Electricity Portfolio Optimization for Large Consumers: Iberian Electricity Market Case Study

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A B S T R A C T

Electricity markets are nowadays flooded with uncertainties that rise from renewable energy applications, technological development, fossil fuel prices fluctuation, among others, and it becomes ever so hard for any industry to contract its future energy needs in a riskless way. This results in lumpy electricity price for consumers who contract their electricity in these markets, making it necessary to come up with risk management tools to help them hedge this risk.
To cope with this behaviour, a portfolio optimization is successfully applied to electricity sector. So, based on the works of (Conejo et al, 2010), a mixed integer programming problem is solved to optimize the electricity portfolio of a decision maker by considering the pool market, forward contracts and self-generation, using the Iberian Electricity market as a market environment. The optimization is done through a multi objective approach of minimizing the expected cost and the value of the Conditional Value-at-Risk (CVaR). Scenario analysis is used to reflect the uncertainty on the price of the pool market.
Three case studies are used to explore how the portfolio evolves with different demand profiles and how to take advantage of the seasonality characteristic of the pool market. The expected cost and CVaR are optimized for each case study, and an analysis of the portfolio for each risk posture is done. It is also proven that the seasonality aforementioned can be taken advantage of.

1. Introduction

1.1. *Motivation and Background*

One of the main issues for any industry is to fulfill its energy needs in the smartest way possible. However, the management of an energy portfolio can prove complex: reducing the cost while following a risk management method, but without ever forgetting to satisfy the demand. This is often a concern in the literature given the many opportunities and the different methods and tools that can be used. In addition, the energy sector is characterized by many risk and opportunities one can take advantage of, given the often technology and renewable energy developments, which open many doors for an energy portfolio manager to take advantage of and diversify. So, the goal is to efficiently use the energy resources available accordingly to certain objectives, such as cost, risk, renewable energy, among other, and perform an optimization in accordance to them.
To contract the energy needed the industries shall resort to electricity markets, where there are a few tools they can use and take advantage of: they can plan the long term future, with forward contracts, they can plan the short term future, by the pool market, or they could use a combination of both. In alternative, the industries can resort as well to contracting their own generation units.

Given all the opportunities and threads present in the energy sector, it is important to for any consumer to best take advantage of them when procuring their energy needs, so an appropriate portfolio optimization would be essential. However, regarding electricity markets literature has paid little attention to the buyer's perspective, and so, the goal of this article is to make a contribution directed to them. To do so a portfolio optimization for a large consumer is conducted, with the objective of both reducing the expected cost and minimizing the risk associated with the portfolio.

1.2. *Objectives*

The main purpose is to develop a mathematical model to address the problem of energy portfolio optimization of a large consumer by considering as objectives the total cost and risk, while integrating the problem inside the environment of an electricity market. For this, intermediate goals have to be achieved: characterization of electricity markets, literature review on the methodologies to be used, problem characterization, gathering data from an electricity market and the mathematical model formulation and implementation.

1.3. *Structure*

On this first chapter, an introduction is made, where the background and general motivation are set, as well as the objectives and article structure. On the second chapter a characterization of the electricity markets is done, followed by a literature review on the methods to be used, on chapter 3. Chapter 4 is where the problem characterization is done. The mathematical formulation is presented in 5, followed by the explanation of how the data was collected for the parameters and how was it fit. On chapter 7 we proceed to the presentation and analysis of case studies, followed by the general conclusions on chapter 8.

2. Electricity Markets

2.1. *Players*

In the electricity sector we have three key players: the private investors, the managers commercializing energy and the planners.
They all interact in the markets but face different problems and uncertainties, changing the way each of them sees and prioritizes certain tools or mechanisms of the markets (Odeh et al, 2018). The investors invest in technology mixes that favor them and maximize their profit, whilst planner seek social welfare maximization. The managers commercialize energy and their biggest concern is to maximize return and minimize risk (Odeh et al, 2018).
Although the private side is nowadays key in electricity markets, their risk management problems are less developed than planners (Odeh et al, 2018).

2.2. *Trading tool and Mechanisms*

Electricity markets worldwide normally offer two types of market structure where energy is traded: spot (or day-ahead, or pool) market and forward (or physical) market. The spot market is the energy traded in the real time and day-ahead market, and the most popular structure is a centralized pool-base auction where the buyers and sellers submit bids (Toczyłowskiet and Zoltowska, 2009). A big drawback of this spot market is its price volatility, being the highest when compared to any other commodities' spot markets (Huisman and Mahieu, 2003).
The physical markets is seen as the way to avoid the risk of price variations in the spot market, where with forward contracts, consumers and suppliers commit on the trading of a specific amount of electricity, at a specific future time against a fixed price (Huisman et al, 2009).
There are several European electricity markets where the players can interact. As example, there is the Iberian electricity Market, which is operated by OMIE and provides electricity trading for Portugal and Spain (OMIE, 2019).

2.3. *Uncertainties*

The energy sector, more particularly the electricity one, is nowadays flooded by numerous sources of uncertainty that rise since the sector itself, historically, is particularly unpredictable. So, to plan on it one must think of what future conditions we might face: how will the fossil fuel prices and demand evolve, will the electricity generation still so hardly depend on fossil fuels, etc. Literature over the last two decades has majorly focused its attention on the uncertainties that come from, for example, fuel prices, demand growth and CO2 prices. However, other factors, such as renewable resources availability, technology development, social opposition and emissions limits, also play a role (Odeh et al, 2018).

3. Literature Review

3.1. *Multi Objective Optimization*

Multi-objective optimization (MOO) is a growing subject in the engineering world today given the conflicting nature of the multiple objectives of nowadays real-world problems. Although the ideal would be to optimize all the objectives at hand all at once, that is generally impossible due to their high number and to the fact that they can easily be in competition, so the optimization process has to search for the best compromise solution (Cui et al, 2017) (Chiandussi et al, 2012).
There are multiple techniques to solve MOO problems. The weighted sum: a MOO problem is solved by giving weights to each objective and transform it into a single objective. The ε-constraint method, proposed by (Chankong and Haimes, 2008), where we pick one objective to be minimized and constrain the remaining objectives to be less or equal to a target value. Goal Programming - it does not pose the question of maximizing multiple objectives, but it rather sets specific goals for each objective and attempts to find them (Caramia and Dell'Olmo, 2008).
Most of the traditional algorithms tackle the MOO problem by transforming it into a single-objective function with the weighted-sum method (Pindoriya et al, 2010). Examples of this application are found in (Liu and Wu, 2007) and (Feng et all, 2007).

3.2. *Portfolio optimization with Downside Risk: Conditional Value-at-Risk*

3.2.1. Portfolio Optimization

The Modern Portfolio theory era was firstly started by (Markowitz, 1952), and has since then become the most common way for investors to deal with expected returns, costs and uncertainty for a large number of problems and industries. The portfolio optimization problem was firstly formulated through looking at the expected return and the risk, which measured the variability of the former, and has found large popularity in the field of finance, focusing the research mostly on this topic (Odeh et al, 2018).
Portfolio optimization has found popularity in the energy sector since it exploits the diversification idea through the "cancellation effect" (Odeh et al, 2018).

3.2.2. Downside Risk: VaR and CVaR

Among the several techniques proposed to tackle the problem of portfolio selection, we have downside risk measures.

On downside risk we look at the negative deviations from the expected return, focusing on the side that brings loss, opposite to the mean variance of (Markowitz, 1952), that evaluates in the same way upside and downside risks (Chong et al, 2013). To address this issue, downside risk measures are introduced, in our study specifically the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR).

Initially introduced by (Rockafellar and Uryasev, 2000), CVaR, for a specific desired level α, refers to the *"conditional expectation of losses in the top* 100(1-α)%", whilst the VaR, for the same α value, refers to the "threshold level for losses in the top 100(1-α)%" (Lim et al, 2011). So CVaR is the average of the worst-case loss scenario for a specific level depending on the scope of the study. Common values considered for α are 0.90, 0.95 and 0.99 (Sarykalin et al, 2008).

CVaR's popularity has grown significantly in the literature and is mostly due to VaR's undesirable properties in certain situations. (Uryasev, 2000).

3.3. *Energy Portfolios*

As previously said, Modern Portfolio theory has seen its popularity growing in the energy sector in the last years, mostly on the planner's side, standing as a widely accepted methodology to solve the long-term investment selection problem in energy planning.

From the private agents, buyers and sellers in a liberalized market need to allocate their electricity among different instruments, such as the day-ahead or real-time markets, and bilateral forward contracts. So, these agents can take advantage of portfolio optimization by diversifying throughout these instruments, as well as by choosing among generation technologies (deLlano-Paz et al, 2017).

As examples of portfolio theory applications to the buyer's perspective, there is the works of (Huisman et al, 2009) and of (Conejo et al, 2010).

3.4. *Scenario Analysis*

Scenarios have become a fundamental part of foresight science and scenario planning is recognized as the most widely used method in the futures field.

Again, since the planner's side is generally more developed, most of scenario analysis application are done on the planner's perspective, which (Park et al, 2016) and (Wang and Li, 2016) are examples of.

From the buyer's perspective we have the works of (Conejo et al, 2010), that built a scenario tree as a tool to reflect the uncertainty shown in the price of the pool market .

4. Problem Characterization

The gap identified on which this article proposes to make a contribution is helping a buyer optimizing his energy portfolio. With that in mind, a mixed integer programming (MIP) problem was formulated with the bi-objective of minimizing the expected cost and the risk. The weighted sums method is used. For risk management Conditional Value-at-Risk is used, given the general consensus around it. A model from the electricity portfolio literature was selected to be the base for the new mathematical model, being that (Conejo et al, 2010).

The consumer is inserted in the environment of an electricity market, being the Iberian Electricity Market (OMIE, 2019) selected for such.

Three sources of electricity are considered: forward contracts, installation of self-generation facilities and pool market trading.

The Pool market trading stands as the factor that brings uncertainty into the decision making. To model this uncertainty, we resort to the use of scenarios. So, the scenario tree in Figure 4.1 was built. In this scenario tree we have 3 branches leaving each node and 3 nodes, resulting in $3^3 = 27$ scenarios. Each branch represents one week, with a total planning horizon of 3 weeks.

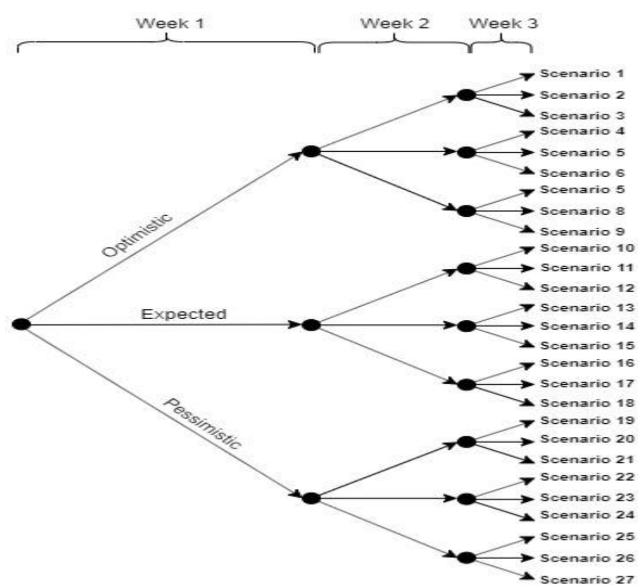

Figure 4.1 – Scenario Tree

On each decision node decisions regarding weekly forward contracts and pool market trading are done, and exclusively on the first node, the decisions regarding 3-Week Contract and self-generation production are taken. So, given the price details of each of the sources, the scenarios and their probability, the demand, the aversion to long term investment from the decision maker and the confidence level for the CVaR calculation, we can obtain the procurement of electricity on each of the sources, the optimized scheduled and the pareto frontier between the expected cost and the CVaR for different risk postures. While subject to the minimization of the expected cost and CVaR value.

5. Mathematical Formulation

Following the literature review, the objective settled is to build a model to optimize an electricity portfolio of a large consumer. So, a mixed integer program (MIP) was built with the multi objective of optimizing the cost and risk.

5.1. *Forward Contract*

The forward contracts are considered to be made for weekly intervals and deliver the same quantity of energy for every hour of the contract horizon. They can be set on single or multiple weeks.
So, there is maximum and minimum of energy available in each contract c for a block b, which is modelled by (1).

$$E^{Cmin}_{c.b} * s^{C}_{c.w} \leq E^{C}_{c.b.w} \leq E^{Cmax}_{c.b} * s^{C}_{c.w} \quad (1)$$

$$\forall c \in N_c, b \in N_b, w \in N_w$$

Where $E^{C}_{c,b,w}$ is the energy contracted in contract c for block b in scenario w, and $s^{C}_{c,w}$ is a binary variable which is equal to 1 if contract c was signed in scenario w, and 0 otherwise. $E^{Cmax}_{c,b}$ and $E^{Cmin}_{c,b}$ are the maximum and minimum amounts of electricity respectively.
The cost of forward contracting is then given by (2).

$$C^{C}_{w} = 7 * \sum_{s}^{N_s} \sum_{t}^{N_t} \left(\sum_{c \in CD_s, CT_t} \sum_{b}^{N_b} E^{C}_{c,b,w} * P^{C}_{c,b} \right) \quad (2)$$

$$\forall w \in N_w$$

Where $P^{C}_{c,b}$ is the price of block b from contract c, and CD_s and CT_t are the set of available contracts in week s and hour t, respectively.
For forward contracts, it is necessary to set a dependency between different scenarios regarding contract decisions. This because scenarios that are equal until a certain decision stage, should have the same decisions made until that stage. To model this non-anticipativity constraints are needed. As done by (Conejo et al, 2010), a matrix A of 0s and 1s is used where A(w, k) is equal to 1 scenario w and w+1 are equal up to stage k, 0 otherwise. So, the matrix's size is (N_w- 1) x (N_k - 1), which in our case is a 26 x 3 matrix. Using A we can formulate the constraint as done in (3).

$$E^{C}_{c.b.w} = E^{C}_{c.b.w+1} \quad (3)$$
$$\forall c \in N_c, b \in N_b, w = 1, \ldots, N_w - 1, \text{if } A(w, K_c) = 1$$

Where K_c is the stage at which a decision on contract c is made.

5.2. *Self-generation facilities*

Self-generation facilities are considered as a source of electricity. Equation (4) is used as to limit the self-generation.

$$E^{Gmin}_{g,b} * s^{G}_{g} \leq E^{G}_{g,b} \leq E^{Gmax}_{g,b} * s^{G}_{g} \quad (4)$$

$$\forall g \in N_g, b \in N_b$$

Where $E^{G}_{g,b}$ is the hourly output of the self-generation facility g. s^{G}_{g} is a binary variable equal to 1 if self-generated facility g is contracted, 0 otherwise. $E^{Gmax}_{g,b}$ and $E^{Gmin}_{g,b}$ are the maximum and the minimum outpour respectively.
The total cost of the self-generation facilities is given by (5), where H is the total number of hours in the planning horizon and $P^{G}_{g,b}$ is the price per energy unit.

$$C^{g} = \sum_{g}^{N_g} H * \sum_{b}^{N_b} P^{G}_{g,b} * E^{G}_{g,b} \quad (5)$$

5.3. *Pool market and energy balance*

For our model the consumer is considered to be a price-taker in the pool market, meaning this that its trades do not influence the market clearing price. There is no real need to constrain the trade in the pool market, so the one constraint needed is to guarantee that the consumer only interacts as a buyer. So, the energy procured from the pool, $E^{P}_{s,d,t,w}$, has to be positive.

$$0 \leq E^{P}_{s,d,t,w} \quad (6)$$

$$\forall w \in N_w, t \in N_t, d \in N_d, s \in N_s$$

The total cost of pool trading is then given by (7), where $P^{P}_{s,d,t,w}$ is the price per energy unit.

$$C^{P}_{w} = \sum_{s}^{Ns} \sum_{t}^{Nt} \sum_{d}^{Nd} E^{P}_{s,d,t,w} * P^{P}_{s,d,t,w} \quad (7)$$

$$\forall\ w \in N_w$$

To meet the demand in each hour of the planning horizon, an energy balance constraint is needed. This equation ensures the energy available per hour from the three different sources: self-generated, forward contracted and pool market, is equal or greater than the demand on that hour. Demand is given by $D_{d,t}$.

$$E^{G}_{g,b} + \sum_{c \in CD_s, CT_t} \sum_{b}^{N_b} E^{C}_{c,b,w} + E^{P}_{s,d,t,w} \geq D_{d,t} \quad (8)$$

5.4. Conditional Value-at-Risk definition

The risk of cost variability is modelled through use of Conditional Value-at-Risk. From chapter 3, we have seen that the CVaR value is approximately the expected cost of the $(1-\alpha) * 100\%$ scenarios with greatest cost. So, from (9) we have the CVaR value to be minimized. ζ and η_w are auxiliary variable for the CVaR calculation defined by (10) and (11). π_w is the probability of each scenario w.

$$\mathrm{CVaR} = \zeta + \frac{1}{1-\alpha} * \sum_{w}^{N_w} \pi_w * \eta_w \quad (9)$$

$$(C_w^P + C^G + C_w^C) - \zeta \leq \eta_w \quad (10)$$

$$\forall\, w \in N_w$$

$$\eta_w \geq 0 \quad (11)$$

$$\forall\, w \in N_w$$

5.5. Objective function

The multi objective is to minimize the cost and the risk of the operation. The two objectives are considered through the use of a single objective function, (12), since a weighted sums method is considered. The way to define the weights for each objective is through the value of the risk aversion factor β, which is used to model the decision maker's propensity towards risk. So, when β = 0, the weight of the cost objective is 100%, but when it is, for example, β = 5, the weight of the cost objective is 1/6 and the weight of the CVaR objective is 5/6.
Here it is presented the aversion to long-term investment factor, λ, which is aimed at countering or favouring the production of self-generated energy depending on the willingness to contract technology with high fixed costs and long life span.
The first part of the objective function (12) makes use of (2), (5) and (7).

$$\min \sum_{w} \pi_w * (C_w^P + \lambda * C^G + C_w^C) + \beta * \mathrm{CVaR} \quad (12)$$

6. Data Collection and Parameter Estimation

Following the model's definition, the second step is to gather data needed for the optimization. For this there was a need to resort to an open data market, so for this estimation the Iberian Electricity Market (OMIE, 2019) was preferred. This data is then fit into the parameters.
To capture the potential statistical patterns associated with seasonality, and so not to lose any information on that regard, a whole year was considered for the studies, from September 2018 to August 2019. As it can be seen in figure 6.1 and 6.2, seasonality is something present in electricity pool markets, therefore there is an optimization opportunity to best take advantage of this seasonality.

6.1. *Definition of Valley, Shoulder and Peak hours*

In figure 6.1, all values from the planning horizon from each hour are averaged and displayed. We can easily see that the pool market prices follow a pattern when it comes to hours of the day and there is a tangible difference between the maximum value (61,02€ at 22h) and the minimum value (48,23€ at 5h). A popular way to categorize the hours of the day according to their average pool price, is through the definition of 3 sets of hours: valley, shoulder and peak hours, which have associated to them a low, medium and high price, respectively. So, we consider the Valley hours to be the ones with an average lower than 54€, the peak the one with an average higher than 58€ and the shoulder the ones in between. We are left with the following sets:

Valley = {1, 2, 3, 4, 5, 6, 16}
Shoulder = {0, 7, 14, 15, 17, 18, 23}
Peak = {8, 9, 10, 11, 12, 13, 19, 20, 21, 22}

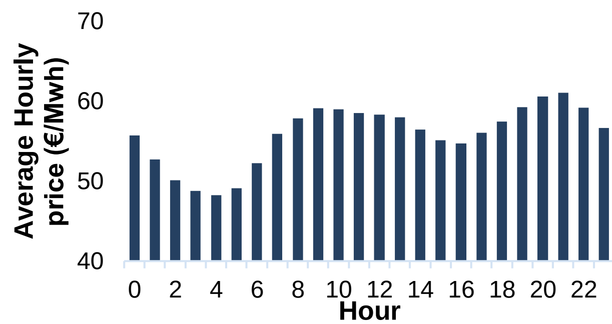

Figure 6.1 - Average hourly price of electricity

6.2. *Scenario generation*

Scenarios are used to model the uncertainty that rises from the pool market prices. In figure 6.2 we see the difference from the average price from each day of the week to the year average.

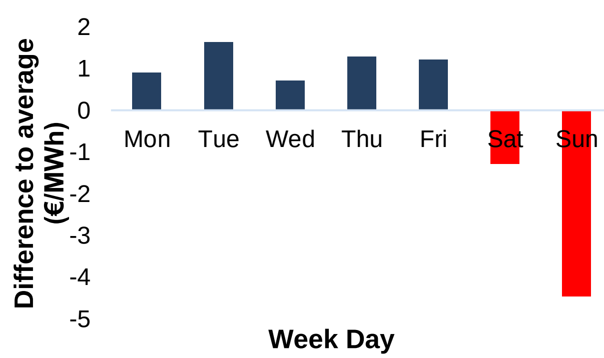

Figure 6.2 – Difference to the year average of each day of the week

We can quickly grasp that there is a statistical relation between the price of the energy and the day of the week, so it is important to consider contiguous weeks in the scenario building. So, for the scenario generation 3 weeks are needed, to represent an optimistic, a pessimistic and an expected week in terms of prices. So, from the 48 weeks of the time considered, 3 weeks are sampled. The week with the highest average price and the one with the lowest are chosen as the pessimistic and optimistic respectively. The expected week was chosen as the one which has the median average value.

So, we look now at the scenario tree in Figure 4.1. The 3 branches leaving each node correspond to the time series for each of the 3 weeks selected for optimistic, expected and pessimistic. This way, the same 3 time series are used to create the 27 scenarios.

To assign probabilities to each of the scenarios we first assign probabilities to each of these 3 weeks among themselves, and then consider for each node a conditioned probability, assuming this way that the probabilities associated with each branch leaving a specific node adds to 1. In the end, for each scenario, we multiply the probability associated with the 3 branches that define that scenario.

The probability for each week is then needed. To obtain them we consider how many weeks, from the 48 available, had close average values to the weeks chosen as optimistic and pessimistic. And we reached the probabilities in Table 1

	Pessimistic Week	Expected Week	Optimistic Week
Probability	0.15	0.58	0.27

Table 6.1 - Probabilities of the 3 reference weeks

6.3. *Forward Contract Data*

The approach chosen for forward contracting was to consider 2 types of contracts: weekly contracts and contracts for the entire time horizon (3-week contracts in this case). The former will be called weekly contracts and the latter 3-Week contracts. In addition, these 2 types will either be set on the 24 hours of the weeks they are settled on, base contracts, or on one of the 3 sets of hours of the day defined above in 6.1. These latter will be called time-of-day contracts. There are therefore 16 contracts considered.

Each contract is divided in 4 blocks, characterized by different prices of electricity, and have a maximum and minimum of electricity available. Each block has 20 MWh available, which adds up to 80 MWh of electricity available per hour in each contract. If a contract is signed it must have a minimum output of 20MWh per hour.

The prices of the blocks are assigned to represent in the best way the behavior of a price-maker, who needs to stay competitive in price and has to deal with risk in Table 6.2. So, prices were assigned to the block 2 of each time-of-day contract according to the prices in each of these times, from there the prices of the other blocks are obtained by subsequently increasing the price by 5% of the original value, for block 3 and block 4, and decreasing 2% for the block 1. The price of the base contracts set is obtained through the prices of the valley, shoulder and peak contracts and the fraction of the day they represent.

	Block 1	Block 2	Block 3	Block 4
Valley	56.84	58	60.90	63.80
Shoulder	58.80	60	63	66
Peak	60.76	62	65.10	68.20
Base	59.05	60.25	63.26	66.28

Table 6.2 – Weekly Forward Contract unit price (€/MWh)

The price of the second block for the 3-week contracts is determined by decreasing the values of the corresponding weekly contract in 2%. The price for the other blocks is calculated following the same methodology as the one used in the weekly contracts.

6.4. *Self-Generation Facility*

As options for the self-generation facility, only a single renewable electricity source is considered.

Instead of considering fixed and variable costs separately, the model considers instead a merge of the two, a levelized electricity price. This is an estimation of the price per energy unit produced throughout the entire expected lifetime of the equipment. This way it is possible to put the long-term investment into perspective, enabling a comparison with the electricity pool market prices, which allows for an analysis to the trade-off of the two.

The data was taken from study conducted by Fraunhofer Institute for Solar Energy Systems (Kost et al, 2018), and we consider the installation of a PV technology. The levelized price is 35.5 €/MWh. This will be the price for the first block of energy. The following blocks' prices are obtained by subsequently increasing in 10% the value of the original block. The price can be seen in Table 6.3. This method's intention is to represent other costs that can arise as the investment grows, such as, for example, the cost of space for the PV panels. Each block has 15 MWh of energy available, and there is a minimum of 15 MWh if this source is chosen.

	Block 1	Block 2	Block 3	Block 4
Price unit	35.5	39.5	42.6	63.8

Table 6.3 – Price of self-generated blocks (€/MWh)

7. Case Studies

To best demonstrate the model's applicability to different situations three case studies are elaborated. For each case study, the electricity demand profile is changed, so to analyze different portfolios.

The same confidence level of 0.95 is used in the CVaR calculation for all cases.

7.1. *Case a): Constant Demand Profile*

On this case study, a constant hourly demand throughout the planning horizon of 200 MWh is considered. On this case the more general aspects of the portfolio options are analyzed.

7.1.1. Aversion factor to long-term generation investment Value Definition

In this first step of the analysis we will tackle the issue of the definition of a value for the aversion factor to long-term generation investment (λ). As a way to somewhat penalize or favor the self-generation facility, since it implies a big initial investment, which can be seen on the eyes of a decision-maker as a risk, this factor is multiplied by the cost obtained from the levelized electricity price. So, a single value for this factor should be chosen to be used throughout this analysis. On the absence of a decision maker, the value of λ will be such that makes the self-generation facility competitive among the other options.
To evaluate the competitiveness, the model is run for different values of β and λ presented in table 7.1.

λ	β				
	0	1	1.5	2	5
1	60	60	60	60	60
1.3	30	45	52.65	60	60
1.6	0	0	0	15	15
2	0	0	0	0	0

Table 7.1 – Self-Generated Electricity for different values of β and λ (MWh)

From table 7.1 we can easily notice that the amount of electricity contracted from the Self-Generation is sensitive to the value of these two factors. The Value of λ = 1.3 was chosen since it was considered the one that makes the self-generation facility more attractive to different risk postures.

7.1.2. Expected Cost vs CVaR

When using CVaR, one of our goals is to observe how the risk aversion factor (β) changes the ratio between the total expected cost and the CVaR value.
To achieves this the model is ran for different risk postures and we analyse how the expected cost and CVaR change for each. The values are presented in table 7.2.

β	Expected Cost	CVaR
0	5.386	7.009
1	5.649	5.874
1.5	5.702	5.831
2	5.732	5.812
5	5.787	5.788

Table 7.2 – Expect Cost and CVaR trade-off for different risk postures (million €)

The solution reached for β = 0 is the one with the lowest expected cost and highest CVaR, and the one for β = 5 is the one with the lowest CVaR and highest expected cost. This is true since when β = 0 the term of the objective function regarding risk is not taken into account, making it the solo objective to minimize the expected cost, and when it is equal to 5, it mainly prioritizes the risk reduction. So, a reduction of the CVaR comes with an increase of the expected cost.
It is particularly relevant to mention the case of β = 1, where there is a decrease of 16.2% of the CVaR and only an increase of 4.88% of the Expected Cost.

7.1.3. Portfolio evolution

While building the portfolio, the model is presented with sources with different prices and, through the value of the Conditional Value-at-Risk, different risk levels associated to them. So, the sources chosen for the portfolio while varying the risk aversion factor will change accordingly. Figure 7.1 shows the evolution of the portfolio of sources for the 3 weeks for different levels of β. The pie charts are made by resorting to the expected value of procurement over all the scenarios. These show four types of sources: pool market, self-production, weekly contracts and 3-Week contracts.
One of the most noticeable result is how the share of the Weekly and 3-Week Contracts grows as the value of β increases, as well as the self-generated electricity, which also grows. This is an expected result since these sources represent more risk-free options. It is important mentioning that the share of 3-Week contracts is always higher than the share of weekly contracts. In the opposite direction, the electricity procured from the pool drops massively as the risk aversion increases, going from an 85% share, to 0% when β = 5.

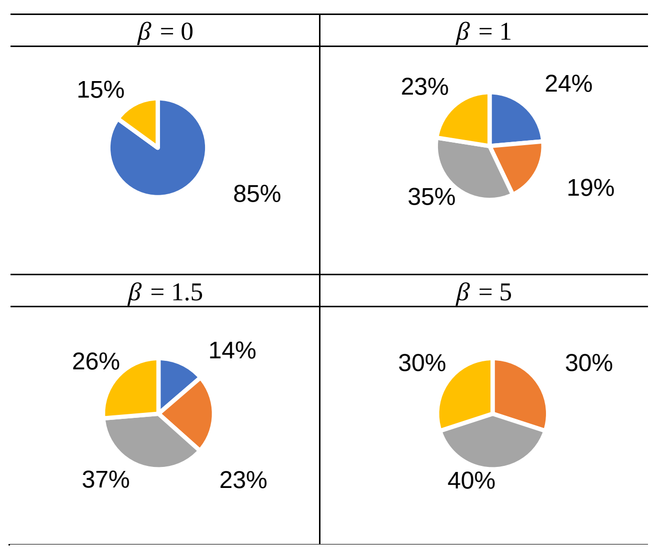

Figure 7.1 – Portfolio evolution

7.1.4. Forward Contracts

Forward contracts and the self-generation facility are a way to hedge risk from the pool market. The contracting of these options changes for different values of risk posture. Specifically, for the forward contracts, it is interesting to look at how the contracting is done from time-of-day contracts and base, or from weekly and 3-Week contracts. With the increment of the risk aversion, there is more energy contracted for peak and shoulder hours than for valley ones, since this latter have higher pool market price, so contracts are done to hedge this risk.

The electricity from forward contracts to valley hours comes mainly from base contracts than from Valley contracts, and the electricity for Shoulder and Peak comes mainly from time-of-day contracts than from base. This happens since the pricing of the base contract is done based on the prices of the 3 times of day, and although it might be advantageous since it can represent a cheaper option in peak and shoulder hours, it makes the decision maker overpay in valley hours, so a trade-off is done in that regard.

7.2. *Case b): Cyclic Demand Profile*

The second case study is of a cyclic daily demand profile. This case study is inspired by the situation described in (Pinto-Varela et al, 2009), where the problem of design of multipurpose batch plant is tackled and tasks are scheduled to fit a 24-hour cycle which repeats itself continuously. For these cases there is an optimization opportunity: optimize the starting time of this 24-hour cycle so to best achieve the objective of the problem: minimize cost and risk.

A mixed demand pattern was used for these 24h, with minimum of 170 MWh, maximum of 250 MWh and average of 200 MWh.

This case study serves the purpose of exploring the seasonality that the pool market prices suffer depending on the hour of the day, which leads to the definition of valley, shoulder and peak hours in chapter 6. Cases will be considered with and without starting time optimization. To simulate no optimization, the starting time is defined to hour 0 of the series.

7.2.1. Modelling the starting time

To model this situation we need a new index, which represents the possible starting times, θ, a new parameter for the demand, where the demand profile along the hours t is shifted for all the possible starting times θ, $\mathrm{ND}_{t,\theta}$, and a binary variable, s_θ which is equal to 1 if θ is the starting time chosen, and 0 otherwise. So, equation (13) is needed to guarantee that only one starting time is chosen and (14) to select the right schedule to be used by energy balance equation (8).

$$\sum_{\theta}^{N_\theta} s_\theta^s = 1 \qquad (13)$$

$$\sum_{\theta}^{N_\theta} (s_\theta^s * \mathrm{ND}_{t,\theta}) = \mathrm{D}_t \qquad (14)$$

Parameter $\mathrm{D}_{d,t}$ of (8) is substituted by variable D_t.

7.2.2. Expected Cost vs CVaR

Again, and for the same reasons stated in sub-chapter 7.1, it is critical to look at how the expected cost and the CVaR value react, their trade-off, to the increment in the aversion factor β. However, for this particular optimization, it is also interesting to analyze what are the practical impacts in this area when an optimization on the starting time is done.

The solutions reached are in line with the expected: the expected cost grows in both scenarios with the increment of the risk aversion factor and the CVaR decreases. From figure 7.2 we see that the values of the expected cost and CVaR on the starting-time optimization are always lower than the ones without optimization, for the same β value, which demonstrates the relevance this optimization has. So, with optimization we can reach the lowest levels of expected cost and CVaR, and without it the highest. The starting time changes for different risk postures (for $\beta = 0$ it was 7h, for $\beta = 1$ it was 6h and 8h for $\beta = 1.5, 2, 5$).

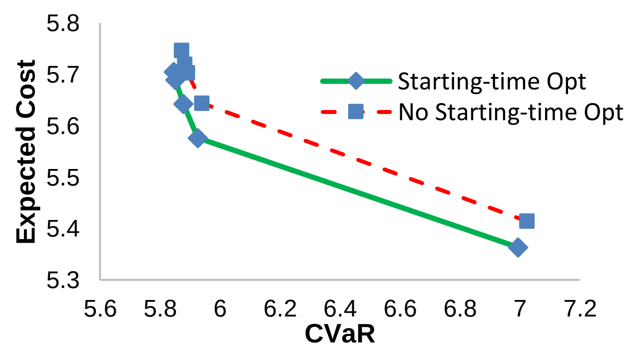

Figure 7.2 – Efficient Frontier for case b (million €)

7.2.3. Procurement differences: valley, shoulder and peak hours

From the case with and without starting time optimization, the share of energy procured from each time of day can change, since we are shifting the demand profile.

So, as expected, the main thing to notice is that the procurement of energy grews in valley hours and decreases for peak hours. This makes sense since the optimization on the starting time avoided matching the hours with higher demanded to the peak hours, and rather matched them with the valley ones. This resulted on a drop of 12.24% in the electricity procured in peak hours, on average from every value of β, and on an increase of 26.5% in the valley hours.

7.3. *Case c): Weekly optimization*

This third case study is a weekly optimization. Similar to case b), the scheduling of activities is optimized according to the objectives of minimizing risk and cost. However, for this case we consider a set of 7 different day demand profiles that have to all be scheduled within the week. The schedule chosen is repeated for the 3 weeks under study. Each of these 7 day demand profiles is defined by a set of 24 values, which define the demand for the 24 hours of a day. To generate each set, 24 values were randomly sampled from a normal distribution. The mean value and standard deviation were changed for each of these 7 sampling procedures. Statistical characteristics of each set are shown in Table 7.3.

	1º	**2º**	**3º**	**4º**	**5º**	**6º**	**7º**
Mean	180	220	250	270	250	220	250
Std. dev.	10	20	20	20	30	40	40
Max	193	256	296	318	324	305	334
Min	263	181	211	228	130	136	185

Table 7.3 – Statistics on the 7 day demand profiles

7.3.1. Model the weekly optimization

To model this situation we need to consider a new index, γ, which is an alias of d, a new parameter, $\mathrm{EED}_{t,\gamma}$, where the 7 day profiles are stored and a new binary variable, $s^s_{d,\gamma}$.
Imagine the binary variable $s^s_{d,\gamma}$ as matrix of 1s and 0, where lines represent the index d and the columns represent the index γ : for the first real day d, the position of the 1 in the first line would denote which time series would fit the first day, and so on. For this it is needed that the sum of each line and each column sums to 1. So, equation (15) and (16) guarantee that.

$$\sum_{\gamma} s^s_{d,\gamma} = 1 \qquad \forall d \in N_d \tag{15}$$

$$\sum_{\gamma} s^s_{d,\gamma} = 1 \qquad \forall \gamma \in N_\gamma \tag{16}$$

Finally, the parameter $D_{d,t}$ is obtained in (17) to be used in (8).

$$D_{d,t} = \sum_{\gamma} s^s_{d,\gamma} * \mathrm{EED}_{t,\gamma} \qquad \forall d \in N_d, t \in N_t \tag{17}$$

7.3.2. Expected Cost vs CVaR

As done for case a) and case b), it is important to see how these alterations can be impactful on the objective. So, a comparison is made with the optimized order and the order set on table 7.8. The trade-off between the two objectives can be seen in figure 7.3.

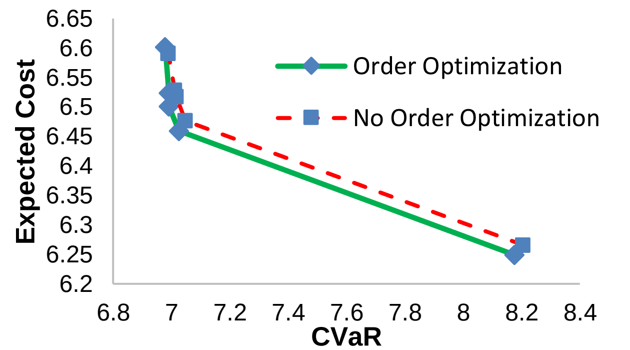

Figure 7.3 – Efficient Frontier for case c (million €)

As expected, in both cases the expected costs rises and the CVaR drops with the increment of β, besides the order optimization produces more favorable trade-offs between the two variables for every β and both the expected cost and CVaR see their lowest value when there is an order optimization and their highest where the is not.

7.3.3. Day-order analysis

The result in terms of schedule of this case study are presented in table 7.4.

β	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
0	2º	1º	6º	5º	3º	7º	4º
1	3º	6º	1º	2º	5º	7º	4º
1.5	2º	6º	1º	3º	5º	7º	4º
2	6º	5º	1º	3º	2º	7º	4º
5	4º	5º	1º	7º	2º	6º	3º

Table 7.4 – Scheduling results of case study c

One of the objectives for this order optimization was to analyze how the price seasonality of the day of the week, which is under analysis in Figure 6.2, could come into consideration in the model when choosing the optimized order of the day-time series from Table 7.4. From Figure 6.2, generally, the days with cheaper electricity are Sunday and Saturday, in that order, and from table 7.10 we see that for almost all risk aversion values, except β = 5, the day-series assigned to each are the 7º and 4º, respectively, which significantly proves the impact seasonality has. This because the 4º series is the one with the highest mean value and is assigned to the day with cheaper electricity, in general, and the 7º time series is the one with the second highest mean value, tied with the 3º and 5º. However, it is the one with the highest standard deviation, and therefore highest maximum,

making it the least favorable among the 3, making sense then that it was assigned to the second cheapest day.
For the remaining days of the week, Monday to Friday, the pattern of price difference is not as relevant, seeable from the figure 6.3, where the days' difference to average was very similar.

8. Conclusions

This dissertation proposes to come up with a mathematical framework to fill the gap identified in the literature: the lack of tools for buyers to interact in electricity markets.
With that in mind a detailed mathematical model for portfolio optimization is developed and tested. From it the portfolio is obtained with specific values for the electricity contracted from each of the three sources considered, the expected cost and Conditional Value-at-Risk. For case studies b) and c), the model also provides information of the optimized schedule of Demand. The output of the model is subject to different values of the risk aversion factor and demand. The problem is formulated as a mixed integer programming problem.
From case a), we see that for risk prone decision makers the expected cost is lower and CVaR higher than for risk averse. Plus, risk prone mostly procure from the pool and risk averse prefer forward contracting and self-generation.
From case b) and c), it was conclusively proven that the seasonality of prices in the Iberian Electricity Market can be used to further optimize the objectives. So, it was seen that if possible, the model will avoid high price hours and prioritize low price ones, in addition, it will also prioritize low price days of the week.
Further work can be developed by building a model which incorporates more decision nodes. For this a shorter-term model could be made, where instead of weekly decisions there are hourly decisions.

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