# Stabilization and path-following control of a flying wing using nonlinear control techniques 

Alda Pombinho Caeiro Caldeira

Thesis to obtain the Master of Science Degree in

## Mechanical Engineering

Supervisor(s): Prof. Alexandra Bento Moutinho<br>Prof. José Raul Carreira Azinheira

## Examination Committee

Chairperson: Prof. Paulo Jorge Coelho Ramalho Oliveira
Supervisor: Prof. José Raul Carreira Azinheira
Member of the Committee: Prof. Duarte Pedro Mata de Oliveira Valério

## Acknowledgments

I would like to reserve my first words to show my sincere gratitude to Professor Alexandra Moutinho and Professor José Raul Azinheira for all their orientation and patience during this thesis. For all the advises, the constant assistance and the time that both dedicated to receive me, thank you.

A special word to my family, specially to my parents and sisters, to show me that the patience and persistence obtains all things.

I would like to thank the constant support of my friends that helped me throughout my college years, and especially to those who, during this time, thoroughly followed my work.

And last but certainly not least, a special word to my sister Madalena who is beginning her journey to become an engineer.


#### Abstract

Nonlinear control techniques stand out for responding to the linear controllers weakness and also because it is possible to design a single nonlinear controller that ensures the validation of a single control action for the entire flight envelope. During this study and within the Innovative High Altitude Balloon for Atlantic Observation (HABAIR) project, the nonlinear model of a flying wing will be analysed and controlled. The linearization of this model for trim conditions over the flight envelope results in the decoupling of the longitudinal and lateral motions. Considering this decoupling, not only a detailed analysis of the flying wing motions over the flight envelope is performed but also a stability analysis. The Flying Wing control is implemented considering three stages: a rate control to stabilise the flight, an attitude control and a flight path control. The control methodologies developed in this thesis are Gain Scheduling and Incremental Nonlinear Dynamic Inversion (INDI). Within the scope of this project is intended to study the gliding flight of the flying wing when it is released from a High Altitude Balloon (HAB). This analysis is based on the simulation results of a path-following mission. During this study the performance of each control approach is analysed, as well as how each one responds in the presence of external perturbations.


## Keywords

Flying Wing; Classical Control; Gain Scheduling; Nonlinear Control; INDI; Path-following.

## Resumo

Técnicas de controlo não linear aplicadas ao controlo de voo têm ganho expressão nas mais recentes investigações no âmbito da aeronáutica. Estas técnicas de controlo destacam-se por responderem às limitações que os controladores lineares apresentam e também por ser possível desenvolver um único controlador não linear que garante a validação de uma única ação de controlo para todo o envelope de voo. Durante este estudo e no âmbito do projecto HABAIR (Innovative High Altitude Balloon for Atlantic Observation) realiza-se a análise e o controlo do modelo não linear de uma asa voadora. A linearização deste modelo para diferentes condições de equilíbrio resulta no desacoplamento dos movimentos longitudinal e lateral. Considerando esta linearização e para todo o envelope de voo, realiza-se não só uma análise detalhada dos movimentos como também uma análise de estabilidade do voo. O controlo da asa voadora é implementado considerando três estágios: controlo das razões angulares, para estabilizar o voo, controlo de atitude e controlo de seguimento. As metodologias de controlo desenvolvidas nesta tese são o escalonamento de ganhos e o controlo não linear incremental da dinâmica inversa, INDI. Ainda no âmbito deste projecto pretende-se estudar o voo planado da asa voadora quando esta é largada de um balão atmosférico de alta altitude HAB. Esta análise basea-se nos resultados de simulação para missões de guiamento. Durante este estudo é ainda analisado de que forma é que cada um dos controladores satisfaz os objectivos definidos e como reage a perturbações.

## Palavras Chave

Asa Voadora; Controlo Clássico; Escalonamento de ganhos; Controlo Não Linear; INDI; Guiamento.

## Contents

1 Introduction ..... 1
1.1 Motivation and Mission ..... 2
1.2 Objectives ..... 4
1.3 Thesis Outline ..... 4
2 State of the Art ..... 5
2.1 Linear Flight Controllers ..... 6
2.2 Nonlinear Flight Controllers ..... 8
2.2.1 Backstepping (BKS) ..... 8
2.2.2 Nonlinear Dynamic Inversion (NDI) ..... 10
3 The Flying-Wing Model ..... 13
3.1 Nonlinear Model ..... 16
3.2 Flying Wing Simulator ..... 19
3.3 Flight Envelope ..... 20
3.4 Trim or Equilibrium Conditions ..... 24
3.5 Flying-Wing Linearized Model ..... 25
3.6 Model Analysis ..... 26
3.6.1 Lateral Model ..... 27
3.6.2 Longitudinal Model ..... 30
3.7 Flight Quality Analysis ..... 33
4 Flying Wing Control ..... 39
4.1 Control Approaches ..... 40
4.1.1 Classical Control Approach ..... 40
4.1.1.A Root Locus ..... 40
4.1.1.B Linear Quadratic Regulator (LQR) ..... 40
4.1.2 Nonlinear Control Approach - INDI ..... 42
4.2 Control Objectives ..... 43
4.3 Stability Augmentation System (SAS) - Stability Augmentation System ..... 44
4.4 Attitude Control ..... 46
4.4.1 Classical Approach: Linear Control ..... 47
4.4.1.A Lateral Model ..... 47
4.4.1.B Longitudinal Model ..... 47
4.4.1.C Nonlinear Model ..... 49
4.4.2 Gain Scheduling ..... 50
4.4.3 Nonlinear Control ..... 53
4.5 Guidance ..... 57
4.5.1 Course Angle Control - $\chi$ ..... 57
4.5.1.A Linear Control ..... 57
4.5.1.B Nonlinear Control ..... 60
4.5.2 Guidance by Waypoints ..... 60
4.5.3 Path-following Performance ..... 62
4.5.3.A Wind Disturbances ..... 63
4.5.3.B Sensor Noise ..... 64
4.5.4 Actuators Request ..... 65
5 Final Conclusion and Future Work ..... 67
A Appendix ..... 75
A. 1 Flight Quality Level ..... 76
B Appendix ..... 81

## List of Figures

1.1 Flying Wing Mission 1: HAB and Unmanned Aerial Vehicle (UAV) ascent; 2: UAV release and descent with parachute; 3: UAV controlled descent ..... 3
2.1 Nonlinear Control Methods - (Adapted from [1]) ..... 8
2.2 NDI representation ..... 10
3.1 North-East-Down (NED) frame and the Body frame or Aircraft Body Centered (ABC) frame. (Adapted from [2]) ..... 14
3.2 ABC referential and Euler angles in the Flying Wing ..... 15
3.3 Aerodynamic Angles - Angle of Attack, $\alpha$, and Sideslip Angle, $\beta$. ..... 16
3.4 Nonlinear Model - Flying Wing Open-Loop Model ..... 19
3.5 Actuator Block ..... 20
$3.6 \mathrm{C}_{L}-\alpha$ ..... 21
3.7 Theoretical Flight Envelope ..... 21
3.8 Flight Envelope ..... 23
3.9 Trim condition for the flying wing model without motor varying the airspeed and setting an altitude. $\quad(\mathrm{h}=500 ; \mathrm{h}=2500 ; \mathrm{h}=5000 \mathrm{~m} ; \mathrm{h}=7500 \mathrm{~m} ; \mathrm{h}=10000 \mathrm{~m} ; \mathrm{h}=12500 \mathrm{~m} ; \mathrm{h}=15000 \mathrm{~m}$; $h=16000 \mathrm{~m} ; \mathrm{h}=17500 \mathrm{~m} ; \mathrm{h}=20000 \mathrm{~m}$.) ..... 24
3.10 Evolution of the poles for $h=5000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from
$V_{t}=20 \mathrm{~m} / \mathrm{s}(\mathrm{X})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{O}) .(\triangle)-V_{t}=25 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=35 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=45 \mathrm{~m} / \mathrm{s}$ ..... 28
3.11 Evolution of the poles for $h=10000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from
$V_{t}=25 \mathrm{~m} / \mathrm{s}(\mathrm{X})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{O}) .(\triangle)-V_{t}=30 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=40 \mathrm{~m} / \mathrm{s}$. ..... 28
3.12 Evolution of the lateral poles for $V_{t}=30 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of 100 mand from $h=1 m(\mathrm{X})$ to $h=10000 m(\mathrm{O}) .(\triangle)-h=500 m ;(\triangle)-h=2500 m ;(\triangle)-$$h=5000 m ;(\triangle)-h=7500 m$29
3.13 Evolution of the lateral poles for $V_{t}=47 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of 500 m and from $h=1 m(\mathrm{X})$ to $h=20000 m(\mathrm{O}) .(\triangle)-h=500 m ;(\triangle)-h=5500 m ;(\triangle)-$ $h=10500 m ;(\triangle)-h=15500 m$ ..... 29
3.14 Evolution of the longitudinal poles for $h=5000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from $V_{t}=20 \mathrm{~m} / \mathrm{s}(\mathrm{O})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{X}) .(\triangle)-V_{t}=25 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=35 \mathrm{~m} / \mathrm{s} ;$ $(\triangle)-V_{t}=45 \mathrm{~m} / \mathrm{s}$ ..... 31
3.15 Evolution of the longitudinal poles for $h=10000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from $V_{t}=25 \mathrm{~m} / \mathrm{s}(\mathrm{X})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{O}) .(\triangle)-V_{t}=30 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=40 \mathrm{~m} / \mathrm{s}$. ..... 31
3.16 Evolution of the longitudinal poles for $V_{t}=30 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of $100 m$ and from $h=1 m(\mathrm{X})$ to $h=10000 m(\mathrm{O}) \cdot(\triangle)-h=500 m ;(\triangle)-h=2500 m ;(\triangle)$ $-h=5000 m ;(\triangle)-h=7500 m$ ..... 32
3.17 Evolution of the longitudinal poles for $V_{t}=47 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of $500 m$ and from $h=1 m(\mathrm{X})$, to $h=20000 m,(\mathrm{O}) .(\triangle)-h=500 m ;(\triangle)-h=5500 m ;(\triangle)$ - $h=10500 m ;(\triangle)-h=15500 m$ ..... 32
3.18 Evolution of the lateral poles for different flight conditions. o - $h=1 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant$ $50 \mathrm{~m} / \mathrm{s} ; \mathrm{o}-h=500 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ; \mathrm{o}-h=1000 \mathrm{~m}$ and $20 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ o- $h=5000 \mathrm{~m}$ and $25 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; о $-h=15000 \mathrm{~m}$ and $30 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; o- $h=20000 \mathrm{~m}$ and $47 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$ ..... 33
3.19 Evolution of the longitudinal poles for different flight conditions. o - $h=1 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant$ $V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ о $-h=500 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ o $-h=1000 \mathrm{~m}$ and $20 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant$ $50 \mathrm{~m} / \mathrm{s} ;-h=5000 \mathrm{~m}$ and $25 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ o $-h=15000 \mathrm{~m}$ and $30 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant$ $50 \mathrm{~m} / \mathrm{s} ; \mathrm{o}-\mathrm{h}=20000 \mathrm{~m}$ and $47 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$ ..... 34
3.20 Dutch Roll Mode poles ..... 36
4.1 Linear control in the outer loop and Inner INDI loop ..... 43
4.2 Flying Wing controller - Inner and outer system loops. ..... 43
4.3 Root-Locus - SAS Lateral Model ..... 45
4.4 Root-Locus - SAS Longitudinal Model ..... 45
4.5 Linear attitude control and stabilization - Lateral Model ..... 46
4.6 Linear attitude control and stabilization - Longitudinal Model ..... 46
4.7 Linear attitude control and stabilization - Nonlinear System ..... 47
4.8 Time Domain Responses for linear controller (figure 4.7). (...) - $\phi_{\text {Ref }}=20^{\circ}$ and $\theta_{\text {Trim }}$;$(-) \phi$ and $\theta$ time domain responses.50
4.9 Time Domain Responses for the system configuration (figure 4.7). (...) - $\phi_{R e f} . V_{t}=15 \mathrm{~m} / \mathrm{s}$and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}(-)$;$V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$.52
4.10 Time Domain Responses for the system configuration (figure 4.7). (...) $-\theta_{\text {Ref }} . V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}(-)$; $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$. ..... 52
4.11 INDI attitude control and stabilization ..... 53
4.12 Time Domain Responses for the system configuration (figure 4.1). (...) - $\phi_{R e f} . V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}(\quad)$; $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$. ..... 56
4.13 Time Domain Responses for the system configuration (figure 4.1). (...) - $\phi_{R e f} . V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}(-)$; $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$. ..... 56
$4.14 \psi$ and $\chi$ time domain responses for a Nonlinear Attitude Control (figure 4.11) and for $h=15000 \mathrm{~m}$ and $V_{t}=35 \mathrm{~m} / \mathrm{s}$. ..... 57
$4.15 \chi$ Control - Closed Loop System block represents the block diagram represented in figure 4.7584.16 Root locus to $\chi(V t=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m})$58
$4.17 \chi$ and $\theta$ time responses for a flight condition of $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$ for a Linear controller. (-) - $\theta_{\text {Trim }}$ and $\phi_{\text {Ref }}=20^{\circ} ;(-)-\theta$ and $\phi$ time responses. ..... 59
$4.18 \chi$ Control - considering the approximation of a coordinated flight for a $\phi-\chi$ transformation. Low Level (LL) INDI defined by the $2^{\text {nd }}$ order transfer function (4.25) ..... 60
$4.19 \chi$ and $\theta$ time responses for a flight condition of $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$ and for a Linear and an INDI controller. (-) - $\theta_{\text {Trim }}$ and $\phi_{\text {Ref }}=20^{\circ} ;(-)-\theta$ and $\phi$ time responses. ..... 60
4.20 Path Following - Waypoints; Segments; Incidence area of each waypoint; $\chi_{\text {Ref }}$ and $\chi$ measurement. ..... 61
4.21 2D and 3D trajectory for a no-wind flight controlled by a linear control approach. Way- points - (X). Flying wing trajectory - (-). ..... 62
4.22 2D and 3D trajectory for a no-wind flight controlled by a nonlinear control approach. Waypoints - (X). Flying wing trajectory - (-). ..... 62
4.23 Path-following considering a $10 \mathrm{~m} / \mathrm{s}$ North wind: INDI control for a $r_{1}$ boundary (...); Linear control for a $r_{1}$ boundary ( - ) Linear control for a $r_{2}$ boundary ( $\ldots$ ). ( $r_{2}>r_{1}$ ) . . ..... 63
4.24 Time domain responses considering the sensor noise effect for Linear and Nonlinear Con- trollers. (... No-Sensor Noise; - Sensor Noise.) ..... 64
4.25 Sensor noise effect on the path-following for a no-wind gliding flight. (-) -INDI control approach; (-) -Linear control approach. ..... 65
4.26 Actuators request during the path-following: $(-)-\delta_{A} ;(-)-\delta_{E} ;(-)-\delta_{T}$ ..... 65

## List of Tables

3.1 Rolling Subsidence Mode - Flight Quality Level ..... 35
3.2 Dutch Roll Mode - Flight Quality Level ..... 35
3.3 Spiral Mode - Flight Quality Level ..... 35
3.4 Short Period Mode - Flight Quality Level ..... 35
3.5 Phugoid Mode - Flight Quality Level ..... 36
4.1 Flight quality level for $h=15000 \mathrm{~m}$ and $V_{t}=35 \mathrm{~m} / \mathrm{s}$ ..... 44
4.2 Flight quality level for $h=15000 m$ and $V_{t}=35 \mathrm{~m} / \mathrm{s}$ - Level 1 ..... 45
4.3 Lateral Model - Uncontrolled and Controlled System ..... 47
4.4 Longitudinal Model - Uncontrolled and Controlled System ..... 49
4.5 Gain Scheduling - Lateral attitude control design. The letter C represents the flight con- dition for which the control was designed. $K_{\phi}$ for each airspeed ..... 51
4.6 Gain Scheduling - $\chi$ control design. The letter C represents the flight condition for which the control was designed. $K_{\chi}$ for each airspeed. The $N / A$ area represents flight conditions that do not belong to the flight envelope. ..... 59
5.1 Controllers' comparison summary ..... 68
A. 1 Flight Quality Level - Lateral Model ( $\mathrm{h}=1 \mathrm{~m}$ - $\mathrm{h}=7500 \mathrm{~m}$ ) ..... 76
A. 2 Flight Quality Level - Lateral Model (h=10000m - h=20000m) ..... 77
A. 3 Flight Quality Level - Longitudinal Model ( $\mathrm{h}=1 \mathrm{~m}-\mathrm{h}=7500 \mathrm{~m}$ ) ..... 78
A. 4 Flight Quality Level - Longitudinal Model ( $\mathrm{h}=10000 \mathrm{~m}-\mathrm{h}=20000 \mathrm{~m}$ ) ..... 79
B. 1 Path-Following Performance - Linear Control Approach ..... 82
B. 2 Path-Following Performance - INDI Control Approach ..... 82

## Acronyms

| SISO | Single-Input/Single Output |
| :---: | :---: |
| LQR | Linear Quadratic Regulator |
| MIMO | Multi-Input/Multi-Output |
| NED | North-East-Down |
| FBL | Feedback Linearization |
| BKS | Backstepping |
| NDI | Nonlinear Dynamic Inversion |
| INDI | Incremental Nonlinear Dynamic Inversion |
| IBKS | Incremental Backstepping |
| ABC | Aircraft Body Centered |
| MAC | Mean Aerodynamic Chord |
| UAV | Unmanned Aerial Vehicle |
| UAVs | Unmanned Aerial Vehicles |
| HAB | High Altitude Balloon |
| SAS | Stability Augmentation System |
| IPMA | Instituto Português do Mar e da Atmosfera |
| GPS | Global Positioning System |
| IMU | Inertial Measurement Unit |
| PP | Polo Placement |

## Nomenclature

## Superscripts

$T \quad$ Transpose

## Subscripts

$0 \quad$ Current observation
$a \quad$ Air
$B \quad$ Body frame
d Desired
$E \quad$ Earth frame
$e \quad$ Error
st Stall
$w \quad$ Wind

Variables description
$\alpha \quad$ Angle of attack
$\beta \quad$ Sideslip angle
$\chi \quad$ Course angle
$\delta_{A} \quad$ Aileron deflection
$\delta_{E} \quad$ Elevator deflection
$\delta_{T} \quad$ Motor Throttle
$\lambda \quad$ Input scaling gain
$\nu \quad$ Virtual control
$\Omega \quad$ Propeller angular speed
$\omega_{n} \quad$ Natural frequency
$\phi \quad$ Roll angle
$\psi \quad$ Yaw angle
$\rho \quad$ Air density
$\theta \quad$ Pitch angle
$\xi \quad$ Damping Factor
$b$ Wing span
c Mean Aerodynamic Chord
$C_{D} \quad$ Drag coefficient
$C_{L} \quad$ Lift coefficient
$C_{l} \quad$ Rolling moment
$C_{m} \quad$ Pitching moment
$C_{n} \quad$ Yawing moment
$C_{Y} \quad$ Sideforce coefficient
$g \quad$ Gravity acceleration
$h \quad$ Altitude
$I_{m} \quad$ Motor current
$J \quad$ Cost function
$L \quad$ Lift
$m \quad$ Inertial mass
$p \quad$ Roll rate
$q \quad$ Pitch rate
$r \quad$ Yaw rate
$S$
$T \quad$ Constant time
$T_{2} \quad$ Time to double
$u \quad$ Longitudinal velocity
$v \quad$ Lateral velocity
$V_{t} \quad$ Airspeed

W Lyapunov function
$w \quad$ Vertical velocity
$X \quad$ Longitudinal force
$Y \quad$ Lateral force
$Z \quad$ Normal force
$\omega \quad$ Angular velocity vector
$\Phi \quad$ Angular position vector

A n-state dynamic matrix

B m-input matrix

D Disturbance vector

E External force

I Inertia matrix

K Gain matrix

M External Moment

P Position vector

Q State weight matrix
q Quaternions

R Input weight matrix
u Input vector

X $\quad$ State vector

D Down position

E East position

N North position

## Introduction

1.1 Motivation and Mission ..... 2
1.2 Objectives ..... 4
1.3 Thesis Outline ..... 4

### 1.1 Motivation and Mission

In the past years, important progresses have been made by the aeronautics industry. Despite those, fixedwing aircraft remain the most common aircraft configurations for aeronautical applications. Furthermore, those aircraft configurations have not changed significantly since the beginning of the flight era. The high confidence given to the conventional aircraft, the well-developed stability and control studies as well as structural reliability are the evidences that support the popularity of using these configurations [3].

A flying wing is a tailless aircraft and represents the simplest design configuration of a flying machine. Compared with the conventional aircraft, flying-wing aircraft has advantages on structural strength and aerodynamic aspects. Its configuration minimizes the drag and the aircraft weight. However, cancelling the tail leads to course stability weakness. Additionally, the coupling between lateral and longitudinal motion is more pronounced on flying wings than that in conventional aircraft [4].

Currently, due to the excellent performance in the slow-to-medium speed range, the interest of using flying wing aircraft to military and commercial applications has increased. Moreover, during the last decade, the widespread development of increasingly advanced Unmanned Aerial Vehicles (UAVs) further increased this design interest.

Recently, with the development of flight control techniques and high performance computational technologies, the maneuverability of flying wing Unmanned Aerial Vehicle (UAV) has attracted attentions [3]. Due to this growing interest in the applicability of a flying wing UAV and under the HABAIR (Innovative High Altitude Balloon for Atlantic Observation: Fostering the Development of a Collaborative Platform for Integrated Aerial and Oceanic Research) project, it is intended to combine a flying wing UAV to respond to this project's aim: the design and the development of an aerial hybrid platform that allows the precise positioning of scientific payloads for atmospheric and/or oceanic monitoring. The proposed solution consists of a High Altitude Balloon (HAB) that will carry a flying wing UAV to be released and guided to a predetermined location [5]. With this intention, the HABAIR mission admits essentially two steps to achieve a better atmospheric and oceanic monitoring with a precise acquisition of data:

- The HAB motion responsible for the ascent in altitude of the payload.
- The flying wing UAV motion responsible for the guided descent and controlled release of probes.

The considered solution provides an economic and viable option of scientific payload transportation to remote areas and the acquisition of data otherwise unreachable. This platform is an innovative solution because it is a cost-effective way of covering an extensive area for different altitudes and velocities [6]. Moreover, it has a high range of applications. Some of the advantages of this data acquisition platform are the atmosphere monitoring, the study of the volcanic activity and the assistance in rescue missions. Considering a typical mission, the flying wing's motion includes essentially three stages. As mentioned
before, the first step is the ascent movement of the flying wing carried by the HAB. At a desired location (defined by altitude or latitude/longitude coordinates), the UAV is released from the HAB. In this transition phase, the UAV descent is supported by a parachute. When the UAV gains stability and lift in its descent, it can then be released from the parachute and start its controlled flight. This study deals with this last phase, developing control solutions to guarantee a waypoint-defined controlled descent of the UAV. During this study is intended to guarantee a guidance by waypoints.


Figure 1.1: Flying Wing Mission 1: HAB and UAV ascent; 2: UAV release and descent with parachute; 3: UAV controlled descent

Besides its structural robustness and simplicity, an important reason for choosing the flying wing aircraft is its higher aerodynamic lift when compared with other fixed-wing aircraft. This capacity allows to save the UAV power during the descent, thus extending its range, by defining a trajectory achievable using only the control surfaces. The motor is used only as last resource, namely for landing or for more (and usually rare) demanding trajectories.

Considering the objective mission, it is notable the high range of flight conditions under consideration in the flying wing control design. The wide flight envelope, the existing nonlinearities and the missiondependent control allocation are some of the challenges faced, requiring that other solutions be considered besides the well known gain-scheduling of linear controllers.

### 1.2 Objectives

Considering the mission and the difficulties mentioned for the flying wing control, this work intends to report and analyse the following points:

- Identify the flight conditions range (altitudes and airspeeds), flight envelope, for the flying wing model;
- Develop a flight quality analysis;
- Obtain a feasible nonlinear controller for the flying wing stabilization that addresses the mentioned challenges and compare its performance with a gain-scheduling control solution under wind disturbances;
- Develop a path-following algorithm to perform a guidance by waypoints of the flying wing descent motion considering a gliding flight.

This report is presented considering the simulation results obtained by the utilization of a given flying wing Matlab ${ }^{\circledR} /$ Simulink ${ }^{\circledR}$ model and whose design is not included in this thesis' objectives.

### 1.3 Thesis Outline

In order to achieve the objectives previously defined, the structure of this thesis is divided into five chapters.

- The first chapter introduce an overview about the problem and mission;
- The second chapter presents a state of the art on linear and nonlinear flight controllers;
- The third, presents a detailed analysis of the flying wing model;
- The fourth suggests a flying wing control in order to stabilize the flight and to guarantee a precise path-following;
- Finally, in the last and fifth chapter the controllers developed to accomplish the objectives are analyzed and compared.


## 2

## State of the Art

Contents

Linear control includes several approaches whose research has been greatly developed over the years. Those have been used successfully for different control problems [7]. Linear control methods require an operation range for which its implementation is considered valid. However, for a large operation range it is necessary to analyse if the presence of nonlinearities in the system affect or not the success of the control performance [8]. Moreover, using linear controllers it is assumed that the system is linearizable. However, there are discontinuities that make this linearization difficult or impossible. Nonlinear controllers are able to handle with the nonlinearities of the system. Also, many control problems involve uncertainties in the model parameters, linear controllers are more sensitive to these uncertainties than the nonlinear controllers [9]. Finally, depending on the system to be controlled linear controllers may present a design more complex than nonlinear controllers and may require high quality actuators and sensors to produce linear behavior in the specified operation range. Nevertheless, linear controllers are the most widely used in automatic flight control systems. Considering this evidence, it is important to analyse the theory that supports the linear and nonlinear controllers [1] [10].

### 2.1 Linear Flight Controllers

Physical systems are nonlinear systems that can be described by nonlinear differential equations. However, it is also possible to approximate a nonlinear system into a linearized system if the nonlinearities of the system does not affect the success of the control performance and if the operation range is reasonably small.

According to Katsuhiko Ogata [11], a linear system assumes that the system response to multipleinput is calculated by treating one input at a time and adding the results. Following this definition a nonlinear system does not verify the principle of superposition, however, in control engineering it is possible to obtain a linear mathematical model for a nonlinear system assuming that the variables deviate only slightly from the operating condition.

Considering the design of linear controllers, it is possible to implement different control approaches within the Classical and Modern control theories. Usually, the Polo Placement (PP) and the Linear Quadratic Regulator (LQR) are the control approaches most used in linear control and for Multi-Input/Multi-Output (MIMO) systems [2]. Moreover, in flight control, the desirable pole locations are stated from the flying qualities specifications. The flight qualities' specifications are setting according to the formalization provided by Donald McLean [9]. Firstly, following this formalization, it is necessary to establish the aircraft class and the flight phase. Then, the flying qualities are specified in terms of parameters such as short period damping, natural frequency of yawing motion and roll subsidence time constant. The conventional design control methods are used in Single-Input/Single Output (SISO) systems and it implementation requires considerable experience and time-consuming in order to achieve a successful design control.

Furthermore, Modern control theory can be implemented to optimize the design of the control system. This theory is characterized as a parameter optimization [12]. Modern control or optimal control has as main objective the determination of the control signals that satisfy the physical constraints and at the same time minimize or maximize the performance index. This control theory deal with MIMO systems.

LQR is an optimal control approach that provides a stable closed-loop system and produce a full state feedback. It aim is the minimization of a quadratic cost function restricted by the system dynamics and based on the states and input weights [13]. The cost function minimization provides an optimal feedback of the states. The cost function shapes the state evolution and the amplitude of the control action. Those parameters have an associated weight: state and input weight. An optimal balance between both weights determine the performance of the control law to achieve the desired control specifications.

As mentioned before, it is possible to control a nonlinear system performing linear control approaches. However, it is important to take into account the operation range of a linear controller. For this mission, the airspeed and the altitude are the parameters that define an equilibrium condition [14]. For this reason, it is important to understand that a single linear controller is not valid for the entire flight envelope. Changing the flight conditions, the control has to change. This solution is given by a gain scheduling on the system. Implementing this technique the flight envelope is divided into several operation regimes and for each one a conventional controller is designed. Establishing this scheduling a satisfactory control performance over the flight envelope is guaranteed. Gain-scheduling is the solution commonly applied in flight control, however, this technique presents some weaknesses: systems with a significant number of nonlinearities require a complex scheduling to reduce the performance degradation between each central solution choose to design the controller; a gain scheduling success depends on the division defined and there is not a specific and systematic approach to define an optimized division; it is a time-consuming technique and requires also an high computational power [15].

In order to improve those weaknesses and to increase the robustness of the control approach the nonlinear controller are designed [16].

### 2.2 Nonlinear Flight Controllers

According to Paul Acquatella [1] among the nonlinear control methods, Nonlinear Dynamic Inversion (NDI) and Backstepping (BKS) are well-known nonlinear control approaches and the most common in flight control problems. In the figure 2.1 it is presented a nonlinear control framework with the division between the most common nonlinear control approaches for Aerospace applications. It is important to empathize that there are more nonlinear control approaches that are able to be implemented to control a nonlinear system: Neural Networks and Fuzzy Control methods are examples of nonlinear controllers. In the Aerospace field, the application of Feedback Linearization (FBL) is commonly referred to as


Figure 2.1: Nonlinear Control Methods - (Adapted from [1])

NDI [1]. Considering the Lyapunov-based control methods it is possible to highlight the BKS control approach [10], [17] as the control approach commonly used for flight control applications.

### 2.2.1 BKS

BKS is a nonlinear control approach substantially modern. It first reference date back from the nineties (1991) [18]. However, it implementation on flight control problems comes years later with the elimination of overparameterization [19] [20].

The BKS control approach provides a systematic methodology based on the Lyapunov theory. Considering this method, it is possible to design a controller that guarantee a global stability [21] [22].

BKS control approach can be summarized by two main ideas intrinsically related: the choice of a Lyapunov function and the design of a feedback control. The control law is calculated together with a Lyapunov function to ensure a global stability at each step. The BKS approach is defined essentially by the following steps [23]:

- Rewrite a state equation in function of a scalar parameter and auxiliary states;
- Choose a Lyapunov function and treating it as a final stage;
- Choose an equation for the scalar parameter that stabilizes the selected Lyapunov function.

To represent the backstepping design approach as presented in [2], it is considered a generic problem with an output $\mathbf{y}$, a positive scalar parameter $a$ and two auxiliary outputs $\mathbf{y}_{1}$ and $\mathbf{y}_{2}$.

$$
\begin{gather*}
\mathbf{y}_{1}=a \mathbf{y}+\dot{\mathbf{y}}  \tag{2.1}\\
\mathbf{y}_{2}=\dot{\mathbf{y}} \tag{2.2}
\end{gather*}
$$

Deriving both equations,

$$
\begin{gather*}
\dot{\mathbf{y}}_{1}=a \dot{\mathbf{y}}+\ddot{\mathbf{y}}  \tag{2.3}\\
\dot{\mathbf{y}}_{2}=\ddot{\mathbf{y}} \tag{2.4}
\end{gather*}
$$

Considering a Lyapunov-candidate-function $W_{0}(x)(2.5)$ globally positive definite and radially unbounded $\left(\|x\| \rightarrow+\infty \Rightarrow W_{0}(x) \rightarrow+\infty\right):$

$$
\begin{equation*}
W_{0}=\frac{1}{2} \mathbf{y}_{1}^{T} \mathbf{y}_{1}+\frac{1}{2} \mathbf{y}_{2}^{T} \mathbf{y}_{2} \tag{2.5}
\end{equation*}
$$

$\dot{W}_{0}$ is defined by,

$$
\begin{equation*}
\dot{W}_{0}=\mathbf{y}_{1}^{T} \dot{\mathbf{y}}_{1}+\mathbf{y}_{2}^{T} \dot{\mathbf{y}}_{2}=(a \mathbf{y}+\dot{\mathbf{y}})^{T}(a \dot{\mathbf{y}}+\ddot{\mathbf{y}})+\dot{\mathbf{y}}^{T} \ddot{\mathbf{y}}=(a \mathbf{y}+2 \dot{\mathbf{y}})^{T}(a \dot{\mathbf{y}}+\ddot{\mathbf{y}})-a \dot{\mathbf{y}}^{T} \dot{\mathbf{y}} \tag{2.6}
\end{equation*}
$$

Considering $\boldsymbol{\Lambda}$ as a positive definite matrix and defining,

$$
\begin{equation*}
a \dot{\mathbf{y}}+\ddot{\mathbf{y}}=-\boldsymbol{\Lambda}(a \mathbf{y}+2 \dot{\mathbf{y}}) \tag{2.7}
\end{equation*}
$$

it is possible to rewrite the equation (2.6) as,

$$
\begin{equation*}
\dot{W}_{0}=-(a \mathbf{y}+2 \dot{\mathbf{y}})^{T} \boldsymbol{\Lambda}(a \mathbf{y}+2 \dot{\mathbf{y}})-a \dot{\mathbf{y}}^{T} \dot{\mathbf{y}} \tag{2.8}
\end{equation*}
$$

Analysing the equation (2.8), it is seen that $\dot{W}_{0}$ is negative definite $\left(\dot{W}_{0}<0\right)$ and for this reason, considering the Lyapunov theory, the system is globally asymptotically stable.

### 2.2.2 NDI

NDI has verified successful results in flight control researches [4], [24], [25], [26], [27]. The NDI may be explained by a simple principle: the nonlinear system is inverted by means of state feedback resulting in linear closed-loop dynamics. A NDI controller eliminates the nonlinearities in the model by cancel them with state feedback. And then, a linear controller is designed considering classical control approach to obtain the desired performance of the closed loop system.

The NDI principles are summarized in the equations (2.9)-(2.11): $x$ is the state vector, $u$ is the control input, $f(x)$ and $g(x)$ are the nonlinear functions of the state vector and $\nu$ is the virtual control input. The equation (2.9) summarizes the affine in control system dynamics. The equation (2.10) represents the inversion step. And finally, the equation (2.11) presents the linear control approach. A NDI controller is represented by the block diagram in figure 2.2.

$$
\begin{gather*}
\dot{x}=f(x)+g(x) \mathbf{u}  \tag{2.9}\\
u=G^{-1}(\nu-f(x))  \tag{2.10}\\
\nu=K_{p}\left(x_{d}-x\right)-K_{d}(\dot{x}) \tag{2.11}
\end{gather*}
$$



Figure 2.2: NDI representation

However, this control approach presents some limitations and weaknesses. Model mismatches and measurement errors reduces the NDI performance. The dynamics of the system has to be well-known in order to implement this methodology. In order to deal with those problems, an Incremental Nonlinear Dynamic Inversion (INDI) approach can be implemented to control a nonlinear system. This approach
decreases the model dependency and demonstrates a robustness performance regarding the uncertainties and measurement errors.

Beside that, this control approach is considerably recent, according to Paul Acquatella [1] it date back from the late nineties with the research performed by Smith [28]. However, the application of this control method in flight control problems comes years later with the Smith Berry researches [29].

For aeronautical applications INDI has stood out as a very powerful control tool. In contrast with regular NDI, this method is inherently implicit because closed-loop dynamics are obtained when the loop is closed through feedback without explicit knowledge of the whole model [30]. An INDI approach requires control increments that are obtained from the sensors data. Using the data that come from the sensors instead of the information that comes from the nonlinear model is verified a reduction on the model dependence.

For those reason, INDI is the nonlinear control approach used during this study to control the flying wing flight. The theory that sustains this control approach is presented in the chapter 4.

## 3

## The Flying-Wing Model

## Contents

3.1 Nonlinear Model ..... 16
3.2 Flying Wing Simulator ..... 19
3.3 Flight Envelope ..... 20
3.4 Trim or Equilibrium Conditions ..... 24
3.5 Flying-Wing Linearized Model ..... 25
3.6 Model Analysis ..... 26
3.7 Flight Quality Analysis ..... 33

In order to describe the dynamic of a Flying Wing it is essential to define the referentials to which the flying wing motion is associated. In this study two referentials are used: The North-East-Down (NED) frame and the Body frame or Aircraft Body Centered (ABC) frame [31].


Figure 3.1: NED frame and the Body frame or ABC frame. (Adapted from [2])

The NED frame is connected to the Earth's surface and it is defined by $x_{E}, y_{E}$ and $z_{E}$ axes (figure 3.1). In this representation, the two first axes $\left(x_{E}\right.$ and $\left.y_{E}\right)$ are tangent to the meridian and to the parallel of the Earth. The $x_{E}$ axis indicates the North and the $y_{E}$ axis indicates the East. This referential will be assumed as inertial. The body frame is the local referential and it is centered on the Flying-Wing's center of gravity. This referential is known as moving referential and it is defined by the $x_{B}, y_{B}$ and $z_{B}$ axes (figure 3.1).

In order to transform one referential to another a rotation has to be performed. $\mathbf{R}_{E}^{B}$ is the transformation matrix that allows this change: from the Earth's referential to the Body's referential (3.1). For this reason, $\mathbf{v}_{B}=\mathbf{R}_{E}^{B} \mathbf{v}_{E}$ and $\mathbf{v}_{E}=\mathbf{R}_{E}^{B^{T}} \mathbf{v}_{B} . B$ and $E$ are the indexes to identify the frames: Body or Earth frame, respectively. $\mathbf{R}_{E}^{B}$ is expressed by quaternions: $q_{0}, q_{1}, q_{2}$ and $q_{3}$. The quaternions notation is more difficult to visualize than the Euler notation, however, it provides a more compact, stable and efficient spacial representation, avoiding singularity problems.

$$
\mathbf{R}_{E}^{B}=\left[\begin{array}{ccc}
q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} & 2\left(q_{1} q_{2}+q_{0} q_{3}\right) & 2\left(q_{1} q_{3}-q_{0} q_{2}\right)  \tag{3.1}\\
2\left(q_{1} q_{2}-q_{0} q_{3}\right) & q_{0}^{2}-q_{1}^{2}+q_{2}^{2}-q_{3}^{2} & 2\left(q_{2} q_{3}+q_{0} q_{1}\right) \\
2\left(q_{0} q_{2}+q_{1} q_{3}\right) & 2\left(q_{2} q_{3}-q_{0} q_{1}\right) & q_{0}^{2}-q_{1}^{2}-q_{2}^{2}+q_{3}^{2}
\end{array}\right]
$$

In order to facilitate the visualization, it is also possible to represent the orientation of the flying wing by Euler notation. For this, it is only necessary to transform the quaternions into Euler angles according
to,

$$
\left[\begin{array}{c}
\phi  \tag{3.2}\\
\theta \\
\psi
\end{array}\right]=\left[\begin{array}{l}
\arctan \left(\frac{\mathbf{R}_{E}^{B}(2,3)}{\mathbf{R}_{E}^{B}(3,3)}\right) \\
\arcsin \left(\mathbf{R}_{E}^{B}(1,3)\right) \\
\arctan \left(\frac{\mathbf{R}_{E}^{B}(1,2)}{\mathbf{R}_{E}^{B}(1,1)}\right)
\end{array}\right]\left[\begin{array}{l}
\arctan \left(\frac{2\left(q_{0} q_{1}+q_{2} q_{3}\right)}{1-2\left(q_{1}{ }^{2}+q_{2}{ }^{2}\right)}\right) \\
\arcsin \left(2\left(q_{0} q_{2}-q_{3} q_{1}\right)\right) \\
\arctan \left(\frac{2\left(q_{0} q_{3}+q_{1} q_{2}\right)}{1-2\left(q_{2}{ }^{2}+q_{3}{ }^{2}\right)}\right)
\end{array}\right]
$$



Figure 3.2: $A B C$ referential and Euler angles in the Flying Wing

In the Euler angles representation, $\phi$ is the rolling angle $(\phi \in[-\pi, \pi]), \theta$ is the pitch angle $\left(\theta \in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$ and $\psi$ is the yaw angle $(\psi \in[0,2 \pi])$.

Finally, it is important to understand the nomenclature used to describe the Flying Wing motion. Also, to describe the movement and to understand the Flying Wing model, the body frame is the referential through that the forces, moments and velocities are referenced to. For this reason, it is possible to describe the vectors bellow that describe the flying wing motion.

- Air Velocity: $\mathbf{V}_{a}=[U V W]_{B}{ }^{T}$
- Angular Velocity: $\omega=[P Q R]_{B}{ }^{T}$
- External Force: $\mathbf{F}=[X Y Z]_{B}{ }^{T}$
- External Moment: $\mathbf{M}=[L M N]_{B}{ }^{T}$
- Position: $\mathbf{P}=[N E D]_{E}{ }^{T}$

Moreover, in order to represent the orientation of the airspeed it is important to represent the graphical representation of the aerodynamic angles: the angle of attack, $\alpha$, and the sideslip angle, $\beta$. As represented in figure $3.3, \alpha$ is the angle of rotation about the $y_{B}$-axis in order to overlap the $x_{B}$-axis plane and the $x_{a}$-axis plane. And finally, $\beta$ is the angle of rotation about the $z$-axis in order to overlap the axis $x_{B}$ and $x_{a}$.

Representing this transformation, the equation (3.3) presents the rotation of wind axes to body axes. According to this nomenclature, the air velocity vector can be expressed in the body referential as represented by the equation (3.4). Here, $V_{t}$ is the air velocity vector norm.

$$
\mathbf{R}_{a}^{B}=\left[\begin{array}{ccc}
\cos \alpha & 0 & -\sin \alpha  \tag{3.3}\\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{array}\right]\left[\begin{array}{ccc}
\cos \beta & -\sin \beta & 0 \\
\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{array}\right]
$$



Figure 3.3: Aerodynamic Angles - Angle of Attack, $\alpha$, and Sideslip Angle, $\beta$.

$$
\mathbf{V}_{a B}=V_{t}\left[\begin{array}{c}
\cos (\alpha) \cos (\beta)  \tag{3.4}\\
\sin (\beta) \\
\sin (\alpha) \cos (\beta)
\end{array}\right]_{B}
$$

### 3.1 Nonlinear Model

The flying wing model is obtained by its dynamics and kinematics equations and the relation between the model inputs and the variables [9] [14] [32]. The nonlinear flying wing model is a function of the actuators input, $u$, the wind disturbances, $D$, and the state variables, $x$, all function of time, t .

$$
\begin{equation*}
\dot{x}=f(x, u, D, t) \tag{3.5}
\end{equation*}
$$

## Flying-Wing dynamics

According to the Newton-Euler formulation it is possible to obtain the dynamic equations of the Flying Wing. In this formulation, the linear and angular moment are distinguished. Firstly, from the inertial referential, NED referential, the moment linear and angular are expressed by (3.6) and (3.7).

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} t}[m \mathbf{V}]_{E}=\mathbf{F} \tag{3.6}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} t}[\mathbf{I} \omega]_{E}=\mathbf{M} \tag{3.7}
\end{equation*}
$$

In the local referential, body frame, the linear and angular moments are expressed by (3.8) and (3.9), respectively.

$$
\begin{gather*}
\frac{\mathrm{d}}{\mathrm{~d} t}[m \mathbf{V}]_{B}+\omega \times[m \mathbf{V}]_{B}=\mathbf{F}  \tag{3.8}\\
\frac{\mathrm{d}}{\mathrm{~d} t}[\mathbf{I} \omega]_{B}+\omega \times[\mathbf{I} \omega]_{B}=\mathbf{M} \tag{3.9}
\end{gather*}
$$

## Flying-Wing kinematics

The position of the flying wing is expressed by $\mathbf{P}$ in the NED frame. The flying wing velocity vector, $\mathbf{V}_{g}$, is represented by the sum of the air and wind velocity vectors, $\mathbf{V}_{a}$ and $\mathbf{V}_{w}$, respectively. Considering the inertial frame, NED, to represent these velocities,

$$
\begin{gather*}
\mathbf{V}_{g_{E}}=\mathbf{V}_{a E}+\mathbf{V}_{w E}  \tag{3.10}\\
\mathbf{V}_{a_{E}}=\mathbf{R}_{E}^{B^{T}} \mathbf{V}_{a B}  \tag{3.11}\\
\frac{\mathrm{~d} \mathbf{P}}{\mathrm{~d} t}=\mathbf{V}_{g_{E}}=\mathbf{R}_{B}^{E} \mathbf{V}_{a}+\mathbf{V}_{w} \tag{3.12}
\end{gather*}
$$

In equation (3.12) $\mathbf{V}_{w}$ is expressed in the NED referential and $\mathbf{V}_{a}$ is expressed in the ABC referential. Here, $\mathbf{R}_{B}^{E}$ represents the referential transformation: ABC to NED.

Considering the matrix $\boldsymbol{\Omega}$

$$
\boldsymbol{\Omega}=\left[\begin{array}{cccc}
0 & P & Q & R  \tag{3.13}\\
-P & 0 & -R & Q \\
-Q & R & 0 & -P \\
-R & -Q & P & 0
\end{array}\right]
$$

and $\lambda=1-\left(q_{0}^{2}+q_{1}^{2}+q_{2}^{2}+q_{3}^{2}\right)$, for a quaternions representation $\left(q_{0}, q_{1}, q_{2}\right.$ and $\left.q_{3}\right)$, it is defined,

$$
\frac{\mathrm{d}}{\mathrm{~d} t} \mathbf{q}=\frac{\mathrm{d}}{\mathrm{~d} t}\left[\begin{array}{l}
q_{0}  \tag{3.14}\\
q_{1} \\
q_{2} \\
q_{3}
\end{array}\right]=\left[\begin{array}{l}
\dot{q}_{0} \\
\dot{q}_{1} \\
\dot{q}_{2} \\
\dot{q}_{3}
\end{array}\right]=-\frac{1}{2} \boldsymbol{\Omega} \mathbf{q}+\lambda \mathbf{q}
$$

## Forces and Moments

At this point, it is relevant to understand the forces to which the flying wing is subjected. The external forces can be gravitational forces, aerodynamic forces and propulsive forces. For this reason, F, the sum of the forces in the flying wing and $\mathbf{M}$, the sum of the moments, are $\mathbf{F}=\mathbf{F}_{g}+\mathbf{F}_{\text {Aero }}+\mathbf{F}_{\text {Prop }}$ and $\mathbf{M}=\mathbf{M}_{g}+\mathbf{M}_{\text {Aero }}+\mathbf{M}_{\text {Prop }}$, respectively. Furthermore, the gravitational force $(\mathrm{mg})$ is represented in the NED frame. However, the forces have to be expressed in the ABC frame. In order to guarantee it, a
referential transformation has to be preformed (3.15).

$$
\begin{equation*}
\mathbf{F}_{g, B}=\mathbf{R}_{E}^{B} F_{g, E} \tag{3.15}
\end{equation*}
$$

Beside that, the aerodynamic forces depend on the flying wing movement related to the air velocity, angular rates and also depend on the control inputs: $\delta_{A}, \delta_{E}$ and $\delta_{T}$.

$$
\begin{equation*}
\mathbf{F}_{\text {Aero }, B}=f\left(V_{t}, \alpha, \beta, p, q, r, \delta_{A}, \delta_{E}, \delta_{T}, \ldots\right) \tag{3.16}
\end{equation*}
$$

Finally, the propulsive forces are influenced by the air velocity, the altitude and by the flying wing motor.

$$
\begin{equation*}
\mathbf{F}_{\text {Prop }, B}=h\left(V_{t}, \alpha, \beta, h, \delta_{T}, \Omega, I m, \ldots\right) \tag{3.17}
\end{equation*}
$$

Looking for each of the force types, firstly, the aerodynamic forces are given by (3.18),

$$
\mathbf{F}_{\text {Aero }}=P_{\text {dynamic }} S \mathbf{R}_{a}^{B}\left[\begin{array}{c}
C_{D}  \tag{3.18}\\
C_{Y} \\
C_{L}
\end{array}\right]
$$

In this equation, $P_{\text {dynamic }}$ is the dynamic pressure,

$$
\begin{equation*}
P_{\text {Dynamic }}=\frac{1}{2} \rho V_{t}^{2} \tag{3.19}
\end{equation*}
$$

$S$ is the flying wing surface, $\mathbf{R}_{a}^{B}$ is the transformation matrix from the wind axis to the body axis and finally, $C_{D}$ is the drag coefficient, $C_{Y}$ is the sideforce coefficient and $C_{L}$ is the lift coefficient. In addition, the Aerodynamic moment is given by:

$$
\mathbf{M}_{\text {Aero }}=P_{\text {dynamic }} S\left[\begin{array}{c}
b C_{l}  \tag{3.20}\\
c C_{m} \\
b C_{n}
\end{array}\right]
$$

In this equation, $C_{l}$ is the rolling moment, $C_{m}$ is the pitching moment and $C_{n}$ is the yawing moment. Also, the coefficients $b$ and $c$ are respectively the flying wing span and the Mean Aerodynamic Chord (MAC). Finally, the propulsive forces and moments are given by:

$$
\begin{gather*}
\mathbf{F}_{\text {Prop }}=\left[\begin{array}{c}
\frac{4}{\pi^{2}} \rho R_{\text {Prop }}{ }^{4} \Omega^{2} C_{T} \\
0 \\
0
\end{array}\right] ;  \tag{3.21}\\
\mathbf{M}_{\text {Prop }}=\left[\begin{array}{c}
-\frac{4}{\pi^{3}} \rho R_{\text {Prop }^{5} \Omega^{2} C_{P}}^{0} \\
0
\end{array}\right] ; \tag{3.22}
\end{gather*}
$$

At this point it is possible to define the derivative of the air velocity and angular rate vectors, (3.23) and (3.24), respectively. In equation (3.24), $\mathbf{I}_{b}$ is the inertia matrix.

$$
\begin{gather*}
\frac{\mathrm{d}}{\mathrm{~d} t} \mathbf{V}=\frac{\mathrm{d}}{\mathrm{~d} t}\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]=\left[\begin{array}{c}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{array}\right]=-\left[\begin{array}{ccc}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{array}\right]\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]+\frac{1}{m}\left(\mathbf{F}_{g}+\mathbf{F}_{\text {Aero }}+\mathbf{F}_{\text {Prop }}\right)  \tag{3.23}\\
\frac{\mathrm{d}}{\mathrm{~d} t} \omega=\frac{\mathrm{d}}{\mathrm{~d} t}\left[\begin{array}{c}
p \\
q \\
r
\end{array}\right]=\left[\begin{array}{c}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{array}\right]=-\left[\begin{array}{ccc}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{array}\right]\left[\begin{array}{l}
p \\
q \\
r
\end{array}\right]+\mathbf{I}_{b}^{-1}\left(\mathbf{M}_{g}+\mathbf{M}_{\text {Aero }}+\mathbf{M}_{\text {Prop }}\right) \tag{3.24}
\end{gather*}
$$

### 3.2 Flying Wing Simulator



Figure 3.4: Nonlinear Model - Flying Wing Open-Loop Model

During this study and to test the control implementation, a Matlab ${ }^{\circledR} / \operatorname{Simulink}^{\circledR}$ mode is used, built taking into account the mathematical formulation previously presented. The simulator block diagram of the flying wing open-loop model is represented in figure 3.4. In this simulation the sensors are considered ideal while the actuator model refers to the control surfaces dynamics. This work only addresses the flying wing missions achievable by control surfaces alone, reason for which the motor is not represented in this model.

## Actuator

In the simulator model, the actuator dynamics for control surface deflections is described by a first order linear model. For each control surface deflection $\left(\delta_{i}\right)$, and for a actuator time constant, $T_{a c t}$, of 0.01 s , the relation between the commanded actuator positions $\left(u_{i}\right)$ and the control surface deflections is:

$$
\begin{equation*}
\dot{\delta}_{i}=-\frac{1}{T_{a c t}} \delta_{i}+\frac{1}{T_{a c t}} u_{i} \tag{3.25}
\end{equation*}
$$

Moreover, each control surface has physical limitations, maximum deflection of $\pm 25^{\circ}$ and a rate limit of $1 \mathrm{rad} / \mathrm{s}$.


Figure 3.5: Actuator Block

## Sensors

The main control and navigation sensors currently used on the flying wing are [33] [34] [35]:

- Global Positioning System (GPS) that provides the inertial position coordinates and velocity;
- Inertial Measurement Unit (IMU) using a combination of accelerometers, gyroscopes, and magnetometers, which provides the roll, pitch and yaw angles, the angular rates and the body linear accelerations;
- And a Pivot tube that provides the airspeed.

In order to consider the influence of the sensors, in the chapter 5 , it is analysed the sensors' noise influence on the flying wing motion.

At this point, taking into account the mathematical formulation that underlies the flying wing model, it is already possible to analyse the conditions, altitude and airspeed, for those the flight can be analysed: the flight envelope.

### 3.3 Flight Envelope

In order to identify the flying wing airspeed range for each altitude it is necessary to introduce the concept of stall velocity. Stall is a condition in aerodynamics such that if the angle of attack exceeds its maximum value, $\alpha_{\text {max }}$, then lift begins to decrease.

This condition is also linked with the consideration of a minimum airspeed for each altitude. It means, there is a minimum velocity that defines the necessary lift for the flight. This minimum velocity is known as stall velocity [9]


Figure 3.6: $\mathrm{C}_{L}-\alpha$


Figure 3.7: Theoretical Flight Envelope

It is possible to identify the boundary for the minimum velocity mentioned above for each altitude $h$ calculating the stall velocity, $V_{s t}$. Firstly, it is seen that the critical angle of attack is reached when it is verified the maximum lift coefficient, $C_{L \max }$. The figure 3.6 represents the relation between the lift
coefficient and the angles of attack for the flying wing model [36]. Here it is seen that the critical angle of attack is approximately $15^{\circ}$. Also, $C_{L}$, the lift coefficient is a dimensionless coefficient given by,

$$
\begin{equation*}
C_{L}=\frac{L}{P_{\text {Dynamic }} S}=\frac{L}{\frac{1}{2} \rho V_{t}^{2} S} \tag{3.26}
\end{equation*}
$$

Here, $L$ is the lift force, $P_{\text {Dynamic }}$ is the dynamic pressure (3.19) and $S$ is the surface area. As seen before, for the maximum elevation coefficient value, the angle of attack is also as high as possible ( $\alpha_{\max }$ ). Consequently, this characterizes the stall condition. $\left(C_{L \max } \Leftrightarrow \alpha_{\max } \Leftrightarrow\right.$ Stall). Moreover, balancing the forces on the flying wing, the lifting force is equal to the gravitational force: $L=m g$. Rewriting the equation (3.26),

$$
\begin{equation*}
C_{L}=\frac{L}{P_{\text {Dynamic }} S}=\frac{m g}{\frac{1}{2} \rho V_{t}^{2} S} \tag{3.27}
\end{equation*}
$$

For this reason, a stall velocity is calculated directly from the equation (3.27) considering $C_{L \max }$,

$$
\begin{equation*}
V_{s t}=\sqrt{\frac{2 m g}{\rho S C_{L \max }}} \tag{3.28}
\end{equation*}
$$

In fact, the air density, $\rho$, is not a constant value. The air density is a function of the altitude, $h$, and for this reason it is possible to specify a minimum airspeed for each altitude of the flying wing.

In figure 3.7, the airspeed boundary is presented. The dash line represents the airspeed boundary for each altitude. The white area in figure 3.7 represents the flight conditions (altitude and airspeed) when the loss of lift occurs. The blue area represents the flight envelope.

As it is expressed in this figure, the airspeed has not only a minimum value but also a maximum value. This maximum airspeed value of $50 \mathrm{~m} / \mathrm{s}$ represents the structural limits of the flying wing, in fact, there is insufficient information so this speed is selected arbitrarily. An airspeed higher than $50 \mathrm{~m} / \mathrm{s}$ is not possible to reach because of the flying wing structure specifications (materials and the structure resistance, for example).

Beside the airspeed limit it is also relevant the determination of the maximum altitude for the flying wing. This maximum altitude is constrained by the maximum deflection of the control surface deflections. A flight condition that require a deflection that exceed the maximum limit of $25^{\circ}$ is excluded for the flight envelope. Introducing this limit in the flight envelope determination is verified a maximum altitude of approximately 20000 m for the flight envelope.


Figure 3.8: Flight Envelope

However, it is also important to emphasise that this boundary is obtained by a theoretical concept. Because of that, it is important to guarantee if the flight envelope previously presented is suitable for this flying wing. For this reason, from the theoretical flight envelope presented in 3.7 it is analysed if those flight conditions have a trim evolution as expected or not. The figure 3.9 presents the expected trim evolution. Testing the theoretical flight envelope, it is seen that it has to be re-established in order to guarantee a trim evolution as expected. The problem at this point is the definition of an algorithm to rectify the flight envelope.

The rectification process developed for this study intends to analyse the boundary calculated by the equation (3.28). Starting from a flight condition on the flight envelope boundary (dash line in the figure 3.7), it is tested if this condition, with an airspeed of $V_{t 1}$ and an altitude of $h_{1}$, has the expected behavior or not. Then, if the behavior is as expected it means that the chosen condition belongs to the flight envelope boundary. Otherwise, it is necessary a flight envelope boundary rectification: considering the same altitude $h_{1}$ it is chosen a new airspeed $V_{t 2}$ - a slightly higher airspeed - $V_{t 2}=V_{t 1}+v$. For this new condition ( $h_{1}$ and $V_{t 2}$ ) the validation procedure is repeated.

Implementing this test algorithm for several conditions of the theoretical flight envelope boundary (figure 3.7) it is possible to re-establish a new flight envelope, the real flight envelope (figure 3.8).

### 3.4 Trim or Equilibrium Conditions

For the model linearization it is necessary the determination of the trim conditions. A trim or equilibrium condition is defined by [3] [14]:

$$
\begin{equation*}
\dot{y}_{\text {Trim }}=C \dot{x}_{\text {Trim }}=0 \tag{3.29}
\end{equation*}
$$

With, $\dot{x}_{\text {Trim }}=f\left(x_{\text {Trim }}, u_{\text {Trim }}\right)$, where $\dot{x}$ is considered here as only function of the states and inputs, $f(x, u)$. However, this trim condition is not solved analytically, it is performed by a numerical procedure: an optimization methodology.

For a straight trim flight, the values for the variables that describe the lateral flight are equal to zero ( $v=p=r=\phi=\psi=\delta_{A}=0$ ), so, in order to obtain a flight in equilibrium conditions, it is necessary to find the values for the longitudinal variables.

It is relevant to look for the flying wing problem with and without motor to understand the trim condition determination. Considering the motor: $w, \Omega, \delta_{E}$ and $\delta_{T}$ are the variables to define in order to obtain a zero value for $\dot{u}, \dot{w}, \dot{q}, \dot{E}, \dot{D}, \dot{\theta}$ and $\dot{\Omega}$. In this case it is also important to set some extra condition: the flying wing has a level flight, it means, $\theta=\alpha$.


Figure 3.9: Trim condition for the flying wing model without motor varying the airspeed and setting an altitude. (h=500; h=2500; h=5000m;h=7500m; h=10000m;h=12500m;h=15000m;h=16000m;h=17500m; $\mathrm{h}=20000 \mathrm{~m}$.)

However in this study and as mentioned before, the control is projected for the case without the motor. In a flight without motor, $\alpha, \theta$ and $\delta_{E}$ are the variables to define in order to obtain a zero value for $\dot{u}, \dot{w}$,
$\dot{q}, \dot{E}$, and $\dot{\theta}$ [2]. In the figure 3.9 it is possible to verify the trim values of $\alpha_{0}, \theta_{0}$ and $\delta_{E 0}$ for the flying wing varying the airspeeds, $V_{t}$, and setting an altitude $h$. The figure 3.9 presents the evolution of the trim values for altitudes of: $h=500 \mathrm{~m} ; h=2500 \mathrm{~m} ; h=5000 \mathrm{~m} ; h=7500 \mathrm{~m} ; h=10000 \mathrm{~m} ; h=12500 \mathrm{~m}$; $h=15000 \mathrm{~m} ; ~ h=16000 \mathrm{~m} ; h=17500 \mathrm{~m}$ and $h=20000 \mathrm{~m}$.

### 3.5 Flying-Wing Linearized Model

Usually, to deal with the complexity of the nonlinear dynamic equations a linearization of the problem is made in order to evaluate and analyse the flying wing dynamics. Typically, it is assumed that each variable is composed by a sum of an equilibrium term, $X_{0}$, and a perturbation term, $x$. Considering a generic variable, $X$, the linearization considers:

$$
\begin{equation*}
X=X_{0}+x \tag{3.30}
\end{equation*}
$$

A nonlinear function, $f$, can be linearized according to Taylor's first order expansion (3.31).

$$
\begin{equation*}
f(X, Y, \ldots)=f\left(X_{0}, Y_{0}, \ldots\right)+\frac{\partial f}{\partial X}\left(X-X_{0}\right)+\frac{\partial f}{\partial Y}\left(Y-Y_{0}\right)+\ldots \tag{3.31}
\end{equation*}
$$

As seen before, the nonlinear function $\dot{x}=f(x, u)$ can be defined by the equation (3.31). Applying the nomenclature used, the nonlinear function can be rewritten to (3.32).

$$
\begin{equation*}
f(x, u) \approx f\left(x_{\text {Trim }}, u_{\text {Trim }}\right)+\frac{\partial f}{\partial x}\left(x-x_{\text {Trim }}\right)+\frac{\partial f}{\partial u}\left(u-u_{\text {Trim }}\right) \tag{3.32}
\end{equation*}
$$

In addiction, $\frac{\partial f}{\partial x}$ for $x=x_{\text {Trim }}$ and for $u=u_{\text {Trim }}$, is known as an A matrix, the dynamic matrix, and $\frac{\partial f}{\partial u}$ for $x=x_{\text {Trim }}$ and for $u=u_{\text {Trim }}$ is known as a B matrix, input matrix.

Assuming that $x=x-x_{\text {Trim }}$ and $u=u-u_{\text {Trim }}$, it is possible to define the flying wing motion by a state space formulation neglecting the disturbances:

$$
\begin{equation*}
\dot{x}=\mathbf{A} x+\mathbf{B} u \tag{3.33}
\end{equation*}
$$

However, this methodology is only possible if the data values for the dynamic model are obtained analytically. But, in fact, the aerodynamic parameters are obtained by lookup tables. For this reason, the process to obtain the matrices mentioned above, dynamic matrix, $\mathbf{A}$, and input matrix, $\mathbf{B}$, has to be numerical instead of analytical. As a consequence, $\mathbf{A}$ and $\mathbf{B}$ are obtained by a finite difference. Each matrix entry, $A_{i j}$ and $B_{i j}$ is calculated by [9],

$$
\begin{equation*}
A_{i j}=\frac{f_{i}\left(\Delta x_{j}\right)-f_{i_{T r i m}}}{\Delta x_{j}} \tag{3.34}
\end{equation*}
$$

$$
\begin{equation*}
B_{i j}=\frac{f_{i}\left(\Delta u_{j}\right)-f_{i_{T r i m}}}{\Delta u_{j}} \tag{3.35}
\end{equation*}
$$

$f_{i}\left(\Delta x_{j}\right)$ and $f_{i}\left(\Delta u_{j}\right)$ are the acceleration terms at the disturbed state and input, respectively. $\Delta x_{j}$ is the perturbation value for the state j and $\Delta u_{j}$ is the perturbation input. Finalized the model linearization, the model decoupling between the longitudinal and lateral motion is a natural consequence of the linearization.

### 3.6 Model Analysis

The set of dynamics and kinematics equations results in 4 matrices equations: $\mathbf{V}, \omega, \mathbf{P}, \Phi$ or 12 real equations with 12 unknown variables: $u, v, w, p, q, r, N, E, D, \phi, \theta$ and $\psi$. The process of modelling and linearization of the dynamics equations and angular kinematics leads to describe the flying wing movement with two decoupled systems: Lateral and Longitudinal system.

Moreover, both models are decoupled into a set of simpler motions. Those are characterized by their frequency and damping ratio, which, in turn, are determined by the value of the associated eigenvalue.

The Lateral Model is defined as presented in (3.36) - (3.38). Usually, the lateral movement of an aircraft is defined by 3 modes: Rolling subsidence mode; Dutch-roll mode; and finally, Spiral mode. The identification of those modes is relevant to characterize the stabilization objectives and to verify if the model is or is not stable.

$$
\begin{gather*}
x_{\text {Lateral }}=[v, p, r, \phi, \psi]^{T}  \tag{3.36}\\
u_{\text {Lateral }}=\delta_{A}  \tag{3.37}\\
\dot{x}_{\text {Lateral }}=A_{\text {Lat }} x_{\text {Lat }}+B_{\text {Lat }} u_{\text {Lat }} \tag{3.38}
\end{gather*}
$$

The Rolling subsidence mode consists in considering only the rotation about the longitudinal axis and to neglect the sideslip angle, $\beta$, and the yaw rate, $r$. This motion is non-oscillatory and it consists of almost pure rolling motion. The Dutch-roll mode is described by a second order approximation. In this mode, the movement in the horizontal plan is considered and the rolling is neglected. It is an oscillatory motion. And the Spiral mode, for this mode the rolling and yawing motion are predominant although the mode is usually unstable. This mode consists on considering the rotation about the longitudinal axis and the rolling angle, $\phi$, as a pure integrator of the rolling ratio, $p, \dot{\phi}=p$. Also, the Spiral mode neglect the sideslip angle, $\beta$, and the yaw rate, $r$.

The Longitudinal Model is defined as presented in (3.39) and (3.41). Usually, the longitudinal movement of an aircraft is defined by 2 modes: Short Period mode and Phugoid mode.

$$
\begin{equation*}
x_{\text {Longitudinal }}=[u, w, q, \theta]^{T} \tag{3.39}
\end{equation*}
$$

$$
\begin{gather*}
u_{\text {Longitudinal }}=\delta_{E}  \tag{3.40}\\
\dot{x}_{\text {Longitudinal }}=A_{\text {Lon }} x_{\text {Lon }}+B_{\text {Lon }} u_{\text {Lon }} \tag{3.41}
\end{gather*}
$$

The Short Period mode consists in neglecting the variations of the longitudinal velocity, $u=0$ and the pitch angle, $\theta=0$, and considering only the equations in $w$ (vertical velocity) and $q$ (pitch rate). This motion has high damping and very short period.

Finally, the Phugoid mode neglect the vertical velocity, $w$, and the pitch rate , $q$, considering only the $u$ (longitudinal velocity) equation and $\theta$ (pitch angle) equation. This motion has low damping and very long period.

### 3.6.1 Lateral Model

Considering the lateral model it characteristic equation is of fifth degree and it is obtained by:

$$
\begin{equation*}
\operatorname{det}\left(A_{L a t} \lambda-A_{L a t}\right)=0 \tag{3.42}
\end{equation*}
$$

Factorizing this equation into the following form: $\lambda(\lambda+a)(\lambda+b)\left(\lambda^{2}+2 \xi_{D} \omega_{D} \lambda+{\omega_{D}}^{2}\right)=0$, it is obtained the dissociation of the characteristic roots of each lateral mode. According to this dissociation, it is identified:

- $\lambda+a=0$ - Corresponds to the Spiral mode;
- $\lambda+b=0$ - Corresponds to the Rolling Subsidence mode;
- $\lambda^{2}+2 \xi_{D} \omega_{D} \lambda+\omega_{D}^{2}=0$ - Corresponds to the Dutch Roll motion.

In conclusion, it is seen that the Spiral mode and the Rolling Subsidence mode are characterized by real poles and the Dutch Roll mode by a complex conjugate pole pair. Moreover, the pole that characterize the Spiral mode is the closest to the origin and sometimes is characterized by an unstable real pole.

At this moment it is important to analyse the poles location over the flight envelope and if the poles location is as expected. Firstly, setting an altitude $h$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$ it is possible to verify the evolution of the poles location as presented in the figures 3.10 and 3.11.


Figure 3.10: Evolution of the poles for $h=5000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from $V_{t}=$ $20 \mathrm{~m} / \mathrm{s}(\mathrm{X})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{O}) .(\triangle)-V_{t}=25 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=35 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=45 \mathrm{~m} / \mathrm{s}$


Figure 3.11: Evolution of the poles for $h=10000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from $V_{t}=$ $25 \mathrm{~m} / \mathrm{s}(\mathrm{X})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{O}) .(\triangle)-V_{t}=30 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=40 \mathrm{~m} / \mathrm{s}$.

Beside that, it is important to understand how is the evolution of the poles with altitude variations by setting an airspeed $V_{t}$. This representation is presented in the figures 3.12 and 3.13.


Figure 3.12: Evolution of the lateral poles for $V_{t}=30 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of 100 m and from $h=1 m(\mathrm{X})$ to $h=10000 m(\mathrm{O}) .(\triangle)-h=500 m ;(\triangle)-h=2500 m ;(\triangle)-h=5000 m ;(\triangle)-$ $h=7500 m$


Figure 3.13: Evolution of the lateral poles for $V_{t}=47 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of 500 m and from $h=1 m(\mathrm{X})$ to $h=20000 m(\mathrm{O}) .(\triangle)-h=500 m ;(\triangle)-h=5500 m ;(\triangle)-h=10500 m ;(\triangle)-$ $h=15500 m$

In the previous figures it is concluded that increasing the airspeed maintaining the altitude (figures 3.10 and 3.11 ), the poles will move away from the origin. Increasing the altitude for a constant airspeed (figures 3.12 and 3.13), the poles will move to the origin direction, once again, through a linear behaviour. In both analyses it is seen that the Spiral mode is characterized by the real pole closest to the origin, this pole varying the airspeed or the altitude is unstable: it belongs to the right half of the complex plane.

The Rolling subsidence mode is characterized by the real pole more distant from the origin. And finally, the Dutch roll mode is characterized by the complex conjugate pole pair. After this analysis, it can be concluded that the poles that characterize the modes of the lateral motion present the expected structure as presented at the beginning of this section.

### 3.6.2 Longitudinal Model

Considering the longitudinal model it characteristic equation is of fourth degree and it is obtained by:

$$
\begin{equation*}
\operatorname{det}\left(A_{L o n} \lambda-A_{L o n}\right)=0 \tag{3.43}
\end{equation*}
$$

Factorizing this equation into the following form: $\left(\lambda^{2}+2 \xi_{p h} \omega_{p h} \lambda+\omega_{p h}{ }^{2}\right)\left(\lambda^{2}+2 \xi_{s p} \omega_{s p} \lambda+\omega_{s p}{ }^{2}\right)=0$, it is obtained the dissociation of the characteristic roots of each longitudinal mode. According to this dissociation, it is identified:

- $\lambda^{2}+2 \xi_{p h} \omega_{p h} \lambda+\omega_{p h}^{2}=0$ - Corresponds to the Phugoid mode.
- $\lambda^{2}+2 \xi_{s p} \omega_{s p} \lambda+\omega_{s p}^{2}=0$ - Corresponds to the Short Period mode.

It is seen that each mode is characterised by a complex conjugate pole pair. The Phugoid mode is characterized by an oscillation of long period, defined by the low frequency of it natural frequency $\omega_{p h}$. The damping of this mode is usually very low, and sometimes it is negative, so that the mode is unstable. The Short Period mode corresponds to a rapid and relatively well-damped motion.

Applying the methodology used in the lateral model analysis, in the next figures represent the poles position changing the airspeed 3.14-3.15 and the altitude 3.16-3.17. Analysing this figures is confirmed a linear behaviour in the evolution of the poles over the flight envelope for an altitude and airspeed variation. Setting an altitude and varying the airspeed (figures 3.14-3.15) it is seen that increasing the airspeed the real pole most distant from the origin will move to the left, the real pole closest to the origin will move to the right. And it is also verified that the complex poles will move to the origin until they become real poles and follow opposite directions: one pole will move to the left and the other pole to the right.

(a) Evolution of the longitudinal poles for $h=$ 5000 m and varying the airspeed

(b) 3.14(a) figure zoom - $-0.16<\operatorname{Re}(z)<$ -0.02

Figure 3.14: Evolution of the longitudinal poles for $h=5000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from $V_{t}=20 \mathrm{~m} / \mathrm{s}(\mathrm{O})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{X}) .(\triangle)-V_{t}=25 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=35 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=45 \mathrm{~m} / \mathrm{s}$

(a) Evolution of the longitudinal poles for $h=$ 10000 m and varying the airspeed

(b) 3.15(a) figure zoom - $-0.4<\operatorname{Re}(z)<-0.05$

Figure 3.15: Evolution of the longitudinal poles for $h=10000 \mathrm{~m}$ and varying the airspeed in intervals of $1 \mathrm{~m} / \mathrm{s}$, from $V_{t}=25 \mathrm{~m} / \mathrm{s}(\mathrm{X})$ to $V_{t}=50 \mathrm{~m} / \mathrm{s}(\mathrm{O}) .(\triangle)-V_{t}=30 \mathrm{~m} / \mathrm{s} ;(\triangle)-V_{t}=40 \mathrm{~m} / \mathrm{s}$.


Figure 3.16: Evolution of the longitudinal poles for $V_{t}=30 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of 100 m and from $h=1 m(\mathrm{X})$ to $h=10000 m(\mathrm{O}) .(\triangle)-h=500 m ;(\triangle)-h=2500 m ;(\triangle)-h=5000 m ;(\triangle)-$ $h=7500 m$


Figure 3.17: Evolution of the longitudinal poles for $V_{t}=47 \mathrm{~m} / \mathrm{s}$ varying the altitude in intervals of 500 m and from $h=1 m(\mathrm{X})$, to $h=20000 m$, (O). ( $\triangle$ )- $h=500 m ;(\triangle)-h=5500 m ;(\triangle)-h=10500 m$; ( $\triangle$ ) - $h=15500 m$

Moreover, setting an airspeed and varying the altitude (figures 3.16-3.17) it is seen that increasing the altitude the initial real pole most distant from the origin will move to the right and the pole closest to the origin will move to the left. And finally, the remaining two real poles converge to a point from which they dissociate into a pair of conjugated poles. The longitudinal model is an unconventional model for this reason it is necessary to verify how is the poles' behaviour for a specific feedback to understand
the correspondence between the longitudinal poles and the longitudinal modes. A pitch rate $(q)$ feedback improves essentially the short period mode response [14]. Implementing this feedback in the longitudinal model it is verified that the complex poles' location changes substantially. Regarding this behavior it is possible to identify the correspondence between the longitudinal poles and the longitudinal modes: the Short period mode is characterised by the complex poles and the Phugoid mode is characterised by the real poles. In the next chapter the influence of the longitudinal model feedback is presented more in detail.

### 3.7 Flight Quality Analysis

Before the control implementation it is relevant to evaluate the flight quality for the flying wing.
The flight qualities' specifications are defined according to the formalization provided by Donald McLean [9]. Firstly, following this formalization, it is necessary to establish the aircraft class and the flight phase. According to the methodology provided, the analysis is proceeded for lateral and longitudinal motion, separately. Moreover, this methodology is characterized by a set of parameters related to the damping ration, natural frequency and time to double. The knowledge of those parameters is important to understand the flying wing response to a specific command or disturbance.


Figure 3.18: Evolution of the lateral poles for different flight conditions. o - $h=1 m$ and $15 m / s \leqslant V_{t} \leqslant 50 m / s$; o- $h=500 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ о $-h=1000 \mathrm{~m}$ and $20 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;-h=5000 \mathrm{~m}$ and $25 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; о $-h=15000 \mathrm{~m}$ and $30 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; о $-h=20000 \mathrm{~m}$ and $47 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$


Figure 3.19: Evolution of the longitudinal poles for different flight conditions. o - $h=1 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant$ $50 \mathrm{~m} / \mathrm{s} ;$ о $-h=500 \mathrm{~m}$ and $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ о $-h=1000 \mathrm{~m}$ and $20 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; $h=5000 \mathrm{~m}$ and $25 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s} ;$ о $-h=15000 \mathrm{~m}$ and $30 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; о-h $h=20000 \mathrm{~m}$ and $47 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$

According to the procedure used for this analysis, the flying wing belongs to the IV state of the aircraft class, aircraft with high manoeuvrability, and an A category for the flight phase: flight nonterminal, rapid manoeuvring, precision tracking and precise control of the flight path. This evaluating process output is the characterization of the flight quality level, level 12 or 3 , where level 1 corresponds to an aircraft with better handling qualities. Furthermore, the control will be implemented to ensure a level 1 flight.

As seen before the lateral model has 3 modes: Rolling subsidence mode, Dutch-roll mode and Spiral mode.

The Rolling subsidence mode is evaluated by the constant time $T$; The Dutch-roll mode is evaluated by the damping coefficient, $\xi$, and the natural frequency, $\omega_{n}$; And finally, the Spiral mode is evaluated by the time to double, $T_{2}$.

Mathematically, the parameters are represented by the expressions (3.44) to (3.48). A pole in the complex plan is expressed by the equation (3.44).

$$
\begin{gather*}
s=-\xi \omega_{n} \pm \omega_{d} j  \tag{3.44}\\
\omega_{d}=\omega_{n} \sqrt{1-\xi^{2}}  \tag{3.45}\\
\omega_{n}=\frac{2 \pi}{T}  \tag{3.46}\\
\xi=\sin (\theta) \tag{3.47}
\end{gather*}
$$

$$
\begin{equation*}
T_{2}=\frac{\ln (2)}{\xi \omega_{n}} \tag{3.48}
\end{equation*}
$$

Previously it was presented how the poles location is influenced by the flight conditions and also it is also identified the poles that characterize each mode.

Figures 3.18 and 3.19 summarizes the evolution of the poles by setting different flight conditions. In blue for an $h=1 \mathrm{~m}$ and for $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; In red for an $h=500 \mathrm{~m}$ and for $15 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; In green for an $h=10000 \mathrm{~m}$ and for $20 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; In yellow for an $h=5000 \mathrm{~m}$ and for $25 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; In cyan for an $h=15000 \mathrm{~m}$ and for $30 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$; In magenta for an $h=20000 \mathrm{~m}$ and for $47 \mathrm{~m} / \mathrm{s} \leqslant V_{t} \leqslant 50 \mathrm{~m} / \mathrm{s}$.

Table 3.1: Rolling Subsidence Mode - Flight Quality Level

| Aircraft Class and Flight Phase | Level 1 | Level 2 | Level 3 |
| :---: | :---: | :---: | :---: |
| IV and A | $T_{\text {Max }}=1 s$ | $T_{M a x}=1.4 s$ | $T_{M a x}=10 s$ |

Table 3.2: Dutch Roll Mode - Flight Quality Level

| Aircraft Class and Flight Phase | Level 1 |  |  | Level 2 |  |  | Level 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IV and A | $\xi$ | $\xi \omega_{n}$ | $\omega_{n}$ | $\xi$ | $\xi \omega_{n}$ | $\omega_{n}$ | $\xi$ |  |
|  | 0.19 | 0.35 | 1 | 0.02 | 0.05 | 0.5 | $0.02-0.4$ |  |

Table 3.3: Spiral Mode - Flight Quality Level

| Flight Phase | Stable or unstable spiral | Level 1 | Level 2 | Level 3 |
| :---: | :---: | :---: | :---: | :---: |
| A | Unstable | $T_{2}>12 s$ | $T_{2}>8 s$ | $T_{2}>5 s$ |
|  | Stable | Level 1 | N/A | N/A |

Table 3.4: Short Period Mode - Flight Quality Level

| Flight Phase | Level 1 | Level 2 | Level 3 |
| :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | $\xi$ | $\xi$ | $T$ |
|  | $0.35<\xi<1.3$ | $0.25<\xi<2$ | $\xi>0.1$ |

Table 3.5: Phugoid Mode - Flight Quality Level

| Level 1 | Level 2 | Level 3 |
| :---: | :---: | :---: |
| $\xi$ | $\xi$ | $T$ |
| $>0.04$ | $>0$ | $>55 s$ |

Firstly, and regarding the formulation used to evaluate the flight quality level, the Rolling Subsidence mode is classified by the constant time value. In the table 3.1 the maximum time value for each level is presented.

In order to guarantee a flight quality level of 1 the real pole that characterize the Rolling Subsidence mode has to have a value smaller than $-2 \pi\left(T=1 s=\xi \omega_{n}=2 \pi\right)$. In fact, this real pole is already in the admissible area, so, a flight quality of level 1 is verified.

For the Dutch Roll mode, the conditions for each level are presented in the table 3.2. The values for $\xi, \xi \omega_{n}$ and $\omega_{n}$ are the parameters to analyse for the flight level definition. The values in the table 3.2 are the minimum values for the parameters $\xi, \xi \omega_{n}$ and $\omega_{n}$. In order to achieve a level 1 , the poles for the Dutch-Roll mode have to be out of the circle that determine the minimum limit for $\omega_{n}$, they have to be on the left of the dash vertical line that determine the minimum limit of $\xi \omega_{n}$. And also, the maximum slope between the poles and the origin is represented by the two dash obliques lines. Setting those limits on the figure 3.20 it is possible to verify that from an altitude higher than 5000 m it is not possible to reach a flight quality of level 1 .


Figure 3.20: Dutch Roll Mode poles

For the Spiral mode and if the analysis is performed for unstable poles, the time to double is the parameter responsible to determine the flight quality level. In fact, in order to establish a level 1 , the
minimum time to double is equals to $12 s$. This limit is represented by a $\xi \omega_{n}$ maximum. If the analysis is performed for stable poles, the flight quality level is directly 1. The Spiral mode pole is unstable, for this reason, to guarantee a level 1, the Spiral mode pole has to have a value smaller than -0.0578 $\left(T=12 s=\xi \omega_{n}=0.0578\right)$. And there are several flight conditions that does not reach this condition.

Moreover, the Longitudinal motion is represented by 2 modes: Short Period Mode and Phugoid Mode. Those modes are evaluated by the damping coefficient.

The Short Period mode is characterized by complex poles. It is verified that increasing the flight altitude the damping coefficient decreases. For this reason, a flight quality of level 1 is not reached for the entire flight envelope.

The Phugoid mode is characterized by the real poles. For this reason, a flight quality of level 1 for the Phugoid mode is verified for the entire flight envelope.

Finally, it is concluded that varying the flight conditions the level 1 for the flight quality may be compromised.

Usually to guarantee a stable flight and to obtain a level 1 for the flight quality a correction of the flying wing dynamics has to be performed. In order improve the dynamic it is implemented a Stability Augmentation System (SAS).

## Flying Wing Control

4.1 Control Approaches ..... 40
4.2 Control Objectives. ..... 43
4.3 SAS - Stability Augmentation System ..... 44
4.4 Attitude Control ..... 46
4.5 Guidance ..... 57

### 4.1 Control Approaches

In order to introduce the control action for flying wing, firstly, it is relevant to present an overview about the control methodologies used in this work. In fact, the control methods can be divided in linear control and nonlinear control approaches.

### 4.1.1 Classical Control Approach

The classical control approaches aggregate different methodologies. Between those methodologies in this study, Root Locus and LQR are the classical control approaches used to control the flying wing flight.

### 4.1.1.A Root Locus

The project of the controller from the desired performance specifications of the system is facilitated by the Root Locus implementation. Root Locus is a graphical representation of the poles for a closed loop system by a gain variation in the system. In this method, the controller is designed from the specifications of the dominant closed loop poles: the desired damping ratio $\xi$ and natural frequency $\omega_{n}$. For this reason, this control approach is a useful tool in order to analyse SISO linear dynamic systems [11].

### 4.1.1.B LQR

LQR is an optimal control approach. The results from this method may be applied in nonlinear systems. In fact, LQR's robustness suggest that the control for nonlinear systems may be achieved assuming that the system is linear [13]. The LQR's aim is to minimize a quadratic cost function (4.1) restricted by the system dynamics (4.2). Typically, the cost function is the energy associated with the state and the input in the system.

$$
\begin{gather*}
J=\int L(x, u, t) d t  \tag{4.1}\\
\dot{x}=A x+B u \tag{4.2}
\end{gather*}
$$

LQR is a cost function based on the state and input weights (4.3). The cost function's minimization provides an optimum feedback (4.4).

$$
\begin{equation*}
J=\frac{1}{2} \int_{0}^{\propto}\left(x^{T} Q x+u^{T} R u\right) d t \tag{4.3}
\end{equation*}
$$

$$
\begin{equation*}
u^{\circ}=-K x \tag{4.4}
\end{equation*}
$$

Examining the cost function J (4.3), the first term shapes the state evolution for a given initial condition $x(0)$ and the second term limits the amplitude of the control action. This weighting is represented by the Q (state weight) and R (input weight) matrices.

The values of Q and R are chosen in order to achieve the desired behaviour in the system. The precision is guaranteed increasing Q. However, increasing Q, the energy associated with the state also increases. Rising the value of $R$, the system precision is jeopardized, however, the amount of energy needed decreases. Attending to this behaviour, it is simple to understand that a jeopardize between the Q and R values must be done in order to choose the weighting for Q and R . The objective is to minimize the energy in the states without using too much control energy.

After the determination of the Q and R matrices and solving the algebraic Riccati equation (4.5) for a symmetric positive definite matrix P the the Kalman gain K may be computed (4.6).

$$
\begin{gather*}
A^{T} P+P A-P B R^{-1} B^{T} P+Q=0  \tag{4.5}\\
K=R^{-1} B^{T} P \tag{4.6}
\end{gather*}
$$

Systematically, in order to design a LQR, the following steps should be followed:

- Definition of the system (4.2).
- Characterization of the cost function, choosing Q and R matrices: state and control weighting matrices, respectively (4.3).
- Resolution of the algebraic Riccati equation (4.5).
- Determination of the Kalman gain K (4.6).
- Implementation of the optimal feedback control in the system (4.4).


### 4.1.2 Nonlinear Control Approach - INDI

Classical control approaches are the methodologies usually used for flight control systems. However, those methods have also weaknesses. In order to deal with the classical control approach limitations: the non inclusion of nonlinearities in the system and the dependence of local linearization points, the nonlinear control strategies are implemented.

INDI considers the incremental dynamics of a nonlinear system and uses the data that comes from the sensors. This method is an improvement of the NDI once INDI decreases the model dependency and increases the controller robustness to uncertainties.

In order to schematize the INDI theory it is considered the state space formulation where $u$ is the system input and $x$ the state [26].

$$
\begin{equation*}
\dot{x}=f(x, u) \tag{4.7}
\end{equation*}
$$

The system dynamics is approximated by incremental observations of the state. In order to apply this strategy it is important to guarantee that the state is observable at the established sampling time. Assuming the sampling time as $T, t=t_{0}+T$, where $t_{0}$ is the previous instant of time.

Considering the first order approximation of $\dot{x}$ from the Taylor series expansion of $\dot{x}=f(x, u)$,

$$
\begin{equation*}
\dot{x} \approx \dot{x}_{0}+\frac{\partial f}{\partial x}_{x_{0}, u_{0}}\left(x-x_{0}\right)+\frac{\partial f}{\partial u}_{x_{0}, u_{0}}\left(u-u_{0}\right) \tag{4.8}
\end{equation*}
$$

Assuming a small sampling time, $x \approx x_{0}$. The incremental term caused by the control input is the only term that is taking into account: the incremental term caused by the system dynamics is neglected [37].

$$
\begin{equation*}
\dot{x} \approx \dot{x}_{0}+\frac{\partial f}{\partial u}_{x_{0}, u_{0}}\left(u-u_{0}\right)=\dot{x}_{0}+B \Delta u \tag{4.9}
\end{equation*}
$$

Also, assuming that $x_{0}, u_{0}$ and $\dot{x}_{0}$ are observable and the sampling time applied is small enough, the state can be given by the $\dot{x}_{0}$ values. This eliminates the urgency of an extensive system modeling what promotes to an increase in robustness for modeling uncertainties. However, INDI would be more sensitive to sensor performance.

Then, the incremental control law can be defined by the equation 4.10, where $\nu$ is the virtual control.

$$
\begin{equation*}
\Delta u=B_{0}^{-1}\left(\nu-\dot{x}_{0}\right) \tag{4.10}
\end{equation*}
$$

Applying the control law above, the relation between the virtual control and the output is linear,

$$
\begin{equation*}
\dot{x}=\nu \tag{4.11}
\end{equation*}
$$



Figure 4.1: Linear control in the outer loop and Inner INDI loop

### 4.2 Control Objectives

To successfully complete the mission defined in chapter 1 the flying wing control is implemented gradually (figure 4.2). Firstly, a SAS intends to increase the flight stability ensuring a flight quality of level 1 for all the lateral and longitudinal modes, this stability is assured by a rate controller. Secondly, an attitude controller is designed in order to improve the system dynamic and to enable the tracking of the roll and pitch angles $(\phi$ and $\theta)$. And finally, the flight path controller is projected to guarantee a precise guidance. Beside that, during this chapter the controller is projected in order to achieve a closed loop system response with a null stationary position error, a settling time lower than $3 s$ and a minimum damping factor, $\xi$, of 0.5 .


Figure 4.2: Flying Wing controller - Inner and outer system loops.

### 4.3 SAS - Stability Augmentation System

As mentioned in the previous chapter, it is necessary to implement a stability augmentation system to improve the system dynamics in order to obtain a flight quality of level 1 . Between the flight conditions analysed in the previous chapter, it is seen that an altitude of $h=15000 \mathrm{~m}$, represented graphically in figures 3.18 and 3.19 by cyan, does not verify a flight quality of level 1 . For this reason, a flight condition of $h=15000 \mathrm{~m}$ and $V_{t}=35 \mathrm{~m} / \mathrm{s}$ is selected to introduce the SAS study.

Table 4.1 identify the poles for each mode, the parameters that guide the flight quality analysis and the conclusion about the flight quality level. It is seen that a level 1 for the Dutch roll mode and Short Period mode is not reached and a SAS has to be performed with this goal, the achievement of a flight quality of level 1 for all the modes. Considering the lateral movement it is necessary to stabilize the

Table 4.1: Flight quality level for $h=15000 \mathrm{~m}$ and $V_{t}=35 \mathrm{~m} / \mathrm{s}$

|  | Mode | Poles | $\xi$ | $\omega_{n}$ | Level |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LATERAL | Rolling Subsidence Mode | -14.7 | 1 | 14.7 | 1 |
|  | Dutch Roll Mode | $0.0417 \pm 10.5 \mathrm{i}$ | -0.004 | 10.5 | 3 |
|  | Spiral Mode | 0.113 | -1 | 0.113 | 1 |
| LONGITUDINAL | Phugoid Mode | -16.1 | 1 | 16.1 | 1 |
|  |  | -0.0968 | 1 | 0.0968 |  |
|  | Short Period Mode | $0.727 \pm 2.21 \mathrm{i}$ | -0.313 | 2.33 | 3 |

Dutch roll mode. In order to achieve this stability a feedback of $p$ and $r$ is performed, the roll and the yaw rates, respectively.

To design the SAS for the lateral movement a Root Locus is implemented using the Matlab $®$ rootlocus tool, rlocus (figure 4.3). The gains obtained considering a negative feedback are: $K_{p}=0.0925$ and $K_{r}=-0.0925$. This feedback stabilize the Dutch roll mode. For this reason, considering this SAS, the lateral movement achieves a flight quality of level 1.

For the Longitudinal model, and as seen before, the Short Period mode has to be stabilized. In order to obtain a level 1 for this mode it is performed a feedback of $q$, the pitch rate. Once again, using the Matlab(®) root-locus tool, a positive feedback of $q$ with a $K_{q}=0.2396$ guarantees a flight quality of level 1 (figure 4.4).


Figure 4.3: Root-Locus - SAS Lateral Model


Figure 4.4: Root-Locus - SAS Longitudinal Model

Table 4.2: Flight quality level for $h=15000 m$ and $V_{t}=35 m / s$ - Level 1

|  | Mode | Poles | $\xi$ | $\omega_{n}$ | Level |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LATERAL | Rolling Subsidence Mode | -32.6 | 1 | 32.6 | 1 |
|  | Dutch Roll Mode | $-3.49 \pm 6.01 \mathrm{i}$ | 0.5 | 6.96 | 1 |
|  | Spiral Mode | -3.75 | 1 | 3.75 | 1 |
| LONGITUDINAL | Phugoid Mode | -112 | 1 | 112 | 1 |
|  |  | -2.36 | 1 | 2.36 |  |
|  | Short Period Mode | -0.176 | 1 | 2.33 | 1 |
|  |  | -0.180 | 1 | 0.180 |  |

The question at this moment is if the previous SAS is valid for all the flight conditions or if it is necessary to define a gain scheduling. To understand how the stability is or is not affected varying the flight conditions in the appendix A the tables A.1, A.2, A. 3 and A. 4 show the poles and the flight quality levels for different flight conditions and for each pole over the flight envelope. As presented in those tables the lateral and longitudinal stability is guaranteed with this feedback design and it is not necessary to design a SAS depending on the flight conditions. At this point the flight stability is already defined and in order to guarantee the reference tracking it is necessary to define the attitude control.

### 4.4 Attitude Control

The attitude controllers have essentially two aims: to improve the dynamic response and to follow the reference precisely. In order to achieve those goals, the system configuration could be with the controller in feedback control or with the controller in servomechanism - error feedback. It is also important to clarify that the error feedback configurations are structured in order to implement the tracking of the rolling angle, $\phi$, or the pitch angle, $\theta$. In those configurations are verified that the attitude control is represented by the external feedback loop and the SAS is represented by the inner loop. The error feedback for the lateral, longitudinal and the nonlinear model are represented in figures 4.5, 4.6 and 4.7, respectively.


Figure 4.5: Linear attitude control and stabilization - Lateral Model


Figure 4.6: Linear attitude control and stabilization - Longitudinal Model


Figure 4.7: Linear attitude control and stabilization - Nonlinear System

### 4.4.1 Classical Approach: Linear Control

### 4.4.1.A Lateral Model

In order to achieve the control objectives, a SISO approach is implemented: a Root Locus methodology. For a flight condition for an altitude of $h=1500 \mathrm{~m}$ and an airspeed of $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and considering this SISO methodology and the inner feedback loop, an external feedback control of $\phi$, roll angle is designed. Using the Matlab® root-locus tool the gain obtained for this system is: $K_{\phi}=0.5186$. As expected introducing this gain in the system it is possible to highlight the improvements in the system dynamic. In table 4.3 the poles location in the complex plan, the damping factor, natural frequency and also the time constant for the uncontrolled and controlled system are presented. Analysing this table, the control applied in the system improves significantly the response. The controlled system guarantees an overshoot minimization and a significant improvement in the time response.

Table 4.3: Lateral Model - Uncontrolled and Controlled System

|  | Poles | $\xi$ | $w_{n}[\mathrm{rad} / \mathrm{s}]$ | $T[\mathrm{~s}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Uncontrolled System (Open Loop System) | -23.3 | 1 | 23.3 | 0.043 |
|  | 0.153 | -1 | 0.153 | -6.53 |
|  | $-2.51 \pm 11.9 i$ | 0.2061 | 12.2 | 0.398 |
| Controlled System | -3.75 | 1 | 3.75 | 0.267 |
|  | $-3.49 \pm 6.03 i$ | 0.501 | 6.96 | 0.287 |
|  | -32.6 | 1 | 32.6 | 0.0306 |

### 4.4.1.B Longitudinal Model

Considering the longitudinal model, a LQR is the methodology implemented. The theory that underlies the LQR methodology was previously presented. However, the real LQR implementation required a more detailed and exhaustive approach. As it was seen before, the determination of Q and R matrices is a crucial step. In order to support this choice and decrease the arbitrariness, the Bryson method is implemented. This method suggests a definition for the weighting of the Q and R matrices. Following
this approach, Q and R are defined as diagonal matrices (4.12) (4.14) and each term is the inverse of the square of the maximum variation for each respective variable during the maneuver (4.13) (4.15).

$$
\begin{align*}
& Q=\operatorname{diag}(Q i)  \tag{4.12}\\
& Q i=\frac{1}{x_{i, \max }^{2}}  \tag{4.13}\\
& R=\operatorname{diag}(R i)  \tag{4.14}\\
& R i=\frac{1}{u_{i, \max }^{2}} \tag{4.15}
\end{align*}
$$

Implementing the Bryson method, firstly, it is necessary to define the maximum variation to each variable in the system. Considering the flying wing, and attending to the variables from the longitudinal model, the maximum variation estimation for each one is:

$$
\begin{gathered}
u_{\max }[\mathrm{m} / \mathrm{s}]=0.1 \times V_{t} \mathrm{~m} / \mathrm{s} \\
w_{\max }[\mathrm{m} / \mathrm{s}]=0.1 \times V_{t} \mathrm{~m} / \mathrm{s} \\
q_{\max }[\mathrm{rad} / \mathrm{s}]=2^{\circ} / \mathrm{s} \times \frac{\pi}{180} \mathrm{rad} /{ }^{\circ} \\
\theta_{\max }[\mathrm{rad}]=2^{\circ} \times \frac{\pi}{180} \mathrm{rad} /{ }^{\circ} \\
\delta e_{\max }[\mathrm{rad}]=0.1 \times 25^{\circ} \times \frac{\pi}{180} \mathrm{rad} / /^{\circ}
\end{gathered}
$$

At this moment, the determination of the Q and R matrices is given by:

$$
\begin{gather*}
Q_{\text {Lon }}=\operatorname{diag}\left(\frac{1}{u_{i, \max }^{2}}, \frac{1}{w_{i, \max }^{2}}, \frac{1}{q_{i, \max }^{2}}, \frac{1}{\theta_{i, \max }^{2}}\right)  \tag{4.16}\\
R_{\text {Lon }}=\frac{1}{\delta e_{i, \max }^{2}} \tag{4.17}
\end{gather*}
$$

However, Bryson's method just supports the initial $Q$ and $R$ attempt. This method is not sufficient to choose the weighting matrices. In fact, a Q and R correction is essential for an optimal system feedback. Taking into account this evidence, and for a specific flight condition, it is necessary to study how much the weighting matrices values should be modified in order to improve the system. Progressively, and to implement this methodology, each input magnitude of the Q and R matrices is increased or decreased and for each modification it is evaluated if the poles localization and the system response are or are not as desired: the changes of the poles localization cannot be severely pronounced and the system response has to met the control objectives. In fact, it is necessary to test several combinations to get some sensibility
and to achieve both conditions. However, after this analysis the Q and R matrices are defined by:

$$
\begin{gather*}
Q_{L o n}=\left[\begin{array}{cccc}
0.03 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 2
\end{array}\right]  \tag{4.18}\\
R_{\text {Lon }}=1000 \tag{4.19}
\end{gather*}
$$

Using the Matlab lqr function and the $Q$ and $R$ matrices (4.18)-(4.19), the next step is the determination of the gain values for a longitudinal attitude control:

$$
\begin{equation*}
K_{L o n}=l q r\left(A_{L o n}, B_{L o n}, Q_{L o n}, R_{L o n}\right) \tag{4.20}
\end{equation*}
$$

Applying this methodology, it is possible to itemize the location of the poles in the complex plan, the damping factor, the natural frequency and also the time constant. In table 4.4 those parameters are presented for the uncontrolled and controlled longitudinal system.

Table 4.4: Longitudinal Model - Uncontrolled and Controlled System

|  | Poles | $\xi$ | $w_{n}[\mathrm{rad} / \mathrm{s}]$ | $T[\mathrm{~s}]$ |
| :---: | :---: | :---: | :---: | :---: |
| Uncontrolled System | -25.7 | 1 | 25.7 | 0.0389 |
|  | -0.169 | 1 | 0.169 | 5.93 |
|  | $-1.3 \pm 2.12 i$ | 0.524 | 2.49 | 0.767 |
| Controlled System | $-0.350 \pm 0.37 i$ | 0.721 | 0.486 | 2.86 |
|  | -2.73 | 1 | 2.73 | 0.366 |
|  | -116 | 1 | 116 | 0.00866 |

### 4.4.1.C Nonlinear Model

At this point, and after the lateral and longitudinal linear models analysis and control, it is relevant to understand how the controllers projected for a linear problem are suitable for a nonlinear scenery. The Nonlinear system implementation requires the consideration of more parameters in its construction. It is necessary to add the initial values for each state $\left(X_{0}\right)$ and for each system input $\left(U_{0}\right)$. In this analysis, the gain values considered, $K_{X}$, were the gain values calculated for the linear control approach.

The Nonlinear system represented by a Nonlinear system block in the block diagram 4.7 has as inputs $\mathbf{X}, \mathbf{U}$ and $\mathbf{D}$ values. $\mathbf{X}$, the state, is composed by $\left[u, v, w, p, q, r,(N E D), q_{0}, q_{1}, q_{2}, q_{3}, \Omega, I_{m}\right]$ where $u, v$ and $w$ are the ground speed in body frame ( ABC ); $p, q$ and $r$ are the angular rates in body frame; $N E D$ is the earth position; $q_{0}, q_{1}, q_{2}$ and $q_{3}$ are the quaternions of rotation from earth to body frame; $\Omega$ is the propeller angular speed; and $I_{m}$ is the DC motor current.
$\mathbf{U},\left[\delta_{A}, \delta_{E}, \delta_{T}\right]$, aileron, elevator and throttle deflections, respectively. And $\mathbf{D}$, wind velocity in NED coordinates, $\left[\omega_{N}, \omega_{E}, \omega_{D}\right]$.

Is the control projected for the linear system adequate for the nonlinearized system? Repeating the
previous control methodology, the time response for a flight condition: $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$ is tested. The figure 4.8 presents this time domain response obtained by the nonlinear closed loop system for $\phi$ and $\theta$ system inputs ( $\phi$ : square input with $\phi_{\text {max Ref }}=20^{\circ} . \theta$ : constant input equals to $\theta_{\text {Trim }}$ ).


Figure 4.8: Time Domain Responses for linear controller (figure 4.7). (...) - $\phi_{\text {Ref }}=20^{\circ}$ and $\theta_{\text {Trim }}$; (-) $\phi$ and $\theta$ time domain responses.

Analysing this time domain response is verified the adequacy of a linear controller for a nonlinear system: a satisfactory reference tracking without overshoot, a settling time equals to $2 s$ and a null steady state error.

However, is this control suitable for the entire flight envelope? To answer to this question, the control designed previously is tested for different flight conditions. And in fact, the controller design must depend on flight condition. This control is not suitable for the entire flight envelope.

To solve this limitation, a gain scheduling is performed to met a full achievement of the control objectives for the entire flight envelope

### 4.4.2 Gain Scheduling

In order to simplify the methodology used to control the flying wing by classical control approaches, the previous analysis was made for a flight condition of: $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$. However, as seen, it is necessary to understand that varying the flight conditions the flying wing control has to change: the set of gains obtained for this flight condition should not be valid for the entire flight envelope.

In fact, the control for the longitudinal model was established by a LQR approach. The determination of the gains for this model, as explained previously, is obtained directly solving the equations (4.5) and (4.6). Or using the lqr Matlab function: $K_{L o n}=\operatorname{lqr}\left(A_{L o n}, B_{L o n}, Q_{L o n}, R_{L o n}\right)$. For this reason, the set of gains for the longitudinal attitude control is already conditioned by the flight conditions. Varying the
flight conditions the matrices $A_{L o n}$ and $B_{L o n}$ change and consequently, $K_{L o n}$ changes too following those variations.

Attending to the attitude control for the lateral model it was seen that the set of gains for the lateral model is obtained by a Root-locus approach and this approach requires more steps and it is not an automatic approach.

However for both linear control approaches it is necessary to understand how sensitive is the control efficiency to the flight condition variations.

In order to study this sensitivity a set of gains is fixed and changing the flight conditions is analysed if the control objectives are or are not verified. After this analysis is concluded that it is necessary to establish different set of gains for different airspeeds. And it is also verified that for the same airspeed and decreasing the altitude, the flying wing performance improves. For this reason, the control is projected for different airspeeds and for the maximum altitude of each airspeed.

Table 4.5 represents the algorithm defined for the gain scheduling. In this table the letter C represents the flight conditions for which the control is projected and here is also represented the gain obtained for $\phi$. It is also seen that for the flight conditions in the table 4.5, the matrices Q (4.18) and R (4.19) satisfy the requirements imposed. So, for these flight conditions, the set of gains of the longitudinal state is calculated directly as shown above in equation (4.20). Figure 4.9 presents this time domain

Table 4.5: Gain Scheduling - Lateral attitude control design. The letter C represents the flight condition for which the control was designed. $K_{\phi}$ for each airspeed

response obtained by the nonlinear closed loop system for $\phi$ and $\theta$ system inputs ( $\phi$ : step input with $\phi_{\max \text { Ref }}=20^{\circ} . \theta:$ constant input equals to $\theta_{\text {Trim }}$ ) and for different flight condition: $V_{t}=15 \mathrm{~m} / \mathrm{s}$ and
$h=100 \mathrm{~m}$ (purple); $V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}$ (blue); $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$ (yellow); $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 m$ (green).


Figure 4.9: Time Domain Responses for the system configuration (figure 4.7).
(...) $-\phi_{R e f}$.
$V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$ $(\quad) ; V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$.


Figure 4.10: Time Domain Responses for the system configuration (figure 4.7).
(...) $-\theta_{\text {Ref }}$.
$V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$
$(\quad) ; V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$.

Figure 4.10 presents this time domain response obtained by the nonlinear closed loop system for $\phi$ and
$\theta$ system inputs ( $\theta$ : step input with $\theta_{\min R e f}=-20^{\circ} . \phi$ : constant input equals to zero) and for different flight condition: $V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}$ (purple); $V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}$ (blue); $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$ (yellow); $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}$ (green).

Considering the previous analysis a full achievement of the control objectives introducing a gain scheduling on the system is guaranteed. A linear control approach implemented in a nonlinear system has a satisfactory performance: a controller projected for a linear model is suitable for the nonlinear model.

However, it is also unquestionable the time-consuming and the sensitivity required in order to project a linear controller. Beside that, it is not possible to design a single controller for the entire flight envelope. To improve the controller design and to provide an alternative solution that does not verify those limitations and difficulties, a nonlinear control approach is implemented.

### 4.4.3 Nonlinear Control



Figure 4.11: INDI attitude control and stabilization

The proposed INDI control solution requires the measurement of all system states and states derivatives. However to design an attitude INDI controller is defined a sub-state $\xi=[p q]^{T}$ to provide a directional control and then allow the path-tracking control of the flying wing. Assuming that the desired dynamic is obtained as a state error feedback with constant gains,

$$
\dot{\xi}=\nu=\left[\begin{array}{l}
\dot{p}  \tag{4.21}\\
\dot{q}
\end{array}\right]=\mathbf{K}_{\mathbf{1}}\left[\begin{array}{c}
\phi_{d}-\phi_{0} \\
\theta_{d}-\theta_{0}
\end{array}\right]-\mathbf{K}_{\mathbf{2}}\left[\begin{array}{c}
p_{0} \\
q_{0}
\end{array}\right]
$$

Rewriting the equation (4.21) for $\dot{\phi} \approx p$ and $\dot{\theta} \approx q$,

$$
\left[\begin{array}{l}
\ddot{\phi}  \tag{4.22}\\
\ddot{\theta}
\end{array}\right]=\mathbf{K}_{\mathbf{1}}\left[\begin{array}{c}
\phi_{d}-\phi_{0} \\
\theta_{d}-\theta_{0}
\end{array}\right]-\mathbf{K}_{\mathbf{2}}\left[\begin{array}{c}
\dot{\phi}_{0} \\
\dot{\theta}_{0}
\end{array}\right]
$$

An INDI controller is designed considering a sample rate high enough and that the actuators present fast dynamics when compared to the system. Increasing the sample rate or decreasing the sampling time, the oscillatory response will decrease and the reference tracking is improved. The control frequency used in this study is 100 Hz , a sampling time of 0.01 s . The assumption of a fast control and high sample rate admit,

$$
\begin{align*}
\phi_{0} & \approx \phi  \tag{4.23}\\
\theta_{0} & \approx \theta \tag{4.24}
\end{align*}
$$

Which correspond to two desired second order responses for $\phi$ and $\theta$.

$$
\begin{gather*}
\frac{\phi}{\phi_{d}}=\frac{K_{1}(1,1)}{s^{2}+K_{2}(1,1) s+K_{1}(1,1)}  \tag{4.25}\\
\frac{\theta}{\theta_{d}}=\frac{K_{1}(2,2)}{s^{2}+K_{2}(2,2) s+K_{1}(2,2)}  \tag{4.26}\\
s^{2}+2 \xi \omega_{n} s+\omega_{n}^{2}=s^{2}+K_{2}(2,2) s+K_{1}(2,2)=s^{2}+K_{2}(1,1) s+K_{1}(1,1)  \tag{4.27}\\
K_{1}(1,1)=K_{1}(2,2)=\omega_{n}^{2}  \tag{4.28}\\
K_{2}(1,1)=K_{2}(2,2)=2 \xi \omega_{n} \tag{4.29}
\end{gather*}
$$

Defining the control input matrix by $\mathbf{B}$

$$
\mathbf{B}=\left[\begin{array}{l}
B_{p}  \tag{4.30}\\
B_{q}
\end{array}\right]
$$

And the control input as $u$,

$$
\begin{equation*}
u=u_{0}+\mathbf{F}^{-1}\left(\dot{\xi}_{d}-\dot{\xi}_{0}\right) \tag{4.31}
\end{equation*}
$$

Being $\mathbf{F}^{-1}=\lambda{B_{0}}^{-1}$, the closed loop system depends on the input scaling gain $(\lambda)$ and on the desired dynamic loop gains. Equation (4.31) shows that, contrary to regular NDI, the INDI allows dealing with systems that are not affine in control and still obtain a valid input-output linearisation based only on high sample rate and fast control assumptions [30].

## Input scaling gain ( $\lambda$ )

Given the incremental nature of the controller, one intuitive solution to reduce control oscillation is to scale the incremental input [38]. The parameter $\lambda$ is an adjustable input scaling gain and varies between 0 and 1. $\lambda$ scales the control action: a low value of $\lambda$ reduces the disturbances effect, however, the reference
tracking gets worse. An input scaling gain of $\lambda=0.6$ is used to design the INDI controller for the flying wing.

## Desired dynamic loop gains

Considering a $\xi=1$ and $T^{2 \%}=\frac{4}{\xi \omega_{n}}=1 \mathrm{~s} \Leftrightarrow \omega_{n}=4 \mathrm{rad} / \mathrm{s}$. According to the equations (4.28) and (4.29), the linear gains, $\mathbf{K}_{1}$ and $\mathbf{K}_{2}$ are defined by

$$
\mathbf{K}_{1}=\left[\begin{array}{cc}
16 & 0  \tag{4.32}\\
0 & 16
\end{array}\right] \mathbf{K}_{2}=\left[\begin{array}{ll}
8 & 0 \\
0 & 8
\end{array}\right]
$$

## Angular accelerations - Filter

Moreover, according to the equation (4.31), the control input depends on the angular acceleration measurements, $\dot{\xi_{0}}=\left[\begin{array}{cc}\dot{p_{0}} & \dot{q_{0}}\end{array}\right]^{T}$. Those angular accelerations must be obtained from the angular rates using a filter which estimates $\dot{p}$ and $\dot{q}$ since no angular acceleration sensor should be available. For this reason, it is implemented a second order washout filter with a damping factor and a bandwidth of: $\xi=1$ and $\omega_{n}=50 \mathrm{~Hz}$, respectively.

$$
\begin{equation*}
\text { Filter }=H(s)=\frac{\omega_{n}^{2} s}{s^{2}+2 \xi \omega_{n} s+\omega_{n}^{2}}=\frac{50^{2} s}{s^{2}+100 s+50^{2}} \tag{4.33}
\end{equation*}
$$

At this point is important to understand how is the system sensitivity by changing the flight conditions.

Repeating the analysis implemented in the previous chapters, figure 4.12 presents the time domain response obtained by the nonlinear closed loop system for $\phi$ and $\theta$ system inputs ( $\theta$ : step input with $\theta_{\text {min Ref }}=-20^{\circ} . \phi:$ constant input equals to zero) and for different flight condition: $V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}$ (purple); $V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}$ (blue); $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$ (yellow); $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 m$ (green).

And figure 4.9 presents the time domain response for $\phi$ and $\theta$ system inputs ( $\phi$ : step input with $\phi_{\max \text { Ref }}=20^{\circ}$. $\theta$ : constant input equals to $\theta_{\text {Trim }}$ ) and for different flight condition: $V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}$ (purple); $V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}$ (blue); $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$ (yellow); $V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}$ (green). Analysing those figures, it is seen that changing the flight conditions the system response remains essentially unchanged.


Figure 4.12: Time Domain Responses for the system configuration (figure 4.1).
(...) - $\phi_{R e f}$.
$V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$
$(\quad) ; V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$.


Figure 4.13: Time Domain Responses for the system configuration (figure 4.1).
(...) - $\phi_{R e f}$.
$V_{t}=15 \mathrm{~m} / \mathrm{s}$ and $h=100 \mathrm{~m}(-) ; V_{t}=20 \mathrm{~m} / \mathrm{s}$ and $h=2500 \mathrm{~m}(-) ; V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=5000 \mathrm{~m}$ $(\quad) ; V_{t}=40 \mathrm{~m} / \mathrm{s}$ and $h=10000 \mathrm{~m}(-)$.

### 4.5 Guidance



Figure 4.14: $\psi$ and $\chi$ time domain responses for a Nonlinear Attitude Control (figure 4.11) and for $h=15000 \mathrm{~m}$ and $V_{t}=35 \mathrm{~m} / \mathrm{s}$.

Until this moment, the flying wing control was designed without the consideration of the lateral state $\psi$, yaw angle. It is seen that this $\psi$ response is affected by a zero in the right half complex plane non minimum phase system. This response is defined by a negative derivative in the origin. This effect represents a physical phenomenon in the lateral movement, the effect of the sideslip. For this reason, a new output is defined, the course angle, $\chi$. This angle is the sum of the yaw angle and the sideslip angle, $\chi=\psi+\beta$. The $\chi$ response considering the sideslip effect does not present a negative derivative in the origin - the influence of the non minimum phase system is canceled.

### 4.5.1 Course Angle Control - $\chi$

The first step in order to follow a reference path defined by waypoints is to guarantee that the flying wing is capable of following the course angle reference. Considering this, in the next subsections the design of a $\chi$ controller is detailed.

### 4.5.1.A Linear Control

Considering a Root Locus approach (figure 4.16) for the system represented in figure 4.15, it is determined a $K_{\chi}$ that guarantees a satisfactory reference tracking. The closed loop system for this case is the closed loop of the system represented in figure 4.7 - the closed loop system controlled by a classical control approach.


Figure 4.15: $\chi$ Control-Closed Loop System block represents the block diagram represented in figure 4.7


Figure 4.16: Root locus to $\chi(V t=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m})$

As seen before, over the flight envelope a gain scheduling must be performed to guarantee the achievement of control objectives. For the same reason, it is necessary to implement a gain scheduling to calculate $K_{\chi}$. Repeating the strategy implemented before, it is necessary to define different $K_{\chi}$ values varying the airspeed. Once again, it is also verified that for the same airspeed and decreasing the altitude, the flying wing performance improves. For this reason, the control is projected for different airspeeds and for the maximum altitude of each airspeed. Figure 4.16 represents the Root locus approach for an airspeed of $35 \mathrm{~m} / \mathrm{s}$. The methodology used to define the different $K_{\chi}$ values varying the flight conditions is summarized in table 4.6.

Table 4.6: Gain Scheduling - $\chi$ control design. The letter C represents the flight condition for which the control was designed. $K_{\chi}$ for each airspeed. The $N / A$ area represents flight conditions that do not belong to the flight envelope.



Figure 4.17: $\chi$ and $\theta$ time responses for a flight condition of $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$ for a Linear controller. $(-)-\theta_{\text {Trim }}$ and $\phi_{\text {Ref }}=20^{\circ} ;(-)-\theta$ and $\phi$ time responses.

Figure 4.17(a) represents the $\chi$ time domain response for a flight condition of $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$. Analysing this figure, a satisfactory reference tracking without overshoot, a settling time equals to $6 s$ and a null steady state error are verified.

### 4.5.1.B Nonlinear Control

Once again, to perform a path-following the $K_{\chi}$ determination for the system obtained by a Nonlinear control approach must be done. The system 4.18 enables the course angle, $\chi$, tracking. Applying a Root Locus approach to define this $K_{\chi}$ value it is obtained a $K_{\chi}=2.65$. Regarding the INDI's characteristics previously seen, the system is not influenced by changing the flight conditions for this reason, changing the flight conditions, the $K_{\chi}$ value does not change and the control objectives are also verified. Figure 4.19(b) it is possible to verify the satisfactory $\chi$ tracking: response without overshoot, a sampling time equals to $6 s$ and a null stationary error.


Figure 4.18: $\chi$ Control - considering the approximation of a coordinated flight for a $\phi-\chi$ transformation. Low Level (LL) INDI defined by the $2^{n d}$ order transfer function (4.25)


Figure 4.19: $\chi$ and $\theta$ time responses for a flight condition of $V_{t}=35 \mathrm{~m} / \mathrm{s}$ and $h=15000 \mathrm{~m}$ and for a Linear and an INDI controller. (-) - $\theta_{\text {Trim }}$ and $\phi_{\text {Ref }}=20^{\circ} ;(-)-\theta$ and $\phi$ time responses.

### 4.5.2 Guidance by Waypoints

The flying wing path as mentioned before is defined by a reference trajectory defined by waypoints. The problem at this point is define a algorithm that guarantees that the flying wing passes by these predefined
conjunct of points - directional lateral control - with a stable longitudinal flight. It is also important to emphasize that in this study it is considered a gliding flight. For this reason, without the motor influence, the flying wing altitude during the flight is not defined [39].

The first step is define a conjunct of waypoints (waypoints with lateral and longitudinal coordinates) in the NED frame. Then, the path between two consecutive waypoints define a trajectory segment. In order to improve the reference tracking it is considered a influence circular area rounding each waypoint (circumferences centered in each waypoint). When the flying wing's position is in the interior of a waypoint influence area, the flying wing should follows the next waypoint - going through the next trajectory segment.

(a) Trajectory defined by waypoints (A,B,C and D). Representation of the trajectory segments between the waypoints. $\chi_{\text {Ref }}$ measurement. The circumferences around each waypoint represent each incidence area: a flying wing position inside the area limited by the circumferences is sufficient to admit that the flying wing has reached the desired waypoint.

(b) $\chi$ measurement - course angle of the current flying wing position. Angle between the North and the flying wing velocity vector.

Figure 4.20: Path Following - Waypoints; Segments; Incidence area of each waypoint; $\chi_{\text {Ref }}$ and $\chi$ measurement.

The desired course angle, $\chi_{\text {Ref }}$ is measured as an angle relative to North, $0^{\circ}<\chi<360^{\circ}$ (angle measured in the clockwise direction). However it is necessary to consider the course angle discontinuity otherwise the flying wing turning movement is not optimised: the rotation angle is not minimized. For example, if the angle relative to North for the waypoint that the flying wing has to reach is $355^{\circ}$ and the actual course is $5^{\circ}$, the flying wing should turn left $10^{\circ}\left(-10^{\circ}\right)$ instead of turn $350^{\circ}$. The graphical representation of these considerations is presented in figures 4.20(a).

### 4.5.3 Path-following Performance



Figure 4.21: 2D and 3D trajectory for a no-wind flight controlled by a linear control approach. Waypoints - (X). Flying wing trajectory - (-).


Figure 4.22: 2D and 3D trajectory for a no-wind flight controlled by a nonlinear control approach. Waypoints - (X). Flying wing trajectory - (-).

As explained in the chapter 1, one of the study's objectives is guarantee that the flying wing with a gliding flight is able to reach several predefined waypoints. To test if this objective is or is not verified, it is defined a specific horizontal trajectory by a set of waypoints. Then, considering that the first waypoint given by the horizontal trajectory is the actual flying wing position, it is presented the flying wing trajectory tracking. Figure 4.21 and 4.22 represent the flying wing flight controlled by a linear and nonlinear control approach, respectively, for a no-perturbed flight. Tables B. 2 and B. 1 represent the instant of time of each waypoint achievement and also the error between the real and the desired lateral and longitudinal flying
wing positions.

### 4.5.3.A Wind Disturbances

Until this point the wind disturbance is not considered. However, the wind effect has to be analysed to understand how is the performance of the controller under real conditions. The flying wing dynamic model is designed from the no-wind assumption. For this reason the wind is considered was a external system perturbation. In fact, one of the most relevant perturbation parameters is the wind. According to Instituto Português do Mar e da Atmosfera (IPMA) the wind is classified by the orientation and the intensity [40]:

- Light wind: wind speed $<4.2 \mathrm{~m} / \mathrm{s}$
- Moderate wind: $4.2 \mathrm{~m} / \mathrm{s}<$ wind speed $<9.8 \mathrm{~m} / \mathrm{s}$
- Strong wind: $9.8 \mathrm{~m} / \mathrm{s}<$ wind speed $<15.4 \mathrm{~m} / \mathrm{s}$
- Super strong wind: $15.4 \mathrm{~m} / \mathrm{s}<$ wind speed $<21 \mathrm{~m} / \mathrm{s}$
- Extreme wind: wind speed $>21 \mathrm{~m} / \mathrm{s}$


Figure 4.23: Path-following considering a $10 \mathrm{~m} / \mathrm{s}$ North wind: INDI control for a $r_{1}$ boundary (...); Linear control for a $r_{1}$ boundary ( - ); Linear control for a $r_{2}$ boundary ( $\ldots$ ). $\left(r_{2}>r_{1}\right)$

Setting different simulations considering a North wind step for different speeds it is concluded that winds with a velocity up to $30 \%$ of the airspeed $\left(\approx 0.3 V_{t}\right)$ do not jeopardize a good attitude reference tracking. For this reason, to guarantee a satisfactory performance the wind has to be light or moderate. This assumption is valid for both controllers. For a flight disturbed by wind and according to the pathfollowing algorithm mentioned before, considering two waypoints, the waypoint A and the waypoint B ,
when the flying wing reaches the proximity of the waypoint A (boundary defined by a circumference centered in A), it is considered that the flying wing is already able to move to the waypoint B. Regarding this assumption and setting a North wind step, it is seen that the flying wing flight controlled by linear controllers is more sensitive to wind perturbations than the flying wing controlled by an INDI controller. (Figure 4.23). Implementing the path-following methodology considering two different conditions for each waypoint boundary: $r_{1}$ and $r_{2}$ with $r_{2}>r_{1}$, and for linear and nonlinear control approaches, it is seen that the entire set of waypoints is just reached for a flight controlled by a linear controller if it is considered a waypoint boundary radius of $r_{2}$ (condition 2). However, performing an INDI control methodology with a tighter boundary a successful path-following is achieved.

### 4.5.3.B Sensor Noise


(a) Time domain responses considering the sensor noise effect - Linear Controller

(b) Time domain responses considering the sensor noise effect - INDI Controller

Figure 4.24: Time domain responses considering the sensor noise effect for Linear and Nonlinear Controllers. (... No-Sensor Noise; - Sensor Noise.)

Considering white noise to simulate the sensors inaccuracies on the path-following performance it is concluded that the presence of inaccuracies on the sensors measures affects considerably both controllers performance. However, both controllers ensure the achievement of the defined set of waypoints as represented in figure 4.25 . Figure 4.24 represents the effect of the sensors on the time response. For the accelerations are added a white noise of $0.1 \mathrm{~m} / \mathrm{s}^{2} \mathrm{rms}$; For the angular rates a $0.1^{\circ} / \mathrm{s} \mathrm{rms}$; For the Euler angles a white noise of $0.3^{\circ} \mathrm{rms}$; And finally, for the velocities a $2.5 \mathrm{~m} / \mathrm{s} \mathrm{rms}$ [37].


Figure 4.25: Sensor noise effect on the path-following for a no-wind gliding flight. (-) -INDI control approach; $(-)$-Linear control approach.

### 4.5.4 Actuators Request



Figure 4.26: Actuators request during the path-following: (-) - $\delta_{A} ;(-)-\delta_{E} ;()-\delta_{T}$

Comparing the actuators request made by each controller during the performance of the predefined path, as represented in the figure 4.26, it is verified a similar actuators request for both control approaches, however, the INDI controller shows a smaller actuators request than the linear controller. Nevertheless, both have a smooth behaviour.

## Final Conclusion and Future Work

Table 5.1: Controllers' comparison summary

|  |  | INDI CONTROLLER | LINEAR CONTROLLER |
| :---: | :---: | :---: | :---: |
| Path-Following <br> Performance | Position errors | + | + |
|  | Tracking smoothness | + | + |
|  | Requested control effort | + | $\pm$ |
| Robustness | Wind | + | - |
|  | Sensors | $\pm$ | $\pm$ |
| Control Design | Code simplicity | + | - |
|  | Design parameter tunning | ++ | - |

Regarding those considerations the flying wing is controlled by a classical controller and an INDI controller. Both controllers, as mentioned, have some advantages and disadvantages. In order to introduce an overall controllers comparison, table 5.1 represents a qualitative evaluation for each controller and for some parameters. The symbols ' + ', ' $\pm$ ' and '-' are used to represent a good, an average and a poor performance, respectively, for each parameter [2]. This overall comparison presents three qualitative parameters under analysis: the path-following performance, the robustness and the control design. In the previous sections, the performed analysis focused primarily on the first two topics: path-following performance and robustness. However, as introduced in the state of the art, one of the major improvements and innovations achieved with nonlinear flight control is the design of it controllers. Using an INDI controller the model dependency is greatly reduced and an independency of the system is verified by changing the flight conditions. For this reason, this control approach does not need a design parameter tuning. However, for a linear controller it is necessary to perform a gain scheduling to deal with the nonlinearities in the model: varying the flight conditions the flying wing control was to change. For this reason, the flight envelope is divided into several operation regimes and for each one of those a linear controller is designed. This gain scheduling is time-consuming and for models with significant nonlinearities it requires an exhaustive and well defined operation regimes division. The fact that this gain scheduling procedure is not required for an INDI control leads to one of the most relevant advantages of this control methodology. Considering the study developed, and reflecting about possible improvements and future works, the following topics enumerate some of the suggestions:

- Implementation of experimental tests to confirm the results obtained by simulation;
- Sensors modeling to perform a more detailed study of the sensors effect on the flight control;
- Consideration of the motor influence on the flying wing flight control;
- Introduction of a more conservative gain scheduling. Divide the flight envelope into more range conditions;
- Design different nonlinear controllers and analyse those controllers contribution on the flying wing flight. According to the researches developed and reported in: [1] [2] [17] [20], it could be relevant
the design of a Incremental Backstepping (IBKS).


## Bibliography

[1] P. Acquatella, "Robust Nonlinear Spacecraft Attitude Control: an Incremental Backstepping approach," Ph.D. dissertation, Delft University of Technology, 2011.
[2] A. Moutinho, "Modeling and Nonlinear Control for Airship Autonomous Flight," Ph.D. dissertation, Instituto Superior Técnico, 2007.
[3] J. Tonti, "Development of a Flight Dynamics Model of a Flying Wing Configuration," Ph.D. dissertation, Sapienza University of Rome, 2014.
[4] J.-g. Li, X. Chen, Y.-j. Li, and R. Zhang, "Control system design of flying-wing UAV based on nonlinear methodology," Defence Technology, vol. 13, pp. 397-405, 2017.
[5] E. C. de Paiva, J. R. Azinheira, J. G. Ramos, A. Moutinho, and S. S. Bueno, "Project AURORA: Infrastructure and Flight Control Experiments for a Robotic Airship," Journal of Field Robotics, vol. 23(3/4), pp. 201-222, 2006.
[6] "Innovative high altitude balloon for atlantic observation project," http://habair.tecnico.ulisboa.pt/ index.html, accessed: 2019-09-25.
[7] H. Lindeberg, "Modelling and Control of a Fixed-wing UAV for Landings on Mobile Landing Platforms," Ph.D. dissertation, KTH Royal Institute of Technology, 2015.
[8] J. J. E. Slotine and W. Li, Applied Nonlinear Control, 1st ed. Prentice-Hall, 1991.
[9] D. McLean, Automatic Flight Control Systems, 1st ed. Prentice-Hall, 1990.
[10] H. K. Khalil, Nonlinear Systems, 3rd ed. Prentice-Hall, 2002.
[11] K. Ogata, Modern control engineering, 5th ed. Prentice-Hall, 2017.
[12] F. Lewis and V. L. Syrmos, Optimal Control. John Wiley \& Sons, 1995.
[13] B. D. Anderson and J. B. Moore, Optimal Control - Linear Quadratic Methods. Prentice-Hall, 1989.
[14] R. C. Nelson, Flight Stability and Automatic Control, 1st ed. McGraw-Hill, 1989.
[15] W. J. Rugh and J. S. Shamma, "Research on Gain Scheduling," Automatica, vol. 36(10), pp. 14011425, 2000.
[16] J. M. Kai, "Nonlinear automatic control of fixed-wing aerial vehicles," Ph.D. dissertation, Université Côte d'Azur, 2018.
[17] P. van Gils, E. van Kampen, C. C. de Visser, and Q. Chu, "Adaptive Incremental Backstepping Flight Control for a High-Performance Aircraft with Uncertainties," in American Institute of Aeronautics and Astronautics, 2016.
[18] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse, "Systematic Design of Adaptive Controllers for FeedbackiLinearizable Systems," in American Control Conference, 1991, pp. 649-654.
[19] J. A. Farrell, M. Polycarpou, M. Sharma, and W. Dong, "Command Filtered Backstepping," in American Control Conference, 2008, pp. 1923-1928.
[20] X. Gong, Y. Bai, C. Peng, C. Zhao, and Y. Tian, "Trajectory Tracking Control of a Quad-rotor UAV Based on Command Filtered Backstepping," in Third International Conference on Intelligent Control and Information Processing, 2012, pp. 179-184.
[21] P. J. Acquatella, E.-J. V. Kampen, and Q. P. Chu, "Incremental Backstepping for Robust Nonlinear Flight Control," in 2nd CEAS Specialist Conference on Guidance, Navigation \& Control, Delft, 2013.
[22] L. Sonneveldt, "Adaptive Backstepping Flight Control for Modern Fighter Aircraft," Ph.D. dissertation, Delft University of Technology, 2010.
[23] L. Sonneveldt, Q. Chu, and J. Mulder, "Adaptive Backstepping Flight Control for Modern Fighter Aircraft," in Advances in Flight Control Systems. InTech, 2011, pp. 23-52.
[24] P. Simplício, M. D. Pavel, E. van Kampen, and Q. Chu, "An acceleration measurements-based approach for helicopter nonlinear flight control using Incremental Nonlinear Dynamic Inversion," Control Engineering Practice, vol. 21, pp. 1065-1077, 2013.
[25] R. Van't Veld, "Incremental Nonlinear Dynamic Inversion Flight Control: Stability and Robustness Analysis and Improvements," Ph.D. dissertation, Delft University of Technology, 2016.
[26] W. van Ekeren, "Incremental Nonlinear Flight Control for Fixed-Wing Aircraft Design and Implementation of Incremental Nonlinear Flight Control Methods on the FASER UAV," Ph.D. dissertation, Delft University of Technology, 2016.
[27] R. K. Yedavalli, P. Shankar, and D. B. Doman, "Robustness study of a dynamic inversion based indirect adaptive control system for flight vehicles under uncertain model data," in Proceedings of American Control Conference, Denver, USA, 2003, pp. 1005-1010.
[28] P. Smith, "A simplified approach to nonlinear dynamic inversion based flight control," in AIAA 23rd Atmospheric Flight Mechanics Conference, 1998, pp. 762-770.
[29] P. Smith and A. Berry, "Flight test experience of a non-linear dynamic inversion control law on the VAAC Harrier," in AIAA Atmospheric Flight Mechanics Conference, 2000, pp. 132-142.
[30] A. Mendes, "Incremental nonlinear control for attitude tracking of a fixed-wing UAV," Master's thesis, Instituto Superior Técnico, 2017.
[31] B. L. Stevens and F. L. Lewis, Aircraft Control and Simulation. John Wiley and Sons, 1992.
[32] J. A. Bautista, A. Osorio, and R. Lozano, "Modeling and Analysis of a Tricopter/Flying-Wing Convertible UAV with Tilt-Rotors," in International Conference on Unmanned Aircraft Systems (ICUAS), 2017.
[33] A. Cho, J. Kim, S. Lee, and C. Kee, "Wind Estimation and Airspeed Calibration using a UAV with a Single-Antenna GPS Receiver and Pitot Tube," IEEE Transactions on Aerospace and Electronic Systems, vol. 47, pp. 109-117, 2011.
[34] T. A. Johansen, A. Cristofaro, K. Sørensen, J. M. Hansen, and T. I. Fossen, "On estimation of wind velocity, angle-of-attack and sideslip angle of small UAVs using standard sensors," in International Conference on Unmanned Aircraft Systems (ICUAS). Trondheim: Centre for Autonomous Marine Operations and Systems, Department of Engineering Cybernetics, Norwegian University of Science and Technology, 2015, pp. 510-519.
[35] S. Leutenegger, A. Melzer, K. Alexis, and R. Siegwart, "Robust State Estimation for Small Unmanned Airplanes," in IEEE Conference on Control Applications (CCA), 2014, pp. 1003-1010.
[36] V. Prisacuriu, I. Circiu, and M. Boscoianu, "Flying Wing Aerodynamic Analysis," 2012.
[37] E. L. Simões da Silva, "Incremental Nonlinear Dynamic Inversion for Quadrotor Control," Master's thesis, Instituto Superior Técnico, 2015.
[38] J. R. Azinheira, A. Moutinho, and J. Carvalho, "Lateral Control of Airship with Uncertain Dynamics using Incremental Nonlinear Dynamics Inversion," in IFAC (International Federation of Automatic Control), 2015.
[39] T. Oliveira, A. P. Aguiar, and P. Encarnacão, "Moving Path Following for Unmanned Aerial Vehicles With Applications to Single and Multiple Target Tracking Problems," IEEE Transactions on Robotics, vol. 32, pp. 1062-1078, 2016.
[40] "Instituto português do mar e da atmosfera," https://www.ipma.pt/pt/index.html, accessed: 2019-9-16.


## Appendix

## A. 1 Flight Quality Level

Table A.1: Flight Quality Level - Lateral Model (h=1m - $\mathrm{h}=7500 \mathrm{~m}$ )


Table A.2: Flight Quality Level - Lateral Model (h=10000m - h=20000m)

| $\begin{gathered} \mathrm{h}(\mathrm{~m}) / \\ \mathrm{V}_{t}(\mathrm{~m} / \mathrm{s}) \end{gathered}$ | 10000 | L | 12500 | L | 15000 | L | 17500 | L | 20000 | L |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | $\begin{gathered} -40 \\ -5.2+6.1 \mathrm{i} \\ -5.2-6.1 \mathrm{i} \\ -2.6 \end{gathered}$ | 1 1 1 1 | N/A |  |  |  |  |  |  |  |
| 30 | -57 | 1 | N/A |  |  |  |  |  |  |  |
|  | -5.8+7.4i | 1 |  |  |  |  |  |  |  |  |
|  | -5.8-7.4i | 1 |  |  |  |  |  |  |  |  |
|  | -2.8 | 1 |  |  |  |  |  |  |  |  |
| 35 | -76 | 1 | -51 | 1 | -33 | 1 | N/A |  |  |  |
|  | $-6.5+8.6 \mathrm{i}$ | 1 | $-4.8+7.4 \mathrm{i}$ | 1 | $-3.5+6 \mathrm{i}$ | 1 |  |  |  |  |
|  | -6.5-8.6i | 1 | -4.8-7.4i | 1 | $-3.5-6 \mathrm{i}$ | 1 |  |  |  |  |
|  | -3 | 1 | -3.3 | 1 | -3.7 | 1 |  |  |  |  |
| 40 | -97 | 1 | -66 | 1 | -43 | 1 | N/A |  |  |  |
|  | $-7.3+9.7 \mathrm{i}$ | 1 | $-5.3+8.5 \mathrm{i}$ | 1 | $-3.8+7.1 \mathrm{i}$ | 1 |  |  |  |  |
|  | -7.3-9.7i | 1 | -5.3-8.5i | 1 | -3.8-7.1i | 1 |  |  |  |  |
|  | -3.2 | 1 | -3.4 | 1 | -3.7 | 1 |  |  |  |  |
| 45 | -120 | 1 | -83 | 1 | -54 | 1 | -91 | 1 | N/A |  |
|  | $-8+11 \mathrm{i}$ | 1 | $-5.8+9.6 \mathrm{i}$ | 1 | $-4.1+8 \mathrm{i}$ | 1 | $-1.3+14 \mathrm{i}$ | 1 |  |  |
|  | -8-11i | 1 | -5.8-9.6i | 1 | -4.1-8i | 1 | -1.3-14i | 1 |  |  |
|  | -3.4 | 1 | -3.5 | 1 | -3.8 | 1 | -3.8 | 1 |  |  |
| 50 | -150 | 1 | -100 | 1 | -67 | 1 | -43 | 1 | -27 | 1 |
|  | $-8.8+12 \mathrm{i}$ | 1 | -6.3+11i | 1 | $-4.4+8.9 \mathrm{i}$ | 1 | $-3.2+7.3 \mathrm{i}$ | 1 | $-2.3+5.8 \mathrm{i}$ | 1 |
|  | -8.8-12i |  | -6.3-11i | 1 | -4.4-8.9i | 1 | -3.2-7.3i | 1 | -2.3-5.8i | 1 |
|  | -3.5 |  | -3.6 | 1 | -3.8 | 1 | -4.2 | 1 | -5 | 1 |

Table A.3: Flight Quality Level - Longitudinal Model (h=1m - h=7500m)

Table A.4: Flight Quality Level - Longitudinal Model (h=10000m - h=20000m)

| $\begin{gathered} \mathrm{h}(\mathrm{~m}) / \\ \mathrm{V}_{t}(\mathrm{~m} / s) \end{gathered}$ | 10000 | L | 12500 | L | 15000 | L | 17500 | L | 20000 | L |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -130 | 1 | N/A |  |  |  |  |  |  |  |
| 25 | -4.1 | 1 |  |  |  |  |  |  |  |  |
| 25 | $-0.15+0.1 \mathrm{i}$ | 1 |  |  |  |  |  |  |  |  |
|  | -0.15-0.1i | 1 |  |  |  |  |  |  |  |  |
| 30 | -180 | 1 | N/A |  |  |  |  |  |  |  |
|  | -5.1 | 1 |  |  |  |  |  |  |  |  |
|  | -0.1+0.071i | 1 |  |  |  |  |  |  |  |  |
|  | -0.1-0.071i | 1 |  |  |  |  |  |  |  |  |
| 35 | -240 | 1 | -170 | 1 | -110 | 1 | N/A |  |  |  |
|  | -6.1 | 1 | -4 | 1 | -2.4 | 1 |  |  |  |  |
|  | $-0.072+0.054 \mathrm{i}$ | 1 | $-0.11+0.055 \mathrm{i}$ | 1 | -0.18 | 1 |  |  |  |  |
|  | -0.072-0.054i | 1 | -0.11-0.055i | 1 | -0.18 | 1 |  |  |  |  |
| 40 | -310 | 1 | -210 | 1 | -140 | 1 | N/A |  |  |  |
|  | -7.1 | 1 | -4.7 | 1 | -2.9 | 1 |  |  |  |  |
|  | -0.054+0.043i | 1 | -0.081+0.044i | 1 | $-0.13+0.017 \mathrm{i}$ | 1 |  |  |  |  |
|  | -0.054-0.043i | 1 | -0.081-0.044i | 1 | -0.13-0.017i | 1 |  |  |  |  |
| 45 | -380 | 1 | -270 | 1 | -180 | 1 | -290 | 1 | N/A |  |
|  | -8 | 1 | -5.4 | 1 | -3.4 | 1 | 0.17 | 1 |  |  |
|  | $-0.042+0.036 \mathrm{i}$ | 1 | $-0.063+0.037 \mathrm{i}$ | 1 | $-0.099+0.02 \mathrm{i}$ | 1 | 0.051 | 1 |  |  |
|  | -0.042-0.036i | 1 | -0.063-0.037i | 1 | -0.099-0.02i | 1 | -0.3 | 1 |  |  |
| 50 | -470 | 1 | -330 | 1 | -220 | 1 | -150 | 1 | -100 | 1 |
|  | -9 |  | -6 | 1 | -3.8 | 1 | -2.3 | 1 | -1.1 | 1 |
|  | $-0.034+0.03 \mathrm{i}$ | 1 | $-0.05+0.031 \mathrm{i}$ | 1 | $-0.078+0.021 \mathrm{i}$ | 1 | -0.073 | 1 | -0.083 | 1 |
|  | -0.034-0.03i | 1 | -0.05-0.031i | 1 | -0.078-0.021i | 1 | -0.18 | 1 | -0.43 | 1 |



Appendix

Table B.1: Path-Following Performance - Linear Control Approach

|  | $\begin{gathered} \left\|N-N_{i}\right\| \\ {[m]} \end{gathered}$ | $\begin{gathered} \left\|E-E_{i}\right\| \\ {[\mathrm{m}]} \end{gathered}$ | $\begin{gathered} t \\ {[s]} \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| A-B | 4.39 | 0.41 | 13.18 |
| B-C | 2.64 | 1.09 | 31.12 |
| C-D | 2.92 | 3.36 | 50.47 |
| D-E | 1.03 | 3.55 | 69.42 |
| E-F | 0.11 | 2.04 | 89.73 |
| F-G | 0.18 | 2.27 | 104.20 |
| G-H | 1.86 | 4.50 | 125.60 |
| H-I | 2.82 | 2.44 | 149.80 |
| I-J | 2.08 | 0.57 | 172.60 |
| J-K | 2.97 | 0.03 | 193.30 |
| K-L | 1.73 | 0.01 | 206.80 |
| L-M | 3.50 | 1.01 | 241.70 |
| M-N | 3.51 | 3.24 | 265.90 |
| $\mathrm{N}-\mathrm{O}$ | 1.66 | 4.18 | 288.70 |
| O-P | 0.15 | 2.21 | 312.10 |
| P-Q | 0.33 | 4.64 | 327.80 |
| Q-R | 0.89 | 2.78 | 351.10 |
| R-S | 1.58 | 1.60 | 401.80 |
| S-T | 2.49 | 0.90 | 431.04 |

Table B.2: Path-Following Performance - INDI Control Approach

|  | $\begin{array}{c}\left\|N-N_{i}\right\| \\ {[\mathrm{m}]}\end{array}$ |  | $\begin{array}{c}\left\|E-E_{i}\right\| \\ {[\mathrm{m}]}\end{array}$ |
| :---: | :---: | :---: | :---: | \(\left.\begin{array}{c}t <br>

{[s]}\end{array}\right]\)

