Instituto Superior Técnico

*Department of Engineering and Management*

# In-store capacity planning – The case of IKEA

Bernardo Nolasco Reis dos Santos Silva

**A B S T R A C T**

Customers nowadays value wide product varieties, fast response times, flexibility and attractive shopping environments, making it more challenging for companies to remain competitive in their markets, particularly traditional retailers, which now compete not only with each other, but also with the new online businesses.

Logistics accounts for a significant amount of companies' costs, consisting on a field with possible improvements. Product allocation plays an important role in logistics, whether in retail stores or warehouses. On the first case, it has the potential to maximize sales. On the last, to reduce operational costs. Although product allocation in warehouses and retail stores is a widely studied problem, the literature does not refer to retail stores that have incorporated self-service warehouses, as is the case of the model of IKEA stores. This gap causes practitioners to develop their own capacity management tools, which lack scientific and empirical proof.

The IKEA Portugal's logistics department uses a proprietary capacity management tool that allocates their products to different commercial areas within the stores and provides information about whether stores have enough capacity to store the product range. Although the tool suggests that some stores do not have enough space to stock the product range, those stores sometimes outperform stores that supposedly have no capacity shortages, suggesting that the tool is not reflecting the stores' reality.

In this context, using data from IKEA, this paper proposes a tool that can be used in this type of retail stores, in order to improve operational performance.

**KEYWORDS:** operations management, warehouse operations, warehouse design, retail store operations, storage location assignment.

## 1. Introduction

The retail industry is going through a disruption phase, with companies taking advantage of the new tools, strategies and technologies that are now available for increasing businesses' performance. On the other hand, customers' expectations about their shopping experience are higher than ever, making it increasingly difficult for companies to compete in crowded markets. Still, despite the growing presence of e-commerce, 90% of retail sales are still done in physical stores (KPMG, 2018).

One of the main expectations of the 21st century customer is that he always finds retail stores' shelves well stocked with products, which can only be achieved with efficient product allocation. Although this subject has been widely studied both in retail and warehousing literature, self-service warehouses have not been approached, which causes practitioners to develop their own proprietary tools to address the problem.

IKEA is a multinational furniture retailing franchise business with stores in 29 countries around the world. The franchisees, in general, have experienced sales growth in the last years, but are aware of the global retail trends and consider that there is still a need to increase efficiency (Inter IKEA Group, 2018). The franchisor provides the franchisees with management tools in order to facilitate and standardize planning and operational activities across the stores. IKEA managers in Portugal have recently started to question the results of one of these tools. Stores are divided into several zones called commercial areas. This tool assigns product references from different product categories to different commercial areas in each store according to the product range specified for each store and its respective forecasted sales and informs the user about the capacity lack or surplus in each of those commercial areas for the chosen range of products. The problem managers have identified is that, some stores that supposedly have a lack or surplus of capacity, perform better in almost every Key Performance Indicators (KPI) than stores that supposedly have no capacity problems. This counterintuitive result has made them doubt the ability of the tool to reflect the stores' realities.

Given the presented facts, this dissertation proposes a capacity management tool that allocates products to commercial areas and sales locations accounting to each store's specificities, with the objectives of increasing store efficiency and providing more accurate data about some chosen KPIs regarding the assignment decisions.

In order to develop this dissertation, a work plan with five steps was used:

i) Interviews with the company's managers – these stakeholders were interviewed and regularly consulted throughout the development of all the work

ii) Literature review – using the Web of Knowledge, relevant academic work on operations management, warehousing and retail stores was reviewed in order to develop the methodologies used to achieve the defined objectives and to leverage the knowledge obtained from other scholar's works.

iii) Methodology and models formulation - by combining the insights from the interviews with the company's managers and the literature review, a methodology composed by a set of models was conceptualized and the models were formulated. The methodology is structed so that the three models are interconnected and work in a sequence in order to solve the retailer's problem.

iv) Data collection and implementation of the models – several extractions of data from the company's databases were made in order to feed the models. By taking advantage of 3 computer tools (MS Excel, RStudio and GAMS), the models were implemented in order to use the data collected and produce the results.

Considering the work developed, the paper proceeds as follows: Section 2 presents the literature reviewed, which focused on the topics of capacity planning in operations management, retail store operations, warehousing and the storage location assignment problem (which is a specific research theme within warehousing). Section 3 briefly presents the problem and details the formulation of the methodology and the 3 individual models that compose it. Section 4 presents the implementation of the models and discusses the results obtained. Section 5 presents the main conclusions of this work and the main guidelines for future work.

## 2. Literature review

The problem of allocating space to products in retail stores is referred to in the literature as the Shelf-Space Allocation Problem while in the case of warehouses it is called the Storage Location Assignment Problem (SLAP).

The SSAP can be defined as the distribution of the limited shelf space in a store among a set of products in a category (Aguiar, 2015) and it usually starts with assigning categories to departments or areas in the store, then to specific locations in the departments' racks, ending with the allocation of an amount of space (number of product facings) for every specific product (Mowrey, Parikh, & Gue, 2018). Space-dependent demand is computed using space elasticity, defined as the ratio of additional sales to additional space. Most shelf space allocation models consider only product width, but there are also planogram-based models, which consider the three dimensions (Mou, Robb, & DeHoratius, 2018). A planogram is a visual representation of a specific part of a category in a store that shows the placement and number of product facings for each individual SKU in those shelves (Aguiar, 2015).

In-store logistics planning consists of planning replenishment and personnel. With inventory dependent demand, the inventory level at the shelves needs to remain high, which does not comply with the traditional inventory management models, because it implies higher holding and/or ordering costs. Specific models for shelf replenishment are developed where, unlike in traditional inventory models, the objective is not minimizing the costs but maximizing the profit (Chang, Teng, & Kumar, 2010). However, as customers arrive at the SF of IKEA with their purchasing decision already made, it is more important to optimize travel distance than to give more visibility to the products, so the problem in hands is more similar to a SLAP.

The literature on the SLAP (published between 2005 and 2017) has recently been reviewed in (Reyes, Solano-charris, & Montoya-torres, 2019). In this review, 71 papers on the SLAP were reviewed and classified according to, among other, three characteristics: resolution methods, performance measures and "restrictions and constraints". The resolution methods can either be exact methods, heuristics, meta-heuristics, simulation, policies and rules, information and technology tools and, finally, multi-criteria methods. Performance measures that can be used in these problems are space and distance, time, operational efficiency, handling costs, infrastructure and human factors. On the third category, "restrictions and considerations", these problems can take into account the capacity and physical conditions of the warehouse, product characteristics, the market (demand, sales and location), logistic resources and the configuration of the operation.

It was considered that the relevant articles for the problem in study should be searched within the "restrictions and considerations" classification than can be seen in Annex G.

Because IKEA products are grouped into categories (the HFBs) and because a large number of very different products (namely in terms of size and shape) are in the stores, papers that approach restrictions based on product characteristics should be of particular interest for this work, particularly the papers that deal with association, correlation and compatibility, which are related to the grouping of products. Association refers to the relationship between different products in turnover or sales volume (e.g. products that are usually sold together), and is approached in (Chiang et al., 2014), (Boysen & Stephan, 2013), (Guerriero et al, 2013) and (Chuang et al, 2010). Correlation takes into account the physical characteristics of the products (e.g. similar materials, dimensions, etc.) and is approached in (Kim & Smith, 2012), (Kovács, 2011), (Bindi et al, 2009) and (Manzini, 2006). Compatibility concerns the relationship between products and storage locations (e.g. problems with shared storage, where multiple SKUs may be assigned simultaneously to the same slots in a warehouse, but for any reason certain combinations of SKUs cannot be assigned to the same slot) and this is approached in (Fumi et al., 2013), (Ene & Öztürk, 2012) and (Chen et al., 2010).

With these papers identified, a table was built (see Annex H) with their classification on "restrictions and considerations" (including all the other types of restrictions and considerations that might be present), performance measures and resolution methods. During the elaboration of the dissertation, this table can be consulted so the most relevant SLAP articles for the problem in study can easily be identified as they are needed.

## 3. Methodology

### 3.1. Problem description and methodology proposal

IKEA's stores have the particularity of being essentially divided into four areas: i) the Showroom, where some products are displayed but not stocked; ii) the Market-Hall, where products are stocked in shelves and are available for customers to pick them like in a traditional retail store; iii) the Self-service warehouse (SF), where customers can pick most of the products that they have decided to buy in the Showroom; iv) The full-services (FS), which is an interdicted warehouse adjacent to the SF where some other products are stocked and the picking is made by the company's employees. On the other hand, since self-service warehouses have not been studied in operations management/management science literature it was decided that this study would address the entire space allocation problem. This corresponds to defining the macro level decisions, that is, defining number of references and volume per product category – which in IKEA receives the nomenclature of Home Furnishing Businesses (HFB) - that ensure a good store performance) and thereafter defining the micro level decisions (allocation of specific products and/or specific sub-categories within each HFB to sales locations within the warehouse).

The sequence of models is to be integrated in a methodological proposal to be adopted for this work which has the structure proposed in Figure 1.

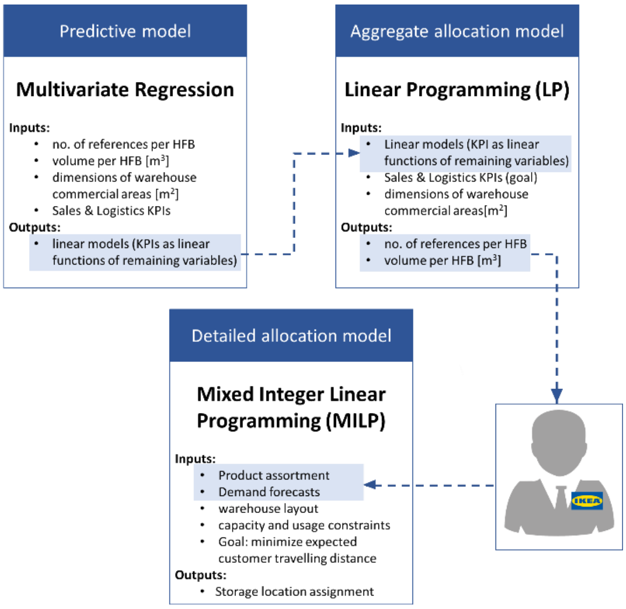


*Figure 1 - Methodology proposed*

The **predictive model** is a regression model that should allow the logistics managers to understand the potential impact of the macro allocation (defining how many different products of each HFB and volume per HFB to be stored in the SF and the FS) on a set of KPIs. The model receives as input the macro allocation and produces probable results for the KPIs.

The **aggregate allocation model** intends to find the best possible solution for the macro allocation problem. It will use the linear equations obtained from the previous model that represent each KPI to formulate a linear programming (LP) model, where the goal is to calculate the combination of volumes and number of references per HFB that jointly optimize the set of KPIs.

**Detailed allocation model** consists in a SLAP that will be formulated and optimized taking into account that the objective is to provide a storage allocation that will minimize the total travelled distance by the customers. This model takes as inputs the product assortment and demand forecasts made by the company's managers, taking into account the results from the previous model, namely the number of references and volume per HFB.

## 3.2. Predictive model

The goal of the **predictive model** is to develop predictive functions that reflect the behaviour of selected logistics KPIs given the warehouses commercial areas space usage (being commercial areas the floor and rack's first level locations in the SF and FS). This model is formulated as multiple regression model.

According to (Rodrigues, 2018) multiple regression can be used to investigate the relationship between a continuous dependent variable ($y_i$), a set of p continuous explanatory variables ($x_{ij}$) and a random error term ($\varepsilon_i$). A multiple regression model is described by a set of $i$ linear equations that can be written as:

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + \ldots + b_p x_{ip} + \varepsilon_i \qquad (3.2.1)$$

The model can also be written in matrix form:

$$y = Xb + \varepsilon \qquad (3.2.2)$$

where y and $\varepsilon$ are vectors of order $(n \times 1)$, $b$ is a vector of order $[(p+1) \times 1]$ and $X$ is a matrix of order $[n \times (p+1)]$, configured as follows:

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \ldots & x_{1p} \\ 1 & x_{11} & x_{11} & \ldots & x_{2p} \\ 1 & x_{11} & x_{11} & \ldots & x_{3p} \\ \vdots & & & & \vdots \\ 1 & x_{n1} & x_{n2} & \ldots & x_{np} \end{bmatrix} \qquad (3.2.3)$$

The vector of 1's in the first column of $X$ corresponds to a dummy variable that is multiplied by the intercept term (i.e., the parameter $b_0$)" (Lattin, Carrol, & Green, 1978).

In order to apply this formulation, the independent variables are the different measures of capacity usage and the independent variable is the KPI for which the predictions are to be made.

In terms of the case study at hand, since the KPIs are calculated on a weekly basis, it was decided that weekly data on capacity usage should be used. This posed a problem because stores' sales location capacity usage is dynamic (products are constantly being sold, moved or replenished). A simplification had to be made: the stores' capacity usage is observed weekly in a random instant of a random day, which is considered as representative of that week. The model is developed following a specific procedure: The development of the model consists on the following procedure:

i) Data preparation – after the extraction from the retailer's database, the data must be cleaned and arranged in tabular form. Also, a portion of the data (the threshold used in this case is 10%) must be removed with random sampling and must not be included in the regression in order to be used later to validate the model;

ii) Outlier detection and removal – some of the observations contain values for the variables that are not commonly seen across the population and that might result either from measurement or data treatment errors, very specific events, either known or unknow that cause some variables to display uncommon values and, finally, observations where its not the values of the individual variables that are uncommon, but their combination. These observations are detecting by analysing each individual variable's density plots and standard deviations or, in the case of the problem being in the combination of values and not in the individual variables, the squared *Mahalanobis* distance, $D^2$.

iii) Ensuring the multivariate analysis assumptions are met - in order to perform multivariate regression there are four statistical assumptions that need to be met: normality, homoscedasticity, linearity and absence of correlated errors (Hair, Black, Babin, Anderson, & Tatham, 1979), although, according to the authors, the normality assumption ca be neglected for more than 200 observations, which is the case. To fulfil the homoscedasticity requirement, each variable is analysed to check for heteroscedastic behaviour (when variables don't present constant variance across their range of values) and specific functions, as suggested in (Tabachnick & Fidell, 2006) and (Howell, 2007) are applied to each individual variable to mitigate this effect. The linearity assumption is not verified in this work, which consists in a limitation, as possible non-linear relationships are underestimated by the model. Lastly, the errors are analysed to ensure there is no non-random pattern in their values.

iv) Variables selection – First, a set of 82 potential capacity variables are selected: the volume of each HFB in the SF (20), the volume of each HFB in the FS (20), the number of references (different products) of each HFB in the SF (20), the number of references of each HFB in the FS (20) and, lastly, the floor areas of the SF and the FS (2). In addition, 6 potential KPIs, that could have a linear relationship with some of the variables, are selected. After this, some of the dependent variables are eliminated because of multicollinearity (the degree to which each the dependent variables can predict each other), which makes the models' results harder to interpret (Hair et al., 1979). According to (Kutner, et al., only models where max{$VIF_k$} < 10 should be accepted. An algorithm was developed to produce only linear models that respected this condition. A a maximum limit for the correlation coefficient $R^2$ between each pair of dependent variables is set and the variable from each pair with the lowest correlation with the dependent variable is removed. Then multiple regression is performed and the alogorithm calculates the VIF for the model, and discards it is lower than 10. Then, the maximum limit allowed for the correlation coefficient between independent variables is lowered and the model is ran again. By specifing an increment on the tollerance and a lower bound, the algorithm repeats this process and returns the best models that have acceptable levels of multicollinearity.

v) Assessing the fit between the model and the data – the F-test, the correlation coefficient $R^2$ and the adjusted $R^2$ of the model are computed and analysed as a first validation.

vi) Checking the statistical significance of each regression coefficient – the significance level α of each regression coefficient is analysed in order to understand the probability of the null hypothesis being true;

vii) Checking the violation of the regression assumptions – The model's errors are compared with a normal distibutution with mean 0 in order to validate that the error term in equation 3.2 can be safely ignored.

## 3.3. Aggregate allocation model

The aggregate allocation model takes as inputs the results of the predictive model (the regression coefficients) and the retailer's desired capacity variables values (already transformed like it is described in sub-section 3.1) to, according to a specified tolerance, set the values for those variables that maximize a KPI (in this case, the modelling is only made considering one KPI). The model is formulated as a Linear Programming (LP) problem.

With this in mind, the indexes for the model are:

$i$ — Set of transformed capacity variables ($i = 1, \dots, n$)

The parameters are:

$u_i$ — Upper bound for the value of the transformed capacity variable i

$l_i$ — Lower bound for the value of the transformed capacity variable i

$b_i$ — Regression coefficient for the transformed capacity variable i

$b_0$ — Constant from the regression model

Finally, the decision variable is:

$x_i$ — Value of the transformed capacity variable i

Consequently, the mathematical formulation is:

$$maximize\ \text{KPI} = b_0 + \sum_{i=1}^{n} b_i x_i \tag{3.2.1}$$

$$subject\ to:$$

$$x_i \geq l_i \quad \forall i = 1, \dots, n \tag{3.2.2}$$

$$x_i \leq u_i \quad \forall i = 1, \dots, n \tag{3.2.3}$$

The objective function (3.2.1) is simply the linear equation obtained from the predictive model. Constraints (3.2.2) and (3.2.3) ensure that the upper and lower bounds for the transformed variables are respected. The values for $u_i$ and $l_i$ are obtained by adding/subtracting 1% to the original assortment variables (number of references per HFB in the SF and volume of references per HFB in the FS) and then applying the transformations to the variables.

## 3.4. Detailed allocation model

The Storage Location Assignment Problem (SLAP) is formulated as Mixed Integer Linear Model (MILP) that assigns quantities of specific products to sales locations. A sales location is defined by the number of the aisle (with aisles numbered from 1 to 42), the number of the section (a section is a portion of the racking as can be seen in Figure 2 a and an aisle can be composed of up to 20 sections) and the number of the level (there is the floor level and up to 5 possible racks).

As the figure suggests, problems can be stored in two ways, either in pallets or in unpalletized form. This means that in this work the SLAP is not a slotting problem, because there is no standardization of the dimensions of the unit loads, as there are products in many different shapes and dimensions.

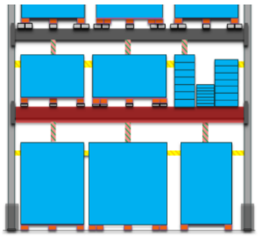

*Figure 2 - IKEA SF racking section with two levels*

The aisles are also not all equal, there are three types of ailsees: EUR ailes (for EUR pallets or products with a length equal to or smaller than and EUR pallet); IKEA ailses (for products that are too large for EUR aisle), which are deeper; IKEA XL aisles, for products with even larger dimensions. Each product is assigned to a specific type of aisle and a matrix of compatibility between products and aisles is built.
Regarding the problem with unpalletized products, fictitious unit loads were defined in order to simplify the problem. Let Figure 3 be a sales location with two different products:

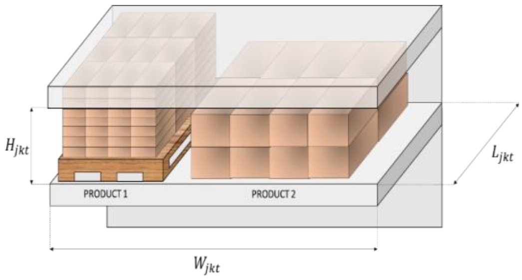


*Figure 3 – Sales location and respective dimensions*

Unpalletized products (Product 2) are placed in the locations forming larger parallelepiped. The optimization of the SLAP intends not only to define where the products should be placed, but also in what quantity. If products are placed in these blocks, then the quantity is defined as the multiplication of the number of products that compose a block in its width, length and height. If it is considered that a product can only be allocated to one type of aisle (remember that the difference between the 3 types of aisles is $L_{ijk}$) and that this allocation is known, then the number of products in the length direction of each big block is known for every product and can be calculated beforehand. It was defined with the logistics managers that there should be space optimization, which means that, for unpalletized products, they should be placed in order to maximize the height usage of the sales locations. Because it is known that there are only 6 levels and that their height is known, then it is possible to know, for each product, which should be the maximum number of product units that fit in height in each level. When it is known how many units of an unpalletized product fit in the product's respective aisle type and depending on the level, this means that the only decision variable needed to model the products is how many products are needed in terms of width. For this reason, fictitious unit loads are defined for each unpalletized product: the unpalletized product's unit load are blocks with 1 product in terms of width, the number of products that fit in the products aisle type in terms of length and the number of products that fit in the level in terms of height (because there are 6 levels, each product has 6 different possible fictitious unit loads). Fictitious unit loads are defined as can be seen in Figure 4 that represents a unit load of Product 2 from Figure 3. Note that height is a two dimension parameter as it depends on the product and on the aisle.

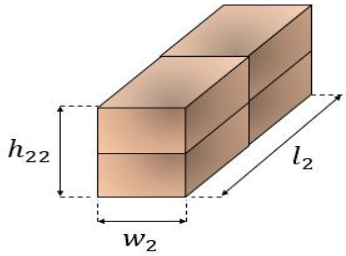


*Figure 4 – An unpalletized product's unit load and its respective dimensions*

In order to formulate the model, two simplifying assumptions are made: each customer purchases only one product and every customer takes the same main route in the SF, so the distance increments associated with the picking of a product are fixed for every location. The mixed-integer formulation is as follows:

The indexes and sets of the model are described as follows:

- n  Set of products (i=1,…,n)
- m  Set of aisles (j=1,…,m)
- k  Set of locations (k=1,…, p)
- t  Set of levels (t=1,…,v)

The parameters of the model are described as follows:

- $a_{it}$  Number of units of product i per unit load when assigned to a location in level t;
- $c_{ij}$  Compatibility between product i and aisle j;
- dij  Distance incremented to customers' route by purchasing product i from location j;
- $e_i$  Weight of one unit load of product i;
- $f_{jkt}$  Weight limit for t^th level of location k from aisle j;
- $h_{it}$  Height requirement of one unit load of product I when located in level t;
- $o_t$  Height capacity of the locations of the t^th level;
- M  Large number used to form a "Big M" type constraints;
- $s_{it}$  Space capacity (area) required by one unit load of product i, when located in level t
- $r_{jkt}$  Space capacity (area) of the t^th level of location k from aisle j;
- $u_i$  Binary parameter equal to 1 if product is stored in a pallet, 0 otherwise;

- $y_i$ Demand (in units) for product i;
- α Reward parameter used in the objective function to ensure consecutively between unit loads of the same product that are stored in different locations;
- β Penalization parameter used in the objective function to ensure that the model assigns the minimum number of locations possible to each product.

And the decision variables are:

- $b_{ij}$ Binary variable equal to 1 if product i is assigned to aisle j, 0 otherwise;
- $w_{it}$ Binary variable equal to 1 if product I is assigned to level t, 0 otherwise;
- $x_{ijkt}$ Binary variable equal to 1 if product i is assigned to the $t^{th}$ level of location k from aisle j, 0 otherwise;
- $q_{ijkt}$ Number of unit loads of product i assigned to the $t^{th}$ level of location k from aisle j;
- $z_{ijkt}$ Binary variable equal to 1 if product i is assigned simultaneously to the location the $t^{th}$ level of location k from aisle j and to the $t^{th}$ level of location (k + 1) from aisle j.

Consequently, the mathematical formulation is as follows:

$$minimize\ (\text{Distance} + \text{Reward} + \text{Penalization}) \tag{3.4.1}$$

$$\text{Distance} = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p}\sum_{t=1}^{v} d_{ij} q_{ijkt} a_{it} \tag{3.4.1a}$$

$$\text{Reward} = -\alpha \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p}\sum_{t=1}^{v} z_{ijkt} * y_i \tag{3.4.1b}$$

$$\text{Penalization} = \beta \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{m}\sum_{k=1}^{p}\sum_{t=1}^{v} x_{ijkt}\right) - 1\right] * y_i \tag{3.4.1c}$$

$subject\ to$:

$$\sum_{j=1}^{m} b_{ij} = 1 \quad \forall i = 1,\dots,n \tag{3.4.2}$$

$$b_{ij} \le c_{ij} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m \tag{3.4.3}$$

$$x_{ijkt} \le b_{ij} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.4}$$

$$\sum_{t=1}^{v} w_{it} = 1 \quad \forall i = 1,\dots,n \tag{3.4.5}$$

$$x_{ijkt} \le w_{it} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.6}$$

$$q_{ijkt} \le M x_{ijkt} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.7}$$

$$\sum_{j=1}^{m}\sum_{k=1}^{p}\sum_{t=1}^{v} a_{it} q_{ijkt} \ge y_i \quad \forall i = 1,\dots,n \tag{3.4.8}$$

$$\sum_{i=1}^{n} q_{ijkt} s_i \le r_{jkt} \quad \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.9}$$

$$x_{ijkt} h_{it} \le o_t \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.10}$$

$$\sum_{i=1}^{n} q_{ijkt} e_{it} \le f_t \quad \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.11}$$

$$z_{ijkt} \ge x_{ijkt} + x_{ij(k-1)t} - 1 \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 2,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.12}$$

$$z_{ijkt} \le x_{ijkt} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 2,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.13}$$

$$z_{ijkt} \le x_{ij(k-1)t} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 2,\dots,p,\ \forall t = 1,\dots,v \tag{3.4.14}$$

$$z_{ijkt} = 0 \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ k = 1,\ \forall t = 1,\dots,v \tag{3.4.15}$$

$$b_{ij} \in \{0,1\} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m \tag{3.4.16}$$

$$w_{it} \in \{0,1\} \quad \forall i = 1,\dots,n,\ \forall t = 1,\dots,v \tag{3.4.17}$$

$$x_{ijkt} \in \{0,1\} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1\dots,v \tag{3.4.18}$$

$$z_{ijkt} \in \{0,1\} \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1\dots,v \tag{3.4.19}$$

$$q_{ijkt} \in \mathbb{N}_0 \quad \forall i = 1,\dots,n,\ \forall j = 1,\dots,m,\ \forall k = 1,\dots,p,\ \forall t = 1\dots,v \tag{3.4.20}$$

The first equation, (3.4.1), is the objective function. The goal is to maximize the distance travelled by customers. As it is assumed that each customer buys only one unit of one product, the total distance travelled by all the customers who visit the SF is simply given by the sum of multiplication of all the individual trips (equal to the total number of units sold per product) by their respective distances, and is expressed in equation (3.4.1a). However, in order to be compliant with the warehouse rules defined by IKEA, there was the need to include two more terms in the objective function which work as soft constraints for the model. The first of these terms, (3.4.1b), rewards the model when products that need to be located in more than one location (because of space constraints) are stored in consecutive locations – this is fundamental as customers must be easily able to find a product, if one or more of the locations are out of stock. The problem with this reward is that the model would tend to overspread the products across more locations than needed, so it was necessary to develop the second term, (3.4.1c), which penalizes the objective function the more locations are assigned to each individual product. The multiplication by $y_i$ both in the penalization and in the reward term ensures that if the model does not respect those two soft constraints, the penalization is always larger than the distance the model could eventually save by violating the overspreading and that the model receives a higher reward for products with higher demands. Both $\alpha$ and $\beta$ must be set to a value higher than the largest distance in the distances matrix, and β should be higher than α, to ensure the model does not overspread the products to receive the connectivity reward. Also, because for some locations the allocated quantity can be higher than the demand (for example, if the demand of a palletized product is 1.1 pallets, the model will allocate 2 full pallets), α and β should actually not only be higher than the highest distance in the matrix, but at least twice as high as that distance. The relationship between α and β is discussed sub-section 5.3.2., which describes the model's implementation.

The first 2 constraints, (3.4.2) and (3.4.3), express the logical conditions/rules of the allocation that dictate the relationships between products and aisles. Constraint (3.4.2) implies that each product can only be allocated to a single aisle. As customers need to know the location of the products they intend to acquire before they arrive at the SF, it is fundamental that each product is only located in one place (else the customers would be confused). Constraint (3.4.3) ensures the compatibility between products and the aisles.

The next 4 constraints ensure that the allocation of products to specific locations follows a set of logical rules. Constraint (3.4.4) ensures that a product can only be assigned to a specific location if it is located to the respective aisle of that location. Constraint (3.4.5) ensures that every product is assigned only to one specific level. Like constraint (3.4.2), this rule exists to ensure the customers can easily find the product they are looking for, but also to make replenishment more efficient. Constraint (3.4.6) states that a product can only be allocated to a specific location if it is allocated to the corresponding level. Constraint (3.4.7) is a "Big M" constraint that ensures that a number of unit loads of a product can only be assigned to a specific location if the product is assigned to that location.

The following constraint, (3.4.8), ensures that there must be enough quantity of every product for its demand to be met. Note that the demand is expressed in product units, thereby the need to multiply the parameter $a_{it}$ (unit load quantity) for the total number of unit loads of product i in the warehouse.

There is limited capacity in each location to store products which cannot be exceed. Constraint (3.4.9) ensures that the area of the locations are not exceeded by the sum of the areas of the unit loads assigned to those locations. The area of the locations is obtained in the data treatment stage by multiplying each location's width by its length. Note that the width and length of the unit loads are not dependent on the level as height is. Constraint (3.4.10) ensures that unit loads of products can only be assigned to levels with a height capacity larger than the height of those unit loads. Constraint (3.4.11) ensures that the weight restrictions of the locations are not exceeded by the weight of the products that are assigned to them.

The next four constraints define the consecutively condition used in the reward term (3.4.1b) of the objective function. If $z_{ijkt}$ is equal to 1 if $x_{ijkt}=x_{ij(k-1)t}=1$ (that is, if the product is stored in two consecutive locations) and equal to 0 otherwise, then, $z_{ijkt}$ can be defined by the relationship:

$$z_{ijkt} \leq x_{ijkt} * x_{ij(k-1)t} \qquad (3.3.21)$$

Constraints (3.4.12), (3.4.13) and (3.4.14) are obtained by the linearization of this relationship. Note that the domain of these equations is different, with k starting in 2 and not 1. This is because there is no location before 1, so $z_{ij1t}$ is always equal to zero, which is expressed by constraint (4.4.15).

The following 5 constraints simply define the domains of the decision variables.

This concludes the formulation of the aggregate allocation model (the third and last model). The following chapter reports the implementation of all the three models and presents the results obtained.

# 4. Models implementation and results

## 4.1. Predictive model

In order to compose the dataset, that is, the set of observations used in the analysis, information about capacity usage and KPIs was extracted, for ten consecutive weeks. This information regarded the capacity usage in 3 stores in Portugal, 13 stores in Spain, 22 stores in France and 13 stores in the United Kingdom. In total, there were 7 different types of files (either Excel tables or dashboards) and 351 single files were extracted. After this, an R program was created using RSutdio that treated each individual file and merged all the information into a single table that constitutes the data set. Each line of the final table corresponds to an observation (that is, a given store in a given week) and each column to a variable. In total, there were 481 observations (481 lines) of 88 variables (88 columns), which correspond to the 82 capacity variables and the 6 KPIs.

First, 48 random numbers between 0 and 481 were generated and the lines corresponding to those numbers were removed and stored to later test the model

The next step was outlier removal. Each variable was normalized in order to have a mean equal to 0 and a standard deviation equal to 1. Every variable for which there were observations that presented an absolute deviation larger than 4 (as recommended in (Hair et al., 1979)) was analysed by plotting its histogram. It was possible to categorize some observations as outliers and these were removed. An example of this process can be seen in Figure 5 below:

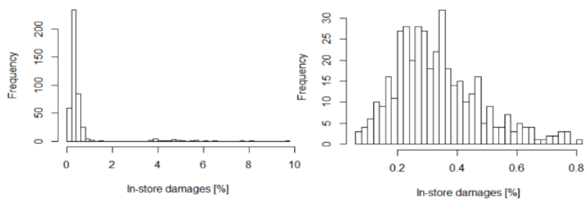


*Figure 5 - Left: Histogram for the variable "In-store damages"; Right: Histogram for the variable " In-store damages without univariate outliers"*

After this the *Mahalanobis* $D^2$ distance was computed for every observation and the observations with $D^2/df$ larger than 2.5 (as suggested by the authors) were removed.

In the end of this process, the dataset without outliers contained 395 observations.

After this, all the variables, both the capacity variables and the KPI were transformed according to the procedure described in section 3.1, in an attempt to prevent heteroscedastic behaviour. 58 variables were transformed as the shape of their density plots wasn't approximately normal. The algorithm was applied to one KPI and returned 20

models. The model with the highest adjusted $R^2$ was chosen. The model's statistics are as follows: The optimal tolerated $R^2$ between variables is 0.8. The models overall and adjusted $R^2$ values are, respectively, 0.723 and 0.703. The VIF is 8.419 and 26 independent variables are included in the model.
The model's error terms were compared with a Normal distribution and it is concluded that the error terms are normally distributed. The model overall is statistically significant at α approximately equal to 0% and only 4 of the included variables are not statistically significant at α smaller than or equal to 5%.
The model was then used to predict the KPI values for the 48 observations in the test sample. The average deviation between the predicted values and the observed values is -1.2 (given that the variable varies between 0 and 100, it is considerably small). This was expected as the mean error should be close to zero as dictated by the regression assumptions. The average of the absolute value of the all deviations (which is of course more relevant because one can grasp for how much the model typically fails) is 10.73. The maximum value of the absolute deviation for the test sample is 73.10. However, the test sample contained observations from stores that were excluded from the model in the outlier detection stage. If these stores are not considered, then the average deviation is -2.18, the average absolute deviation is 10.25 and the maximum deviation found is 33.25. Because this is still a large value, it was assessed which percentage of predictions resulted in absolute deviations lower than 10, which corresponded to 54% of the predictions.

### 4.2. Predictive model

In order to stipulate the upper and lower bounds for the model's the data from one IKEA store was used (store 001 wasn't used as it was removed when its observations were classified as outliers). A random day was picked, and it was considered that the values for the capacity variables were the defined range. A dataset was built which contained the lower bound values (99% of each capacity variable, rounded to the nearest integer) and the upper bound values (101% of each capacity variable, rounded to the nearest integer). Only the capacity variables that are included in the predictive model were included. After this, the respective transformations were applied to every value. By this stage one has to be careful in defining which values to consider as lower and upper bounds for each variable. This is because, for some variables, because of the transformations applied, higher values of the variable imply lower values of its transformed version.
The model was ran in a Windows 10 64-bit operating system with an AMD A6-9220 RADEON R4 2.5GHz processor, 8,0 GB RAM and 5 cores. The optimal solution was found within 0.015 seconds. The predicted value for the KPI was around 13% higher than the observed value.
The model was formulated as a single objective LP. In reality, because the objective function is just the sum of the capacity variables multiplied by the regression coefficients, this means that the model simply sets the capacity variables to either the allowed lower bounds (in the case of the capacity variables with negative regression coefficients) or to the allowed upper bounds (in the case of the capacity variables with positive regression coefficients).
This model could be of more interest if more than one KPI was used, making it a multi-objective optimization problem.
As for the result of the optimization, although it appears better than the observed value, that apparent improvement can be explained by the weak predictive accuracy of the predictive model. In fact, the difference between the predicted and real values is similar to the average absolute deviation of the predictive model, so it cannot be concluded that this approach would optimize the KPI.
Nevertheless, if the predictive model had been proven to have a very high predictive accuracy, then the results of this model would certainly be of use.

### 4.3. Detailed allocation model

The distances of every location were measured using the plant of the SF. Then, all the sections were numbered and individually analysed in order to understand which levels each one had present and also to eliminate some sales locations that were being used only to display products a or to store very specific products that are not included in this work. All the measures of the sales locations were identified through an IKE proceeding document and the information on the 2680 products was divided in different spreadsheets (one spreadsheet per parameter).
The mixed-integer formulation was implemented in GAMS and the solver used was CPLEX. The computer used in the optimization runs on a Windows 10 64-bit operating system with an Intel(R) Xeon(R) CPU X5680 3.33GHz processor, 24.0 GB RAM and 16 cores. GAMS stopped even before finishing reading the data because of memory shortage, which implied that a different approach had to be used.
In order to shorten the amount of data used in each run, the problem was divided into four-sub problems. The objective of this division was to divide the products into four groups, so that the software needed to read less data in each run. In order to divide the products into sub-groups an ABC analysis on the products was conducted.
For practicality, GAMS was first set to run for at most two hours. GAMS ran for A products and the problem was infeasible. The restriction that caused the problem to be infeasible was the restriction that stated that each product should be assigned to a single aisle. This restriction was then removed as the reward parameter on the objective function already works as a soft constraint.
the parameters were defined as α=55 m and β=6600 m and the model was run. Table1 summarizes the model's statistics. The total distance that customers would need to travel to consume all the products in the SF was also computed and its value is 1,349,155.5 m. The file with the allocation was then inspected in order to understand the extent to which the soft constraints worked as supposed.
As the optimal solution wasn't reached, the results present some limitations. For the 666 A products the model allocated 1,472 locations. The model used practically every aisle to allocate products (40 aisles were used), but only 3 levels (t1, t2 and t5). 64 products had unit loads allocated to more than one location. For those products, in 29 cases each products' unit loads were assigned to a single aisle and in 13 cases the

unit loads of each product were assigned to consecutive locations.

*Table 1 - Detailed allocation model's statistics*

| | |
|---|---|
| Computation time (s) | 7204.77 |
| Optimal solution | - 3,735,925 |
| Solution found | 67,461,026 |
| Number of iterations | 1,647,204 |
| Relative gap | 100% |
| Number of rows from reduced LP | 823,944 |
| Number of columns from reduced LP | 548,216 |
| Number of non-zero entries from reduced LP | 2,363,666 |

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After this, an attempt was made to allocate the B products and the problem was integer infeasible.

Overall, the results of the aggregate allocation model are inconclusive.

There a set of problems related to the data used that can influence the results. The first is the definition of the length, width and height measures for the products. Although this data was available, there was no certainty on the position of the products on the shelves, which means that for some products the height, length and width considered might not reflect their true placement position on the sales locations.

The demand of the products is another problem. Because there was no available data on the demand, a parameter, defined by the retailer as the appropriate quantity of the product to store in each location, was used as the demand. The problem with this approach is that the retailer's parameter already accounted for the expected demand of the products and probably surpassed it. The model is formulated so that each product has at least enough quantity in its assigned sales locations in order to satisfy the demand, but as mentioned, in practice the quantity assigned was such that it was at least as large as the retailer's parameter. This means that the real demand might have been too largely surpassed in the quantities assigned.

There are two plausible reasons to explain this infeasibility: i) The way that the demand was calculated might be responsible by the allocation of excessive quantities to the products. The parameter that was defined here as the demand was, in fact, a quantity that the retailer calculates as the quantity that should (according to its procedures) be on the shelves. In calculating this quantity, it is possible that the retailer already accounts for possible variations in demand, which means that the parameter should be higher. As in the formulation integer numbers of unit loads are allocated in such a way that the retailers' parameter is exceeded, it is possible that the true demand is by far exceeded and that the capacity of the warehouse is consequently being exceeded too ii) Another explanation has to do with the fact that the model, by allocating the A products, might allocate too many products for example to the locations of the level t1, making it impossible for the B products that can only fit in height in those locations to find available space. Maybe some A products that could be stored in levels with lower height capacities are taking the needed space for the higher products from the B category.

## 5. Conclusions and future work

The retailer uses proprietary capacity management tools and human expertise in order to manage the capacity usage in its stores. There were two areas with room for improvement: i) the macro allocation problem, which consists in defining the appropriate variety and volumes per product category and HFB and knowing the impacts of the allocations; ii) the micro allocation problem, or detailed allocation, which consists in assigning sales space to each individual product, in particular in the self-service warehouse, in order to improve the customers' experience.

In order to tackle this problem with a holistic perspective, a methodology was proposed that consisted of three models that could allow the solution of these problems in an interconnected way, where each models output would produce the results that would allow the retailer to manage the whole process of capacity allocation.

A methodology was proposed supported on three models that were implemented using MS Excel, RStudio and GAMS.

The overall objective of this work, which was to implement the methodology and to solve the retailer's problem was not fulfilled. However, the methodology and the models developed can serve as a base for future work on the subject.

As it wasn't possible to fully apply the methodology as the store used for the first two models was different from the store used for the last two models, it would be interesting to apply the proposed methodology, that is, the sequence of models (regardless of the formulation of the individual models) to a new case-study and compare the results obtained with the methodologies used by the practitioners.

The predictive model, using the same types of capacity variables, could be applied to a new case-study with more data, in order to allow to apply clustering methodologies and to include non-linear relationships in order to achieve higher levels of predictive accuracy. It would also be relevant to test the evolution of the model's results when the amount of available data periodically increases. One of the main potentialities of the predictive model is that it can always yield better results the more data is fed to it.

For the aggregate allocation model, it would be relevant to have more than one KPI to optimize and to model the problem as a multi-objective MIP. Multi-Criteria Decision Making could be used to assign weights to each KPI, and the model could produce solutions according to the decision makers' preferences.

Lastly, the detailed allocation model could be reformulated, namely the definition of the fictitious unit loads. The way these are defined in this work caused an over dimensioning of the needed quantities to store. Instead of using the sales space quantity defined by the retailer, it would be more interesting to calculate the necessary quantities. For that, a product activity profile should be carefully developed to study the demand variability. In this way, data based statistics could be developed to compute average demand and standard deviation of demand. With this information, one could define a stock quantity in sales location that ensures that during the opening hours of the store no replenishment is required. For example, allocate stock quantities equal to the average daily demand plus one standard deviation or more, if one wishes to ensure that more products are not replenished during the day. Also, the objective function has significant room for improvement. In this work, the distance was calculated by multiplying the quantity allocated to each location by the location's distance. Although in most cases this quantity is proportional to the products' demands, the most correct procedure would be to multiply the demand (which corresponds to the real number of picks) by the locations' distances. However, a problem arises for the products that, for the products that need more than one location, to accurately model the travelled distance it would be needed to know the fraction of the demand that would be met by each location where a product is located, which is practically impossible. A good approximation should be to consider that each location satisfies the total demand for the product divided by the number of locations to which it is allocated to. There is a consequence of this approach: the model would be non-linear, as a parameter would be divided by a decision variable. This means that, to accurately model the problem, it should be formulated as a Mixed Integer Non-Linear Programming problem. Finally, there is the problem with the computational capacity required to handle the large amount of data required and to solve the system of equations that is generated when the number of products is high as in this case. Maybe a heuristic procedure would produce more practical results than the exact method that was used.

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