Electrical instability in pumps as turbines (PATs): Transient Stability and Energy Storage

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Declaration of conformity

I declare that the present document is an original work of my making and that it complies with the requirements of the code of conduct and good practices of the university of Lisbon.
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Figure 1
Resumo

Para a utilização de PATs isoladas da rede elétrica é necessário usar um tipo de gerador autoexcitado e, ao mesmo tempo, que seja economicamente viável em aplicações de pequena potência. Neste caso optou-se por usar uma máquina de indução auto-excitada (SEIG), devido a ter um baixo custo e existir no mercado em várias potências. Este tipo de máquina pode funcionar tanto como motor ou gerador, no entanto sem estar ligado à rede, requer o auxílio de condensadores em paralelo de forma a ter energia reativa suficiente para gerar um campo magnético. Esta configuração tem o nome de gerador de indução auto-excitado (SEIG).

Este trabalho tem como objetivo a criação de um modelo computacional, que permite simular o comportamento transitório tanto do SEIG como o da PAT a funcionarem em conjunto. Deste modo, o modelo foi validado usando dados experimentais obtidos em laboratório. À posteriori, tendo o modelo validado, este foi usado para calcular os limites do sistema e as várias configurações possíveis, bem como o efeito que a PAT tem no SEIG e vice-versa.

Key-Words: Bomba como Turbina, Gerador de Indução Autoexcitado, Redes de Distribuição de Água
Abstract

In order to use PATs in an off-grid application, it is necessary to use a specific type of generator that can self-excite and at the same time, it has to be economically sustainable for low power applications. In this case, a self-excited induction generator (SEIG) was chosen due to its low cost and being on the market widely available with many range of power. This type of machine can work either as a motor or a generator, however, without being connected to the grid, it is necessary to have shunt capacitors, so it has enough reactive power to self-excite.

This work’s objective is to create a computational model, which can simulate the transient behaviour of both the SEIG and PAT working together. This way, the model was validated using experimental data taken from the laboratory. After having a validated model, this one was used to compute the system’s limits and its different configurations. It was also used to see how the systems PAT and SEIG affect each other.

**Keywords:** Pump as turbine, Induction generator self-excited, Water distribution networks
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Nomenclature

\( C \) Capacitance value of the excitation system (SEIG) [F]
\( EFF \) Efficiency of the system [%]
\( H \) Head [mwc]
\( HP \) Hydraulic power [W]
\( I_a \) Armature current (DC machine) [A]
\( I_f \) Field current (DC machine) [A]
\( I_m \) Magnetization current (Induction machine) [A]
\( I_r \) Rotor phase current (Induction machine) [A]
\( I_s \) Stator phase current (Induction machine) [A]
\( L_L \) Load inductance [H]
\( L_m \) Magnetization inductance (Induction machine) [H]
\( L_r \) Rotor inductance (Induction machine) [H]
\( L_s \) Stator inductance (Induction machine) [H]
\( N \) Rotational speed [RPM]
\( N_s \) Synchronous frequency [RPM]
\( PF \) power factor
\( P_a \) Induction machine active power [W]
\( P_mec \) Mechanical power [W]
\( P_m \) Friction losses [W]
\( Q \) Flow [l/s]
\( Q_s \) Induction machine reactive power [VAR]
\( R_L \) Electric load resistance [\( \Omega \)]
\( R_a \) Armature's resistance (DC machine) [\( \Omega \)]
\( R_f \) Field's resistance (DC machine) [\( \Omega \)]
\( R_r \) Rotor resistance (Induction machine) [\( \Omega \)]
\( R_s \) Stator resistance (Induction machine) [\( \Omega \)]
\( T_H \) Hydraulic torque [\( N \, m \)]
\( T_{ele} \) Electromagnetic torque [\( N \, m \)]
\( U_a \) Armature voltage (DC machine) [V]
\( U_f \) Field voltage (DC machine) [V]
\( X_L \) Electric load inductive reactance [\( \Omega \)]
\( X_c \) Capacitor reactance [\( \Omega \)]
\( X_r \) Rotor reactance (Induction machine) [\( \Omega \)]
\( X_s \) Stator reactance (Induction machine) [\( \Omega \)]
\( \beta \) Angle between rotor and \( dq \) frame [Rad]
\( \beta_0 \) Viscosity coefficient [\( Nm/(rad/s) \)]
\( \kappa_\phi \) DC machine magnetic field constant
\( \omega \) Angular frequency of the machine [Rad/s]
\( \omega_r \) Angular frequency of the rotor [Rad/s]
\( \omega_s \) Synchronous frequency [Rad/s]
\( \phi \) Magnetic flux [Wb]
\( \rho \) Water density [\( Kg/m^3 \)]
\( \theta \) Angle between stator axis and \( dq \) reference frame [Rad]
\( \theta_r \) Angle between stator axis and rotor axis [Rad]
\( f \) Induction generator frequency [Hz]
\( g \) Gravitational acceleration [\( m/s^2 \)]
\( p \) Number of pairs of poles of the induction machine
\( s \) Induction machine slip
# Glossary

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>BEP</td>
<td>Best efficiency point.</td>
</tr>
<tr>
<td>BH</td>
<td>Backhead.</td>
</tr>
<tr>
<td>DMA</td>
<td>District metered areas.</td>
</tr>
<tr>
<td>ER</td>
<td>Electric regulation.</td>
</tr>
<tr>
<td>HPG</td>
<td>Hydroelectric power generation.</td>
</tr>
<tr>
<td>HR</td>
<td>Hydraulic regulation.</td>
</tr>
<tr>
<td>IG</td>
<td>Induction generator.</td>
</tr>
<tr>
<td>IST</td>
<td>Instituto Superior técnico.</td>
</tr>
<tr>
<td>PAT</td>
<td>Pump as turbine.</td>
</tr>
<tr>
<td>PVR</td>
<td>Pressure reduction valve.</td>
</tr>
<tr>
<td>REDAWN</td>
<td>Reducing Energy Dependency in Atlantic area Water Networks.</td>
</tr>
<tr>
<td>SEIG</td>
<td>Self-excited induction generator.</td>
</tr>
<tr>
<td>WDN</td>
<td>Water distribution network.</td>
</tr>
<tr>
<td>WDS</td>
<td>Water distribution systems.</td>
</tr>
<tr>
<td>WTS</td>
<td>Water transmission systems.</td>
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</table>
Chapter 1

Introduction

1.1 Motivation

With global warming becoming a real threat, an effort to improve renewable energy sources is being made, either by improving a system’s efficiency or by harnessing other sources of power. Moreover, companies prefer to use all their resources as this represents an increase in revenue. With this in mind, hydroelectric power generation (HPG) has been a developing field of technology since it has a small environmental impact and can be used in many scenarios.

In HPG a turbine is commonly used, however, because the turbine has to be tailored to that specific case it might end up being a costly solution, and in some cases, like rural areas or small companies, there is only a small capital available. The implementations costs can be reduced using a pump as a turbine (PAT), or reverse mode, since pumps are widely available in the market and in a lot of formats and shapes. On the other hand, the PAT has a lower efficiency when compared to the turbines. The PAT is a flexible system that can either be used as a pump or as a turbine. The PATs are considered to be a good approach where the water supply is relatively constant, such as water transmission systems (WTS).

The idea of having a pump as turbine started during the 20th century with researchers such as Stepanoff [1], Knapp [2], that in 1941 published the complete pump characteristics for a few pump designs based on experimental investigations. Although there was a cost reduction with the use of a PAT, the technology that allowed its proper use was still not developed enough. Contrary to the turbines, the PAT does not provide any control over the hydraulic system (pressure or flow), hence the control comes from the electric generation system or hydraulic valves in series/parallel. In recent years significant developments have been made in the area of energy Systems Electronics, which made sensors, converters and more equipment easily available in the market.

The aim of this work is related to an idea for a new application for PAT that has recently emerged and consists of using it to generate energy in water distribution systems (WDS). In some nodes of WDS, there is a high pressure in the pipes, which has to be reduced, not only because it can damage the pipes but also because in most cases it is not convenient to have high-pressure water coming out of the
taps. The current solution consists in installing pressure reduction-valves in the system (PRVs) [3],[4] and [5], however all the energy is dissipated and wasted. Using hydropower devices instead, could be an attractive investment, and it could lead not only to energy savings, but also reduction in costs of maintenance. Studies in recent years have been proposing this type of system and try to achieve the best performances with new solutions. The solution proposed is to have an isolated system from the grid, that can then use the recovered energy to power any appliances or store it for future use. The idea is that the system does not need to be connected to the grid since, in some cases, the pressure reduction needs to happen in remote locations.

The generator chosen to recover the mechanical energy from the PAT is the induction machine. This machine is widely available, is simpler and is less expensive than many others. Modeling both a PAT and an induction machine has not yet been made and there is not much information regarding on how to model a induction machine using \(dq\) transformation. Both systems have an influence on each other, which turns it into a complex system.

### 1.2 Objectives

The thesis is inside of a part of a big project called Reducing Energy Dependency in Atlantic area Water Networks (REDAWN), which aims to foster the adoption of hydropower energy recovery technology. It is an European project and has partners from all the Europe. In this specific case, Instituto Superio Tècnico (IST) and the universitat politècnica de València have working together in the application of PAT in rural areas.

This project is divided into 4 phases, represented in figure 1.1. The first part has been done by Bernardo [6] and focused on the analysis of the PAT and SEIG system’s steady-state behavior under different loads. Part two of the project is separated into two tasks. The first one, this thesis, is to model the PAT using experimental data and modeling the generator using \(dq\) coordinates, to create a transient model of the PAT+SEIG system. Then, after having validated the system, simulations are done to test the system’s limits and how it responds to sudden changes. The second part consists in modeling series/parallel PAT+SEIG system, which is other thesis’ scope.

The proposed solution consists in having an induction generator mechanically connected to the PAT, as seen in figure 1.2. This type of electrical machine needs reactive power to produce its magnetic field and become excited. When connected to the electrical grid, the induction machine gets its reactive power from there, however, when there is no electrical grid, a source of reactive power is needed, which is obtained by using shunt capacitors. This project was done using a bank of capacitors, star-connected, in parallel with the induction machine.
This type of solution requires that the capacitors supply the induction machine with enough reactive power. Therefore, certain conditions have to be met in order for it to work properly, otherwise the generator might not self-excite. For this project, a computational model had to be developed since its complexity does not allow to predict its behaviour using other methods. First, the Induction machine model was developed by using a DC motor as prime mover, fig. 1.3. After being sure the SEIG (self-excited induction machine) model had been validated correctly, the DC motor was replaced by the PAT,
in the computational model. The thesis objectives are listed below:

1. Develop a computational model for the SEIG with a DC motor.
2. Validate the DC+SEIG model with experimental tests. Stedy-state and transient regime.
3. Develop a computational model for the SEIG with the PAT.
4. Validate the PAT+SEIG model with experimental tests.
5. Predict the system’s behavior, its limits and how to mitigate risks, using the final computational model.

Figure 1.3: Representation of the overall System setup of DC+SEIG.

1.3 Thesis Outline

The thesis is divided into 6 chapters, described as follows:

- Chapter 1 provides an introduction and overview of the work.
- In chapter 2 a brief review of the state of the art about self-excited induction generators and pumps as turbines is done.
- In chapter 3 the first computational model with a DC motor is described and validated with both simulations and experimental tests.
- In chapter 4 the second computational model with the PAT is described and validated with both simulations and experimental data.
- In chapter 5 the model is used to predict the system’s behavior and limits.
- In chapter 6 some conclusions of the thesis are made and also some comments on future work.
Chapter 2

State of Art: PAT + SEIG systems

The energy from water distribution systems (WDS) can be harnessed using a turbine. Turbines convert either the kinetic or potential energy into mechanical power. WDS with turbines in energy generation is not new and have been in use for the a long time. However, turbines are made and tailored for a specific application, thus they become expensive. There is other solution which is using a pump as a turbine (PAT) or in reverse mode. Pumps do not work exactly like turbines, however they are widely available in a variety of shapes and sizes, which makes them cheaper. The disadvantage of this solution is that PATs have a lower efficiency than turbines. The PATs are coupled to an electric generator. The most commonly used is the induction machine since it is also widely available and is cost-efficient. The induction generator is usually connected to the grid and for stable points of operation does not require much control. However, for stand-alone systems or systems with different demands, such generators are not able to maintain a constant voltage or frequency. The voltage profile depends on the speed and torque given to the generator’s mechanical shaft, and it depends also on the type of load at its terminals. To control the voltage profile, power electronic circuit is used with both conversion and inversion process.

2.1 PAT designs and regulations

The PAT’s characteristics curves of head, efficiency and power change depending on the working mode as exemplified in figure 2.1. The curves are plotted for a constant speed. The efficiency curve shape changes from one mode to another and the best efficiency point (BEP) is represented also in the plot.

In water transmission systems (WTS) the pressure remains almost constant and so the the use of a PAT can be advantageous and is easy to implement because there is not much need for a control. With water distribution systems (WDS) the situation is more complicated as the pressure and flow in the pipes might depend on the demand. This creates a scenario where some control and flexibility is needed to control at least either the pressure or the flow rate. In most cases, in a WDS, the pressure is kept constant while the flow changes according to the demand.
The excessive pressure in pipes and etc, can cause damage to the infrastructures and so pressure reduction valves (PRV) are used. The valves match the flow of gas through the regulator to the demand for gas placed upon it, whilst maintaining a constant output pressure. Solutions have been presented to replace the PRV for a system capable of both reducing the pressure and harnessing the energy dissipated. Installation of a PAT aims to produce energy along with managing/keeping the downstream pressure to a desired level. According to Patelis [8], the installation of PAT reduces the amount of water saved, but in contrast it provides the ability to generate energy. Figure 2.2 is an example of a case study in Greece, where the water saved and energy recovered in one year are shown. The simulation was done using three different solutions: Pressure reduction valves (PRV); A pump as turbine (PAT) capable of providing 7.5KW mechanically; District Metered Areas (DMA), which divides the networking into different areas without any solution.

![Characteristic curves for a pump and a turbine operating modes](image)

Figure 2.1: Characteristic curves for a pump and a turbine operating modes [7]

![Water saved and energy recovered annually in a case study](image)

Figure 2.2: Water saved and energy recovered annually in a case study [8] (predicted from model simulation).
2.1.1 Regulation and control PAT designs

An important issue when analysing the economical convenience of this application of PAT is the power available in the specific node of the WDN chosen to install the PAT. In WDSs, unlike the WTSs, the flow rate and head drop present high variations depending on the user demands, and so the available hydraulic power changes. Some designs to control the PAT have been proposed, but two important are hydraulic regulation (HR) \[9\] and electrical regulation (ER) \[10\]. With a regulation system is possible to obtain the best efficiency point for different working points, hence producing more energy.

In figure 2.3 both methods are represented. HR consists in having a parallel or in series valves which regulate the pressure and adjusts the working conditions to a more suitable area. The ER consists in having an converter between the energy generator and the grid, which is able to change the generator’s working frequency.

![Illustrative scheme of hydraulic and electrical regulation modes, adapted from \[10\].](image)

In figure 2.4 it is represented how both methods affect the PAT curves. The HR keeps the same rotational speed, by changing both the differential pressure and the flow. The ER changes the rotational speed while keeping the differential pressure constant, therefore changing the flow as well.

Both the ER and HR show interesting results, although the HR has higher efficiencies \[6\]. For values of backhead (BH) between 23m and 37m, both methods present similar efficiencies.
2.2 SEIG applications

Using an induction generator in isolated locations has been an area of study for some time, however in the last years it has become a more interesting subject because of the electronics development. SEIG generators are being increasingly utilized in stand-alone generation systems that employ wind or hydro power. In order for the generator to function it is necessary to inject reactive power, which can be made with capacitors or batteries. If a battery is used, some sort of control is necessary, while with capacitors, with the right value, it can self-excite.

The major drawbacks in the use of self excited induction generators are the poor voltage and frequency regulations under prime mover speed and load perturbations. The generated terminal voltage and the output frequency, depends on the excitation capacitance, the three-phase induction machine parameters, the electrical passive load and the prime mover speed [11].

Computational models of a SEIG can help predict its behavior and may give an insight in how to better control it. Even though, tremendous literature is available on modeling a self-excited induction generator, there is a shortage of $dq$ models. In [12] a matlab model is developed, where the magnetization inductance, $L_m$, is taken into account. The authors conclude that, the capacitance value has to be well chosen and that the magnetization inductance variation cannot be neglected.

2.2.1 SEIG control solutions

Some solutions have been proposed and tested along the years to control the voltage profile and efficiency. Most of these solutions are applied to wind power generation. The wind turbine can be compared to using PAT, as it also provides mechanical power to the induction generator. In the paper by Sowndarya [13], a diode bridge rectifier, buck-boost converter and the Voltage Source Inverter ($120^\circ$ mode of operation) is used to control the voltage. The diagram is represented in figure 2.5,
Figure 2.5: Block diagram for voltage control of a SEIG, using a diode bridge, a converter and a inverter, taken from [13].

Another paper made by Taoufik [14] presents a solution, where a fuzzy logic controller is used to regulate the voltage, fig.2.6. This controller uses a switching unit that injects reactive power into the system when needed, with the help of an extra bank of capacitors.

Figure 2.6: Block diagram for voltage control of a SEIG using a fuzzy control, adapted from [14].

2.3 Previous work done in this project

A thesis has been made by Bernardo [6], in order to better understand the working principles of a PAT+SEIG system. In his thesis, a induction generator isolated from the grid and a PAT were used. The work focused on understanding how the parameters of the PAT and SEIG change and the relation between them. A model was developed to estimate the minimum necessary capacitance needed for the induction machine to self-excite. The model was based on the electric equivalent circuit of the induction machine, where the admittances were used to estimate the capacitance.

The parameters of the induction machine are not always equal and they can change with the magnetic flux, temperature, voltage and frequency. Therefore, experimental tests using a DC motor instead of the PAT were done, which allowed to plot the curves how the variables change. The most important
was how the magnetization induction changes with the magnetic flux $E/f$, represented in figure 2.7.

![Figure 2.7: Magnetic inductance variation with E/f, taken from [15].](image)

Experimental tests with the PAT+SEIG were also done, which allowed to plot the head-flow curves for different speeds and also to estimate the PAT efficiency. The maximum efficiency of the PAT, SEIG and global were found to be 45%, 65% and 27% respectively.

The experimental tests also resulted in having the curves of required capacitance needed for the induction machine to self-excite. In figure 2.8 is represented the capacitance required as function of rotational speed of the generator, for specific resistive loads, experimentally and using the computational method.

![Figure 2.8: Minimum capacitance- In continuous lines, the values obtained using the simulation diagram and in dashed lines the ones obtained using the computational method.](image)
Chapter 3

DC motor + SEIG Modeling

In this work, the final objective is to model the induction machine, the turbine and their connections, however, in order to model both systems, the induction machine was modelled first with a DC machine. The DC motor replaces the PAT and provides the necessary mechanical power to the shaft. The DC motor was chosen because it is simple and its behavior is well known. The figure 3.1 shows the SEIG and DC machine's connections. The DC motor is connected to the SEIG via a shaft. The generator is then connected to a bank of capacitors and a resistive electric load. The capacitors provide the necessary reactive power for the induction machine to be excited, while the resistance represents the electric load requested to the PAT+SEIG system.

The induction machine is operating as a generator, and it was modelled using d-q coordinates. This option allows for a more straightforward computation and, most importantly, it allows to model the transitory regime.

The connection between the two machines was modelled using mechanical equations that relate the torque and the speed.

Figure 3.1: Representation of the overall system setup with a DC motor as prime mover.
3.1 Induction machine

The induction machine is made with a stator (stationary) and a rotor (moving part). The stator is made of distributed coils wrapped around the iron core and connected in a specific manner, to create a rotating magnetic field. In this specific case, the induction machine has three-phases and three pole pairs. In one rotation, the rotor passes through the same group of coils three times. The coils are physically spread across the stator and are supplied with a three-phase current, which creates a rotating magnetic field. In this work the induction machine has a squirrel cage rotor. The rotor is composed of conductor bars displayed along the rotor's cylindrical surface and are short-circuited at its terminals, figure 3.2.

Induction machines are asynchronous speed machines, operating below synchronous speed when motoring and above synchronous speed when generating. In motor mode, the magnetic field produced by the stator induces a current in the rotor and, thus, induces tangential forces in the rotor, making it to rotate. To work as a generator, a reactive current must circulate first in the stator windings in order to create a small rotating magnetic field with the synchronous rotational speed. This will induce currents in the rotor and, with the external torque introduced in the rotor, if the rotor exceeds the synchronous velocity (generator mode) it will convert the mechanical power in its axis to electrical in the stator.

3.1.1 Equivalent electrical circuit

The equivalent circuit of an induction machine is represented in picture 3.3. The circuit represents only one phase of the machine and AC power is needed in order for it to function. The stator coils are represented by a resistance, $R_s$, and an inductance, $L_s$. The rotor is also represented by a resistance, $R_r$, and an inductance, $L_r$, whose value can be considered to be equal to $L_s$. The magnetic field is represented by a shunt inductance, $L_m$ and resistance, $R_m$. This values are obtained through no load and blocked rotor tests. Finally, the $\frac{R_r(1-s)}{s}$ represents the mechanical power provided or given to the machine.

In the analysis of the SEIG the following assumptions were made:
1. Only the magnetizing reactance $X_m$ is assumed to be affected by the magnitude of the magnetic flux [15] and all other parameter are assumed to be constant.

2. Stator and rotor leakage reactance ($L_s$ and $L_q$) are assumed to be equal.

3. Core loss in the machine is neglected.

Figure 3.3: Induction machine electrical equivalent, working as a generator.

The induction machine is either self-excited or externally excited if connected to the grid. In grid-connected operation, the reactive power is drawn from the network, and it has a fixed frequency and voltage. If self-excited, shunt capacitors are used to provide the necessary reactive power.

Assuming the voltage drop in the stator is small when compared to the total voltage, $V_s$, the flux in the air gap is given by equation 3.1c. This assumption is not valid for low voltages and frequencies.

$$U_s = \frac{d\psi}{dt} \quad (3.1a)$$

$$\ddot{U}_s = j\omega\dot{\psi} \quad (3.1b)$$

$$|\psi| \propto \frac{U_s}{f} \quad (3.1c)$$

### 3.1.2 dq model

The process of self-excitation is a transient phenomenon and it cannot be described by using the induction machine electrical equivalent circuit. For a better analysis, the abc-dq0 transformation is used.

In the d-q model there are only two axes and both of them can rotate synchronously or not with the rotor and stator references. This means that, instead of using the 3-phase sinusoidal components, the machine can be characterized by two DC components, d and q. In this case, the stationary reference
frame was used, thus the quantities will vary in time because the reference frame does not rotate. However, instead of three-phase variables, now there are two-phase variables for the rotor and other two for the stator. These new variables correspond to the previous quantities in direct and quadrature reference frame.

The matrix used to transform abc coordinates to d-q is given by the Park’s transformation in 3.2, where $\lambda = 2\pi/3$ is the phase angle. The park transformation must be applied to both the stator and rotor quantities. When applying to the stator $\gamma = \theta$, where $\theta$ is the angle between the stator axis and the d-q reference frame. For the rotor, $\gamma = \beta$, where $\beta$ is the angle between the rotor axis and the reference d-q frame. Using a stationary reference, $\theta = 0$ and $\beta = -\theta_r$. Figure 3.4 serves to illustrate the relation between the reference frames and the angles for an arbitrary d-q axis.

$$\begin{bmatrix} d \\ q \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \cos(\gamma) & \cos(\gamma - \lambda) & \cos(\gamma + \lambda) \\ \sin(\gamma) & \sin(\gamma - \lambda) & \sin(\gamma + \lambda) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$ (3.2)

![Figure 3.4: D, Q axes superimposed onto a three-phase induction motor.](image)

Using the stationary reference, the respective angular speeds between d-q axis and the stator phase,
\( \omega \), and between the d-q axis and the rotor, \( \omega_r \), are defined in eq. 3.3 and 3.5.

\[
\begin{align*}
\omega &= \frac{d}{dt}\theta = 0 \\
\beta &= \theta - \theta_r = -\theta_r \\
\frac{d}{dt}\beta &= -\frac{d}{dt}\theta_r = -\omega_r
\end{align*}
\] (3.3)

By dividing the stator and rotor induction coefficients in the leakage and mutual inductances, 3.6 and 3.7, the stator and rotor magnetic fluxes can be written in d-q coordinates, as presented in 3.8 to 3.11.

\[
\begin{align*}
L_s &= \lambda_s + L_M \\
L_r &= \lambda_r + L_M
\end{align*}
\] (3.6)

**Flux linkage equations on stator:**

\[
\begin{align*}
\Psi_{sd} &= L_s i_{sd} + L_m i_{rd} \\
\Psi_{sq} &= L_s i_{sq} + L_m i_{rq}
\end{align*}
\] (3.8)

**Flux linkage equations on rotor:**

\[
\begin{align*}
\Psi_{rd} &= L_r i_{rd} + L_m i_{sd} \\
\Psi_{sq} &= L_r i_{rq} + L_m i_{sq}
\end{align*}
\] (3.9)

Therefore, after the computation of the stator and rotor fluxes, their currents can be computed as in 3.12 to 3.15.

**Current equations on stator:**

\[
\begin{align*}
i_{ds} &= \Psi_{ds} - L_M i_{dr} \\
i_{qs} &= \Psi_{qs} - L_M i_{qr}
\end{align*}
\] (3.12)

**Current equations on rotor:**

\[
\begin{align*}
i_{dr} &= \Psi_{dr} - L_M i_{ds} \\
i_{qr} &= \Psi_{qr} - L_M i_{qs}
\end{align*}
\] (3.13)

The voltages equations are obtained by using expression 3.16, which results from the application of Faraday’s law in the three phase model and then, by using park’s transformation and some algebra,
expression 3.19 is obtained. Equations 3.17 represent the transformation between the quantities from d-q frame to three-phase.

\[ U_{abc} = R I_{abc} + \frac{d}{dt} \Psi_{abc} \tag{3.16} \]

\[ U_{dq} = [P] U_{abc} \Rightarrow U_{abc} = [P]^{-1} U_{dq} \tag{3.17a} \]

\[ I_{dq} = [P] I_{abc} \Rightarrow I_{abc} = [P]^{-1} I_{dq} \tag{3.17b} \]

\[ \Psi_{dq} = [P] \Psi_{abc} \Rightarrow \Psi_{abc} = [P]^{-1} P \Psi_{dq} \tag{3.17c} \]

Replacing the above equations in 3.16, 3.18 is obtained

\[ [P(\lambda)]^{-1} U_{dq} = R [P(\lambda)]^{-1} I_{dq} + \frac{d}{dt} ([P(\lambda)]^{-1} \Psi_{dq}) \tag{3.18} \]

Multiplying by the transformation matrix \([P]\), results in 3.19. Note that this equation is generic, for the stator or for the rotor. Now, this must be applied to the stator and rotor with the respective value of \(\lambda\), as described in previously.

\[ U_{dq0} = R I_{dq0} + \frac{d}{dt} \Psi_{dq0} + [P(\lambda)] \frac{d}{dt} ([P(\lambda)]^{-1} \Psi_{dq}) \tag{3.19} \]

**Voltage equations on stator** \([P(\lambda = \theta)] = [P(\theta = 0)]\)

\[ [P(0)] \frac{d}{dt} ([P(0)]^{-1} \Psi_{dq}) = [P(0)] \frac{d}{dt} ([P(0)]^{-1}) + \frac{d}{dt} \Psi_{dq} \tag{3.20} \]

\[ [P(0)] \frac{d}{dt} ([P(0)]^{-1}) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} (\theta = 0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.21} \]

By applying the results from 3.21 the expressions 3.22 and 3.23 for the voltage in the stator are obtained in 3.22 and 3.23.

\[ v_{ds} = R_s i_{ds} + \frac{d}{dt} \Psi_{ds} + \frac{d(\theta = 0)}{dt} \Psi_{qs} = R_s i_{ds} + \frac{d}{dt} \Psi_{ds} \tag{3.22} \]

\[ v_{qs} = R_s i_{qs} + \frac{d}{dt} \Psi_{qs} - \frac{d(\theta = 0)}{dt} \Psi_{ds} = R_s i_{qs} + \frac{d}{dt} \Psi_{qs} \tag{3.23} \]

**Voltage equations on rotor** \([P(\lambda = \theta)] = [P(\beta = -\theta_r)]\)
\[ [P(\beta)] \frac{d}{dt} ([P(\beta)^{-1}] \Psi_{dq}) = [P(\beta)] \frac{d}{dt} ([P(\beta)^{-1}] + \frac{d}{dt}\Psi_{dq}) \quad (3.24) \]

\[ [P(\beta)] \frac{d}{dt} ([P(\beta)^{-1}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{d(-\theta_r)}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times -\omega_r \quad (3.25) \]

In the rotor, the coils are short-circuited, so the voltage \( V_{dr} \) and \( V_{qr} \) are zero.

\[ v_{dr} = 0 = R_r i_{dr} + \frac{d}{dt}\Psi_{dr} - \omega_r \Psi_{qr} \quad (3.26) \]

\[ v_{qr} = 0 = R_r i_{qr} + \frac{d}{dt}\Psi_{qr} + \omega_r \Psi_{dr} \quad (3.27) \]

Finally, the electromagnetic torque at the machine’s axis is obtained by using the magnetic energy principle, eq. 3.28. The force is given by the derivative of the energy in respect to the angle.

\[ T_{ele} = \frac{d\omega_M}{d\theta} \frac{1}{2} \frac{dL_i^2}{d\theta} \quad (3.28) \]

After some algebra it is possible to obtain expression 3.29. The expression was taken from [16].

\[ T_{ele} = \frac{3}{2} p \left( \Psi_{ds} i_{qs} - \Psi_{qs} i_{ds} \right) \quad (3.29) \]

### 3.1.3 Magnetic inductance

Previous to this work, a master thesis was made by Bernardo [6], where experimental tests were made to the induction machine available at the laboratory to characterize its electric parameters and to verify how they change according to the magnetic flux operating point.

An important conclusion from the previous work was that the magnetization inductance, \( L_m \), has an important role in the machine’s operating point, and it changes with the applied magnetic flux, figure 3.5. The tests were done for several frequencies. In this work, the curve used for 50 Hz since it is the nominal speed of the induction machine. The magnetization inductance, \( L_m \), curve was defined as function of the magnetic flux, \( \phi_m \), eq. 3.30. During the operation of the SEIG, the \( L_m \) value changes in real-time.

\[ L_m = 0.0219 \Psi^3 - 0.8093 \Psi^2 + 0.5531 \Psi + 0.53 \quad (3.30) \]
3.1.4 Complete model of induction machine

All equations were implemented in matlab and are represented in figure 3.6. The "flux's" block uses equations from 3.8 to 3.11 in order to compute the stator and rotor fluxes. The "Lm" block first computes the ratio E/f, that is an image of the magnetic flux, and then uses the curve in figure 3.5 to define the specific magnetization inductance value. The "Currents" block uses the equations from 3.12 to 3.15 to compute the currents in the stator and rotor. Finally, the "Eletromagnetic Torque" block uses equation 3.29 to compute the torque produced by the induction machine. The park's transformation was done using the transformation matrix discussed before.

The inputs of the SEIG model are represented by the red color and they are the stator voltages in abc reference coming from the capacitors bank and the rotor speed, $\omega_r$. The outputs are represented by the blue color and they are the stator currents in abc reference, that will go for the capacitors and resistive loads, and the electromagnetic torque that will be coupled to the PAT mechanical shaft.
Figure 3.6: Induction machine model in Matlab. The inputs are represented by the red squares and the outputs by the blue squares.
3.2 SEIG + electrical load

In order for the induction machine to work without being connected to the grid it is necessary and external auxiliary source of reactive power. The SEIG is composed of the induction machine plus capacitors banks. Although the load is not necessary for it to function it has a great influence in the system.

3.2.1 Capacitors and electrical load

The capacitors are responsible for providing the necessary reactive power to the induction machine, which is given by the expression 3.31. Without the capacitors, when the induction machine is running, a small remnant voltage appears at the stator terminals due to the asymmetry of the rotor’s magnetic core. To introduce the capacitors in the simulation model expression 3.32b was included.

\[ Q = \omega CU^2 \]  
\[ i = C \frac{dU}{dt} \]  
\[ I(s) = CsU(s) \equiv U(s) = \frac{I(s)}{Cs} \]

The electrical load is in parallel with the capacitors and the induction generator, and it can have or not and inductive component. The load is governed by equation 3.33. When the induction machine excites and a voltage appears at its terminals, it is also applied to the electrical load. The current created by the voltage applied to the load is then subtracted to the current in the capacitors. If the load has an inductive component, the capacitors must provide enough reactive power for the induction machine and also for this load component.

\[ U = Ri + L \frac{di}{dt} \]  
\[ U(s) = RI(s) + LsI(s) \equiv I(s) = \frac{U(s)}{R + sL} \]

3.2.2 Complete model of the SEIG + Load

In figure 3.7 the complete model is presented. Since it is a three-phase system, all the equations previously discussed in this section are applied to one phase and is necessary to apply it for the remaining phases. The inputs of the system are the rotor speed and the values of capacitor and resistive and inductive load. The rotor speed is converted into the electrical speed by multiplying it by the number of pole pairs. In the "capacitor” block, the remnant voltage of the machine is added, which makes it possible for the SEIG to start. The "capacitors “ block uses the equation 3.32b and is responsible for the
reactive power, while the "Load" block uses the equation 3.33 and is responsible for the active power that will be asked to the induction generator (it can also request reactive power if an inductive load is added). Finally, the block "Induction machine" uses all the equation discussed in section 3.1.2. In the figure it is possible to see the "feedback loop" of the load, where its current is then subtracted to the current in the capacitors. The output of the SEIG model is the electromagnetic torque produced in its rotor.

![SEIG complete model in matlab (simulink). Rotor speed is the input and the T_{ele} is the output.](image)

**Figure 3.7:** SEIG complete model in matlab (simulink). Rotor speed is the input and the $T_{ele}$ is the output.

### 3.3 Voltage build-up

To better understand the physics about the start-up of the induction machine, a brief explanation will be done and along with some factors, which were not taken into consideration.

The induction generator will self-excite, using the external capacitor, only if the rotor is asymmetric, thus producing the required remnant induced voltage at the stator’s terminals. In the model it was assumed that the inductances in the stator and in the rotor were equal and also that both had the same effect in each other. However, in reality, this is not true as they are dependent on the angle of rotation. Because there are asymmetries, the term $dL/dt \neq 0$ leading to an induced voltage in the stator terminals, which is proportional to the rotor angular speed, $\omega_r$. In order to know how the remnant voltage varies with the speed, some laboratory tests were made. Results are shown in figure 3.8. Since the machine was left without a load, according to Faraday’s law, the voltage increases with the speed. As it can be seen the voltage relation with speed is linear.

The curve used for the model is represented by equation 3.34.

$$V_s = 0.0009N_r + 0.0045 \quad (3.34)$$
Due to this small voltage, the capacitors are able to have a small charge. By charging, they give reactive current to the machine. Now the electric energy supplied (from capacitors) is higher than the reactive energy demanded by the machine (due to inductances), leading to an increase in voltage. With an increase in voltage the capacitors charge-up even more and so on, until it stabilizes, when the electric and magnetic energy are equal. So, in stand-alone mode of operation, it is necessary for the induction generator to be operated in the saturation region. This guarantees one and only one intersection between the magnetization curve (image of the $L_m$ coefficient) and the capacitor reactance line, as well as output voltage stability under load as seen in the figure 3.9.

3.4 DC motor model

In order to validate the model of the SEIG, a DC motor was used as the prime mover, whose objective was to provide the induction machine with enough power and torque. In order for it to work, a voltage is applied to the stator (excitation circuit), which creates a constant magnetic field. Then a voltage is also applied to the rotor (armature circuit) and due to the Lorentz force law, the rotor begins to rotate. Due to the existence of commutator rings in the rotor, the voltage applied on the rotor is always changing polarity, which makes the rotor to be always rotating.
In this case, a motor with separated excitation field was used. This is a simpler setup and has an easier equivalent circuit than the induction machine. The circuit can be seen in Fig. 3.10. Both the excitation circuit and the armature circuit can be simply described as a resistance and an inductance. For the excitation field the voltage will be constant, therefore the inductance can be removed.

The equivalent model is done using the electric and torque equations 3.35 and 3.36. The $U_a$ is the armature voltage, the $I_a$ is the armature current, the $R_a$ is the armature resistance, $K_\Phi$ is a constant that represents the flux’s strength and the $\omega_r$ is the rotor speed. $T_L$ is the load torque (in this case is the SEIG electromagnetic torque), $J$ is the moment of inertia and $\beta\omega_r$ is the mechanical torque caused by its rotation, being $\beta$ the viscosity coefficient.

$$
\begin{align*}
U_a &= R_a i_a + L_a \frac{di_a}{dt} + K_\Phi \omega_r \\
T_{ele} &= I_a K_\Phi \\
J \frac{d\omega_r}{dt} &= T_{ele} - T_L - \beta_0 \omega_r
\end{align*}
$$

Although the $K_\Phi$ can be taken from the nameplate by using the equations above, some trials were made in order to obtain the constant. This was preferable because it can change with time and it depends on the voltage applied to the excitation circuit. Moreover, the resistance changes with the temperature, thus so does the current of the excitation circuit. Nonetheless, the current, $I_f$, was aimed to be always at its nominal value the The armature resistance was measured with a multimeter, while the inductance was measured with a oscilloscope and measuring the phase difference.

- $R_a = 6.5 \, \Omega$
- $L_a = 0.0918 \, \text{mH}$
3.5 Inertia of the system

In order to determine experimentally the $J$ and $\beta$ a speed-down test was done to the SEIG+DC machine setup. To perform this, the rotational speed curve was needed. For this purpose the DC machine can be used as a tachometer by leaving its terminal in an open circuit. For this purpose the machine has to be running as generator and the induction machine as motor. If $I_a = 0$, then, from 3.35, $U_a = K_\Phi \omega_r$, which means the voltage is proportional to the rotor speed. To find the proportional constant, the rotor speed is measured, as well as the voltage. In this case for a 985 RPM the voltage was 6.24 V, meaning the constant is 157.82. Now, it is possible to obtain the speed transient by multiplying the DC machine’s armature voltage with the constant. For this test the induction machine was connected to the grid and working as a motor, while the DC machine was working as generator, although without any electrical load. The excitation field circuit had a current of 1.67 A (nominal current). Since turning on the system, without any control, involves having high currents, turning off the machine was preferable for the test and the same results are obtained with both methods. The transitory curve of the voltage, which is proportional to the rotational speed, can be seen in 3.11.

![Figure 3.11: Voltage at DC machine’s terminals after power being turned off.](image)

To determine the coefficients, the definition of kinetic energy for angular momentum was used, shown in expression 3.37. To obtain the mechanical power equation in 3.38, the previous equation is derived in time. Finally, the inertia coefficient, $J$, is given by expression 3.39..

\[
E_m = \frac{1}{2}J\omega_r^2 + \frac{1}{2}Mv.v \tag{3.37}
\]

\[
P_m = \frac{d}{dt}\left(\frac{1}{2}J\omega_r^2\right) = J\left(\frac{\pi}{30}\right)^2 N_n \left(\frac{dN}{dt}\right)\bigg|_{N=N_n} \tag{3.38}
\]

\[
J = \frac{P_{mn}}{\left(\frac{\pi}{30}\right)^2 N_n \left(\frac{dN}{dt}\right)\bigg|_{N=N_n}} \tag{3.39}
\]

The derivative $\left(\frac{dN}{dt}\right)\bigg|_{N=N_n}$ is obtained by using the speed curve in figure 3.11 and its value is 146.6. The $P_{mn}$ term is the losses due to friction right before deceleration, which is the power supplied to the
shaft via the induction motor, without the Joule losses inside the induction machine. By knowing the resistance values, the voltage and the current at each phase was possible to compute the mechanical power supplied to the shaft, which was found to be $P_{\text{mec}} = 34.5 \text{W}$. The viscosity coefficient, $\beta$ can be obtained by using equation 3.40.

$$P_m = \beta \omega^2$$  \hspace{1cm} (3.40)

Final coefficients :

- $J = 0.022 \text{Kgm}^2$
- $\beta_0 = 0.003247 \text{[Nm/(rad/s)]}$

### 3.6 Mechanical and magnetic losses

The mechanical losses are related to the bearings, air resistance and friction in the shaft and naturally the change along with speed. However, by doing some experimental tests, it was discovered that the overall losses, which also include the magnetic ones, do not change much over time, for the range of speeds in this application.

To discover the total losses in the system, tests were done using the DC machine as motor, while the induction machine was not connected to anything. The values were measured in steady-state and for different speeds. The DC machine was excited to its nominal values and the armature circuit was connected to the grid and then a three-phase rectifier. By applying the formulas presented in eq. 3.4 it is possible to know the torque the DC motor is producing since it is directly correlated to the current. With the voltage, current and resistance in the armature it is possible to compute the $K\Phi$, and then it is possible to compute the electromechanical torque produced. Since the induction machine is not connect to anything, it can be assumed the torque produced is only used to compensate the losses. The tests for different speeds are expressed in table 3.1.

<table>
<thead>
<tr>
<th>N [RPM]</th>
<th>Ia [A]</th>
<th>Ua [V]</th>
<th>$K\Phi$</th>
<th>$T_{ele}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>529</td>
<td>0.20</td>
<td>95.8</td>
<td>1.700</td>
<td>0.34</td>
</tr>
<tr>
<td>629</td>
<td>0.20</td>
<td>113.2</td>
<td>1.696</td>
<td>0.34</td>
</tr>
<tr>
<td>700</td>
<td>0.21</td>
<td>126.3</td>
<td>1.700</td>
<td>0.36</td>
</tr>
<tr>
<td>816</td>
<td>0.22</td>
<td>147</td>
<td>1.700</td>
<td>0.37</td>
</tr>
<tr>
<td>915</td>
<td>0.22</td>
<td>164.5</td>
<td>1.699</td>
<td>0.37</td>
</tr>
<tr>
<td>990</td>
<td>0.22</td>
<td>177.7</td>
<td>1.697</td>
<td>0.37</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 3.1: Experimental data with no load
3.7 Verification and Validation

To prove the model's accuracy, experimental tests had to be made. The previous model made by Capelo et al. [15] used a different method than in this work, therefore, this new model has to be validated as well.

The model with d-q coordinates was validated first starting the system and comparing the experimental steady-state values with the ones obtained from the simulation. The next step was to validate the model during transient regime. This was done by comparing the voltage at the SEIG terminals. The first transient tests were done by connecting and disconnecting the capacitors from the induction machine. One consisted in having the SEIG excited and disconnecting the capacitors, while on the other the SEIG had a rotating speed and then the capacitors were connected.

Tests with the electric load were also done. The SEIG was already excited and rotating at a constant speed and then the electrical resistance was connected or disconnected. The resistances used were of 600Ω or 300Ω.

3.7.1 Experimental setup

Firstly, it is important to point out that all the tests were made by setting the system for the current nominal value of the induction generator (1.6A).

The excitation in the DC machine was aimed to be at its nominal value, in other words, the magnetic field created was aimed to be constant and thus the $K_\phi$ was almost constant as well. It was impossible to be completely constant because the resistance of the excitation circuit is high and its value changes with the temperature, therefore the more time the machine works, the more the temperature increases.

In the circuit used can be seen in figure 3.12, where a DC power source with 220 V was used along and a three-phase AC power source (electric grid) with 230V and 50Hz were used. In this case, the DC power source was used to excite the DC machine, along with a resistor and the AC power was used for the DC machine stator with the help of a rectifier. This implementation allows for easier control over the excitation circuit and guarantees an almost constant magnetic field.

The induction machine and DC motor are connected via the shaft and can be seen in figure 3.13. The nameplates of the DC motor and Induction generator are represented by tables 3.3 and 3.2 respectively, where the rated values of the machines are represented.

The induction machine and DC motor are connected via the shaft and can be seen in figure 3.13. The nameplates of the DC motor and Induction generator are represented by tables 3.3 and 3.2 respectively, where the rated values of the machines are represented.
Figure 3.12: Experimental setup circuit with DC machine. DC motor, Induction generator, Bank of capacitors, wattmeter and resistances.

Figure 3.13: DC machine and induction machines in experimental setup.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>50 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>550 W</td>
</tr>
<tr>
<td>Current</td>
<td>1.6/2.8A (Y/Δ)</td>
</tr>
<tr>
<td>Phase Voltage</td>
<td>230/400 (Y/Δ)</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.74</td>
</tr>
<tr>
<td>Speed</td>
<td>910 RPM</td>
</tr>
</tbody>
</table>

Table 3.2: Induction motor nameplate

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Current</th>
<th>Speed</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator</td>
<td>230 V</td>
<td>5.2 A</td>
<td>1500</td>
</tr>
<tr>
<td>Motor</td>
<td>220 V</td>
<td>4.8-5.5 A</td>
<td>1500-3000</td>
</tr>
<tr>
<td>Excitation circuit</td>
<td>220</td>
<td>0.67</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3.3: DC motor nameplate
In figure 3.14 some material used in the laboratory can be seen. Other materials used only in certain setups are listed below, as well as some explanation of what each equipment does and why some equipment had to be used.

- Current probes
- Three-phase switch
- Auto-transformer
- Three-phase AC-DC diode converter

![Experimental setup with three-phase resistance (star connected), bank of capacitors, tachometer, cables, wattmeter, voltage transformer, and oscilloscope](image)

**Voltage transformer 400V-20V**

The voltage transformer was used only for safety reasons. Although the oscilloscope could handle a voltage of 300V, it would trigger the breaker since it is connected to a grounded power source (Electrical grid). This happens because the terminals in the induction motor are referenced to and float above the ground potential, meaning there will be a fault current. The other solution would be to disconnect the ground from the oscilloscope, but if an internal fault in the equipment does happen it would represent a hazardous for anyone who touches it.

**Bank of capacitors**

Both banks of capacitors had 7 possible levels, the first one with each level having an increment of 10 nF from the previous one and the second bank the increments were of 16 nF each level. To have a wide range of possibilities they were connected in parallel, so that their values add-up.
**Electric load resistance**

The three-phase resistor represent a pure resistance. The values are changed also in steps and range from 60 $\Omega$ to 600 $\Omega$. In this case resistance were connected in star.

**Auto-transformer**

The auto-transformer was also used for safety reasons since it isolates the system from the grid. It also allows to have control over the voltage source easily, which was needed to start the motor slowly, otherwise, the initial current would surpass the motor limits and blow its fuse.

**AC-DC converter**

The converter was needed in order to convert the AC voltage to DC to use in the DC motor. It is a converter simply made of 6 diodes and it has some ripples, though they do not affect the overall stability of the system.

### 3.7.2 Turning On/Off capacitors

The system was implemented, as shown in figure 3.1, using a switch to connect/disconnect the capacitor bank while the SEIG is operating as generator. In figures 3.15(a), 3.15(b), 3.16(a) and 3.16(b) it is possible to see that the experimental data is corroborating the simulation. The permanent regime values in the simulation are very close to the real ones, having a maximum 5% difference from the experimental ones. During the transitory behaviour both the frequency and the voltage are similar to the experimental data, however, there is some small variance. This might happen because some elements are not constant during the transitory regime, mainly the magnetic inductances of both machines and the $K_\Phi$ of the DC motor. Moreover, the hysteresis in the iron it is not being taken into account and in fact, it increases the magnetic flux in the machine, because it stays magnetized. A higher flux means a higher voltage. Also, it is important to point out that, if there were no resistances, the transitory regime would have taken more time. With resistors always connected, the energy stored is always being dissipated. More tests were made with other capacitances and speeds and all of them show the same results.

When looking at the figures 3.16(a) and 3.16(b), is possible to see that in the simulation, the voltage drops to zero while in the experimental setup, there is an AC voltage, even though the capacitors were disconnected. This is the remnant voltage talked in section 3.3.
Figure 3.15: Turning on capacitors.

Figure 3.16: Turning off capacitors after system reaching stability.
3.7.3 Changing the electric resistance

The implementation of this test was made by suddenly removing or connecting the three-phase resistance, which was also made with a switch, although now the capacitors were always connected. Since the machine was always at nominal current and the variance in the resistance was not a lot, the induction machine would never stop being excited. By analysing all figures 3.17 to 3.20 it is possible to see that both the simulation and the experimental results are very similar. However, there are some differences in frequency for the cases where the load is disconnected. Disconnecting the electric load has different implications, which the model cannot predict. This discrepancy is more visible for lower resistance values because having a lower resistace means the system is being asked for more active power. This differences do not affect the validation of the results as the model still predicts the system behavior with a small error in frequency.

When the load is connected there will be a current flowing through the circuit, which means there will a voltage drop in the induction machine windings. The opposite happens when disconnecting the load and the voltage drop disappears.

![Figure 3.17: Connecting a resistance of 600 Ω at 633 RPM.](image)

![Figure 3.18: Connecting a resistance of 300 Ω at 848 RPM.](image)
Figure 3.19: Disconnecting a resistance of 600 Ω at 634 RPM.

Figure 3.20: Disconnecting a resistance of 300 Ω at 510 RPM.
Chapter 4

PAT+SEIG modeling

The system is composed of a pump working as a turbine, and an induction machine working as self-excited induction generator (SEIG), which connects to the pump via a mechanical shaft. Inputs for the PAT+SEIG model are a constant differential pressure for the PAT, the electrical load of the SEIG and the value of the capacitors. The model’s development had to go through several iterations until being completed. Those iterations consisted in different curves that had to be tuned with experimental data, different approaches regarding the mechanical losses, whether they were dynamic or not and finally the implications of whether or not modelling the water pipes. The final model was validated presenting sufficiently accurate results for the overall system dynamics. Despite the deviations, it is representative of what happens in the system and how the different components interact with each other.

Figure 4.1: Representation of the overall system setup with PAT as prime mover
4.1 Pump as turbine (PAT) model

Micro-hydropower generation has the potential to be used in many applications, usually by pairing a turbine and an induction generator. The problem with micro-hydro is the cost of turbine, which increases the initial investment and makes it almost impossible for small communities or locations. The solution is using pumps since they can work in reversed mode, which is the same as a turbine. PATs are reaction water turbine, which behave in a similar manner to that of a Francis turbine. They are available in many shapes and sizes making it a cheaper solution. However, PATs do have a lower efficiency than turbines and no control.

The approach was to develop a hydraulic model for the PAT based on the experimental curves, which were taken for different speeds and hydraulic power. The curves were then represented in matlab as functions. To define the PAT, experimental tests were done to obtain the Head Vs Flow as well the efficiency Vs Head Vs Speed. The curves were done using previous experiments that had been made, in which the data of the flow, head and speed were taken. The PAT efficiency was estimated considering the overall efficiency and the one from SEIG, in other words, knowing the SEIG and mechanical losses was possible to deduce the turbine efficiency, since both the hydraulic power and the electrical power (delivered by the generator) are known.

In figure 4.2 the Q-H curve is represented for different speeds. As the rotationary speed increases, the curve shifts up. The runway curve is also shown, which represents the speed of the turbine with full flow and with no load. The turbines are designed to withstand the mechanical load forces at this speed and it is usually supplied by the manufacturer.

![Image](image.jpg)

**Figure 4.2: Flow-Head curves of the PAT for different rotational speeds**

The figure 4.3 shows the efficiency of the PAT, which depends on the rotational speed and head. The figures 4.5 and 4.4 show the previous plot from different perspectives. The experimental values were taken for speeds of 440, 735 and 830 RPM. Then the curves for the values of 1000, 1100 and 1200 RPM were computed by using the affinity laws. The curve was interpolated with a linear method and no extrapolation was done.
Since there is only a certain amount of experimental data from the turbine, due to technical constraints in the laboratory, it is not possible to have the efficiency curve for all its range. In some cases, both in simulation and experimentally, the turbine works outside that range. The problem was solved by also using the affinity laws for these individual cases [18]. Using a reference point and some algebra,
it is possible to estimate the efficiency for other values of head and speed. The reference point used was from the dataset of 830 RPM, because it was the closest data to the nominal functional point of the SEIG. The flow is not used for efficiency since it is already being taken into account when the H-Q curve is used. In other words, the flow is directly correlated to the head of the turbine. It was chosen to show the efficiency curve as function of the head, and not the flow, because the input of the model is the differential pressure in the PAT.

The model for the PAT+SEIG system was modelled in matlab simulink. The pump is modelled through two main functions, whose code can be seen in the appendix C. After having the input differential pressure at the PAT terminals, P, the rotationary speed of the SEIG, Nr, (which will result from the interconnection of the PAT, SEIG, capacitors and electrical load) is then possible to compute the head with equation 4.1. \( \rho \) is the water density \((1000 \text{kg/m}^3)\) and \(g\) is the standard acceleration due to gravity \((9.81 \text{m/s})\). After defining the head is then possible to use the curves in figure 4.2 and compute the flow. The curves are represented by a general equation, which is given by the formula in 4.2. The formula is then invered to define the flow as function of the head.

\[
H = \frac{P}{\rho g} \tag{4.1}
\]

\[
H = A Q^2 + B \alpha Q + C \alpha^2 \tag{4.2}
\]

A, B and C are the parameters of a second-degree function that defines the PAT H-Q curve in figure 4.2. \( Q \) and \( H \) represent the Flow and head in \([\text{m}^3/\text{s}]\) and \([\text{m.w.c.}]\) respectively. \( \alpha \) is the speed \( N_r \) divided by the reference speed \( N_{ref} \), which models an equation for each speed, as the curve depends on the rotational speed of the machine at a given moment. The \( N_{ref} \) is the speed at which the values of the curve A,B and C were estimated.

With both, the flow and the head at the pump is then possible to compute the hydraulic power, \( P_h \), and hydraulic torque, \( T_h \), using formulas 4.3 and 4.4 respectively.

\[
P_h = P Q \tag{4.3}
\]

\[
T_h = \frac{60 P_h}{2 \pi N_r} \tag{4.4}
\]

In figure 4.6 the PAT is represented in simulink, where the X,Y,Z inputs are the necessary matrix’s needed to represent the efficiency grid. They are respectively the head, speed and efficiency matrix’s. The “eff_map_of_PAT” block obtains from the figure 4.3, the efficiency related to the specified H and Nr. The block “Pressure (dp,Nr)” receives the pressure and the speed and computes the flow from the invered form of the equation 4.2. The hydraulic power and hydraulic torque are also obtain from the last block.
**4.2 PAT + SEIG system**

The system is made of 2 main components, the PAT and SEIG. The last one was previously explained in detail in chapter 3. The PAT transforms the kinetic energy and pressure of a fluid to mechanical energy in the shaft, while the SEIG produces electric power without being connected to the grid. In order to not be connected to the grid, the system has to produce its own reactive power and this done through the use of capacitors.

The complete system has four inputs, which are the initial speed, the differential pressure in the PAT, the capacitors value and the resistance value. The program outputs the torques, the voltages, current, rotational speed, PAT efficiency and power.

In figure 4.7 is possible to see all system and how the components are connected. The PAT and the SEIG are coupled via mechanical shaft, represented by the "mechanical coupling" subsystem. This coupling presents some losses, experimentally they were found to be in the order of $0.1 \text{ Nm}^{-1}$.
4.3 Validation

To validate the developed model, a set of experimental tests were done. Results from both experimental tests and simulations were compared and analyzed. Some adjustments to the model had to be made in order to adapt the developed model to the experimental tests. The temperatures effect on the resistances, both inside and outside of the SEIG were taken into account, as well the inertia coefficient. Several simulations were made and with each simulation, the temperature and the inertia coefficient were adjusted. This adjustments were done since it was not possible to measure their values during the laboratory, besides the values used in the computation model are based in real life scenarios and experiments. This means that, for temperature it is normal to be around 80°C and as to the inertia, its variation only affect the transitory time, which makes it easy to adjust the value, when comparing both simulation and experimental data. The final parameters that were found to guarantee the closer results were to have the SEIG temperature at 80°C and the load resistance temperature at 20°C. The difference in temperature from the SEIG to the load is because the load resistances were being cooled by an auxiliary fan. From the analysis of experimental transient tests to the PAT+SEIG system, the inertia coefficient was found to be around 0.012 $Kg m^{-3}$.

4.3.1 Experimental Setup

The same induction machine and the same material was used in this setup as in section 3.7.1. The hydraulic turbine, considered in this work was an Etanorm 32-125 KSB 4.8, which had a rated speed equal to 1020 rpm and a rated flow of 3.9l.

![PAT used in the experimental setup](image)

The overall hydraulic system was setup as in figure 4.9. The PAT was connected to the induction machine via the shaft. The “flow control tank” is used to reduce the hydraulic variations in the system and keep the piped with water, otherwise if the PAT had a sudden variation in flow, it would create a cascade of events leading to a failure. In this setup two control flow tanks were mounted since just one has shown inadequate and did not prevent a failure. The “recirculating pump” keeps the water flowing though the system. The “pressure tank” is filled with compressed air and keeps the fluid with a constant Head. The “Flowmeter” measured the flow rate in l/s. There were also two pressure sensors, one after the PAT and another before the PAT, which allowed to know the pressure difference in the PAT.
In figure the following elements are shown: PAT (1); Induction machine (2); Recirculating Pump (3); control flow tank 1 (4); control flow tank 2 (5); pressure sensor (6); oscilloscope (7); three-phase resistors (8); capacitor’s bank (9); pressure tank (10); flowmeter (11);

It is important to mention that in the experimental setup there is an error associated with all the instruments, however there is a more significant error when talking about the hydraulic sensors. Because there is an inertia associated with the hydraulic fluid, there are more fluctuations. This means the fluid takes more time to stabilize and thus the readings will need more time as well. The water tank is there to guarantee that, the subsystems function properly with variations in the water flux. However, the water tank adds a lag to the hydraulic system. The "recirculating pump" is controlled on and off manually, meaning there is also a small human error associated with the setup.
4.3.2 Comparison between simulation and experimental data

The experimental data taken from tests is presented in appendix B.2, however the mean errors, from the experimental data comparing to the simulation ones, are listed in table 4.1, as well the max error value. The comparison between the simulation and experimental values are from the steady state. These tests were performed within the ranges listed below:

- Capacitance - From 10 uF to 120 uF
- Resistance - From 200 Ω
- Head - From 5.2 m.w.c to 8 m.w.c
- Rotational speed - From 800 RPM to 1200 RPM

The comparison between the simulation and experimental values are from the steady state. These tests were performed within the ranges listed below:

Table 4.1: Mean errors and max values between experimental and simulated results, in steady-state.

<table>
<thead>
<tr>
<th></th>
<th>∆Global efficiency</th>
<th>∆U RMS</th>
<th>∆I RMS</th>
<th>∆P h</th>
<th>∆flow</th>
<th>∆Head</th>
<th>∆N r</th>
<th>∆P ele</th>
<th>∆S</th>
<th>∆power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean values</td>
<td>7.1 %</td>
<td>5.1 %</td>
<td>6.1 %</td>
<td>5.1 %</td>
<td>5.1 %</td>
<td>0.2 %</td>
<td>6.8 %</td>
<td>6.9 %</td>
<td>8.5%</td>
<td>5.7%</td>
</tr>
<tr>
<td>Max values</td>
<td>17.2 %</td>
<td>15.5 %</td>
<td>18.6 %</td>
<td>9.1 %</td>
<td>9.2 %</td>
<td>0.2 %</td>
<td>12.7%</td>
<td>15.4%</td>
<td>24.9%</td>
<td>17.6%</td>
</tr>
</tbody>
</table>

The results validate the system model since the mean values of the errors are below 10%, which is an acceptable value for a system like this, with hydraulic, mechanical and electrical submodels.

In table 4.2 some simulations are displayed as they represent a general example of the totality of the tests made. Tests 1 and 2 were done with an electrical load, tests 3, 4 and 5 were done without any electrical load. Finally the last two simulations represent specific cases, where the system was functioning at a certain point, then the load or capacitance changed and the system stabilized with other values. The values after the change are the ones presented in the table. Also the inertia coefficient was calibrated with these specific tests as the load and capacitance variation transitory allowed for a better visualization and comparison.

Table 4.2: Comparison between experimental and simulation results, in steady state.

<table>
<thead>
<tr>
<th>Test</th>
<th>Experimental Data</th>
<th>Comparison with simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hydro Power(W)</td>
<td>U error</td>
</tr>
<tr>
<td>With Load</td>
<td>243</td>
<td>7%</td>
</tr>
<tr>
<td>With Load</td>
<td>408</td>
<td>16%</td>
</tr>
<tr>
<td>Without Load</td>
<td>160</td>
<td>0%</td>
</tr>
<tr>
<td>Without Load</td>
<td>126</td>
<td>9%</td>
</tr>
<tr>
<td>Without Load</td>
<td>103</td>
<td>27%</td>
</tr>
<tr>
<td>Load variation</td>
<td>342</td>
<td>8%</td>
</tr>
<tr>
<td>Capacitors variation</td>
<td>343</td>
<td>12%</td>
</tr>
</tbody>
</table>

| Mean                  | 12%               | 14%     | 2%      | 29%               |

In table 4.2 the rotational speed has very little error, which indicates the mechanical losses are well adjusted since they have a significant influence on the speed.
The transitory regime in the laboratory was also compared with the simulation. From figure 4.11 to 4.14 different system situations are shown. They represent what can or might happen in case the system is implemented.

4.3.3 System start-up without and with electrical load

The figures 4.11 and 4.12 were tests without and with electric load respectively. The first test was done using a $C = 17.36 \, \mu F$, $R = 0 \, \Omega$ and $dP = 0.57 \, \text{Bar}$. The second test was done with $C = 34.72 \, \mu F$, $R = 300 \, \Omega$ and $dP = 0.58 \, \text{Bar}$. The values used in simulations were the same used in the experimental setup and in both cases the machine was turning at a constant speed without any capacitance and then the capacitors were connected to the generator via a switch. Since the capacitors are not connected, there is no reactive power, hence no voltage is either. When comparing both of them, it is possible to assert that the transitory regime with load is more oscillatory than without load. It means having a load when the system starts, makes it more keen to fail, either by not starting at all or by not stabilizing.

![Figure 4.11: Experimental Data and simulation of system exciting without Electric load.](image1)

![Figure 4.12: Experimental Data and simulation of system exciting with Electric load.](image2)

4.3.4 Resistance variation

In the next test, the PAT+SEIG system was already running with a PAT head of 7.46 mwc and with the electrical parameter $R = 600 \, \Omega$ and $C = 17.36 \, \mu F$. After reaching the steady state the value of the
resistance was decreased in 50% to $R = 300\Omega$, which can be seen if figure 4.13. The rotationary speed increased in 6%. Decreasing the resistance mimics the increase of load requested from the consumer to the SEIG.

4.3.5 Capacitance variation

In figure 4.14, after reaching steady-state, with a $H = 7.46$ mwc, $R = 300$ Ω and $C = 17$ uF the capacitance value was increased in 100%, to 34 uF. Although there was a transitory in the voltage, its steady state value is very similar to the initial one. However, the rotationary speed increased in 29%. Comparing with the resistance variation, the capacitance has much more influence on the speed.

From the figures above, it is possible to see that, during the transitory, there is a bigger voltage difference when the system gets excited than when there is a variation in load or capacitance. This is because, when the system has to excite, the variations in the system’s quantities are of higher values, like the rotational speed, that instead of changing 5%, changes around 20%. These big fluctuations lead to have bigger error during the transitory periods.
4.4 Frequency analysis

In the figures from 4.11 to 4.14, it is possible to observe that the electrical frequencies of the voltages curves in the simulation are different from the experimental tests since the sinusoidal waves never sync.

To help understand the difference between the simulations and the experiments, the frequency for the cases of a capacitor and resistance variation, in table 4.2 were computed and are represented in figures 4.15 and 4.16 respectively. The frequency was computed by first doing the absolute value of the voltage and then measuring the time between each peak. The frequency is then given by \( f = \frac{2}{T} \), being \( T \) the period. It is multiplied by two because the time difference is measured between the peaks of the wave's absolute value. Due to the experimental data being sampled, the frequency always varied between two values, therefore it had to be smoothed.

Figures 4.15 and 4.16 show that one reason for the deviation between transient behaviors is also related to the error between steady-state conditions. Nevertheless, the average deviation between experimental and simulation frequencies is about 7%. Also, the error before and after the variation remains constant.

An important observation, when comparing the capacitance and resistance variations is that, while the rotational speed as an error of 1% and 2% respectively, the frequency has an error of around 7% for both cases. The difference is explained because the slip of the induction machine in module, is actually smaller experimentally than in the simulation. This conclusion is done because for a induction machine to be working as a generator, the rotor has to have a bigger speed than the electrical frequency. Since the rotational speed is very similar both experimentally and in the simulation and the electrical frequency is smaller in simulation than experimentally, then the slip ,in module, is bigger in the simulations. The difference in slip might be because the rotor parameters have an error and further tests and measurements would have to be made.

![Figure 4.15: Computed frequency of both experimental and simulation data for a capacitor variation.](image)

Although there is not more experimental data, when comparing to the simulation, it is reasonable to assert that something is causing the oscillatory movement in the experimental setup. The movement might be because of the behaviour of the hydraulic system that is not being taken into account. The hydraulic buffer and pipes influence the system and cause it to need more time to stabilize.
These cases show that the computational model can predict and simulate the system with enough accuracy, though some differences can be seen, mostly in frequency and peak values.

In order to compare the transient regime in frequency, other test was made while ensuring that the simulation starts with the same frequency value as the experiment. Having no starting error is important because otherwise it might influence how the system behaves. For example, starting with a higher value will probably ensure that the curve has a higher inclination. In figure 4.17 a resistance variation was used and the same values as in 4.16 were used, although the capacitor was decreased slightly in order to have the same starting frequency. In this case the simulation behaves almost identically as in the laboratory test, however it is not as abrupt and the peak is farther away. Also the simulation peak value has a difference of -1.5% to the experimental one.

Figure 4.16: Computed frequency of both experimental and simulation data for a resistance variation.

Figure 4.17: Computed frequency of both experimental and simulation data for a resistance variation with no starting error.
Chapter 5

Simulations and system predicted behavior with complete model

The number of different physics and devices presented on this system create a complex and highly dependent system. One small change in one subsystem can affect significantly the other, and lead to a cascade of events. Moreover, each subsystem has its own stable operation zone, which, if the operation falls outside this zone, might implicate a destabilization of the overall system. In a real scenario, as in the experimental setup present in the hydraulic laboratory, the hydraulic system has limited hydraulic power to provide, not only because is a closed loop but also because of the vessel’s pressure capacity, the power of the recirculating pumps and, of course, the pipes and buffer limits.

In an industrial application, the system must be designed for a specific range of operation, by choosing the right equipment’s size. Although most of the equipment can be personalized in a certain way and have different characteristics, the most practical and most straightforward methodology to change the operation point is to change the capacitors’ value and also the electric load (change the electric resistances). These, alongside with the PAT pressure, will define the working points of the overall system.

However, this system is also susceptible to external variations, such as pressure variation, which might be impossible to control or predict. These variations can influence the system in a harmful way either by increasing the current or its rotational speed to values that the system cannot withstand. There is also the case where hydraulic power may drop, through the sudden reduction of pressure, decreasing the SEIG rotational speed which could lead to a lack of reactive power to proper excite the SEIG.

The results for sudden changes on this complex system may not be predicted without a simulation of the overall system. This happens because having variations leads to different operating points, and each point is unique and produces different results. For example, increasing the pressure does not necessarily imply an increase in active power produced by the SEIG as the speed will increase, which increases the voltage, which, in turn, increases reactive power produced by the capacitors.

In a real practical case, different types of scenarios and variations may occur due to external factors, influencing the system performance. For example, one sudden increase of the electric load of the SEIG will influence the working point of the PAT. Between these cases it is necessary to know which of those
may become harmful to the system as well as to know how or if it is possible to mitigate worst scenarios. It is in this context that several scenarios with a sudden change in the capacitance, resistance and PAT pressure are simulated to analyze its impact in the global system behavior.

5.1 Baseline Solution

The simulation model in section 4 was developed to estimate the behavior of the overall system, for different working points, with different speeds and pressure. It is also used to estimate transient behaviors of the system under different scenarios of sudden changes. In the laboratory, it was not possible to reach the nominal current of the SEIG due to hydraulic limitations (not enough hydraulic power to reach the rated SEIG point). The maximum current obtained in experimental tests was about 0.8 A, which was the same obtained in the simulation, under the same conditions. The temperature of the resistance loads were considered to be at room temperature and the SEIG windings’ temperature was considered to be 80°C.

All the values of voltage and current, in the following tables, are in root mean square values. Although the frequency is not directly represented in the results, it is directly proportional to the rotational speed by a factor of three and has a minor difference around 5 % and 10 %. The factor of three is due to the number of pair of poles in induction machine. In this case, the induction generator has three pair of poles, meaning in one mechanical rotation, there will be three electrical rotations. The existing difference between the electrical frequency and rotational speed is because of the slip, s. The relationship is given by the equation 5.1. The efficiency, Eff, presented in the following tables is the efficiency of the all system (PAT+SEIG).

\[
Freq = \frac{3}{60} \times N_r
\]

5.1.1 Capacitance variation

In this simulation case, the PAT+SEIG system was started with a specific pressure, a resistive load and capacitance until reaching its steady-state conditions. After reaching steady-state, the capacitor was then changed from its initial value to different values, from -50% to 50% in steps of 10%. The complete set of values are presented in table A.1, in annex, and also shown if figure 5.1. In the figure it is possible to see the variations of the quantities when the capacitance is changed. With a 50% reduction in the capacitance value, the generator halted and stopped generating power, since no longer had the necessary reactive power to excite itself.

For a constant electric resistance, the system behaves as expected: reducing the capacitor value increases the system speed, N, and vice-versa because it depends on apparent power the machine is producing. From an electric point of view, the current, Is, is much more influenced by the capacitor change than the voltage, Us. The current decreases as the capacitor decreases but remains almost constant when increasing the capacitor value.
The voltage behaves differently than expected since no matter how the capacitance is reduced or increased, never increases itself. Furthermore, according to the induction law, represented in equation 5.2, the voltage should be highly dependent on the machine’s speed. Since the voltage is not varying, it means the flux, \( \phi \), is the one changing. The flux is given by \( \frac{E}{f} \) as explained in section 3.1.1.

\[
U = \frac{d \psi}{dt} = j\omega \phi
\]  

(5.2)

One interesting thing is that increasing or decreasing the capacitor does not necessarily mean the other values will decrease or always increase in the same way. For example, increasing the capacitance in 30 % increases the reactive power in 7.4 %, while increasing the capacitance in 40 % increases the reactive power in only 4.1 %. The same happens to the other quantities like the voltage and current, however the same does not happen with the rotationary speed. This happens because the relationships between variables are not linear, i.e., the outputs are a combination of complex variables, which react differently to each variation, which is connected to different reactive power points. This phenomenon is much more significant when decreasing the capacitance.

The efficiency of the system is quite low, because the system is not operating in the nominal point (as stated: not enough hydraulic power available in the laboratory), where it is more efficient.

![Figure 5.1: System quantities variation when the capacitance is changed. Nr: rotation speed; Us: Voltage; Is: phase current; Pa: active power; S: Apparent power; Q: Reactive power.](image)

### 5.1.2 Resistance variation

In this simulation case, the PAT+SEIG system was started with a specific pressure of 0.724 bar, a resistive load of 230 \( \Omega \) and capacitance of 35 uF until reaching its steady-state conditions. After reaching steady-state, the three-phase resistance was then changed from its initial value to different values, ranging from -50\% to 50\% in steps of 10\%. The results are presented in table 5.1, where the system
halted with a resistance variation of -50% (first value of the table).

Changing the electric load resistance shows that it is not possible to increase the active power, however the reactive power changes almost in the same proportion as the resistance. Decreasing the resistance value is equivalent to asking the SEIG for more active power since according to Ohm’s law, for the same voltage, the current will increase. Because of the PAT, the SEIG does not have enough torque to generate the necessary active power. This will have an effect in the SEIG’s quantities.

The power factor is difficult to predict since it depends on the inductances and capacitances of the system, which might change over time.

<table>
<thead>
<tr>
<th>∆ RL</th>
<th>HP (W)</th>
<th>Nr (RPM)</th>
<th>Us (V)</th>
<th>Is (A)</th>
<th>Pa (W)</th>
<th>Q (VA)</th>
<th>PF</th>
<th>∆ Us</th>
<th>∆ Is</th>
<th>∆ Pa</th>
<th>∆ Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40%</td>
<td>97</td>
<td>1945</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
<td>-110</td>
<td>-22.9%</td>
<td>-8.9%</td>
<td>-15.5%</td>
</tr>
<tr>
<td>-30%</td>
<td>341</td>
<td>1055</td>
<td>61.0</td>
<td>0.71</td>
<td>67</td>
<td>0.52</td>
<td>-22.9%</td>
<td>-9.9%</td>
<td>-29.9%</td>
<td>-39.9%</td>
<td>-40.3%</td>
</tr>
<tr>
<td>-20%</td>
<td>350</td>
<td>975</td>
<td>69.4</td>
<td>0.74</td>
<td>76</td>
<td>0.49</td>
<td>-12.4%</td>
<td>-4.6%</td>
<td>-30.6%</td>
<td>-24.6%</td>
<td>-28.2%</td>
</tr>
<tr>
<td>-10%</td>
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<td>929</td>
<td>74.8</td>
<td>0.76</td>
<td>79</td>
<td>0.46</td>
<td>-5.5%</td>
<td>-2.3%</td>
<td>-23.2%</td>
<td>-19.3%</td>
<td>-19.9%</td>
</tr>
<tr>
<td>0</td>
<td>357</td>
<td>899</td>
<td>79.2</td>
<td>0.77</td>
<td>80</td>
<td>0.43</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>10%</td>
<td>359</td>
<td>878</td>
<td>83.1</td>
<td>0.79</td>
<td>81</td>
<td>0.41</td>
<td>5.0%</td>
<td>2.4%</td>
<td>1.8%</td>
<td>8.8%</td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>360</td>
<td>862</td>
<td>86.5</td>
<td>0.81</td>
<td>80</td>
<td>0.38</td>
<td>9.2%</td>
<td>4.5%</td>
<td>0.2%</td>
<td>17.1%</td>
<td></td>
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<tr>
<td>30%</td>
<td>361</td>
<td>850</td>
<td>90.4</td>
<td>0.82</td>
<td>81</td>
<td>0.36</td>
<td>14.2%</td>
<td>6.0%</td>
<td>1.9%</td>
<td>25.1%</td>
<td></td>
</tr>
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<td>40%</td>
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<td>841</td>
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<td>0.84</td>
<td>79</td>
<td>0.34</td>
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<td>-0.2%</td>
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<tr>
<td>50%</td>
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<td>0.85</td>
<td>78</td>
<td>0.32</td>
<td>20.9%</td>
<td>10.2%</td>
<td>-1.6%</td>
<td>40.0%</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Resistance variation from -50% to 50%

5.2 Simulations with an increased size PAT

The hydraulic setup in the laboratory was not correctly dimensioned for the hydraulic power of the PAT-SEIG system because it could not produce enough pressure to reach the rated torque of the SEIG. The maximum hydraulic power delivered to the PAT was 400 W (hydraulic) which, due to the system’s overall efficiency, resulted into an electric power load of around 120 W, much lower than the rated power of the induction machine (500 W). Although the machine reaches speeds higher than 1200 RPM’s, it never reaches the 1.6A rated current and, therefore, never reaching the induction machine best efficiency point. When the induction generator requests more active power than the one the PAT is capable to provide, the torque load from the SEIG is higher than the PAT one, thus it will drive power from the inertia of the system, decreasing the speed. When the speed diminishes, the induced voltage also decreases, resulting in less load power, stabilizing the PAT-SEIG system in a new working point, with lower active power.

In the previous simulations, to reach higher values of active power, the speed had to go up to values much higher than the rated ones, making it unpractical to test experimentally. Doing tests with rotationals speeds much higher than the rated ones would increase the toll on the system, not only decreasing its lifetime but also by damaging it. To reach the rated values of the electric machine, the PAT model was changed so it could produce more mechanical torque, allowing the induction machine to produce higher currents and active power. This was done by increasing the head and flow relation by a factor of 3. In
other words, for the same flow as before, now there is three times more head or differential pressure in the pump. This allows for a higher mechanical torque. In a real scenario not only the PAT would have to change but also the hydraulic setup would have to change. The efficiency curve was also shifted in order to have the same efficiency profile, but now for higher values of head.

In figures 5.2(a) and 5.2(b) are presented the head and hydraulic power curves in function of the PAT flow, for both the original curve and the new one. The hydraulic power of the original PAT would have never been enough to reach the induction machine’s rated point. In a practical scenario, the system would have been designed to guarantee the most optimal situation, therefore the simulations done with the enhanced model are much more meaningful since they represent the most probable scenario.

![Flow Vs Head curves](image1.png)

![Flow Vs Hydraulic power curves](image2.png)

Figure 5.2: Comparison between the old PAT curves and the new ones.
5.2.1 Capacitance variation

In the table 5.2 are shown the results for a capacitance variation, after the system reaching the steady-state condition. The steady-state conditions were obtained for a PAT pressure of 2.1 Bars, a capacitance value of 35 uF and an electric resistance of 200 Ω.

Results show the speed increasing with the decrease of capacitance value, as expected [15]. The active power has small changes even for large changes of the capacitor, however, the reactive power greatly increases as the speed decreases. As consequence of increase of the reactive power, the reactive part of the current also increases, thus increasing its effective value. The higher values of the currents lead to higher losses in the electrical machine, thus decreasing its efficiency and the global one.

<table>
<thead>
<tr>
<th>ΔC</th>
<th>HP (W)</th>
<th>Nr (RPM)</th>
<th>US (V)</th>
<th>Is (A)</th>
<th>Pa (W)</th>
<th>Q (VA)</th>
<th>PF</th>
<th>Eff</th>
<th>∆Us</th>
<th>∆Is</th>
<th>∆Pa</th>
<th>∆Q</th>
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<td>-30%</td>
<td>-14%</td>
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<td>-23%</td>
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<td>1173</td>
<td>157.4</td>
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<td>-592</td>
<td>0.52</td>
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<td>5%</td>
<td>-11%</td>
<td>8%</td>
<td>-11%</td>
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<td>6%</td>
<td>-4%</td>
<td>11%</td>
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<td>22%</td>
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<td>903</td>
<td>149.4</td>
<td>2.07</td>
<td>334</td>
<td>-866</td>
<td>0.36</td>
<td>21%</td>
<td>-1%</td>
<td>26%</td>
<td>1%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table 5.2: Simulation data with capacitance variation at current’s nominal value

A fundamental observation is the fact that the hydraulic power also changes according to the capacitance. Although the PAT and SEIG are two different systems, since they are connected through the mechanical shaft, they both influence each other. For example, by reducing the reactive power injected in the SEIG, the speed increases and figure 5.3 shows that the flow will decrease as long as the differential pressure remains constant, which is assumed to be the case. In reality there may be a small change in pressure, however, the hydraulic setup is meant to keep a constant pressure. By decreasing the flow, the hydraulic power also decreases, as exemplified in Figure 5.2(b), converging the overall system into a new working point.

The power factor is inversely proportional to the capacitance variation because, if the voltage does not change, the reactive power increases with the capacitor value. By decreasing the capacitance, according to the equation 5.3, the current decreases, for the same voltage and electric frequency. The capacitors are responsible for the reactive power injection in the SEIG, and by decreasing it, the proportion of active power to reactive power increases, which is expressed by the power factor. At the same time, the reactive component of the current decreases, resulting in a lower current effective value. This decreases the power losses inside the electrical machine and, thus, increases the overall efficiency of the system.

\[ I_c = C \frac{dU_c}{dt} \equiv T_c = j\omega U_c C \quad (5.3) \]
Decreasing the capacitance seems advantageous, however it depends on the application because it may also decrease the amount of active power and, of course, the speed also increases. Increasing the capacitors does not guarantee an increase in active power as can be seen by the simulations in table 5.2, however it always results in an increase of current. If the SEIG was already set for its rated current, increasing the current will result in the overload of the SEIG. Depending on the current overload, the machine may break down or get damaged.

### 5.2.2 Resistance variation

In this section, simulations were done with constant pressure and capacitance. After reaching the steady state with pressure of 2.1 Bar, capacitance of 35 uF and resistance of 200, the resistance was changed to more/less 30% of its previous value. Changing the resistance can be seen as changing the electrical load of the induction generator. The active power is the power lost in the resistance, which is given by equation 5.5. For a constant voltage, the power increases as the resistance decreases, however, in this complex system, more variables play an important role in the working point and, so, decreasing the resistance may not be associated to increasing the power load.

\[ P = I^2R \]  

(5.5)

Table 5.3 shows the results for the variation of the electric resistance. As shown in the previous
simulation 5.2.1, the active power remains almost constant, even when changing the resistance value. This is due to the change of the speed of the electrical machine, where decreasing the value of the resistance leads to a different curve of capacitors to excite the machine, as shown in Fig. 5.4. Therefore, for the same capacitance value, if the resistance is decreased the speed of the machine increases. At the same time, from the PAT point of view, the Q-H curve shifts to higher values of head, and the working flow point reduces, figure 5.3. With a constant head and a reduction of the flow, the hydraulic power decreases, decreasing the available power for the PAT-SEIG system.

The main difference occurs for the voltage, current and power factor. The voltage and current increases with the electrical resistance, while the power factor decreases.

![Speed Vs Capacitance](image)

Figure 5.4: Speed Vs Capacitance. Example of how both quantities vary with the resistance. Picture taken from [15]

<table>
<thead>
<tr>
<th>ΔR_L</th>
<th>HP (W)</th>
<th>Nr (RPM)</th>
<th>Us (V)</th>
<th>Is (A)</th>
<th>Pa (W)</th>
<th>Q (VA)</th>
<th>PF</th>
<th>Eff</th>
<th>ΔUs</th>
<th>ΔIs</th>
<th>ΔPa</th>
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<td>1190</td>
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<td>1.544</td>
<td>295</td>
<td>466</td>
<td>0.54</td>
<td>23%</td>
<td>-19.4%</td>
<td>-5.5%</td>
<td>-8.2%</td>
<td>-28.1%</td>
</tr>
<tr>
<td>-20%</td>
<td>1412</td>
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<td>1.613</td>
<td>334</td>
<td>565</td>
<td>0.51</td>
<td>24%</td>
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<td>4.0%</td>
<td>-12.9%</td>
</tr>
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</tr>
<tr>
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<td>1.660</td>
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<td>722</td>
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<td>6.9%</td>
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<td>310</td>
<td>827</td>
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<td>13.8%</td>
<td>7.2%</td>
<td>-3.5%</td>
<td>27.5%</td>
</tr>
</tbody>
</table>

Table 5.3: Simulation data with Resistance variation at current’s nominal value

5.2.3 Pressure variation

In a real case scenario, one external influence to the PAT-SEIG system that might happen is a transient of pressure. If the pressure drops, the PAT delivers less power to the SEIG and this may loose power or
might not have enough reactive power to continue operating, on the other hand, if the pressure increases this may lead to an increase of the SEIG’s current over its rated value. In table 5.4 the simulations were done varying the pressure from 2.1 Bars to a range of 60% less and to 30% more. A resistance of 200 $\omega$ and a capacitance of 35 uF were used.

Results show this expected behavior: when the pressure decreases the available hydraulic power decreases and the SEIG decreases its output power. The voltage and current drops as well as the active and reactive power. The global efficiency also starts dropping for pressure variations higher than -20%. For a variation of -60%, the SEIG reaches its operation limit, almost without enough excitation and almost no active power. Although every output is affected by pressure variation, the active power is the most affected. While the current and voltage fluctuate almost in the same way as the pressure, the active power fluctuates in a more significant way. When the pressure increases 10%, the active power increases 30% and then stabilizes at the value of +50%. Increasing the pressure may lead to short-term increase of active power, however, the current passes its rated value, which wears down the machine and can also damage it severely. A +10% variation in the pressure leads to a surplus of 17% in the rated current, which the machine can only withstand for a few minutes. Higher values of increase of pressure may not be tolerated by the SEIG.

When comparing the pressure variation with the variation of the resistance and capacitance, the first one has more impact on the system than the others. For pressure drops, the system has at least a threshold of 50%, which is acceptable and in the worst case the generator halts. For an increase in pressure, the case is more concerning, as a 10% pressure increase means 20% increase in current. Depending on the conditions of the system and the source of mechanical power, it might need to have a more significant current threshold and thus a reduced capability. For example, if the hydraulic system is prone to have pressures variations, then the system has to be projected for a lower capacity of the SEIG in order to withstand slow transients of pressure. Spikes of brief periods in pressure were not taken into account because, although they might have some implications in the pipes or the hydraulic system, they would not influence the generator or turbine because of the inertia. The spikes in question exist when closing and opening valves. The simulation of the hydraulic system and consequences to it goes outside of the scope of the thesis.

<table>
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<th>$\Delta$ P</th>
<th>HP (W)</th>
<th>Nr (RPM)</th>
<th>Us (V)</th>
<th>Is (A)</th>
<th>Pa (W)</th>
<th>Q (VA)</th>
<th>PF</th>
<th>Eff</th>
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<th>$\Delta$Is</th>
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<td>19.1%</td>
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<td>-74.5%</td>
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<td>21.1%</td>
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<td>56.5%</td>
</tr>
</tbody>
</table>

Table 5.4: Simulation data with pressure variation at current’s nominal value
5.2.4 Capacitance variation with inductive load

It ought to be noted that, having an inductive load will affect the reactive power of the SEIG, hence the capacitance variation was the only scenario being taken into account. The simulations with varying resistance and pressure will be affected by an inductive load. However, having an inductive load will not change how both quantities fluctuate but rather the limits at which the system halts.

Now it is analyzed when an inductive load is connected to the SEIG output, i.e. not only a resistance, but a resistance in series with an induction load. A power factor around 0.75 is used, which is usually seen in-home equipment, and it can be used in a general case scenario.

In table 5.5 the results for a capacitor change are presented. During the -60% increase of capacitance simulation the machine halted, however, it got excited again. The machine is at its limit because it has lost most active power and also because the current and voltage do not follow the trend, in other words, both quantities have a big jump in value comparing to the previous simulation of -50 %.

When comparing with the results from 5.2.1 (capacitance variation with a resistive load), the SEIG is more influenced by the capacitance variation. This is because the inductive load consumes reactive power from the capacitors, which results in less reactive power available for the SEIG. This reduces the SEIG excitation and decreases the active power output.

<table>
<thead>
<tr>
<th>∆C</th>
<th>HP (W)</th>
<th>Nr (RPM)</th>
<th>Us (V)</th>
<th>Is (A)</th>
<th>Pa (W)</th>
<th>Q (VA)</th>
<th>PF</th>
<th>Eff</th>
<th>∆Us</th>
<th>∆Is</th>
<th>∆Pa</th>
<th>∆Q</th>
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<td>207.2</td>
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<td>347</td>
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<td>-6.9%</td>
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<td>-8.5%</td>
<td>13.1%</td>
<td>-5.6%</td>
<td>7.4%</td>
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<tr>
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<td>-8.3%</td>
<td>8.1%</td>
</tr>
<tr>
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<td>950</td>
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<td>-12.9%</td>
<td>19.1%</td>
<td>-10.4%</td>
<td>8.4%</td>
</tr>
</tbody>
</table>

Table 5.5: Simulation data with capacitance variation at current's nominal value with inductive load

In figure 5.5 both currents with and without inductive load are represented. They are both very similar in shape, but the values are lower for the inductive case. This means the capacitance can be increased more without endangering the induction machine. It also means that reducing the capacitance will have a greater effect in the current.

In figure 5.6 the active powers with and without inductive load are represented. With an inductive load is more difficult to have an increase in active power since it lowers the curve when comparing to the original one. An inductive load will make the control of the system more difficult because the active power will always decrease from its original value.

In figure 5.7 the reactive powers with and without inductive load are represented. When comparing to the other variables above is possible to see that now the inductive load has an impact on the shape of the curve and they are not similar. The increase in capacitance with inductive load does not have a big influence as the resistive load.
5.2.5 Halting generator

Comparing both simulations of capacitance and resistance variations, there is a pattern, which is increasing the capacitance or the resistance has less influence on the system than decreasing them. This effect is because when increasing these quantities, the SEIG is exceeding its rated operating point since, in both cases, it still has enough reactive power to work. This may be a problem for the SEIG integrity.

In addition, there are also consequences when decreasing greatly these values, as seen in figure 5.8. In this simulation the system was in steady state with constant values of capacitance, resistance and pressure. Then the capacitance was decreased in 70%. When decreasing the capacitance, the reactive power is also being decreased. The SEIG requires a minimum of reactive power in order to work and to generate the electric voltages. In the case of decreasing the resistance, the SEIG is less sensitive, nonetheless the same can happen since the reactive power is also being decreased.
There is another similar case to the one presented, which can be seen in Figure 5.9, where the generator does not halt completely and gets in an oscillatory loop. In this case, the system was in steady-state as in figure 5.8, however the capacitance was only decreased in 56%. The system halts, but as the capacitors still have some charge left and the rotational speed increases, the generator excites again, however, is not being supplied with enough reactive power for that kind of torque and so it halts again, and so on. This scenario is a never-ending cycle, which only wears down the machine and increases the risk of a component breaking. Another problem is that the flow of the hydraulic system (water) will be oscillatory as well, which in turn can create turbulences in the hydraulic system. The computation model assumes that the pressure is constant, but in reality the pressure will vary due to pressure drop in pipes. During one experiment, this situation happened and the hydraulic system failed. Moreover, the different pressures help to increase the cavitation in the system, which leads to surface erosion and components failure in the long run. Cavitation is when the static pressure in a given location falls below the vapor pressure of a liquid, which in this case is water.

Both these situations can also happen with a pressure drop. From the simulations and assuming the model is behaving like the real system, it is possible to infer that the capacitance, the pressure and
the resistance cannot be decreased more than 50%, 30% and 60% respectively. These limits can be applied in the case where the current is at its nominal value, however, as the current gets lower, the limits also decrease because less reactive power is being produced.

Figure 5.9: Phase voltage when induction generator halts and stays in a oscillatory regime.

5.3 Current overload mitigation

In a real application the PAT+SEIG system is prone to external variations, and it would probably require some control in exceptional cases. The system is mainly subject to different pressures or electric load variation.

The limitations of the system were analyzed, and in this section, solutions for mitigating problems will be presented. In the following simulations, the induction machine has a current 25% above the rated one (2A). The overload current can be the case where either there was an increase in pressure or the electric load changed. Mitigating this scenario by changing pressure has not been taken into account because it might not be an option, and it goes outside of the master thesis scope. Changing the resistance and capacitor was taken into consideration since there can be an actual resistor connected in parallel or series with the electric load.

5.3.1 Capacitor variation

Other simulations were done while changing the capacitance, only this time, in the initial reference point, the current has a higher value than the rated one. In table 5.6 are shown the results for a change in the capacitor value. It is possible to see the capacitor varying from -45 % to 10%. In order to reduce the current in 25% is necessary to decrease the capacitance in 45 %. The rotational speed is also affected, being increased to a maximum of 20%, which also increases the electrical frequency in 20%. The change in frequency may or not have some implications depending on what is connected to the machine. For example, if there is a rectifier and a battery at the generator's terminals, then it will not be affected by the frequency variation.
With a bank of capacitors, a simple controller and a switch is possible to mitigate the risk of overloading the generator. Moreover, since the power is almost constant it means the controller does not have to control the electric load.

<table>
<thead>
<tr>
<th>∆C</th>
<th>HP (W)</th>
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<th>Pa (W)</th>
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<th>PF</th>
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<td>-13.6%</td>
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Table 5.6: Simulation of reducing capacitance with current over nominal current

5.3.2 Resistance variation

In table 5.7 are shown the simulations were the resistance changes from -40% to +10%. The electric load was not increased more, because it would only increase the current even more as it can be seen by the 5% and 10% trials. When a -40% variation is achieved the generator halts, thus it is not possible to change the resistance further and at this point the current is still too high.

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<th>Pa (W)</th>
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<th>PF</th>
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<th>ΔIs</th>
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<td>-7.2%</td>
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<td>0.0%</td>
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<td>5.0%</td>
<td>2.1%</td>
<td>-5.0%</td>
<td>9.9%</td>
</tr>
</tbody>
</table>

Table 5.7: Simulation of reducing capacitance with current over nominal current

From the simulations, it is possible to assert that using this method is only possible to decrease the current in 20% maximum and it requires a -35 % resistance variation. Moreover, reaching this amount of variation increases the risk of the machine halting and stopping producing any power. It is safer to have a small variation in order to decrease the current. It is not necessary to decrease the resistance more than 15% as the current stays almost constant until a-35% is reached, which for this scenario can be considered the limit. This method does not allow for an increase in power however, can be used to make small adjustments to the current and bigger ones to the voltage. In this case, decreasing the resistance, is not enough to decrease the current.
5.4 Flow analysis

The SEIG has an influence in the PAT system has seen before. Therefore, the electric system can be used to change the PAT parameters if needed, like the water flow, Q. By changing the rotor speed the Head-flow curves moves up and down. In this case, the differential pressure is constant, hence only the water flow will change.

From the previous simulations the water flow was also logged and in the next figures, it is possible to see how the flow changes when there is a resistance or capacitance variation. In figure 5.10, the capacitance is changed, while the SEIG is at half nominal current. The simulations were done using the same resistance of 200 Ω and different pressures were used. In figure 5.11 the electric resistance was changed while keeping the same capacitor of 35 uF.

![Figure 5.10: Flow variation with capacitance variation simulations.](image1)

Figure 5.10: Flow variation with capacitance variation simulations.

![Figure 5.11: Flow variation with resistance variation simulations.](image2)

Figure 5.11: Flow variation with resistance variation simulations.

High current means the SEIG is at its rated value (1.6A), while low current is arround 0.6A. It is
intuitive that having a higher current is equivalent to having a higher flow because the SEIG will have more mechanical energy. Although the shape of the curves are identical in the two cases, it is possible to see that, the capacitance variation has a more significant influence on the flow than the resistance. This happens because the capacitance also has more effect on the speed. The simulations show that is not possible to have a significant amount of control over the water flow while increasing the variables (resistance and capacitance). This is because the rotational is also not affected in a significant manner. It is possible to have some control when decreasing the flow, however for significant changes, the system has to approach its operational limits. For example, in figure 5.10, in order to decrease the flow from 8 l/s to 4 l/s, it is necessary to decrease the capacitance in 50%. This amount of variation will affect the reactive power in the SEIG and the system will become more vulnerable to variations.

5.5 Conclusions

The simulations show that the model is capable of predicting the system’s behavior and how will the system react to external changes. In general, the resistance can be used to change the voltage to a maximum of 20% before a system failure occurs. The capacitors can be used to change the current until a maximum of 30%, however they will also affect significantly the reactive power, which changes the rotational speed of the machine and electrical frequency.

It was observed that the limits to which the resistance or capacitance can be changed, do not depend on the value of the current/electric load. The resistances cannot go below the 35%, while the capacitance cannot go below -50%, otherwise the system will halt. Having an inductive load does alter the system’s limits, however it is not significant. The inductive load also changes how the SEIG’s active and reactive power behave while changing the capacitance.

Increase in pressure represents the most hazardous scenario since it will make the SEIG surpass its rated values, both the current and speed. After the machine is excited an increase in pressure is the only way of increasing the SEIG’s active power and if the pressure goes below 60%, the system will halt since the SEIG will not be able to produce enough reactive power.

In conclusion, if there is a harmful external event of electric load or pressure variation, there is possible way to mitigate it, although somekind of control system is necessary. If the active power asked to the system changes overtime, then a control in pressure is necessary as the electric controls cannot change effectively the power output.
Chapter 6

Conclusions

The European commission set as an objective for 2020 the increase of energy saving in 20%. New ways of harnessing energy and optimize the existing energy sources have been studying throughout the recent years. One field that has been seen as an opportunity for energy recovery is the micro hydraulic power generation. PATs can be used to harness power and at the same time recover energy that would have been lost if using reduction pressure valves.

Although several approaches for self-excited induction machines had been proposed, this work represents one of the first to model both the electrical and hydraulic system together. This improvement allows for better control of the overall system as well as having a prediction model, which allows mitigating risks and failures. The work was developed by first creating a model of a self-excited induction generator connected to a DC motor, for validation purposes. Then instead of a DC motor, the PAT is used, where some fluids dynamics are taken into account. In both cases, the models were validated and tested with experimental data, which allowed to calibrate the model and its parameters properly. Finally, as theoretical exercise, the PAT was modified in order for the SEIG to achieve its rating values. With this "new" system it was possible to predict and to understand the behaviour of a PAT+SEIG system in real case scenario. The ability to predict the system’s behavior allows for a better fitting of all the components for each application. Moreover, it is possible to predict what the best operation point of the PAT+SEIG system, considering the hydraulic and electrical inputs.

During testing and simulation of the PAT + SEIG, it became clear that the PAT and SEIG were correctly sized for each other. Either the hydraulic setup had to be changed or either the SEIG had to be changed for a lower power one. The PAT rated power was too small for the induction machine and could not provide the necessary power needed in order to reach the SEIG’s nominal value, meaning the machine could never achieve its best efficiency. The PAT was resized, in a theoretical manner, to better suit the SEIG demand, which allowed the modelling and testing of a real case scenario.

The simulation of changing capacitors, resistances and pressure enabled to understand how an electric load might affect the system and what could be done to mitigate the risks. Changing the capacitance is the best option for varying the current and has little effect on the active power. However, it comes with the cost of having a more significant effect on speed. On the other hand, changing the resistance affects
more the voltage than the current. Moreover, it has less effect on the speed and active power than the capacitance. Finally, the threshold to decrease the capacitance is higher than the one for the resistance.

With the computational model, it is possible to know how much the pressure can change and how it affects the system. Pressure variation affects significantly the active power generated, and it might cause a problem in the load or machine. Pressure control would be more challenging to implement, therefore changing the capacitance would be a viable option to stop breakdowns from happening.

The model predicts that, the system’s active power, when functioning at a certain point, to remain stable, which allows for a stable power generation. However, the speed will always change with fluctuation, and if the speed needs to be kept constant, then a sophisticated control will be required.

In essence, the model of both the hydraulic and electric systems proves to be essential for a practical scenario. Due to the external agents, it would not be possible to have an implemented system without any control or at least it would have to have some failure safeguard. Results from the developed model are important when designing a control methodology for the PAT+SEIG system.

### 6.1 Future Work

It is clear that, unless there is an ideal hydraulic system without external factors (pressure variation, etc.), the PAT+SEIG system will need to have some sort of control. In order for the system to be resilient to external factors and flexible enough to have multiple best operation points, a control system must be developed. A field oriented control (FOC) for the SEIG can be one option, taking into consideration all the limits of the system, as well as the influence of the PAT in the SEIG, both studied in this work.

For the FOC to be implemented, electronic components will be needed. The capacitors have to be replaced with batteries, which can be then used by the electronics to control and regulate the voltage, load and speed of the SEIG. The control can be made by injecting the necessary reactive power into the SEIG and some additional rules to reduce the impact in the PAT operating point.
Bibliography


Appendix A

Equipment sheets

- **Tektronix TDS 2001/2012C Oscilloscope**
  - **Brand:** Tektronix
  - **Model:** TDS 2001/2012C
  - **Analog Bandwidth:** 100 MHz
  - **Sample Rate:** 2 GS/s
  - **Record Length:** 2.5k Point
  - **Analog Channels:** 2

- **Power Logger Fluke 1735**
  - **Model:** Fluke 1735 three-phase power logger
  - **Memory:** 4MB flash memory – 3.5MB for measuring data
  - **Sample Rate:** 10.24 kHz
  - **V-RMS wye resolution:** 0.1V
  - **Operating Error:** ±0.5% of measured value +10 digit
  - **A-RMS resolution:** 0.01A
  - **Operating Error:** ±1% of measured value +10 digit
• Three-phase bank of resistances Oficel

Brand: Oficel
Range: 40 – 600 Ω

• Capacitor banks Esselte Studium

Brand: ESSELTE STUDIUM
Power: 3.2 kVar
Frequency: 50 Hz
Maximum capacitance: 3.60 μF
Connections:
Δ - 3.220 V ; 1.2 - 8.4 A
Y - 3.380 V ; 0.7 – 4.9 A
/// - 220V ; 2.1 – 14.5 A

Brand: ESSELTE STUDIUM
Power: 5.1 kVar
Frequency: 50 Hz
Maximum capacitance: 3.115 μF
Connections:
Δ - 3.220 V ; 1.9 - 13.4 A
Y - 3.380 V ; 1.1 – 7.7 A
/// - 220V ; 3.3 – 23.1 A
### SIEMENS

**Data sheet for three-phase Squirrel-Cage-Motors**

**Ordering data:**
- 1LA7083-6AA10-Z
- A23

**Remarks:**

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<td>Rated speed</td>
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<td>Rated torque</td>
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<td>5.8 Nm</td>
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### Mechanical data:

- Sound pressure level 50Hz/60Hz: 84 dB(A) / 88 dB(A)
- Moment of inertia: 0.0017 kg*m²
- Bearing DE: 6004 2Z C3
- Bearing NDE: 6004 2Z C3
- Type of bearing: Ball bearings pre-lubricated DE (standard)
- Condensate drainage holes: No
- Regreasable device: No
- Lubricants: ISO VG 220
- Grease lifetime/lubrication interval: 40000 h
- Quantity of grease for relubrication: 16 kg
- Internal starting terminal: No
- Coating: Special paint finish 8504-7002-3225 dark-gray

### Environmental conditions:

- Ambient temperature: -20 °C - +40 °C
- Altitude above sea level: 1000 m
- Standards and specifications: IEC, DIN, ISO, VDE, EN

### General data:

- Frame size: 000 N
- Type of construction: (0) M 43 / (1) I 87 / (1) P 85
- Weight in kg. without optional accessories: 10.00 kg
- Frame material: Aluminium
- Degree of protection: IP 55
- Method of cooling, TEFC: I (Standard)
- Insulation: 145 (F or 130D)
- Vibration class: (Standard)
- Duty type: 51 - continuous duty
- Direction of rotation: Bi-directional

### Terminal box:

- Material of terminal box: Aluminium
- Type of terminal box: YH 033
- Contact screw thread: M4
- Max. cross-sectional area: 1.50 mm²
- Cable diameter from ... to ...: 3.00 mm - 16.00 mm
- Cable entry: 1xM20x1.5 - 1xM16x1.5
- Cable gland: 2 plugs

### Special design:

- A23 Motor temperature sensing with integrated KTY 84-130 temperature sensor
Appendix B

Tables

B.1 Simulation data of capacitance variation with low current

B.2 Table Experimental values with PAT
<table>
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Table B.1: Simulation of capacitance variation with low current (laboratory values)
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</table>

Mean 6.1% 5.1% 5.1% 0.2% 6.8% 6.9% 8.5% 21%
Appendix C

Matlab code

C.1 Efficiency function

```matlab
function eff = effPAT(X,Y,Z,Nr,H)

Nref = 830;
H = H - 3.6644*2;
if Nr<min(min(X))
    Nr=min(min(X));
elseif Nr>max(max(X))
    Nr=max(max(X));
end

if H<min(min(Y))
    H=min(min(Y));
elseif H>max(max(Y))
    H=max(max(Y));
end

a=interp2(X,Y,Z,Nr,H);

if isnan(a)
    Href = H*(Nref/Nr)ˆ2;
nref= interp2(X,Y,Z,Nref,Href);
    if isnan(nref)
        if Href >= 7.93
            nref = 0.43; % maximo de eficincia obtida
```
elseif Href <= 2.8
    nref=0.27; % minimo de eficiencia
end
end
eff = nref*(H/Href)*((Nref/Nr)^2);
else
    eff = a;
end

C.2 Flow function

function [Q, Th, Ph] = funcPAT(u)

rho=997; % water density kg/(m^3)
g=9.81; % gravity acceleration m/(s^2)

% turbine parameters
Nref= 1050; % rpm

% The following parameters are of the function: H=alpha^2*A+alpha*B+Q+C*Q
A=3.6644*3;
B=-694.45;
C=314560;

% New function
Nr=u(1); % in rpm
dP=u(2); % in Pa
H=dP/(rho*g);
alpha=Nr/Nref;

%outputs
if H<0
    Q=0;
else
    if H<alpha^2*A-(alpha*B)^2/(4*C)
        Q=alpha*B/(2*C)/(alpha^2*A-(alpha*B)^2/(4*C))*H; %alpha^2*A-(alpha*B)^2/(4*C);
    else
...
Q = (-alfa*B + ((alfa*B)^2 - (4*C*(alfa^2*A-H)))^(1/2))/(2*C);

end

end

p = dP; % pressure
Ph = p*Q; % Hydraulic power

if Nr==0 % this is needed because for Nr=0, the other equation cannot be computed.
    Th=0;
else
    Th = Ph/(Nr^2*pi/60);
    if Th<0
        Th = 0;
    end
end
end