# Stabilization and path-following control of a flying wing using nonlinear control techniques

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### Abstract

Nonlinear control techniques stand out for responding to the linear controllers weakness and also because it is possible to design a single nonlinear controller that ensures the validation of a single control action for the entire flight envelope. During this study and within the Innovative High Altitude Balloon for Atlantic Observation (HABAIR) project, the nonlinear model of a flying wing will be analysed and controlled. The linearization of this model for trim conditions over the flight envelope results in the decoupling of the longitudinal and lateral motions. Considering this decoupling, not only a detailed analysis of the flying wing motions over the flight envelope is performed but also a stability analysis. The Flying Wing control is implemented considering three stages: a rate control to stabilise the flight, an attitude control and a flight path control. The control methodologies developed in this thesis are Gain Scheduling and Incremental Nonlinear Dynamics Inversion (INDI). Within the scope of this project is intended to study the gliding flight of the flying wing when it is released from an High-Altitude Balloon (HAB). This analysis is based on the simulation results of a path-following mission. During this study the performance of each control approach is analysed, as well as how each one responds in the presence of external perturbations.

Keywords: Flying Wing, Classical Control, Gain Scheduling, Nonlinear Control, INDI, Path-following

### 1. Introduction

### 1.1. Motivation and Mission

In the past years, important progresses have been made by the aeronautics industry. Despite those, fixed-wing aircraft remain the most common aircraft configurations for aeronautical applications. Furthermore, those aircraft configurations have not changed significantly since the beginning of the flight era. The high confidence given to the conventional aircraft, the well-developed stability and control studies as well as structural reliability are the evidences that support the popularity of using these configurations [22].

A flying wing is a tailless aircraft and represents the simplest design configuration of a flying machine. Compared with the conventional aircraft, flying-wing aircraft has advantages on structural strength and aerodynamic aspects. Its configuration minimizes the drag and the aircraft weight. However, cancelling the tail leads to course stability weakness. Additionally, the coupling between lateral and longitudinal motion is more pronounced on flying wings than that in conventional aircraft [10].

Currently, due to the excellent performance in the slow-to-medium speed range, the interest of using flying wing aircraft to military and commercial applications has increased. Moreover, during the last decade, the widespread development of increasingly advanced Unmanned Aerial Vehicles (UAVs) further increased this design interest.

Recently, with the development of flight control techniques and high performance computational technologies, the maneuverability of flying wing Unmanned Aerial Vehicle (UAV) has attracted attentions [22]. Due to this growing interest in the applicability of a flying wing UAV and under the HABAIR (Innovative High Altitude Balloon for Atlantic Observation: Fostering the Development of a Collaborative Platform for Integrated Aerial and Oceanic Research) project, it is intended to combine a flying wing UAV to respond to this project's aim: the design and the development of an aerial hybrid platform that allows the precise positioning of scientific payloads for atmospheric and/or oceanic monitoring. The proposed solution consists of a High-Altitude Balloon (HAB) that will carry a flying wing UAV to be released and guided to a predetermined location [6]. With this intention, the HABAIR mission admits essentially two steps to achieve a better atmospheric and oceanic monitoring with a precise acquisition of data:

- The HAB motion responsible for the ascent in altitude of the payload.
- The flying wing UAV motion responsible for the guided descent and controlled release of probes.

The considered solution provides an economic and viable option of scientific payload transportation to remote areas and the acquisition of data otherwise unreachable. This platform is an innovative solution because it is a cost-effective way of covering an extensive area for different altitudes and velocities [1]. Moreover, it has a high range of applications. Some of the advantages of this data acquisition platform are the atmosphere monitoring, the study of the volcanic activity and the assistance in rescue missions. Considering a typical mission, the flying wing's motion includes essentially three stages. As mentioned before, the first step is the ascent movement of the flying wing carried by the HAB. At a desired location (defined by altitude or latitude/longitude coordinates), the UAV is released from the HAB. In this transition phase, the UAV descent is supported by a parachute. When the UAV gains stability and lift in its descent, it can then be released from the parachute and start its controlled flight. This study deals with this last phase, developing control solutions to guarantee a waypoint-defined controlled descent of the UAV. During this study is intended to guarantee a guidance by waypoints. Besides its structural robustness and simplicity, an important reason for choosing the flying wing aircraft is its higher aerodynamic lift when compared with other fixed-wing aircraft. This capacity allows to save the UAV power during the descent, thus extending its range, by defining a trajectory achievable using only the control surfaces. The motor is used only as last resource, namely for landing or for more (and usually rare) demanding trajectories.

Considering the objective mission, it is notable the high range of flight conditions under consideration in the flying wing control design. The wide flight envelope, the existing nonlinearities and the mission-dependent control allocation are some of the challenges faced, requiring that other solutions be considered besides the well known gain-scheduling of linear controllers.

## 1.2. Objectives

Considering the mission and the difficulties mentioned for the flying wing control, this work intends to report and analyse the following points:

• Identify the flight conditions range (altitudes and airspeeds), flight envelope, for the flying wing model;

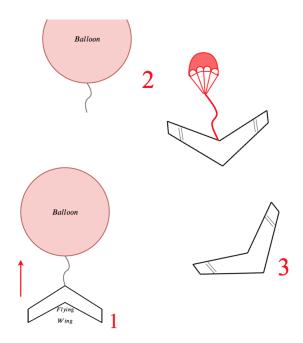


Figure 1: Flying Wing Mission 1: HAB and UAV ascent; 2: UAV release and descent with parachute; 3: UAV controlled descent

- Develop a flight quality analysis;
- Obtain a feasible nonlinear controller for the flying wing stabilization that addresses the mentioned challenges and compare its performance with a gain-scheduling control solution under wind disturbances;
- Develop a path-following algorithm to perform a guidance by waypoints of the flying wing descent motion considering a gliding flight.

Linear control includes several approaches whose research has been greatly developed over the years. Those have been used successfully for different control problems [11]. Linear control methods require an operation range for which its implementation is considered valid. However, for a large operation range it is necessary to analyse if the presence of nonlinearities in the system affect or not the success of the control performance [18]. Moreover, using linear controllers it is assumed that the system is linearizable. However, there are discontinuities that make this linearization difficult or impossible. Nonlinear controllers are able to handle with the nonlinearities of the system. Also, many control problems involve uncertainties in the model parameters, linear controllers are more sensitive to these uncertainties than the nonlinear controllers [12]. Finally, depending on the system to be controlled linear controllers may present a design more complex than nonlinear controllers and may require high quality actuators and sensors to produce linear behavior in

the specified operation range. Nevertheless, linear controllers are the most widely used in automatic flight control systems. Considering this evidence, it is important to analyse the theory that supports the linear and nonlinear controllers [2] [9].

### **Linear Flight Controllers**

In control engineering it is possible to obtain a linear mathematical model for a nonlinear system assuming that the variables deviate only slightly from the operating condition. Considering the design of linear controllers, it is possible to implement different control approaches within the Classical and Modern control theories. Usually, the Polo Placement (PP) and the Linear Quadratic Regulator (LQR) are the control approaches most used in linear control and for Multi-Input/Multi-Output (MIMO) systems [14]. As mentioned before, it is possible to control a nonlinear system performing linear control approaches. However, it is important to take into account the operation range of a linear controller. For this mission, the airspeed and the altitude are the parameters that define an equilibrium condition [15]. For this reason, it is important to understand that a single linear controller is not valid for the entire flight envelope. Changing the flight conditions, the control has to change. This solution is given by a gain scheduling on the system. Implementing this technique the flight envelope is divided into several operation regimes and for each one a conventional controller is designed. Establishing this scheduling a satisfactory control performance over the flight envelope is guaranteed. Gain-scheduling is the solution commonly applied in flight control, however, this technique presents some weaknesses: systems with a significant number of nonlinearities require a complex scheduling to reduce the performance degradation between each central solution choose to design the controller; a gain scheduling success depends on the division defined and there is not a specific and systematic approach to define an optimized division; it is a timeconsuming technique and requires also an high computational power [16].

In order to improve those weaknesses and to increase the robustness of the control approach the nonlinear controller are designed [8].

# Nonlinear Flight Controllers

According to Paul Acquatella [2] among the non-linear control methods, Nonlinear Dynamic Inversion (NDI) and Backstepping (BKS) are well-known nonlinear control approaches and the most common in flight control problems.

The BKS control approach provides a systematic methodology based on the Lyapunov theory. Con-

sidering this method, it is possible to design a controller that guarantee a global stability [3] [21].

NDI has verified successful results in flight control researches [10], [17], [24], [23]. The NDI may be explained by a simple principle: the nonlinear system is inverted by means of state feedback resulting in linear closed-loop dynamics. A NDI controller eliminates the nonlinearities in the model by cancel them with state feedback. And then, a linear controller is designed considering classical control approach to obtain the desired performance of the closed loop system. However, this control approach presents some limitations and weaknesses. Model mismatches and measurement errors reduces the NDI performance. The dynamics of the system has to be well-known in order to implement this methodology. In order to deal with those problems, an Incremental Nonlinear Dynamic Inversion (INDI) approach can be implemented to control a nonlinear system. This approach decreases the model dependency and demonstrates a robustness performance regarding the uncertainties and measurement errors.

Beside that, this control approach is considerably recent, according to Paul Acquatella [2] it date back from the late nineties with the research performed by Smith [19]. However, the application of this control method in flight control problems comes years later with the Smith Berry researches [20].

For aeronautical applications INDI has stood out as a very powerful control tool. For this reason, INDI is the nonlinear control approach used during this study to control the flying wing flight. An INDI approach requires control increments, those increments are obtained from the sensors data. Using the data that come from the sensors instead of the information that comes from the nonlinear model, it is verified a reduction on the model dependence.

### 2. Flying Wing Model

The flying wing model is obtained by its dynamics and kinematics equations and the relation between the model inputs and the variables [12] [15] [5]. The nonlinear flying wing model is a function of the actuators input, u, the wind disturbances, D, and the state variables, x, all function of time, t. In a generic way, the dynamic system is represented by the function (1). The time is also defined by t.

$$\dot{x} = f(x, u, D, t) \tag{1}$$

During this study and to test the control implementation, a  $Matlab^{\textcircled{o}}/Simulink^{\textcircled{o}}$  mode is used, built taking into account the mathematical formulation. The simulator block diagram of the flying wing open-loop model is represented in figure 2. In this simulation the sensors are considered ideal while the actuator model refers to the control surfaces dynamics. This work only addresses the flying

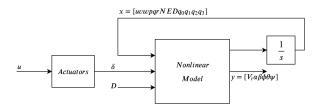


Figure 2: Nonlinear Model - Flying Wing Open-Loop Model

wing missions achievable by control surfaces alone, reason for which the motor is not represented in this model.

### 2.1. Flight Envelope

One of the most important steps in this study is the specification of the flight envelope. In order to identify the flying wing airspeed range for each altitude it is necessary to introduce the concept of stall velocity. Stall is a condition in aerodynamics such that if the angle of attack exceeds its maximum value,  $\alpha_{max}$ , then lift begins to decrease.

This condition is also linked with the consideration of a minimum airspeed for each altitude. It means, there is a minimum velocity that defines the necessary lift for the flight. This minimum velocity is known as stall velocity [12].

$$V_{st} = \sqrt{\frac{2mg}{\rho SC_{Lmax}}} \tag{2}$$

Moreover, the structural limits of the flying wing defines it maximum airspeed value. There is insufficient information so this speed is selected arbitrarily. Both speed limits define the theoretical flight envelope (blue area in figure 3). However, it is also important to emphasise that this boundary is obtained by a theoretical concept. Because of that, it is important to guarantee if the flight envelope previously presented is suitable for this flying wing (2). For this reason, it is analysed if those flight conditions have a trim evolution as expected or not. Testing the theoretical flight envelope, it is seen that it has to be re-established in order to guarantee a trim evolution as expected. The rectification process developed for this study intends to analyse the boundary calculated by the equation (2). Starting from a flight condition on the flight envelope boundary (dash line of the theoretical flight envelope (figure 3), it is tested if this condition, with an airspeed of  $V_{t1}$  and an altitude of  $h_1$ , has the expected behavior or not. Then, if the behavior is as expected the chosen condition belongs to the flight envelope boundary. Otherwise, it is necessary a flight envelope boundary rectification: considering the same altitude  $h_1$  it is chosen a new airspeed  $V_{t2}$  - a slightly higher airspeed -  $V_{t2} = V_{t1} + v$ .

For this new condition  $(h_1 \text{ and } V_{t2})$  the validation procedure is repeated.

Implementing this test algorithm for several conditions of the theoretical flight envelope boundary (figure 3) it is possible to re-establish a new flight envelope, the real flight envelope (grey area in figure 3).

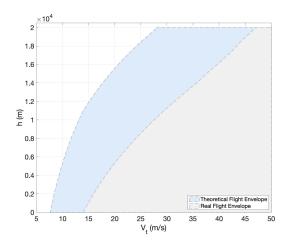


Figure 3: Flight Envelope

### 2.2. Trim or Equilibrium Conditions

For the model linearization it is necessary the determination of the trim conditions. A trim or equilibrium condition is defined by [22] [15]:

$$\dot{y}_{Trim} = C\dot{x}_{Trim} = 0 \tag{3}$$

With,  $\dot{x}_{Trim} = f(x_{Trim}, u_{Trim})$ , where  $\dot{x}$  is considered here as only function of the states and inputs, f(x, u). However, this trim condition is not solved analytically, it is performed by a numerical procedure: an optimization methodology.

For a straight trim flight, the values for the variables that describe the lateral flight are equal to zero ( $v = p = r = \phi = \psi = \delta_A = 0$ ), so, in order to obtain a flight in equilibrium conditions, it is necessary to find the values for the longitudinal variables.

For a flight without motor,  $\alpha$ ,  $\theta$  and  $\delta_E$  are the variables to define in order to obtain a zero value for  $\dot{u}$ ,  $\dot{w}$ ,  $\dot{q}$ ,  $\dot{E}$ , and  $\dot{\theta}$  [14].

### 2.3. Flying-Wing Linearized Model

Usually, to deal with the complexity of the nonlinear dynamic equations a linearization of the problem is made in order to evaluate and analyse the flying wing dynamics. Typically, it is assumed that each variable is composed by a sum of an equilibrium term,  $X_0$ , and a perturbation term, x. Considering a generic variable, X, the linearization considers:

$$X = X_0 + x \tag{4}$$

A nonlinear function, f, can be linearized according of an aircraft is defined by 2 modes: Short Period to Taylor's first order expansion (5).

$$f(X,Y,...) = f(X_0,Y_0,...) + \frac{\partial f}{\partial X}(XX_0) + \frac{\partial f}{\partial Y}(YY_0) + ...$$

As seen before, the nonlinear function  $\dot{x} = f(x, u)$ can be defined by the equation (5). Applying the nomenclature used, the nonlinear function can be rewritten to (6).

$$f(x,u) \approx$$

$$f(x_{Trim}, u_{Trim}) + \frac{\partial f}{\partial x}(x - x_{Trim}) + \frac{\partial f}{\partial u}(u - u_{Trim})$$
(6)

In addiction,  $\frac{\partial f}{\partial x}$  for  $x = x_{Trim}$  and for  $u = u_{Trim}$ , is known as an **A** matrix, the dynamic matrix, and  $\frac{\partial f}{\partial u}$  for  $x = x_{Trim}$  and for  $u = u_{Trim}$  is known as a  $\ddot{\mathbf{B}}$  matrix, input matrix.

Assuming that  $x = x - x_{Trim}$  and  $u = u - u_{Trim}$ , it is possible to define the flying wing motion by a state space formulation neglecting the disturbances:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \tag{7}$$

However, this methodology is only possible if the data values for the dynamic model are obtained analytically. But, in fact, the aerodynamic parameters are obtained by lookup tables. For this reason, the process to obtain the matrices mentioned above, dynamic matrix, A, and input matrix, B, has to be numerical instead of analytical. As a consequence, **A** and **B** are obtained by a finite difference. Each matrix entry,  $A_{ij}$  and  $B_{ij}$  is calculated by [12],

$$A_{ij} = \frac{f_i(\Delta x_j) - f_{i_{Trim}}}{\Delta x_j} \tag{8}$$

$$B_{ij} = \frac{f_i(\Delta u_j) - f_{i_{Trim}}}{\Delta u_j} \tag{9}$$

 $f_i(\Delta x_i)$  and  $f_i(\Delta u_i)$  are the acceleration terms at the disturbed state and input, respectively.  $\Delta x_i$ is the perturbation value for the state j and  $\Delta u_i$ is the perturbation input. Finalized the model linearization, the model decoupling between the longitudinal and lateral motion is a natural consequence of the linearization.

The Lateral Model is defined as presented in (10) and (12). Usually, the lateral movement of an aircraft is defined by 3 modes: Rolling subsidence mode; Dutch-roll mode; and finally, Spiral Mode.

$$x_{Lateral} = [v, p, r, \phi, \psi]^{T}$$
 (10)

$$u_{Lateral} = \delta_A$$
 (11)

$$\dot{x}_{Lateral} = A_{Lat} x_{Lat} + B_{Lat} u_{Lat} \tag{12}$$

The Longitudinal Model is defined as presented in (13) and (15). Usually, the longitudinal movement mode and Phugoid mode.

$$x_{Longitudinal} = [u, w, q, \theta]^{T}$$
 (13)

$$u_{Longitudinal} = \delta_E$$
 (14)

$$\dot{x}_{Longitudinal} = A_{Lon} x_{Lon} + B_{Lon} u_{Lon} \tag{15}$$

### 2.4. Flight Quality Analysis

Before the control implementation it is relevant to evaluate the flight quality for the flying wing. The flight qualities' specifications are defined according to the formalization provided by [12]. Firstly, following this formalization, it is necessary to establish the aircraft class and the flight phase. According to the methodology provided, the analysis is proceeded for lateral and longitudinal motion, separately. Moreover, this methodology is characterized by a set of parameters related to the damping ration, natural frequency and time to double. The knowledge of those parameters is important to understand the flying wing response to a specific command or disturbance. According to the procedure used for this analysis, the flying wing belongs to the IV state of the aircraft class, aircraft with high manoeuvrability, and an A category for the flight phase: flight non-terminal, rapid manoeuvring, precision tracking and precise control of the flight path. This evaluating process output is the characterization of the flight quality level, level 1 2 or 3, where level 1 corresponds to an aircraft with better handling qualities.

Analysing how those parameters change over the flight envelope and for each model, it is concluded that varying the flight conditions the level 1 for the flight quality may be compromised.

For this reason, a Stability Augmentation System (SAS) has to be performed with this goal, the achievement of a flight quality of level 1 for all the modes.

For the lateral modes a feedback of the roll and vaw rates is performed. And for the longitudinal modes a feedback of the pitch rate. Those feedback operations are designed considering a Root locus approach.

At this point the flight stability is already defined and in order to guarantee the reference tracking it is necessary to define the attitude control.

### 3. Flying Wing Control

### 3.1. Attitude Control

To achieve control objectives, two controllers are designed. The first is designed considering linear control approaches and the second is designed considering nonlinear control approaches. Finally, the attitude controllers are designed in order to follow references of  $\theta$  (pitch angle) and  $\phi$  (rolling angle).

### 3.1.1 Linear Control

Considering a stability augmentation system, Stability Augmentation System (SAS), in the inner loop of the attitude controller, it is defined a gain to the pitch angle and to the roll angle,  $K_{\theta}$  and  $K_{\phi}$ (figure 5). Those gains are projected for the linear and decoupled lateral and longitudinal models, respectively. The approach used to calculate the  $K_{\phi}$ follows the Root-Locus approach: placing of dominant closed-loop poles at the desired location. The approach used to calculate the  $K_{\theta}$  follows the LQR approach: minimization of a quadratic cost function restricted by the system dynamics and based on the states and input weights. However, it is also important to understand that varying the flight conditions the flying wing control has to change: a single set of gains is not valid for the entire flight envelope. For this reason, it is necessary to introduce a gain scheduling on the system: varying the flight conditions, the control gain is modified.

The figure 4 presents this time domain response obtained by the nonlinear closed loop system for  $\phi$  and  $\theta$  system inputs ( $\phi$ : step input with  $\phi_{maxRef} = 20^{\circ}$ .  $\theta$ : constant input equals to  $\theta_{Trim}$ ) and for different flight condition:  $V_t = 15m/s$  and h = 100m (purple);  $V_t = 20m/s$  and h = 2500m (blue);  $V_t = 35m/s$  and h = 5000m (yellow);  $V_t = 40m/s$  and h = 10000m (green)).

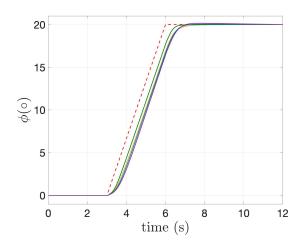


Figure 4: Linear Attitude Control -  $\phi$  time domain response

# 3.1.2 Nonlinear Control: INDI - Incremental Nonlinear Dynamic Inversion

The proposed INDI control solution requires the measurement of all system states and states derivatives. However to design an attitude INDI controller is defined a sub-state  $\xi = [pq]^T$  to provide a directional control and then allow the path-tracking

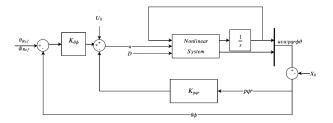


Figure 5: Attitude Control - Nonlinear Model (Tracking of  $\phi$ )

control of the flying wing. Assuming that the desired dynamic is obtained as a state error feedback with constant gains,

$$\dot{\xi} = \nu = \begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \mathbf{K_1} \begin{bmatrix} \phi_d - \phi_0 \\ \theta_d - \theta_0 \end{bmatrix} - \mathbf{K_2} \begin{bmatrix} p_0 \\ q_0 \end{bmatrix}$$
 (16)

Rewriting the equation (16) for  $\dot{\phi} \approx p$  and  $\dot{\theta} \approx q$ ,

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} = \mathbf{K_1} \begin{bmatrix} \phi_d - \phi_0 \\ \theta_d - \theta_0 \end{bmatrix} - \mathbf{K_2} \begin{bmatrix} \dot{\phi}_0 \\ \dot{\theta}_0 \end{bmatrix}$$
(17)

An INDI controller is designed considering a sample rate high enough and that the actuators present fast dynamics when compared to the system. Increasing the sample rate or decreasing the sampling time, the oscillatory response will decrease and the reference tracking is improved. The control frequency used in this study is 100Hz, a sampling time of 0.01s. The assumption of a fast control and high sample rate admit,

$$\phi_0 \approx \phi \tag{18}$$

$$\theta_0 \approx \theta$$
 (19)

Which correspond to two desired second order responses for  $\phi$  and  $\theta$ .

$$\frac{\phi}{\phi_d} = \frac{K_1(1,1)}{s^2 + K_2(1,1)s + K_1(1,1)} \tag{20}$$

$$\frac{\theta}{\theta_d} = \frac{K_1(2,2)}{s^2 + K_2(2,2)s + K_1(2,2)} \tag{21}$$

$$s^{2} + 2\xi\omega_{n}s + {\omega_{n}}^{2} = s^{2} + K_{2}(2,2)s + K_{1}(2,2)$$

$$s^{2} + 2\xi\omega_{n}s + {\omega_{n}}^{2} = s^{2} + K_{2}(1,1)s + K_{1}(1,1)$$

$$K_1(1,1) = K_1(2,2) = \omega_n^2$$
 (22)

$$K_2(1,1) = K_2(2,2) = 2\xi\omega_n$$
 (23)

Defining the control input matrix by B

$$\mathbf{B} = \begin{bmatrix} B_p \\ B_q \end{bmatrix} \tag{24}$$

And the control input as u,

$$u = u_0 + \mathbf{F}^{-1}(\dot{\xi_d} - \dot{\xi_0}) \tag{25}$$

Being  $\mathbf{F}^{-1} = \lambda B_0^{-1}$ , the closed loop system depends on the input scaling gain  $(\lambda)$  and on the desired dynamic loop gains. The equation (25) shows that, contrary to regular NDI, the INDI allows dealing with systems that are not affine in control and still obtain a valid input-output linearisation based only on high sample rate and fast control assumptions [13].

# Input scaling gain $(\lambda)$

Given the incremental nature of the controller, one intuitive solution to reduce control oscillation is to scale the incremental input [4]. The parameter  $\lambda$  is an adjustable input scaling gain and varies between 0 and 1.  $\lambda$  scales the control action: a low value of  $\lambda$  reduces the disturbances effect, however, the reference tracking gets worse. An input scaling gain of  $\lambda=0.6$  is used to design the INDI controller for the flying wing.

### Desired dynamic loop gains

Considering a  $\xi = 1$  and  $T^{2\%} = \frac{4}{\xi \omega_n} = 1s \Leftrightarrow \omega_n = 4rad/s$ . According to the equations (22) and (23), the linear gains,  $\mathbf{K}_1$  and  $\mathbf{K}_2$  are defined by

$$\mathbf{K}_1 = \begin{bmatrix} 16 & 0\\ 0 & 16 \end{bmatrix} \mathbf{K}_2 = \begin{bmatrix} 8 & 0\\ 0 & 8 \end{bmatrix} \tag{26}$$

### Angular accelerations - Filter

Moreover, according to the equation (25), the control input depends on the angular acceleration measurements,  $\dot{\xi_0} = \left[\dot{p_0} \ \dot{q_0}\right]^T$ . Those angular accelerations must be obtained from the angular rates using a filter which estimates  $\dot{p}$  and  $\dot{q}$  since no angular acceleration sensor should be available. For this reason, it is implemented a second order washout filter.

$$Filter = H(s) = \frac{\omega_n^2 s}{s^2 + 2\xi\omega_n s + \omega_n^2}$$
 (27)

Moreover, changing the flight conditions the system response remains essentially unchanged. Figure 6 presents the time domain response obtained by the nonlinear closed loop system for  $\phi$  and  $\theta$  system inputs ( $\theta$ : step input with  $\theta_{minRef} = -20^{\circ}$ .  $\phi$ : constant input equals to zero) and for different flight condition:  $V_t = 15m/s$  and h = 100m (purple);  $V_t = 20m/s$  and h = 2500m (blue);  $V_t = 35m/s$  and h = 5000m (yellow);  $V_t = 40m/s$  and h = 10000m (green).

### 3.2. Path-Following

The flight mission requires that the flying wing be able to follow a defined path. A desired tracking has

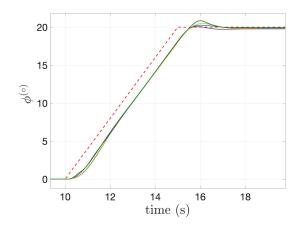


Figure 6: INDI Attitude Control -  $\phi$  time domain response

to be achieved indirectly by means of a controlled change of flying wing course,  $\chi$ .

The flying wing path is defined by waypoints. So, it is defined an algorithm that guarantees that the flying wing passes by these predefined conjunct of lateral and longitudinal coordinates in the North-East-Down (NED) frame - directional lateral control - with a stable longitudinal flight: considering two waypoints, the waypoint A and the waypoint B, when the flying wing reaches the proximity of the waypoint A (boundary defined by a circumference centered in A), it is considered that the flying wing is already able to move to the waypoint B. The

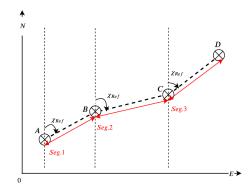


Figure 7: Trajectory defined by waypoints (A,B,C and D). Representation of the trajectory segments between the waypoints.  $\chi_{Ref}$  measurement. The circumferences around each waypoint represent each incidence area: a flying wing position inside the area limited by the circumferences is sufficient to admit that the flying wing has reached the desired waypoint.

desired course angle,  $\chi_{Ref}$  is measured as an angle relative to North,  $0^{\circ} < \chi < 360^{\circ}$  (angle measured in the clockwise direction). However it is necessary

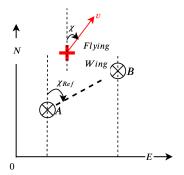


Figure 8:  $\chi$  measurement - course angle of the current flying wing position. Angle between the North and the flying wing velocity vector.

to consider the course angle discontinuity otherwise the flying wing turning movement is not optimised: the rotation angle is not minimized. For example, if the angle relative to North for the waypoint that the flying wing has to reach is 355° and the actual course is 5°, the flying wing should turn left 10°  $(-10^{\circ})$  instead of turn 350°. The graphical representation of these considerations is presented in figures 7. Defined the methodology to implement the path-tracking on the system and as mentioned before, the desired tracking is achieved indirectly by means of a controlled change of flying wing course,  $\chi$ . So it is necessary to implement the course angle control for the both controllers: Linear and Nonlinear flight controllers.

### 3.2.1 Linear Flight Controller

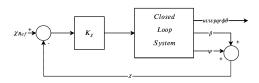


Figure 9:  $\chi$  control - Closed Loop System block represents the block diagram represented in the figure 5

Considering a Root Locus approach for the system represented in figure 9, it is determined a  $K_{\chi}$  that guarantees a satisfactory reference tracking. The closed loop system for this case is the closed loop of the system represented in the figure 5 - the closed loop system controlled by a classical control approach. As seen before, over the flight envelope a gain scheduling must be performed to guarantee that the control objectives are achieved. For the same reason, a gain scheduling for  $K_{\chi}$  has also to

be obtained: different  $K_{\chi}$  varying the flight conditions.

### 3.2.2 Nonlinear Flight Controller

The system represented in figure 10 enables the course angle,  $\chi$ , tracking. It is applied a Root Locus approach to define the  $K_{\chi}$ . Regarding the INDI's characteristics,  $K_{\chi}$  is not influenced changing the flight conditions.

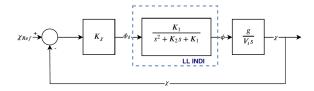


Figure 10:  $\chi$  Control - considering the approximation of a coordinated flight for a  $\phi$ - $\chi$  transformation. Low Level (LL) INDI defined by the  $2^{nd}$  order transfer function (20)

### 4. Results

The figures 11 and 12 represents the flying wing flight controlled by a linear and nonlinear control approach, respectively, for a no-perturbed flight and for a specific path defined by waypoints.

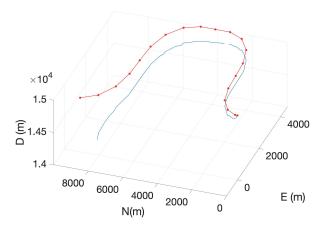


Figure 11: 3D trajectory for a no-wind flight controlled by a linear control approach.

Waypoints - (\*). Flying wing trajectory - (—).

### 4.1. Wind Disturbances

Setting different simulations considering a North wind step for different speeds it is concluded that winds with a velocity up to 30% of the airspeed ( $\approx 0.3V_t$ ) do not compromise a good attitude reference tracking. For this reason, to guarantee a satisfactory performance the wind has to be light or

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rabic r.	Controllers'	Comparison	Summa y

		INDI CONTROLLER	LINEAR CONTROLLER
Path-Following Performance	Position errors	+	+
	Tracking smoothness	+	+
	Requested control effort	+	±
Robustness	Wind	+	-
	Sensors	土	±
Control Design	Code simplicity	-	+
	Design parameter tuning	++	-

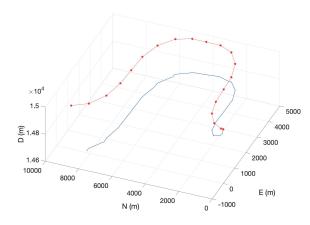


Figure 12: 3D trajectory for a no-wind flight controlled by a nonlinear control approach. Waypoints - (\*). Flying wing trajectory - (—).

moderate. This assumption is valid for both controllers.

### 4.2. Sensor Noise

Considering white noise to simulate the sensors inaccuracies on the path-following performance it is concluded that the presence of inaccuracies on the sensors measures affects considerably both controllers performance. However, both controllers ensure the achievement of the defined set of way-points.

### 4.3. Actuators Request

Comparing the actuators request made by each controller during the performance of the predefined path it is verified a similar actuators request for both control approaches, however, the INDI controller shows a smaller actuators request than the linear controller. Nevertheless, both have a smooth behaviour.

## 5. Conclusions

Previously, it was discussed some of the advantages and disadvantages of each control approach that have been used on this study. Regarding those considerations it was designed the control of the flying wing considering a classical controller and an INDI controller. Both controllers, as mentioned, have some advantages and disadvantages. In order to introduce an overall controllers comparison, table 1 represents a qualitative evaluation for each controller and for some parameters. It is used the symbols '+', '±' and '-' to represent a good, an average and a poor performance, respectively, for each parameter [14]. This overall comparison presents three qualitative parameters under analysis: the path-following performance, the robustness and the control design.

As introduced in the state of the art, one of the major improvements and innovations achieved with nonlinear flight control is the design of it controllers.

Using an INDI controller the model dependency is greatly reduced and it is verified an independency of the system by changing the flight conditions. For this reason, this control approach does not need a design parameter tuning. However, for a linear controller it is necessary to perform a gain scheduling to deal with the nonlinearities in the model: varying the flight conditions the flying wing control was to change. For this reason, the flight envelope is divided into several operation regimes and for each one of those a linear controller is designed. This gain scheduling is time-consuming and for models with significant nonlinearities it requires an exhaustive and well defined operation regimes division. The fact that this gain escalation procedure is not required for an INDI control leads to one of the most relevant advantages of this control methodology.

Considering the study developed, and reflecting about possible improvements and future works, the following topics enumerate some of the suggestions:

- Implementation of experimental tests to confirm the results obtained by simulation;
- Sensors' modeling to perform a more detailed study of the sensors' effect on the flight control;
- Consideration of the motor's influence on the flying wing flight control;
- Design different nonlinear controllers and analyse those controllers contribution on the flying wing flight. According to the researches developed and reported in: [2] [14] [7], it could be

relevant the design of a incremental backstepping controller.

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