crank pin displacement in local z axis

ACRONYMS

DOF	Degree of Freedom
SDOF	Single Degree of Freedom
MDOF	Multi-degree of Freedom
COG	Center of Gravity
CR	Center of Resistance
CAE	Computer Aided Engineering
SPT	Standard Penetration Test
CPT	Cone Penetration Test
SCPT	Seismic Cone Penetration Test
SASW	Spectral Analysis of Surface Wave
CU	Consolidated Undrained
OCR	Over-consolidation Ratio
FEA	Finite Element Analysis

1 INTRODUCTION

Structures supporting vibrating machines require a specific design in order to minimize the negative effects of vibrations. The performance (as well as safety and stability) of machines depend largely on their design, manufacturing and ultimately on the interaction with the environment. Therefore, machine foundations shall be designed in such a way that the dynamic forces caused by the machines are transmitted to the soil through the foundation avoiding all kinds of harmful effects.

Excessive vibrations adversely affect persons, buildings and equipment. The following problems are associated with excessive vibrations: personal discomfort; damage to structures and equipment; excessive maintenance costs; forced shutdowns resulting in business interruption losses and safety hazards (and possibly hazardous materials releases).

Risk mitigation of excessive vibrations foresees the control of the frequency and amplitude of the machine vibrations. As a first iteration the design of foundations for control of vibrations considers increasing the mass of the foundation and/or strengthening the soil beneath the foundation base by using piles. This methodology is however costly resulting in significant overdesign. Alternatively, a more detailed vibration analysis can be performed considering a proper characterization of the soil in the vicinity of the foundation. Thus, it is of great importance that the soil parameters are accurately determined serving as input to the differential equation solution that describes the vibratory motion.

The source of excitation, in this case mechanical vibration, should be properly characterized, regarding operating speed, magnitude of the dynamic forces (unbalance mass) and nature of the excitation. Different types of machines, such as rotating, reciprocating and impulsive, generate different dynamic forces (in magnitude, direction and frequency).

It's up to the design engineer to evaluate, in line with the machine design parameters and the geotechnical context at the location of the machine foundation, the best design to avoid excessive vibration, beyond the limits of acceptance. Such measures include either the stiffening (over-tuning) of the foundation or the opposite, namely under-tuning of the foundation. In different circumstances (where the above is not effective to control the vibrations) other counter-measures can be of application, such as using vibration absorbers (isolation).

Both mitigation measures described in the previous paragraph allow a design that will not compromise the operation of the machine and not disturb the environment (people) in the vicinity of the machine. Limits of acceptance are properly defined, either by national standards, suppliers of equipment and ultimately, good practice. Such guidelines impose boundaries to avoid any damage of the machine and any of its components (for long term operation) as well as people discomfort (mainly, plant operators).

This subject is not widely present in common standards or covered in the academic domain. Thus, the design approaches and acceptance criteria are not well defined. This document intends to give an overview of possible approaches to the problem with the current state-of-the-art.

2 DYNAMIC ANALYSIS

2.1 FUNDAMENTALS OF VIBRATION THEORY

The following sources are considered when developing this chapter: [1], [2], [3].

The dynamic behaviour of the machine foundation/structure system is directly related to the theory of vibration. Normally the reinforced concrete foundation provides the required inertial mass so that foundations can be treated as massive and rigid. Since the machine is often a rigid element, its combination allows a simplification in the dynamic analysis

If the equipment is placed on a rigid foundation and the position of all parts of the system can be described by a single variable at any time, the system can be represented by a SDOF model, with a considerable degree of accuracy (e.g. pure vertical motion of a rigid system can be treated with a SDOF model). In most cases, however, the horizontal motion is coupled with the rocking motion. This results from the fact that the system's center of gravity (COG) is not at the same level as the center of resistance (CR). In such cases, at least a two degree of freedom model is required to characterize the coupled response of the system. Other cases require a six DOF model to accurately represent the dynamic behavior of a machine foundation system. Nevertheless, the six DOF system can be simplified in instances where the system's COG is coincident with the CR. In such cases the system can be characterized by two DOF systems (rocking coupled with translation in both plan directions) and two SDOF models (vertical motion and torsional motion about the vertical axis).

2.1.1 SINGLE DEGREE OF FREEDOM (SDOF) SYSTEMS

A system is single degree of freedom when its motion is constrained to one direction only. The case where a concentrated mass acts on a spring (and damper) element, as in Figure 1, allows the development of fundamental relations of general applicability.

2.1.1.1 Free Vibration with viscous damping

If a dynamic system experiences a transient motion without external excitation, such state is designated as free vibration. This state, in case damping is proportional to the vibration velocity, is described by the differential equation:

$$m.\ddot{u} \quad + \quad c.\dot{u} \quad + \quad k.u \quad = \quad 0 \qquad (2\text{-}1)$$

Inertial Force	Damping Force	Spring Force

Where:

u = displacement

$\dot{u}$ = velocity

$\ddot{u}$ = acceleration

m = mass

c = damping constant

k = spring stiffness.

The terms in (2-1) represent forces standing in equilibrium:

$m.\ddot{u}$ = inertial force

$c.\dot{u}$ = viscous damping force

$k.u$ = elastic restoring (spring) force.

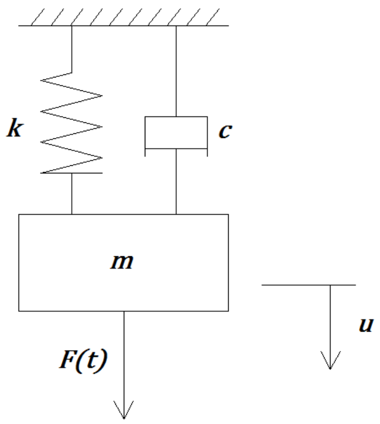


Figure 1 - Dynamic system with a single degree of freedom (SDOF oscillator)

In case the differential equation (2-1) is represented by an exponential type of solution (in case damping is present, solution is complex):

$$u = e^{st} \tag{2-2}$$

in which *s* is a constant that will be determined later. Substituting Eq. (2-2) in Eq. (2-1):

$$\left(s^2 + \frac{c}{m}.s + \frac{k}{m}\right).e^{st} = 0 \tag{2-3}$$

Resulting (e^{st} is non-zero, for solution to exist):

$$s^2 + \frac{c}{m}.s + \frac{k}{m} = 0 \tag{2-4}$$

This gives two values of *s*,

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \tag{2-5}$$

and the general solution of the equation of motion can be written as follows:

$$u = A.e^{s_1 t} + B.e^{s_2 t} \tag{2-6}$$

in which *A* and *B* are arbitrary constants depending upon the initial conditions of motion.

If the radical in Eq. (2-5) is zero, the damping is said to be *critical damping* c_c, and:

$$\left(\frac{c_c}{2m}\right)^2 = \frac{k}{m} = p^2 \quad \text{with} \quad c_c = 2mp \tag{2-7}$$

Considering: *p*, the natural frequency of the system.

The ratio of actual damping c to critical damping c_c, is defined as the damping factor ξ:

$$\xi = c/c_c \tag{2-8}$$

Coming:

$$\frac{c}{2m} = \frac{c}{c_c}\frac{c_c}{2m} = \xi p \tag{2-9}$$

Substituting the previous relations into Eq. (2-5):

$$s_{1,2} = p\,.\left[-\xi \pm \sqrt{(\xi^2 - 1)}\right] \tag{2-10}$$

The nature of the resulting motion depends upon the values of roots s_1 and s_2, and thereafter on the magnitude of damping (in terms of critical damping) present in the system. Three different cases of interest are considered.

2.1.1.1.1 Case 1: ξ>1 - Over Damped System

When ξ>1, both s_1 and s_2 are real and negative and u (Eq. (2-6)) decreases as *t* increases (never changing sign). The radical inside Eq. (2-10) is positive. There are no oscillations and the equilibrium position is reached at a relatively slower rate compared to a critically damped system. When an initial displacement is given to this system, the mass is pulled back by the springs and all the energy is absorbed by the dampers (before the mass returns to the initial position). Structural systems normally do not have damping higher than the critical damping. A typical solution for ξ>1 is shown in Figure 2, below:

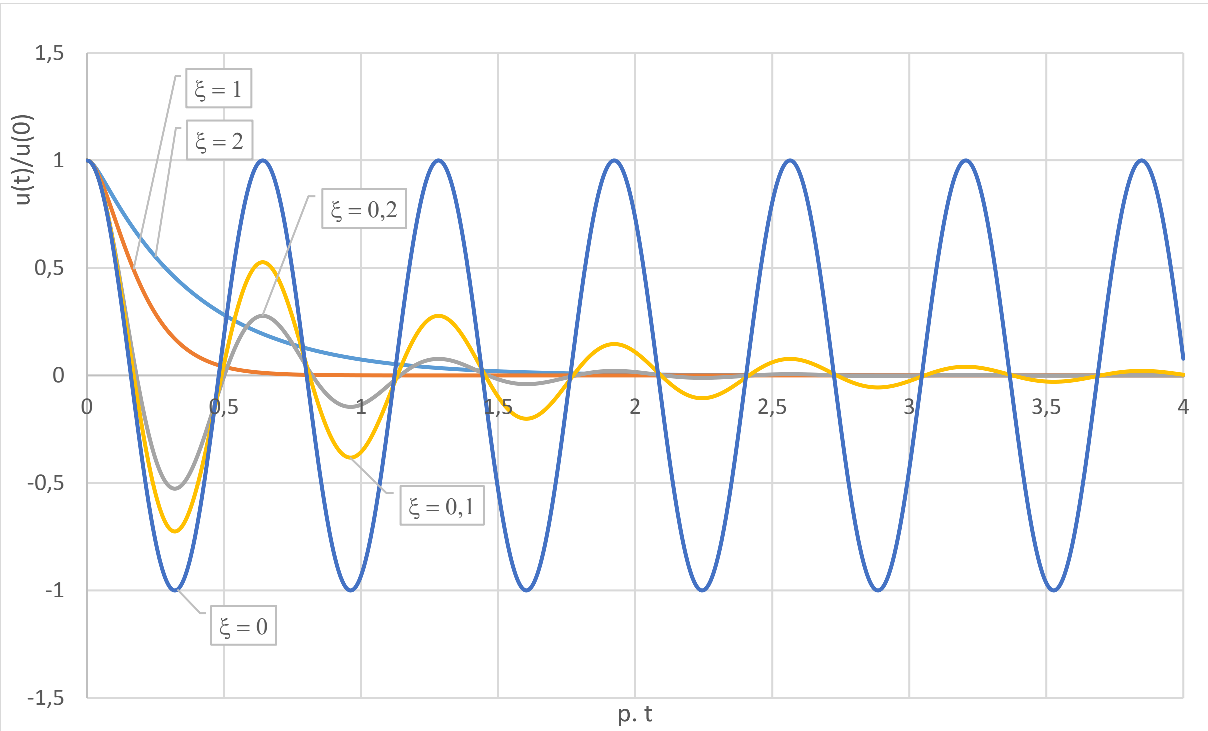


Figure 2 - Free Vibration Response for Undamped System $\xi = 0$, Under-Damped $\xi < 1$, Critically Damped $\xi = 1$ & Over-Damped $\xi > 1$

2.1.1.1.2 Case 2: ξ=1 - Critically Damped System

When ξ=1, Eq. (2-10) gives $s_1 = s_2 = -p$. The solution becomes

$$u = (A + B). e^{-p\,t} \tag{2-11}$$

In this case u decreases as t increases but never changes (no oscillation). It is designated as critically damped and ξ=1 is the minimum value of damping for no oscillations in the system.

A typical solution for ξ=1 is shown in Figure 2, above.

2.1.1.1.3 Case 3: ξ<1 - Under-Damped System

In case damping is less than the critical damping (ξ<1), the values of s_1 and s_2 (Eq. (2-10)) are determined according the following expression:

$$s_{1,2} = p\,.\left[-\xi \pm i\sqrt{(1-\xi^2)}\right] \tag{2-12}$$

And the general solution turns out to be:

$$u = A.exp\left[\left(-\xi + i\sqrt{1-\xi^2}\right).p\,t\right] + B.exp\left[\left(-\xi - i\sqrt{1-\xi^2}\right).p\,t\right] \tag{2-13}$$

or

$$u = A.exp(-\xi p\,t)exp\left(i\sqrt{1-\xi^2}p\,t\right) + B.exp(-\xi p\,t)exp\left(-i\sqrt{1-\xi^2}p\,t\right) \tag{2-14}$$

$$u = exp - (\xi p\,t)\left[\left(A\,cos\sqrt{1-\xi^2}p\,t + iA\,sin\sqrt{1-\xi^2}p\,t\right) + \left(B\,cos\sqrt{1-\xi^2}p\,t - iB\,sin\sqrt{1-\xi^2}p\,t\right)\right] \tag{2-15}$$

$$u = exp(-\xi p\,t)\left(C\,cos\sqrt{1-\xi^2}\,p\,t + D\,sin\sqrt{1-\xi^2}p\,t\right) \tag{2-16}$$

With C equal to A + B, and D equal to i(A- B).

The damped natural frequency p_d of the system becomes:

$$p_d = p\,\sqrt{1-\xi^2} \tag{2-17}$$

Equation (2-16) can then be written as

$$u = U_0 exp\left(\frac{-\xi p\,t}{\sqrt{1-\xi^2}}\right)sin(p\,t + \phi) \tag{2-18}$$

with U_0 and ϕ being arbitrary constants depending upon the initial conditions.

A typical solution for ξ<1 is shown in Figure 2.

2.1.1.2 Forced Vibration

A forced vibration results when an external transient load F(t) acts on the mass, extending the differential Eq. (2-1) by a force term on the right-hand side:

$$m.\ddot{u} \quad + \quad c.\dot{u} \quad + \quad k.u \quad = \quad F \tag{2-19}$$

The free vibration, in this case, is only one part, the homogeneous solution, on which the force-dependent, particular solution is superimposed. In case the time function of the force is harmonic, the particular solution becomes particularly simple.

A machine with rotating parts in operation is an example of a harmonic loading. The constant-load excitation is characterized with a harmonic force as following:

$$F = F_0 \sin \omega t \tag{2-20}$$

In which ω is the frequency of excitation.

The differential equation (2-19) of forced motion of a damped SDOF system then becomes:

$$m.\ddot{u} + c.\dot{u} + k.u = F_0 \sin \omega t \tag{2-21}$$

The solution to this equation is

$$u = U_0 \sin(\omega t - \phi) \tag{2-22}$$

Substituting Eq. (2-22) in Eq. (2-21):

$$F_0 = \sqrt{(k - m\omega^2)^2 + (c\omega)^2} U_0 \tag{2-23}$$

Or

$$U_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{2-24}$$

And

$$\phi = \tan^{-1} \frac{c\omega}{k - m\omega^2} \tag{2-25}$$

Equation (2-24) can be expressed in non-dimensional terms as follows:

$$U_0 = \frac{F_0/k}{\sqrt{(1 - m\omega^2/k)^2 + (c\omega/k)^2}} \tag{2-26}$$

F_0/k is equal to the static displacement u_{st} of the system under the action of F_0.

Considering that:

$$\frac{m\omega^2}{k} = \left(\frac{\omega}{p}\right)^2 = r^2 \tag{2-27}$$

in which *r* is the frequency ratio, and

$$\left(\frac{c\omega}{k}\right)^2 = \left(\frac{c}{c_c}\frac{c_c\omega}{mp^2}\right)^2 = \left(\frac{c}{c_c}\frac{2m\,p\,\omega}{mp^2}\right)^2 = (2\xi r)^2 \tag{2-28}$$

Results:

$$U_0 = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \tag{2-29}$$

If there is no damping present, i.e., ξ = 0, undamped amplitude U_0 is given by

$$U_0 = \frac{F_0/k}{1-r^2} \tag{2-30}$$

The following expression defines the magnification factor (also designated as non-dimensional amplitude):

$$\frac{U_0}{u_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = M \tag{2-31}$$

Similarly,

$$\phi = \tan^{-1}\frac{2\xi r}{1-r^2} \tag{2-32}$$

Figure 3 is a plot of Magnification factor *M*[1] versus frequency ratio *r;* Figure 4 is a plot of the phase angle ϕ versus frequency ratio *r*. As shown in both figures, the two parameters *M* and ϕ are functions of the frequency ratio, *r*, and the damping coefficient, ξ.

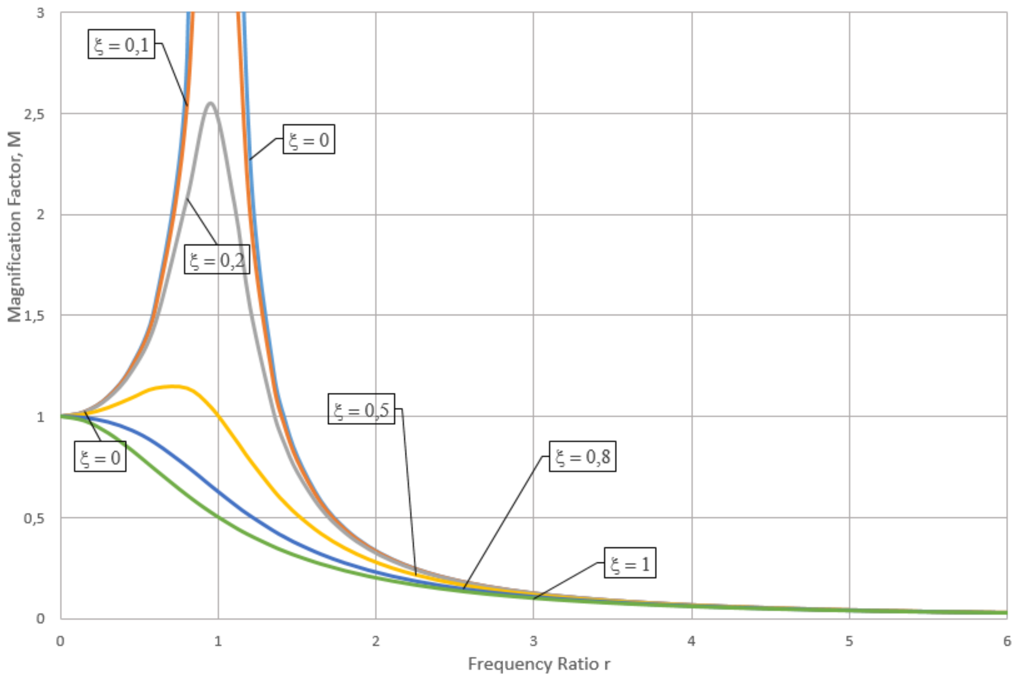


Figure 3 - Magnification factor (M) versus frequency ratio (r)

[1] In this case the Magnification factor M is related to displacements. It can also be related to other parameters, such as velocity, acceleration, etc.

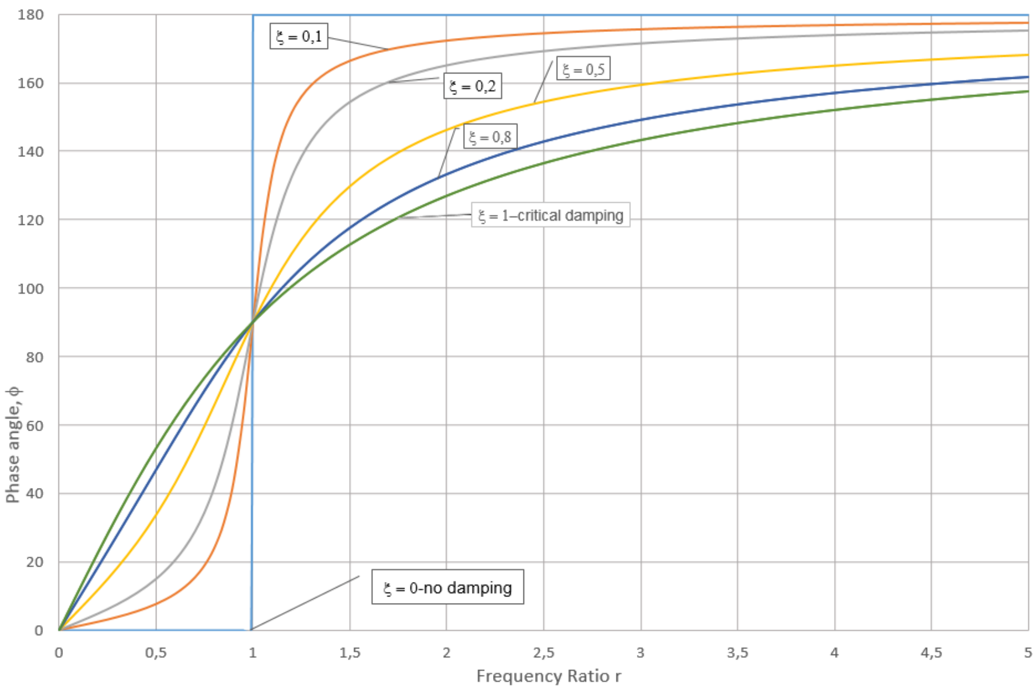


Figure 4 – Phase angle (ϕ) versus frequency ratio (r)

Effect of Frequency Ratio r for a Particular Case (ξ = 0)

From Eq. (2-31): In case *r* = 0, *M* = 1; in case *r* = 1, *M* $\rightarrow \infty$; in case *r* $\rightarrow \infty$, *M* = 0.

When *r* is equal to 1, resonance occurs, and the amplitude tends to be infinite. When *r* is less than 1, the phase angle ϕ is zero; when *r* is bigger than 1, the displacement u is in phase with the exciting force, F_0 , and ϕ is equal to 180°.

Effect of Damping ξ

In case damping is present, a reduction of the resonant amplitudes to finite values occurs. When damping increases, the peak of the magnification factor shifts slightly to lower frequencies. This happens because maximum amplitudes occur in damped vibrations when the forcing frequency ω equals the system's damped natural frequency, p_d (Eq. (2-17)) - slightly smaller than the undamped natural frequency, *p*.

If *r* is equal to 1, the phase angle ϕ is 90° for all values of damping, except when ξ is equal to 0. In case *r* is less than 1, the phase angle is less than 90°; otherwise, when *r* is bigger than 1, the phase angle is bigger than 90°.

The maximum amplitude of motion when r is equal to 1 and ξ is greater than 0 is expressed by Eq. (2-33):

$$U_0 = \frac{u_{st}}{2\xi} = \frac{F_0}{2\xi k} = \frac{F_0}{c\omega} \tag{2-33}$$

The steady-state solution characterized by Eq. (2-26) includes the influence of the forcing function. It is important in most of the practical cases. The motion described by Eq. (2-18) is called transient motion of the system. These oscillations die out in a short interval of time (in the first few cycles) if significant damping is present.

2.1.2 MULTI-DEGREE OF FREEDOM (MDOF) SYSTEMS

Except for cases described in the previous chapter 2.1.1 or in cases not described herein of two degree-of-freedom systems, the analysis of structures/foundations supporting vibrating machines demands a more complex model such as a three or more degrees of freedom system. In these cases, the SDOF system normally is not adequate to represent the analysis of machine foundations. In such cases, it's common to resort to numerical models, where the machine, structure and stiffness of the supporting medium (soil or piles) is accurately represented/characterized. The dynamic analysis of a MDOF system is performed resorting to a Modal Analysis, where the responses in the normal modes (each treated as independent one-degree systems) are determined separately, and then superimposed to provide the total response.

2.1.2.1 Free Vibration

Without external excitation, a MDOF system vibrates in its *natural modes* – as demonstrated for a SDOF oscillator. The differential equation of motion for free vibration is written in real matrix-form:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = 0 \qquad (2\text{-}34)$$

Where:

[M] = Mass matrix

[C] = Damping matrix

[K] = Stiffness matrix

{u} = Vector of displacement

In this case, in opposition to the SDOF oscillator, the single variables are replaced by vectors and matrices.

For a frequency analysis, the damping term is set to zero:

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \qquad (2\text{-}35)$$

The solution to this equation can be represented by:

$$\{u\} = \sum_{j=1}^{n} \left\{\Psi_j\right\} e^{i\,p_j\,t} \qquad (2\text{-}36)$$

$\left\{\psi_j\right\}$ = vector of the j-th mode at frequency p_j

Resulting:

$$\left([K] - p_j^2[M]\right)\left\{\Psi_j\right\} = \{0\} \qquad (2\text{-}37)$$

Solving Eq. (2-37) one obtains the natural frequencies and mode shapes. With these results it is possible to evaluate if the frequencies associated with the excitation forces generated by operation of the machine are in the same range of the natural frequencies of the system (verification of resonance condition).

2.1.2.2 Forced Vibration

For a forced vibration the differential equation of free vibrations is modified including a forcing function vector {*F(t)*}:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \qquad (2\text{-}38)$$

The solution can be obtained:

- in the frequency domain
- by modal analysis
- by direct integration in the time domain.

The Eq. (2-38) is solved for $\{u\}$, the displacements of the machine-foundation system as a function of time. The forced response analysis can take the form of a harmonic frequency analysis or a time history analysis. The selection of method depends on the forcing functions to be evaluated. For the analysis of rotating or reciprocating machinery foundations, the harmonic analysis method is used extensively to determine the steady-state response of a linear structure to a set of given harmonic loads.

The results of the forced response analysis include displacements, velocities and accelerations. These results are then compared against allowable limits for acceptance.

3 DESIGN MACHINE PARAMETERS

Vibrations induced by machines depend mainly on the type of machine exerting the excitation. Therefore, three types of machines can be defined according to its motion: rotating, oscillating (or reciprocating) and impacting. In case of a rotating and reciprocating type, a periodic time-dependent loading function develops and is transferred to the structure/foundation. According to *Bhatia* [1], a machine consists of a drive machine, a driven machine and a coupling device, within the field of foundation/structural design.

Certain machine parameters (geometrical and performance) are required for the design of the foundation or structure supporting the vibrating machine. Ideally, this information should be provided by the machine manufacturer. The required machine data (referring to the drive and driven machine as well as to the coupling device) should include the following:

- Outline drawing (footprint of the machine, base frame details and holding down bolts) of machine assembly and data-sheets
- Functions of the machine
- Weight of the machine and its rotor components
- Location of center of gravity both vertically and horizontally
- Speed ranges of the machine and its components or frequency of unbalanced primary and secondary forces
- Magnitude and direction of the excitation (unbalanced) forces both vertically and horizontally and their points of application
- Height of the centerline of the rotor from the machine base frame
- Limits imposed on the foundation with respect to differential deflection between points on the plan area of foundation
- Forces generated under Emergency and Faulted conditions e.g. Short Circuit Forces
- Foundation requirements

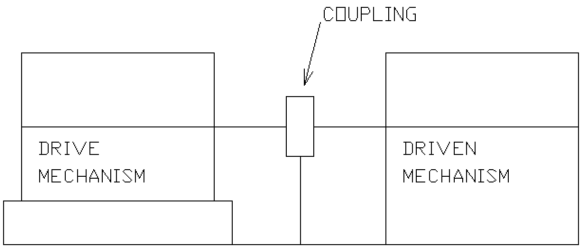


Figure 5 – Typical rotary machine

3.1 MACHINE PARAMETERS FOR A RECIPROCATING MACHINE – CASE STUDY

Examples of reciprocating machines are for instance: internal combustion engines, steam engines, piston-type pumps and compressors, and other similar machines having a crank mechanism. The operation of a reciprocating machine results in primary (rotating) and secondary (oscillating) forces in the direction of the piston motion. The rotating component derives from the eccentric hinging of the connecting rod to the crankshaft. Thus, the basic form of a reciprocating machine consists of a piston that moves within a cylinder, a connecting rod, a piston rod and a crank (Figure 6). The crank rotates with a constant angular velocity, converting the reciprocating motion into rotary motion. The operating speeds of reciprocating machines are usually smaller than 1200 rpm.

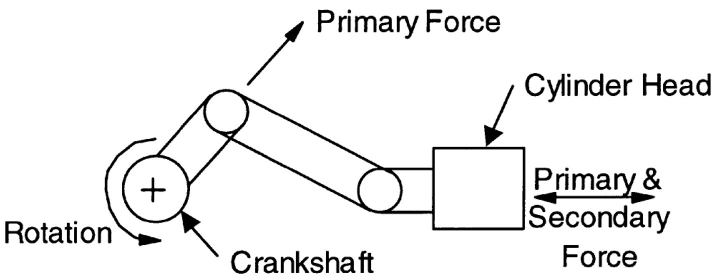


Figure 6 - Reciprocating machine diagram – from ACI 351-3R [4]

3.1.1 DYNAMIC LOADS

The design of a foundation/structure supporting a reciprocating machine should always consider the dynamic loads exerted by the operation of the equipment. The computation of these loads is extremely important for the definition of the vibration amplitudes, velocities and accelerations.

3.1.1.1 Unbalanced forces and moments

A scheme of a single crank mechanism is shown in Figure 7, illustrating the concept of an engine producing both primary and secondary forces. The mechanism includes a crank rod, a connecting rod and a piston. The piston moves within the guiding cylinder, the crank rod of length *r* rotates about the crankshaft, and the connecting rod of length *L* has both linear motion (at one end) and rotating motion at the other end. Considering that the connecting rod is attached to the piston at point P and to the crank at point C, the motion of the mechanism produces oscillation of the wrist pin (P) and rotation (circular path) of the crank pin (C). Thus, the mechanism produces primary and secondary dynamic forces.

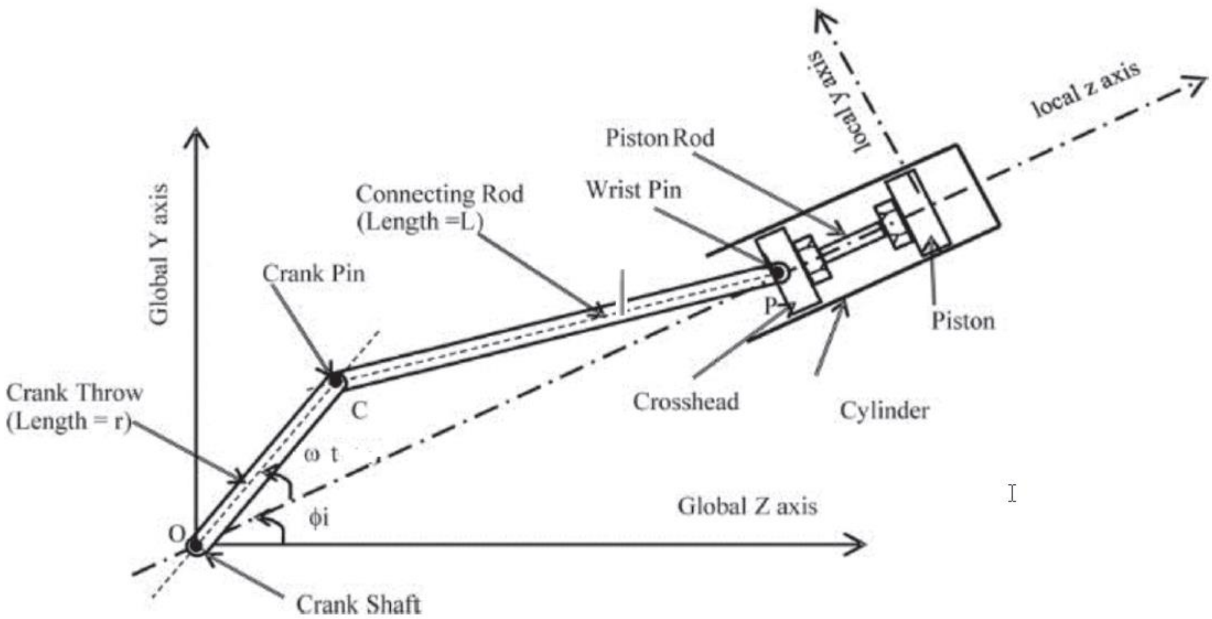


Figure 7 – Crack mechanism – from ACI 351-3R [4]

The acceleration of the piston along its axis can be determined assuming that the crank rotates at a constant angular velocity ω. Considering the piston displacement toward de crankshaft as z_P, an expression derives at any time t [3]:

$$z_P = \left(r + \frac{r^2}{4L}\right) - r\left(\cos\omega t + \frac{r}{4L}\cos 2\omega t\right) \tag{3-1}$$

The velocity and acceleration expressions for the motion of the piston are obtained from the first and second derivates of the displacement with respect to time:

$$\dot{z}_P = r\omega\left(\sin\omega t + \frac{r}{2L}\sin 2\omega t\right) \tag{3-2}$$

$$\ddot{z}_P = r\omega^2\left(\cos\omega t + \frac{r}{L}\cos 2\omega t\right) \tag{3-3}$$

One of the two terms of the above expressions varies with the frequency of the rotation, ω, and is designated as the primary term; the other, that varies at twice the frequency of rotation, 2ω, is defined as the secondary term.

A similar exercise can be done for the motion of the rotating parts of the crank (at the crank pin C):

$$y_C = -r\sin\omega t \tag{3-4}$$

$$\dot{y}_C = -r\omega\cos\omega t \tag{3-5}$$

$$\ddot{y}_C = r\omega^2\sin\omega t \tag{3-6}$$

$$z_C = r(1 - \cos\omega t) \tag{3-7}$$

$$\dot{z}_C = r\omega\sin\omega t \tag{3-8}$$

$$\ddot{z}_C = r\omega^2\cos\omega t \tag{3-9}$$

Being:

y_C = crank pin displacement in local Y-axis, in. (mm);

z_C = crank pin displacement in local Z-axis, in. (mm).

Assuming the mass of the piston plus a part of the connecting rod (usually 1/3) as the reciprocating mass m_{rec} concentrated at point P and the mass of the crank plus the remainder of the connecting rod as the rotating mass m_{rot} concentrated at point C, the unbalanced forces are obtained as:

Parallel to piston movement

$$F_z = m_{rec}\ddot{z}_P + m_{rot}\ddot{z}_C \tag{3-10}$$

$$F_z = (m_{rec} + m_{rot})r\omega^2\cos\omega t + m_{rec}\frac{r^2\omega^2}{L}\cos 2\omega t \tag{3-11}$$

The first term of the above equation is designated as primary force and the second as secondary force.

Perpendicular to piston movement:

$$F_Y = m_{rot}\ddot{y}_C = m_{rot}r\omega^2\sin\omega t \tag{3-12}$$

The above expression only has a primary component.

In cases where the reciprocating machine has multiple cylinders, manufacturers attempt to diminish the unbalanced forces. It can be achieved by balancing the rotating part (using small *r* and masses) or by having one crank rotating counterclockwise while another is rotating clockwise. Despite the mitigation measures implemented to reduce the unbalanced forces it is not possible to eliminate them completely.

In the case study (see chapter 9) the reciprocating machine has multi cylinders and the piston pairs are not directly opposed (having a horizontal offset). This results in some torsional forces designated as horizontal primary couple and horizontal secondary couple (Figure 8). The relative force created depends on the offset distance "D" between the center lines of the opposing throws. Torsional primary and secondary couples derive from the primary and secondary forces resultant from the motion of each cylinder.

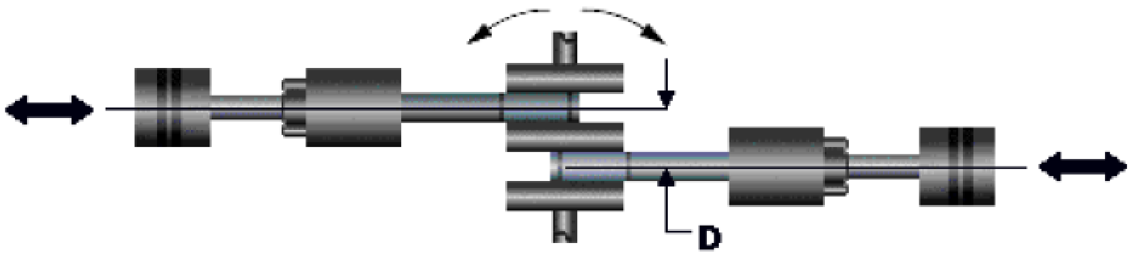


Figure 8 – Horizontal primary and second couple

3.1.1.2 Torque load variations

As previously mentioned, a machine comprises a drive mechanism, a driven mechanism and a coupling device. The drive machine can be either integrated or separated. If separated (e.g. separate reciprocating engine, electric motor, gas or steam turbine), drive and driven machine are coupled through a gear box. In these circumstances, the drive mechanism produces a net external drive torque on the driven machine. The torque is equal in magnitude and opposite in direction on the driver and driven machine.

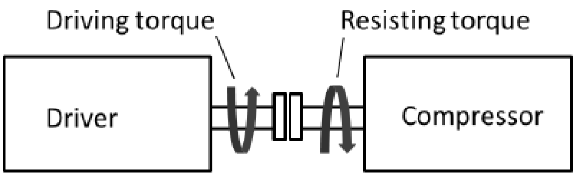


Figure 9 – Driving and resisting torque of a driver and a compressor, respectively

The normal torque (also called drive torque) is often computed as [4]:

$$NT = \frac{9550 P_S}{f} \tag{3-13}$$

With,

NT = normal torque (N.m)

P_S = power being transmitted by the shaft at the connection (kilowatts)

f = operating speed (rpm)

3.1.1.3 Machine unbalance load (possible rotor unbalance)

A rotor generates dynamic unbalance forces due to the centrifugal force produced by the rotor mass, m_r, having a certain eccentricity, e, and rotating with a certain frequency, ω. The magnitude of the dynamic force is:

$$F = m_r e \omega^2 \quad (3\text{-}14)$$

Normally, the rotating machines are balanced to an initial balance quality that can be defined according to the manufacturer's procedures or the purchaser's specifications. In absence of information from the machine manufacturer regarding the eccentricity of the rotating mass, a balance criterion from the *ISO 1940* [5] and *ASA/ANSI S2.19* [6]can be defined to allow the determination of the dynamic force amplitude due to the machine unbalance. ISO 1940 [5] and ASA/ANSI S2.19 [6] define balance quality in terms of a constant $Q = e\omega$. The magnitude of the dynamic force becomes:

$$F = m_r Q \omega S_f / 1000 \quad (3\text{-}15)$$

With:

m_r – rotating mass

Q – normal balance criteria according ASA/ANSI S2.19

ω – circular operating frequency of the machine (rad/s);

S_f – service factor, used to account for increased unbalance during the service life of the machine, generally greater than or equal to 2

Table 3-1 - Balance quality grades for selected groups of representative rigid rotors (excerpted from ANSI/ASA S2.19) [6]

Balance quality guide	Product of eω, in./s (mm/s)	Rotor types—general examples
G1600	63 (1600)	Crankshaft/drives of rigidly mounted, large, two-cycle engines
G630	25 (630)	Crankshaft/drives of rigidly mounted, large, four-cycle engines
G250	10 (250)	Crankshaft/drives of rigidly mounted, fast, four-cylinder diesel engines
G100	4 (100)	Crankshaft/drives of fast diesel engines with six or more cylinders
G40	1.6 (40)	Crankshaft/drives of elastically mounted, fast four-cycle engines (gasoline or diesel) with six or more cylinders
G16	0.6 (16)	Parts of crushing machines; drive shafts (propeller shafts, cardan shafts) with special requirements; crankshaft/ drives of engines with six or more cylinders under special requirements
G6.3	0.25 (6.3)	Parts of process plant machines; centrifuge drums, paper machinery rolls, print rolls; fans; flywheels; pump impellers; machine tool and general machinery parts; medium and large electric armatures (of electric motors having at least 3-1/4 in. [80 mm] shaft height) without special requirement
G2.5	0.1 (2.5)	Gas and steam turbines, including marine main turbines; rigid turbo-generator rotors; turbo-compressors; machine tool drives; medium and large electric armatures with special requirements; turbine driven pumps
G1	0.04 (1)	Grinding machine drives
G0.4	0.015 (0.4)	Spindles, discs, and armatures of precision grinders

3.1.1.4 Compressor gas loads

Reciprocating motion causes the increase of pressure inside the cylinder. This raise of pressure creates a reaction force both on the head and crank end side of the cylinder alternating as gas flows to and from each end. The gas force applied to the piston rod equals the instantaneous difference between the pressure force acting on the head (P_{head}) and crank end of the piston (P_{crank})(Figure 10). These forces are to be resisted within the frame of the equipment and are normally only considered locally for design of the anchor bolts (and machine frame/components). Therefore, they are not accounted for the design and dynamic analysis of the foundation.

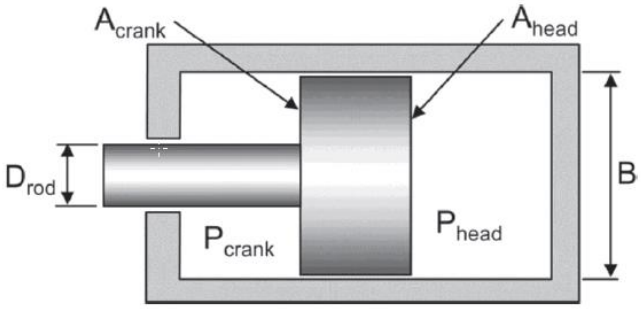


Figure 10 – Schematic diagram of double-acting compressor cylinder and piston, with A_{head}, A_{crank} = head and crank areas [4].

3.1.2 FORCES DUE TO EMERGENCY AND FAULTED CONDITIONS

During its life cycle, every machine withstands abnormal forces that are caused by malfunction. This includes generator short-circuit, turbine loss-of-blade/loss-of-bucket or a bowed rotor in a steam turbine. Such conditions are designated as Emergency and Faulted Conditions (also termed as catastrophic conditions). Catastrophic loads are of high magnitude and usually applied as pseudo-static loads despite being time dependent. Therefore, they are considered only for strength design, being applied vertically or horizontally at the bearings of the machine shaft. In case of an electrical short-circuit-induced torque load, the emergency drive torque is estimated by applying a magnification factor to the normal torque. These loads should be provided by the machine manufacturer.

4 FOUNDATION PARAMETERS

The following sources are considered when developing this chapter: [7] [3], [8], [9] and [10]

Foundations for equipment transmitting dynamic loads can be either rigid mat foundations, flexible raft foundations or deep foundations (piles and piers). The quality of the soil, the magnitude of the loads, geometry requirements and vibration performance criteria will define which type should be selected. In case of rigid mat foundations, several sources provide procedures to define the soil springs and damping constants, to be used in a model of six degree of freedom possible vibration/excitation modes. Barkan (1962)[11], Richart et al (1970)[12], Novak and Beredugo (1972) [13] and Dobry and Gazetas (1986)[14] are examples of that. The later have computed the spring and damping constants depending on a dimensionless frequency factor a_0, such that the impedances are defined in function of the excitation frequency. When a foundation becomes larger so that it is classified as flexible, it is required to model it as a series of discrete elements (with at least bending capability), connected by springs and damping elements and supported by soil springs and damping elements. The soil spring constant calculated for the total foundation should be adjusted to the joint regarding the bordering areas for a rigid mat. This analysis requires computer aided engineering (CAE) for the dynamic analysis. On the other hand, deep foundations have a different behavior than direct foundations requiring special methods for evaluation of the spring and damping constants. Prediction of the response of pile groups is often complicated since a rigid cap is positioned above the pile heads adding surface stiffness and damping to the pile group. Novak [7], Grigg [15], Beredugo [13], Poulos [16] and Gazettas [14] among others have conducted extensive investigation of single piles and pile groups subjected to dynamic loading.

4.1 SOIL SPRINGS AND DAMPING CONSTANTS FOR DEEP FOUNDATIONS – CASE STUDY

Whenever the soil conditions are such that direct foundations cause permanent settlements under dynamic loads or when there is a need to increase the foundation frequency, the solution of deep foundations shall be preconized to support the vibrating machine. It is of common knowledge that pile foundations subjected to dynamic loads: decrease the geometric damping, increase the resonance frequency (and may also increase the natural frequency) and influence the deformation near resonance. On the other hand, when laterally loaded, the dynamic response can be adverse and uncertain to estimate. Dynamic pile analysis has been deeply investigated by Novak. The basic theory was developed in 1974 and is accepted by current standards (e.g. *ACI 351-3R* [4]) or presented in several references (Das [9], Bowles [10], Arya [8], Prakash [3], …). The studies were developed for single piles and pile groups for uncoupled vertical, horizontal and rocking motion. The solutions consider fully embedded elastic piles interacting with uniform soil. The pile head is considered fixed on the pile cap. The pile tip is considered fixed for horizontal and rocking motion; for vertical motion it can be either fixed (end-bearing piles) or relaxed (short friction piles). The stiffness and damping resulting from this solution are frequency dependent. However, under certain conditions there is low dependency on the excitation frequency. Thus, approximate frequency independent expressions have been developed for both impedances. According to Novak and Grigg [15] the shear modulus to be used for the analysis of laterally loaded piles should be a reduced value considering the action of the soil against the pile. It is noted by *Arya*[8], that piles should be limited to a static load of 50% of the static capacity of the soil thus avoiding that the soil around the pile resonates with the pile with its consequent plunging. The pile cap should be embedded in a proper backfill to benefit from the increased damping, due to the embedment [8].

4.1.1 VERTICAL MOTION

The spring constant for a single end-bearing pile is given by the following equation (after Novak [7]):

$$K_z^1 = \frac{E_p A}{r_0} f_{18,1} \tag{4-1}$$

Considering: E_p, the Young modulus of the pile material; A, the cross-sectional area of the pile; r_0, the equivalent radius of the pile and $f_{18,1}$, a stiffness factor given as function of rations of pile penetration (length) L to radius r_0 and V_S/V_C (shear velocity in soil above tip/compression wave velocity in pile). In case combined friction and end bearing piles, *Arya* [8] suggests defining the impedances assuming end-bearing piles.

The geometric damping constant for vertical motion is defined as:

$$c_z^1 = \frac{E_p A}{V_s} f_{18,2} \tag{4-2}$$

Considering: V_S, the shear wave velocity of the soil through which the pile is driven; $f_{18,2}$, a damping factor given as function of rations of pile penetration (length) L to radius r_0 and V_S/V_C (shear velocity in soil above tip/compression wave velocity in pile). The compression wave velocity in pile is defined as:

$$V_C = \sqrt{\frac{E_p}{\rho_P}} \tag{4-3}$$

Considering: E_p, the Young modulus of the pile material; ρ_P, the density of the pile material.

The expressions defined for stiffness and damping constants are valid for a domain of the dimensionless frequency factor, a_0, between 0.05 and 0.8 (*Arya* [8]), with a_0 defined below:

$$a_0 = \frac{\omega r_0}{V_s} \tag{4-4}$$

$f_{18,1}$ and $f_{18,2}$ are obtained in Figure 11:

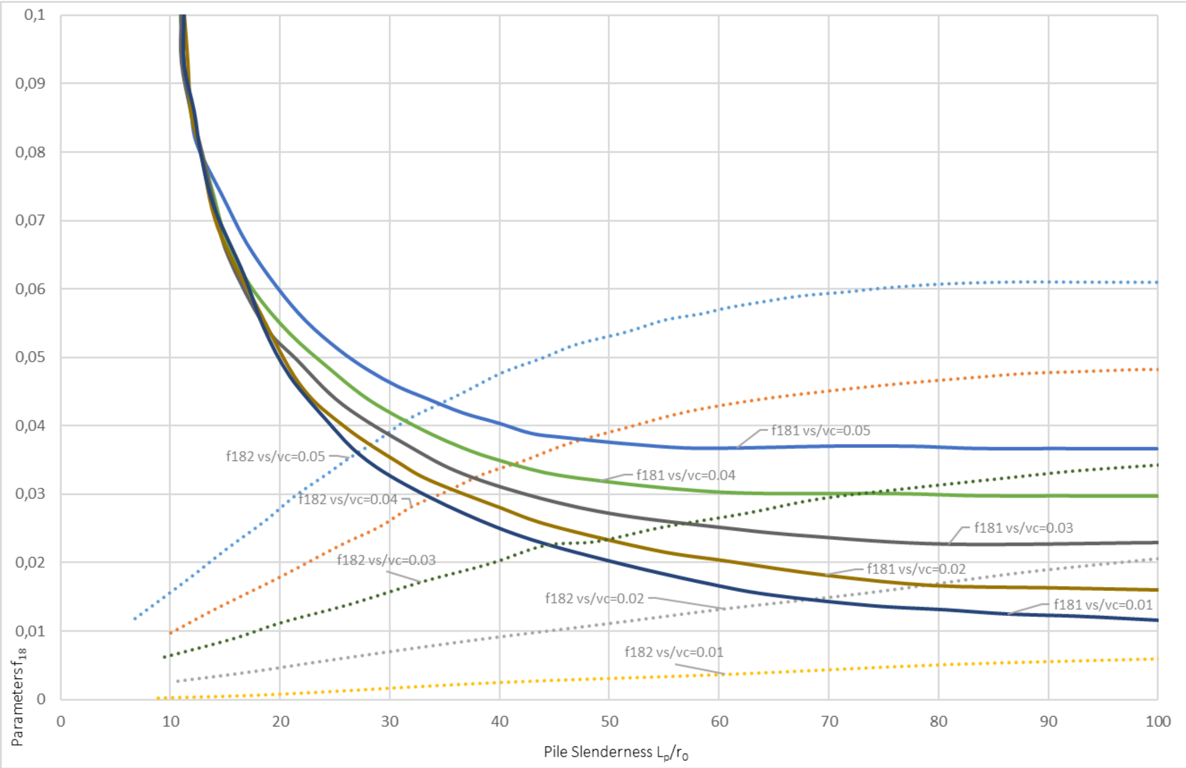


Figure 11 – Stiffness and damping factors for vertically excited fixed-tip concrete piles (reproduced after Novak [7]*)*

4.1.1.1 Pile group interaction applied to vertical motion

Piles are usually set in groups. The group stiffness and damping, unless the piles are widely spaced, will not be the sum of the individual stiffnesses and damping constants. *ACI 351-3R* [4] suggests neglecting the pile group interaction in case the spacing between piles reaches 20 diameters. Other authors like *Bowles* suggests 6 diameters spacing between piles as a boundary to neglect the pile group interaction. According to Novak, the deflection factors proposed by Poulos for groups of statically loaded piles can also be applied to a pile group subject to steady-state vibration. *ACI 351-3R* [4] states that *"pile group stiffness using static interaction coefficients may be used to estimate dynamic pile group stiffness if the frequency of interest is low. If the dimensionless frequency $a_o < 0.1$, or if the frequency is much less than the natural frequency of the soil layer (…), then this approach should provide a reasonable estimate of pile group stiffness."*

Therefore, the pile group stiffness subject to vertical motion is defined as:

$$K_z^G = \frac{\sum_1^N K_z^1}{\sum_1^N \alpha_A} \quad , \quad N \geq 2 \tag{4-5}$$

With α_A, being the axial displacement interaction factor for a typical reference pile in the group relative to itself and to all other piles in the group, assuming the reference pile and all other piles carry the same load and *N* the number of piles in the pile group. The damping constant is obtained in the same way:

$$c_z^G = \frac{\sum_1^N c_z^1}{\sum_1^N \alpha_A} \quad , \quad N \geq 2 \tag{4-6}$$

α_A can be obtained from the below Figure 12 and Figure 13:

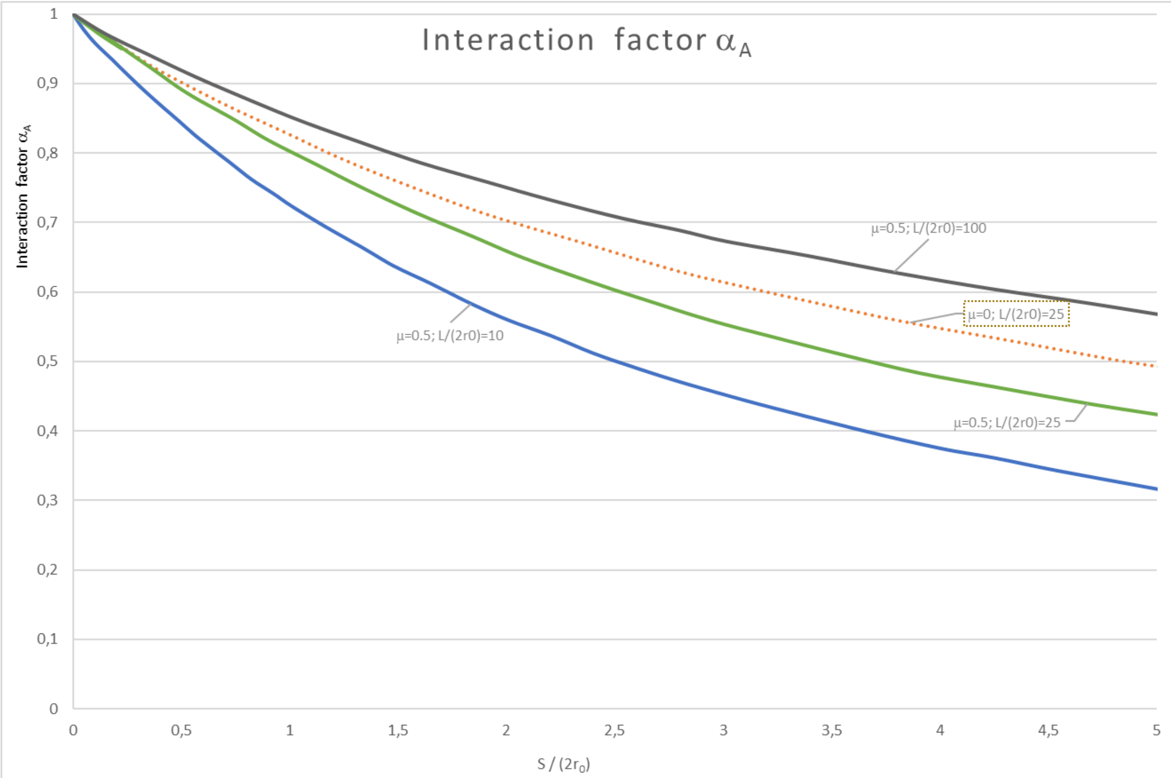


Figure 12 – Axial displacement influence factor as a function of pile length and spacing and Poisson's ratio, μ (reproduced after Poulos[16])

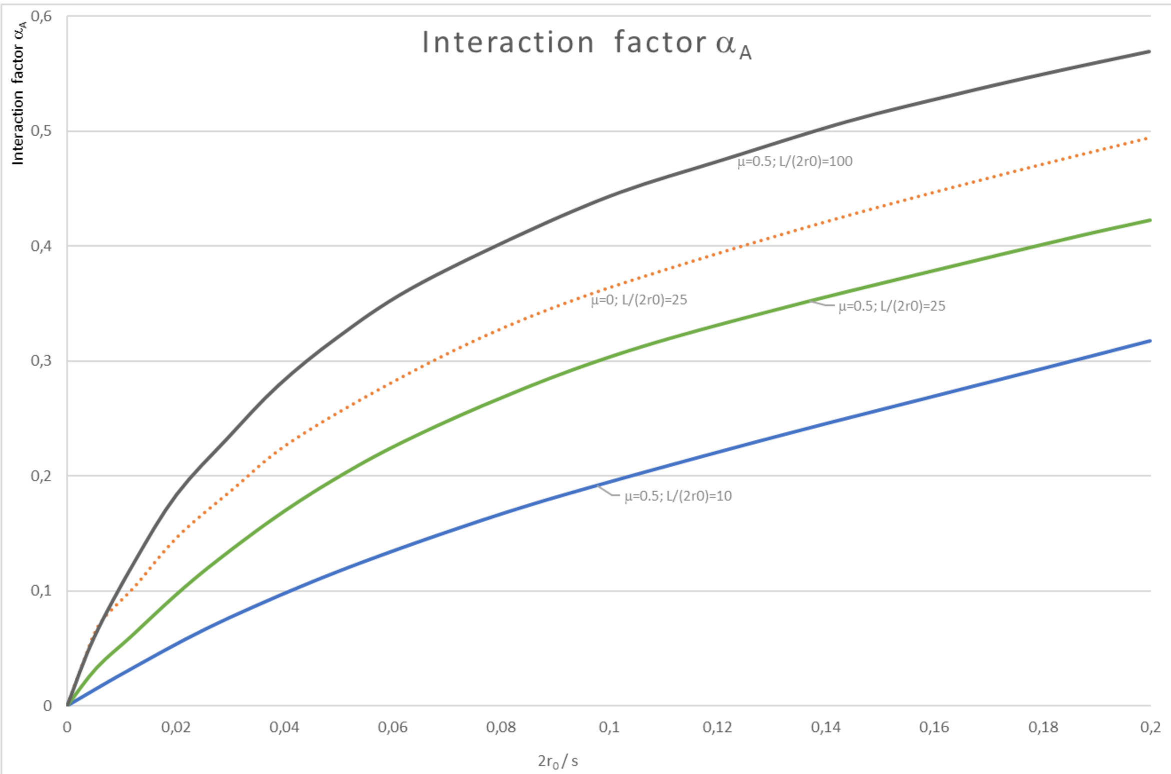


Figure 13 – Axial displacement influence factor as a function of pile length and spacing and Poisson's ratio, μ (reproduced after Poulos [16])

4.1.1.2 Embedment of the pile cap

The contribution of the embedment of the pile cap regarding the overall stiffness and damping constant of the foundation on piles is not negligible. That contribution is more effective when a dense granular backfill is used to embed the pile cap. The increase of damping is significant in circumstances where the pile cap is embedded. Expressions for stiffness and geometric damping were developed by Novak and Beredugo [13]:

$$K_z^f = G_s D_f \overline{S_1} \tag{4-7}$$

$$c_z^f = D_f r_0 \sqrt{G_s \gamma_s / g}\ \overline{S_2} \tag{4-8}$$

With D_f being the depth of the embedment of the cap, r_0 the equivalent radius of the cap, G_s and γ_s the shear modulus and the total unit weight of the backfill, respectively, and $\overline{S_1}$ and $\overline{S_2}$ constants given in Table 4-1 (μ_s – Poisson's ratio of the backfill soil).

Table 4-1 - Frequency Independent Constants for Embedded Pile Cap with Side Resistance (after Novak and Beredugo) [13]

μ_s	$\overline{S_1}$	$\overline{S_2}$	$\overline{S_{u1}}$	$\overline{S_{u2}}$	$\overline{S_{\varphi1}}$	$\overline{S_{\varphi2}}$
0	2,7	6,7	3,6	8,2	2,5	1,8
0,25	2,7	6,7	4	9,1	2,5	1,8
0,4	2,7	6,7	4,1	10,6	2,5	1,8

4.1.2 HORIZONTAL MOTION

The approach regarding horizontal motion is similar to the solution applied to vertical motion. The stiffness and damping constants for a single pile are obtained from the expressions below:

$$K_x^1 = \frac{E_p I}{r_0^3} f_{11,1} \tag{4-9}$$

$$c_x^1 = \frac{E_p I}{r_0^2 V_s} f_{11,2} \tag{4-10}$$

With I being the moment of inertia of the pile cross-section about a centroidal axis perpendicular to the direction of translation and $f_{11,1}$ and $f_{11,2}$ factors for fixed-head piles obtained from Table 4-2.

Table 4-2 - Stiffness and damping factors for horizontal motion, pure rocking and cross-stiffness (after Novak [7])

μ_s	V_s/V_p	$f_{11,1}$	$f_{11,2}$	$f_{7,1}$	$f_{7,2}$	$f_{9,1}$	$f_{9,2}$
0,4	0,01	0,0036	0,0084	0,202	0,139	-0,0194	-0,0280
	0,03	0,0185	0,0438	0,349	0,243	-0,0582	-0,0848
	0,05	0,0397	0,0942	0,450	0,314	-0,0970	-0,1410
0,25	0,01	0,0032	0,0076	0,195	0,135	-0,0181	-0,0262
	0,03	0,0166	0,0395	0,337	0,235	-0,0543	-0,0793
	0,05	0,0358	0,0850	0,435	0,304	-0,0905	-0,1321

4.1.2.1 Pile group interaction applied to horizontal motion

Similar to the vertical motion for a group of piles the stiffness and damping constants are obtained:

$$K_x^G = \frac{\sum_1^N K_x^1}{\sum_1^N \alpha_L} \quad , \ N \geq 2 \tag{4-11}$$

$$c_x^G = \frac{\sum_1^N c_x^1}{\sum_1^N \alpha_L} \quad , \ N \geq 2 \tag{4-12}$$

Considering: α_L, the displacement factor for lateral motion defined in the same way as α_A. α_L is defined in Figure 14 (after Poulos [16]):

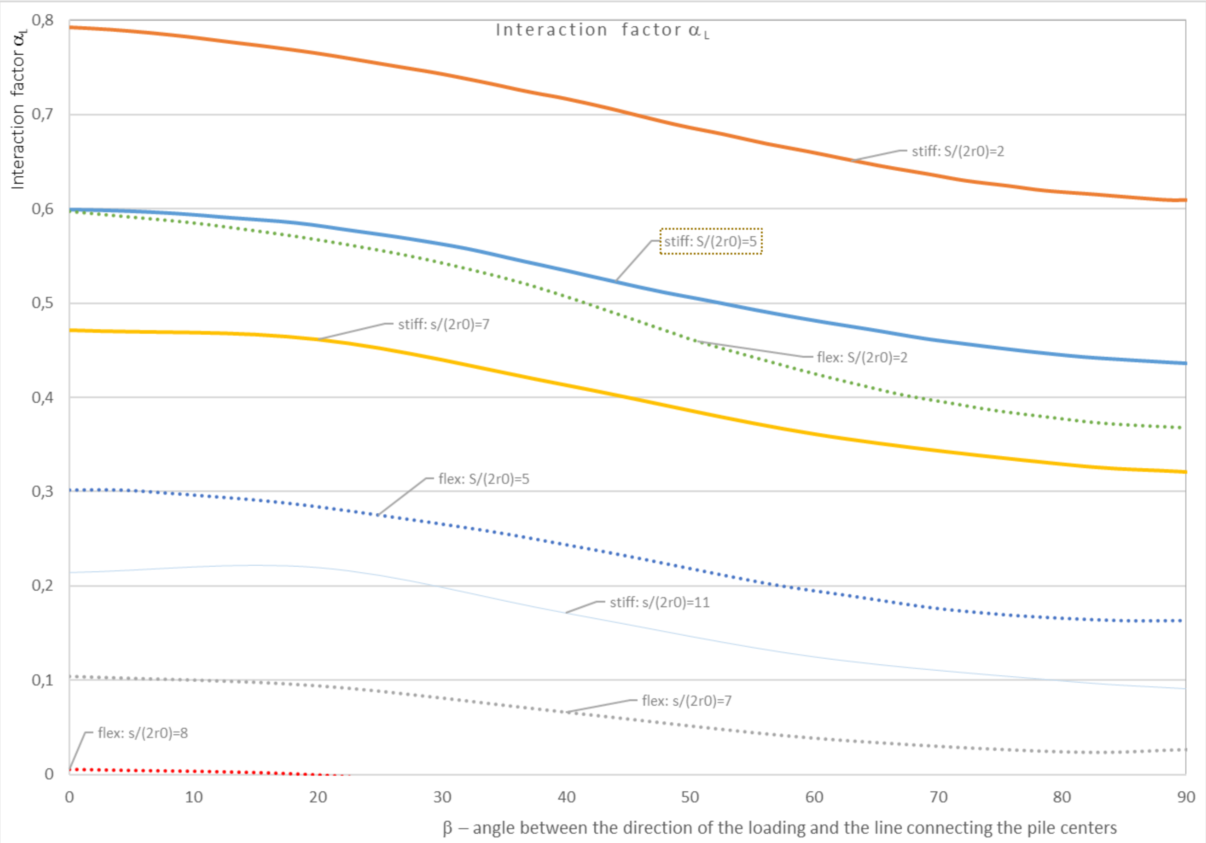


Figure 14 – Lateral displacement influence factor as a function of pile spacing, direction of the load and flexibility of the pile (reproduced and adapted after Poulos [16])

The flexibility of the piles is evaluated according following expression:

$$K_R = \frac{(EI)_{PILE}}{2[G(1+\mu)]_{SOIL} L^4} \tag{4-13}$$

With $K_R = 10$ (stiff pile); $K_R = 10^{-5}$ (flexible pile)

Poulos [16] graphical solution for α_L considers the soil Poisson's ratio equal to 0.5 and a ratio of pile length to diameter ($L/(2r_0)$) equal to 25. It considers only two cases of pile spacing ($S/(2r_0)$ equal to 2 and 5). For different pile spacing the curves were extrapolated.

4.1.2.2 Embedment of the pile cap

The stiffness and damping constants due to the embedment of the pile cap with regard to horizontal motion are obtained from the following expressions:

$$K_x^f = G_s D_f \overline{S_{u1}} \tag{4-14}$$

$$c_x^f = D_f r_0 \sqrt{G_s \gamma_s / g}\, \overline{S_{u2}} \tag{4-15}$$

With $\overline{S_{u1}}$ and $\overline{S_{u2}}$ being factors defined in Table 4-1.

4.1.3 UNCOUPLED ROCKING MOTION

The below expressions for stiffness and geometric damping constants were developed by Novak for the pure (uncoupled) rocking motion for single piles:

$$K_\varphi^1 = \frac{E_p I}{r_0} f_{7,1} \tag{4-16}$$

$$c_\varphi^1 = \frac{E_p I}{V_s} f_{7,2} \tag{4-17}$$

With *I* being the moment of inertia of the pile cross-section about the axis of rotation and $f_{7,1}$ and $f_{7,2}$ factors for fixed-head piles obtained from Table 4-2.

As per *Arya* [8], "*group action in pure rocking is not as prevalent as in the translational modes*", and therefore interaction factors are not included in the solution for pile group.

4.1.3.1 Embedment of the pile cap

The stiffness and damping constants due to the embedment of the pile cap with regard to rocking motion are obtained from the following expressions:

$$K_\varphi^f = G_s r_0^2 D_f \overline{S_{\varphi 1}} + G_s r_0^2 D_f [(\delta^2/3) + (z_c/r_0)^2 - \delta(z_c/r_0)] \overline{S_{u1}} \tag{4-18}$$

$$c_\varphi^f = \delta r_0^4 \sqrt{G_s \gamma_s / g} \{\overline{S_{\varphi 2}} + [(\delta^2/3) + (z_c/r_0)^2 - \delta(z_c/r_0)] \overline{S_{u2}}\} \tag{4-19}$$

With $\overline{S_{\varphi 1}}$ and $\overline{S_{\varphi 2}}$ being factors defined in Table 4-1, $\delta = D_f/r_0$ and x_r and z_c are defined in Figure 15.

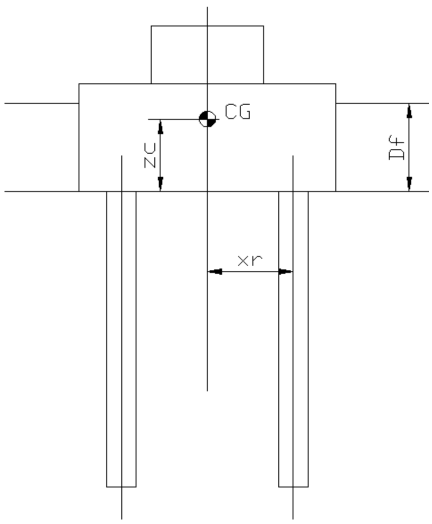


Figure 15 – Definition of x_r and z_c for determination of pile cap embedment impedances

5 GEOTECHNICAL CONSIDERATIONS – DYNAMIC SOIL PROPERTIES

The following sources are considered when developing this chapter: [10], [8], [4] and [17]

The dynamic analysis of foundations supporting vibrating machines depends directly on the soil behavior under dynamic stress. As seen in Chapter 4, the defined foundation stiffness and damping constants, depend on the following soil properties: dynamic shear modulus, Poisson's ratio and damping of soil. The unit weight (or soil density) is also required to define the relationship between the dynamic shear modulus and the shear wave velocity. These properties are likely impacted by the existence of water table variations. That aspect should be well assessed when computing the dynamic soil properties. Dynamic loads generated by machines, in opposition to blast or earthquake loading, generate low amplitudes of dynamic motion, and therefore low strains in the soil (strains less than 10^{-5}). Hence, the dynamic properties are defined for this strain level, particularly.

5.1 DYNAMIC SHEAR MODULUS

From the expressions defined in Chapter 4.1 one concludes that the dynamic impedances, stiffness and damping, have a strong dependency on the dynamic shear modulus, G_{max}. This parameter represents the slope of the shear stress versus the shear strain curve. The plot of the curve is not linear as most of the soils respond to shear strains in a combination of elastic and plastic strain. The dynamic shear modulus varies inversely with the strains level, being very high for low strain levels. There are three methods to obtain the dynamic shear modulus: in situ determination, laboratory determination and correlation with other soil properties. Due to ingrained variations of the dynamic shear modulus it is of good practice to consider an upper and a lower level of the dynamic shear modulus for the computation of the dynamic impedances (besides computation done for the G_{max} determined by the selected method).

5.1.1 IN SITU DETERMINATION

In situ determination is normally the appointed method to obtain the dynamic shear modulus. The parameters measured are elastic wave velocities that are subsequently related to other parameters for determination of the dynamic shear modulus. Hence, this is considered an indirect determination. The data is measured at the location of interest. Three types of elastic waves can be transmitted through soil:

- Compression (or P, primary) waves: transmitted through soil by a volume change associated with compressive and tensile stresses;
- Shear (or S, secondary) waves (usually the wave of interest): transmitted through soil by distortion associated with shear stresses in the soil;
- Surface (or Rayleigh) waves: occur at the ground surface having components both perpendicular and parallel to the surface.

Compression waves are the fastest of the three stress waves; shear waves are slower than compression waves; Rayleigh waves are 10% slower than the shear waves.

The following are the most common methods for field measuring wave velocities:

- The cross-hole method;
- The down-hole method;
- Seismic cone penetration test;
- The up-hole method;
- Seismic reflection (or refraction);
- Spectral analysis of surface wave (SASW).

The cross-hole method as shown in Figure 16 considers the drilling of two vertical boreholes, at a known distance apart from each other and at a certain depth. A shock-producing device (or small blast) is placed in one hole and a sensor device placed in the other hole (side or bottom). An impulse signal (shock or blast) is generated in one hole and subsequently the time the shear wave takes to travel from the sock producing device to the sensor device is measured. The travel time divided by the distance defines the shear wave velocity. It can be used to determine G_{max} at different depths.

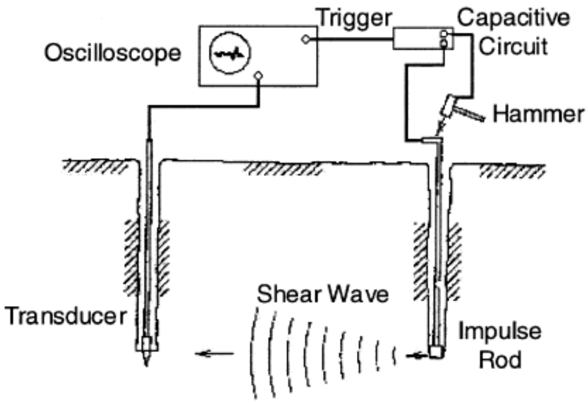


Figure 16 - Schematic of cross-hole technique (from ACI 351-3R [4])

The down-hole method as shown in Figure 17 considers only one vertical borehole. A shock device is placed at the ground surface at a known distance away from the borehole. A shock detector is placed in the bottom of the borehole (at a known depth). A shock is applied, and subsequently the time the shear wave takes to travel from the signal generator to the sensor is measured. As with the cross-hole test, the travel time divided by the distance defines the shear wave velocity. The detector device can be placed at several depths with the signal generator located at different distances from the borehole each time, until a reasonably average value of V_S is obtained.

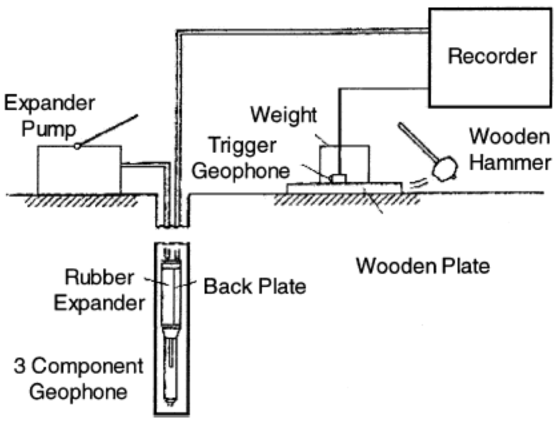


Figure 17 - Equipment and instrumentation for down-hole survey (from ACI 351-3R [4]).

The seismic cone penetration test (SCPT) considers a small seismometer inside the cone penetrometer. The cone is pushed to a certain depth with subsequent application of a shock using a hammer and striking plate. This test is like the down-hole test in terms of theoretical approach. This method does not require a borehole.

The up-hole method is similar to the down-hole method but considering, the signal generator placed in the borehole and the sensor placed at the ground surface instead. Other methods such as seismic reflection or SASW are not described herein but can be found in *Briaud* (2013) [17].

Dynamic shear modulus is obtained from the following expression:

$$G_{max} = \rho(V_S)^2 \tag{5-1}$$

Considering: G_{max}, the dynamic shear modulus of the soil, (Pa); V_S, the shear wave velocity of the soil, (m/s); ρ, soil mass density, (kg/m^3).

5.1.2 LABORATORY DETERMINATION

The shear modulus can be estimated from Laboratory tests such as the resonant-column tests (described in some detail in ASTM D 4015). Laboratory test are not as accurate as field measurements since there is a possibility of sample disturbance. They are used mostly as a validation methodology of field tests, for circumstances that require a higher level of confidence in the results (e.g. nuclear power plants).

5.1.3 CORRELATION TO OTHER SOIL PROPERTIES

Dynamic shear modulus can also be obtained by correlations (empirical) with other soil properties [17]. Several expressions for G_{max} have been formulated. Hardin and Drnevich (1972)[18] and Hardin (1978)[19] proposed for all types of soils (after *Briaud (2013)* [17]):

$$\frac{G_{max}}{P_a} = \frac{625}{0.3 + 0.7e_v^2}(OCR)^k\left(\frac{\sigma'_M}{P_a}\right)^n \tag{5-2}$$

Considering: P_a, the atmospheric pressure; e_v, the void ratio; *OCR,* the over-consolidation ratio; σ'_M, the mean effective normal stress; and *k* and *n*, exponents. *Jamiolkowski et al.* (1991) [20] proposed:

$$\frac{G_{max}}{P_a} = \frac{625}{e_v^{1.3}}(OCR)^k\left(\frac{\sigma'_M}{P_a}\right)^n \tag{5-3}$$

The exponent *n* is usually taken equal to 0.5, and k is given in Table A 1 (see Annex). *Seed and Idriss* (1970)[21] proposed the following expressions for sands:

$$\frac{G_{max}}{P_a} = 22.4K_{2,max}\left(\frac{\sigma'_M}{P_a}\right)^{0.5} \tag{5-4}$$

Considering: $K_{2,max}$, the modulus number given in Table A 2 for sands (see Annex). For gravel, $K_{2,max}$, is higher than for sands, ranging between 80 and 180. Kramer (1996)[22] , considers relating G_{max} to the OCR and the undrained shear strength s_u measured in a CU triaxial test, for fine-grained soils (Table A 3 in Annex). G_{max} can also be obtained correlating with the results of in situ tests, such like CPT and SPT. Ohta and Goto (1976) [23] and Seed et al. (1986)[24] proposed, for sands:

$$\frac{G_{max}}{P_a} = 447(N)^{0.33}\left(\frac{\sigma'_M}{P_a}\right)^{0.5} \tag{5-5}$$

Considering: *N*, the SPT blow count corrected for 60% of maximum energy and corrected to 1000 kPa of pressure. Similar correlations were done with the CPT. For quartz sands, Rix and Stokoe (1991)[25] proposed a correlation with the CPT point resistance q_c:

$$\frac{G_{max}}{P_a} = 290\left(\frac{q_c}{P_a}\right)^{0.25}\left(\frac{\sigma'_M}{P_a}\right)^{0.375} \tag{5-6}$$

Mayne And Rix (1993)[26] proposed, for clay:

$$\frac{G_{max}}{P_a} = 100\left(\frac{q_c}{P_a}\right)^{0.695} e_v^{-1.13} \tag{5-7}$$

The correlation methods are usually the least appropriate methods and should be used for preliminary design. The best way to obtain the dynamic shear modulus is through field tests. Some common values of G_{max} are given in Table A 4 (see Annex).

5.2 POISSON'S RATIO

Poisson's ratio μ, is used to calculate the dynamic impedances - soil stiffness and damping. It can be determined from the field measured values of wave velocities traveling through the soil – for soils above the water table.

$$\mu = \frac{1}{2}\left(\frac{(V_C/V_S)^2 - 2}{(V_C/V_S)^2 - 1}\right) \tag{5-8}$$

Considering: V_C, the compression wave velocity of the soil (m/s), and V_S, the shear wave velocity of the soil (m/s).

The Poisson's ratio is often estimated, as to obtain it in laboratory can be rather difficult. There is low dependence of the stiffness and damping of a foundation regarding the variations of the Poisson's ratio in the range of values common for dry or partial saturated soils (0.25 to 0.4). In contrast, the dynamic vertical and rocking responses of a foundation are very sensitive to Poisson's ratios greater than 0.4.

If the Poisson's ratio value is not provided in the geotechnical report, values from 0.25 to 0.40 should be used for cohesionless granular (free-draining) soils, and from 0.33 to 0.5 for cohesive (impermeable) soils. For fully saturated soils below the water table, a value of approximately 0.5 should be used. A practical approach whenever the value of Poisson's ratio is not available is to consider (based on the in situ predominant soil type), a Poisson's ratio equal to 0.33 for cohesionless soils and 0.40 for cohesive soils.

5.3 DAMPING OF SOIL

Damping is defined as energy dissipation that opposes free vibrations of a system. Damping of soil assumes the form of geometric and material damping.

5.3.1 MATERIAL DAMPING

The material component represents the energy dissipated by the soil during the vibrations induced by the machine-foundation system. The following mechanisms contribute to material damping: friction between soil particles, strain rate effects and nonlinear soil behavior. Material or hysteretic damping is defined by the following expression:

$$D = \frac{W_D}{4\pi W_S} \tag{5-9}$$

Considering: W_D, the energy dissipated in one cycle of loading; W_S, the maximum strain energy stored during the cycle. The area inside the hysteresis loop is W_D and the area of the triangle is W_S.

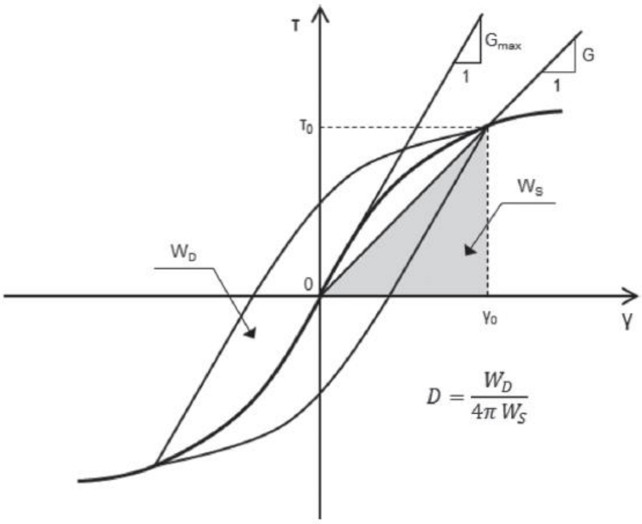


Figure 18 - Hysteresis loop for one cycle of loading showing G_{max}, G, and D. (from ACI 351-3R [4]).

The material damping is frequency-independent, not being modeled using a viscous dashpot. In order to incorporate the material damping ratio (D_m) in the dynamic calculations for machine foundations, the shear modulus, *G,* is replaced by the complex shear modulus, *G*,* defined below:

$$G^* = G(1 + 2iD_m) \tag{5-10}$$

The low strain or minimum material damping (D_{min}) should be used for the design of machine foundations, even though material damping increases with the shear strain. The minimum material damping (D_{min}) can be obtained in laboratory (e.g. resonant column), being however a difficult process. The laboratory estimations can be replaced by the consideration of the low-strain minimum soil damping ratio equal to 1 percent or less.

5.3.2 RADIATION DAMPING

Radiation damping (also termed geometry damping) is a mechanism where energy seems to be lost due to wave propagation. It reflects the dissipation of elastic energy inherent to a foundation vibrating in an elastic half-space (even when the half-space has no material damping). The vibration of the foundation results in elastic waves (body and surface waves) with significant energy content, that propagate continuously away from the structure. Consequently, there is dissipation of energy, because the energy carried by the waves is lost in the half-space medium. This causes a decrease of the vibration - due to damping.

Radiation damping is frequency-dependent, being usually modeled using a damper/dashpot (defined in Chapter 4.1).

6 PRELIMINARY DESIGN METHODOLOGIES

The following sources are considered when developing this chapter: [10], [8] and [4]

Different approaches to deal with the design of a dynamically loaded structure have been developed along the time. From very simplistic analysis, assuming rules of thumb (adopting geometries), to more detailed analysis considering equivalent static loading computations, to the current state of the art, on which detailed dynamic analysis are performed.

Regardless of the method appointed, the dynamic response of the foundation shall be properly assessed such that it meets the acceptance limits regarding vibration criteria. The following steps comprise the design of a machine foundation (*As per ACI 351-3R* [4]):

- Developing a preliminary size for the foundation using rule-of-thumb approaches or equivalent static loading method, past experience, machine manufacturer recommendations, and other available data;
- Calculation of the vibration parameters: natural frequency, amplitudes, velocities, and accelerations, for the preliminarily sized foundation using a detailed dynamic analysis;
- Checking of the computed parameters against accepted limits or project specific vibration performance criteria;
- Implementation of appropriate modifications (if required) in the foundation design to diminish vibration responses for compliance with the specific vibration performance criteria (and cost);
- Verification of the structural integrity of the concrete foundation and machine-mounting system;
- In case the dimensions are excessive regarding common state of the art sizing or layout (for operations purpose), evaluate if vibration mitigation measures like vibration isolation are required or beneficial.

From a cost point of view, reducing vibrations at a design stage is always less expensive than applying corrective measures at a later stage when the machine is in operation.

The following design guidelines are normally considered to mitigate the risk of machine vibrations (either from the point of view of the designer of the foundation, or from the point of view of the machine manufacturer):

- The machine should be properly balanced, and the speed should be adjusted to avoid dynamic forces of high magnitude;
- Case by case, the machine can be isolated from the foundation through special mountings like springs and flexible foundation mats;
- The machine foundation geometry should be defined such that the dynamic response is minimized (e.g. the machine should be positioned close to the ground to reduce rocking vibrations; the center of gravity of the machine should be close by the center of foundation rigidity (5% maximum eccentricity is normally established);
- Mass and stiffness of the machine foundation should be defined closely. The increase of mass beyond certain limits might have a harmful effect in the decrease of the vibration response. Normally certain ratios of mass of the foundation to the mass of the machine are defined as a starting point. If required,

an increase of stiffness can be performed recurring to deep foundations (piles or caissons);

- The machine foundation should be Isolated from the surrounding equipment and structures, materialized with an expansion joint. For sensitive surrounding items, other measures can apply, such as pile or trench barriers.

The design procedure of a machine foundation starts with the selection and assessment of the foundation type. The foundation is then evaluated regarding size and location. The foundation type is mainly governed by the local geotechnical conditions and layout operation requirements. The machine characteristics, such as foot print, weight, magnitude of dynamic forces and operating speed govern the size of the foundation. The location is mainly defined according to local environmental conditions and layout operation requirements. It is therefore necessary that certain information is available before the start of the foundation design, comprising the three types of parameters described: Soil, Machine and Environment.

The following data should be available for machine foundation design:

- Machine characteristics and machine-foundation performance requirements:
  - Weight of the machine and its moving components
  - Information regarding the center of gravity in both vertical and horizontal dimensions
  - Functions of the machine
  - Operating speed ranges of the machine
  - Magnitude and direction of the unbalanced forces and moments
  - Vibration acceptance criteria regarding displacements and velocities limits (occasionally acceleration as well)
- Geotechnical information
  - Allowable soil-bearing capacity
  - Effect of vibration on the soil - in case there is settlement or liquefaction risk
  - Classification/type of soil (soil stratigraphy)
  - Modulus of subgrade reaction
  - Dynamic soil shear modulus
- Environmental conditions
  - Existing vibration sources like existing vibrating equipment, vehicular traffic, or construction activities
  - Human susceptibility to vibration or vibration-sensitive equipment
  - Flooding, high water table, seismic hazard, wind, snow, environmental and operation temperature variations

Rule-of-thumb is the first approach when starting the design of a machine foundation. With the design parameters described above gathered, the geometry of the foundation can start being defined according to some state-of-the-art rules. This method considers the definition of the foundation block (resting on the soil or on piles) with sufficient mass so that the vibration is attenuated and absorbed by the foundation and soil system. Machines supported on elevated structures are out of the boundaries of application of this methodology, requiring instead a dynamic analysis. This methodology returns acceptable results for rotating and reciprocating machines, considering it as a first attempt to size the foundation. The smaller the machine and the unbalanced dynamic forces, the better the accuracy of the results obtained. For machines as the one defined in the case study, Chapter 9, the procedure is used only for preliminary sizing. Therefore, a detailed dynamic analysis should be performed to analyze the response of the foundation. Normally, for machines above 2270 kg, a detailed analysis is required (*ACI351-3R-18* [4]).

A common rule adapted by engineers for machinery on block-type foundations is to make the mass of the foundation block at least three times the mass of a rotating machine and at least five times the mass of a reciprocating machine. In case of pile-supported foundations, the ratios are reduced. The foundation block mass, including pile cap, is defined with at least 2.5 times the mass of a rotating machine and at least with four times the mass of a reciprocating machine. These ratios include both moving and stationary parts of the machine as compared with the mass of the concrete foundation block.

Another common practice is to define a foundation of a certain weight such that the resultant of lateral and vertical loads (static loads) falls within the middle one-third of the foundation base. This meaning that the eccentricity of the vertical load should not cause lack of compression under any part of the foundation. In case of pile-supported foundations, the piles should be equally loaded. Therefore, a limited maximum eccentricity of 5% is usually considered between the center of gravity of the combined machine foundation system and the center of resistance (center of stiffness) of the foundation (or pile-supported foundation). Besides the structural requirements the geometry of the foundation should be sufficient to accommodate the equipment, operation and maintenance areas. The minimum width of the foundation should be at least 1.5 times the vertical distance from the machine centerline to the bottom of the foundation block, this to guarantee a better distribution of soil pressures underneath the foundation. It is of good practice, to limit the allowable soil-bearing pressure for dynamic machines to one-half of allowable soil bearing pressure for foundations supporting static equipment. The same principle applies to pile-supported foundation, on which the pile capacity is limited to 50% for static loading.

A thickness criterion is usually adapted by design engineers. The minimum thickness of the foundation block is considered to be one-fifth of its width (short side), one-tenth of its length (long side), or 0.6 m, whichever produces the largest foundation thickness. This rule is defined mainly to enforce that the foundation behaves as a rigid body. However, such rule is not of generic usage since a thinner section may be sufficient for soft soils, while a thicker section might be required for stiffer soils. Nevertheless, foundations not behaving as rigid can always be analyzed by other means, such as finite element analysis.

Another important aspect to consider during the preliminary design phase, is to avoid that the dynamically loaded foundation is placed on top of a building foundation or in such locations where the dynamic effects can be transferred to the building foundations.

7 MITIGATION MEASURES FOR MACHINE-INDUCED VIBRATIONS

The following sources are considered when developing this chapter: [1], [2] and [3]

Different types of machines produce specific vibration for which specific countermeasures apply. Often, rotating and reciprocating machines are treated the same way. On the other hand, impacting machines require special measures specific for the intermittent motion. The most effective and common mitigation measure for vibration treatment is separating the frequencies by tuning. It might be also required to apply other special measures like vibration absorption and isolation. Regardless, the mitigation of vibration should be applied at its source, keeping the unbalance masses low. The prevention of vibrations from being transmitted to the vicinity of the vibrating machines is commonly termed as active isolation. In contrast, the action of preventing external vibrations from reaching an object is designated as passive isolation.

7.1 MACHINES WITH ROTATING AND OSCILLATING PARTS

In most cases of rotating and oscillatory (reciprocating) motion, adjusting the fundamental frequency of the supporting foundation (or structure) so that it is not coincident with the operating frequency nor with higher harmonic is effective. For different types of motion and operating frequency different measures apply. *Bachmann and Ammann, 1987* [2], suggest the following categories below for differentiated vibration mitigation measures:

Group 1:

- Low to medium operating frequency (operating speed = 1 to 600 rpm, f = 0.02 to 10 Hz);
    - e.g. reciprocating pumps and compressors, weaving machines, rotary presses, etc.

Group 2:

- Medium to high operating frequency (operating speed = 300 to 900 rpm, f = 5 to 15 Hz);
    - e.g. large diesel engines, blowers, certain weaving machines, etc.

Group 3

- High operating frequency (operating speed > 1000 rpm, f > 15 to 20 Hz);
    - e.g. turbines, small diesel engines, centrifugal separators, vibrators, etc.

7.1.1 FREQUENCY TUNING

The main drivers affecting the natural frequency of the foundation are the machine characteristics, the in-situ soil characteristics and the foundation type. The design parameters of the three drivers have a certain level of uncertainty that might affect the confidence level of the computed results of natural frequencies, amplitudes and forces. For example, soil data might be impacted by the soil test methods, type and quality of instrumentation, effect of embedment, influence of the ground water table, among other aspects. On the other hand, design machine parameters can vary due to late changes in machine design. Due to these variations opposing the actual design data, the computed natural frequencies shall be kept sufficiently away from the operating speeds avoiding resonance phenomena. Based on practical experience, a frequency margin of 20% is usually adapted, this meaning that the ratio of combined natural frequencies of the foundation and machine is kept 20% away from the operating speeds and harmonics from the machine. The 20% margin can be plotted in the Figure 19, for the variation of the Magnification factor, *M,* with the frequency ratio, *r.* The 20% margin guarantees a substantial reduction in the magnification factor (even for zero damping), ensuring a reduction in the amplitudes of vibration (and forces, velocities and accelerations).

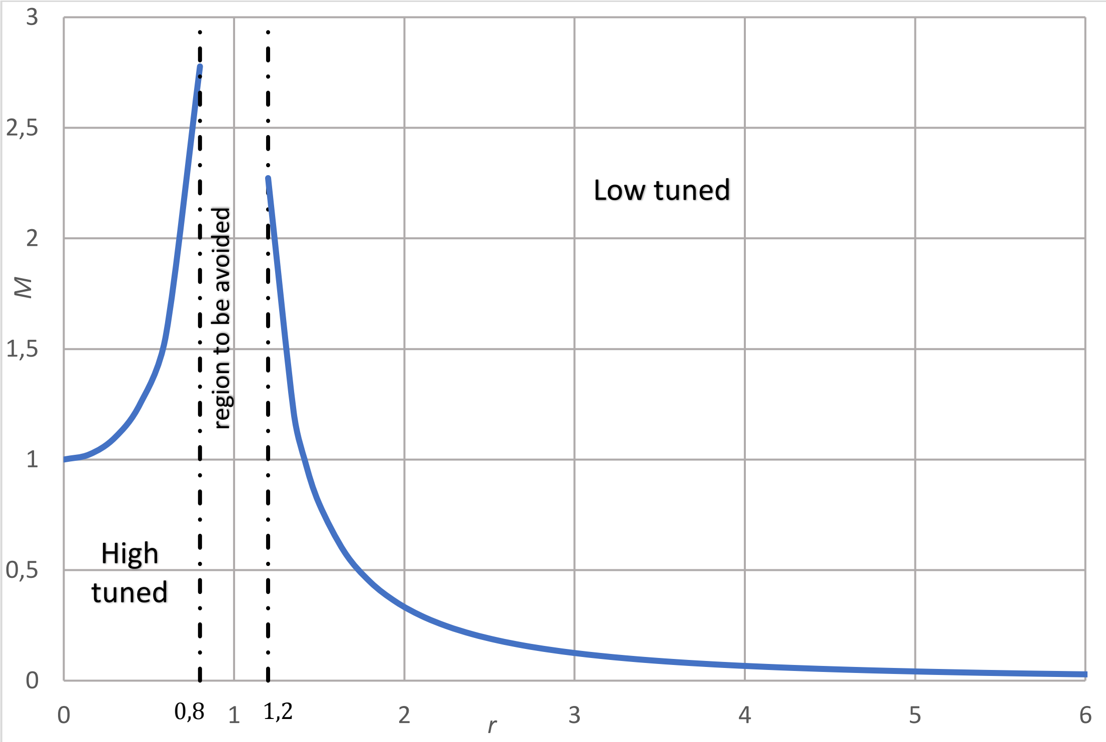


Figure 19 - Magnification Factor M vs. Frequency Ratio r ± 20% Frequency Margin Region

Rotating and reciprocating machines normally allow low tuning and as well as high tuning. The following parameters are therefore important for the frequency tuning to avoid resonance phenomena:

- the dominant natural frequencies of the structure and of the member immediately under the machine, considering spring-damper elements or a base slab (designated machine base, in Figure 20)
- the frequencies of the dominant dynamic load components (operating frequency of the machine and relevant higher harmonics).

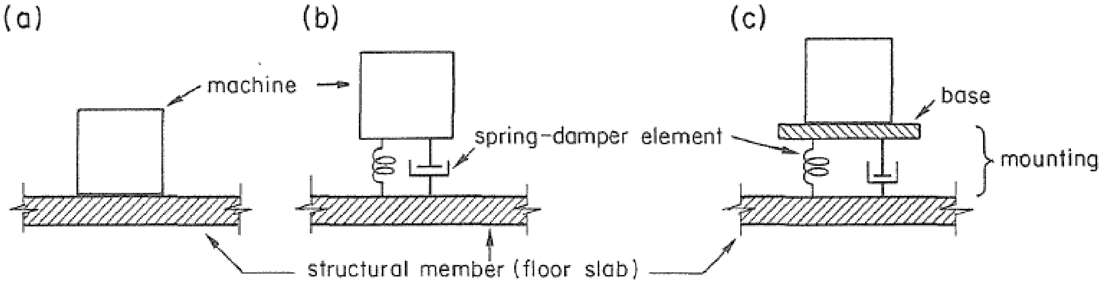


Figure 20 - Low-tuning of machinery with rotational or oscillatory motion:(a) direct mounting on a structural member (b) support on springe-damper elements (c) mounting on a vibration-isolated base [2]

In case the fundamental frequency f_1 of the isolated system is substantially below the operating frequency f_b of the machine, the system is considered under-tuned. On the contrary, if the fundamental frequency f_1 of the isolated system is considerably above the highest frequency component with a significant contribution to the dynamic loading (higher harmonics of the operating frequency), the system is considered over-tuned.

7.1.1.1 Theoretical Background

The load excitation (quadratic[2] or constant) exerted by a machine on its base can be visualized as composed of a spring force and a damping force. In case a damped spring mass system, with mass m, stiffness *k* and damping *c*, is subjected to dynamic excitation, the following two cases can be considered [1]:

a. Dynamic Excitation Force $F_E(t)$ is applied at the mass and the Transmitted Force at the base (foundation) is $F_T(t)$ as shown in Figure 21 (A).
b. Dynamic Excitation Force $F_E(t)$ is applied at the base (foundation) and the Transmitted Force at the mass is $F_T(t)$ as shown in Figure 21 (B).

The main objective is to have a small force transmitted from the mass to the foundation (case A) or from the foundation to the mass (case B).[1]

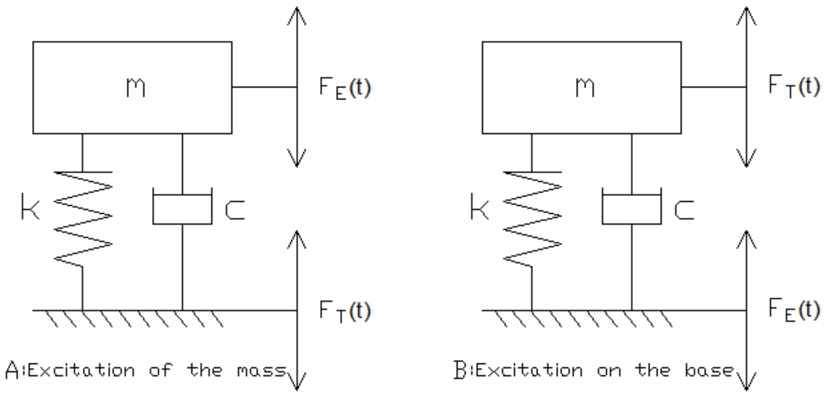


Figure 21 - SDOF Spring Mass System: (A) excitation on mass; (B) excitation at base (reproduced from Bhatia, 2008 [1]*)*

The Transmissibility Ratio, Tr, is defined as the ratio of the transmitted force to the excitation force:

$$T_r = \frac{F_T(t)}{F_E(t)} \quad (7\text{-}1)$$

For vibration isolation the objective is that the transmitted force is the lowest possible. This meaning that the term, T_r, should be minimum, knowing that T_r depends upon the dynamic response of SDOF system.

This transmissibility T_r is plotted in Figure 22 for various damping ratios.

[2] Quadratic excitation is always related to out-of-balance forces (e.g. rotating-mass excitation, such as $F=m_r e\omega^2 \sin(\omega t)$)

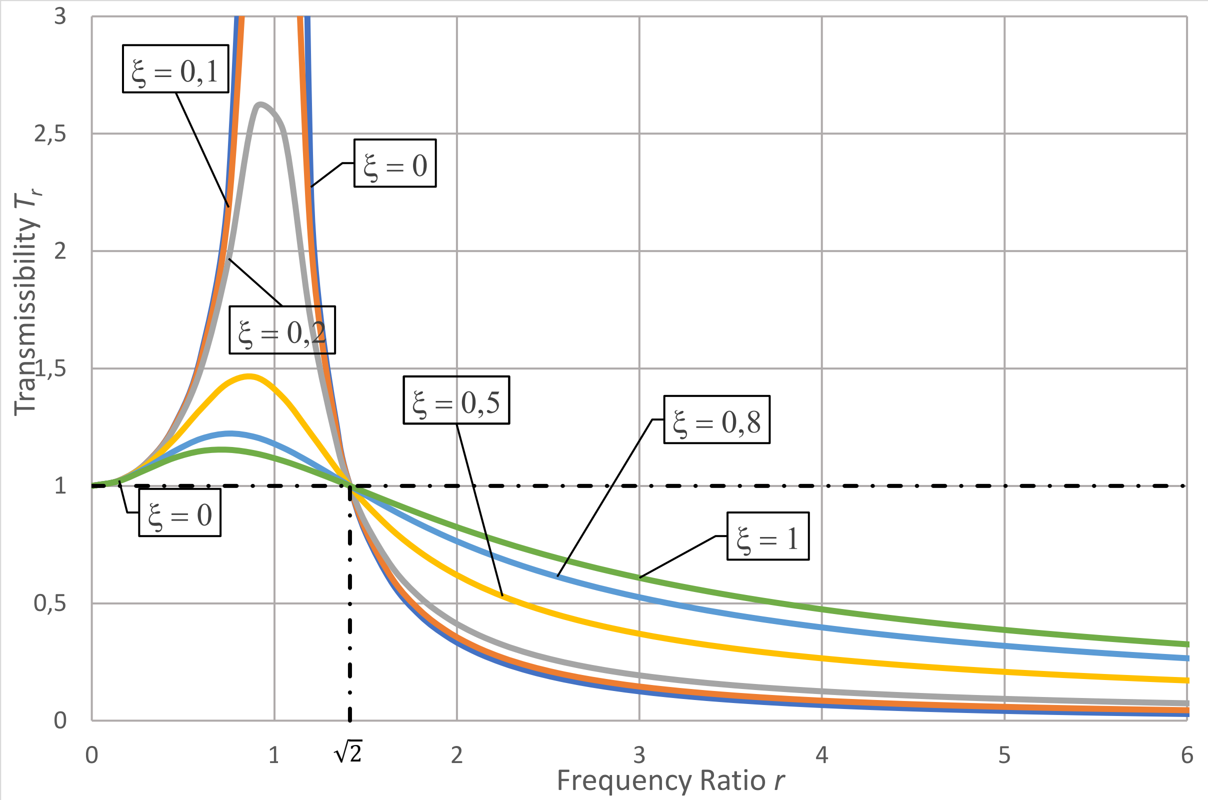


Figure 22 - Transmissibility for various damping ratios as depending on the tuning ratio between operating and structural frequency

Considering the case A in Figure 21 where the dynamic excitation Force $F_E(t)$ is applied at the mass and the Transmitted Force $F_T(t)$ at the base (foundation), two distinct situations may arise: the dynamic force being externally applied or internally generated by the machine itself:

- If the dynamic force is externally applied:

The constant excitation force is:

$$F_E(t) = F_0 \sin \omega\, t \tag{7-2}$$

In such case the maximum transmitted force $F_T(t)$ to the support is:

$$F_T = F_0 \frac{\sqrt{1 + (2r\xi)^2}}{\sqrt{(1 - r^2)^2 + (2r\xi)^2}} \tag{7-3}$$

Considering ξ, the damping constant and $r = \omega/p$, the frequency ratio.

And the Transmissibility Ratio T_r comes as:

$$T_r = \frac{F_T}{F_E} = \frac{F_T}{F_0} = \frac{\sqrt{1 + (2r\xi)^2}}{\sqrt{(1 - r^2)^2 + (2r\xi)^2}} \tag{7-4}$$

- If the dynamic force is internally generated:

The quadratic excitation force is:

$$F_E(t) = F_0 = m_r\ \mathrm{e}\omega^2 \qquad (7\text{-}5)$$

$$F_T = m_r\ \mathrm{e}\omega^2 \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{(1-r^2)^2+(2r\xi)^2}} \qquad (7\text{-}6)$$

With: m_r, rotating mass; ω, circular operating frequency of the machine (rad/s); and e, eccentricity of the unbalanced mass to axis of rotation at operating speed.

Considering case B:

The maximum value of transmitted force is given by the following equation:

$$F_T = -m\ \ddot{y}_g^{\,2} \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{(1-r^2)^2+(2r\xi)^2}} \qquad (7\text{-}7)$$

Considering the dynamic excitation force as follows:

$$F_E(t) = F_0 = -m\ \ddot{y}_g^{\,2} \qquad (7\text{-}8)$$

Considering: $\ddot{y}_g$, the ground acceleration

In any of the cases, the dynamic force being applied on the mass or applied at the base - the transmitted force remains the same for the same system characteristics of the SDOF system. The transmissibility is the same for passive isolation, and therefore can be used for both kinds of isolation – as demonstrated in equation (7-7).

Instead of the transmissibility ratio T_r one may prefer to work with the complementary part, which is the isolation efficiency, η, defined as:

$$\eta = 1 - T_r = 1 - \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{(1-r^2)^2+(2r\xi)^2}} \qquad (7\text{-}9)$$

From the above equation it is clear that a reduced value of the Transmissibility Ratio T_r provides a high value of the Isolation Efficiency, η. With the previous relationships, the Isolation Efficiency, η, can be computed for different values of frequency ratio (for $r \geq 2$), r, and different values of isolator damping, ξ:

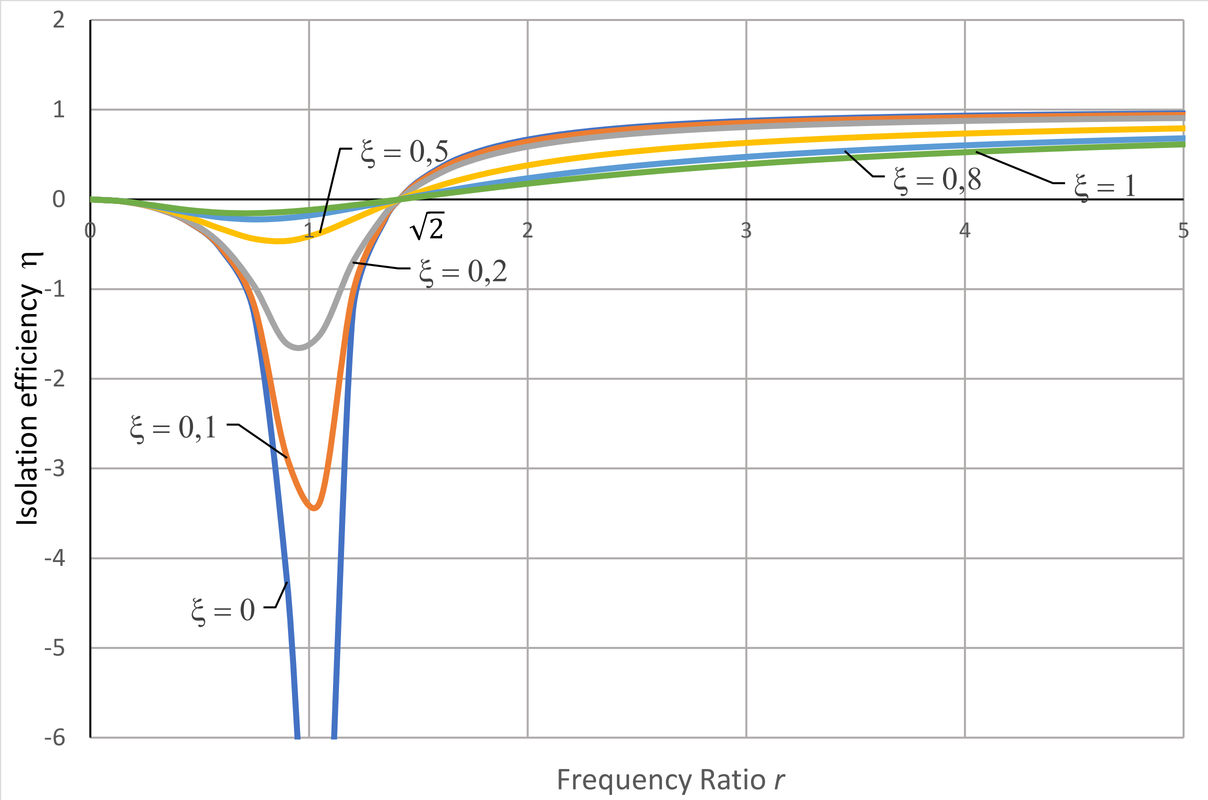


Figure 23 - Isolation Efficiency η vs Frequency Ratio r for different Damping Values of ξ

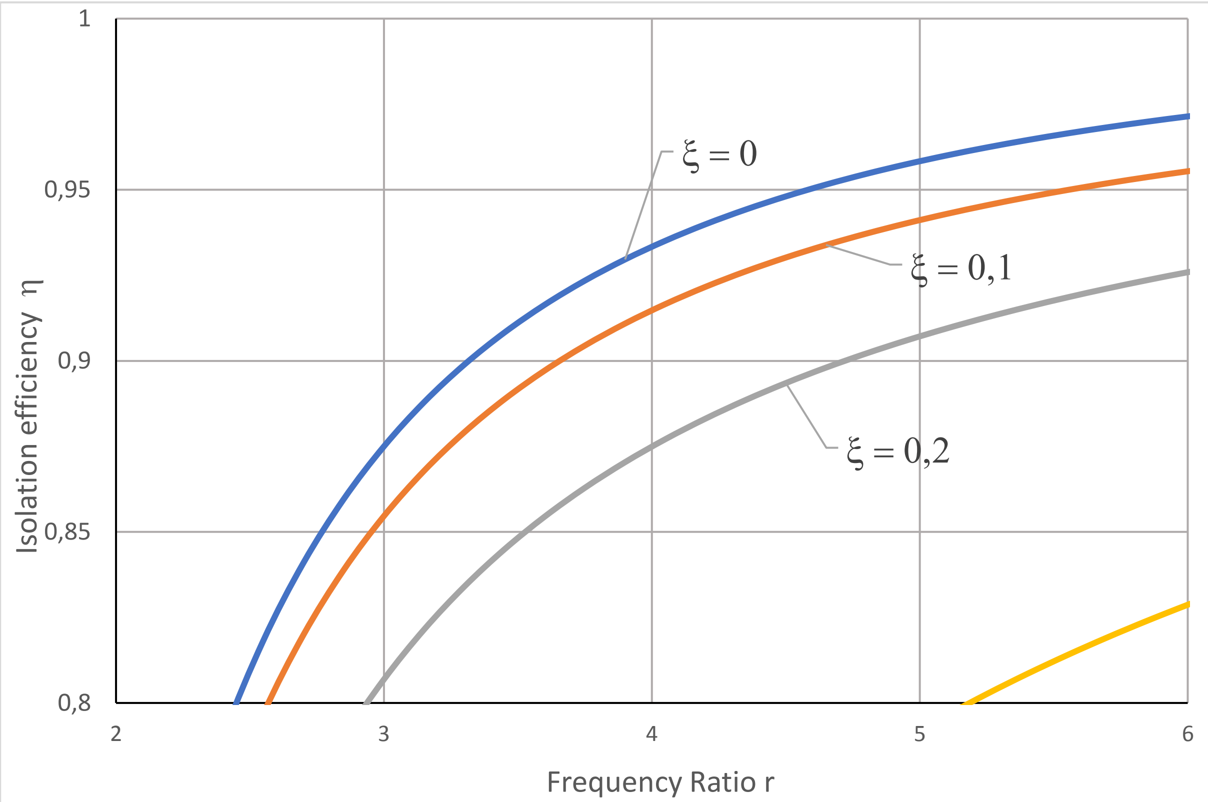


Figure 24 - Isolation Efficiency η > 80% vs Frequency Ratio r > 2

The transmissibility T_r and the Isolation efficiency η, plotted in Figure 22 and Figure 23, respectively, for various damping ratios elucidate the following:

- Transmissibility Ratio T_r < 1 for frequency ratio r > $\sqrt{2}$
- For effective isolation, the value of the frequency ratio *r* should be as high as possible
- The natural frequency of the isolated system should be the lowest possible regarding the forcing frequency
- Influence of damping is advantageous only up to the tuning ratio $\omega/p \leq \sqrt{2}$
- For frequency ratio greater than $\sqrt{2}$, T_r decreases with decrease in the damping value (T_r is lower for zero damping compared to 10% damping). This meaning, damping reduces the isolation effect.
- For frequency ratio smaller than $\sqrt{2}$, T_r > 1, which is not desirable. It is recommended that the frequency ratio be at least equal to two in all cases of vibration isolation.

Normally, for machine foundation applications, the objective is to have an isolation above 85 % (so that it is effective). This means the frequency ratio shall be bigger than 2, as shown in Figure 24. In the same figure, one can see that a maximum isolation efficiency of η = 97 % is achieved for a frequency ratio, r = 6. In case damping is present the value would be slightly reduced.

In case damping is negligibly small and for $\omega/p > \sqrt{2}$:

$$T_r = \left| \frac{1}{1 - \left(\frac{\omega}{p}\right)^2} \right| = \left| \frac{1}{1 - r^2} \right| = \left| \frac{p^2}{\omega^2 - p^2} \right| \quad \text{(7-10)}$$

The eigenfrequency $p = \sqrt{k/m}$ of the isolated system can be related through $m = G/g$ to the combined weight of machine and base and therefore to the static deflection produced in the spring-damper elements:

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{u_{st}} . \frac{2 - (1 - T_r)}{1 - (1 - T_r)}}. \quad \text{(7-11)}$$

with:

- *f* frequency of the dynamic loading
- *g* acceleration of gravity (9.81 m/s²)
- u_{st} static displacement under the weight of machine and base.

In case the excitation frequency is known (*f*), the necessary support displacement (δ) to achieve any desired level of isolation efficiency η (1-T_R) can be determined, assuming that the isolators have little damping (Figure 25):

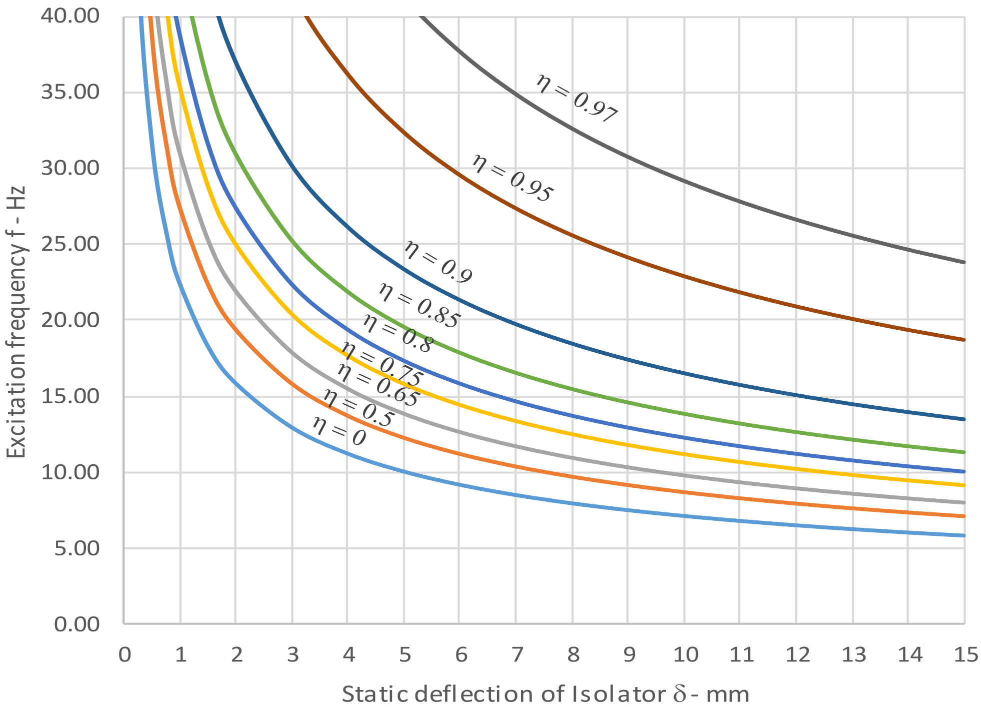


Figure 25 - Efficiency of vibration isolation depending on the static displacement due to machine weight and operation frequency

7.1.1.2 Low Tuning

High-speed machines should have a foundation under-tuned, this meaning that the natural frequency of the foundation is below the operating speed of the machine. Nevertheless, under-tuned foundations will always experience resonance during every startup and shutdown of the machine due to its natural frequency being less than the operating speed. Such condition is designated as transient resonance and computation of amplitudes during transient resonance is therefore necessary.

Low tuning reduces the reaction forces considerably. Still, the resulting soft support yields large displacements even under static load. Certain conditions must be fulfilled for low tuning to be effective (after *Bachmann and Ammann, 1987* [2]):

- The operation speed of the machine should be more than 4 to 6 Hz;
- Higher structural eigenfrequencies must not match the dominating harmonic of the dynamic load;
- The soft base of the machine must not cause manufacturing/operation problems;
- During start-up and shut-down of the machine, crossing the fundamental and higher natural frequencies of the base system, large vibrations shall be avoided.

The first condition imposes a limitation on the degree of softening of the base (Figure 21) which can be achieved. The limit is frequently reached with a stiffness resulting in an isolation-system frequency of about 1.5 to 3 Hz – in case the minimum operating frequency is about 4 to 6 Hz. This assuming a tuning ratio of around 2 is achieved. Nevertheless, a tuning ratio between 2.5 and 4, results in smaller reaction forces.

The second condition avoids resonance phenomena in the higher frequencies of the base.

The third condition is a production requirement, since a softer support yields larger displacements of the machine.

The fourth condition can be guaranteed by specific instructions on start-up and shut-down of the machine, and with braking devices in the slowing-down phase or enhanced damping of the base.

From the three machine frequency groups defined in chapter 7.1, the third machines group, corresponding to machines with high operating frequencies, is most suited for low tuning. For a tuning ratio near the lower limit (2.0 to 2.5), low tuning can also be applied to the second group of medium to high-frequency machines.

There are three possible ways to materialize low tuning of a foundation (according *Bachmann and Ammann, 1987)*:

i. Direct mounting of the machine on the structural member (Figure 20a)

Since the structure or its structural members have a limited flexibility, this type of solution is suitable only for high-speed machines (group 3). In case the structure or structural members are designed to achieve a considerable flexibility (providing a softer response: 2 to 3 Hz), the resulting displacement amplitudes might then be high enough to critically affect its operation by the personnel and the functioning of the machine.

ii. Mounting of the machine on spring(-damper) elements (Figure 20b)

In this case the frame of the machine shall be designed sufficiently stiff in order to equalize the differences in motion of the individual support points. Additionally, the basic vibration behavior of the machine must remain unchanged (e. g. without introduction of new torsional vibrations). This type of solution is usually defined by the machine manufacturer, since the spring(-damper) elements can easily be adapted in case of inefficient isolation. The spring (-damper) elements depend on the machine characteristics (dimensions, mass, genuine stiffness, operating frequency, etc.). Examples of spring(-dampers) are steel springs, elastomeric or rubber pads, and air suspension.

iii. Mounting on a vibration-isolated base (Figure 20c)

This applies for machines with:

- high magnitude of dynamic loads;
- insufficient stiffness for isolation;
- very small mass which would demand extremely soft springs for low tuning
- very eccentric centre of mass.

The additional mass of the base allow for the inclusion of stiffer spring (-damper) elements than in case of direct support, hence reducing the vibration amplitudes. The additional vibration-isolated mass should have at least the same weight as the mass of the machine alone.

Despite being very advantageous for isolation, mounting on a machine base is relatively costly. Special care shall be given to ensure sufficient stiffness of the base itself.

7.1.1.3 High Tuning

For low speed machines it is desirable to have over-tuned foundation/supporting structures, this meaning keeping its vertical natural frequency above the operating speed of the machine. For such foundations/structures one should check the possibility of resonance with higher harmonics of the machine. It is desirable, though not essential, to avoid resonance with the higher harmonics. Where that is not fulfilled, high vibration due to resonance with component frequencies corresponding to twice and thrice the operating speed of the machine may occur.

The sole solution for high tuning is to provide a rigid connection to the structure. Such solution is considered in cases in which low tuning is not possible. The requirements for high tuning are:

- The highest relevant load component shall not have a frequency bigger than 20 Hz.
- Machine vibrations are to be kept small according to productions' requirements.

The first condition reflects the fact that providing high-stiffness to a foundation/structure, even if possible, will have a direct impact in cost, layout and execution. The tuning ratio shall be increased by an additional safety factor, to cover any uncertainty resultant from the computational prediction of the structural frequency. The lower bound to the fundamental frequency of the structure or the respective structural member can be defined as:

$$f_1 \geq S_f \, n_{max} f \, \frac{p}{\omega}. \tag{7-12}$$

with:

f_1 - required fundamental structural frequency

f - operating frequency of the machine (rpm/60)

n_{max} - order of the highest relevant harmonic of the loading

p/ω - reciprocal of the tuning ratio

S_f - safety factor (usually taken as 1.1 to 1.2).

The expected frequency of stiffly designed structures shall take into consideration the flexibility of the ground. In certain conditions it might reduce considerably the combined frequency of the soil-structure system.

Figure 22 shows that the forces under high tuning always have a factor $T_r > 1$ regarding the rigid base (equivalent to the static case). For machines part of the group 1 (lowest frequency), high tuning is the preferred measure for vibrating mitigation.

7.1.2 VIBRATION ABSORPTION AND ISOLATION

In certain situations, where the manufacturer's permissible amplitudes of motion are much smaller than those considered normally for uninterrupted machine operation, the adjustment of the mass (inertia) or of the rigidity (by adjusting the contact area or considering a solution on piles) might result ineffective. In such cases, the incorporation of absorbers on the foundation might be the solution. Different types of absorbers are used for this purpose. Regarding the material type, they can be made of rubber, cork, felt, neoprene pads or steel springs. Pneumatic absorbers can also be used for this purpose.

In other cases where the amplitudes of vibration of the machine are within the acceptable limits for the performance of the machine, the resulting vibrations may affect the performance of other equipment and be harmful to people or nearby structures. The foundation becomes a source of wave generation in the soil mass. The harmful effects will always depend on the operating frequency of the machine, the amplitude of motion of the foundation, and the nature of the soil. In these situations, a wave screening such as pile or trench barriers can be considered. This solution can be applied either around the source of vibration (active isolation) or around the item (structure or equipment) required to be protected against vibrations (passive isolation). This applies normally to sensitive equipment that need to be isolated against vibration caused by machine operation or caused by traffic. It usually applies to seismographs.

There are circumstances in which remedial measures might need to be applied to existing foundations. That is the case when, due to continued operation of the equipment, there is an increase of the unbalance loads of the rotating mass. Also, in case of deterioration of the rotor bearings; changes in soil conditions (such as water table fluctuations subsequent to construction of the foundation) or simply thanks to poor design or construction of the foundation

In such cases the measures comprise stabilization of the soil (increasing the rigidity of the base) or structural intervention: either by increasing the foundation or by creating additional slabs attached to the foundation (particularly effective for rocking motion).

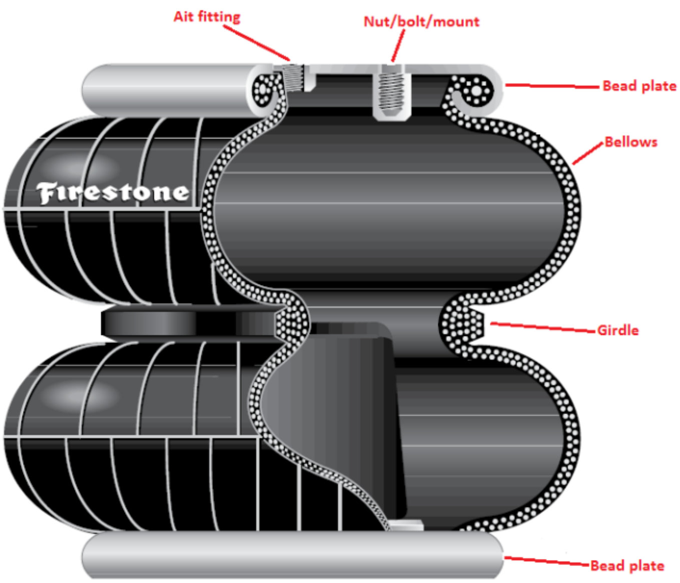


Figure 26 - A typical convoluted air spring. (Airmount® isolator by Firestone – extracted from https://www.firestoneip.com)

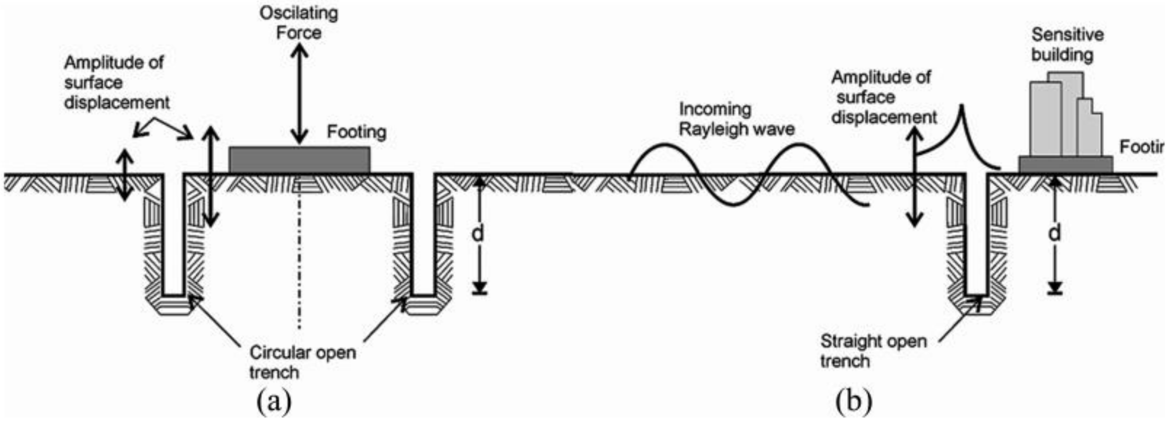


Figure 27 - Vibration isolation system: (a) active and (b) passive systems (Woods, 1968)

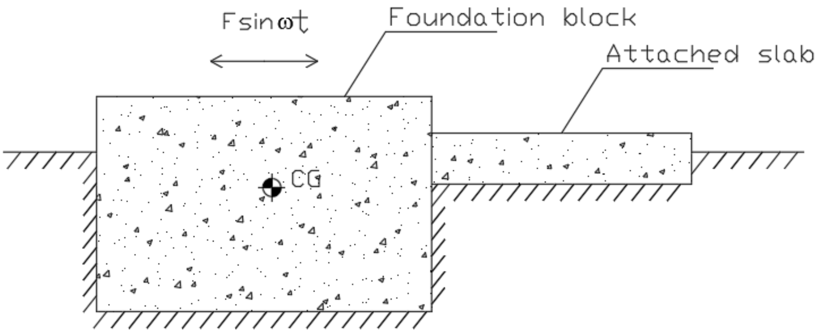


Figure 28 - Use of special slabs in reducing vibrations (Reproduced after Prakash and Puri, 1988 [3]*).*

8 ACCEPTANCE CRITERIA

8.1 GENERAL ASPECTS

The following sources are considered when developing this chapter: [2] and [4]

A machine foundation subject to dynamic loading should be capable of withstanding the dynamic loads and remain within accepted limits of vibration displacements, velocities and accelerations. The calculated vibration magnitudes (of accelerations, velocities and displacements) shall be evaluated to acknowledge whether the vibration effects can be tolerated. The vibration effects should also be limited in the areas around the equipment where sensitive equipment and structures might be located, or personnel need to work on a regular basis. Performance criteria are normally based on vibration amplitudes at key points on or around the equipment and foundation system. Limits of displacement are usually based on peak-to-peak amplitudes measured in mils (0.001 in.) or microns (10^{-6} m). Limits of velocity are often based on either peak velocities or root-mean-square (rms) velocities in units of inch per second or millimeter per second. Displacement criteria are often frequency dependent with bigger motions tolerated at slower speeds. Velocity criteria may be frequency dependent but are almost always independent. Acceleration criteria can be both frequency dependent or independent. For equipment that does not operate at a constant speed, but instead across a range of speeds, speed variations shall be considered in the foundation design.

Criteria for vibrations is usually defined regarding the following aspects (*Bachmann and Ammann, 1987)*:

- overstressing of structural members (deformation, fatigue, strength);
- physiological effects on people (mechanical, acoustic, optical);
- stop of production processes (problems of product tolerances, etc.) and overstressing of machinery (deformation, fatigue, strength).

The classification of the various vibration effects described above results in the following division:

- structural criteria;
- physiological criteria;
- production-quality criteria.

The acceptance criteria are of difficult definition, and for certain effects there isn't a full agreement on the boundaries to be respected (physiological effects), implying a considerable range of discretion. On the other hand, boundaries for vibrations in structures or equipment are easier to define.

Vibration limits can be defined as physical quantities such as:

- displacement amplitude;
- velocity amplitude;
- acceleration amplitude.

Alternatively, vibration limits can be defined with empirically derived quantities (e. g. KB intensity in *DIN 4150:*

«Vibrations in civil engineering - Part 2: Effects on people in buildings», Sept. 1975).

Some types of equipment operate at a constant speed while other types operate across a range of speeds. The effect of these speed variations should be considered in the foundation design.

8.2 PRODUCTION-QUALITY CRITERIA

Either in an industrial or scientific environment, the following effects may arise due to vibration beyond acceptable limits:

- Impact on production regarding quality of the manufactured goods (eg. problems of tolerance on lathes, milling machines, weaving machines, extrusion presses);
- reduced performance of devices (e. g. electron microscope, computer);
- damaging of the machine (eg. wear on shaft bearings, excessive deformation, fatigue and strength limits of machinery parts or at supports).

The vibrations limits applied for equipment are usually defined for specific types of machines. Criteria are defined according to the type of motion (rotating, oscillatory or impacting) and operation speed. In most cases, the machine limits are set by the equipment manufacturer jointly with the owner (or machine operator). The main objective is to avoid damage to the equipment but as well to ensure long-term performance.

In case of rotating equipment (such as fans, pumps and turbines), vibration displacements are limited at the bearings of the rotating shaft. Vibrations beyond limits increase maintenance requirements and can lead to the failure of the bearings. This type of equipment is commonly furnished with sensors and alarm devices to monitor vibrations.

In case of reciprocating equipment, the vibration limits are often higher than the ones of rotating machines, but the dynamic generated forces are also higher. The motions are limited at the bearing locations. Vibrations generated by reciprocator compressors are often monitored in relation to the foundation as a measure of the foundation and machine-mounting condition and integrity.

A variety of standards developed by governmental entities (industrial standards), engineering companies or plant owners are defined based on several researches on machinery vibration. Whenever vibrating limits are not defined by the equipment manufacturer, the recommendations from ISO 10816-1, Blake (1964), and Baxter and Bernhard (1967) are often followed.

8.2.1 STANDARD ISO 10816

ISO 10816 has seven parts dedicated to the evaluation of machinery vibration by measurements on the nonrotating parts. The first part, ISO 10816-1, provides general guidelines. The remaining parts provide specific criteria for different machine types. These standards are mainly used for in-place measurements and not so much as a design reference. Vibration criteria is defined in terms of rms velocity. In case of a complex vibration signal, the rms velocity basis provides a comprehensive measure of vibration severity and can be correlated to likely machine damage. When the vibration signal is characterized by a simple frequency, the rms velocities can be multiplied by $\sqrt{}2$ to obtain the corresponding peak velocity criteria. For these cases, the displacement, Δ, can be obtained by the following expression (considering ω, the operating frequency of the machine):

$$\Delta = \frac{v_{peak}}{\omega} = \sqrt{2}\frac{v_{rms}}{\omega}. \tag{8-1}$$

The displacement values calculated are zero-to-peak displacements but can be doubled to obtain peak-to-peak values. Parameters such as velocity, displacement or acceleration (rms, zero-to-peak and peak-to-peak) can only be related among each other, in case the motion is defined by a pure harmonic.

ISO 10816-1 establishes the following classification areas regarding the magnitude of vibration measured:

- Zone A: vibration typical of new equipment
- Zone B: vibration normally considered acceptable for long-term operation
- Zone C: vibration normally considered unsatisfactory for long-term operation
- Zone D: vibration normally considered severe enough to damage the machine

The remaining parts of the ISO 10816 (2 to 7) determine the limits between the zones defined above to specific machinery.

- ISO 10816-2: criteria for large, land-based, steam-turbine generators sets rated over 50 MW
- ISO 10816-3: in-place evaluation of general industrial machinery nominally over 15 kW and operating between 120 and 15000 rpm.
- ISO 10816-4: evaluation criteria for gas-turbine-driven power generation units (excluding aircraft derivatives) operating between 3000 and 20000 rpm
- ISO 10816-5: criteria for machine sets in hydropower facilities and pumping plants
- ISO 10816-6: criteria for reciprocating machines with power ratings over 134 horsepower (100 kW)
- ISO 10816-6: evaluation of vibration on rotodynamic pumps for industrial applications with nominal power above 1 kW

8.2.2 STANDARD PIP STC01015

The PIP STC01015 [27](PIP - Process Industry Practices) is a general structural design standard for application in process industry facilities. This guideline establishes some criteria specific to vibrating machines. PIP STC01015 considers a maximum velocity of movement during steady-state normal operation limited to 0.12 inch (3.0 mm) per second for centrifugal machines and to 0.15 inch (3.8 mm) per second for reciprocating machines (in case of lack of the manufacturer's vibration criteria for the equipment). The standard also defines an interval of resonance to be avoided: *"Structures and foundations that support vibrating equipment shall have a natural frequency that is outside the range of 0.80 to 1.20 times the exciting frequency."* Other rules-of-thumb already defined in previous chapters are enforced as well.

8.2.3 OTHER REFERENCES

8.2.3.1 Blake (1964), Standard Vibration Chart

Blake (1964) standard vibration chart is considered a reference for vibration criteria and is adapted by some industries and companies. The chart applies to process equipment with performance ratings from “No Faults (typical of new equipment)” to “Dangerous (shut it down now to avoid danger)”. It was initially developed to aid plant personnel in assessing field installations and determining maintenance plans. For different types of equipment, different service factors apply, making it of general application. The chart considers vibration displacement (in. or mm) rather than velocity and covers speed ranges from 100 to 10000 rpm. The service factors are defined in Annex C.

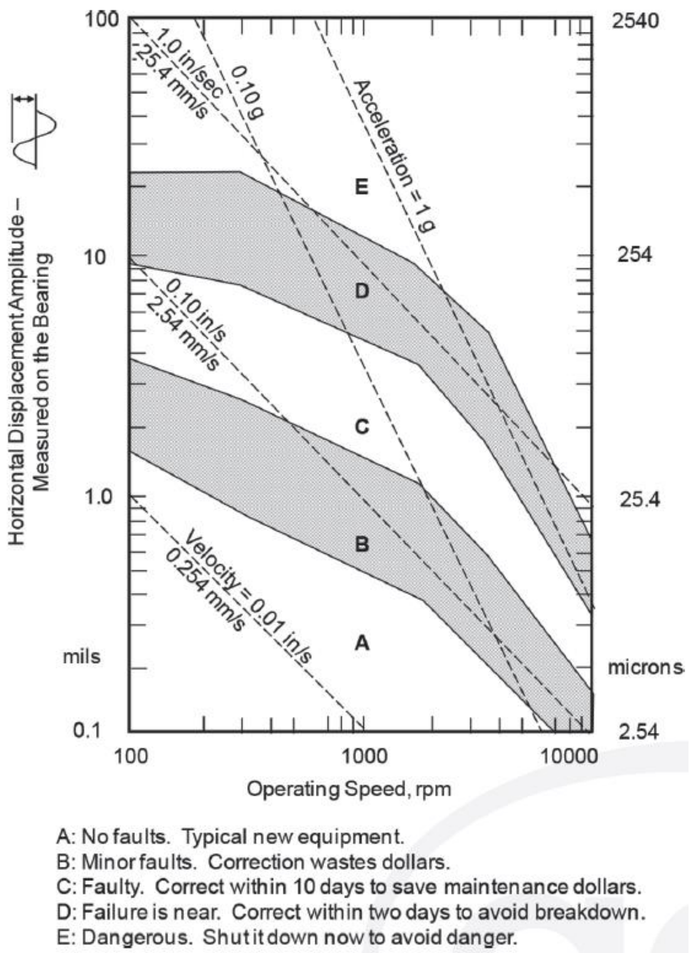


Figure 29 - Vibration performance of rotating machines (After Blake, 1964 as modified by Arya, O’Neill and Pincus 1979 [8]*)*

8.2.3.2 Baxter and Bernhard (1967), General Machinery Vibration Severity Chart

The chart developed by Baxter and Bernhard (1967) considers more general vibration tolerances. As Blake’s chart, it was initially developed to help in the plant maintenance operations. It’s again a criterion of severity, rating from “Extremely Smooth” to “Very Rough”. The chart defines severity plotted as displacement versus vibration frequency so that the various categories are differentiated along lines of constant peak velocity. The chart defined below is computed in a table in Annex C.

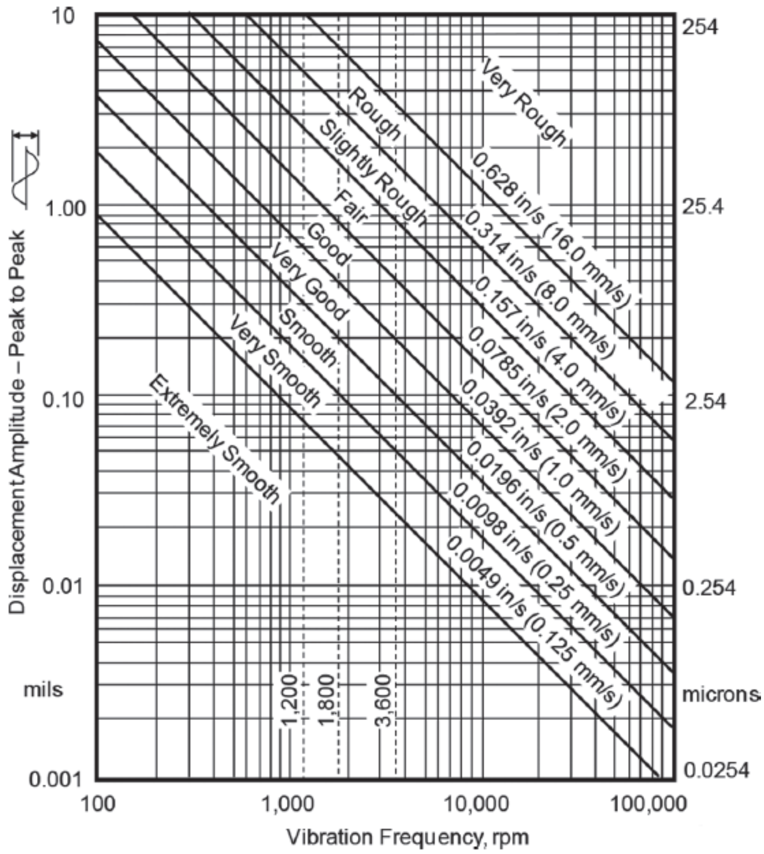


Figure 30 – – General Machinery Vibration Severity Chart (After Baxter and Bernhard, 1967 [28])

8.3 PHYSIOLOGICAL CRITERIA

The human sensitivity to mechanical vibrations is ambiguous and subjective. The human reaction to a vibration source deeply depends on circumstances. Someone that works at an office has a different perception from someone that operates a machine. Thus, the personnel expectation and needs directly influence the physiological criteria.

The following parameters influence the human sensitivity:

- position of the affected person (standing, sitting, lying);
- direction of incidence with respect to the human spine;
- activity of the affected person (resting. walking, running);
- community (existence of fellow-sufferers);
- age;
- sex;
- frequency of occurrence and time of day;
- duration of decay (damping).

The intensity of perception depends on the physical vibration parameters:

- displacement amplitude;
- velocity amplitude;

- acceleration amplitude;
- duration of effect (exposure time);
- vibration frequency.

According to *ACI351-3R-18 (2018)* [4], *"in the design of foundations for dynamic equipment, most engineering offices do not consider human perception to vibrations, unless there are extenuating circumstances, such as proximity to office or residential areas".*

In the chapters below, a few relevant codes are described covering the criteria for physiological vibration effects.

8.3.1 STANDARD DIN 4150 PART 2

DIN 4150-2 defines boundaries for allowable vibrations on the basis of perception as dependent on location (residential, light industrial) and time of day (daytime or nighttime). The standard considers the effects of vibrations from mostly external sources on people for a range of frequencies from 1 to 80 Hz. Measured vibration quantities (displacement, velocity, acceleration) are used as input for an empirically derived intensity of perception, called «KB value»:

$$KB = d\frac{0.8f^2}{\sqrt{1+0.032f^2}} \tag{8-2}$$

Considering: d, the displacement amplitude (mm) and f, the vibration frequency (Hz)

KB can be formulated in terms of velocity, *v*, or acceleration, *a*, of the vibration (if harmonic):

$$d = \frac{v}{2\pi f} = \frac{a}{4\pi^2 f^2} \tag{8-3}$$

The determined KB value of the examined vibration is later compared with the reference value in the code according to:

- use of the building;
- frequency of occurrence;
- duration of effects;
- time of day of occurrence.

8.3.2 STANDARD ISO 2631

ISO 2631 (Parts 1, 2, 4, and 5) provides guidance for human exposure to whole-body vibration and considers three different levels of human discomfort and duration of exposure: "reduced comfort boundary", "fatigue-decreased proficiency boundary" and "exposure limit". The frequency of the accelerations also impacts fatigue and proficiency. The limits (on accelerations) provided in tables and diagrams depend on the direction of incidence to the human body, using a reference coordinate system with the z-axis in the direction of the human spine. The criteria are defined in terms of an effective acceleration, a_{eff}, which is defined as the root-mean-square value over the exposure time T:

$$a_{eff} = \sqrt{\frac{1}{T}\int_0^T a^2(t)\,dt} \tag{8-4}$$

8.4 STRUCTURAL CRITERIA

Vibrations induced by man, machinery, traffic or construction works, can cause deformations and damage in structural members and particularly in nonstructural elements, such as:

- cracking of concrete structural elements;
- aggravation of existing cracking in structural members and nonstructural elements;
- collapse of equipment or cladding.

Continuous vibration may also cause problems of fatigue and overstress in principal load-bearing members.

Acceptance criteria must consider the following parameters:

- type and quality of the structural materials (especially its ductility);
- type of construction;
- properties of the foundation;
- main dimensions of the principal load-bearing members;
- age of the structure;
- duration of the vibration effects;
- characterization of vibration (frequency, etc.).

The amount of expected structural damage can be derived from Richart's chart (Figure 31) depending on several parameters (vibration velocity, displacement and acceleration, both varying with the excitation frequency). Most regulations, however, define their limits in terms of velocity.

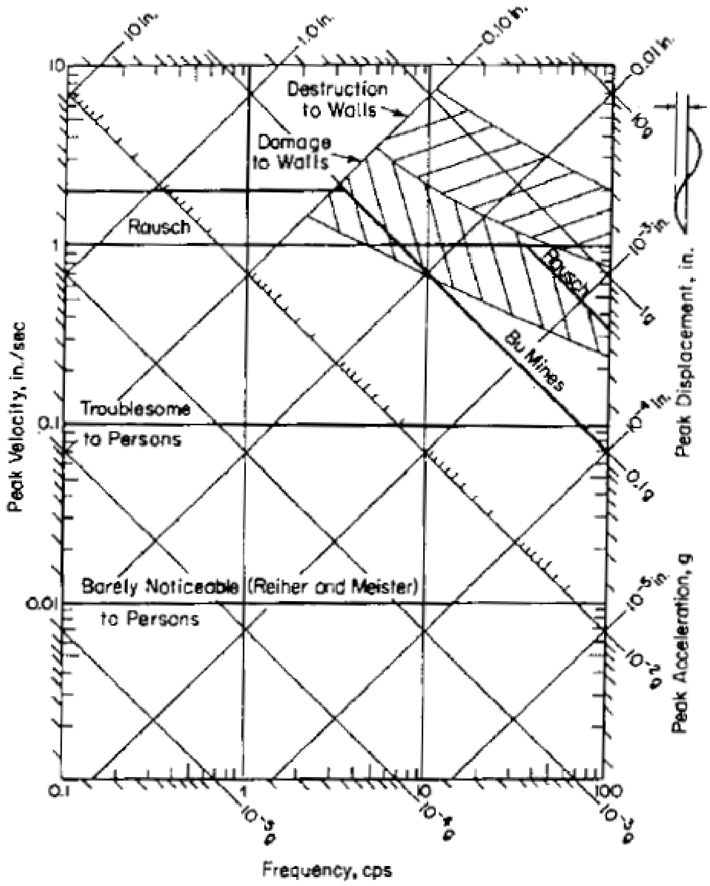


Figure 31 - Response spectra for allowable vibration at facility (After Richart, Hall and Woods, 1970)

8.4.1 STANDARD DIN 4150 - PART 3

Standard DIN 4150 – Part 3 considers effects on buildings and structural members resulting from an internal or external source of vibration. In this case vibration velocities or stresses due to dynamic loads are to be compared with the given criteria. This standard considers mostly the parameter vibration velocity (mostly measured) and is used for the following type of excitation:

- short-term structural vibrations (transient)
- steady-state structural vibrations
- steady-state vibrations, particularly of floor slabs.

Transient vibrations, such as blasting operations, pile driving, etc., are to be limited in terms of maximum foundation velocities. The following boundaries can be seen in Figure 32: 20 to 50 mm/s (from f ≤ 10 Hz to f = 100 Hz) to avoid severe damage, 5 to 20 mm/s to avoid slight damage, and 3 to 5 mm/s for particularly sensitive environments. For steady-state structural vibrations, floor slabs in particular, 10 mm/s is considered admissible, even though it does not entirely prevent slight damage like cracking.

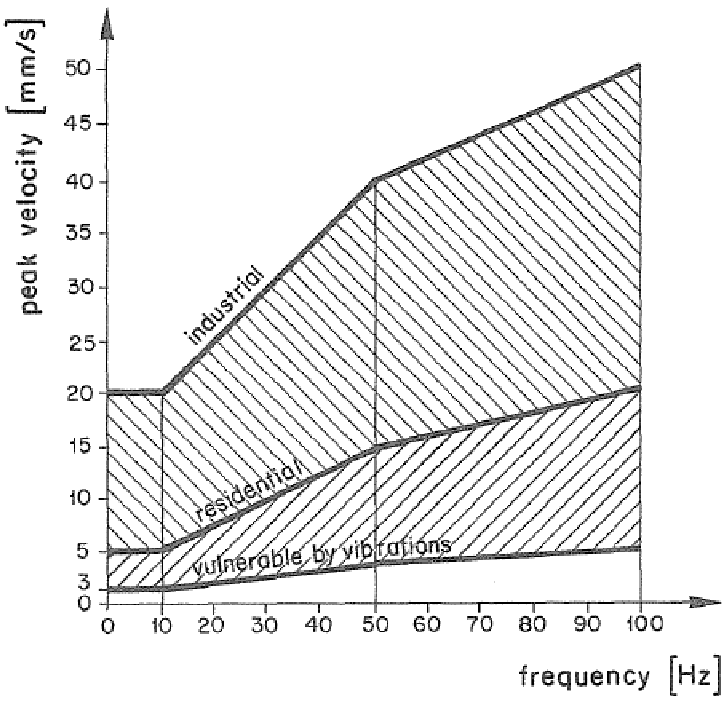


Figure 32 - Bounds on the foundation vibration velocities for building categories of DIN 4150 Part 3.

9 CASE STUDY – PILE CAP SUPPORTING A RECIPROCATING MACHINE

In order to better understand the approach for design of foundations supporting vibrating machines, a case study is presented, with a practical application of the preceding chapters. The object of analysis and design is a foundation supporting a reciprocating machine (compressor). The machine is composed by a driver – electrical engine, a driven machine (compressor with two cylinders) and a coupling device (flywheel coupling). The poor local soil resistance characteristics demand the consideration of a foundation supported on piles.

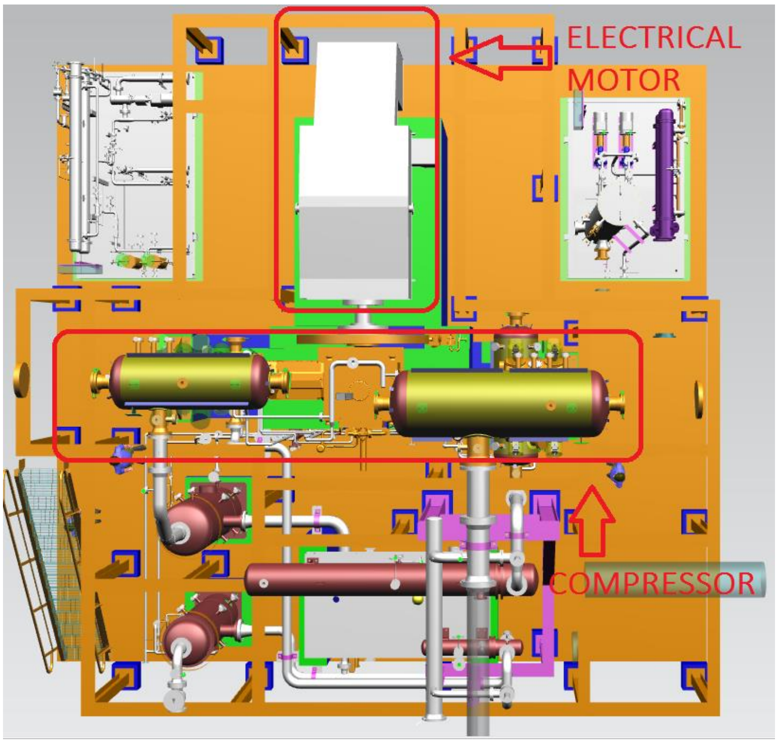


Figure 33 – Layout arrangement – Reciprocating machine, Compressor – Top View

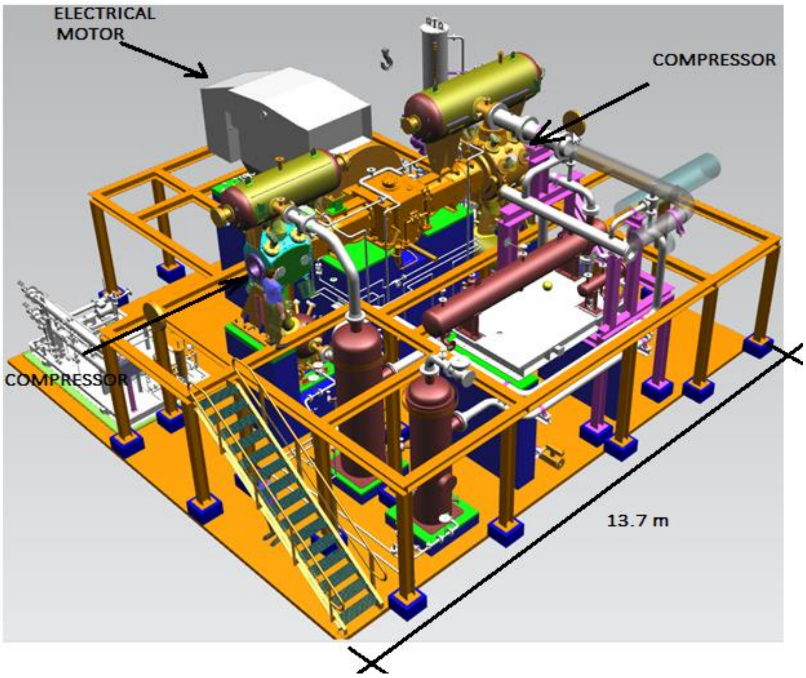


Figure 34 – Layout arrangement – Reciprocating machine, Compressor – 3D View

9.1 GEOTECHNICAL CONSIDERATIONS

In order to define the pile impedances, certain local geotechnical parameters need to be determined based on in situ tests. In this case a Seismic Cone Penetration Test (SCPT) (similar to the Down Hole Test) was performed at the piling location. The following results were obtained regarding the parameter *Vs*, shear wave velocity. Other parameters such as the soil density and Poisson ratio were assessed by other means (correlations from CPT's or references from local codes).

Table 9-1 – Parameters determined by Soil Investigation –SCPT & Parameters computed to determine the dynamic impedances

Parameters determined by Soil Investigation - SCPT					Parameters computed to determine the dynamic impedances				
		V_S	ρ	G_{max}	h_i/v_i	$\Sigma h_i/v_i$	ρ_{avg}	$V_{s\ avg}$	$G_{max\ avg}$
depth	depth	shear wave velocity	soil density	dynamic shear modulus			average soil density	average shear wave velocity	average dynamic shear modulus
(m) mv	(m) TAW	(m/s)	(t/m^3)	(MPa)			(t/m^3)	(m/s)	(MPa)
1,5	0,2	97	1,6	15,1	0,002	0,002			
2,5	-0,8	124	1,8	27,3	0,004	0,011	2,6	93,32	22,2
3,5	-1,8	71	1,7	8,4	0,007	0,023	2,1	88,20	16,6
4,5	-2,8	80	1,7	10,7	0,006	0,036	2,0	84,08	13,9
5,5	-3,8	106	1,7	18,8	0,005	0,046	1,9	86,43	14,1
6,5	-4,8	141	1,7	33,0	0,004	0,053	1,8	93,72	16,2
7,5	-5,8	145	1,6	34,8	0,003	0,060	1,8	99,22	17,8
8,5	-6,8	189	1,9	66,4	0,003	0,067	1,8	105,08	20,0
9,5	-7,8	191	1,7	63,5	0,003	0,072	1,8	111,42	22,5
10,5	-8,8	186	1,8	61,6	0,003	0,077	1,8	116,53	24,6
11,5	-9,8	257	1,9	123,9	0,002	0,082	1,8	122,69	27,4
12,5	-10,8	325	2,0	208,5	0,002	0,085	1,8	129,74	30,8
13,5	-11,8	338	2,0	229,6	0,002	0,088	1,8	136,80	34,5
14,5	-12,8	395	2,0	318,1	0,001	0,090	1,9	143,83	38,5
15,5	-13,8	413	2,0	349,0	0,001	0,093	1,9	150,74	42,6
16,5	-14,8	377	2,1	291,6	0,001	0,095	1,9	157,24	46,6
17,5	**-15,8**	**411**	**2,1**	**357,0**	**0,001**	**0,098**	**1,9**	**163,35**	**50,7**
18,5	-16,8	410	2,1	354,8	0,001				
19,5	-17,8	401	2,1	335,6	0,001				

The shear wave velocity, V_S, and Dynamic Shear Modulus, G_{max}, for a certain depth/pile length were determined according to the following expressions:

$$^{3}V_{S,L} = \frac{L}{\sum_{i=1}^{N} \frac{h_i}{V_i}} \quad (9\text{-}1)$$

$$V_{S,16} = \frac{16}{0,098} = 163,35\ m/s \quad (9\text{-}2)$$

$$G_{\max L} = \rho_{L\ avg} V_{s,L}^2 \quad (9\text{-}3)$$

$$G_{\max 16} = 1,9 * 163,35^2 = 50,7\text{MPa} \quad (9\text{-}4)$$

The definition of the length of the pile was done according to resistance and stiffness requirements. The later was more decisive in the choice of reaching deeper soil layers.

9.2 MACHINE PARAMETERS AND UNBALANCED FORCES

Most of the machine parameters are indicated by the compressor manufacturer. The following information is provided:

- General arrangement drawings of all the equipment, indicating self-weight and operation weight, position of the center of gravity, layout of hold down anchor bolts;
- Static loading diagram with all the equipment mass and relative distance to a reference point;
- The dynamic forces resultant from the operation of the compressor (steady-state excitation);
- Frequency of operation of the compressor and motor;
- Power and rotor weight of the motor.

Based on the information provided the dynamic forces are computed and then included in the detailed dynamic analysis – in this case a finite element analysis (in the time domain).

The following information (relevant for the dynamic analysis) is provided by the equipment manufacturer:

[3] Expression from Eurocode 8 - average shear wave velocity $v_{s,30}$ (for 30 m depth) - adapted for the pile length

REFERENCE COORDINATES SYSTEM

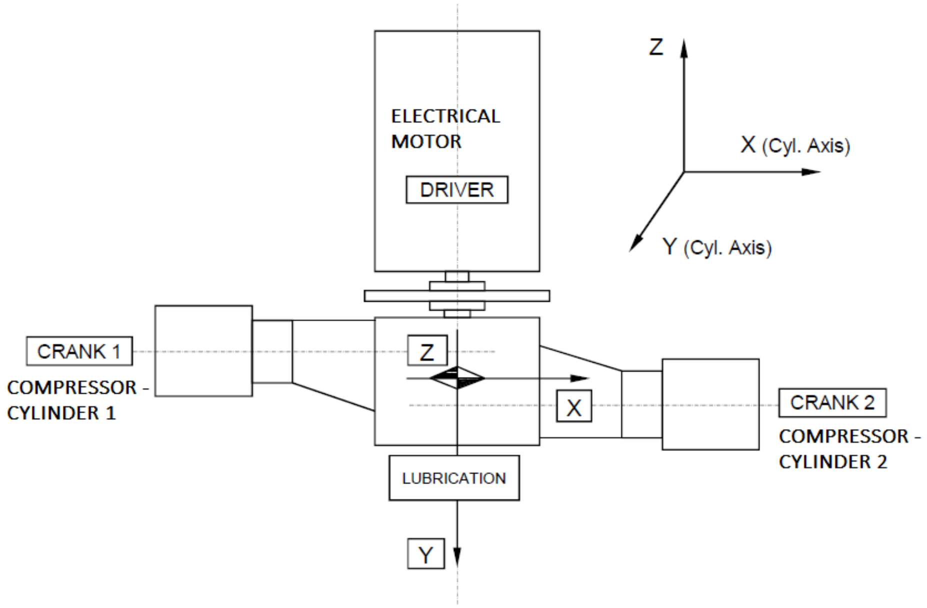
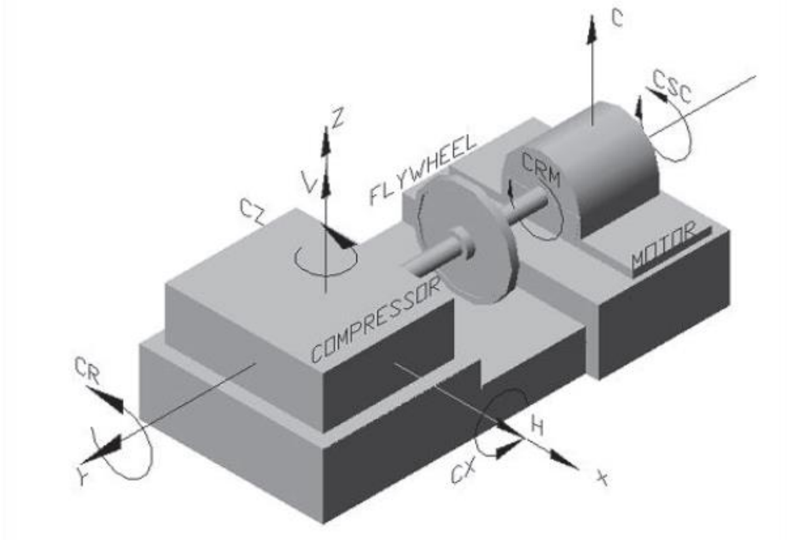


Figure 35 – Manufacturer input - Reference coordinate system for resulting overall dynamic forces (from the compressor)



CSC = (1) Nm
C = (1) N
(1) = Refer to electric motor drawings

X,Y and Z axes origin is indicated with a diamond (◆) on the foot print or installation drawing

H = Horizontal Force
V = Vertical Force
CX = Moment with axis-moment X
CZ = Moment with axis-moment Z
CRM = Mean compressor brake torque
CR = Moment due to the compressor torque reaction
CSC = Motor reaction torque in biphase short circuit (oscillating, duration shorter than 1 sec.)
C = Force caused by possible rotor unbalance (rotating with frequency equal to the driver rotation speed)

Figure 36 – Manufacturer input – Dynamic and Static forces due to operation and emergency shutdown of the machine

ROTATION SPEED = 370 RPM

COMPRESSOR UNBALANCED INERTIA FORCES AND MOMENTS (**)

HORIZONTAL PRIMARY	FORCE H'	19074	N	PHASE A	180.00	DEG
HORIZONTAL SECONDARY	FORCE H"	4087	N	PHASE A	0.01	DEG
VERTICAL PRIMARY	FORCE V'	697	N	PHASE A	(*)	DEG
VERTICAL SECONDARY	FORCE V"	0	N	PHASE A	0.00	DEG
HORIZONTAL PRIMARY	MOMENT CZ'	55693	N*m	PHASE A	180.00	DEG
HORIZONTAL SECONDARY	MOMENT CZ'	12865	N*m	PHASE A	0.00	DEG
VERTICAL PRIMARY	MOMENT CX'	4345	N*m	PHASE A	90.00	DEG
VERTICAL SECONDARY	MOMENT CX'	0	N*m	PHASE A	0.00	DEG

CR - MOMENT DUE TO THE COMPRESSOR TORQUE REACTION

FOURIER ANALYSIS OF OSCILLATING PORTION

FREQ. (Hz)	MAGNITUDE (Nm)	PHASE (DEG)
6.2	4507.2	139.3
12.3	18585.5	-157.1
18.5	3530.1	-66.6
24.7	7547.2	-166.7
30.8	2807.1	20.4
37.0	2066.7	39.1

CRM - MEAN VALUE (EQUIVALENT TO THE DRIVER MEAN TORQUE) = 25550 Nm <1>

(*) UNBALANCE DUE TO CONSTRUCTION TOLERANCE
PHASE ANGLE IS UNPREDICTABLE
(**) PRIMARY FORCE/MOMENTS FREQUENCY 6.2 Hz
SECONDARY FORCE/MOMENTS FREQUENCY 12.3 Hz

Figure 37 – Manufacturer input – Dynamic and Static forces due to operation and emergency shutdown of the machine

9.2.1 DYNAMIC FORCES - UNBALANCED FORCES AND MOMENTS

The dynamic forces shown in Figure 36 and Figure 37 characterize the rotating and oscillatory motion of the steady-state excitation generated by the reciprocating machine (compressor). The primary forces (or of 1^{st} order) are due to the rotating motion and have the same frequency of operation as the driver/compressor. On the other hand, the secondary forces (or of 2^{nd} order) are due to the oscillatory (or reciprocating) motion of the piston.

Table 9-2 – Machine parameters of the reciprocating machine (steady-state excitation)

operating speed	370	rpm
operation frequency of the machine, f	6,17	Hz
Angular Velocity, $\omega = 2\pi\phi$	38,75	rad/Sec
	2220	deg/sec
Period of the machine, T (cycle time)	0,162	Sec
Time step: time/10 T_Δ = Cycle	0,0162	Sec

Table 9-3 – Dynamic forces (unbalanced forces) due to operation of the reciprocating machine (steady-state excitation)

Unbalanced Forces, F_x (H) (kN)		Unbalanced Forces, F_z (V) (kN)		Unbalanced Moment, M_z (Cz) (kNm)		Unbalanced Moment, M_x (Cx) (kNm)	
1st Order	2nd Order	1st Order	2nd Order	1st Order	2nd Order	1st Order	2nd Order
F_x = 19.07 * cos(ωt - 180)	F_x = 4.09 * cos(2ωt - 0,01)	F_z = 0.70 * cos(ωt - 0,01)	F_z = 0 * cos(2ωt-0),	M_z = 55.69 *cos(ωt - 180)	M_z = 12.87 *cos(2ωt - 0)	M_x = 4.35 *cos(ωt - 90)	M_x = 0 *cos(2ωt - 0)

The motor unbalanced force caused by possible rotor unbalance (rotating with frequency equal to the driver rotation speed) is defined assuming the mass of the rotor, an eccentricity (mass unbalance) corresponding to a balance quality grade G16 (eω = 16 mm/s) and a Service Factor equal to 2:

$$F = \frac{m_r Q\omega S_f}{1000} = \left(5081 * 16 * \frac{370}{60} * 2\pi * 2\right)/1000 = 6{,}30\ kN \qquad (9\text{-}5)$$

$$F_z = 6.30 * \cos(\omega t)\ (kN) \qquad (9\text{-}6)$$

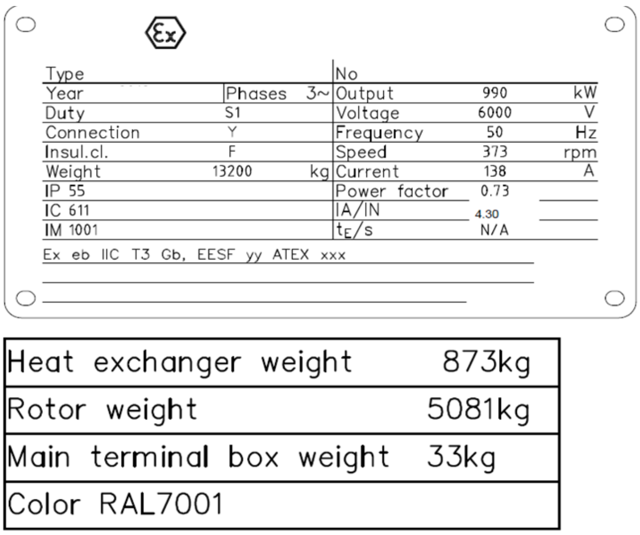


Figure 38 – Manufacturer input – Tag plate - Motor

9.2.2 DYNAMIC FORCES – TORQUE LOAD VARIATIONS

Since the drive mechanism is non-integral (electric motor), it produces a net external drive torque on the driven machine. The torque is equal in magnitude and opposite in direction on the driver and driven machine. The following values define the moment due to the compressor torque reaction varying in time according to the manufacturer's input:

Table 9-4 – Dynamic forces (Moment) due to the compressor torque reaction

Moment due to the compressor torque reaction (CR)					
My = 4.51 *cos(ωt -139.3), kN-m	My = 18.59 *cos(2ωt +157.1), kN-m	My = 3.53 *cos(3ωt +66.6), kN-m	My = 7.55 *cos(4ωt +166.7), kN-m	My = 2.81 *cos(5ωt -20.4), kN-m	My = 2.07 *cos(6ωt -39.1), kN-m

On the other hand, the motor develops a torque moment (also called drive torque) as defined below:

$$NT = \frac{9550 P_S}{f} = \frac{9550 * 990}{370} = 25{,}552\ kNm \tag{9-7}$$

Considering: *NT*, normal torque (kNm); *PS*, power being transmitted by the shaft at the connection (kilowatts) and *f*, operating speed (rpm).

The drive torque moment has opposite direction regarding the torque moment developed in the compressor (driven machine).

$$My = 25.55 * \cos(\omega t)\ (kNm) \tag{9-8}$$

9.2.3 SHORT CIRCUIT FORCES

Short circuit in motors causes short circuit forces of a considerable magnitude. Therefore, its consideration for the strength design of the foundation is required. This force is considered pseudo-static. The manufacturer furnished these forces as shown below (CSC = M_{sc} = 108,3*1,4 = 151,62 kNm):

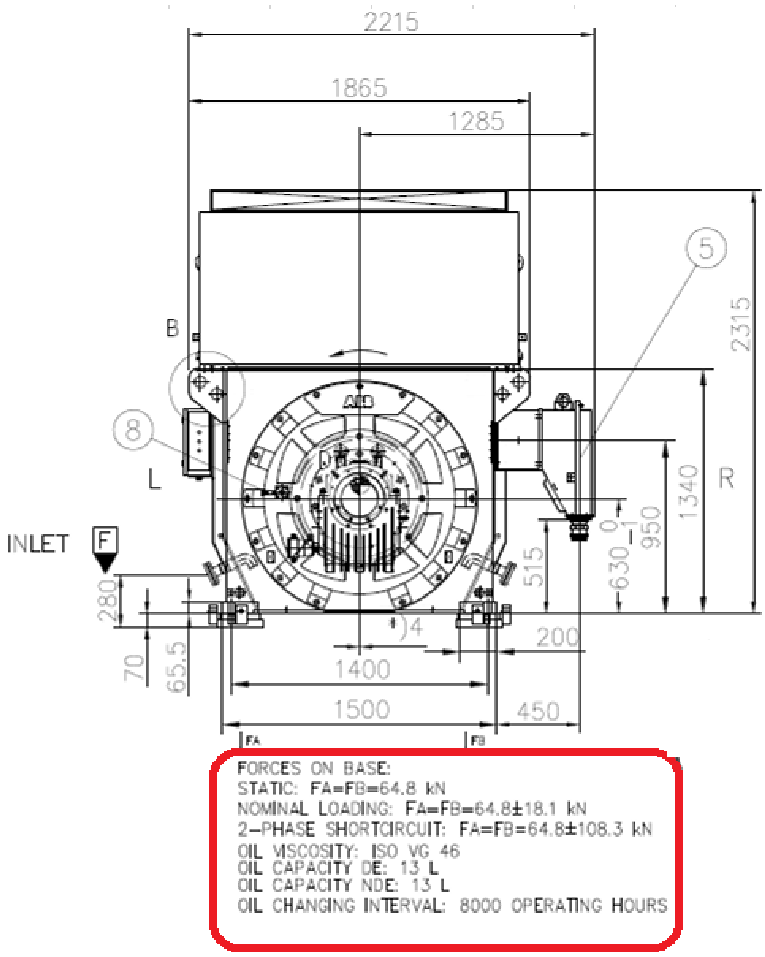


Figure 39 – Manufacturer input – Short circuit forces – Motor

9.3 PRELIMINARY DESIGN

Based on the layout operation requirements, the machine characteristics regarding mass and its distribution and the *in situ* geotechnical characteristics a first attempt is performed to define the foundation geometry. As described previously, the soil stratigraphy at the location of this equipment is of poor resistance to allow direct foundations. Thus, the decision of using deep foundations (piles) is obvious. According to the rules of thumb defined in the previous chapter, the mass of the pile cap for a reciprocating machine shall be at least four times the mass of the reciprocating machine. At the same time, the length and width of the foundation and the piling layout is defined such that the combined center of gravity of the foundation-machine system coincides with the center of stiffness of the pile group with a margin of 5%. This requirement minimizes the torsional effects. A minimum width/length in plan of at least 1.5 times the vertical distance from the machine centerline to the bottom of the foundation block is considered for the plan dimensions. Besides the above stated rules, care is taken in the definition of the piling layout and pile diameter in such a way that 50% of the pile capacity is not exceeded in static design conditions, allowing low pile stresses. The thickness of the pile cap is defined such that the piles are properly anchored to the pile cap. The pile cap is kept isolated from the remaining production unit by an expansion joint of 30 mm, avoiding any propagation of vibration to the surrounding equipment.

Considering the type of machine (reciprocating) and the operation speed, the foundation is defined such that its frequency is considerably above the excitation frequency. Therefore, the pile cap is over-tuned regarding the vibrating machine. Considering the operation speed of the machine, isolation is not desirable.

With the parameters defined in Table B 5 (see Annex B) and Table B 6 (see Annex B) the overall center of gravity and mass ratio is determined:

Total Weight (machine + pile cap)	$W = W_{e1} + W_{e2} = 9032{,}63$ kN	(9-9)
Total Mass (machine + pile cap)	$m = m_{a1} + m_{a2} = 920{,}76$ kN-s2/m	(9-10)
Distance of CG in x-direction	$\overline{x_c} = (m_i x_{i1} + m_i x_{i2})/(m_{a1} + m_{a2}) = 6{,}383\text{m}$	(9-11)
Distance of CG in y-direction	$\overline{y_c} = (m_i y_{i1} + m_i y_{i2})/(m_{a1} + m_{a2}) = 6{,}519\text{m}$	(9-12)
Distance of CG in z-direction	$\overline{z_c} = (m_i z_{i1} + m_i z_{i2})/(m_{a1} + m_{a2}) = 1{,}478\text{m}$	(9-13)
Ratio $m_{Foundation\text{-}Machine}/m_{Machine}$	$Ratio = (m_{a2})/(m_{a1}) = 11{,}33 > 4, OK$	(9-14)

The ratio of mass of the foundation to the mass of the machine is higher than the minimum good practice values and therefore is not object of concern. This results from the enlargement of the foundation in plan due to operation requirements. The pile cap is defined with a geometry of 12.9m by 13.7 m and 1.5m thickness. Thirty-six piles are disposed within this area with minimum spacing of $3^*\Phi_{pile}$. A pile diameter bigger than 660 mm is defined according to stiffness requirements. The geometry and layout of piles is shown in Figure 40 to Figure 42.

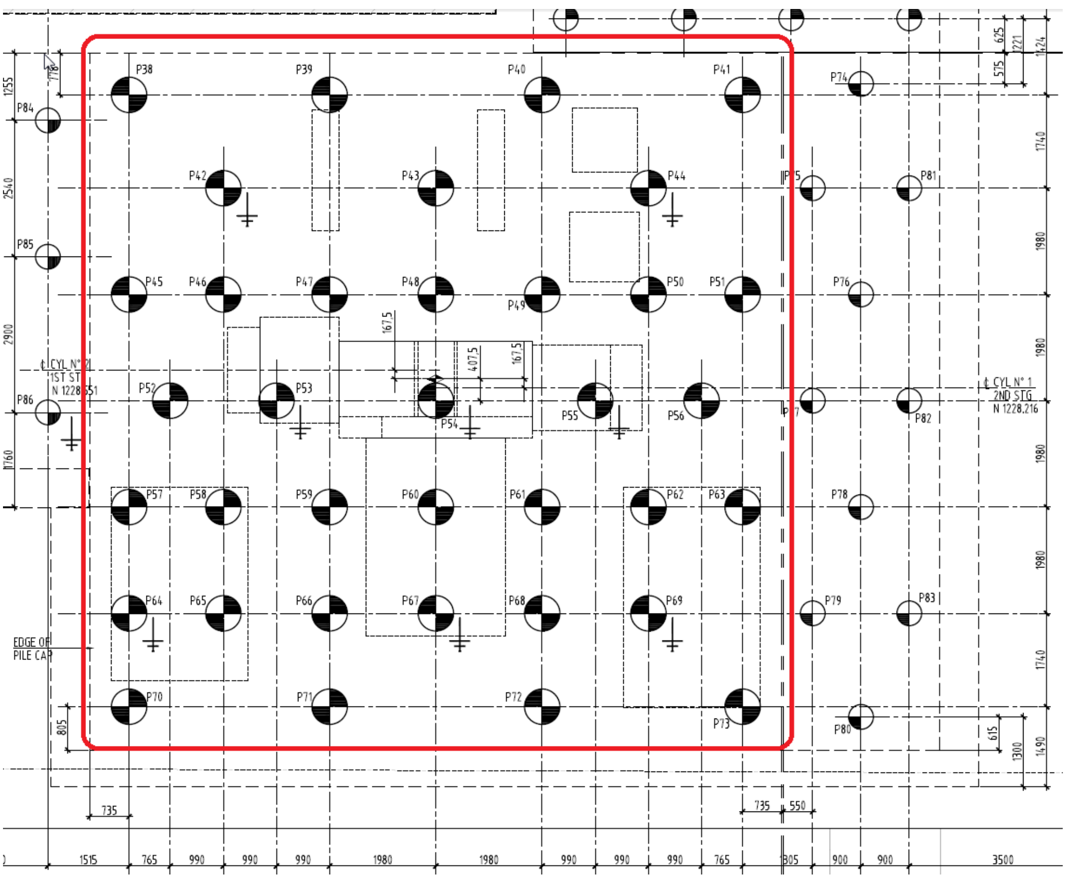


Figure 40 – Planview – Piling plan

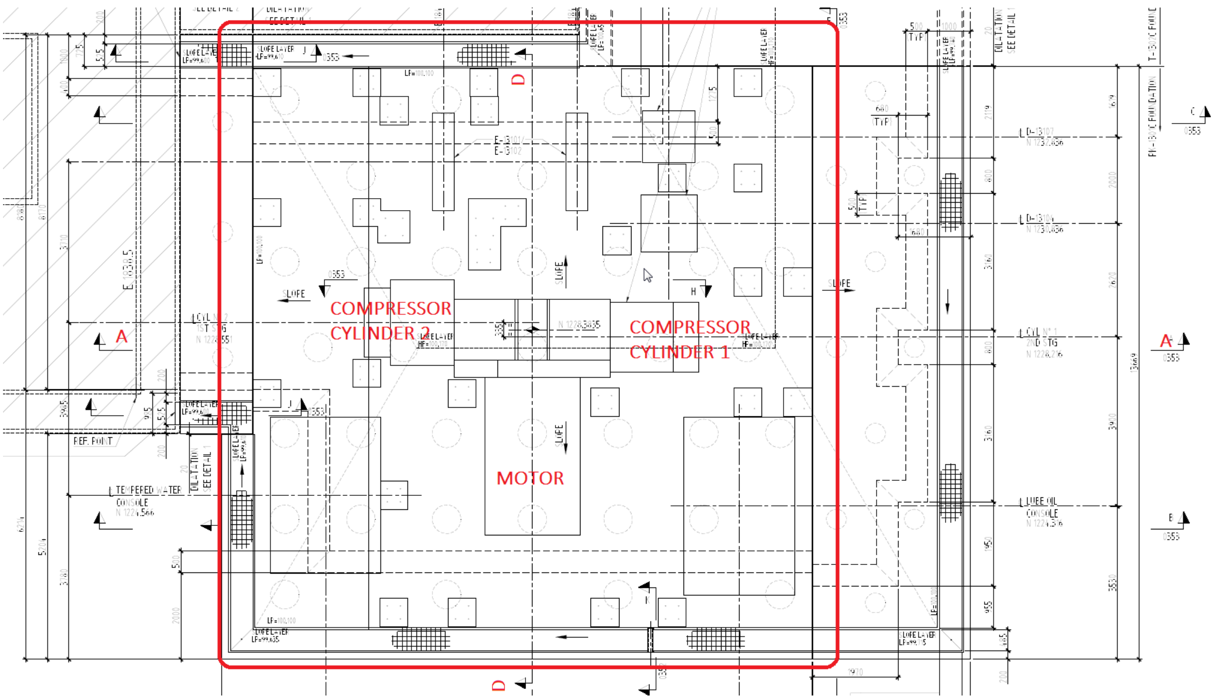


Figure 41 – Planview – Geometry definition of pile cap and equipment plinths

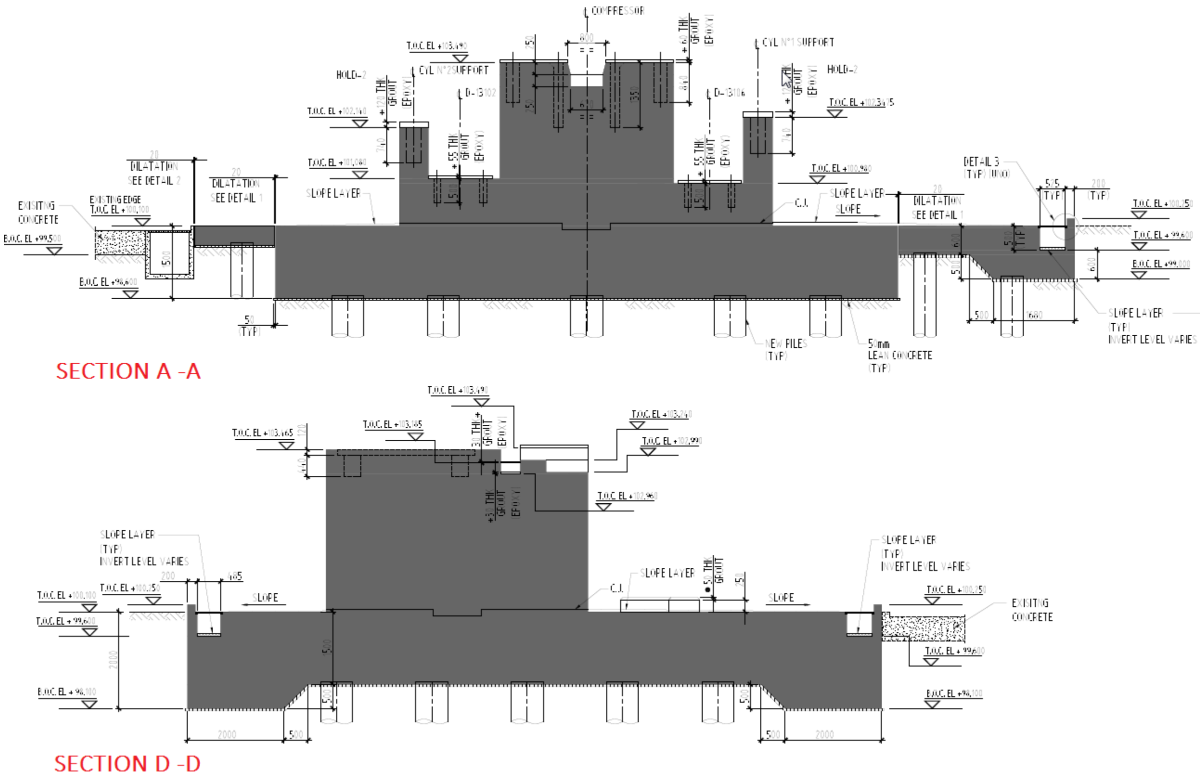


Figure 42 – Section-cuts A-A and D-D– Geometry definition of pile cap and equipment plinths

The plan-view eccentricities between the center of gravity of the combined machine-foundation system and the center of resistance (center of stiffness) of the pile group are calculated being less than 5 percent of the plan dimensions of the foundation:

Centroid of pile group:

$$\overline{x_p} = \frac{\sum_1^N x_i I_{xi}}{\sum_1^N I_{xi}} = 5{,}874\ m \quad (9\text{-}15)$$

With,

$$\sum_1^N x_i I_{xi} = 1{,}97\ m5 \quad (9\text{-}16)$$

$$\sum_1^N I_{xi} = 0{,}335\ \text{m4} \quad (9\text{-}17)$$

$$\overline{y_p} = \frac{\sum_1^N y_i I_{yi}}{\sum_1^N I_{yi}} = 6{,}030\ m \quad (9\text{-}18)$$

With,

$$\sum_1^N y_i I_{yi} = 2{,}022\ m^5 \quad (9\text{-}19)$$

$$\sum_{1}^{N} I_{yi} = 0{,}335\ m^4 \quad (9\text{-}20)$$

Resulting in eccentricities below the 5% limit:

$$^{4}\ e_{cx} = \left|\overline{x_p'} - \overline{x_c}\right| / L_x = 1{,}75\% < 5\% \quad (9\text{-}21)$$

$$^{5}\ e_{cy} = \left|\overline{y_p'} - \overline{y_c}\right| / L_y = 2{,}11\% < 5\% \quad (9\text{-}22)$$

9.4 PILE IMPEDANCES (STIFFNESS AND DASHPOT)

The definition of the pile impedances in this case study is done following the approximate procedures developed by Novak, as described in the previous chapter.

Despite being an approach where both spring and damping constants are frequency dependent, simplified expressions were developed as well by Novak, that under certain conditions, are frequency independent. Since this case study fits in that domain of validity, decision is taken to use those expressions, for simplicity.

Few aspects are considered for the computation of the impedances:

- The shear dynamic modulus is diminished in about 30% for the computation of the impedances for horizontal motion (for the reasons previously described);
- Torsional stiffness is disregarded according to Arya, O'Neill, Pincus [8] : "torsional moments applied to the structure are resisted almost completely by couples produced by lateral reactions at the pile heads";
- Impedances due to the embedment of the pile cap are considered for vertical, horizontal and rocking motion;
- Pile group interaction is considered using static interaction coefficients, referenced by Arya, O'Neill, Pincus [8] and developed by Poulos. According to ACI 351-3R [4] "If the dimensionless frequency a0< 0.1 (…) then this approach should provide a reasonable estimate of pile group stiffness." – about the static interaction coefficients;
- Group Interaction factors are not considered for rocking motion according to Arya, O'Neill, Pincus [8]: "group action for rocking motion is not as prevalent as in the translational modes";
- Horizontal motion and rocking motion are defined for one direction of motion and for the smaller plan dimension (conservative assumption) for simplicity (the pile cap is close to a square shape and the center of stiffness of the pile group is almost coincident with the center of gravity of the pile cap);
- Piles are considered as end bearing piles for computation of vertical stiffness; according to geotechnical design: 70% tip resistance and 30% friction resistance (shaft). As per Arya, O'Neill, Pincus [8]: if

[4] Since $\overline{x_p}$ is defined from the centerline of a corner pile (P41), a translation to the corner of the foundation shall be considered for the determination of the overall center of resistance of the pile group: $\overline{x_p'} = \overline{x_p} + e_{x,pile}$, with $e_{x,pile}$, = 0.735 m -pile edge distance)

[5] Exercise done for direction x is valid for direction y, with $e_{y,pile}$, = 0.778 m -pile edge distance)

combined friction and end bearing piles, fixed tip piles impedance factors apply;

- Piles are considered flexible, as demonstrated later, for computation of pile group interaction factors for horizontal motion.

The following geotechnical, machine and foundation parameters are used for the calculation of the impedances:

Table 9-5 – Foundation parameters (Pile and Pile Cap) for computation of impedances (stiffness and dashpot)

Foundation parameters - Pile			
Pile length	L_p	16	m
Pile diameter	Φ_{Pile}	660	mm
Pile radius	r_0	330	mm
	L_p/r_0	48,48	OK - Lp/r0>25
Area	A_{pile}	0,34	m^2
	E_p	35,9	GPa
	I_p	0,009314	m^4
	E_p * I_p	334154,70	$kN.m^2$
	γ_p	25,00	kN/m^3
	V_C	3751,45	m/s
	μ_p	0,20	
	J	0,01863	m^4
	Piles subjected to vertical vibration	end-bearing piles	
	Piles subjected to horizontal motion	flexible piles	
K_R =	10 Stiff Pile		
K_R =	10^{-5} Flexible Pile		
K_R =	3,9E-05		

Foundation parameters - Pile Cap			
	Lx	12,9	m
	Ly	13,7	m
	A_{cap}	177	m^2
	E_p	35,9	GPa
	$m_{pile\ cap\&plinths}$	967,39	ton
Equivalent radius of the pile cap	r_0	7,50	m
	x_r	6,57	m
	z_c	1,38	m
	Nr Piles	36	
Backfill	G_s	18000	kPa
	γ_s	18	kN/m^3
	D_f	0,75	m

Table 9-6 – Machine parameters for computation of impedances (stiffness and dashpot)

Machine Parameters

	operating speed	370	rpm	
	f	6,2	Hz	
operation frequency of the machine	$\omega=2\pi f$	38,75	rad/s	
dimensionless frequency parameter	a_0	0,0783		OK - $0,05>a_0>0,8$
	$m_{machine}$	74,69	ton	

Table 9-7 – Geotechnical parameters for computation of impedances (stiffness and dashpot)

Geotechnical parameters

	γ	18,635	kN/m^3
Density of soil	ρ	1,900	kN.s^2/m^4
in the range of 0.3 to 0.5	μ_s	0,3	
	G_{max}	50707	kPa
	E_{max}	131837	kPa
	V_s	163,35	m/s

Table 9-8 – Foundation-Machine parameters for computation of impedances (stiffness and dashpot)

Foundation-Machine parameters

V_s/V_p	0,044	
γ/γ_p	0,75	
E_p/G	707,517	
$m_c=m_{machine}+m_{pile\ cap}=$	1042,078	ton

Following Arya, O'Neill, Pincus [8], the pile impedances are computed using the parameters defined in Table 9-5, Table 9-6, Table 9-7 and Table 9-8.

Table 9-9 – Pile impedances – Single pile: Stiffness and damping (after Novak (1974) [7])

Motion	Spring K_i^1	Damping c_i^1
Vertical	$K_z^1=\frac{E_pA}{r_0}f_{18,1}=$ 1270,934 MN/m	$c_z^1=\frac{E_pA}{V_s}f_{18,2}=$ 3,261 MN.s/m
Horizontal	$K_x^1=\frac{E_pI}{r_0^3}f_{11,1}=$ 282,788 MN/m	$c_x^1=\frac{E_pI}{r_0^2V_s}f_{11,2}=$ 1,378 MN.s/m
Rocking	$K_\varphi^1=\frac{E_pI}{r_0}f_{7,1}=$ 415,592 MN.m/rad	$c_\varphi^1=\frac{E_pI}{V_s}f_{7,2}=$ 0,592 MN.m.s/rad
Cross-stiffness/damping	$K_{x\varphi}^1=\frac{E_pI}{r_0^2}f_{9,1}=$ -259,211 MN/rad	$c_{x\varphi}^1=\frac{E_pI}{r_0V_s}f_{9,2}=$ -0,762 MN.s/rad

Table 9-10 – Impedances factors – Single pile: Stiffness and damping factors (after Novak (1974) [7])

Motion	Stiffness factor	Damping factor
Vertical	$f_{18,1}$ = 0,0342	$f_{18,2}$ = 0,0434
Horizontal	$f_{11,1}$ = 0,0304	$f_{11,2}$ = 0,0734
Rocking	$f_{7,1}$ = 0,4104	$f_{7,2}$ = 0,2894
Cross-stiffness/damping	$f_{9,1}$ = -0,0813	$f_{9,2}$ = -0,1196

Table 9-11 – Embedded cap Impedances: Stiffness and damping (after Novak and Beredugo [13])

Motion	Spring K_i^f	Damping c_i^f
Vertical	$K_z^f = G_s D_f \bar{S_1}$ = 36,450 MN/m	$c_z^f = D_f r_0 \sqrt{G_s \gamma_s / g}\,\overline{S_2}$ = 6,850 MN.s/m
Horizontal	$K_x^f = G_s D_f \overline{S_{u1}}$ = 54,450 MN/m	$c_x^f = D_f r_0 \sqrt{G_s \gamma_s / g}\,\overline{S_{u2}}$ = 9,816 MN.s/m

Rocking

$$K_\varphi^f = G_s r_0^2 D_f \overline{S_{\varphi 1}} + G_s r_0^2 D_f [(\delta^2/3) + (z_c/r_0)^2 - \delta(z_c/r_0)]\overline{S_{u1}} = 1956{,}343 \text{ MN.m/rad}$$

$$c_\varphi^f = \delta r_0^4 \sqrt{G_s \gamma_s / g}\{\overline{S_{\varphi 2}} + [(\delta^2/3) + (z_c/r_0)^2 - \delta(z_c/r_0)]\overline{S_{u2}}\} = 113{,}942 \text{ MN.m.s/rad}$$

Table 9-12 – Embedded cap Impedance factors: Stiffness and damping factors (after Novak and Beredugo [13])

Motion	Stiffness factor	Damping factor
Vertical	$\overline{S_1}$ = 2,700	$\overline{S_2}$ = 6,700
Horizontal	$\overline{S_{u1}}$ = 4,033	$\overline{S_{u2}}$ = 9,600
Rocking	$\overline{S_{\varphi1}}$ = 2,50	$\overline{S_{\varphi2}}$ = 1,800

The pile group interaction is considered according to the static solution of Poulos, for vertical and horizontal motion. In both cases an interaction factor is calculated assuming any of the 36 piles as reference pile. The average value of the interaction factor is considered for the computation of the pile group stiffness (corner piles and side piles have less stiffness/damping reduction (interaction).

For the layout of 36 piles supporting the pile cap and machine the pile group interaction factors are obtained (see Annex B - Table B 7).

With:

α_{Aj} = the interaction factor for vertical motion describing the contribution of the j^th pile to the displacement of the reference pile (that is, α_1 =1)

α_{Lj} = the interaction factor for horizontal motion describing the contribution of the j^th pile to the displacement of the reference pile (that is, α_{L1} =1)

The values of the pile group impedances are determined assuming the average static interaction factors defined in Table 9-13:

Table 9-13 – Pile group static interaction factors for vertical and horizontal motion

Total nr of piles	**36**		
Reference Pile	**$\Sigma\alpha_A$**	**$\Sigma\alpha_{Lx}$**	**$\Sigma\alpha_{Ly}$**
Σ	437,62	126,03	127,34
Average	**12,16**	**3,50**	**3,54**

Table 9-14 – Pile group impedances for vertical and horizontal motion: stiffness and damping

Motion	Spring K_i^G	Damping c_i^G
Vertical	$K_z^G = \frac{\sum_1^N K_z^1}{\sum_1^N \alpha_A} + G_s\, D_f \overline{S_1} =$ 3800,255MN/m	$c_z^G = \frac{\sum_1^N c_z^1}{\sum_1^N \alpha_A} + D_f\, r_0 \sqrt{G_s \gamma_s / g}\, \overline{S_2} =$ 16,507MN.s/m
Horizontal	$K_x^G = \frac{\sum_1^N K_x^1}{\sum_1^N \alpha_L} + G_s\, D_f \overline{S_{u1}} =$ 2932,570MN/m	$c_x^G = \frac{\sum_1^N c_x^1}{\sum_1^N \alpha_L} + D_f r_0 \sqrt{G_s \gamma_s / g}\, \overline{S_{u2}} =$ 23,844MN.s/m

As stated previously, pile group interaction is not considered for the rocking motion. Therefore, the individual stiffness and damping is assigned for the individual piles. For vertical and horizontal motion, the values calculated in Table 9-14 are divided by the number of piles and assigned to the viscous dampers supports in the finite element analysis (FEA).

9.5 DETAILED ANALYSIS (FEA) AND RESULTS

A detailed dynamic analysis is performed by means of a finite element analysis. The software used for this calculation is Robot Structural Analysis from Autodesk. The finite element analysis is done resorting to solid finite elements (cuboid elements, in particular) to facilitate the inclusion of eccentricities resulting from the large thicknesses of the structural elements. Solid finite elements have only translation degrees of freedom - no rotation stiffness in the nodes. Restraining rotation or modelling rotation stiffness requires the definition of rigid links or fictitious rigid bars, connecting at least 3 non-collinear nodes of the solid and subsequent application of rotation stiffness to the master node of the rigid link (or a selected node of the fictitious bars). The solid finite elements dimensions are limited to 0,50 m per face.

The reinforced concrete pile cap is modelled according to the geometry shown in Figure 41 and Figure 42. This meaning that the mass and stiffness is correctly distributed according to the geometry. Following the recommendation from ACI 351 3R [4] the concrete is defined with a modified modulus of elasticity. Established relationships suggest that the ratio of dynamic to static modulus can vary from 1.1 to 1.6, with significant variation with age and strength. Thus, a higher dynamic modulus of elasticity is used, instead of the static modulus of elasticity. It was defined according to the following formula from ACI 318:

$$E_D = 6550 * \sqrt{f_{ck}} \quad (MPa) \tag{9-23}$$

The piles are defined as elastic viscous-damper supports according to the piling layout shown in Figure 40. The impedance (stiffness and dashpot) is defined in 9.4 and is modelled at the pile head connected to the pile cap. The supports are modelled with rigid links connecting the solid (rotation free nodes), in the limits of the pile diameter, and a master node (pile head) where the rotation restriction is applied (due to the fixation of the pile in the pile cap).

The static and dynamic equipment is modelled with rigid links connecting the supports of the equipment to its center of gravity (master node). Therefore, the mass is lumped at its center of gravity and the equipment is considered infinite stiff. For the dynamic equipment, compressor and motor, the axis of the rotor is modelled at its centerline as defined in the manufacturer drawings (regarding the top of concrete of the plinth). The bearing pedestals of the rotor are modelled with rigid links.

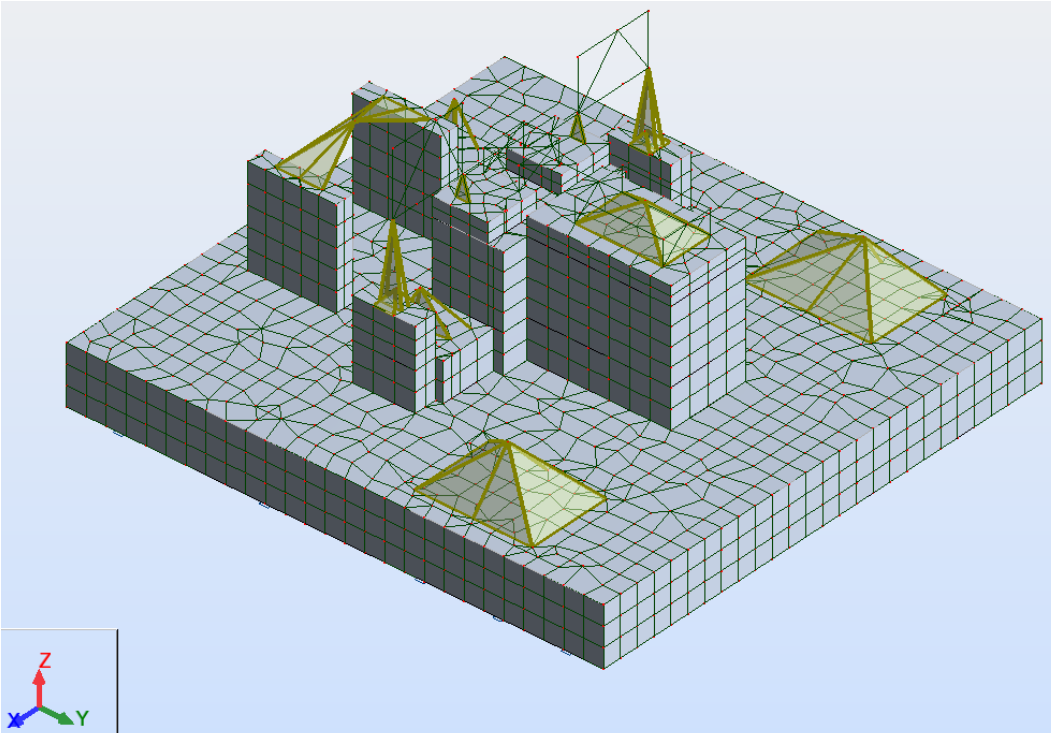


Figure 43 – Finite Element Analysis – Volumetric finite elements - Pile cap and equipment plinths supporting a reciprocating a machine

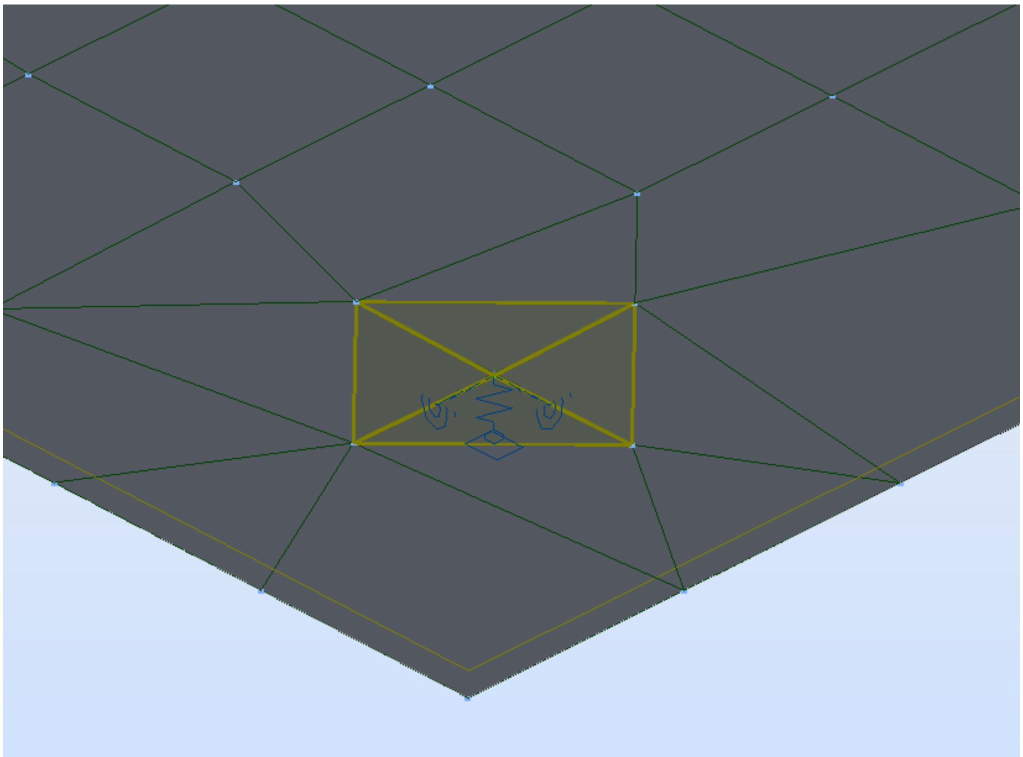


Figure 44 – Finite Element Analysis – Volumetric finite elements – Detail – Modelling of the Pile head elastic-viscous support

9.5.1 RESULTS: FREE VIBRATION RESPONSE

A modal analysis is performed, for which a fundamental frequency of 7,62 Hz is obtained. The fundamental mode of vibration corresponds to an horizontal motion in direction y. The 2nd and 3rd mode of vibration correspond to horizontal motion in x (7,77 Hz) and vertical motion (10,87 Hz), respectively. The deformed shape for the first three modes of vibration can be seen in the Figure 45 to Figure 47:

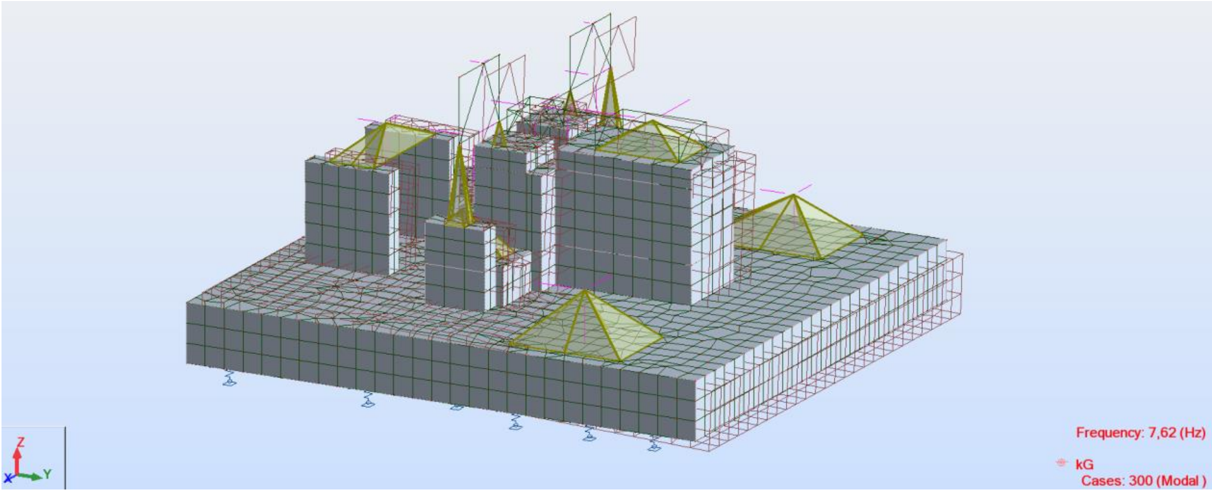


Figure 45 – Finite Element Analysis – Modal Analysis: Vibration Mode 1 – Translation in Y

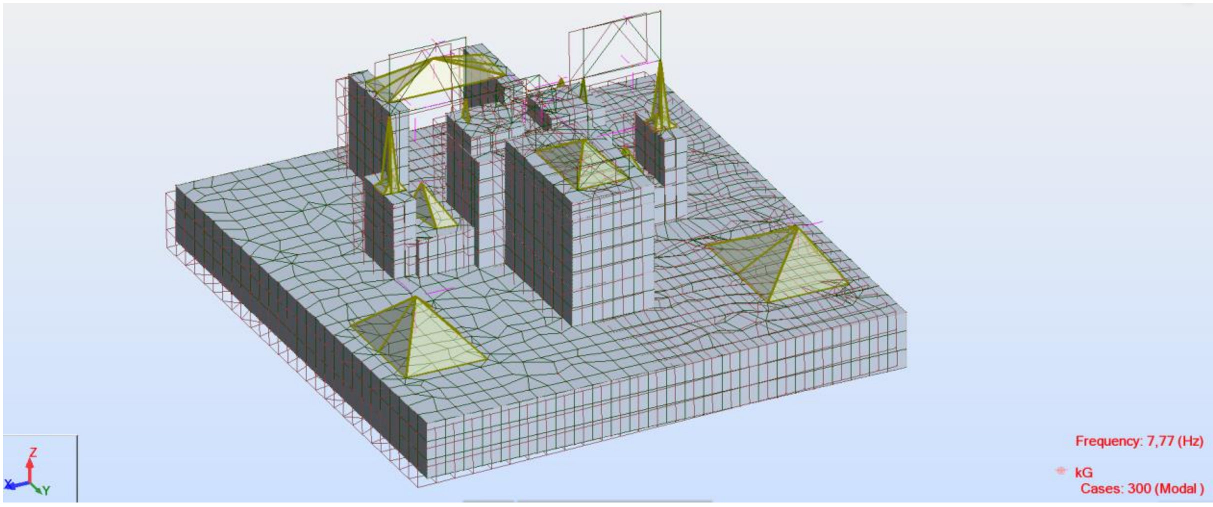


Figure 46 – Finite Element Analysis – Modal Analysis: Vibration Mode 2 – Translation in X

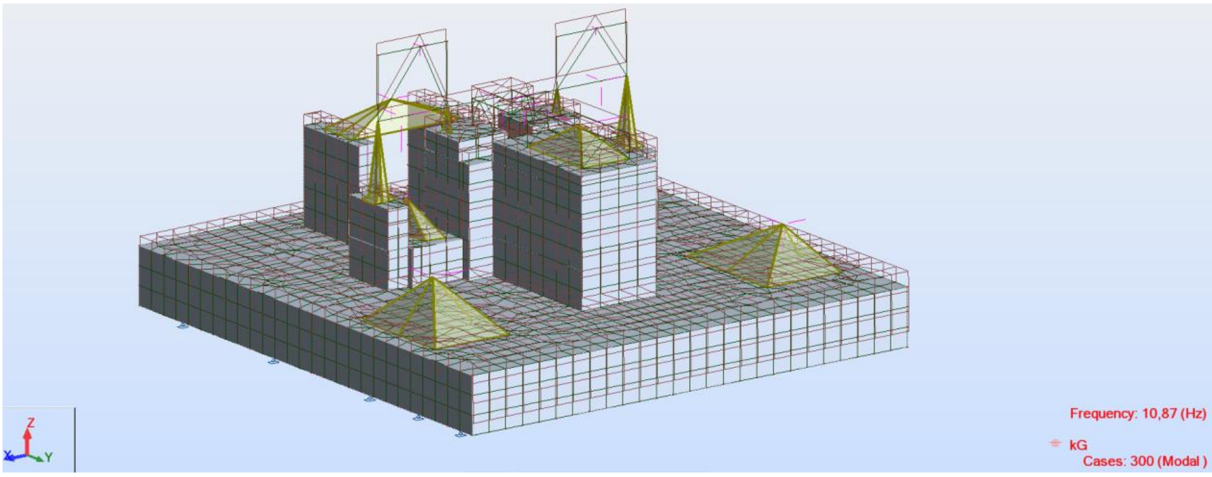


Figure 47 – Finite Element Analysis – Modal Analysis: Vibration Mode 3 Translation in Z

9.5.1.1 Check of resonance

With the natural frequencies of the free vibration response it is possible to compute the frequency ratio (p/ω) by relating the operating speed of the equipment (dynamic excitation frequency) to the natural frequencies of the foundation. As shown in Table 9-15 the frequency of the foundation differs from the operating speed of the equipment by a margin of more than 20%. This limitation is applied to prevent resonance conditions from developing within the dynamic soil-foundation-equipment.

Table 9-15 – Vibration modes, frequencies, mass participation and frequency ratio

Modal Analysis /Mode	Freq, *p* (Hz)	Per, T (s)	Cur.mas.UX (%)	Cur.mas.UY (%)	Cur.mas.UZ (%)	p / ω	
300/1	7,62	0,13	3,03	84,42	0,03	1,236	**OK**
300/2	7,77	0,13	89,57	3,16	0,03	1,260	**OK**
300/3	10,87	0,09	0,16	0,18	98,59	1,763	**OK**
300/4	11,93	0,08	0,19	12,12	0,89	1,935	**OK**
300/5	13,17	0,08	7,04	0,1	0,43	2,136	**OK**
300/6	16,39	0,06	0	0,01	0	2,658	**OK**
		Σ	99,99	99,99	99,97		

9.5.2 RESULTS: FORCED VIBRATION RESPONSE

9.5.2.1 Steady State Response

Dynamic forces defined in 9.2.1 and 9.2.2 are applied at a steady state operating frequency as a time history function. The dynamic forces derived from the operation of the compressor and the motor are applied simultaneously. The amplitudes of vibration are computed at the compressor bearing levels and at the base of the pedestals (top of concrete of the plinths)) – as highlighted in Figure 48.

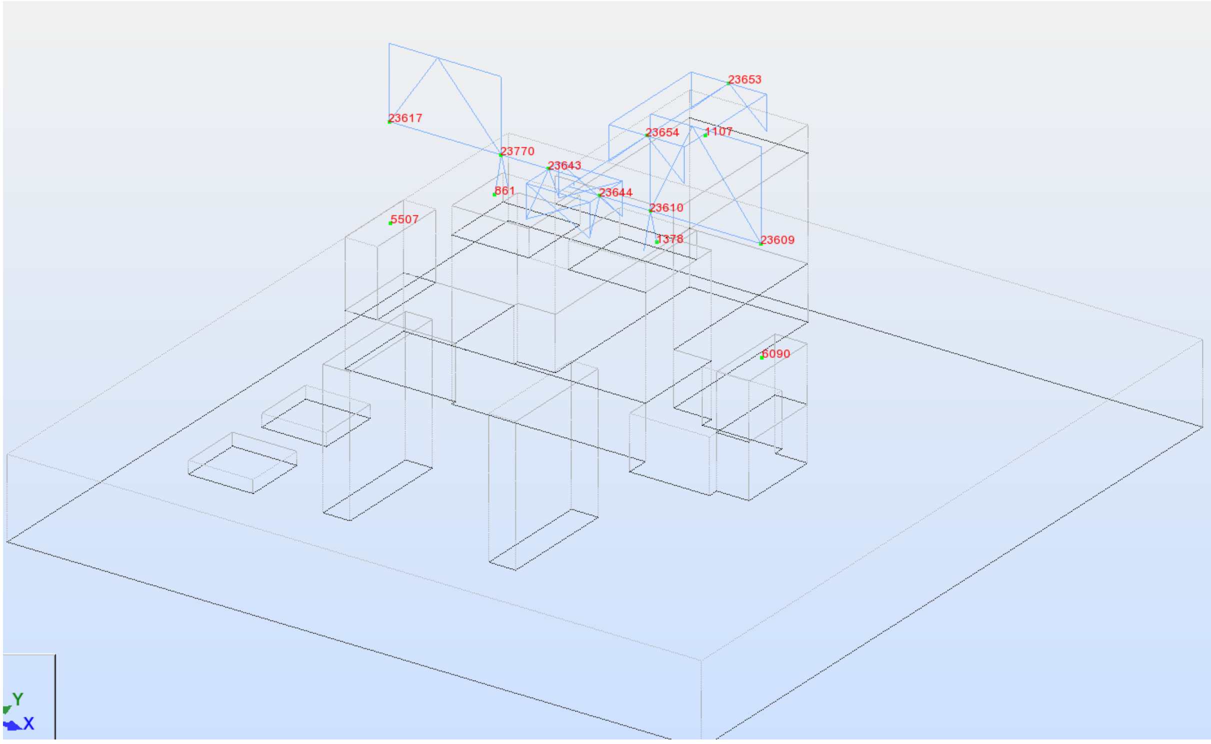


Figure 48 – Nodes object of check of amplitudes of vibration highlighted (labeled) (at the bearing levels and at the base of the pedestals)

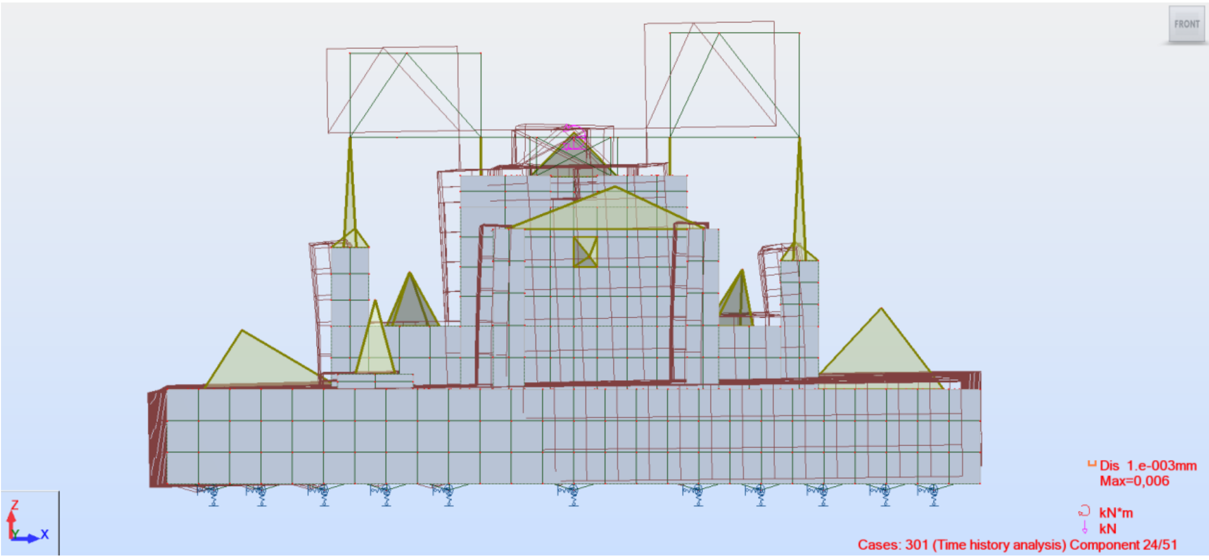


Figure 49 – Time history analysis – Deformed shape at a random time step – Side View XZ

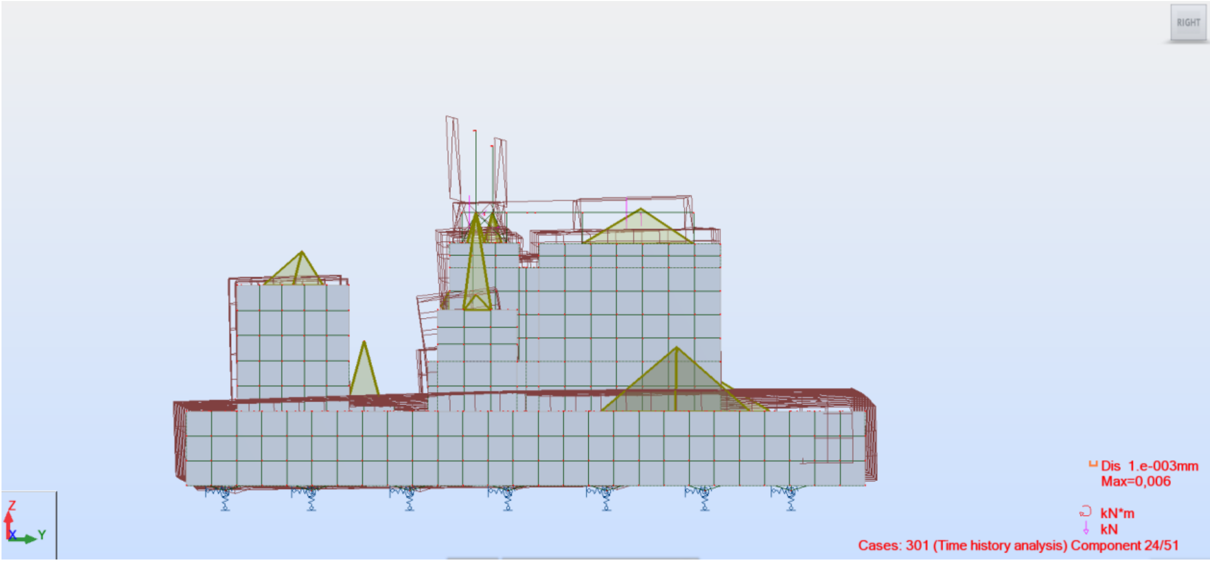


Figure 50 – Time history analysis – Deformed shape at a random time step – Side View YZ

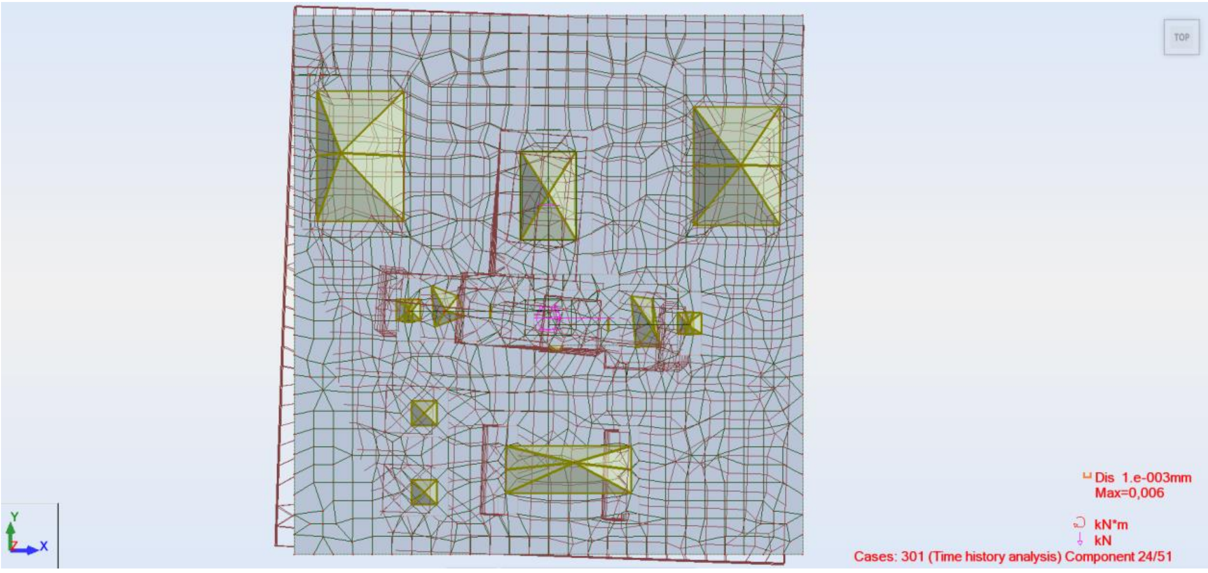


Figure 51 – Time history analysis – Deformed shape at a random time step – Top View

The resulting amplitudes of vibration and velocities in the time domain are plotted below for the singular nodes highlighted in Figure 48:

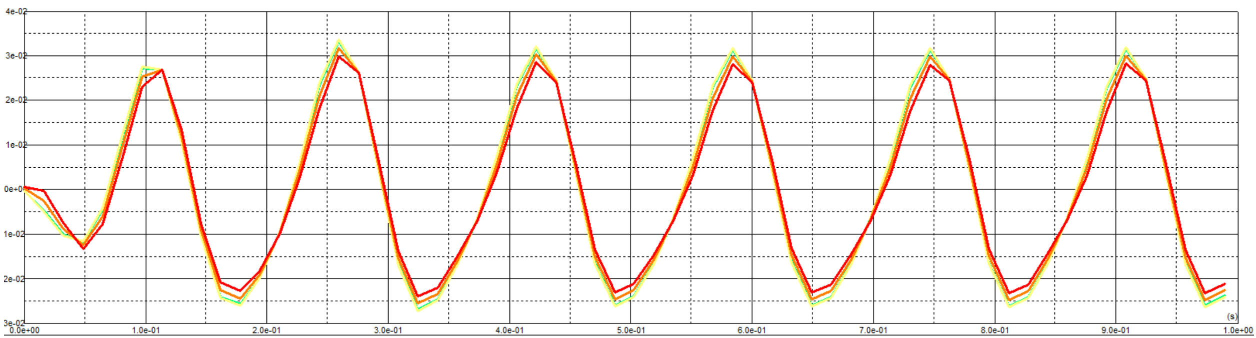


Figure 52 – Time history analysis – Amplitude of vibration (mm)– Direction X : Max displacement ~30 micron

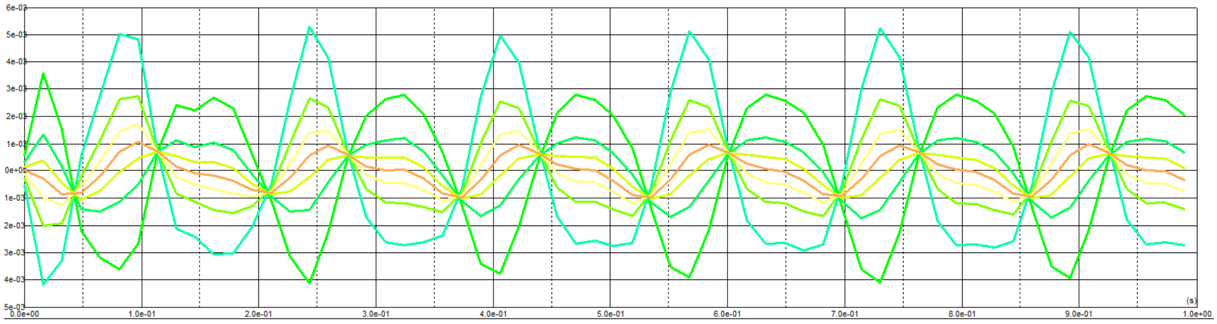


Figure 53 – Time history analysis – Amplitude of vibration (mm)– Direction Y: Max displacement ~5.2 micron

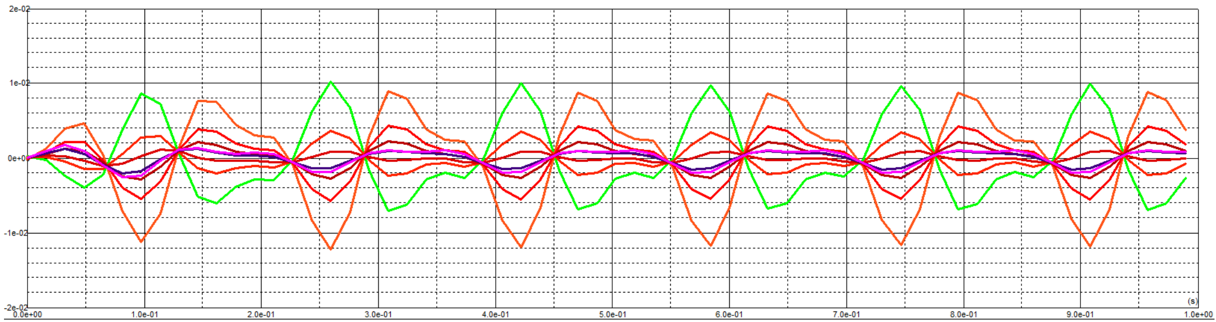


Figure 54 – Time history analysis – Amplitude of vibration (mm) – Direction Z: Max displacement ~12 micron

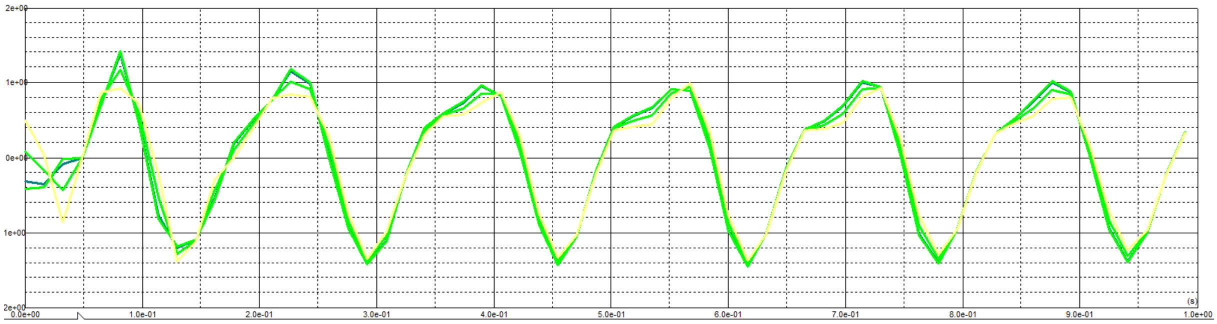


Figure 55 – Time history analysis – Velocity (mm/s) – Direction X: Max velocity ~1.4 mm/s

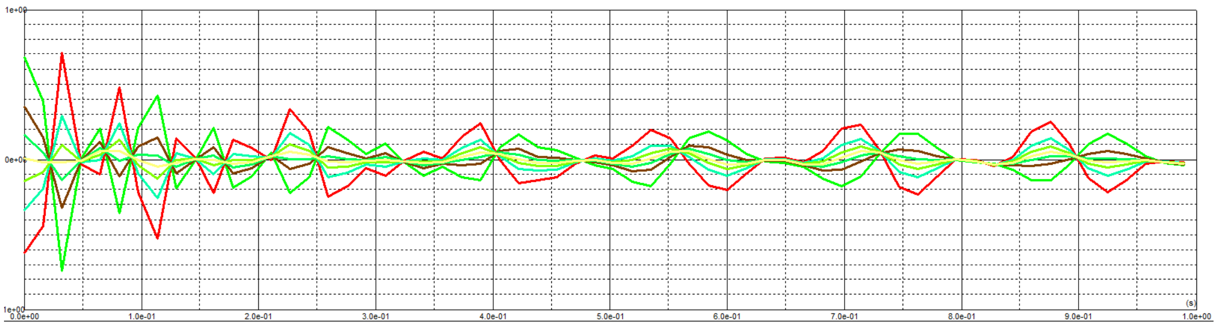


Figure 56 – Time history analysis – Velocity (mm/s) – Direction Y: Max velocity ~0.7 mm/s

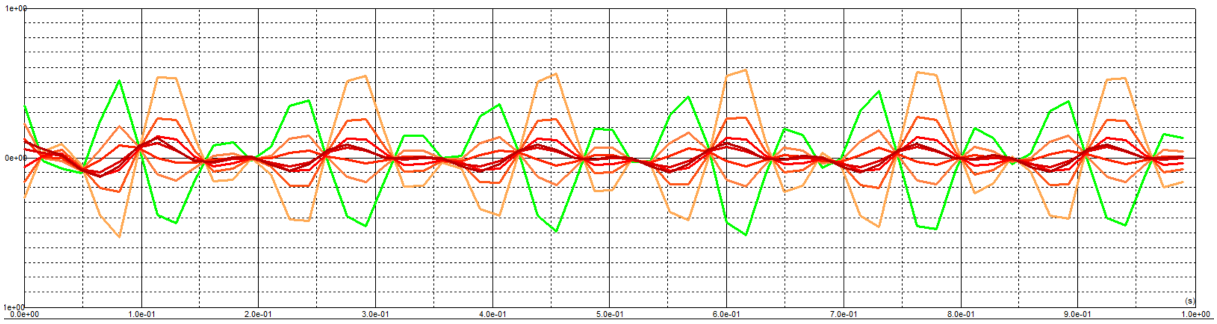


Figure 57 – Time history analysis – Velocity (mm/s) – Direction Z: Max velocity ~0.6 mm/s

9.5.2.2 Transient Vibration Response

During the start-up and shut-down of the reciprocating machine, this transient excitation crosses the natural frequencies of the pile cap. At each foundation frequency, the machine foundation system remains in transient resonance till passing the transient excitation frequency. This results in enhanced amplitudes that should be checked as well.

In the absence of a function defining the start-up and shut-down of the machine in the time domain, a sweep analysis in the frequency domain can be performed. In such case, the dynamic forces characterizing the steady-state excitation can be used for a frequency response analysis, in a range of frequency from 1 Hz to 30 Hz (with increment of 0.25 Hz). Since the magnitude of the forces used derive from an operation frequency, the amplitudes computed for the different frequencies shall be scaled by the square of the ratio of the resonant frequency to the operating frequency.

The transient resonant amplitudes for a specific location (bearing of the compressor) are plotted below for direction X (worst case according results from the time history analysis):

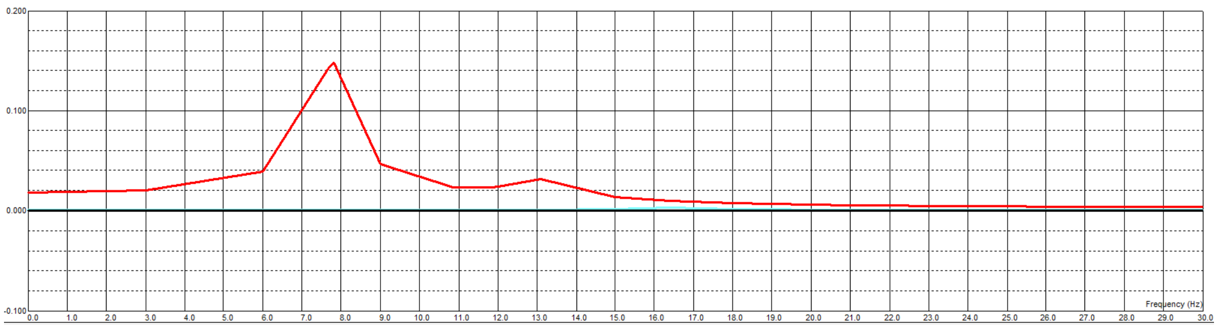


Figure 58 – Frequency response functions – transient response – Amplitude vibration (mm) – Direction X: Max displacement ~145 micron

The maximum displacement is obtained for a frequency of ~7.6 Hz - matching the natural frequency of the pile-supported foundation, as expected.

9.6 ACCEPTANCE CRITERIA

9.6.1 MACHINE LIMITS

The limits of vibration are normally defined by the machine manufacturer. In this case, since there is no information regarding the boundaries of the acceptance criteria, recommendations from Blake (1964) and Baxter and Bernhard (1967) are followed. Most of these studies apply to rotating equipment. However, in absence of other references, they can be applied to reciprocating machines.

According the time history analysis the maximum displacements and velocities occur for the direction X. The maximum peak displacement for this direction is around 30 microns and the maximum velocity for the same direction is around 1.4 m/s. Baxter and Bernhard's chart defines limits for peak to peak displacements. For that reason, the determined displacement values from the time history analysis shall be multiplied by 2 to assess the vibration performance (plotted in Figure 59).

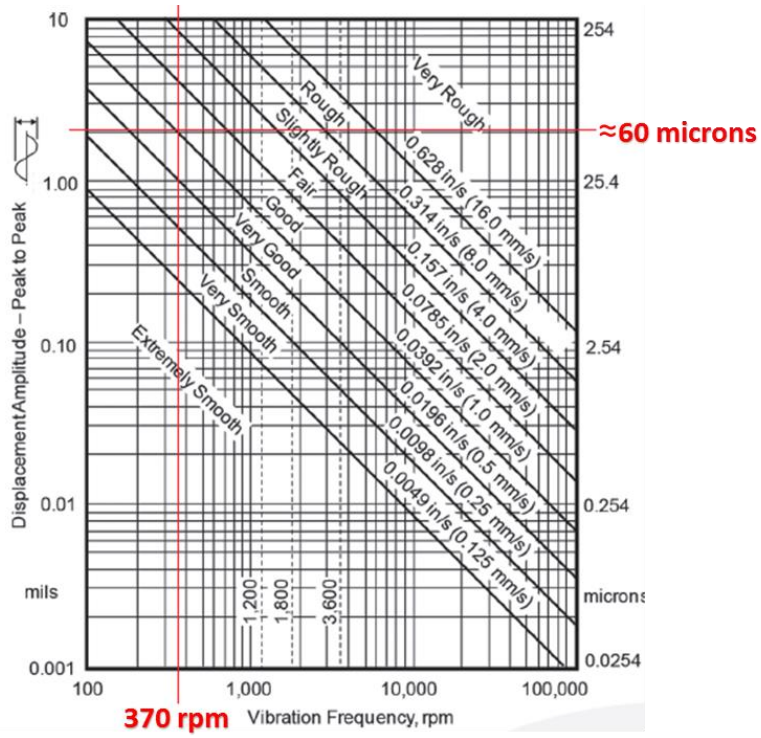


Figure 59 – Vibration performance criteria from Baxter and Bernhard 1967 [28], General Machinery Vibration Severity Chart) – plot of case study (double peak displacements)

The amplitude vibrations of the case study reciprocating machine almost fit in the boundary of the "very good" vibration criteria according to Baxter and Bernhard's chart. The velocity is slightly above the recommended to fit within "very good" classification

On the other hand, Blake's chart considers peak displacements. The values obtained can be plotted directly in the chart (in Figure 60). It is classified as "Minor faults. Correction waste dollars" in terms of vibration performance.

Figure 61 [a), b)] displays several corporate standards [4] among the criteria referenced above. As can be seen, corporate standards are usually more stringent than general standards.

Regarding the transient vibration response, looking at the values of the peak amplitude displacement shown in Figure 58 of around 145 micron one would say that the pile-supported foundation does not meet any of the vibration criteria used for the steady-state verification. However, since the operating speed of the machine is below the natural frequencies of the pile-supported foundation-machine system, it is certain that the start-up and shut-down of the machine will not go above the operating speed of the machine, corresponding to 6.17 Hz (or 370 rpm) thus not posing a problem.

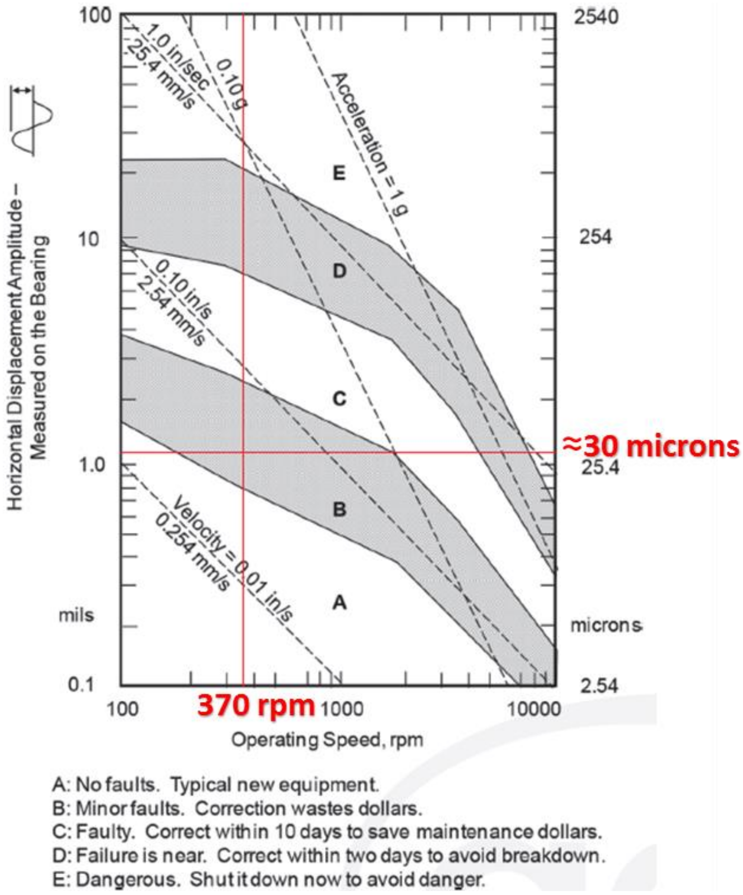


Figure 60 – Vibration performance criteria from Blake 1964 as modified by Arya, O'Neill and Pincus 1979 [8], *Vibration Criteria for rotating machinery – plot of case study*

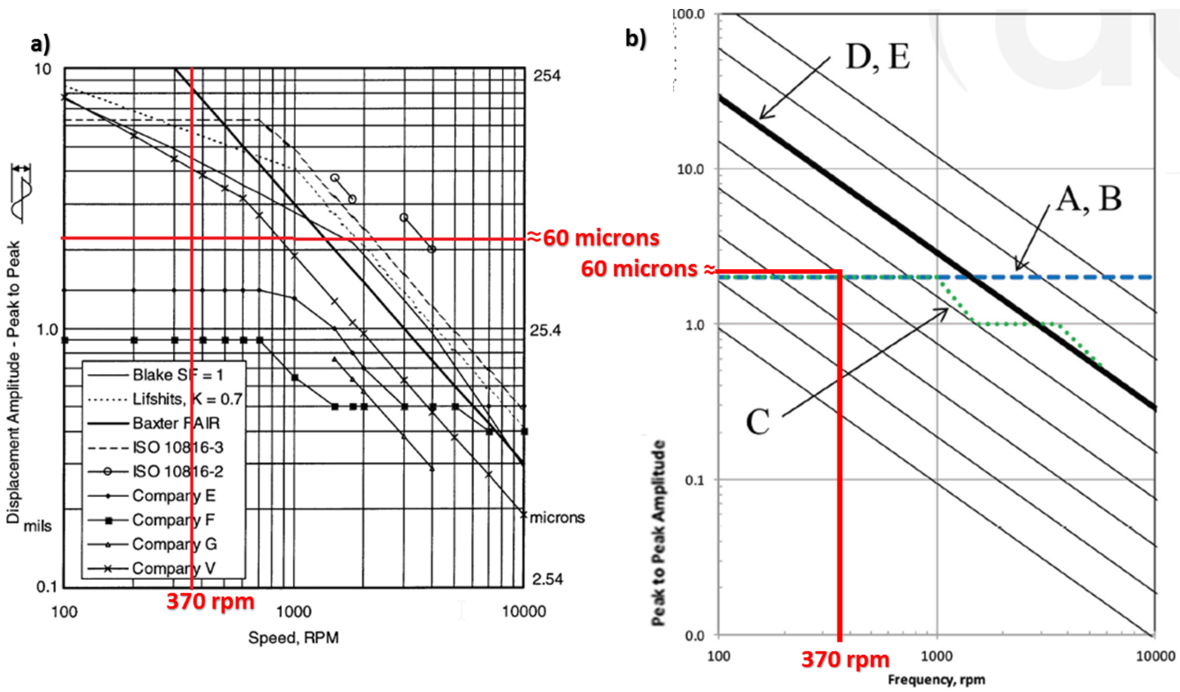


Figure 61 – a) Comparison of permissible displacements from several standards (ACI351-3R) [4] – plot of case study (double peak displacements); b) Comparison of permissible displacements (mils) for five company standards (ACI351-3R) [4] (specific for reciprocating machines) – plot of case study (double peak displacements)

9.6.2 PHISIOLOGICAL LIMITS

The Reiher-Meister chart plotted in Figure 62 is used to evaluate the human perception and sensitivity to vibration.

As described in the Chapter 9.6.1, the maximum displacement obtained from the time history analysis corresponds to direction X. Accordingly, a peak displacement of around 30 microns is plotted in the Reiher-Meister chart (the chart considers peak displacements).

As shown for this case study the source of vibration from the reciprocating machine is almost "Easily Noticeable to Persons", which is acceptable.

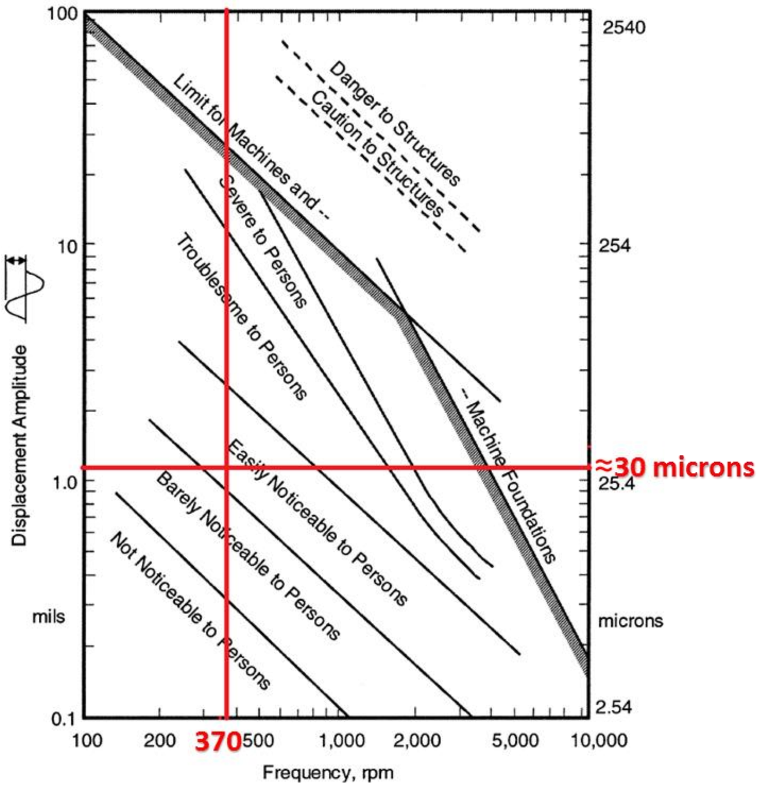


Figure 62 – Reiher-Meister chart (Richart et al. 1970 [12])– plot of case study

10 FINAL CONSIDERATIONS

Machines such as the reciprocating compressor presented in this document are a source of vibration that may have a harmful effect on people, structures or on the machine itself. Therefore, it is essential that the vibration effects are kept at acceptable levels so as not to compromise the health of working personnel neither the integrity of the structures and equipment. This requires a deep understanding of the dynamic behavior of the foundations and structures supporting vibrating machines, so that problems of vibrations can be anticipated.

In the previous chapters a methodology is described for the design of foundations and structures supporting vibrating machines, from a point of view of a design engineer. It is the intention of this document to provide a practical guideline with the current state-of-the-art regarding this subject. Such specific topics are usually out of the radar of the general standards commonly used by practitioner engineers or even from the academic domain. For this reason, it is difficult to define design approaches and establish well defined boundaries of acceptance, in what concerns the design of machine foundations/structures.

Having a methodology and well-defined boundaries of acceptance, it is also important that all the data gathered to perform the design is reliable. This meaning, that the soil parameters should be well assessed through suitable field tests and the communicated mechanical data should be complete and clear. The later, often leads to extensive communication with the manufacturer.

The design methodologies, at present day, are not the same as at the time some reference technical documents on this domain were published. This meaning one should adapt the same notions used in the past to the tools used nowadays. It is nevertheless important that the selected methodology is followed closely with respect to the main principles of the theory of vibrations.

The present document focuses on one approach to a broad topic that can have many variants. Certain subjects are to be studied in detail for future developments. The same case study can be formulated varying some specific parameters and methodologies:

- Machine parameters:
  - Varying the type of motion (rotating or impulsive) and operation speed
- Foundation parameters:
  - Varying the foundation type (flexible or rigid foundation resting on soil) or considering an elevated supporting structure
- Methodologies:
  - Varying the approach to access the dynamic impedances (1 - Frequency dependent approach (e.g. Gazetas model); 2 - Pile group stiffness using dynamic interaction)
  - Varying the approach to estimate the interaction between soil and structure (combined model Soil/pile and machine/foundation - Soil represented as continuum)

REFERENCES

1. Bhatia, K., Foundations for Industrial Machines: Handbook for Practising Engineers. 2009: CRC.
2. Bachmann, H. and W. Ammann, Vibrations in structures: induced by man and machines. Vol. 3. 1987: Iabse.
3. Prakash, S. and V.K. Puri, Foundations for machines: analysis and design. 1988: Wiley-Interscience.
4. Report on Foundations for Dynamic Equipment Reported by ACI Committee 351, in ACI 351.3R-18. 2018, American Concrete Institute.
5. Standard, C., ISO 1940. Mechanical vibration–Balance quality requirements for rotors in a costant state, 2005: p. 1-3.
6. America, A.S.o., ASA/ANSI S2.19:1999—Mechanical Vibration— Balance Quality Requirements of Rigid Rotors, Part 1:Determination of Permissible Residual Unbalance. 1999.
7. Novak, M., Dynamic stiffness and damping of piles. Canadian Geotechnical Journal, 1974. 11(4): p. 574-598.
8. Arya, S.C., M.W. O'Neill, and G. Pincus, Design of structures and foundations for vibrating machines. 1979: Gulf Pub Co.
9. Das, B. and G. Ramana, Principles of Soil Dynamics, Cengage Learning. 2011, Stanford, USA.
10. Bowles, L., Foundation analysis and design. 1996: McGraw-hill.
11. Barkan, D.D., Dynamics of bases and foundations. 1960, McGraw-Hill.
12. Richart, F.E., J.R. Hall, and R.D. Woods, Vibrations of soils and foundations. 1970.
13. Novák, M. and Y.O. Beredugo, Vertical vibration of embedded footings. Journal of Soil Mechanics & Foundations Div, 1972. 98(Sm 11).
14. Dobry, R. and G. Gazetas, Dynamic response of arbitrarily shaped foundations. Journal of geotechnical engineering, 1986. 112(2): p. 109-135.
15. Novak, M. and R. F. Grigg, Dynamic experiments with small pile foundations. Canadian Geotechnical Journal, 1976. 13(4): p. 372-385.
16. Poulos, H.G. and E.H. Davis, Pile foundation analysis and design. 1980.
17. Briaud, J.-L., Geotechnical engineering: unsaturated and saturated soils. 2013: John Wiley & Sons.
18. Hardin, B.O. and V.P. Drnevich, Shear modulus and damping in soils: design equations and curves. Journal of Soil Mechanics & Foundations Div, 1972. 98(sm7).
19. Hardin, B.O. The nature of stress-strain behavior for soils. in From Volume I of Earthquake Engineering and Soil Dynamics--Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, June 19-21, 1978, Pasadena, California. Sponsored by Geotechnical Engineering Division of ASCE in cooperation with:. 1978.
20. JAMIOLKOWSKI, M. Design parameters from theory and practice. in Proc. of the Inter. Conf. on Geotech. Eng. for Coastal Development (Geo-coast'91). 1991.
21. Seed, H.B., Soil moduli and damping factors for dynamic response analysis. EERC, 1970.
22. Kramer, S.L., Geotechnical Earthquake Engineering . 1996, Prentice Hall.: New York.
23. OHTA, Y., Estimation of S-wave velocity in terms of characteristic indices of soil. Butsuri-Tanko, 1976. 29(4): p. 34-41.
24. Seed, H.B., et al., Moduli and damping factors for dynamic analyses of cohesionless soils. Journal of geotechnical engineering, 1986. 112(11): p. 1016-1032.
25. Rix, G.J. and K.H. Stokoe, Correlation of initial tangent modulus and cone penetration resistance. Calibration chamber testing, 1991: p. 351-361.
26. Mayne, P. and G. Rix, G max-qc relationships for clays. Geotechnical Testing Journal, 1993. 16(1): p. 54-60.
27. PIP, Structural design criteria. 2007.
28. Baxter, R. and D. Bernhard. Vibration tolerances for industry. in MECHANICAL ENGINEERING. 1967. ASME-AMER SOC MECHANICAL ENG 345 E 47TH ST, NEW YORK, NY 10017.

ANNEXES

A. GEOTECHNICAL CONSIDERATIONS

Table A 1 – Over-consolidation Exponent k, reproduced from Briaud (2013) [17]

Plasticity Index	Value of k
0	0.00
20	0.18
40	0.30
60	0.41
80	0.48
100	0.50

(After Hardin and Drnevich 1972; Kramer 1996)

Table A 2 – Values of $K_{2,max}$, reproduced from Briaud (2013) [17]

Void ratio	$K_{2, max}$	Relative density (%)	$K_{2, max}$
0.4	70	30	34
0.5	60	40	40
0.6	51	45	43
0.7	44	60	52
0.8	39	75	59
0.9	34	90	70

(After Seed and Idriss 1970; Kramer 1996.)

Table A 3 – Values of G_{max}/s_u, reproduced from Briaud (2013) [17]

	Overconsolidation ratio, OCR		
Plasticity index	1	2	5
15–20	1100	900	600
20–25	700	600	500
035–45	450	380	300

(After Kramer 1996.)

Table A 4 – Common values of G_{max} for different soils based on shear wave velocity, reproduced from Briaud (2013) [17]

Type of Soil	Small-Strain Shear Wave Velocity, v_s (m/s)	Initial Shear Modulus, G_{max} (MPa)
Soft clay	40–90	3–14
Firm clay	65–140	7–36
Loose sand	125–270	29–144
Dense sand and gravel	270–400	72–360

B. CASE STUDY

Table B 1 – Dynamic forces (unbalanced forces) due to operation of the reciprocating machine (steady-state excitation)

			Unbalanced Forces, F_x (H)		Unbalanced Forces, F_z (V)		Unbalanced Moment, M_z (Cz)		Unbalanced Moment, M_x (Cx)	
			1st Order	2nd Order	1st Order	2nd Order	1st Order	2nd Order	1st Order	2nd Order
Sr.No	Time period (Sec)	$t * \omega$	F_x = 19.07 * cos(ωt - 180), kN	F_x = 4.09 * cos(2ωt -0,01), kN	F_z = 0.70 * cos(ωt -0,01), kN	F_z = 0 * cos(2ωt- 0), kN	M_z = 55.69 *cos(ωt -180), kN-m	M_z = 12.87 *cos(2ωt -0), kN-m	M_x = 4.35 *cos(ωt -90), kN-m	M_x = 0 *cos(2ωt -0), kN-m
1	0,00000	0	-19,074	4,087	0,697	0,000	-55,693	12,865	0,000	0,000
2	0,01622	36	-15,431	1,264	0,564	0,000	-45,057	3,976	2,554	0,000
3	0,03243	72	-5,894	-3,306	0,216	0,000	-17,210	-10,408	4,132	0,000
4	0,04865	108	5,894	-3,307	-0,215	0,000	17,210	-10,408	4,132	0,000
5	0,06486	144	15,431	1,262	-0,564	0,000	45,057	3,976	2,554	0,000
6	0,08108	180	19,074	4,087	-0,697	0,000	55,693	12,865	0,000	0,000
7	0,09730	216	15,431	1,264	-0,564	0,000	45,057	3,976	-2,554	0,000
8	0,11351	252	5,894	-3,306	-0,216	0,000	17,210	-10,408	-4,132	0,000
9	0,12973	288	-5,894	-3,307	0,215	0,000	-17,210	-10,408	-4,132	0,000
10	0,14595	324	-15,431	1,262	0,564	0,000	-45,057	3,976	-2,554	0,000
11	0,16216	360	-19,074	4,087	0,697	0,000	-55,693	12,865	0,000	0,000
12	0,17838	396	-15,431	1,264	0,564	0,000	-45,057	3,976	2,554	0,000
13	0,19459	432	-5,894	-3,306	0,216	0,000	-17,210	-10,408	4,132	0,000
14	0,21081	468	5,894	-3,307	-0,215	0,000	17,210	-10,408	4,132	0,000
15	0,22703	504	15,431	1,262	-0,564	0,000	45,057	3,976	2,554	0,000
16	0,24324	540	19,074	4,087	-0,697	0,000	55,693	12,865	0,000	0,000
17	0,25946	576	15,431	1,264	-0,564	0,000	45,057	3,976	-2,554	0,000
18	0,27568	612	5,894	-3,306	-0,216	0,000	17,210	-10,408	-4,132	0,000
19	0,29189	648	-5,894	-3,307	0,215	0,000	-17,210	-10,408	-4,132	0,000
20	0,30811	684	-15,431	1,262	0,564	0,000	-45,057	3,976	-2,554	0,000
21	0,32432	720	-19,074	4,087	0,697	0,000	-55,693	12,865	0,000	0,000

Table B 2 – Dynamic forces (unbalanced forces) due to operation of the motor (steady-state excitation)

			Unbalanced Forces, F_z (C)
			1st Order
Sr.No	Time period (Sec)	t * ω	$F_z = 6.30 * \cos(\omega t)$, kN
1	0,00000	0	6,300
2	0,01622	36	5,097
3	0,03243	72	1,947
4	0,04865	108	-1,947
5	0,06486	144	-5,097
6	0,08108	180	-6,300
7	0,09730	216	-5,097
8	0,11351	252	-1,947
9	0,12973	288	1,947
10	0,14595	324	5,097
11	0,16216	360	6,300
12	0,17838	396	5,097
13	0,19459	432	1,947
14	0,21081	468	-1,947
15	0,22703	504	-5,097
16	0,24324	540	-6,300
17	0,25946	576	-5,097
18	0,27568	612	-1,947
19	0,29189	648	1,947
20	0,30811	684	5,097
21	0,32432	720	6,300

Table B 3 – Dynamic forces (Moment) due to the compressor torque reaction

Sr.No	Time period (Sec)	t * ω	Moment due to the compressor torque reaction (CR)					
			M_y = 4.51 *cos(ωt - 139.3), kN-m	M_y = 18.59 *cos(2ωt +157.1), kN-m	M_y = 3.53 *cos(3ωt +66.6), kN-m	M_y = 7.55 *cos(4ωt +166.7), kN-m	M_y = 2.81 *cos(5ωt -20.4), kN-m	M_y = 2.07 *cos(6ωt -39.1), kN-m
1	0,00000	0	-3,417	-17,121	1,402	-7,345	2,631	1,604
2	0,01622	36	-1,037	-12,169	-3,514	4,922	-2,631	-2,064
3	0,03243	72	1,739	9,600	0,770	-0,618	2,631	1,735
4	0,04865	108	3,851	18,102	3,039	-3,921	-2,631	-0,744
5	0,06486	144	4,492	1,588	-2,648	6,963	2,631	-0,531
6	0,08108	180	3,417	-17,121	-1,402	-7,345	-2,631	1,604
7	0,09730	216	1,037	-12,169	3,514	4,922	2,631	-2,064
8	0,11351	252	-1,739	9,600	-0,770	-0,618	-2,631	1,735
9	0,12973	288	-3,851	18,102	-3,039	-3,921	2,631	-0,744
10	0,14595	324	-4,492	1,588	2,648	6,963	-2,631	-0,531
11	0,16216	360	-3,417	-17,121	1,402	-7,345	2,631	1,604
12	0,17838	396	-1,037	-12,169	-3,514	4,922	-2,631	-2,064
13	0,19459	432	1,739	9,600	0,770	-0,618	2,631	1,735
14	0,21081	468	3,851	18,102	3,039	-3,921	-2,631	-0,744
15	0,22703	504	4,492	1,588	-2,648	6,963	2,631	-0,531
16	0,24324	540	3,417	-17,121	-1,402	-7,345	-2,631	1,604
17	0,25946	576	1,037	-12,169	3,514	4,922	2,631	-2,064
18	0,27568	612	-1,739	9,600	-0,770	-0,618	-2,631	1,735
19	0,29189	648	-3,851	18,102	-3,039	-3,921	2,631	-0,744
20	0,30811	684	-4,492	1,588	2,648	6,963	-2,631	-0,531
21	0,32432	720	-3,417	-17,121	1,402	-7,345	2,631	1,604

Table B 4 – Dynamic forces (Moment) due to the motor torque

			Unbalanced Forces, M_y (CRM)
			1st Order
Sr.No	Time period (Sec)	t * ω	My = 25.55 * cos(ωt), kN
1	0,00000	0	25,550
2	0,01622	36	20,670
3	0,03243	72	7,895
4	0,04865	108	-7,895
5	0,06486	144	-20,670
6	0,08108	180	-25,550
7	0,09730	216	-20,670
8	0,11351	252	-7,895
9	0,12973	288	7,895
10	0,14595	324	20,670
11	0,16216	360	25,550
12	0,17838	396	20,670
13	0,19459	432	7,895
14	0,21081	468	-7,895
15	0,22703	504	-20,670
16	0,24324	540	-25,550
17	0,25946	576	-20,670
18	0,27568	612	-7,895
19	0,29189	648	7,895
20	0,30811	684	20,670
21	0,32432	720	25,550

Table B 5 – Centre of Gravity and Static moment of Mass for the Machine

Item Nr	Component	Weight W	Mass m	Centre of Gravity xi	yi	zi	Static Moment of Mass mi*xi	mi*yi	mi*zi
		kN	ton	m	m	m	t-m	t-m	t-m
1	Compressor frame	84,45	8,61	6,38	5,95	5,45	54,88	51,25	46,92
2	Crank 1 Distance Piece	5,20	0,53	4,28	6,12	5,50	2,27	3,24	2,92
3	Crank 1 Cylinder	53,35	5,44	3,50	6,12	5,50	19,04	33,28	29,91
4	Crank 2 Distance piece	5,20	0,53	8,48	5,79	5,50	4,49	3,07	2,92
5	Crank 2 Cylinder	77,00	7,85	9,31	5,79	5,50	73,08	45,41	43,17
6	Flywheel	45,92	4,68	6,38	6,82	5,50	29,84	31,90	25,75
7	Coupling	3,70	0,38	6,38	6,99	5,50	2,40	2,64	2,07
8	Main Driver	132,00	13,46	6,38	9,11	5,58	85,83	122,55	75,03
9	Suction Vol. Bottle	22,80	2,32	8,68	5,74	7,16	20,17	13,33	16,63
10	Discharge Vol. Bottle	13,60	1,39	9,05	5,69	3,41	12,55	7,89	4,72
11	Suction Vol. Bottle Drain Tank	1,00	0,10	7,90	5,05	2,98	0,81	0,51	0,30
12	Suction Vol. Bottle	15,60	1,59	3,72	6,10	6,84	5,92	9,69	10,87
13	Discharge Vol. Bottle	9,40	0,96	3,77	6,12	3,37	3,61	5,87	3,22
14	Suction Vol. Bottle Drain Tank	1,00	0,10	4,90	5,04	2,98	0,50	0,51	0,30
15	Distance Piece Drain Tank	0,60	0,06	5,65	5,13	2,65	0,35	0,31	0,16
16	1st Packing Gas Recovery Tank	0,60	0,06	7,08	5,13	2,65	0,43	0,31	0,16
17	Barring Device	1,50	0,15	7,64	5,10	4,80	1,17	0,78	0,73
18	Locking Device	1,00	0,10	5,86	5,07	4,26	0,60	0,52	0,43
19	Cylinder Support	9,00	0,92	9,88	5,79	3,85	9,06	5,31	3,53
20	Cylinder Support	9,00	0,92	2,91	6,12	4,05	2,67	5,61	3,72
21	Demister	13,00	1,33	3,23	3,50	3,59	4,27	4,64	4,76
22	Demister	15,70	1,60	3,23	1,50	3,53	5,16	2,40	5,65
23	Gas Cooler	23,10	2,35	6,94	2,65	5,13	16,33	6,24	12,08
24	Gas Cooler	3,95	0,40	8,22	1,46	5,03	3,31	0,59	2,03
25	Cooler Baseplate	26,00	2,65	6,95	2,08	4,32	18,42	5,50	11,44
26	Lube Oil Console	63,00	6,42	1,26	10,11	2,44	8,06	64,91	15,67
27	Cooling Water Console	96,00	9,79	11,18	9,86	2,79	109,39	96,46	27,31
Total		732,67	74,69				494,61	524,74	352,41
		We1	ma1				mixi1	miyi1	mizi1

Table B 6 – Centre of Gravity and Static moment of Mass for Pile Cap

Item Nr	Lx	Ly	Lz	Weight W	Mass m	Centre of Gravity xi	yi	zi	Static Moment of Mass mi*xi	mi*yi	mi*zi
	m	m	m	kN	ton	m	m	m	t-m	t-m	t-m
1	12,900	13,700	1,500	6627,375	675,573	6,375	6,360	0,750	4306,78	4296,65	506,68
2	2,200	3,660	3,365	677,375	69,049	6,175	8,882	3,183	426,38	613,30	219,78
3	3,600	1,800	3,450	558,900	56,972	6,375	6,152	3,225	363,20	350,49	183,74
4	0,800	0,867	-0,500	-8,670	-0,884	6,375	5,686	5,200	-5,63	-5,03	-4,60
5	0,800	0,533	-0,250	-2,665	-0,272	6,375	6,387	5,075	-1,73	-1,74	-1,38
6	2,800	0,400	-0,225	-6,300	-0,642	5,975	6,852	5,063	-3,84	-4,40	-3,25
7	0,800	0,400	-0,405	-3,240	-0,330	7,575	6,852	5,153	-2,50	-2,26	-1,70
8	0,600	1,600	2,370	56,880	5,798	2,825	6,120	2,685	16,38	35,48	15,57
9	0,600	1,600	2,160	51,840	5,284	9,959	5,785	2,580	52,63	30,57	13,63
10	1,450	1,600	0,930	53,940	5,498	3,850	6,120	1,965	21,17	33,65	10,80
11	1,485	1,980	1,030	75,713	7,718	8,918	5,785	2,015	68,83	44,65	15,55
12	1,300	1,300	0,300	12,675	1,292	3,225	3,500	1,650	4,17	4,52	2,13
13	1,200	1,200	0,300	10,800	1,101	3,225	1,500	1,650	3,55	1,65	1,82
14	0,500	2,250	2,600	73,125	7,454	5,345	2,070	2,800	39,84	15,43	20,87
15	0,500	2,250	2,600	73,125	7,454	8,425	2,070	2,800	62,80	15,43	20,87
16	2,550	4,100	0,100	26,138	2,664	1,600	10,020	1,550	4,26	26,70	4,13
17	2,550	3,600	0,100	22,950	2,339	11,150	9,770	1,550	26,08	22,86	3,63
Total				8299,96	846,07				5382,37	5477,96	1008,28
				We2	ma2				mixi2	miyi2	mizi2

Table B 7 – Pile group static interaction factors for vertical and horizontal motion[6]

Total nr of piles **36**

Reference Pile		$\Sigma\alpha_A$	$\Sigma\alpha_{Lx}$	$\Sigma\alpha_{Ly}$
P41	1	10,01	1,79	1,85
P40	2	11,04	2,39	2,34
P39	3	11,07	2,39	2,34
P38	4	10,07	1,79	1,85
P44	5	11,61	3,05	3,27
P43	6	12,27	3,32	3,28
P42	7	11,67	3,05	3,27
P51	8	11,45	2,91	2,99
P50	9	12,48	3,78	3,93
P49	10	12,99	4,19	3,96
P48	11	13,20	4,05	3,95
P47	12	13,04	4,19	3,96
P46	13	12,57	3,78	3,93
P45	14	11,57	2,91	2,99
P56	15	12,27	3,43	3,79
P55	16	13,15	4,33	4,54
P54	17	13,56	4,35	4,47
P53	18	13,25	4,36	4,59
P52	19	12,43	3,52	3,98
P63	20	11,58	3,05	3,07
P62	21	12,70	4,08	4,23
P61	22	13,33	4,60	4,52
P60	23	13,55	4,64	4,54
P59	24	13,43	4,74	4,60
P58	25	12,93	4,38	4,55
P57	26	11,91	3,35	3,57
P69	27	12,01	3,55	3,57
P68	28	12,73	4,12	3,95
P67	29	12,93	4,32	3,88
P66	30	12,84	4,36	4,06
P65	31	12,35	4,08	3,89
P64	32	11,50	3,11	3,21
P73	33	10,25	1,95	1,95
P72	34	11,59	2,84	2,94
P71	35	11,70	3,00	3,04
P70	36	10,62	2,27	2,48
	Σ	437,62	126,03	127,34
	Average	**12,16**	**3,50**	**3,54**

[6] Values indicated in green, blue and red correspond, respectively, to corner, perimetral and inner piles.

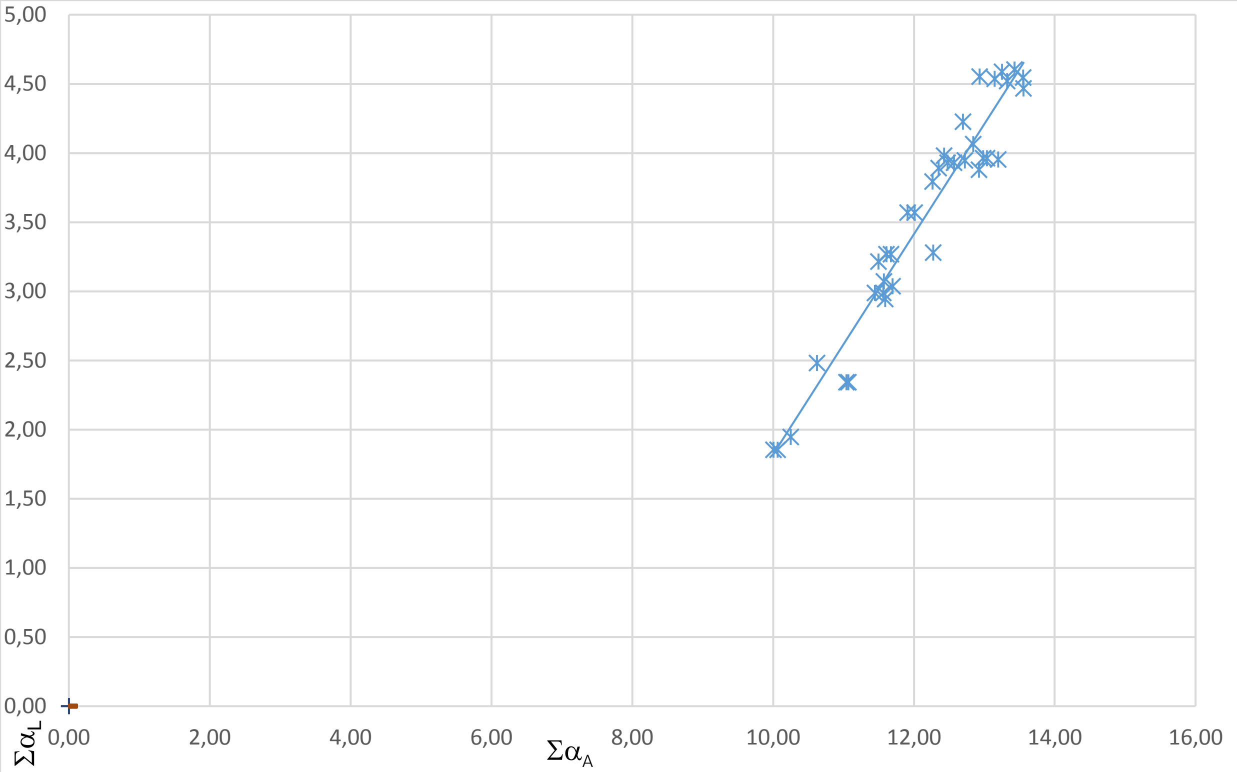


Figure B 1 - Pile group static interaction factors for vertical and horizontal motion after Poulos (any of the 36 piles as reference pile – Case Study)

C. ACCEPTANCE CRITERIA

Table C 1 - Service Factor[7] (After Blake, 1964)

Single-stage centrifugal pump, electric motor, fan	1
Typical chemical processing equipment, noncritical	1
Turbine, turbogenerator, centrifugal compressor	1.6
Centrifuge, stiff-shaft[8]; multistage centrifugal pump	2
Miscellaneous equipment, characteristics unknown	2
Centrifuge, shaft-suspended, on shaft near basket	0.5
Centrifuge, link-suspended, slung	0.3

Table C 2 - General Machinery Vibration Severity Data (After Baxter and Bernhard, 1967)

Horizontal Peak Velocity, in./sec (mm/s)	Machine Operation
< 0.005 (< 0.127)	Extremely smooth
0.005 - 0.010 (0.127 – 0.245)	Very smooth
0.010 – 0.020 (0.254 – 0.508)	Smooth
0.020 – 0.040 (0.508 – 1.016)	Very good
0.040 – 0.080 (1.016 – 2.032)	Good
0.080 – 0.160 (2.032 – 4.064)	Fair
0.160 - 0.315 (4.064 – 8.00)	Slightly rough
0.315 – 0.630 (8.00 – 16.00)	Rough
> 0.630 (> 16.00)	Very rough

[7] Effective vibration = measured single amplitude vibration, in inches, multiplied by the service factor. Machine tools are excluded. Values are for bolted-down equipment; when not bolted, multiply the service factor by 0.4 and use the product as the service factor. Caution: Vibration is measured on the bearing housing, except as stated.
[8] Horizontal displacement on basket housing