Implementation of an alpha-beta Coordinates Speed Observer for a Squirrel Cage Induction Machine Using an Adaptive Scheme

Alexandre Miguel Ribeiro Monteiro

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Supervisor(s): Prof. Paulo José da Costa Branco
Prof. João Filipe Pereira Fernandes

Examination Committee
Chairperson: Prof. Célia Maria Santos Cardoso de Jesus
Supervisor: Prof. Paulo José da Costa Branco
Member of the Committee: Prof. Alexandra Bento Moutinho

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“There’s nothing so practical as good theory”

Kurt Lewin
Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.
Acknowledgments

This project would not be possible without the help I was given.

I would like to express my deepest gratitude to my supervisor, Master Francisco Silva, for his huge dedication to this thesis, as well as for the time spent in guiding me in the right direction always with wisdom and kindness.

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Not least in importance, it is my privilege to thank my family, both my parents and grandparents, for all their love and trust which were a pillar in my academic and personal education.
Resumo

A presente dissertação surge da necessidade da implementação de um controlo vectorial de um motor de indução "sensorless", i.e., com recurso a um observador de velocidade, ao invés de um sensor. Esta implementação decorre com o propósito de ser usado num veículo, o Fiat Seicento Elettra. Para tal, primeiramente foi abordado o estudo de um observador recorrendo a uma bancada experimental, com um motor de indução mecanicamente acoplado a um gerador DC, simulando a carga mecânica.

Primeiramente, foi desenvolvido o modelo em espaços de estados do motor de indução, tendo sido feito o seu estudo prévio. De seguida, projetou-se e desenvolveu-se os circuitos de sensorização, bem como o processamento dos dados e do observador. Sendo que para estes dois últimos, foi usado um Arduino Duo. Por fim, foram realizados testes, ligação directa primeiramente, e com uso de um inversor em segundo, sendo que cada configuração foi testada em vazio e em carga, para a validação do observador, juntamente com as suas simulações.

Os resultados experimentais mostram, que, com a presente implementação, o observador não consegue estimar corretamente a velocidade de arranque do motor, principalmente quando é realizado em ligação directa, devendo-se ao facto de haver uma imprecisão nas estimações das variáveis de estado do sistema. Os resultados foram validados através de simulações e concluiu-se que o aumento da frequência de amostragem conseguiria diminuir o erro de estimação. Desta forma, um sistema com um processamento mais rápido é necessário para o desenvolvimento deste observador.

Palavras-chave: Motor de indução, Inversor, Observador, Controle sem sensor.
Abstract

This Master thesis appears from the requirement of design and implementation of a vector torque control with a sensorless speed controller for a squirrel-cage induction machine. The results achieved are to be further developed for the electric powertrain of the Fiat Seicento Elettra vehicle belonging to the VIENA project.

The linearized state space model of the induction machine was first developed, the machine parameters determined, and the machine simulated. The voltage and current sensing circuits were designed and developed, as well as the data processing and the speed observer. For these last two steps, an Arduino Duo was used. Lastly, experimental tests were performed, with a direct connection to the power grid, and also using an inverter in between, with and without load, to validate the observer, together with the simulation.

Accordingly, the study of the observer was made by using an experimental bench, with an induction motor mechanically coupled to a DC generator, simulating the mechanical load. The experimental results show that this observer implementation can not correctly estimate the speed evolution during the motor starting when connected direct to the power grid, due to the fact that the state variables are not being accurately estimated. This results were confirmed with simulation results, and it is concluded that a higher sampling frequency would reduce the estimation errors. In this way, a faster processing unit is necessary for the development of this observer.

Keywords: Speed estimation, Sensorless control, Observer, Induction machine.
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Nomenclature

Greek symbols

$\chi$  Bulk susceptibility.

$\mu_0$  Permeability of free space.

$\omega_r$  Electric rotor angular speed.

$\omega_s$  Electric stator angular speed.

$\omega_{mec}$  Mechanical stator angular speed.

$\phi$  Flux.

Roman symbols

$e$  Estimation error.

$u$  Input signal.

$x$  System’s state.

$y$  Output signal.

$\vec{B}$  Magnetic flux density.

$\vec{H}$  Magnetic field strength.

$f$  Frequency.

$f_{cut}$  Cutoff frequency.

$f_{sa}$  Sample frequency.

$I$  Electric current.

$J$  Inertia.

$L$  Self inductance.

$l$  Flux leakage.

$M$  Mutual inductance.
\( n_{\text{sinc}} \)  Synchronous speed.

\( p \)  Number of pair of poles.

\( R \)  Electrical resistance.

\( s \)  Slip.

\( T \)  Torque.

\( T_{\text{sa}} \)  Sampling time.

\( V \)  Electric potential difference (Simple).

\( W_{\text{ele}} \)  Electric Energy.

\( W_{\text{mag}} \)  Magnetic Energy.

\( W_{\text{mec}} \)  Mechanical Energy.

\( X \)  Electrical reactance.

\( Z \)  Electrical impedance.

**Subscripts**

\( \text{arm} \)  DC machine armature circuit.

\( n \)  Nominal value.

\( r \)  Rotor component.

\( s \)  Stator component.

\( sr \)  Stator and Rotor.

**Superscripts**

\( \alpha, \beta, \omega \)  Two-axe stationary frame.

\( a, b, c \)  Natural frame.

\( d, q, o \)  Two-axe non stationary frame.

\( T \)  Transpose.
Chapter 1

Introduction

1.1 Motivation

One of the present day challenges is how to address car mobility in a more sustainable manner. Certainly there are many alternatives trying to address this problem, but none is as popular as the electric car. Electric cars are said to be zero-emission vehicles (ZEVs) and therefore a more sustainable option. Although the environmental impact is a controversial topic, there are some electric car’s aspects that make them a better choice than internal combustion engine (ICE) vehicles, such as less noise and less pollution making with zero direct emissions. At the time being, we observe an increasing number of electric cars on the streets which are seen as a viable alternative to the typical ICE vehicle.

Regarding the electric car and in particular the Fiat Seicento Elettra, the car in study, it is mandatory to measure the motor’s power signal and speed in order to perform a speed control loop in the system. However, and given its electromechanical dynamics, a very appealing approach would be to use its electrical quantities to estimate the motor’s mechanical quantities such as speed and torque. This approach dismisses the need of using speed sensors, making the physical system more compact and simple.

Meanwhile, in order to drive the Fiat car again, some of the original components have been replaced.
1.2 Brief introduction vehicle powertrain

The Fiat Seicento, Figure 1.1, was a city car introduced in the year of 1997 [5] by the Fiat company, powered by a three-phase induction machine with a single squirrel rotor. Along with the electric motor, it also requires batteries. Since the motor must be supplied with an alternated voltage, there must be an inverter to convert the DC voltage supplied by the batteries to an AC three-phase voltages. Figure 1.2, shows the electric the electric power train scheme of the car. Despite the role of the inverter been taken lightly when compared with motor and the batteries, it is important to account that this element is responsible to control the whole system by adjusting the applied voltages to the motor, so the speed value fits with the expected behaviour from the driver when the gas pedal is pressed. Thereafter, the motor will produce a torque in the shaft connected to the differential where each side is connected to the car’s wheels.

As stated in Section 1.1, this car is equipped with new components, different from the original ones, namely the inverter and a 600 V batteries.

![Figure 1.1: Fiat power train representation - Source [3].](image1)

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Induction Machine</td>
</tr>
<tr>
<td>2</td>
<td>Differential</td>
</tr>
<tr>
<td>3</td>
<td>Inverter</td>
</tr>
<tr>
<td>4</td>
<td>Accelerator</td>
</tr>
<tr>
<td>5</td>
<td>Braking</td>
</tr>
<tr>
<td>6</td>
<td>Gear selector</td>
</tr>
</tbody>
</table>

![Figure 1.2: Power train.](image2)

![Figure 1.3: Motor Nominal points.](image3)

<table>
<thead>
<tr>
<th>$V_s[V]$</th>
<th>$f_s[Hz]$</th>
<th>$I_s[A]$</th>
<th>RPM</th>
<th>$T[N.m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>76</td>
<td>76</td>
<td>157</td>
<td>2200</td>
<td>65</td>
</tr>
<tr>
<td>121</td>
<td>220</td>
<td>90</td>
<td>6500</td>
<td>22</td>
</tr>
<tr>
<td>121</td>
<td>305</td>
<td>88</td>
<td>9000</td>
<td>16</td>
</tr>
</tbody>
</table>
The general scheme of the electric power train, is presented in Figure 1.2, where the induction motor, Figure 1.7, is controlled by the Siemens inverter, Figure 1.4, which is powered by the 600 V batteries, represented in Figure 1.5. Finally, this induction motor has three nominal points, for three distinct operating speed, which are presented in Figure 1.3.
The more conventional way of making the speed control is with a speed sensor, in order to implement a simple feedback loop, with real time measurements of the speed, such as presented in Figure 1.8, where the internal elements which are a physical part of the car’s elements are highlighted in green, while the auxiliary elements are highlighted in blue. Finally it is also necessary use electric sensors for voltage and current measurement in each motor phase [13]. Alternatively, it is possible to implement a sensorless controller using and speed observer, as shown in Figure 1.9, where the mechanical speed would be estimated using the electrical quantities present in the motor, instead of measuring.

Figure 1.8: Hierarchy structure original.  
Figure 1.9: Hierarchy structure sensorless.

Regarding the speed observer, the most used method to estimated the speed of an induction motor is an Adaptive Speed Observer [1], [10] and [6], this method, the induction motor is treated as an linear system, in which the state variables $x$ are the stator currents and the rotor fluxes or currents, and the applied voltage $u$ is the input signal. However, this linear system is constantly being adjusted by the estimated speed value, which is not considered to be a system’s state but it changes the differential equations coefficients. Hence a determined function $Ax + Bu$, can be used to compute the predicted state from the previous state, while another function $L(y)$ working in feedback loop, adjusts the predicted state with the actual state measurement $y$.

Another approach to solve the problem is to implement an Extended Kalman Filter - EKF [8] and treat the system like a non-linear system, since the describing differential equations are not linear independent from each other, if the speed were considered as a state variable. In order to implement this technique, the non-linearities in the dynamic and the observation model must be smooth, so that it is possible to expand $f(x)$ and $l(x)$ in Taylor Series and approximate this way the forecast and the next estimate of $x$, the EKF is also called as the First Order Filter, since it is only considered the jacobian of $f(x)$ and the higher order terms are considered negligible.
1.3 Objectives

Although the system in focus is Fiat power train, the implementation of the observer has been made in a testing bench induction motor, in order to evaluate the observer performance, while the implementation in the Fiat power train will be done in future works. Using a induction motor dynamic model, the observer will be designed in order to dismiss the need of additional sensors. To implement the observer is necessary to collect the instantaneous values of the electrical quantities of the motor such as currents of each phase and voltages, in addition to this is also necessary to know the values of parameters of the equivalent circuit of the machine in study. After this analyses, the electrical aspects of the inverter will be consider, since this is a key element in system.

1.4 Thesis Outline

The present thesis is entirely focused in the study of the induction motor, which is element accountable for producing the mechanical power in the fiat's power-train.

The chapter 2 begins with the motor's analyses in a stationary state operation regime, which is a operating particular case, where it is possible to used an equivalent circuit to describe the system. In this chapter it is also presented the dynamic operation regime of an induction motor, which is a key step of the development of the speed observer. Finally it is present the adaptive observer model and its first set of simulations. The chapter 3 focus on the implementation of the observer and its hardware choices, the present chapter ends with a new set of simulations more specific to the real system. Finally in chapter 4 is presented the observer results produced by the prototype developed.
Chapter 2

Induction motor

An induction motor, also known as an asynchronous motor, is an electric alternating current (AC) machine capable to produce rotation motion by electromagnetic induction from the magnetic field created in the stator winding. An induction motor can have two types of rotor, wound rotor, which are easier to control the torque and the speed but less efficient, or a squirrel-cage type, which are harder to controller. In the present application, the rotor used is a squirrel-cage which makes the motor self-starting, more reliable and more efficient [11].

2.1 Mathematical representation

In order to make a mathematical representation of the motor, some assumptions are made:

- Three identical stator windings, one for each phase, are arranged along the stator and with a displacement of $120^\circ$ between consecutive windings with the same direction and number of pairs of poles $p$, Figure 2.1.

- Although short-circuit rotor induction machines have a squirrel cage rotor, this rotor can be represented by three windings similar to the stator arrangement, in which their axes are also spaced apart from $120^\circ$. However, the rotor windings have a rotation speed equal to $\omega_{mec}$, Figure 2.2.
The following assumptions are considered for the present scenario:

1. Hypothesis of magnetic linearity, the material is characterized by having a proportionality relation between the magnetization and the magnetic field \( \mathbf{M} = \chi \mathbf{H} \). Thus it is possible to write the following relation:

\[
\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi \mathbf{H}) = \mu \mathbf{H} \tag{2.1}
\]

2. Joule losses in the iron are neglected;

3. The electric source feeds the machine with sinusoidal distribution voltages;

4. The magneto-motive force has a sinusoidal distribution, the lumped parameters is only valid for sinusoidal qualities.

5. There is no change in temperature during operation, the change of temperature changes the electric parameters of the system.

6. The skin effect on conductors are negligible;
2.1.1 Stationary operating regime

Under this set of conditions previously enumerated, we can only assume the machine is operating in stationary regime if we add another conditions in order to use the equivalent circuit:

- There is no variation in the value of any input (no variation of the AC amplitude or its frequency), or outputs of the machine, hence no acceleration is felt in the machine.

When a three phase voltage system is applied to the stator of the machine, a rotating magnetic flux in the air-gap is created, which will induce an electromotive force in the rotor squirrel-cage bars. Since all bars are short-circuited, currents are in the rotor. The frequency of these currents is dependent on the mechanical speed and the frequency of the voltage source.

On a per-phase analysis, the stationary induction machine has a lumped parameter equivalent circuit as depicted in Figure 2.3.

![Figure 2.3: Equivalent circuit of transformer supplying load impedance $Z'_2$; ($X'_i$ = referred to 1st side).](image)

Even though the induction machine resembles a transformer, it is important to note that physically they are much different because the machine transforms electric power into movement. The motor is strongly dependent on the difference between the rotting speed of the magnetic flux and the rotor speed, which is the reason why when in "locked rotor condition" the frequency of the electric qualities in the rotor is the same as in the stator. On the other hand, when the rotor is rotating at the synchronous speed there are no currents in the rotor. The difference between the synchronous speed $n_s$ and the rotating speed of the rotor $n_r$, express in percentage of the first one, is called slip $s$, as seen in (2.2). Since the stator is powered by three electric currents which periodically reverses direction with $T = 1/f$, the electric angular speed of the stator is $\omega_s = 2\pi f$.

$$s = \frac{\omega_r}{\omega_s}$$  \hspace{1cm} (2.2)

where the electric angular speed of the rotor, $\omega_r$ is

$$\omega_r = \omega_s - p\omega_{mec}$$  \hspace{1cm} (2.3)
And finally the synchronous speed

\[
\omega_{sinc} = \frac{60}{2\pi} \frac{\omega_s}{p} = \frac{60}{p} \left[ \frac{f}{p} \right] \quad [\text{r.p.m}]
\] (2.4)

![Figure 2.4: Equivalent circuit of induction machine in steady state regime.](image)

The values presented in the Table 2.1 are the equivalent circuit parameters for the Fiat's induction motor. They were obtained using two different approaches. In the first column, the values were obtained using the auto-tuning capability of the Siemens inverter, in which the program itself determines the following values when the motor is in operation. The values in the second column were obtained experimentally by the conventional locked rotor and no load test. Even though both values are generically similar, there are a significant discrepancy between fluxes leakage values both rotor and the stator. Regarding the chosen values, it was chosen to use the ones obtained by the inverter, since they are what the inverter "sees".

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Physical Interpretation</th>
<th>Inverter Values</th>
<th>Experimental Values - source [7]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_s )</td>
<td>Stator copper losses</td>
<td>9.679 mΩ</td>
<td>6.196 mΩ</td>
</tr>
<tr>
<td>( l_s )</td>
<td>Stator flux leakage</td>
<td>95 µH</td>
<td>14 µH</td>
</tr>
<tr>
<td>( R_c )</td>
<td>Iron losses</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( M )</td>
<td>Linking flux</td>
<td>1.461 mH</td>
<td>1.252 mH</td>
</tr>
<tr>
<td>( R_r )</td>
<td>Rotor copper losses</td>
<td>10.754 mΩ</td>
<td>6.196 mΩ</td>
</tr>
<tr>
<td>( l_r )</td>
<td>Rotor flux leakage</td>
<td>86 µH</td>
<td>14 µH</td>
</tr>
<tr>
<td>( p )</td>
<td>Number of pair of poles</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Scalar command (constant V/f command)**

In a practical sense, when driving, there are a wide range of speeds that you want your car to operate, from the lowest speeds, for maneuvers, to high speeds for driving in highways. In this way, conventional combustion engines use the gearbox to adjust this wide range of speeds to a narrower range of rotations within the stable and, expectedly, most efficient speed-torque region of the ICE. On the other hand, with an electric motor, the speed command is achieved in a different manner. Specifically, for an induction
machine, the way to do so is to use scalar command, also called the $V/f$ command. When using an induction motor.

In order to vary the speed operation of an induction machine, it is necessary to vary the synchronous speed $n_s$ and with (2.4), it is clear that it can be done by changing the frequency of the applied voltage. However, the voltage at each frequency must be adjusted so that the ratio $V/f$ is kept constant up to base speed, which maintains the flux of the machine constant and equal to its nominal value. These are therefore good open-loop characteristics, because the speed is held fairly well from no load until full load operation.

To understand why the ratio $V/f$ must be held constant, consider the following: in the stator we have

$$V_s = R_s I_s + \frac{d\phi}{dt} \quad (2.5)$$

where $V_s$ is the stator voltage, $R_s$ and $I_s$ are the stator winding resistor and stator current, respectively and $\phi$ is the magnetic flux. Neglecting the voltage drop in the stator and expressing the voltage in a phasor quantity (which is possible since it is in steady-state regime), we have

$$|\phi| \propto \frac{|V_s|}{f} \quad (2.6)$$

This means that, as long as the $V/f$ ratios is kept constant, the magnitude of the magnetic flux $\phi$ remains constant. This maintains the motor in the same operating point, maintaining the torque-current relation the same, regardless of the input electric frequency.

For low frequency region it is not possible to neglect the voltage drop in the stator, since the stator voltage and resistance drop are in the same order. To overcome this, the $V/f$ ratio can be increased when at low frequencies in order to maintain the same flux magnitude.

Using a $V/f$ command, the speed-torque map of the induction motor can be made, which is shown in Figure 2.5. This can be grouped in three different regions. The first region, called ‘constant torque region’, which means that for frequencies up to base speed, the maximum possible torque which the motor can deliver is independent of the set speed. After this region, there is a ‘constant power region’, where the flux is reduced and the motor has to operate with higher slips, due to the reduction of flux. In higher speeds it is sometimes possible to operate, but the power produced by the motor starts to decrease because the rotor current will be decreased, and also the input power will decrease, as shown in the figure 2.6.

<table>
<thead>
<tr>
<th>Table 2.2: Motor’s Nameplate.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s$ [Hz]</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>76</td>
</tr>
<tr>
<td>220</td>
</tr>
<tr>
<td>305</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\omega_n$</th>
<th>$T_n$</th>
<th>Rated $P$</th>
<th>Max $P$</th>
<th>Max $\omega$</th>
<th>Max $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2200 rpm</td>
<td>65 N.m</td>
<td>15 kW</td>
<td>30 kW</td>
<td>10k rpm</td>
<td>130 N.m</td>
</tr>
</tbody>
</table>
Finally and in order to produce the three distinct regions presented in the figure 2.5, the applied stator voltage and frequency ratio $V/f$, should have the following graphical representation as shown in figure 2.7.
Figure 2.7: V/f curve.

2.1.2 Dynamic operating regime

Natural Reference Modeling abc

For a three-phase induction machine, the stator can be depicted as being composed of three-windings, one for each phase, which have the following electrical variables:

\[
[V_s] = \begin{bmatrix} v_a^s \\ v_b^s \\ v_c^s \end{bmatrix}, \quad [I_s] = \begin{bmatrix} I_a^s \\ I_b^s \\ I_c^s \end{bmatrix}, \quad [\phi_s] = \begin{bmatrix} \phi_a^s \\ \phi_b^s \\ \phi_c^s \end{bmatrix}
\] (2.7)

Referring to the rotor, we can describe the same quantities in the same way:

\[
[V_r] = \begin{bmatrix} v_a^r \\ v_b^r \\ v_c^r \end{bmatrix}, \quad [I_r] = \begin{bmatrix} I_a^r \\ I_b^r \\ I_c^r \end{bmatrix}, \quad [\phi_r] = \begin{bmatrix} \phi_a^r \\ \phi_b^r \\ \phi_c^r \end{bmatrix}
\] (2.8)

Due to the similarity of stator and rotor windings, their resistances are

\[
[R_s] = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}, \quad [R_r] = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix}
\] (2.9)

and, since the losses in the iron of the machine are negligible, the following relationship can be written between the voltage applied to the winding and the flux in the stator and the rotor respectively:

\[
\begin{align*}
[V_s] &= [R_s][I_s] + \frac{d[\phi_s]}{dt} \\
[V_r] &= [R_r][I_r] + \frac{d[\phi_r]}{dt}
\end{align*}
\] (2.10)
Assuming that the motor is operating in the linear region, the flux $\phi$ is proportional to the stator and rotor currents, being the proportionality constants the self and mutual inductances of the stator and rotor windings, and between the stator and rotor windings, respectively. The self induction of each stator and rotor winding can be decomposed into two variables: the main induction $(l_p)$, that makes the link between the stator and the rotor windings, and the induction that ends up getting lost in the path and does not perform any interaction between the two circuits $(l_{s\sigma})$.

\[
\begin{align*}
    l_s &= l_{sp} + l_{s\sigma} \\
    l_r &= l_{rp} + l_{s\sigma}
\end{align*}
\]  

(2.11)

Since magnetic saturation is not considered, the flux becomes an amount linearly dependent on the different currents of the system. However, it is important to remember that we are in the presence of 6 electrical circuits, 3 of which are in the stator and the remaining 3 are in the rotor that is movable relative to the stator:

$$\phi^a_s = l_s i^a_s + M_s i^b_s + M_s i^c_s + M_{sr} \left( i^a_r \cos(\theta) + i^b_r \cos \left( \theta - \frac{2\pi}{3} \right) + i^c_r \cos \left( \theta - \frac{4\pi}{3} \right) \right)$$  

(2.12)

The mutual inductance between any two phases in the stator, displaced $120^\circ$, can be written as follows:

$$M_s = l_{sp} \cos \left( \frac{2\pi}{3} \right) = -\frac{1}{2} l_{sp}$$  

(2.13)

Relatively to the inductance between the stator and the rotor, a constant $M_{sr}$, which represents the maximum inductance sensed between a rotor element and a stator element can be deferred. This situation is verified when the axes of each winding are aligned due to the rotor movement:

\[
\begin{align*}
    M_{sr} \cos \left( \vec{e}_{sa}, \vec{e}_{ra} \right) &= M_{sr} \cos(\theta) \\
    M_{sr} \cos \left( \vec{e}_{sa}, \vec{e}_{rb} \right) &= M_{sr} \cos \left( \theta - \frac{2\pi}{3} \right) = M_{sr} \cos \left( \theta + \frac{4\pi}{3} \right) \\
    M_{sr} \cos \left( \vec{e}_{sa}, \vec{e}_{rc} \right) &= M_{sr} \cos \left( \theta - \frac{4\pi}{3} \right) = M_{sr} \cos \left( \theta + \frac{2\pi}{3} \right)
\end{align*}
\]  

(2.14)

where $\theta$ is the electric angle between the rotor phase and its homonym in the stator.

Thus, the equations of the flux in each phase in the stator is obtained:

\[
\begin{pmatrix}
    \phi^a_s \\
    \phi^b_s \\
    \phi^c_s
\end{pmatrix} =
\begin{pmatrix}
    l_s & M_s & M_s \\
    M_s & l_s & M_s \\
    M_s & M_s & l_s
\end{pmatrix}
\begin{pmatrix}
    i^a_s \\
    i^b_s \\
    i^c_s
\end{pmatrix}
+ M_{sr} [R(\theta)]
\begin{pmatrix}
    i^a_r \\
    i^b_r \\
    i^c_r
\end{pmatrix}
\]  

(2.15)

where $[R(\theta)]$ is the rotation matrix of the rotor that relates the quantities in the rotor with the quantities
in the stator:

\[
[R(\theta)] = \begin{bmatrix}
\cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\
\cos(\theta - \frac{2\pi}{3}) & \cos(\theta) & \cos(\theta - \frac{4\pi}{3}) \\
\cos(\theta - \frac{4\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta)
\end{bmatrix}
\]  
(2.16)

The mutual induction between two elements of the rotor, displaced 120°, can be defined by the following equation, as it turned out in the stator circuit:

\[
M_r = l_{rp} \cos \left( \frac{2\pi}{3} \right) = -\frac{1}{2} l_{rp}
\]  
(2.17)

In the rotor, we have the following equations to determine the flux in each phase

\[
\begin{bmatrix}
\phi_a^r \\
\phi_b^r \\
\phi_c^r
\end{bmatrix} =
\begin{bmatrix}
l_r & M_r & M_r \\
M_r & l_r & M_r \\
M_r & M_r & l_r
\end{bmatrix}
\begin{bmatrix}
i_a^r \\
i_b^r \\
i_c^r
\end{bmatrix}
+ M_{sr}[R(\theta)]^T
\begin{bmatrix}
i_a^r \\
i_b^r \\
i_c^r
\end{bmatrix}
\] 
(2.18)

However, since we are dealing with a balanced three-phase system, we can describe the third current in a combination of the first two

\[
\begin{cases}
i_a^s + i_b^s + i_c^s = 0 \\
i_a^r + i_b^r + i_c^r = 0
\end{cases}
\] 
(2.19)

obtaining the following flux equation:

\[
\phi_s^a = i_a^s (l_r - M_s) + M_{sr} \left[ i_a^r \cos \theta + i_b^r \cos \left( \theta - \frac{2\pi}{3} \right) + i_c^r \cos \left( \theta - \frac{4\pi}{3} \right) \right]
\] 
(2.20)

With

\[
\begin{cases}
L_s = l_s - M_s = \frac{3}{2} l_{sp} + l_s \sigma \\
L_r = l_r - M_r = \frac{3}{2} l_{rp} + l_r \sigma
\end{cases}
\] 
(2.21)

one can finally write the expressions for the flux of the stator and rotor

\[
\begin{bmatrix}
\phi_s^a \\
\phi_s^b \\
\phi_s^c
\end{bmatrix} =
\begin{bmatrix}
L_s & 0 & 0 \\
0 & L_s & 0 \\
0 & 0 & L_s
\end{bmatrix}
\begin{bmatrix}
i_a^s \\
i_b^s \\
i_c^s
\end{bmatrix}
+ M_{sr}[R(\theta)]
\begin{bmatrix}
i_a^r \\
i_b^r \\
i_c^r
\end{bmatrix}
\] 
(2.22)

\[
\begin{bmatrix}
\phi_r^a \\
\phi_r^b \\
\phi_r^c
\end{bmatrix} =
\begin{bmatrix}
L_r & 0 & 0 \\
0 & L_r & 0 \\
0 & 0 & L_r
\end{bmatrix}
\begin{bmatrix}
i_a^r \\
i_b^r \\
i_c^r
\end{bmatrix}
+ M_{sr}[R(\theta)]^T
\begin{bmatrix}
i_a^s \\
i_b^s \\
i_c^s
\end{bmatrix}
\] 
(2.23)
and it is also possible to write the flux, function of the currents of the system

\[
\begin{bmatrix}
\phi_s \\
\phi_r
\end{bmatrix} =
\begin{bmatrix}
L_s & [M_{sr}(\theta)] \\
[M_{sr}(\theta)^T] & L_r
\end{bmatrix}
\begin{bmatrix}
I_s \\
I_r
\end{bmatrix}
\]  \tag{2.24}

with

\[
L_s =
\begin{bmatrix}
L_s & 0 & 0 \\
0 & L_s & 0 \\
0 & 0 & L_s
\end{bmatrix} ;
L_r =
\begin{bmatrix}
L_r & 0 & 0 \\
0 & L_r & 0 \\
0 & 0 & L_r
\end{bmatrix} ;
[M_{sr}(\theta)] = M_{sr}[R(\theta)]
\]

Now that the expressions for the flux are made, (2.10) can be written as

\[
\begin{bmatrix}
[V_s] \\
[V_r]
\end{bmatrix} =
\begin{bmatrix}
[R_s] & 0 \\
0 & [R_r]
\end{bmatrix}
\begin{bmatrix}
I_s \\
I_r
\end{bmatrix} +
\frac{d}{dt}
\begin{bmatrix}
[L_s] & [M_{sr}(\theta)] \\
[M_{sr}(\theta)^T] & [L_r]
\end{bmatrix}
\begin{bmatrix}
I_s \\
I_r
\end{bmatrix}
\]  \tag{2.25}

For the calculation of the torque, one must go for the energy balance equations. For an electromechanical system, if neglecting iron losses, we have the following:

\[
\frac{d}{dt} W_{mec} = \frac{d}{dt} W_{ele} - \frac{d}{dt} W_{mag}
\]  \tag{2.26}

In which the variation of electromagnetic energy \( dW_{ele} / dt = [I]^T [V] \) can be written as follow:

\[
\frac{d}{dt} W_{ele} = [I_s]^T [L_s] \frac{d}{dt} [I_s] + [I_s]^T M_{sr} \frac{d}{dt} ([R(\theta)] [I_s]) + [I_r]^T [L_r] \frac{d}{dt} [I_r] + [I_r]^T M_{sr} \frac{d}{dt} ([R(\theta)] [I_s])
\]  \tag{2.27}

After the product of the derivatives has been developed, the following expression can be obtained:

\[
\frac{dW_{ele}}{dt} = [I_s]^T [L_s] \frac{d[I_s]}{dt} + [I_s]^T [L_r] \frac{d[I_r]}{dt} + [I_s]^T M_{sr} \frac{d[R(\theta)]}{dt} [I_s] + M_{sr}[I_s]^T [R(\theta)] \frac{d[I_s]}{dt} + [I_r]^T M_{sr} \frac{d[R(\theta)]}{dt} [I_s] + M_{sr}[I_r]^T [R(\theta)] \frac{d[I_s]}{dt}
\]  \tag{2.28}

where:

\[
\frac{d[R(\theta)]}{dt} = \frac{d\theta}{dt}
\begin{bmatrix}
\sin \theta & \sin (\theta - \frac{4\pi}{3}) & \sin (\theta - \frac{2\pi}{3}) \\
\sin (\theta - \frac{2\pi}{3}) & \sin \theta & \sin (\theta - \frac{4\pi}{3})
\end{bmatrix}
\]

\[
\frac{d[R(\theta)]}{dt} = \frac{d\theta}{dt}
\begin{bmatrix}
\sin \theta & \sin (\theta - \frac{4\pi}{3}) & \sin (\theta - \frac{2\pi}{3}) \\
\sin (\theta - \frac{2\pi}{3}) & \sin \theta & \sin (\theta - \frac{4\pi}{3})
\end{bmatrix}
\]  \tag{2.29}

When there is no variation of the angular position, we have the following expression that quantifies the magnetic energy of the system:

\[
W_{mag} = \frac{1}{2} [I_s]^T [L_s] [I_s] + \frac{1}{2} [I_r]^T [L_r] [I_r] + [I_s] M_{sr} \frac{d[R(\theta)]}{dt}^T [I_r]
\]  \tag{2.30}

Deriving the previous expression, the following expression is obtained:
\[
\frac{dW_{mag}}{dt} = [I_s]^T[I_s] \frac{d}{dt}[I_s] + [I_r]^T[I_r] \frac{d[I_r]}{dt} + \frac{d[I_s]^T}{dt} \text{M}_{sr}[R(\theta)][I_R] + [I_s]^T \text{M}_{sr} \frac{d[R(\theta)]}{dt}[I_r] + [I_s]^T \text{M}_{sr}[R(\theta)] \frac{d[I_r]}{dt}
\] 

Equation (2.31)

Then, using the expression that relates the energy balance (2.26) along with the two equations (2.28) and (2.31), the following relation is obtained:

\[
\frac{dW_{mec}}{dt} = [I_s]^T \text{M}_{sr} \frac{d[R(\theta)]}{dt}[I_r]
\]

Equation (2.32)

Finally, it is known that the mechanical power produced is described by the following expression:

\[
\frac{dW_{mec}}{dt} = T \frac{d\theta}{dt}
\]

Equation (2.33)

in which it is possible to determine the equation that quantifies the value of the torque generated by the machine, using equations (2.32), (2.33) and (2.29), where \( p \) is the number of pairs of poles.

\[
T = -p[I_s]^T \text{M}_{sr} \frac{d[R(\theta)]}{dt}[I_r]
\]

Equation (2.34)
Two Axis Stationary Reference Modeling $\alpha\beta$

In order to simplify the mathematical resolution of the system, we will now introduce a mathematical transformation called Clarke Transformation. The present transformation makes possible to reduce the number of state variables, since the present system is a balanced three-phase system. The natural \( abc \) coordinates are three space vectors in which the third one results in the algebraic composition of the first two, that is, only two of them are linearly independent. In this way, the same system can be described by using new orthogonal axes $\alpha\beta$, thus dismissing the need of a third axis, since this system is balanced. Although there are three current values, its sum must be zero, since there is no return. As already mentioned, the new pair of coordinates is orthogonal and by definition aligned with axis $\vec{e}_{sa}$ in the stator, that is, $\vec{X} = X_{a}\vec{e}_{sa} + X_{b}\vec{e}_{sb} + X_{c}\vec{e}_{sc} = X_{\alpha}\vec{e}_{s\alpha} + X_{\beta}\vec{e}_{s\beta}$.

![Figure 2.8: Vector representation.](image)

Thus, to change from the standard \( abc \) reference to the new $\alpha\beta$ reference, the \( abc \) values must be multiplied by a base change matrix $[C_{\alpha\beta}]$, this transformation is known as Concordia transformation.

$$
\begin{align*}
[X_{\alpha\beta\gamma}] &= [C_{\alpha\beta}] [X_{abc}] \\
[X_{abc}] &= [C_{\alpha\beta}]^T [X_{\alpha\beta\gamma}] 
\end{align*}
$$

(2.35)

Where

$$
[C_{\alpha\beta}] = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
$$

(2.36)

and

$$
[C_{\alpha\beta}]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & 0 & \frac{1}{\sqrt{3}} \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix} = [C_{\alpha\beta}]^T
$$

(2.37)

It is important to note that the three stator quantities $\vec{V}_s$, $\vec{I}_s$, $\vec{\phi}_s$ are rotating fields which circulate with electric frequency, this means, a complete rotation of the field corresponds to a complete period of one of the waves of the voltage, where it is obtained that $\omega_s = 2\pi f$ rad/s.

On the other hand, in the rotor the quantities $\vec{V}_r$, $\vec{I}_r$, $\vec{\phi}_r$, are also rotating, but with an angular speed
of $\omega_r$. This is due to the fact that the rotor is in motion and consequently the quantities that are induced by it correspond to the difference between its own speed and the speed of the field that it sees:

$$\omega_r = \omega_s - p\omega_{inc}$$  \hspace{1cm} (2.38)

Applying the basic transformations to the equations that relates the flux and the current of the machine (2.25):

$$[C_{\alpha\beta}]^{-1} \begin{bmatrix} \phi_s^{\alpha\beta\gamma} \\ \phi_r^{\alpha\beta\gamma} \end{bmatrix} = \begin{bmatrix} [L_s] & [M_{sr}(\theta)] \end{bmatrix} [C_{\alpha\beta}]^{-1} \begin{bmatrix} I_s^{\alpha\beta\gamma} \\ I_r^{\alpha\beta\gamma} \end{bmatrix}$$  \hspace{1cm} (2.39)

As seen previously, $[C_{\alpha\beta}]^{-1} = [C_{\alpha\beta}]^T$

$$[C_{\alpha\beta}]^T \begin{bmatrix} \phi_s^{\alpha\beta\gamma} \\ \phi_r^{\alpha\beta\gamma} \end{bmatrix} = \begin{bmatrix} [L_s] & [M_{sr}(\theta)] \end{bmatrix}^T [C_{\alpha\beta}]^T \begin{bmatrix} I_s^{\alpha\beta\gamma} \\ I_r^{\alpha\beta\gamma} \end{bmatrix}$$  \hspace{1cm} (2.40)

Hence:

$$\begin{bmatrix} \phi_s^{\alpha\beta\gamma} \\ \phi_r^{\alpha\beta\gamma} \end{bmatrix} = \begin{bmatrix} [C_{\alpha\beta}]^T[L_s][C_{\alpha\beta}]^T & [C_{\alpha\beta}]^T[M_{sr}(\theta)][C_{\alpha\beta}]^T \\ [C_{\alpha\beta}]^T[M_{sr}(\theta)]^T[C_{\alpha\beta}]^T & [C_{\alpha\beta}]^T[L_r][C_{\alpha\beta}]^T \end{bmatrix} [C_{\alpha\beta}]^{-1} \begin{bmatrix} I_s^{\alpha\beta\gamma} \\ I_r^{\alpha\beta\gamma} \end{bmatrix}$$  \hspace{1cm} (2.41)

Where:

$$[L_s^c] = \begin{bmatrix} l_s - M_s & 0 & 0 \\ 0 & l_s - M_s & 0 \\ 0 & 0 & l_s + 2M_s \end{bmatrix} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \end{bmatrix}$$  \hspace{1cm} (2.43)

$$[L_r^c] = \begin{bmatrix} l_r - M_r & 0 & 0 \\ 0 & l_r - M_r & 0 \\ 0 & 0 & l_r + 2M_r \end{bmatrix} = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \end{bmatrix}$$  \hspace{1cm} (2.44)

$$[M_{sr}(\theta)] = \frac{3}{2} M_{sr}[R_c(\theta)]$$  \hspace{1cm} (2.45)
\[
[R_c(\theta)] = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 0
\end{bmatrix}
\] (2.46)

Where:
- Cyclic stator inductance \( \equiv L_s = l_s - M_s; \)
- Cyclic rotor inductance \( \equiv L_r = l_r - M_r; \)
- Mutual cyclic inductance between a stator phase and a rotor phase, when they are aligned \( \equiv M = \frac{3}{2}M_{sr}; \)
- Homopolar inductance for stator or rotor \( L_{x\gamma} = l_{x\gamma} + 2M_{x\gamma}. \)

Applying the bases transformation to the differential equations (2.10) that describe the variation of the flux according to the state variables:

\[
\begin{align*}
[C_{\alpha\beta}]^T \frac{d}{dt} [\phi_{s}^{\alpha\beta\gamma}] &= [C_{\alpha\beta}]^T [V_{s}^{\alpha\beta\gamma}] - [R_{s}] [C_{\alpha\beta}]^T [I_{s}^{\alpha\beta\gamma}] \\
[C_{\alpha\beta}]^T \frac{d}{dt} [\phi_{r}^{\alpha\beta\gamma}] &= [C_{\alpha\beta}]^T [V_{r}^{\alpha\beta\gamma}] - [R_{r}] [C_{\alpha\beta}]^T [I_{r}^{\alpha\beta\gamma}]
\end{align*}
\] (2.47)

Stator:
\[
\begin{align*}
\frac{d\phi_{s}^{\alpha}}{dt} &= v_{s}^{\alpha} - R_{s}i_{s}^{\alpha} \\
\frac{d\phi_{s}^{\beta}}{dt} &= v_{s}^{\beta} - R_{s}i_{s}^{\beta} \\
\frac{d\phi_{s}^{\gamma}}{dt} &= v_{s}^{\gamma} - R_{s}i_{s}^{\gamma}
\end{align*}
\] (2.48)

Rotor:
\[
\begin{align*}
\frac{d\phi_{r}^{\alpha}}{dt} &= v_{r}^{\alpha} - R_{r}i_{r}^{\alpha} \\
\frac{d\phi_{r}^{\beta}}{dt} &= v_{r}^{\beta} - R_{r}i_{r}^{\beta} \\
\frac{d\phi_{r}^{\gamma}}{dt} &= v_{r}^{\gamma} - R_{r}i_{r}^{\gamma}
\end{align*}
\] (2.49)

Likewise, using the equation of the torque (2.34) in natural coordinates and applying the bases transformation:

\[
T = p \left( [C_{\alpha\beta}]^T [I_{s}^{\alpha\beta\gamma}] \right) \frac{d}{d\theta} M_{sr}(\theta) \left( [C_{\alpha\beta}]^T [I_{r}^{\alpha\beta\gamma}] \right)
\] (2.50)

\[
T = p \frac{3}{2} [I_{s}^{\alpha\beta\gamma}]^T M_{sr} \frac{d}{d\theta} [R_{c}(\theta)] [I_{r}^{\alpha\beta\gamma}]
\] (2.51)

Where:
\[
\frac{d}{d\theta} [R_{c}(\theta)] \equiv \frac{d}{d\theta} [R_{c}(\theta)] = \begin{bmatrix}
\sin \theta & \cos \theta & 0 \\
-\cos \theta & \sin \theta & 0 \\
0 & 0 & 0
\end{bmatrix}
\] (2.52)
\[ M = \frac{3}{2} M_{sr} \] (2.53)

Where we can imply the following expression for the value of the torque produced:

\[ T = pM \left[ \left( i_\alpha^2 i_\alpha^2 - i_\beta^2 i_\beta^2 \right) \cos \theta + \left( i_\alpha^\beta i_\alpha^\beta + i_\beta^\beta i_\beta^\beta \right) \sin \theta \right] \] (2.54)
Modelling in a non-stationary two axis reference (dqo)

In order to further simplify the calculations, the Park transformation will be applied, and this new axes system turns out to be a general case of the $\alpha\beta\gamma$ axes system, since in the previous case the axes were stationary and now they will be animated by a rotation movement. In this way, the three alternating sinusoidal voltages applied to the stator will be seen as three DC voltages of amplitude proportional to their effective value (when the rotation speed of the axes is equal to the frequency of the voltages).

In order to study the system using this new coordinate system, we have to use a rotation matrix $[R(\psi)]$, responsible for converting the variables described in ($\alpha\beta\gamma$) to the new coordinates in ($dqo$), where the angle $\psi$ varies according to time.

$$[R_p(\psi)] = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.55)$$

The product of the two basis transformation matrices (Concordia and Rotation) define the Park transformation, whose aim is to transport the electric quantities in the two different references (stator and rotor) to the same reference.

$$\begin{cases} [X^{dqo}] = [R_p(\psi)][X^{\alpha\beta\gamma}] \\ [X^{\alpha\beta\gamma}] = [C_{\alpha\beta}][X^{abc}] \end{cases} \quad (2.56)$$

The Park transformation is expressed by the following matrix product:

$$[X^{dqo}] = [P(\psi)][X^{abc}] \quad (2.57)$$

Where:

$$[P(\psi)] = [R_p(\psi)][C_{\alpha\beta}] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \psi & \cos \left(\psi - \frac{2\pi}{3}\right) & \cos \left(\psi - \frac{4\pi}{3}\right) \\ -\sin \psi & -\sin \left(\psi - \frac{2\pi}{3}\right) & -\sin \left(\psi - \frac{4\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (2.58)$$

$$[P(\psi)]^{-1} = [P(\psi)]^T \quad (2.59)$$

Applying the transformation the equations of state variation of the system, stator and rotor respectively:

$$\begin{cases} \frac{d}{dt}([P(\theta_s)][\phi_s^{dqo}]) = [P(\theta_s)]^T[V_s^{dqo}] - [R_s][P(\theta_s)]^T[I_s^{dqo}] \\ \frac{d}{dt}([P(\theta_r)][\phi_r^{dqo}]) = [P(\theta_r)]^T[V_r^{dqo}] - [R_r][P(\theta_r)]^T[I_r^{dqo}] \end{cases} \quad (2.60)$$

Simplifying the mathematical equations:
\[
\begin{align*}
\mathbf{P}(\theta_s)^T \frac{d}{dt} ([P(\theta_s)]^T \phi_s^{dqo}) + \mathbf{\phi}_s^{dqo} &= \mathbf{V}_s^{dqo} - [R_s] \mathbf{I}_s^{dqo} \\
\mathbf{P}(\theta_r)^T \frac{d}{dt} ([P(\theta_r)]^T \phi_r^{dqo}) + \mathbf{\phi}_r^{dqo} &= \mathbf{V}_r^{dqo} - [R_r] \mathbf{I}_r^{dqo}
\end{align*}
\] (2.61)

Where:

\[
\mathbf{P}(\psi) \frac{d}{dt} ([P(\psi)]^T) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \psi
\] (2.62)

Writing the differential equations that govern the system in coordinates dqo:

Stator:

\[
\begin{align*}
\frac{d}{dt} \mathbf{\phi}_s^{dqo} &= \mathbf{v}_s^{dqo} - [R_s] \mathbf{i}_s^{dqo} + \omega_s \mathbf{\phi}_s^{dqo} \\
\frac{d}{dt} \mathbf{\phi}_r^{dqo} &= \mathbf{v}_r^{dqo} - [R_r] \mathbf{i}_r^{dqo} - \omega_r \mathbf{\phi}_r^{dqo}
\end{align*}
\] (2.63)

Rotor:

\[
\begin{align*}
\frac{d}{dt} \mathbf{\phi}_s^{dqo} &= \mathbf{v}_s^{dqo} - [R_s] \mathbf{i}_s^{dqo} + \omega_s \mathbf{\phi}_s^{dqo} \\
\frac{d}{dt} \mathbf{\phi}_r^{dqo} &= \mathbf{v}_r^{dqo} - [R_r] \mathbf{i}_r^{dqo} - \omega_r \mathbf{\phi}_r^{dqo}
\end{align*}
\] (2.64)

Determine the relationship between the currents of the system and the flux, using equation (2.25):

\[
\begin{bmatrix} [P(\theta_s)]^T \mathbf{\phi}_s^{dqo} \\ [P(\theta_r)]^T \mathbf{\phi}_r^{dqo} \end{bmatrix} = \begin{bmatrix} [L_s] & [M_{sr}(\theta)] \\ [M_{sr}(\theta)]^T & [L_r] \end{bmatrix} \begin{bmatrix} [P(\theta_s)]^T \mathbf{I}_s^{dqo} \\ [P(\theta_r)]^T \mathbf{I}_r^{dqo} \end{bmatrix}
\] (2.65)

\[
\begin{bmatrix} \mathbf{\phi}_s^{dqo} \\ \mathbf{\phi}_r^{dqo} \end{bmatrix} = \begin{bmatrix} [P(\theta_s)] [L_s] [P(\theta_s)]^T \\ [P(\theta_r)] [M_{sr}(\theta)]^T [P(\theta_r)]^T \\ [P(\theta_s)] [L_r] [P(\theta_s)]^T \\ [P(\theta_r)] [L_r] [P(\theta_r)]^T \end{bmatrix} \begin{bmatrix} \mathbf{I}_s^{dqo} \\ \mathbf{I}_r^{dqo} \end{bmatrix}
\] (2.66)

\[
\begin{bmatrix} \mathbf{\phi}_s^{dqo} \\ \mathbf{\phi}_r^{dqo} \end{bmatrix} = \begin{bmatrix} [L_s^p] & [M_{sp}(\theta)] \\ [M_{sp}(\theta)]^T & [L_s^o] \end{bmatrix} \begin{bmatrix} \mathbf{I}_s^{dqo} \\ \mathbf{I}_r^{dqo} \end{bmatrix}
\] (2.67)

Where:

\[
[L_s^p] = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{so} \end{bmatrix}
\] (2.68)
\[ [L_r^p] = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_{ro} \end{bmatrix} \]  
(2.69)

\[ [M_{sr}^p] = [P(\theta_s)]^T [M_{sr}(\theta)] [P(\theta_r)] = \frac{3}{2} M_{sr} [R_r] \]  
(2.70)

\[ [R_r] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]  
(2.71)

Since \( M = \frac{3}{2} M_{sr} \) the equation can be written as:

\[ \begin{bmatrix} \phi_s^d \\ \phi_r^d \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix} \begin{bmatrix} i_s^d \\ i_r^d \end{bmatrix} \]  
(2.72)

\[ \begin{bmatrix} \phi_s^q \\ \phi_r^q \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix} \begin{bmatrix} i_s^q \\ i_r^q \end{bmatrix} \]  
(2.73)

And on the other hand, it is known than:

\[ \begin{bmatrix} i_s^d \\ i_r^d \end{bmatrix} = \frac{1}{L_r L_s - M^2} \begin{bmatrix} L_r & -M \\ -M & L_s \end{bmatrix} \begin{bmatrix} \phi_s^d \\ \phi_r^d \end{bmatrix} \]  
(2.74)

\[ \begin{bmatrix} i_s^q \\ i_r^q \end{bmatrix} = \frac{1}{L_r L_s - M^2} \begin{bmatrix} L_r & -M \\ -M & L_s \end{bmatrix} \begin{bmatrix} \phi_s^q \\ \phi_r^q \end{bmatrix} \]  
(2.75)

To compute the torque, (2.34):

\[ T = p \left( [P(\theta_s)]^T [I_{dqo}^s] \right)^T \frac{dM_{sr}(\theta)}{d\theta} [P(\theta_r)]^T [I_{dqo}^r] \]  
(2.76)

Simplifying the mathematical equation using the equation (2.70):

\[ T = p [I_{dqo}^s]^T \frac{3}{2} M_{sr} \frac{R_p}{d\theta} [I_{dqo}^r] \]  
(2.77)

\[ T = p M (i_s^q i_r^d - i_s^d i_r^q) \]  
(2.78)
Breaking apart the components of the torque in dq:

\[ T = T_r^q + T_r^d \]  

(2.79)

Where:

\[ T_r^q = \frac{pM}{L_r} i_q^d \phi_r^d \]  

(2.80)

\[ T_r^d = -\frac{M}{L_r} i_d^q \phi_r^q \]  

(2.81)

Using the state equations for the stator (2.63) and rotor (2.87) of the machine, and since the rotor windings are in short circuit, there is no voltage applied to them.

\[
\begin{align*}
\frac{dv_d^s}{dt} &= R_s i_d^s - \omega_s \phi_d^s + \frac{d\phi_d^d}{dt} \\
\frac{dv_q^s}{dt} &= R_s i_q^s + \omega_s \phi_q^s + \frac{d\phi_q^d}{dt} \\
0 &= R_r i_d^r - \omega_r \phi_d^r + \frac{d\phi_d^r}{dt} \\
0 &= R_r i_q^r + \omega_r \phi_q^r + \frac{d\phi_q^r}{dt} 
\end{align*}
\]  

(2.82)

Then, in order to obtain fluxes in the rotor and currents in the stator, it will be necessary to replace all the fluxes referring to the stator and currents referring to the rotor present in (2.83) for, equations that allows the transformation of currents into fluxes (2.72 and 2.73) and vice versa (2.74 and 2.75).

\[
\begin{align*}
\frac{dv_d^s}{dt} &= R_s i_d^s - \omega_s \left( \sigma L_s i_q^s + \frac{M}{L_r} \phi_r^d \right) + \left( \sigma L_s \frac{di_d^q}{dt} + \frac{M}{L_r} \frac{d\phi_d^d}{dt} \right) \\
\frac{dv_q^s}{dt} &= R_s i_q^s + \omega_s \left( \sigma L_s i_d^s + \frac{M}{L_r} \phi_r^d \right) + \left( \sigma L_s \frac{di_q^d}{dt} + \frac{M}{L_r} \frac{d\phi_q^d}{dt} \right) \\
0 &= R_r i_d^r - \omega_r \phi_d^r + \frac{d\phi_d^r}{dt} \\
0 &= R_r i_q^r + \omega_r \phi_q^r + \frac{d\phi_q^r}{dt} 
\end{align*}
\]  

(2.83)

\[ \sigma = 1 - \frac{M^2}{L_r L_s} \]  

(2.84)

\[ \omega_r = \omega_s - p\omega_{\text{mec}} \]  

(2.85)

Electrical differential equations of the system:

\[
\begin{bmatrix}
\frac{di_d^s}{dt} \\
\frac{di_q^s}{dt} \\
\frac{d\phi_d^d}{dt} \\
\frac{d\phi_q^d}{dt}
\end{bmatrix} =
\begin{bmatrix}
\frac{M^2 R_s + L_r^2 R_r}{\sigma L_s L_r} & \omega_s & \frac{MR_s}{\sigma L_s L_r} & \frac{M}{\sigma L_s L_r} p\omega_{\text{mec}} \\
-\omega_s & -\frac{M^2 R_s + L_r^2 R_r}{\sigma L_s L_r} & \frac{M}{\sigma L_s L_r} p\omega_{\text{mec}} \\
-\frac{M R_s}{L_r} & 0 & \frac{M R_r}{\sigma L_s L_r} p\omega_{\text{mec}} & \omega_s - p\omega_{\text{mec}} \\
0 & M \frac{R_r}{L_r} & -\omega_s + p\omega_{\text{mec}} & -\frac{R_r}{L_r}
\end{bmatrix}
\begin{bmatrix}
i_d^s \\
i_q^s \\
\phi_d^d \\
\phi_q^d
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & 0 & 0 \\
0 & \frac{1}{\sigma L_r} & 0 & 0 \\
0 & 0 & \frac{1}{\sigma L_s} & 0 \\
0 & 0 & 0 & \frac{1}{\sigma L_r}
\end{bmatrix}
\begin{bmatrix}
\frac{di_d^s}{dt} \\
\frac{di_q^s}{dt} \\
\frac{d\phi_d^d}{dt} \\
\frac{d\phi_q^d}{dt}
\end{bmatrix}
\]  

(2.86)

Finally, it is possible to describe the remaining system’s mechanical variables, such as speed and electric torque, using the following equations 2.87:
\[
\begin{aligned}
\frac{dw_{acc}}{dt} &= \frac{1}{J}(T_e - T_{load} - T_{losses}) \\
T_e &= \frac{3}{2} \frac{pM}{L_p} (\phi^d_i q^q_s - \phi^q_i q^d_s)
\end{aligned}
\] (2.87)
2.2 New motor parameters

In order to implement the observer, it was chosen to implement the model in a bench testing motor, before attending to do so in the car’s motor, and leaving that work still to be done in future projects. It was decided to do this intermediate step, because the car’s motor is a more complex and harder multi-system and consequently its a more complex system to control and test. Regarding this new induction motor, it is necessary to determine the electric parameters, as presented in the section 2.1, table 2.1, regarding the car’s motor. Only after determining the equivalent circuit parameters, it is possible to implement the observer in this motor.

No-Load test:

In this first test, a DC machine attached to the same shaft of the induction machine is used to drive the set of machines at the synchronous speed, so the slip seen by the induction machine would be exactly zero and no currents inducted in the rotor so in this test a DC machine was used attached to the same shaft, driven by other machine, it is possible to obtain the values of the equivalent inductance of the stator and mutual inductance as well as the loses dude to the current flowing through the stator wires, since the losses in the iron was assumed to be neglectable.

Locked rotor test:

Contrary to the previous test, in this one the slip must be equal to one, meaning the rotor is locked however this time, instead of applying the nominal voltage, it is going to be the nominal current. In these circumstances it is possible to neglect the magnetization branch and obtain the values of the equivalent circuit [9].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Load</th>
<th>Locked Rotor</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a[V]$</td>
<td>229.2</td>
<td>54.6</td>
<td>231.5</td>
</tr>
<tr>
<td>$V_b[V]$</td>
<td>228.2</td>
<td>52.5</td>
<td>230.7</td>
</tr>
<tr>
<td>$V_c[V]$</td>
<td>230.1</td>
<td>54.9</td>
<td>232.6</td>
</tr>
<tr>
<td>$I_a[A]$</td>
<td>2.99</td>
<td>4.66</td>
<td>3.94</td>
</tr>
<tr>
<td>$I_b[A]$</td>
<td>2.92</td>
<td>4.84</td>
<td>3.96</td>
</tr>
<tr>
<td>$I_c[A]$</td>
<td>2.96</td>
<td>4.79</td>
<td>4.02</td>
</tr>
<tr>
<td>$Power_a[W]$</td>
<td>-4</td>
<td>128</td>
<td>576</td>
</tr>
<tr>
<td>$Power_b[W]$</td>
<td>-4</td>
<td>132</td>
<td>596</td>
</tr>
<tr>
<td>$Power_c[W]$</td>
<td>16</td>
<td>133</td>
<td>601</td>
</tr>
<tr>
<td>$Reactive_a[var]$</td>
<td>691</td>
<td>219</td>
<td>710</td>
</tr>
<tr>
<td>$Reactive_b[var]$</td>
<td>666</td>
<td>216</td>
<td>695</td>
</tr>
<tr>
<td>$Reactive_c[var]$</td>
<td>679</td>
<td>226</td>
<td>720</td>
</tr>
<tr>
<td>freq.</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>RPM</td>
<td>1500</td>
<td>0</td>
<td>1453</td>
</tr>
</tbody>
</table>
Table 2.4: Induction Machine test, at 50Hz.

With the set of the three tests it is possible to to determine the value of each single component of the electric equivalent circuit, table 2.5, however the iron losses were assumed to be neglectable, so the model studied previously agrees, reason why the value of the resistance $R_m$ is equal to infinite.

Table 2.5: Induction Machine equivalent circuit parameters.

<table>
<thead>
<tr>
<th>Eq. Circuit</th>
<th>$R_s$</th>
<th>$l_s$</th>
<th>$R_m$</th>
<th>$M$</th>
<th>$R_r$</th>
<th>$l_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ind. Mach.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nº 17</td>
<td>3.34 Ω</td>
<td>15.7 mH</td>
<td>+∞</td>
<td>232 mH</td>
<td>2.45 Ω</td>
<td>15.7 mH</td>
</tr>
</tbody>
</table>
2.3 Observer

In order to design a speed observer, as represented in Figure 2.9, for the present induction motor, it is necessary to use the state space equations presented in the previous Section 2.1.2. With (2.86) it is possible to emulate the behavior of the system, when subjected to a certain input signal.

![Observer representation](image)

Figure 2.9: Observer representation.

With (2.86), it is possible to design a Luenberger observer [12] that estimates the value of motor’s electric state space variables and then, in a second part and using that values estimated previous, it is possible to estimate the mechanical speed of the motor. Hence, and in order to estimate this values, it is necessary to know the applied voltages to the motor, and the stator currents measured by the sensors. In these equations, the electric quantities are described in $dq_0$ coordinates, known as a rotating frame, which is a general case for a $\alpha\beta$ stationary frame. The error between the currents of the stator measured and their estimations will guide the observer, while providing estimations for all the variables, due to the mathematical representation of the system behavior.

For a continuous time linear system:

\[
\begin{aligned}
\frac{dx}{dt} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}
\]  

(2.88)
where

\[
A = \begin{bmatrix}
-\frac{M^2 R_s + L_s^2 R_s}{\sigma L_s L_r} & -\omega_s & \frac{M R_s}{\sigma L_s L_r} & \frac{M}{\sigma L_s L_r} p \omega_{mec} \\
-\frac{M^2 R_s + L_s^2 R_s}{\sigma L_s L_r} & 0 & \frac{M R_s}{\sigma L_s L_r} & \frac{M}{\sigma L_s L_r} p \omega_{mec} \\
M R_s & 0 & -\omega_s + p \omega_{mec} & -\frac{R_s}{L_r} \\
0 & M R_s & \omega_s - p \omega_{mec} & -\frac{R_s}{L_r}
\end{bmatrix}; \quad x = \begin{bmatrix}
i_\alpha \\
i_\beta \\
i_\phi \\
i_{\phi r}
\end{bmatrix}; \quad B = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 \\
0 & \frac{1}{\sigma L_s} \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

(2.89)

\[
y = \begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix}; \quad C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}; \quad D = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\]

(2.90)

and matrix \(A\) is computed using the \(\alpha\beta\) reference frame (\(\omega_s = 0\))

Accordingly, as represented in the Figure 2.9, the observer looks as:

\[
\frac{dx}{dt} = Ax + Bu + L(\hat{y} - y)
\]

\[
\hat{y} = Cx
\]

(2.91)

Where the superscript \(\hat{.}\) means the estimation of the physical quantity, and the \(L\) is the observer gain matrix, that must be carefully chosen so the system remains stable.

\[
\hat{x} = \begin{bmatrix}
i_\alpha \\
i_\beta \\
i_\phi \\
i_{\phi r}
\end{bmatrix}; \quad u_s = \begin{bmatrix}
v_\alpha \\
v_\beta \\
v_\phi \\
v_{\phi r}
\end{bmatrix}
\]

(2.92)

In order to estimate the motor's rotating speed, it is necessary to measure the stator currents, which are easily accessible and also the applied voltages values, which is our input signal. Then it is necessary to ensure that the present system is observable, meaning that any possible sequence of states can be determined, in finite time, using only the outputs. Consequently using the observability index \([O]\), it is possible to check whether the system is observable or not.

The definition for the observability is as follows: if the rank of \([O]\) is equal to the rank of the state matrix \(k = rank[A]\), then the system is observable. otherwise it is not.

\[
\begin{cases}
rank[O] = k, & \text{Observable} \\
rank[O] < k, & \text{Unobservable}
\end{cases}
\]

(2.93)
The matrix \( [O] \) is defined as

\[
[O] = \begin{bmatrix}
CA^0 \\
CA^1 \\
\vdots \\
CA^{k-1}
\end{bmatrix} \Rightarrow \begin{bmatrix}
C \\
CA^1 \\
\vdots \\
CA^3
\end{bmatrix}_{[8 \times 4]} \Rightarrow \text{rank}[O] = 4 \Rightarrow \text{Observable}
\] (2.94)

To sum up, and regarding the system observability, it is possible to determine any state space combination of the present system using only the stator currents.

### 2.3.1 Adaptive Scheme

Despite unknowing one of the parameters of the matrix \( A \) the mechanical speed \( \hat{\omega}_{mec} \), it is possible to estimate the states \( \hat{x} \) and also the mechanical speed all together. Accordingly, the adaptive scheme can be used, and the speed estimation, will be used to fill in the missing values of the matrix \( A \). This has been researched using the work in [1] and [6].

The observer is written as the following state space equation:

\[
\frac{d}{dt} \hat{x} = \hat{A}x + Bu_s + L(I_s - I_s)
\] (2.95)

Towards determining the values assigned to the matrix \( L \), derived by the adaptive scheme, it is necessary to use the Lyapunov’s Therorem. Writing down the expression of the estimation error of the states:

\[
e = x - \hat{x}
\] (2.96)

\[
\frac{d}{dt} e = \frac{d}{dt} x - \frac{d}{dt} \hat{x}
\]

\[
= Ax + Bu_s - \hat{A}x - B\hat{u}_s - L(I_s - I_s)
\]

\[
= Ax - \hat{A}x + LC(x - \hat{x})
\]

\[
= (A + LC)e - [\hat{A} - A]\hat{x}
\] (2.97)

where

\[
[\hat{A} - A] = \begin{bmatrix}
0 & 0 & 0 & c\Delta\omega_{mec} \\
0 & 0 & -c\Delta\omega_{mec} & 0 \\
0 & 0 & 0 & -\Delta\omega_{mec} \\
0 & 0 & \Delta\omega_{mec} & 0
\end{bmatrix}
\] (2.98)
\[ \Delta \omega_r = \omega_{mec} - \omega_{mec}; \quad c = \frac{M}{L_s L_r - M^2}; \]

Now, we define the following Lyapunov function candidate:

\[ V = e^T e + \frac{(\dot{\omega}_{mec} - \omega_{mec})^2}{\lambda} \]  \hspace{1cm} (2.99)

where \( \lambda \) is a positive constant.

Deriving the function candidate \( V \)

\[ \frac{d}{dt}V = e^T [(A + LC)^T + (A + LC)] e - 2e\Delta \omega_{mec} (e_s^\alpha \dot{\phi}_r^\beta - e_s^\beta \dot{\phi}_r^\alpha) + 2\Delta \omega_{mec} \frac{d}{dt} \frac{\dot{\omega}_{mec}}{\lambda} \]  \hspace{1cm} (2.100)

where \( e_s^\alpha = I_s^\alpha - \hat{I}_s^\alpha \) and \( e_s^\beta = I_s^\beta - \hat{I}_s^\beta \).

When the matrix \( L \) is chosen so the first equation term \( [(A + LC)^T + (A + LC)] \) is negative definite, the second and third term on the right hand side must cancel each other, so that (2.100) is negative definite

\[ \frac{d}{dt} \dot{\omega}_{mec} = c\lambda (e_s^\alpha \dot{\phi}_r^\beta - e_s^\beta \dot{\phi}_r^\alpha) \]  \hspace{1cm} (2.101)

Since the motor speed can shift quite quickly, an integral and proportional adaptive scheme is used, so the system's response can be as improved as possible. Therefore, the estimated speed is determined by

\[ \dot{\omega}_{mec} = K_{P_{\text{es}}} (e_s^\alpha \dot{\phi}_r^\beta - e_s^\beta \dot{\phi}_r^\alpha) + K_{I_{\text{es}}} \int_0^t (e_s^\alpha \dot{\phi}_r^\beta - e_s^\beta \dot{\phi}_r^\alpha) \, dt \]  \hspace{1cm} (2.102)

whereas both variables \( K_{P_{\text{es}}} \) and \( K_{I_{\text{es}}} \) are a positive gain.

Regarding the value assign to the matrix \( L \), there were chosen so the observer poles are proportional to those of the induction machine, according to the reference [6].

\[
[L] = \begin{bmatrix}
L_1 & -L_2 \\
L_2 & L_1 \\
L_3 & -L_4 \\
L_4 & L_3 \\
\end{bmatrix} = \begin{bmatrix}
L_1 & 0 \\
0 & L_1 \\
L_3 & 0 \\
0 & L_3 \\
\end{bmatrix} + \begin{bmatrix}
0 & -L_2 \\
L_2 & 0 \\
0 & -L_4 \\
L_4 & 0 \\
\end{bmatrix} \]  \hspace{1cm} (2.103)
where

\[ L_1 = (k_L - 1) \left( \frac{1}{\sigma T_s} + \frac{1}{\sigma T_r} \right) \]  
\[ L_2 = \frac{(k_L - 1)}{2\pi} \hat{\omega}_{mec}(p)^\dagger \]  
\[ L_3 = (k_L^2 - 1) \left[ \left( \frac{1}{\sigma T_s} + \frac{1}{\sigma T_r} \right) \frac{L_m}{L_r} - \frac{L_m}{T_r} \right] + \frac{\sigma}{L_m} \left( \frac{1}{\sigma T_s} + \frac{1}{\sigma T_r} \right) (k_L - 1) \]  
\[ L_4 = -(k_L - 1) \frac{\sigma}{L_m} \frac{L_m}{L_r} \hat{\omega}_{mec}(p)^\dagger \]  

\(^1\)It was empirically verified, that the observer had a better performance when index \(L_2\) and \(L_4\) were not multiplied by the number of pair of poles \(p\), so the matrix \([L]\) has this small modification.

where \(k_L\) is a empirical positive gain, while \(T_r = L_r/R_r\), \(T_s = L_s/R_s\) and \(\sigma = 1 - L_m^2/L_s L_r\). When \(k_L = 1.1\)

\[
\begin{bmatrix}
19.0430 & -0.0159 \hat{\omega}_{mec} \\
-0.0159 \hat{\omega}_{mec} & 19.0430 \\
1.1992 & 0.0028 \hat{\omega}_{mec} \\
-0.0028 \hat{\omega}_{mec} & 1.1992
\end{bmatrix}
\]

(2.105)

The induction motor poles can be determined by computing the eigenvalues of matrix \([A(\hat{\omega}_{mec})]\), which four values are presented in Figure 2.10 and 2.12 as a function of the mechanical speed increases. Meanwhile and regarding the observer poles, it can be computed by solving the eigenvalues of the matrix \([A(\hat{\omega}_{mec}) + L(\hat{\omega}_{mec})C]\), which values are represented in Figure 2.11 and 2.13.
2.4 Matlab Simulation

Once completed the theoretical study of the system, a set of simulations will be done, so that it is possible to predict the system behavior. In the simulations, we will start with the simplest system, i.e. no inverter situation, and then progress to a more complex situation with the inverter integration in the motor supply system.

2.4.1 Purely sinusoidal three-phase system

In order to investigate the behaviour of the observer, a simulation of an induction machine was made in Matlab. The system, represented in Figure 2.14 is divided in two parts: on the left hand side is placed a representation of the physical system, where both electric excitation and the induction machine are located. In the right hand side, is located the observer itself.

In the first sets of simulations, the electric excitation of the induction motor, is represented in the Figure 2.15, where the voltages are a three-phase system purely sinusoidal. This case portrays the motor directly connected to the grid and no inverter is used in this part. On the other hand, the parameters
adopted is this simulation are the following presented table 2.6.

Table 2.6: Simulation parameters physical system.

<table>
<thead>
<tr>
<th>Electric Grid</th>
<th>Induction Motor Electric</th>
<th>Induction Motor Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>freq.</td>
<td>Amp.</td>
<td>$R_s$</td>
</tr>
<tr>
<td>50Hz</td>
<td>230Vrms</td>
<td>3.34Ω</td>
</tr>
</tbody>
</table>

Regarding the observer part, illustrated in the Figure 2.16, it consists of the implementation of the computations of the acquired data from the physical model. Starting with the set of the values of the start up of the machine, with both voltages and currents present in the machine’s stator, this electric quantities represented in abc time domain are firstly converted to $\alpha\beta$ reference. Meanwhile, the matrix $A$ and $L$ are update accordingly with the speed estimation from the last iteration. Thereupon, it is possible to compute the state variation using the equation (2.95) and system’s state from the last iteration. Finally,
and after updating the system’s state it is possible to compute the new speed estimation using the equation (2.102). Repeating all this iterative process for each sample set of acquired data from the physical model. The parameters adopted in the construction of the observer are those presented in the table 2.7.

<table>
<thead>
<tr>
<th>Motor Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed [rpm]</td>
</tr>
<tr>
<td>1600</td>
</tr>
<tr>
<td>1400</td>
</tr>
<tr>
<td>1200</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>800</td>
</tr>
<tr>
<td>600</td>
</tr>
<tr>
<td>400</td>
</tr>
</tbody>
</table>

Table 2.7: Simulation parameters observer system.

<table>
<thead>
<tr>
<th>Induction Motor</th>
<th>System’s constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>$L_s^*$</td>
</tr>
<tr>
<td>3.34Ω</td>
<td>247.7mH</td>
</tr>
</tbody>
</table>

* The stator self inductance $L_s$ and rotor self inductance $L_r$ are different from leakage stator flux $l_s$ and leakage rotor flux $l_r$, presented in the table 2.6. However, using the mutual inductance $M$, it is easy to convert one into the other.

The observer produced data, speed estimation and the electric torque estimation, will be compared against the real quantities produced by the physical model, so that the observer’s convergence can be verified, as well as its behavior during the transitory.

The first graph presented in Figure 2.17 is regarding the speed and it is easily identified the good performers of the observer, where only in the first few moments of the engine start it is noticeable a small deviation between the actual speed and its estimation however, the observer quickly picks up that deviation and both values assume identical values even when it is applied a different mechanical load in the motor shaft.

![Motor Speed Graph](image)

Figure 2.17: Observer performance - Speed.

In Figure 2.18, regarding the electric torque produced by the motor, it is not noticeable any deviation between values except during its initial acceleration, where the estimation value is still slightly lagging behind the real value.

Finally, in this last two, it is presented the values of the stator current errors, alpha and beta respec-
tively. These values are responsible for guiding the observer states, causing all states to shift, so this value can reduce. Thus, it can be seen that during the system’s transitory, such as staring the motor figure 2.19, or less expressively when changing the torque load, this values increase quite significantly however, a moments later it starts to reduce, and gets confined in a region close to zero.

Figure 2.18: Observer performance - Torque.

Figure 2.19: Error fluctuation - Starting the motor.
2.4.2 Implementation of the inverter

When the motor is powered by an inverter system, as depicted in Figure 2.20, in order to change its operating speed, it is injected harmonics in the system since the voltage is no longer purely sinusoidal, Figure 2.21.

![Simulated inverter system](image1)

**Figure 2.20: Simulated inverter system.**

![Produced voltage by the inverter](image2)

**Figure 2.21: Produced voltage by the inverter.**

In this situation figure 2.22, 2.23 and 2.24 the observer behaves equally well as in the previous case. However, there are some oscillation in the stationary values, due to the presence some harmonics in the current generated by the abrupt variations of the applied voltage.
Figure 2.22: Observer performance - Speed with inverter.

Figure 2.23: Observer performance - Torque with inverter.
Figure 2.24: Error fluctuation with inverter.
Chapter 3

Implementation

The present chapter is dedicated to the implementation of the speed observer in an Arduino's board. Starting with the choice of the Arduino Board, since the electric department possessed already two types of Arduino boards, dismissing the need of purchasing a new board. A new simulation will be made according to the specifications of the implemented system. Finally and as already mentioned in section 2.2, the observer will be implemented in a test bench with an induction motor mechanically linked to a DC machine, instead of the car's motor. This testing induction motor is mechanically coupled to a DC machine, this machine will operate as a generator when testing the observer in order to produce a mechanical load on the motor shaft. This was because at the time of testing, the car was unavailable, and on the other hand, the use of the DC machine made the testing phase much more conclusive, since it was possible to know with great accuracy the value of the mechanical load applied to the motor. Reproducing the same environment, in which the car's motor have to operate.

3.1 Observer hardware specifications

In order to implement the observer using the Arduino hardware, it is necessary to sample consecutive values of the all voltages applied to the motor stator as well as all currents. With these six values of electric quantities present in the stator, it is possible to estimate the value of the torque generated by the machine and also estimate the dynamics of the mechanical speed.

In order to acquire the values of the signals, it is necessary to point out some of the limitations of the hardware that is being used. First of all the operating voltage of the Arduino is 3.3 V, reason why no voltage applied to a port can be negative or exceed this value. One of the reasons why the Arduino Due was chosen regarding the Arduino Uno, it is its higher clock frequency to be about 6 times faster at 84 MHz. This is an important factor because one needs to sampling the electric signals as fast as possible and between consecutive samples one has to process a significant number of computations. Another important factor to have chosen this hardware is its higher analog inputs resolution, it can provide a 12 bits of resolution, while the Arduino Uno version only provides a 10 bits resolution, and it represents 4 times more resolution in each measurement of both currents and voltages. This main comparison
between the two boards is presented in Table 3.1.

<table>
<thead>
<tr>
<th>Table 3.1: Arduino Uno VS Due.</th>
<th>Arduino Board</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UNO</td>
</tr>
<tr>
<td>Working Voltage [V]</td>
<td>+5.0</td>
</tr>
<tr>
<td>Clock Speed [MHz]</td>
<td>16</td>
</tr>
<tr>
<td>Analog Input: [N° Pins]</td>
<td>6</td>
</tr>
<tr>
<td>Resolution [N° levels]</td>
<td>1024</td>
</tr>
<tr>
<td>Flash Memory [KB]</td>
<td>32</td>
</tr>
</tbody>
</table>

3.1.1 Stator voltage signals acquisition

In the present application, the induction motor is set to operate at $230V_{RMS}$, meaning that each amplitude phase is set to alternate between $\pm 230\sqrt{2} \approx \pm 330V$, while the operating voltage of the arduino is $+3.3V$. Since, there is the power signal and the data signal, with every different voltage value (3.3V and 330V), an electric isolation component was implemented to separate power signals from data signals, so the sensitive electronics would not be damaged. This insulation element is the AMC 1300, a reinforced isolated precision amplifier, with a differential voltage input and outputs a voltage with a DC offset and a constant gain. The DC offset is a key part because the arduino does not allow any negative voltage, so this ampop ensures that even the most negative voltage applied to the input will translate in to a positive value at the output. The input is set to vary between $\pm 250mV$, although the linear transformation extents to $\pm 350mV$, with the output saturating for any value outside of this range. So, in order to meet this specification of the input signal, a voltage filter with an attenuation gain was implement. The first resistance, with the largest value of $361k\Omega$, was chosen so the current flowing through the filtering system, would not exceed half of the nominal dissipating value of the larger one. In this case, we can ensure that the set of resistances will not warm up and therefore the resistances values will remain constant, during operation.

In the following Figure 3.1, it is represented the circuit diagram for acquisition of the three voltages applied to the motor.
Voltage filter

Since the applied voltage is expected to have a large set of high frequency harmonics due to the presence of the inverter, two first order filters were implemented in the voltage acquisition circuit. The first one, was placed before the isolation ampop AMC1300, while the second one was placed just before the Arduino’s port, as indicated in figure 3.1. The aim with these two filters is to eliminate the higher frequency harmonics before converting the analog signal to digital. Hence, it is critical to prevent any aliasing from happening. So to do this, every harmonic higher than the $f_s/2$ must be eliminated from the analog signal but maintaining the characteristics of the original signal. In order to keep the circuit simple, it was chosen a first order filter for both filters. However, a first order filter is not a very selective filter, eliminating at -20 dB/dec after the pole, thus it is important to place the cut frequency as soon as possible.

Regarding to the first filter, connected directly the voltage signal coming from the inverter to the AMC1300 it is necessary to introduce a gain to the original signal, ensuring that the signal would not overcome the $\pm 250 \text{ mV}$ limit as a maximum operating value. The voltage filter must be designed to match those limits and also to limit the size of the pass-band of the signal with a cutoff frequency, which can be determined by using the 3.1 using the time constant $\tau_f$ of the filter:

$$f_{cut} = \frac{1}{2\pi \tau_f}$$  \hspace{1cm} (3.1)

The first order filter shown in the figure 3.2 is composed by a series of two resistances R1, R2 and a capacitor C connected in parallel with the lower one, R2. Finally, the third resistance represents the input resistance of the ampop that will be connected to that filter.

Developing the previous electrical system, the following equations are obtained:
Applying the Laplace transform to (3.2) and making some algebraic manipulations it is possible to reach to the following equation in frequency domain.

\[
\frac{u_{\text{out}}(s)}{u_{\text{in}}(s)} = \frac{1}{1 + \frac{R_1 (\frac{R_2}{R_3} + 1)}{R_2 C} + \frac{R_1 R_3}{R_3}} = K_{f1} \frac{1}{\tau_{f1}s + 1} \tag{3.3}
\]

In 3.3 \(K_{f1}\) is the filter gain, and \(\tau_{f1}\) the time constant, being given by:

\[
K_{f1} = \frac{R_2 R_3}{R_1 R_3 + R_1 R_2 + R_2 R_3} = 6.8427 \times 10^{-4}
\]
\[
\tau_{f1} = \frac{R_1 R_3 C}{R_1 R_3 + R_1 R_2 + R_2 R_3} = 2.4702 \times 10^{-4} \Rightarrow * f_{cut1} = 644.3 \text{ Hz}
\]

* Using cutoff frequency equation (3.1).

It is important to note that in every filter there is a phase delay that starts to be noticeable one decade before the cutoff frequency. Since that cutoff frequency is place a decade away from nominal frequency of the induction machine, this phase delay will be neglected.
In order to verify the filtering effect of the circuit in the voltage signal, a simulation was made. In this simulation a 50Hz three-phase alternated voltage was created using an inverter with a switching frequency of 8 kHz and two voltages stages. In the single side amplitude spectrum shown in Figure 3.3, it is possible to compared both signals, before and after filtering. Although there are still unwanted harmonic frequencies in the filtered signal, those harmonics are barely noticeable even compared with the main harmonic, demonstrating the effectiveness of the projected filter.

![Inverter Voltage Single-Sided Amplitude Spectrum](image)

Figure 3.3: Voltage Signal Spectrum.

The AMC 1300 is a reinforced isolation precision amplifier, and it is used to isolate the power signal used to feed the electric motor from the data signal used by Arduino. This electronic component is power by two different DC power sources, in which the constant voltage can be chosen between +3.3V and +5.5V for each side. It outputs a voltage up to +3V, while the output current should not exceed the 10mA value. Accordingly, it is also necessary to implement a first order filter after Ampop, in order to make the system more selective, as shown in the figure 3.1. A 360Ω resistance has been chosen, so the maximum value of the current during the charge of the capacitor will not exceed its maximum value of 10mA, the variation of the current value while a capacitor is being charged is used

\[
i(t) = \frac{u}{R} e^{-\frac{t}{RC}}
\]  

(3.4)

Being the most critical case the initial instant when the current reaches the maximum value:

\[
i_{Max} = i(0^+) = \frac{u}{R_{Min}} \Rightarrow R_{Min} = \frac{u}{i_{Max}} = 300\Omega
\]  

(3.5)

That is a suitable value for the resistance must be greater than 300Ω. Sizing a first order filter, using (3.6)
\documentclass{article}
\usepackage{amsmath}
\usepackage{graphicx}
\usepackage{subcaption}

\begin{document}

\begin{equation}
    u_{in}(t) - u_{out}(t) = Ri(t)
    \quad u_{in}(t) - u_{out}(t) = RC \frac{du_{out}}{dt}
\end{equation}

And applying the Laplace transformation to (3.6)

\begin{equation}
    u_{in}(s) - u_{out}(s) = RCs u_{out}(s)
    \quad \frac{u_{out}(s)}{u_{in}(s)} = \frac{1}{RCs + 1} = K \frac{1}{\tau s + 1}
\end{equation}

Where $K_{f2}$ is the filter gain, and $\tau_{f2}$ the time constant, equal to:

\begin{align*}
    K_{f2} &= 1 \\
    \tau_{f2} &= RC
\end{align*}

Finally and due to the low selectivity of the filter, its cutoff frequency is going to be set to as soon as possible at 90Hz, where the inverter is not expected exceed this value as a fundamental harmonic, (3.1), it possible to compute the value of the capacitor as:

\begin{equation}
    f_{cut} = \frac{1}{2\pi CR \Rightarrow C} = \frac{1}{2\pi f_{cut} R} = 5 \mu F
\end{equation}

\begin{figure}[h]
\centering
\subcaptionbox{Filter at AMC1300 input}{\includegraphics[width=0.4\textwidth]{bode_input.png}} \hspace{1cm} \subcaptionbox{Filter at AMC1300 output}{\includegraphics[width=0.4\textwidth]{bode_output.png}}
\caption{Bode Diagrams.}
\end{figure}

\end{document}
Voltage and Current acquisitions tests

As was previously mentioned, the AMC1300 has two power supplying terminals one for each signal side. Regarding the low power signal, side it was chosen the Arduinino’s $+3.3 \text{ V}$ port, because not only the total current consumed by three identical ampops is $7.2 \text{ mA}$, which is much lower than the maximum value of $800 \text{ mA}$ supported by the source, also ensuring the common ground between this three signals and the rest of the Arduino’s incoming data. In the another hand, it is necessary to find out another DC powering source to supply the high power AMC1300 side, so both sides remain galvanic isolated.

A simple way to power AMC 1300 high power side would be to use an AC/DC convert, in which would rectify the voltage and set it to $+5 \text{ V}$. However and due to some fluctuations and noise in the ground reference, the data acquisition did not perform as expected and the voltage signal acquired oscilates and has some deformations, as shown in the figure 3.5.

![Figure 3.5: Shape of the voltage signal when AMC1300 powered by an AC/DC](image)

In order to over come the fluctuations in the ground reference, instead of using an AC/DC convert connected to the grid, a set of batteries was chosen, composed by three cells of $+1.5 \text{ V}$ connected in series adding up $+4.5 \text{ V}$, in which is an allowed supply voltage value for the AMC1300. Finally and regarding the voltage, two acquisition tests were made so it was possible to infer if the system was working correctly and if its measurements was any accurate. The first test, shown in figure 3.6(a) was simply applying the signal from the electric grid directly to the input of the acquisition system. As expected in the output side of the acquisition system a signal with the same shape as the voltage input but multiplied by a scaler factor and with a DC offset, preventing the signal from having any negative value in any instant. In the figure 3.6(a), it is possible to see the final result of the signal acquisition. That, due to some digital calculations, a constant gain was multiplied and the DC offset create by the ampop removed.

In the second test, in order to check the good operation of the sytem, figure 3.6(b), it is possible to see a small delay in the response of the $119 \text{ kHz}$ harmonic signal, as excepted by the data-sheet,
however this delay of less than 2.5 µs is negligible when compared with the 20ms time period of the nominal frequency.

### 3.1.2 Stator current acquisition

In order to measure the values of the current, it was used a set of three current transducers one for each phase, in which this particular model has a full range of ±80 A, and outputs a voltage proportional to the current passing through the device, when the current is equal to −80 A the voltage value is +0.5 V and in the other hand, when it is +80 A the output is equal to +4.5 V, although the maximum values excepted to the machine’s stat is around 25 A. In the transducers output a voltage divider of 2/3 was implement, because the maximum value of the voltage allowed by the arduino is +3.3 V and not the +4.5 V generated by the probe. Other factor relevant to size the voltage divider was the total resistance seen by the current transducer, according to the data-sheet of the device the load resistance should be higher than 2 kΩ, so the total resistance of the voltage divider is approximately 3 kΩ.

The full represented the circuit diagram for acquisition of the current for each stator phase of the motor, is present in the figure 3.7, and finally the connections of each current sensor LEM is represented in the figure 3.8.
3.1.3 Current acquisition and calibration

Concerning the calibration of the current acquisition system, a very simple circuit was implemented using a DC power source and a high power electric resistance. Hence, a known set of different values of the current passing through the LEM was achieved by changing the voltage of the power source, in the figure 3.9 shows the relation between current and its corresponding LEM voltage level, since the voltage applied to the Arduino (ranging from 0 to 3.3V) is discretized and converted to levels ranging from 0 to 4095. Finally the best fitting line was draw according to the points as shown in plot.

<table>
<thead>
<tr>
<th>Current [A]</th>
<th>Equation (Level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Red (a)</td>
<td>0.0478 (x) – 98.325</td>
</tr>
<tr>
<td>Phase Green (b)</td>
<td>0.0481 (x) – 98.426</td>
</tr>
<tr>
<td>Phase Yellow (c)</td>
<td>0.0480 (x) – 98.917</td>
</tr>
</tbody>
</table>

Table 3.2: Linear Transformation.
In order to verify the correct operation of the system, the stator currents of the induction machine was acquired while the output voltage of the system was being measured by the oscilloscope, as showed in the figure 3.10 and where it is possible to see two stator currents.
3.1.4 Real Speed acquisition

At last it is also necessary to measure the value of the speed of the rotor, only this way it is possible to validate the data that is being produced by the observer. Hence it was used a small speed encoder, this encoder composed by a plastic disc with a shape of a cogwheel placed in the shaft of the motor. The way this encoder implemented in Arduino works is very simple, an emissor and receptor, placed in a stationary frame, enables the Arduino to count this alternations of the teeth of the disc, and the number of alterations during an accrued pre-set time interval is multiplied by a proportional gain in order to convert this number in a RPM quantity. This proportional gain can be easy obtain by putting the machine rotating at a known speed and dividing this value by the number of alternations counted by the Arduino during one time interval. This speed acquisition set is presented in the figure 3.11.

![Diagram of speed encoder circuit](image)

Figure 3.11: Speed encoder circuit.
3.1.5 Observer data processing

There are two main tasks that need to be performed in parallel by the Arduino. On the one hand, it is necessary to estimate the speed of the system, and in the other hand it is also necessary to measure the real speed of the motor. In Figure 3.14, in blue is highlighted the consecutive steps required to be performed, regarding the speed estimation, within one iteration. In the same figure in green is highlighted the real speed acquisition which is an interruption routine from the main one.

In each iteration the Arduino takes around 0.69 ms in order to do all the computations which are the following:

1. Reading the voltage and current in each motor phase;
2. Compute the increments of state variables;
3. Update the state variable;
4. Compute the increment of mechanical speed;
5. Update the matrices \([A]\) and \([L]\), using the new mechanical speed.

In addition to this, it is also necessary to:
• Counting the number of revolutions of the motor's shaft - Interruption code;

• Sending a 80 bit communication word, as seen in figure 3.13, containing the computed data.

<table>
<thead>
<tr>
<th></th>
<th>Check Sync</th>
<th>Data to transfer</th>
<th>Sub-Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0xAA</td>
<td>0xBB</td>
<td></td>
</tr>
<tr>
<td>Size [Byte]</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Estimated Speed</td>
<td>2</td>
<td>Estimated Torque</td>
<td>2</td>
</tr>
<tr>
<td>Real Speed</td>
<td>2</td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>

Communication word*

*An extra 2 bits for each byte transferred

Figure 3.13: Communication word.

It was verified, figure 3.14, that on average the total time for one iteration it is slightly higher than 0.68 ms. Thus, it was chosen a sampling time $T_{sa} = 0.72$ ms, in order to give a 5% safety margin from the maximum value.

Figure 3.14: Required time for each iteration.
3.2 Simulation adapted to arduino limitations

In the previous section 3.1.5, it was introduced the processing time needed for the arduino, to complete all computations necessary at each iteration. This physical limitation has an important impact on speed observer’s performance, since the sampling frequency can no longer be any arbitrary number, and it as to satisfy the arduino maximum processing time. Therefore, a sampling time $T_{sa} = 0.72 ms$ means that the sampling frequency must be equal to $f_{sa} = 1389 Hz$. So it has been necessary to repeat simulations of section 2.4 to verify any significant perturbation in the results.

<table>
<thead>
<tr>
<th>Induction Motor</th>
<th>System’s constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>$L_s$</td>
</tr>
<tr>
<td>3.34 Ω</td>
<td>247.7 mH</td>
</tr>
</tbody>
</table>

* With this new sample frequency it was necessary to adjust the gain value, so the system does not get unstable.

After reducing the observer sampling frequency, due to its computational limitation, it is possible to verify major changes in the observer performance. Foremost, in figure 3.15 one verifies that in steady-state the speed is not accurate, and there is an offset deviation. In the figure 3.16 torque observed signal had the same offset deviation from the steady state values. It is possible to verify that the value of the state, necessary to compute the torque value, are not being determined in a very accurate meter.
In the last figure 3.17, it is possible to verify the evolution of the error during the simulation, and in this simulation the error is not circumscribed in a region near zero. The error region depends on sampling frequency, the higher the frequency, the smaller the circumscribed error region.
In this last simulation 3.18, 3.19 and 3.20, the maximum sampling frequency 1389 Hz, allowed by the Arduino was applied and on the other hand, the voltage distortion introduced by the inverter, was also considered as well as the two first-order filters ($f_{c1} = 644$ Hz and $f_{c2} = 90$ Hz) implemented in the voltage acquisition signal before the Arduino.

Lastly, it is important to refer that the data acquired by the current acquisition circuit must be delayed
Figure 3.20: Error fluctuation, $f_{sa} = 1389$ Hz, with inverter.

by 3 samples, because there is no filter in this electric quantity unlike the voltage, and both signals must be computed with the respective instant.

In a first order filter:

$$f_{cut} = \frac{1}{2\pi C_{eq} R_{eq}}.$$  \hspace{1cm} (3.9)

For the $f_{cut} = 644.3$ Hz filter:

$$\begin{cases} C_{eq} = C = 1 \mu F \\ R_{eq} = 247 \Omega \\ \tau_{f644} = R_{eq} C_{eq} = 0.247 \text{ ms} \end{cases}$$  \hspace{1cm} (3.10)

For the $f_{cut} = 90$ Hz filter:

$$\begin{cases} C_{eq} = C = 5 \mu F \\ R_{eq} = R = 360 \Omega \\ \tau_{f90} = R_{eq} C_{eq} = 1.800 \text{ ms} \end{cases}$$  \hspace{1cm} (3.11)

$$Delay = \frac{\tau_{total}}{T_{sa}} = \frac{\tau_{f644} + \tau_{f90}}{T_{sa}} = \frac{2.047}{0.72} = 2.843 \approx 3 \text{ samples}$$  \hspace{1cm} (3.12)

This new physical layout is represented in the figure 3.21.

However the current delay applied in the prototype was only 1 sample instead of 3 samples. Because the current acquisition system is also responsible for introducing a delay in the signal, 400 ns according to the data sheet. And the current delay should be such that in steady state, the electronic torque is approximately zero when there is no load applied to the motor.

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It was verified, by comparing this set of simulations with the previous one made in section 2.4, that the reduction of the observer sampling frequency \( f_{sa} \) increased the deviation of the estimation value of the state variables, and consequently made the speed estimation worse. Thus, a sampling frequency sweep was made, in order to evaluate the offset torque error evolution with the sampling frequency. In figure 3.22, it is possible to check that the torque estimation error tends to zero when sampling frequency tends to infinity, according to the trend-line equation \( 165477 \times (f_{sa})^{-1.15} \) where \( R^2 = 0.994 \). Unfortunately, the maximum sampling frequency allowed by current hardware is only 1389 Hz which will lead us to a 40% torque estimation error.
Chapter 4

Results

In the present chapter, the prototype developed was tested. The set of tests started with the simplest configuration directly connected to the grid, and only after in the second set of tests the inverter was implemented in the system.

4.1 Test without inverter

For the first test, the induction motor was directly connected to the electric grid, and no mechanical load was applied to its shaft.

![No Inverter, No Load](image)

Figure 4.1: No inverter, no load test

In figure 4.1 and 4.2, it is possible to check that the speed difference between its estimation and real measurement are very close from each other, with an error below 1%, however the estimation value is
always estimated below, and does not converge to the accurate value.

Lastly in figure 4.3, it is represented the motor starting, where it is possible to verify the initial torque necessary to speed up the motor, and when the motor stop accelerating that value returned to zero, approximately.
4.2 Test without inverter and with load

A direct current DC machine, which shaft is linked to the induction motor shaft, was set as a generator with its armature winding connected to a set of resistances. The induction motor in the other hand, was directly connected to the electric grid, so the motor when rotating had not only to overcome the system’s inertia but also to produce the necessary torque. In the middle of the test the field excitation of the DC machine was removed, ceasing the mechanical load. This DC motor has a $K_T = 0.8$, so its applied mechanical torque is equal to

$$T_{load} = K_T I_{arm} \ [N.m] \quad (4.1)$$

where $I_{arm}[A]$ is the DC machine armature current.

![No Inverter, Load](image)

Figure 4.4: No inverter test with mechanical load - Speed.

In figure 4.4, it is possible to verify that there is a deviation in the applied torque value and its estimated value, computed by the observer. This deviation is approximately 36%, taking into account that the system’s fiction torque is neglectable. This value in the order of 40% was already anticipated during the system’s simulation, figure 3.22. This event proves that the state variables are not being determined accurately, since the torque is computed by using all state space variables, as shown in equation 2.87. This error, justifies the error also verified in the motor speed estimation, which uses the motor’s rotor flux and also the current error, as seen in equation 2.102.

In the figure 4.5 is represented the speed and electric torque evolution in another test equal to figure 4.4.
4.3 Test with inverter

In this test, figure 4.6, a inverter with a linear command $V/f$ was introduced between the electric grid power source and the induction machine, and no extra mechanical load was produced. The motor speed was alternated by the changing of the stator voltage frequency.
4.4 Test with inverter and mechanical load

Finally in this set of tests, the invert was used and also the DC machine to generate a mechanical torque in the motor shaft, combining the two tests done before.

![Figure 4.7: Inverter and load test - High speed.](image)

![Figure 4.8: Inverter and load test - Torque step.](image)
In the figure 4.7, 4.8 and 4.9, the electric torque and motor speed evolution are represented however, both estimations have a deviation from its real value, $\approx 1\%$ for the speed and $\approx 40\%$ for the torque, in addition the torque estimation presents a high frequency oscillation.
In the figure 4.10 and 4.11, it is represented the estimation values of the speed and the torque produced by the motor where the speed deviation is around 2%.
Chapter 5

Conclusions

The purpose of this thesis was to study the induction motor and to develop a speed observer, which by measuring the motor’s input stator voltages and stator currents signals would estimate the motor’s speed and torque.

It was initially expected that the implemented system could replicate the same results as it is presented in the Section 2.4. However, during the system’s hardware implementation it was noticeable that some compromises needed to be made, such as the reduction of the sample frequency of the system and consequently the implementation of some filtering capabilities in the voltages acquisition circuit. The lower sample frequency, due to the limited computational power of the Arduino board, which must finish all computations necessary between samples. The filter needed is present in order to prevent aliasing from happening in the electric voltage signal, since the inverter introduces a lot of high order frequencies in the voltage signal. Notwithstanding, due to the implementation commitments, it was obvious that they played a pivotal role in the general outcome of the prototype and therefore alter the expected initial results which were very promising therefore, it was not possible to achieve the proper system performance which did not permit a convergence to the accurate value. Eventually, the final system produces state estimations which are not accurate enough, materializing in a 40% error in the torque estimation, and a 2% maximum deviation of the speed.

Nevertheless, this work greater achievement was the implementation of the adaptive observer system in a real time processing hardware, allowing an instant measurement of the speed and torque (estimated) in other words, emulating a speed and torque sensor. The results obtained are not far from the real ones, and the estimated values would have been better, if the observer’s sampling frequency were higher.

5.1 Future work

As it was clear from the results presented in the previous chapter 4, there are some aspects that can be reevaluated and improved.

1. Evaluating the possibility of implementing the observer using another approach, just as the Ex-
tented Kalman Filter, which is more reliable implementation. In an EKF implementation the closed loop function, responsible for adjusting the forecast value with the measurement value, are determine in an optimally manner, considering the impressions in the estimation values and also noise in the electric measurements. Where, according to [8]:

$$[A_{EKF}] = \begin{bmatrix} A(\omega_r) & 0 \\ 0 & 0 \end{bmatrix}; \quad [x_{EKF}] = \begin{bmatrix} \phi^d_s \\ \phi^q_s \\ \omega_r \end{bmatrix}; \quad [B_{EKF}] = \begin{bmatrix} B \\ 0 \end{bmatrix}; \quad [C_{EKF}] = \begin{bmatrix} C & 0 \end{bmatrix}$$ (5.1)

2. **Adaptive observer implementation.**

(a) Using a more selective voltage filter, capable to prevent aliasing, but does not affect the other harmonies that can be sampled by the system.

(b) Increase the sampling frequency of the observer. By implementing a more efficient communication system, and using a more powerful processor in the hardware.

(c) Implementing a $dqo$ frame instead of using $\alpha\beta$ frame. This implementation will help the system to converge, since in steady state the input signal, stator voltage, will be a constant value. However, this requirement demands the observer to know the inverter applied frequency, increasing the complexity of the system.

3. Implement the overall circuit in a Printed Circuit Board - PCB. (Even though the observer performance will not change, the physical system will be more solid.)

### 5.1.1 Implementation of the observer in $dqo$ frame

<table>
<thead>
<tr>
<th>Induction Motor</th>
<th>System’s constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>$L_s$</td>
</tr>
<tr>
<td>3.34 Ω</td>
<td>247.7 mH</td>
</tr>
</tbody>
</table>

Table 5.1: Simulated observer parameters in $dqo$.

In Figures 5.1, 5.2 and 5.3, it is presented a simulation of the observer using $dqo$ reference and a 10 kHz sample frequency. And the reference rotating.
Figure 5.1: Speed performance $k_l = 1.1$, $f_{sa} = 10\, kHz$.

Figure 5.2: Torque performance $k_l = 1.1$, $f_{sa} = 10\, kHz$. 
Figure 5.3: Error fluctuation $k_l = 1.1$, $f_{sa} = 10 \text{ kHz}$. 


Appendix A

Arduino Code

Listing A.1: C++ code using listings

```c
#include "Arduino.h"
#include "stdio.h"
#include "stdint.h"

typedef struct __attribute__((packed)) _DATA {
    byte syncAA;
    byte syncBB;
    short real; // 2-byte data-type
    short est; // 2-byte data-type
    short tor; // 2-byte data-type
} DATA;

union packet {
    DATA values;
    byte byte_array[sizeof(DATA)]; // DATA para o PC
};

void add(void);

// ========================= Variaveis para a medicao da vel.:
volatile byte number_inter = 0, k = 0;
int n;
volatile bool sample = false, data = false;
short vell = 0;
short rpm = 0;
short torque = 0;
double sample_t = 0.000720;
```
union packet my_data;

// ============ Variaveis para a Corrente:
double I_offsetr, I_offsetg, I_offsety;
double I_mr, I_mg, I_my, I_br, I_bg, I_by;
short I_r = 0, I_g = 0, I_y = 0, I_r1 = 0, I_g1 = 0;
short I_y1 = 0, I_r2 = 0, I_g2 = 0, I_y2 = 0, I_r3 = 0, I_g3 = 0, I_y3 = 0;
double I_R, I_G, I_Y;
double Ia, Ib;

// ============ Variaveis para a tensao:
double U_offsetr, U_offsetg, U_offsety;
double U_mr, U_mg, U_my, U_br, U_bg, U_by;
short U_r, U_g, U_y;
double U_R, U_G, U_Y;
double Ua, Ub;

// =============== Counter and compare values:
const uint16_t t1.comp = 13125;

// =============== Matrizes observer:
double Matrix_A[4][4];
double State_Space[4][1] = {
    {0},
    {0},
    {0},
    {0}
};
double dState_Space[4][1];
double Matrix_A1[4][4] = {
    {−180.5389, 0, 304.6903, 0},
    {0, −180.5389, 0, 304.6903},
    {2.2947, 0, −9.8910, 0},
    {0, 2.2947, 0, −9.8910}
};
double Matrix_A2[4][4] = {
    {0, 0, 0, 61.6096},
    {0, 0, −61.6096, 0},
    {0, 0, 0, −2},
    {0, 0, 2, 0}
};
double Matrix_B = 32.8895;
double Matrix_K[4][2];
double Matrix_K1[4][2] = {
    {19.0430, 0},
    {0, 19.0430},
    {1.1992, 0},
    {0, 1.1992}
};

double Matrix_KK[4][2] = {
    {0, -0.0159},
    {0.0159, 0},
    {0, 0.0028},
    {-0.0028, 0}
};

// ================================================= Observer values :
double delta_a, delta_b;
int Kpes = 0; // Ganho variavel, vel. mecanica
int Kles = 300; // Ganho variavel vel. mecanica
double wmec = 0;
double dwmec;

// ==============================================================

void setup()
{
    my_data.values.syncAA = 0xAA;
    my_data.values.syncBB = 0xBB;
    my_data.values.real = 0;
    my_data.values.est = 0;

    Serial.begin(115200);
    Serial.begin(250000);
    pinMode(52, INPUT); // INTERRUPT
    pinMode(A0, INPUT); // CORRENTE RED
    pinMode(A1, INPUT); // CORRENTE GREEN
    pinMode(A2, INPUT); // CORRENTE YELLOW
    pinMode(A3, INPUT); // TENSAO RED
}
```c
105  pinMode(A4, INPUT);  // TENSAO GREEN
106  pinMode(A5, INPUT);  // TENSAO YELLOW
107  analogReadResolution(12);
108  attachInterrupt(digitalPinToInterrupt(52), add, CHANGE);
109
110  /* turn on the timer clock in the power management controller */
111  pmc_set_writeprotect(false);  // disable write protection for pmc registers
112  pmc_enable_periph_clk(ID_TC7);  // enable peripheral clock TC7
113
114  /* we want wavesel 01 with RC */
115  // CLOCK3 = 84MHz/32
116  TC.Configure(TC2, 1, TC_CMRR_WAVE | TC_CMRR_WAVSEL_UP_RC |
117  | TC_CMRR_TCCLKS_TIMER,CLOCK3);
118  TC.SetRC(TC2, 1, 1890);  //0.72ms
119  TC.Start(TC2, 1);
120
121  // enable timer interrupts on the timer
122  TC2->TC_CHANNEL[1].TC.IER=TCIER_CPCS;  // IER = interrupt enable register
123  TC2->TC_CHANNEL[1].TC.IDR=TCIER_CPCS;  // IDR = interrupt disable register
124
125  /* Enable the interrupt in the nested vector interrupt controller */
126  /* TC4_IRQn where 4 is the timer number + timer channels (3) + */
127  /* + the channel number =(1*3)+1) for timer1 channel1 */
128  NVIC_EnableIRQ(TC7_IRQn);
129  // ============== PARA AS MEDICOS
130  // Corrente
131  I.offsetr = 0;
132  I.offsetg = 0;
133  I.offsety = 0;
134
135  I.mr = 0.0478;  // Calibracao da corrente
136  I.mg = 0.0481;
137  I.my = 0.048;
138
139  for (n = 0; n < 1000; n++) {
140    I.offsetr = 0.0010 * analogRead(A0) + I.offsetr;
141    I.offsetg = 0.0010 * analogRead(A1) + I.offsetg;
142    I.offsety = 0.0010 * analogRead(A2) + I.offsety;
143```
\textbf{I}_{\text{br}} = - \text{I}_{\text{mr}} \times \text{I}_{\text{offset}}
\textbf{I}_{\text{bg}} = - \text{I}_{\text{mg}} \times \text{I}_{\text{offset}}
\textbf{I}_{\text{by}} = - \text{I}_{\text{my}} \times \text{I}_{\text{offset}}

\text{U}_{\text{offset}}r = 0;
\text{U}_{\text{offset}}g = 0;
\text{U}_{\text{offset}}y = 0;

\text{U}_{\text{mr}} = 0.2878; // Calibracao da tensao
\text{U}_{\text{mg}} = 0.2874;
\text{U}_{\text{my}} = 0.2876;

\textbf{for} (\text{n} = 0; \text{n} < 1000; \text{n}++) \{
\text{U}_{\text{offset}}r = 0.0010 \times \text{analogRead(A3)} + \text{U}_{\text{offset}}r;
\text{U}_{\text{offset}}g = 0.0010 \times \text{analogRead(A4)} + \text{U}_{\text{offset}}g;
\text{U}_{\text{offset}}y = 0.0010 \times \text{analogRead(A5)} + \text{U}_{\text{offset}}y;
\}
\text{U}_{\text{br}} = - \text{U}_{\text{mr}} \times \text{U}_{\text{offset}}r;
\text{U}_{\text{bg}} = - \text{U}_{\text{mg}} \times \text{U}_{\text{offset}}g;
\text{U}_{\text{by}} = - \text{U}_{\text{my}} \times \text{U}_{\text{offset}}y;
\}
\textbf{void} \text{loop}() \{
\textbf{if} (\text{sample}) \{ // ===== Adquirir Corrente a cada 0.72ms
\textbf{for} (\text{n} = 0; \text{n} < 1000; \text{n}++) \{
\text{I}_{\text{r1}} = \text{I}_{\text{r0}};
\text{I}_{\text{g1}} = \text{I}_{\text{g0}};
\text{I}_{\text{y1}} = \text{I}_{\text{y0}};
\}
\text{I}_{\text{R}} = \text{I}_{\text{mr}} \times \text{I}_{\text{r1}} + \text{I}_{\text{br}};
\text{I}_{\text{G}} = \text{I}_{\text{mg}} \times \text{I}_{\text{g1}} + \text{I}_{\text{bg}};
\text{I}_{\text{Y}} = \text{I}_{\text{my}} \times \text{I}_{\text{y1}} + \text{I}_{\text{by}};
\}
\}
\textbf{void} \text{loop}() \{
\textbf{if} (\text{sample}) \{ // ===== Adquirir Tensao a cada 0.72ms
\text{U}_{\text{r}} = \text{analogRead(A3)};
\}
"
U_g = analogRead(A4);
U_y = analogRead(A5);

U_R = U_mr * U_r + U_br;
U_G = U_mg * U_g + U_bg;
U_Y = U_my * U_y + U_by;

// =========== Transformacao em coordenadas Alfa e Beta ===========
Ia = 0.8165*I_R - 0.4082*I_G - 0.4082*I_Y;
Ib = 0.7071*I_G - 0.7071*I_Y;

Ua = 0.8165*U_R - 0.4082*U_G - 0.4082*U_Y;
Ub = 0.7071*U_G - 0.7071*U_Y;

// ==============================================================
// ================= Calculo da Matriz A =======================
for (int row = 0; row < 4; row++)
{
    for (int col = 0; col < 4; col++)
    {
    }
}
// ==============================================================
// ================= Calculo da Matriz L =========================
for (int row = 0; row < 4; row++)
{
    for (int col = 0; col < 2; col++)
    {
    }
}
// ==============================================================
delta_a = ia - Ia;
delta_b = ib - Ib;
dia = (Matrix_A[0][0])*ia + (Matrix_A[0][2])*ma + (Matrix_A[0][3])*mb + Matrix_B * Ua + Matrix_K[0][0] * delta_a + Matrix_K[0][1] * delta_b;
dib = (Matrix_A[1][1])*ib + (Matrix_A[1][2])*ma + (Matrix_A[1][3])*mb + Matrix_B * Ub + Matrix_K[1][0] * delta_a + Matrix_K[1][1] * delta_b;


```c
222 dma = (Matrix_A[2][0]) * ia + (Matrix_A[2][2]) * ma + (Matrix_A[2][3]) * mb +
223      + Matrix_K[2][0] * delta_a + Matrix_K[2][1] * delta_b;
224
225 dmb = (Matrix_A[3][1]) * ib + (Matrix_A[3][2]) * ma + (Matrix_A[3][3]) * mb +
226      + Matrix_K[3][0] * delta_a + Matrix_K[3][1] * delta_b;

228 i_a = i_a + dia * sample_t;
229 i_b = i_b + dib * sample_t;
230 ma = ma + dma * sample_t;
231 mb = mb + dmb * sample_t;
233
dwmc = (mb * (-delta_a) - ma * (-delta_b));
235 wmc = dwmc * (Kles * sample_t) + wmc;
236 vel = wmc + Kpes * dwmc;
237 rpm = (short) (round (vel * 95.493)); // x9.5493 para rpms x10 para MATLAB
238 torque = (short) (round (187.3234 * (i_b * ma - i_a * mb))); // x1.873 para [N.m] e x100 para matlab
239 // x1.873 para [N.m] e x100 para matlab
240
241 // ================ Adquirir o valor da velocidade a cada 21.6ms
242     k++;
243     // a cada 30*0.72ms a cada 21.6ms a uma media de valor medido
244     if (k == 30){
245         k = 0;
246         vell = (short) (number_inter * 14960 / 64.6749); // x10 para MATLAB
247         number_inter = 0;
248         
249         my_data.values.real = vell;
250         my_data.values.est = rpm;
251         my_data.values.tor = torque;
252         Serial.write(my_data.byte_array, sizeof(DATA)); // Mandar para o PC
253     }
254     sample = false;
255 }
256 }
257 void TC7_Handler(){
258     TC_GetStatus(TC2, 1);
259     sample = true;
260     TC_SetRC(TC2, 1, 1890); // 0.72 ms
```
261  
262  void add() {
263    number_inter++;  // Adiciona 1 para a medicação da velocidade do motor
264  }
Appendix B

Technical Datasheets

In this Appendix it is possible to check the main data sheets from the isolation amp AMC1300, used in the voltage acquisition circuit, and the current transducer LTSR 25-NP, used for measuring the current value respectively. Finally, the last page are dedicated to the Arduino Due technical specifications.
AMC1300x Precision, ±250-mV Input, Reinforced Isolated Amplifier

1 Features
- ±250-mV Input Voltage Range Optimized for Current Measurement Using Shunt Resistors
- Low Offset Error and Drift:
  - AMC1300B: ±0.2 mV (max), ±3 µV/°C (max)
  - AMC1300: ±2 mV (max), ±4 µV/°C (max)
- Fixed Gain: 8.2
- Very Low Gain Error and Drift:
  - AMC1300B: ±0.3% (max), ±50 ppm/°C (max)
  - AMC1300: ±1% (max), ±50 ppm/°C (typ)
- Low Nonlinearity and Drift: 0.03%, 1 ppm/°C (typ)
- 3.3-V Operation on High-Side (AMC1300B)
- System-Level Diagnostic Features
- Safety-Related Certifications:
  - 7071-V Package Reinforced Isolation per DIN V VDE V 0884-11: 2017-01
  - 5000-V RMS Isolation for 1 Minute per UL1577
- High CMTI on AMC1300B: 140 kV/µs (typ)

2 Applications
- Shunt-Resistor-Based Current Sensing In:
  - Motor Drives
  - Frequency Inverters
  - Uninterruptible Power Supplies

3 Description
The AMC1300 is a precision, isolated amplifier with an output separated from the input circuitry by an isolation barrier that is highly resistant to magnetic interference. This barrier is certified to provide reinforced galvanic isolation of up to 5 kV_RMS according to VDE V 0884-11 and UL1577. Used in conjunction with isolated power supplies, this isolated amplifier separates parts of the system that operate on different common-mode voltage levels and protects lower-voltage parts from damage.

The input of the AMC1300 is optimized for direct connection to shunt resistors or other low voltage-level signal sources. The excellent performance of the device supports accurate current control resulting in system-level power savings and, especially in motor control applications, lower torque ripple. The integrated common-mode overvoltage and missing high-side supply voltage detection features of the AMC1300 simplify system-level design and diagnostics.

The AMC1300 is offered with two performance grade options: the AMC1300B is specified over the extended industrial temperature range of –55°C to +125°C, and the AMC1300 for operation at –40°C to +125°C.

Device Information

<table>
<thead>
<tr>
<th>PART NUMBER</th>
<th>PACKAGE</th>
<th>BODY SIZE (NOM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMC1300</td>
<td>SOIC (8)</td>
<td>5.85 mm × 7.50 mm</td>
</tr>
</tbody>
</table>

(1) For all available packages, see the orderable addendum at the end of the data sheet.

Simplified Schematic
Current Transducer LTSR 25-NP

For the electronic measurement of currents: DC, AC, pulsed..., with galvanic separation between the primary circuit and the secondary circuit.

### Electrical data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{PN}$</td>
<td>25 At</td>
</tr>
<tr>
<td>$I_{PM}$</td>
<td>0 ... ±80 At</td>
</tr>
<tr>
<td>$I_p$</td>
<td>250 At</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>2.5 ±0.625 × $I_p/I_{PN}$ V @ $I_p = 0$</td>
</tr>
<tr>
<td>$V_{ref}$</td>
<td>Reference voltage (internal reference), Ref$_{OUT}$ mode 2.5 ± V</td>
</tr>
<tr>
<td>$V_{ref}$</td>
<td>Reference voltage (external reference), Ref$_{IN}$ mode 1.9 ... 2.7 V</td>
</tr>
<tr>
<td>$G$</td>
<td>25 mV/A</td>
</tr>
<tr>
<td>$N_S$</td>
<td>2000</td>
</tr>
<tr>
<td>$R_L$</td>
<td>≥ 2 kΩ</td>
</tr>
<tr>
<td>$C_L$</td>
<td>500 pF</td>
</tr>
<tr>
<td>$R_{IM}$</td>
<td>50 Ω</td>
</tr>
<tr>
<td>$TCR_{IM}$</td>
<td>Temperature coefficient of $R_{IM}$ &lt; 50 ppm/K</td>
</tr>
<tr>
<td>$U_C$</td>
<td>Supply voltage (±5 %) 5 V</td>
</tr>
<tr>
<td>$I_C$</td>
<td>Current consumption @ $U_C = 5 V$ Typical 28 + $I_s$ ($V_{out}/R_L$) mA</td>
</tr>
</tbody>
</table>

### Accuracy - Dynamic performance data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$ Accuracy @ $I_p$, $T_A = 25 ^\circ C$</td>
<td>±0.2 %</td>
</tr>
<tr>
<td>$\varepsilon_L$ Linearity error</td>
<td>&lt; 0.1 %</td>
</tr>
<tr>
<td>$TCV_{out}$ Temperature coefficient of $V_{out}$ @ $I_p = 0$</td>
<td>−40 ... +85 °C 37.5 ppm/K</td>
</tr>
<tr>
<td>$TCG$ Temperature coefficient of $G$</td>
<td>−40 ... +85 °C 50 ppm/K</td>
</tr>
<tr>
<td>$V_{OM}$ Magnetic offset voltage @ $I_p = 0$, after an overload of 3 × $I_{PN}$</td>
<td>±0.5 mV</td>
</tr>
<tr>
<td>$TCV_{ref}$ Temperature coefficient of $V_{ref}$ @ $I_p = 0$</td>
<td>−10 ... +85 °C 50 ppm/K</td>
</tr>
<tr>
<td>$t_{fu}$ Reaction time @ 10 % of $I_{PN}$</td>
<td>&lt; 100 ns</td>
</tr>
<tr>
<td>$t_s$ Step response time to 90 % of $I_{PN}$</td>
<td>&lt; 400 ns</td>
</tr>
<tr>
<td>$BW$ Frequency bandwidth @ 0 ... −0.5 dB</td>
<td>DC ... 100 kHz</td>
</tr>
<tr>
<td></td>
<td>−0.5 ... 1 dB</td>
</tr>
</tbody>
</table>

### Features
- Closed loop (compensated) current transducer using the Hall effect
- Unipolar supply voltage
- Insulating plastic case recognized according to UL 94-V0
- Compact design for PCB mounting
- Incorporated measuring resistance
- Extended measuring resistance
- Access to the internal voltage reference
- Possibility to feed the transducer reference from external supply.

### Advantages
- Excellent accuracy
- Very good linearity
- Low temperature drift
- Optimized response time
- Wide frequency bandwidth
- No insertion losses
- High immunity to external interference
- Current overload capability.

### Applications
- AC variable speed drives and servo motor drives
- Static converters for DC motor drives
- Battery supplied applications
- Uninterruptible Power Supplies (UPS)
- Switched Mode Power Supplies (SMPS)
- Power supplies for welding applications.

### Application domain
- Industrial.
The Arduino Due is the newcomer microcontroller board in the Arduino boards family. It's the first board based on a 32 bit processor (Atmel SAM3X8E ARM Cortex-M3 MCU), which improves all the standard Arduino functionalities and adds many new features.

The arduino DUE offers 54 digital input/output pins (of which 16 can be used as PWM outputs, with selectable resolution), 12 analog inputs with 12 bits of resolution, 4 UARTs (hardware serial ports), two DAC (digital to analog converter) outputs, an 84 MHz crystal oscillator, two USB connections, a power jack, an ICSP header, a JTAG header, and a reset button.

The Due has two micro USB connectors: one intended for debugging purposes and a second one capable of acting as a USB host, allowing external USB peripherals such as mouse, keyboards, smartphones, etc. to be connected to the Arduino Due.

More information will be soon on line at the page http://arduino.cc/ArdueDUE

### Technical Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microcontroller</td>
<td>AT91SAM3X8E</td>
</tr>
<tr>
<td>Operating Voltage</td>
<td>3.3V</td>
</tr>
<tr>
<td>Input Voltage (recommended)</td>
<td>7-12V</td>
</tr>
<tr>
<td>Input Voltage (limits)</td>
<td>6-20V</td>
</tr>
<tr>
<td>Digital I/O Pins</td>
<td>54 (of which 16 provide PWM output)</td>
</tr>
<tr>
<td>Analog Input Pins</td>
<td>12</td>
</tr>
<tr>
<td>Analog Outputs Pins</td>
<td>2 (DAC)</td>
</tr>
<tr>
<td>Total DC Output Current on all I/O lines</td>
<td>130mA</td>
</tr>
<tr>
<td>DC Current for 3.3V Pin</td>
<td>800 mA</td>
</tr>
<tr>
<td>DC Current for 5V Pin</td>
<td>theoretical 1A, realistic 800 mA</td>
</tr>
<tr>
<td>Flash Memory</td>
<td>512 KB all available for the user applications</td>
</tr>
<tr>
<td>SRAM</td>
<td>96 KB (64 + 32 KB)</td>
</tr>
<tr>
<td>DataFlash</td>
<td>2 Mbit (250 KB)</td>
</tr>
<tr>
<td>Clock Speed</td>
<td>84 MHz</td>
</tr>
</tbody>
</table>