Computational Analysis of the Thermoelastic Damping

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Abstract

Quantifying damping phenomena is of utmost importance, once they have a crucial influence on every dynamic system. However, depending on each situation, these phenomena can be associated with different objectives. For instance, in most micro-mechanical systems the goal is to dissipate the least energy, whereas in large structures the suppression of undesirable vibration can be advantageous. A possible model to describe structural damping is the thermoelastic one, as an alternative to the commonly used hysteretic one. In this work a finite element program was developed in order to evaluate the thermoelastic damping of thin models. A serendipity shell element was formulated through Reissner Mindlin’s plate theory, three-dimensional heat conduction and linear thermoelasticity, based on certain assumptions. The consequent computational implementation was described in order to enlighten the user regarding the allocation of the variables and the general functioning of the program, which was validated with simple problems of elasticity and heat-transfer to study the reliability of the meshes. Finally, the thermoelastic damping was evaluated for three cases, beginning with a silicon micro-beam that has already been studied, moving on to an aluminium beam and concluding with an aluminium plate. Although the plate results slightly diverted from the theoretical models, the beam ones were satisfactory.

Keywords: Thermoelastic Damping, Finite Element Analysis, Thin Structures

1. Introduction

Evaluating structural damping phenomena is a crucial matter given its strong influence on any dynamic system. Thus, there is a need to develop precise models to predict it. One of the most used is the hysteretic damping model, which is great for solving systems in the frequency domain but impossible to use in the time domain. An alternative model is the thermoelastic one, based on the transfer of dissipated mechanical energy into thermal energy (and vice-versa). Despite its possible usage in transient systems, the formulation is composed of partial differential equations, being difficult to find analytical solutions. Thus, a finite element approach would be an appropriate method to make use of the thermoelastic model.

The principle behind the thermoelastic damping is related with the link between the thermal state of a material with its elasticity. Since deformations induce temperature variations (and vice-versa), deformation gradients cause temperature gradients. Thus, an irregular deformation field gives rise to irreversible heat fluxes within a body, meaning a fraction of the mechanical energy is lost by conversion into thermal energy.

The first author to study the thermoelastic damping phenomenon was Zener [1]. Studying flexural modes of vibration of beams, the author evaluated the inverse of the quality factor, reaching the expression:

\[ \frac{1}{Q_F} = \frac{E\alpha^2 T_0}{C_e} \left( \frac{\omega \tau}{1 + (\omega \tau)^2} \right) \]  

where \( E, \alpha, T_0 \) and \( C_e \) are the Young’s Modulus, the linear thermal expansion coefficient, the equilibrium temperature and the specific heat per unit volume, respectively. The terms \( \omega \) and \( \tau \) refer respectively to the angular frequency and the thermal relaxation constant (associated with the most relevant mode [2]), given by:

\[ \tau = \frac{C_e h^2}{k \pi^2} \]  

where \( h \) and \( k \) are respectively the thickness of the beam and the material thermal conductivity.

Later on, the exact expression for the inverse of the quality factor was deduced by Lifshitz and Roukes [2] (L-R), deviating only slightly from Zener’s expression. The analytical expression is:

\[ \frac{1}{Q_F} = \frac{E\alpha^2 T_0}{C_e} \left( \frac{6}{E^2} \frac{\sinh(\xi_\omega) + \sin(\xi_\omega)}{\xi_\omega^3 (\cosh(\xi_\omega) + \cos(\xi_\omega))} \right) \]  

where \( \xi_\omega = \frac{\omega h}{k} \).
where the newly introduced dimensionless variable, $\xi_\omega$, is given by:

$$\xi_\omega = h \sqrt{\frac{\omega C_p}{2k}}$$  \(4\)

Figure 1 shows the plot of the loss angle versus $\xi_\omega$ and analyzes the agreement with both Zener’s and L-R’s theories.

Interestingly, from figure 1 the theories are in great agreement (with a percentage error of approximately 1%) until the maximum of the function, that takes place at $\xi_\omega \approx 2.225$ [2]. From then on, the deviation of the Zener’s approximation from the exact solution is amplified, growing to 13% when $\xi_\omega \approx 11$.

2. Formulation

In order to evaluate the thermoelastic damping, a finite element software was developed. The theory behind the shell element is shown in the present section.

2.1. Elasticity

In the present work only elastic isotropic materials were considered, following Hooke’s law. Since mostly thin structures are to be studied, a plane stress condition was assumed. However, shear stresses were not disregarded due to the followed plate theory. Thus, the constitutive law is given by [3]:

$$\sigma = [C_\sigma] \{\varepsilon\}$$  \(5\)

where $\sigma$ and $\varepsilon$ are respectively the stress and strain vectors, and $C_\sigma$ is the elasticity matrix, computed through:

$$C_\sigma = \frac{E}{2(1+\nu)} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & \frac{\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & \frac{\nu}{2(1-\nu)} \end{bmatrix}$$  \(6\)

where $E$ and $\nu$ are the Young’s Modulus and Poisson coefficient, respectively.

In order to consider shear deformation, the Reisser-Mindlin plate theory was chosen. Two additional variables join the problem statement to include membrane behavior. Following the orientation presented in figure 2, the strains are related with the displacements through [4]:

$$\begin{align*}
\varepsilon_{xx} &= \frac{\partial u_x}{\partial x} + \frac{z}{h} \frac{\partial \phi_y}{\partial x} \\
\varepsilon_{yy} &= \frac{\partial v_y}{\partial y} - \frac{z}{h} \frac{\partial \phi_x}{\partial y} \\
\gamma_{xy} &= \frac{\partial u_x}{\partial y} + \frac{\partial v_y}{\partial x} + \frac{z}{h} \left( \frac{\partial \phi_y}{\partial y} - \frac{\partial \phi_x}{\partial x} \right) \\
\gamma_{xz} &= \frac{\partial w_z}{\partial x} + \phi_y \\
\gamma_{yz} &= \frac{\partial w_z}{\partial y} - \phi_x
\end{align*}$$  \(7\)

where $\varepsilon_{xx}, \varepsilon_{xy}$ are the strains, $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ are the distortions, $u_x, v_y, w_z$ are the displacements in $x, y, z$ direction, respectively, and $\phi_x, \phi_y$ are the rotations over $x$ and $y$ axes, respectively.

2.2. Heat Transfer

The main assumptions considered to treat the heat transfer problem were the disregarded radiation, isotropic thermal conductivity and inexistence of thermal inertia.

To govern the heat conduction within the material, the formulation follows the Fourier’s Law [5]:

$$q = -k \nabla \theta$$  \(8\)

where $q$ is the heat flux vector and $k$ is the material thermal conductivity. The variable $\theta$ is used to described temperature variations, defined by:

$$\theta = T - T_0$$  \(9\)

where $T_0$ is the initial and equilibrium temperature of the material.

The finite element has two temperature variables per node to distinguish variations due to horizontal motions (membrane behavior) from vertical ones (plate behavior). Therefore, $\theta_0$ (neutral plane temperature) and $\theta_s$ (surface temperature) were introduced, and they are related with the expression:

$$\theta(z) = \theta_0 + \frac{2z}{h} \left( \theta_s - \theta_0 \right)$$  \(10\)

where $h$ is the plate thickness and $z = 0$ corresponds to the neutral plane position.
2.3. Thermoelastic coupling

This is the part of the formulation that is responsible for the conversion of mechanical energy into thermal, and vice-versa. The thermoelastic coupling comes from the definition of total strain:

\[ \varepsilon_{\text{total}} = \varepsilon_{\text{mechanical}} + \varepsilon_{\text{thermal}} \]  

(11)

where the thermal strain is governed by:

\[ \varepsilon_{\text{thermal}} = \alpha \theta \]  

(12)

where \( \alpha \) is the linear thermal expansion coefficient.

Knowing that Hooke’s Law (equation 5) is related with the mechanical strain, rewriting the constitutive law with the total and thermal strain:

\[ \sigma = C_\sigma \{ \varepsilon_{\text{total}} - \alpha \theta \delta \} \]  

(13)

where the purpose of the auxiliary vector \( \delta \) is to indicate that the thermal expansion only affects the strains \( \varepsilon_{xx} \) and \( \varepsilon_{yy} \).

The second term in equation 13 gives rise to the thermoelastic coefficient, defined as:

\[ \beta = \frac{E \alpha}{1 - \nu} \]  

(14)

2.4. Problem statement

The strong form of the thermoelastic comes in part from the balance of forces:

\[ \nabla \cdot \sigma = \rho [I_\rho] \ddot{\mathbf{u}} \]  

(15)

where the superscript ` denotes the second time rate, \( \mathbf{u} \) is the displacements vector, \( \rho \) is the density and \( I_\rho \) is a matrix that introduces the rotational inertia:

\[ I_\rho = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 \end{bmatrix} \]  

(16)

The entropy balance is:

\[ T_0 \dot{s} = -\nabla \cdot \mathbf{q} \]  

(17)

where \( \dot{s} \) is the rate of entropy per unit volume. Some authors [6, 7] have defined entropy as:

\[ s = \beta \varepsilon + \frac{C_\varepsilon}{T_0} \theta \]  

(18)

Thus, the thermoelectric problem in the strong form is:

\[ \begin{cases} \nabla \cdot (C_\sigma \varepsilon - \beta \theta) = \rho [I_\rho] \ddot{\mathbf{u}} \\ T_0 \beta \dot{\varepsilon} + C_\varepsilon \dot{\theta} = -\nabla \cdot \mathbf{q} \end{cases} \]  

(19)

The expected Dirichlet boundary conditions are, for the kinematic case:

\[ u_x |_{x=x_0} = u_0 \]  

(20)

and for the thermal case:

\[ \theta |_{x=x_0} = \theta_0 \]  

(21)

On the other hand, the Neumann boundary conditions for the kinematic case are:

\[ \sigma |_{x \in A_0} = \sigma_0 \]  

(22)

and for the thermal case:

\[ q |_{x \in A_0} = q_0 \]  

(23)

2.5. The Finite Element

For this work, a the rectangular quadratic serendipity shell element was chosen. It presents 8 nodes which are numbered according to figure 3.

![Figure 3: The Serendipity element (adapted from [8])](image)

After weakening the problem with the chosen shape functions and assembling the finite element, one reaches the form:

\[ \begin{bmatrix} M_\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} K_\varepsilon & K_{\varepsilon \theta} & 0 \\ K_{\varepsilon \theta} & K_{\theta \theta} & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} f_\sigma \\ f_q \end{bmatrix} \]  

(24)

where \( M, C, K \) denote respectively the mass, damping and stiffness matrices, \( f \) stands for the load vector and the superscript ` corresponds to the nodal value.
2.6. Time-integration technique

The time-integration technique chosen to implement was the average accelerations version of the Newmark method. In order to solve the system in the dynamically, one must discretize the time span into small time steps. This way the system is tested for each time frame, computing the residual value [4]:

$$ R = [M] \ddot{U} + [C] \dot{U} + [K] U - F $$

(25)

where $R$ is the residual vector, meant to be null to achieve dynamic equilibrium.

If the residual fails to be null at a given time frame, a correction to the solution must be added:

$$ \Delta U = -J^{-1} R $$

(26)

where $J$ is the Jacobian matrix, computed through:

$$ J = M \frac{4}{\Delta t^2} + C \frac{2}{\Delta t} + K $$

(27)

2.7. Thermoelastic damping evaluation

Similarly to Serra and Bonaldi [7], the thermoelastic damping was analyzed by solving the system given in equation 24 in the frequency domain. This transforms equation 24 into:

$$ (-\omega^2[M] + i\omega[C] + [K]) U_0 e^{i(\omega t + \Phi)} = f_0 e^{i \omega t} $$

(28)

where $i$ is the imaginary unit, $\omega$ is the excitation frequency and $\Phi$ is the phase.

Solving equation 28 gives both the amplitude and the phase of the global displacements and temperature vector, which are crucial to determine the damping factor at a given frequency.

One of the possibilities of calculating the thermoelastic damping coefficient is to acquire several solutions for a range of frequencies and fit the resulting Bode plots with the analytical expressions. Not only was the impact of the boundary conditions on the damping phenomenon studied, but also one dynamic response was recorded to visualize the mechanical energy dissipation.

3. Results

The thermoelastic damping was assessed for three different models, starting with a silicon micro-beam, moving on to an aluminium beam and finishing with an aluminium plate. The solutions are compared with the analytical values given by aforementioned expressions. Not only was the impact of the boundary conditions on the damping phenomenon studied, but also one dynamic response was recorded to visualize the mechanical energy dissipation.

3.1. Silicon micro-beam

The first studied model was the object of Serra and Bonaldi’s work [7], which is a silicon micro-beam with the following geometry 1:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>57mm</td>
<td>10mm</td>
<td>92µm</td>
</tr>
</tbody>
</table>

Table 1: Geometry of the Silicon micro-beam [7]

Having the material properties, at 300K, shown in table 2.

<table>
<thead>
<tr>
<th>E</th>
<th>162.4</th>
<th>GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$2.54 \times 10^{-6}$</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>145</td>
<td>Wm$^{-1}$K$^{-1}$</td>
</tr>
<tr>
<td>$C_e$</td>
<td>$1.65663 \times 10^6$</td>
<td>Jm$^{-3}$K$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2330</td>
<td>kgm$^{-3}$</td>
</tr>
<tr>
<td>$T_0$</td>
<td>300</td>
<td>K</td>
</tr>
</tbody>
</table>

Table 2: Properties of Silicon at 300K [7]

3.2. Clamped-free condition

Firstly the model was analyzed with a clamped-free boundary condition, having an approximate natural frequency of 38Hz. A Convergence study was performed by recording the loss angle at the natural frequency for different meshes. The study is shown in figure 4.

The result in figure 4 is the percentage error from L-R’s analytical value. The mesh quickly converges to errors below 1%, and so a mesh with 150 elements was chosen (3 in $y$ direction and 50 in the $x$ direction). Interestingly the difference between Zener’s theory and the reference one is noticeable, being close to 1%.
To perturb this model, a sinusoidal distributed force was applied at the tip of the beam, as an attempt to suppress torsion modes. The results of the loss angle for a range of frequencies is depicted in figure 5.

Observing figure 5, the agreement between the numerical results and the theoretical curves is quite visible. There is an instability phenomenon that takes place from the peak value on. This might have to do with a mesh refinement problem, where the curvature of the beam is too intense for the mesh to be able to depict it. To further investigate this instability the amplitude field was plotted for the natural frequency and for a very high one. The result is depicted in figure 6.

Figures 6.a and 6.b represent a time frame of the vibrating structure at a given frequency. As it can be seen, the mesh is insufficiently discretized in the $x$ direction to solve the system at a very high frequency, as opposed to a low one. This explains that the instability seen in figure 5 is of numeric nature.

### 3.2.1 Clamped-clamped condition

Moving on to the clamped-clamped boundary conditions, maintaining the geometry and properties, the equivalent convergence study is presented in figure 7 for a the natural frequency (approximately $243\,Hz$).

It is clear to see that the convergence was not so successful in this case. As figure 7 shows, the minimum error from L-R's analytical value is 10%. However, the same mesh as before was considered.

Following the same methodology as before, a distributed sinusoidal force was applied in this case at the center (at $x = a/2$). The loss angle computation is presented in figure 8.

In figure 8 the numerical instability is again found. The slight increase in the curve difference can be explained through the lack of convergence of the solution due to the demanding boundary conditions.

The silicon micro-beam presents in general very low values for the loss angles and for that reason a dynamic simulation would take too long for the
damping phenomenon to be visible. Thus, aluminium models will be discussed further ahead with greater dimensions.

4. Aluminium beam

The second model to be studied is an aluminium beam, with the mechanical properties shown in table 3 and geometry presented in table 4.

Table 3: Properties of Aluminium 2024 T4 [9]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>73.1</td>
<td>GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$24.1 \times 10^{-6}$</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>121</td>
<td>Wm$^{-1}$K$^{-1}$</td>
</tr>
<tr>
<td>$C_e$</td>
<td>$2.4325 \times 10^6$</td>
<td>Jm$^{-3}$K$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2780</td>
<td>kgm$^{-3}$</td>
</tr>
<tr>
<td>$T_0$</td>
<td>300</td>
<td>K</td>
</tr>
</tbody>
</table>

Table 4: Geometry of the Aluminium beam

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.6m</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.02m</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>1mm</td>
<td></td>
</tr>
</tbody>
</table>

The model was chosen to have a clamped-free (cantilevered beam) condition with adiabatic edges. Since this model is more slender than the previous one, no convergence study was performed and the mesh was maintained at 150 elements. Computing the loss angle for similarly as before, the results are presented in figure 9.

Analyzing figure 9, the results are quite satisfactory. Despite the graph looking better than the ones shown for the silicon micro-beam, the shift in the peak frequency must be noticed. This makes the mesh instability phenomenon further away from the peak.

In order to find the damping coefficient through a different method, the Bode diagrams associated with the tip of the beam’s solution were studied. The goal is to curve fit the results in a vicinity of the system’s natural frequency narrow enough not to comprise the influence of other resonance frequencies. The method was applied to the smoother curve which is the phase plot. The Bode plot is presented in figure 10.

Since the model presents a quite small damping coefficient, the vicinity of the natural frequency of the model is fairly narrow to detect the Bode plots changes. A rather tight frequency increment was used ($\Delta \omega = 10^{-3}$ rad/s) in order to fit the real curve as close as possible. The analytical expression of the phase change for one degree-of-freedom is [10]:

$$\Phi = \arctan \left( \frac{2\xi r}{1 - r^2} \right)$$

$$r = \frac{\omega}{\omega_n}$$ (30)

where $\xi$ is the damping coefficient and $\omega_n$ is the system’s natural frequency.
Using the fitting tool from Matlab® (cftool), the results are presented in figure 11:

![Figure 11: Matlab®’s curve fitting of the phase plot](image)

Observing figure 11, the correlation coefficient is acceptably close to the unity ($R^2 = 0.9468$), which means the fitting is quite adequate. The main result from this fitting is the damping factor ($\xi_{fit}$ which is coefficient $s$ from figure 11):

$$\xi_{fit} = 6.96 \times 10^{-5} \quad (31)$$

Comparing the result with the analytical one ($\xi_{LR}$, from L-R’s expression), the quality factor relates with the usual damping factor through [10, 11]:

$$\xi_{LR} = \frac{QF^{-1}}{2} = 7.6325 \times 10^{-5} \quad (32)$$

$$\xi_{LA} = 7.6764 \times 10^{-5} \quad (33)$$

Despite not being the most practical method to obtain the damping coefficient, the curve fit of the Bode diagram shows reasonable results with an associated error of 8.81% from the theoretical one. The approximation to a second order system is apparently reasonable in this narrow region. Nevertheless, this error is still greater than the directly computed one ($\xi_{LA}$), which is 0.575%.

5. Aluminium plate

Finally, the last model is an aluminium plate, having the same material properties as the previous model and the geometry described in table 5.

The geometry was set to be:

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75m</td>
<td>0.75m</td>
<td>2mm</td>
</tr>
</tbody>
</table>

5.1. Clamped sides

In the first situation, the sides of the plate are clamped and adiabatic. To validate the mesh, a convergence study to the computed loss angle was performed. In this case, the mesh evolves at the same rate in both directions ($n^2$). The results are depicted on figure 12.

![Figure 12: Convergence study of the fully clamped plate](image)

The convergence of the solution is not so successful, as it can be seen from figure 12. The model seems to require a finer mesh than the one studied for the solution to have the typical asymptotic behavior. Once again, the clamped condition presents a difficulty in the computation for being rather demanding. Moreover, in this case the complication is spread to four sides, which requires the mesh to have a refinement that is impracticable in the academic environment. For that reason, the study was carried out with a mesh with 196 elements (14 elements in each direction).

![Figure 13: Loss angle of the fully clamped plate](image)

Figure 13 depicts the computed loss angle for the clamped plate. The red circle represents the system natural frequency (approximately $207\text{rad/s}$).

As it can be seen from figure 13, the program seems to be over-estimating the loss angle. In fact, the computed values are roughly twice the expected analytical ones, which may imply the error might be related with the boundary conditions or geometry (which did not happen in the last case). An-
other source of error is the convergence of the solution which was proven not to be complete. However, the results still demonstrate a similar behavior with the analytical ones, despite having a significant relative error. The numeric instability takes place once again, this time at a lower frequency (since this mesh is less refined when compared with the beam ones), approximately from $10^4\text{rad/s}$ on.

Nevertheless, it is interesting to plot the dynamic response of the system when subjected to an impact transverse pressure. Since this model has a damping peak in a region close to its natural frequency (easily revealing the damping phenomenon), this was the chosen system whose dynamic response is plotted.

Figure 14: Dynamic response of the center of the fully clamped plate

The thermoelastic damping is clear to see from figure 14. Even though the simulation seems to be of too small duration, it was meant to calculate seventy periods of motion.

The graph shown in figure 14 could be studied to obtain the damping coefficient with the logarithmic decay of the deflection. Nonetheless, since it is composed of various sinusoidal functions, the method would be tainted with the higher modes of vibration. To ensure the method would only evaluate the natural mode of vibration, the system would need to be enforced to vibrate at it and then released. This technique would require significant computational effort while having a questionable precision.

The time increment chosen was 2% of the natural frequency’s period. It was seen that this value is acceptable for second order systems with one degree of freedom. However, that is not the case with the studied plate, since the impact force triggers more than one mode of vibration (with multiple frequencies). This makes the dynamic plot not so reliable thanks to the time discretization not being able to depict the higher frequencies. Figure 15 demonstrates a closer view to identify the various frequencies present in the deflection.

Figure 15: Closer view at the mid deflection of the fully clamped plate

Despite the damping phenomenon not being so traceable from figure 15, the graph shows the influence of different modes of vibration in the motion of the plate. As it was mentioned, the time step is not small enough to correctly depict the high frequencies, but the corresponding amplitudes are not so significant. The most important amplitude is associated with the natural frequency of the system (as expected) and its response appears to be fairly well described.

5.2. Other boundary conditions

To further investigate the overestimation of the thermoelastic damping in plates, the same model with different boundary conditions was studied to check their influence on the computed quality factor. Figure 16 shows the computed loss angle for a fully pinned plate (SSSS) and a clamped-free-clamped-free (CFCF) one.

Figure 16: Loss angle for different boundary conditions; (a) SSSS (b) CFCF

From both figure 16.a and 16.b the difference from the analytical models remains approximately constant. This implies that the divergence of results is not related with the boundary conditions. In fact, it might not be an error, since the program uses a different theory from the analytical one.
This difference could be revealing itself on models that have a higher aspect ratio (in the case of the plate $a/b = 1$). On systems that consider three-dimensional deformations, curvatures along $y$ axis appear, including plates and wide beams (such as the model for the CFCF case). The system presents additional curvatures that are not considered in the analytical models once they only consider flexural motions of beams (neglecting any changes across $y$ axis). This additional curvature is visualized for the case of CFCF in figure 17.

Figure 17: Amplitude plot of the CFCF’s for $\omega = 100\,\text{rad/s}$

The phenomenon visualized in figure 17 scales with the aspect ratio, which is the reason for it not to appear in the case of the beam. To fully study this divergence of results, the loss angle was evaluated for different widths at a frequency of $100\,\text{rad/s}$, for the case of the CFCF plate. The study is depicted in figure 18.

Figure 18: Loss angle of the CFCF plate for different widths

The divergence of the results is now clear to see from figure 18 that it has origin on the width of the plate. The mesh refinement in the $y$ direction also pays a role in the results, but the models all seem to to present the same behavior for large widths (close to the length). The computation of the loss angle seems to have an inverted peak at a specific width (approximately 0.2m). This could be related with lateral phenomenons that analytical models do not comprise. Given the variation of results with the meshes used, the source of the divergence could also be a numeric error. Nevertheless, the calculations of the thermoelastic damping do not seem to have a problem for small widths (high aspect ratio beams), since for refined meshes the solution gets closer to the analytical value.

6. Conclusions

In terms of calculating the thermoelastic damping the results met the analytical values in the beam cases. A mesh instability phenomenon was detected for the computation of the loss angle for high frequencies which compromised the results. This means the mesh has to be carefully chosen in order to be reliable. Nevertheless, the first natural frequency does not take, in the analyzed cases, a significant value and is fairly distant from the mentioned instability.

A curve fitting to Bode diagrams was experimented for the aluminium beam to obtain the damping factor. This different methodology demonstrated acceptable results, though being quite unpractical.

The analytical values for the loss angle in the plate case were not that successful. The differences were justified for the additional curvatures across $y$ axis (influencing the thermoelastic effect) which are not considered in the analytical models. However, the plate results are not quite reliable since the mesh convergence did not succeed so well. The convergence of the solution seemed to need an even finer mesh (which would take too long to compute).

When analyzing the dynamic response of the plate, the choice of the time increment was still found to be a factor that influenced the results. Despite the motion relative to the system’s natural frequency was quite well depicted, the other vibrations seemed to lack precision. Nevertheless, the motion associated with higher frequencies was of much smaller amplitudes when compared with the natural ones.

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References


