Extended abstract

Assessment of Stress Field Models

Design models for deep beams subject to indirect loads

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ABSTRACT

Stress field modelling is a technique for analysis, design and detailing of structural concrete elements and have been widely used, essentially, in discontinuity zones, i.e. zones where the Bernoulli principle is not valid. These models intend to simulate the load path within a structural element by establishing the equilibrium of internal forces and stresses to check the safety of a given region. Since stress field models are based on the plasticity theory, in particular the static theorem, compatibility is not directly included in the analysis and therefore it is doubtful whether the structure has sufficient ductility to fit the model chosen for the safety check and at the same time ensures proper behavior in service.

Regarding service behavior, in many cases it is proven that a model which has the main compressions and tensions according to the elastic stress trajectories, considering the need for detailed reinforcement, guarantees good behavior in service. However, it is usual to have doubts whether the model chosen for design is the most appropriate.

In this context, this thesis intends to study the behavior of a deep-beam subject to a suspended point load, as is the case of a secondary beam with an indirect support on the main deep-beam.

1. INTRODUCTION

In the late XIX century Wilhelm Ritter introduced the classic lattice model that allows for the consideration of the shear effect for reinforced concrete beams. Later these models were improved by Emil Mörsch.

Before Ritter, engineers thought that the stirrups functioned by connecting the upper and the lower part of a beam, under compression and tension respectively, through a bolt effect (similar to two overlapping wooden beams connected by plates). Ritter's model and Mörsch's lattice showed that the stirrups worked tensile like the longitudinal reinforcement, an innovative idea for the time. It is well known that the Bernoulli hypothesis and the truss analogy are currently the basis for the design and safety verification of structural concrete elements. However, there are regions, called discontinuity (D) regions, where static and / or geometric discontinuities occur, requiring a different approach due to the nonlinearity of deformations and, consequently, stresses.

2. DISCONTINUITY REGIONS

In a structure various types of discontinuities may occur which may have a geometric nature, for example in areas where there is a variation in the height of a beam or near an opening. Alternatively, these discontinuities can have a static nature, through point loads or prestressing. They may also occur in a combination of the two, ie geometric and static in nature.

Load pathing within a structure can be viewed in the form of stress trajectories in an elastic analysis (Figure 1). As soon as there is cracking in the concrete, the load path is adapted, and the stress trajectories change accordingly.

These load paths can be analyzed in a simple and swiftly manner through the use of strut and tie models, where the struts represent the compressive stresses and the ties the tensile forces that develop inside the strut, supported by the steel reinforcement.

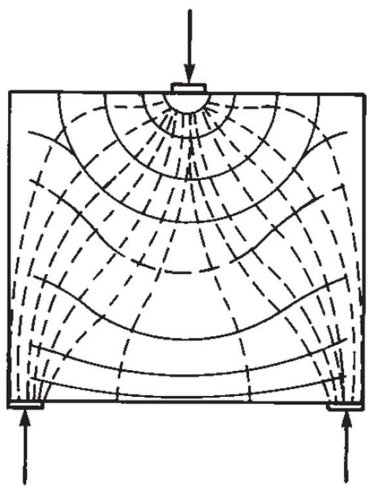

Figure 1 Stress fields inside a deep beam subject to a point load

3. MODELLING AND DESIGN OF DISCONTINUITY REGIONS (D-REGIONS)

3.1 Strut and Tie Models (STM)

Strut and tie modelling is a method for the analysis and design of structural concrete elements. These are equilibrium-based models whose aim is to translate the load trajectories within the region in study.

Schlaich et al. in 1987 proposes a general methodology based on the strut and tie models for the structural analysis of the regions where the Bernoulli principle isn't applicable, which is the case for D-Regions. Strut and tie models consist of a set of struts, which define the compressions that develop inside the structure, a set of ties that represent the steel reinforcement and nodes that represent areas where there are stress concentrations, these last ones must be analyzed closely. For this purpose, several methods can be used for the definition of the strut and tie models, the best currently being the load path method.

3.2 Load Path Method

This method consists in the idealization of the path taken by the forces from the point where they are applied to the support. The force component associated with the path remains constant throughout its path.

Figure 2 presents an example of modeling through the Load Path Method. The load path can be traced by connecting the two byproducts of the distributed load 'p' with the support reactions using curvilinear trajectories. The load paths begin and end at the corresponding center of gravity of the boundary pressure diagram, crossing the shortest possible path. The loads' curvilinear path generates non-vertical stresses, which, for the sake of simplicity, can be considered horizontal, and which generate compression (F_c) and tensile (F_t) forces that need to be considered in the construction of the strut and tie model.

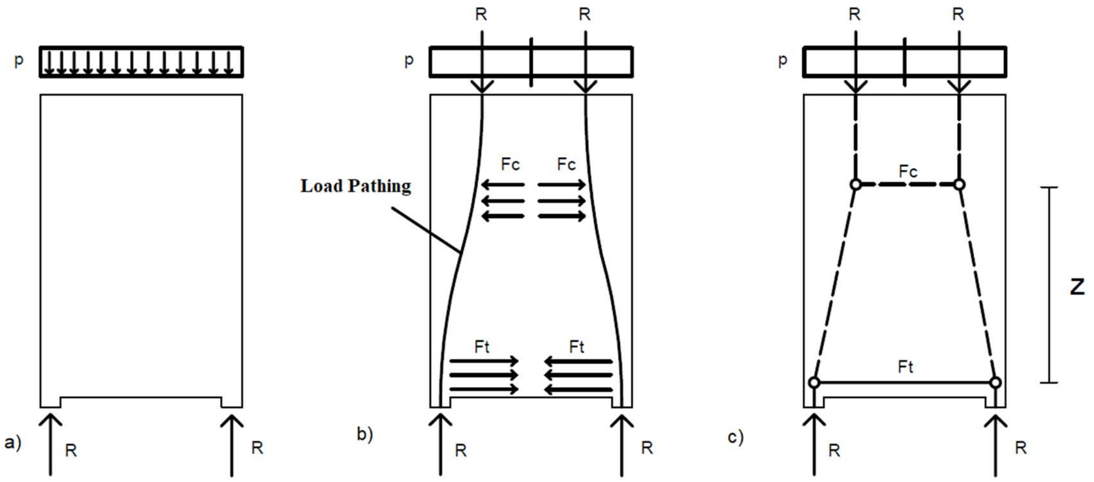

Figure 2 Application of the Load Path Method for the design of a Strut and Tie Model

3.3 Strut and Tie Models for Deep Beams subject to indirect point loads

This kind of situations occur, for example, when connecting a beam to a deep beam, the latter being subjected to stress concentration in a small area. Therefore, it is necessary to model the behavior of the structure in relation to the concentrated loading for the bearing area.

Struts and ties in a strut and tie model direct the load in the longitudinal direction of the point where it is applied, and its deviation should not be elevated. Thus, two mechanisms of load suspension can be studied in order to direct it to the supports, which can be evidenced by a simple elastic finite element analysis as shown in Figure 3.

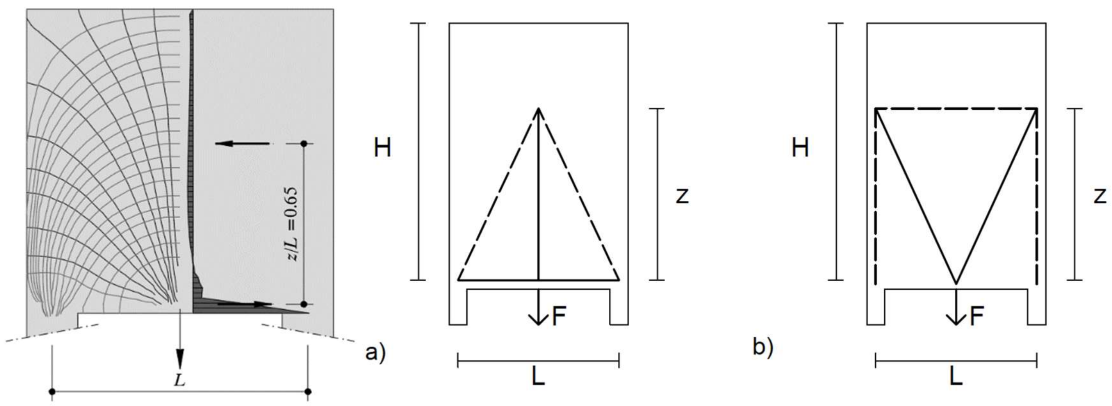

Figure 3 STM for a deep beam subject to an indirect point load and the respective elastic trajectories

4. MODELLING TROUGH A NON-LINEAR FINITE ELEMENT ANALYSIS

4.1 Introduction

This chapter introduces a nonlinear numerical analysis using the finite element method, developed by Ruiz, Muttoni, 2007, and also by Miguel Ferreira through the EvalS program (doctoral work in progress).

This method overcomes some of the difficulties of the SFM and models based on non-cracked elastic analysis and only needs a set number of physical parameters compared to other more refined FE models using other parameters that have an increased difficulty related to their quantification and interpretation (Based on Ruiz, Muttoni, 2007).

4.2 Formulation

Initially, two assumptions are made for the behavior of structural concrete which are quite usual in stress field modeling: The principal stress directions are parallel to the principal strain directions (Figure 4, adapted from Ruiz, Muttoni, 2007); and the tensile strength of concrete is neglected.

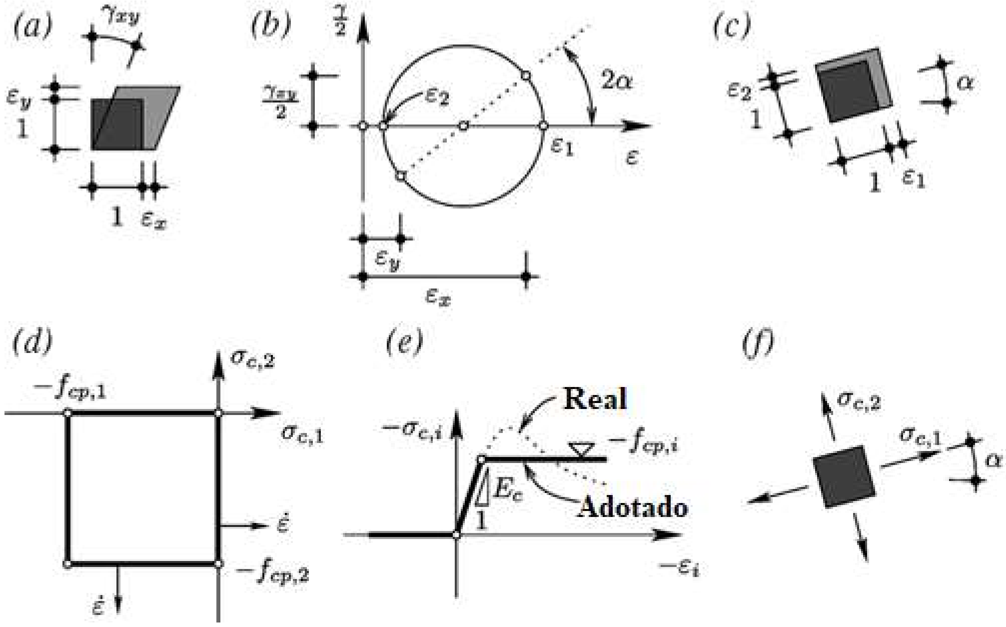

Figure 4 Principles for concrete modeling:
a) Deformations;
b) Mohr's circle and principal deformations;
c) Deformation in the principal directions;
(d) Yield surface adopted for plane stress with an associated flow law;
e) Elastic perfectly plastic response adopted for structural concrete;
f) Principal directions assumed for stresses

Regarding the constitutive relations the behavior of the concrete is assumed to be elastic (with elastic modulus E_c, independent of the phenomena caused by the transverse deformations) until it reaches its compressive strength and later perfectly plastic [Based on Vecchio et al., 1994].

The compressive strength of the concrete is corrected by the parameter η (εj) dependent on the transverse deformation and may be calculated in different ways. In the past several studies have been carried out by different authors leading to different models but with similar results. In this work, Vecchio & Collins (1986)'s equation is applied.

Concrete's finite elements were simulated with CSTs ('Constant Strain Triangles'), defined by triangles with constant deformations and stresses in its domain (Figure 5, adapted from Ruiz, Muttoni, 2007).

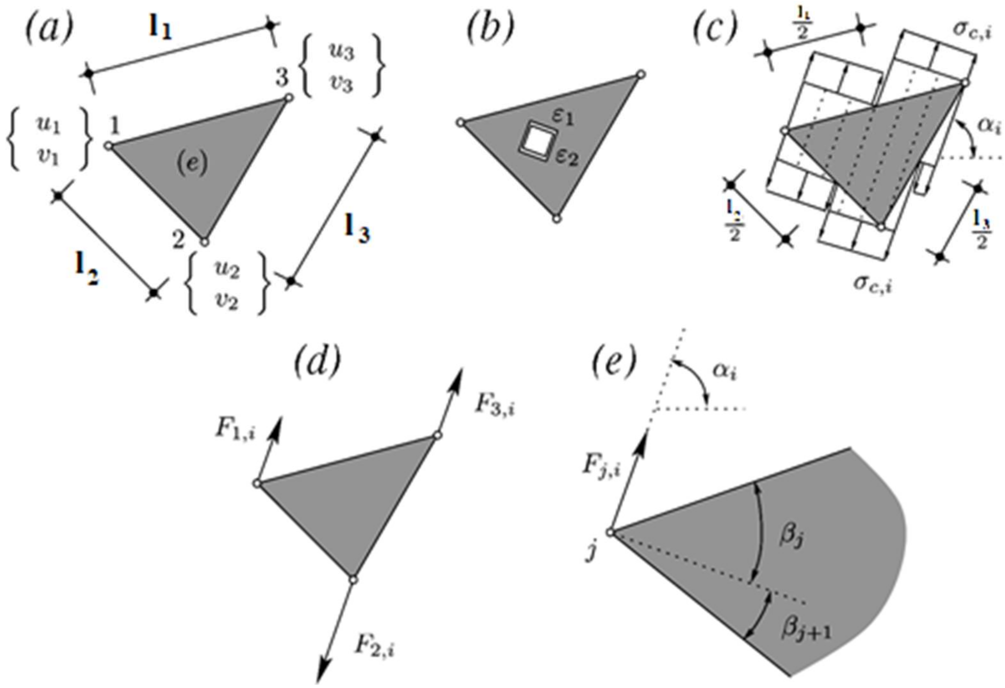

Figure 5 CST ('Constant Strain Triangle') Element
(a) Field of nodal displacements;
b) Field of constant deformation;
c) Stress field assumed for the principal directions;
d) Nodal forces
;e) Angle β in each node j referring to the main directions.

The steel reinforcement was modeled using bar elements (Figure 6) through a uniaxial response with a bilinear elasto-plastic law. The effects of 'Tension Stiffening' (increased axial stiffness of the cracked tie due to the surrounding concrete) were considered. This phenomenon has a significant impact on service behavior.

Its bilinear response is defined by the strength of the f_{yd} material, the elastic modulus Es and the post yield modulus Eh (provided for in Eurocodes 2 and 3). For the consideration of the 'Tension Stiffening' phenomenon the surrounding concrete area of each reinforcement and the corresponding steel and concrete (tensile), f_{yd} and f_{ctm} resistances are respectively applied.

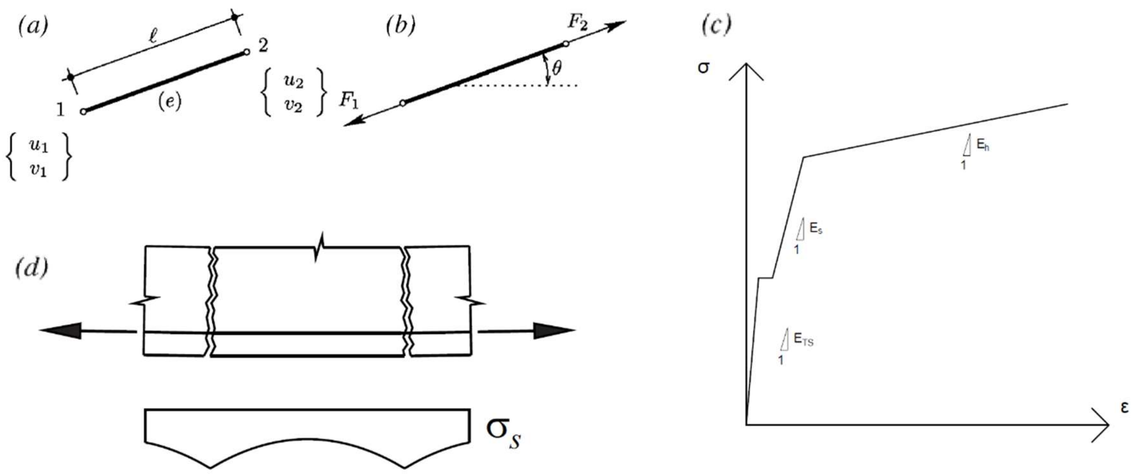

Figure 6 Steel Element Modeling:
(a) Displacement Field; (Adopted by Ruiz, Muttoni, 2007)
b) Nodal forces; (Adopted by Ruiz, Muttoni, 2007)
c) Material behavior with 'Tension Stiffening';
d) Tension Stiffening phenomenon in a cracked concrete tie rod. (Adopted image of Marti, 2013)

The global nonlinear equation system is then solved using the modified Newton-Raphson convergence method. This method differs from the Newton-Raphson method by keeping the stiffness matrix constant throughout the iterations in order to decrease the computational effort.

4.3 Deep beam subject to an indirect point load

Deep beams with suspended point loads have two, previously identified, distinct load paths or a combination of both. Loads can be transmitted to the supports by lifting the load with stirrups or vertical reinforcement, or with inclined reinforcement (see Figure 3).

The objective of this thesis is to verify the designer's freedom for selecting models that consider the suspension of concentrated loads, with deviation from the elastic models while maintaining an adequate service behavior.

The elastic (non-cracked) model for concentrated load suspension suggests that both vertical and diagonal paths have similar characteristics. Thus, this chapter aims to study the suitability of each, and various combinations of the two.

Five case studies were developed in which the variable 'k_d' (corresponding to the amount of load carried by the vertical reinforcement in the strut and tie model) took values between 0 and 1. The detailing for each case was based on these simple STMs, from which the amount of reinforcement adopted was defined.

4.4 Overview of the behavior of the main reinforcement

The stress progression in the main reinforcement can be an important tool for the assessment of the most suitable models for the design of a particular D-Region. Indeed, stress level in the reinforcement is directly related to cracking, these analyzes allow the designer to identify a good or a poor service behavior. The following illustrations show the combined stress results at each load step on the diagonal (Figure 7) and vertical reinforcements (Figure 8) for each case study (indicated by the respective variable k_d).

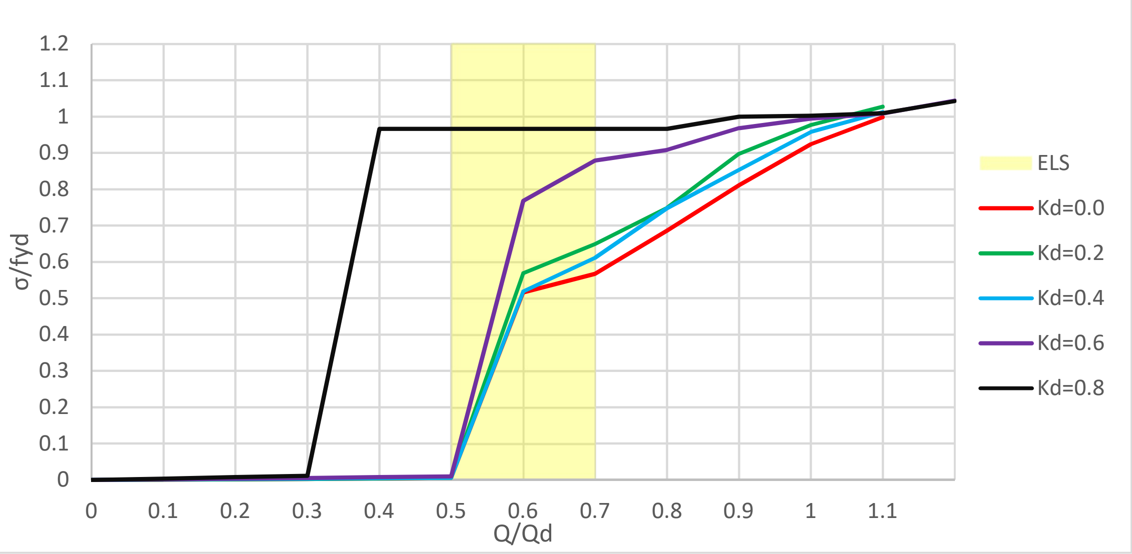

Figure 8 Stress levels on the diagonal steel reinforcement (for each load step and case study)

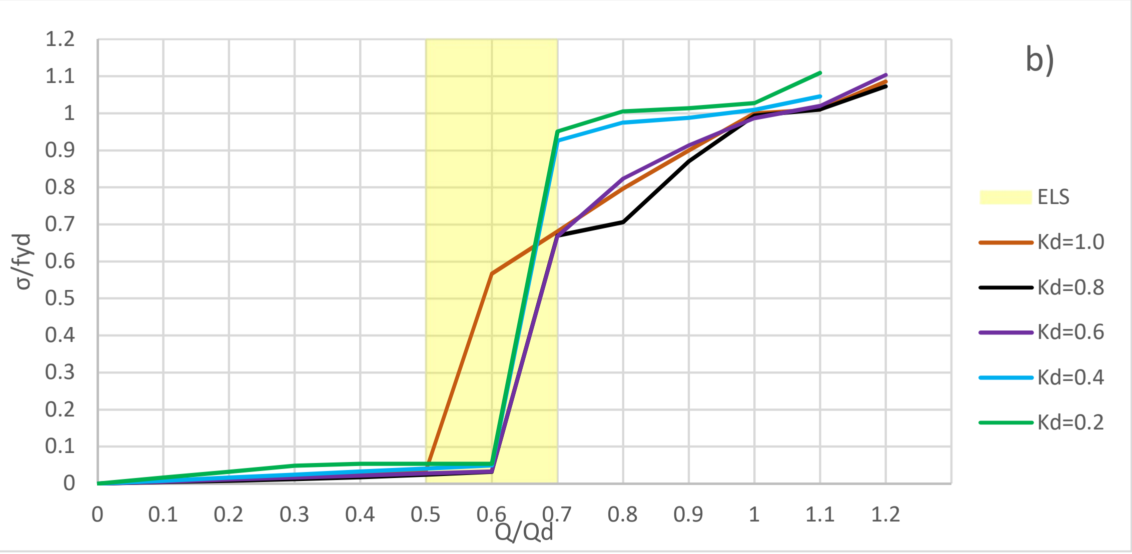

Figure 7 Stress levels on the vertical steel reinforcement (for each load step and case study)

Comparing both graphs (referring to Figures 7 and 8), it can be concluded that the analyzes with the best service behavior are the ones with 60% and 40% of the load carried by the vertical ties (in the strut and tie model). This is expected since these are similar to the elastic trajectories.

Analyzes in which the load was considered to be carried on in only one direction (vertical or diagonal) showed acceptable stresses in service, however, most of the distributed reinforcement ties reached their yield strength.

Figure 9 shows the stress evolution in the horizontal reinforcement for each case study and at each load step. The results demonstrate that the tensile forces in the support were not relevant. In the model with full suspension of the loading in the vertical direction, a value of 220 MPa was obtained for the design load (Q / Qd = 1.0), which suggests the tensile forces in the support are distributed along a reasonable height and are being balanced together by the distributed reinforcement (with horizontal orientation). Thus, it is concluded that the horizontal reinforcement should be distributed in several horizontal ties in order to better balance out these forces.

It is important to note that the reinforcement adopted in all of the analyzes was always greater than the minimum required.

In contrast, the tensile forces arising from the diagonal reinforcement spacing (horizontal space left between reinforcements, near the load) are quite relevant, especially with the consideration of a higher percentage of load traveling along this path (diagonally). In the analysis without explicit vertical reinforcement, very high values were obtained in service in the horizontal reinforcement (reaching 420 MPa, very close to the yield of the material). In the remaining case studies the stresses within service loads remained within reasonable limits, with a maximum of 250 MPa for the analysis with k_d = 0.2.

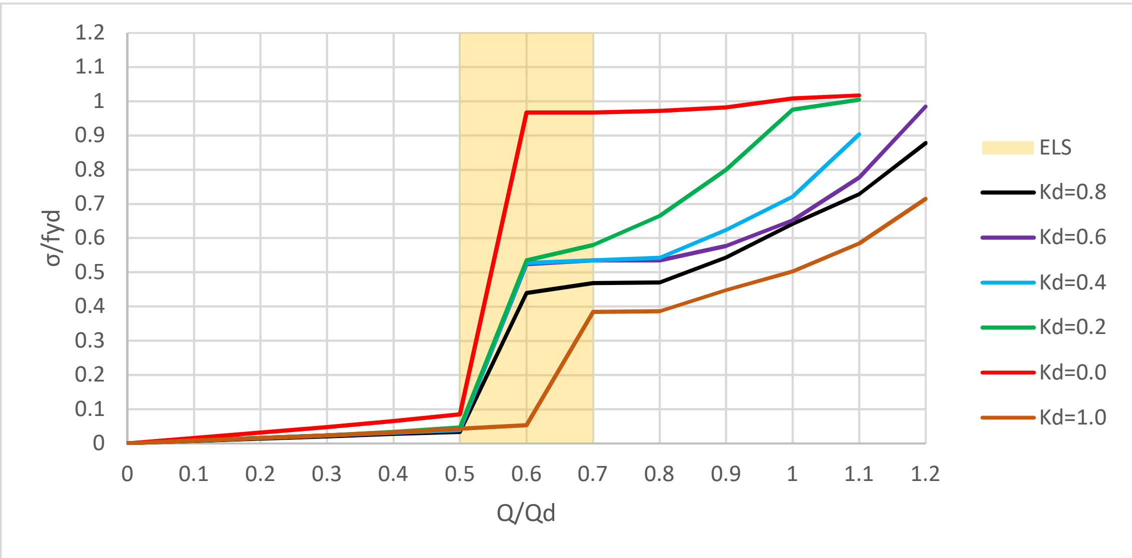

Figure 9 Stress levels on the horizontal steel reinforcement (for each load step and case study)

5. FINAL REMARKS

The main objective of this thesis was to evaluate the *designer's freedom* for the selection of design models to be applied to deep beams subject to indirect point loads. This was studied, varying the vertical and diagonal reinforcement, in order to draw some important conclusions about the relevance of each of the load paths that aim to suspend the load applied in a very small area.

Across all analyzes, there is a high contribution of the distributed reinforcement to the structure behavior, especially in the service limit state, suggesting that the adoption of a distributed reinforcement derived from the main reinforcement (obtained through the strut and tie model) may prove to be a very effective option in the design of this kind of structures. In this context, Lourenço & Almeida (2013) suggested in their studies that the designer can choose one of the available paths (vertical or diagonal) and adopt about 15% of the main reinforcement as a distributed reinforcement to improve service behavior.

Through evaluation of each of the studied models, it is concluded that the diagonal path prevails over the vertical path, suggesting that a ratio according to elastic trajectories (approximately 58% of the load routed diagonally) should demonstrate a more adequate service behavior of all reinforcement ties.

Results show that after cracking of the concrete, there are important redistributions related to an increase or decrease in the inner lever arm (z/L). Near the design load, the inner lever arm matched the one used in the strut and tie model across all case studies.

Concerning the horizontal reinforcement, results show that the tensile forces generated in the support are carried on by the distributed reinforcement ties, suggesting the first should be adopted distributed along a reasonable zone. On the other hand, the tensile forces caused by the diagonal reinforcement spacing (horizontal space left between reinforcements, near the load) showed to be very far more concerning and must be duly taken into account in the strut and tie model used for detailing.

Finally, through the analysis of loads carried on by each of the paths (diagonal and vertical), it is noted that the stresses in service in the main reinforcement ties using the strut and tie model lead to slightly conservative values, thus meaning it can be used for service limit state checks.

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