Analytical methods for high speed impact behaviour of long rods

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Abstract

High speed and hypervelocity impact studies on long rods have been a subject of interest since the 1960's. They are considered one of the most lethal threats as a consequence of the very deep penetration depth that these projectiles reach. The development of analytical models is particularly relevant as they provide a comprehensive understanding of the physical phenomena and serve as an introductory approach to the problem of impact behavior. The research into this type of impact has produced several one-dimensional analytical models that have proven to be quite successful in predicting the penetration depth of a given ideal impact event. Where the research lacks is in models that describe non-ideal impact, that is, angled impact, which is the type of impact most likely to be encountered in real life scenarios. This work reviews the models that have historically been used to describe normal impact and some that have been proposed for angled impact. The normal impact models are then compared to experimental data and their validity confirmed, to subsequently be applied as part of the reviewed angled impact methods. The angled impact models are validated against experimental data regarding yawed impact onto oblique targets - their merits and limitations are discussed.
Keywords: analytical models, angled impact, yaw, obliquity, long rods

1. Introduction

The mechanics of penetrating objects have been extensively studied throughout the years with a substantial emphasis on military applications. The usual shapes and sizes of projectiles with significance for military purposes, e.g. small diameter cylinders and spheres, are considerably different than space debris fragments, which can have any number of irregular geometries and vary in size. Nevertheless, this research can provide concepts and models that can be expanded and modified to study the penetration of fragments in space.

Analytical models concerning angled impact are of particular importance for these efforts since they present an initial approach to predict the damages and implications of an impact at hypervelocities. Fragmentation events resulting from explosions and collisions project debris that presents irregular rotation during flight and can hit a target at multiple different orientations. The studies presented in this work have shown that any angle presented at impact, whether related to the projectiles trajectory or to the targets obliquity in respects to the impending object, can significantly alter the consequences of the event, both in terms of penetration depth and crater size. Furthermore, knowing that any impact occurrence can lead to new fragmentation, thus further aggravating the space debris population problem, a better understanding of hypervelocity impacts in space is necessary to predict these episodes and prevent future accidents.

Cour-Palais [7] studied hypervelocity projectile impact, analysing if Whipple shields, which are the shields used to protect spacecraft from collisions, could prevent non-spherical projectiles including plates, discs, cylinders and rods from damaging the spacecrafts. The conclusions were that while these shields were effective in preventing damage from spherical projectiles, they most often did not prove to successfully protect against these non-spherical projectiles, whose shapes were more consistent with orbital debris fragments.

This work deals with the impact of eroding long rods into semi-infinite targets, which has been a subject of intense research since the 1960's. These types of projectiles are metal cylinders with lengths that are a minimum of 10 times larger than their diameters. When they impact targets at high speeds, typically from 1000 m/s, they are eroding during the impact. These impacts are considered as one of the most lethal threats, as a consequence of the very deep penetration depth that the projectiles reach [15].

Throughout the years, analytical and empirical models have been proposed to describe these events. These efforts focus on reviewing one-

dimensional penetration models that facilitate the complex physical phenomena and can be easily implemented. This study concerns models for both normal and angled impact, with the introduction of yaw and oblique angles to the impact events. These will be validated against experimental data from literature with the purpose of identifying each of their merits and faults.

2. Background

A straightforward way of describing penetration analytically is to employ one-dimensional models. While being rudimentary and simple, they can provide accurate results and serve as an introductory approach to the problem. All models consider that the penetration phase of the rods happens at a constant speed and can therefore be considered steady state. The targets are all deemed semi-infinite and can therefore accommodate any penetration depth without being perforated.

2.1. Normal impact

This section presents prominent analytical and empirical models that describe long rod penetration for normal impact.

2.1.1 Hydrodynamic Theory

The hydrodynamic theory is used in cases of high speed impact, where the impact velocity is great enough to create pressures so high, that the strength of the projectile and target materials are trivial in comparison, in such a way that materials behave like fluids.

Formulating the Bernoulli equation for the dynamic pressures at the interface between the rod and the target, where V is the impact velocity, U is the penetration velocity and ρ_p and ρ_t the projectile's and target densities respectively, one obtains

$$\frac{1}{2}\rho_p(V-U)^2 = \frac{1}{2}\rho_t U^2 \tag{1}$$

Relating the depth of the crater, $P = U\Delta t$, with the time it takes for the projectile to erode, $L/(V-U)$, results in the following relation, which describes the penetration depth according to the hydrodynamic theory [5].

$$\frac{P}{L} = \frac{U}{U-V} = \sqrt{\frac{\rho_p}{\rho_t}} \tag{2}$$

2.1.2 Modified Hydrodynamic Theory

Between 1966 and 1967 both Alekseevskii [1] and Tate [16] proposed a modified version of the hydrodynamic theory that took into account the strengths of projectile and target materials. These strengths need to be overcome in order to reach the stagnation point. This results in the following modified Bernoulli equation, where Y_p is the projectiles strength factor, related to it's dynamic yield strength, and R_t is the target's resistance to penetration.

$$\frac{1}{2}\rho_p(V-U)^2 + Y_p = \frac{1}{2}\rho_t U^2 + R_t \tag{3}$$

The strength factor Y_p and R_t are related to the material's dynamic yield strengths, σ_y, in the following manner, where E_t is the targets Young modulus.

$$Y_p = 1.7\sigma_{yp} \tag{4}$$

$$R_t = \sigma_{yt}\left((\frac{2}{3}) + \ln\left(\frac{0.57E_t}{\sigma_{yt}}\right)\right) \tag{5}$$

If one considers the following relations

$$\mu = \sqrt{\frac{\rho_t}{\rho_p}} \quad and \quad A = \frac{2(R_t - Y_p)(1-\mu^2)}{\rho_t} \tag{6}$$

and solves equation (3) for U and then applies the same erosion rate that was applied for the hydrodynamic theory, one obtains

$$\frac{P}{L} = \frac{U}{U-V} = \frac{1}{\mu}(\frac{\mu\sqrt{V^2+A} - V}{\mu V - \sqrt{V^2+A}}) \tag{7}$$

2.1.3 Empirical approach to the problem

In 1992 Anderson et al. [3] gathered experimental results from a multiple researches across different decades in their penetration mechanics database. Most of the reported data consisted of tungsten alloy projectiles with length to diameter ratios of at least 10 impacting steel targets ($\rho_t = 7.85g/cm^3$). The researchers performed multiple non-linear regressions to fit the data meeting these criteria and arrived at an empirical relation that fitted most of this data reasonably well. This relation is the expression (8) below, where S_t is the ultimate static tensile strength of the target

$$\frac{P}{L} = 1.121(\frac{\rho_p}{\rho_t})^{0.3704}\tanh(B) \tag{8}$$

with

$$B = 0.0175\left(\frac{L}{D}\right)^{-0.302}\left(\frac{\rho_p V^2}{S_t}\right)^{1.137} \tag{9}$$

This empirical relation applies in cases where the projectiles erode, which Anderson et al. determined as when the condition $\rho_p V^2/S_p > 20$, with S_p as the ultimate static tensile strength of the projectile, is met, otherwise the projectile penetrate the target as a rigid body.

2.2. Angled impact

The introduction of an angle to the impact of a long rod can drastically change the outcome of the event, leading to a decrease in penetration. Introducing yaw angles to incoming rods and altering a targets geometry in order to induce an oblique impact are ways to diminish penetration, therefore very useful in military armor design.

There are two angles that define a projectiles impact, the yaw angle and the oblique angle, α and θ respectively. The yaw angle describes the angle between the rod's axis of revolution and its velocity and is generally divided into vertical yaw, in the vertical plane to the target, and horizontal yaw, the off-plane yaw. The obliquity of the impact is defined by the angle between the the rods velocity and the normal to the target plane, as depicted schematically in figure 1.

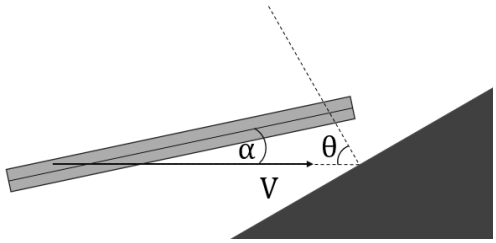

Figure 1: Angled long rod impact

2.2.1 Yawed impact

Contrasting with normal impact, in yawed impact, the penetration depth P is not only related to the length of the rod, but also to it's diameter D, and proportionally inverse to the sine of the impact angle α [6].

Recognizing these fundamental notions, Luttwak [10] ventured into further developing these considerations, and arrived at an analytical method to describe yawed impact,

$$P(\alpha) = \frac{\gamma k P(0)(1 + \sin 2\alpha)}{\frac{L}{D}\sin\alpha + \gamma k exp(-\frac{\alpha}{\alpha_c})} \tag{10}$$

where γ is a constant to correct the values, k is the crater diameter ratio and α_c is the critical yaw angle, meaning the angle at which the rod's tail start to hit the crater wall, widening it. Parameters k and α_c are given by equations 11 and 12.

$$k = \frac{D_h}{D} \cong \frac{V}{(1 + \sqrt{\frac{\rho_t}{\rho_p}})\sqrt{\frac{2Y_t}{\rho_t}}} \tag{11}$$

$$\alpha_c = \sin^{-1}\frac{kD}{L} \tag{12}$$

The factor $(1 + \sin(2\alpha))$ in equation (10) was introduced to account for the shape of the craters of yawed impact, which as opposed to the cylindrical craters caused by normal impact, have a triangular shape. In the denominator, the $exp(-\frac{\alpha}{\alpha_c})$ part of the equation serves as a means to correct the equation for small angles, lower than α_c.

2.2.2 Oblique and yawed impact

The introduction of oblique targets requires a distinction between positive yaw and negative yaw and positive obliquity and negative obliquity, since this difference will change the outcome of the impact considerably. The direction of the yaw is now important in regards to the targets obliquity, because it can either make the angle at which it is impacting even more pronounced, effectively increasing its obliquity, or make the rod more perpendicular in regards to the target.

In 2003 Luttwak [11] expanded on his yawed rod model and adapted it for impacts onto oblique targets. The distinction of positive and negative angles calls for the replacement of the variable α with it's absolute value $|\alpha|$ in equation (12) and the inclusion of a term to account for the obliquity, $\frac{\cos\theta}{\cos\theta+\alpha}$, which in case of positive obliquity decreases the penetration for positive yaw, and increase it for negative yaw, as intended, and affects the crater volume accordingly. The inclusion of this term results in equation (13) to describe yawed oblique impact along the projectile's line of sight.

$$P(\alpha) = \frac{\gamma k P(0)(1 + \sin(2|\alpha|))}{\frac{L}{D}\frac{\sin|\alpha|\cos\theta}{\cos(\theta+\alpha)} + \gamma k exp(-\frac{|\alpha|}{\alpha_c})} \tag{13}$$

3. Implementation

These one-dimensional models presented above are sufficient to determine normal and angled impact penetration onto semi-infinite targets.

The software for the implementation of the analytical models was developed using Microsoft Excel and it's programming language Visual Basic for Applications, resulting in a widely available tool capable of calculating a large number of different scenarios of hypervelocity impact, which is available for download on Github.

3.1. Design Requirements

The primary objective of this software tool is to generate the results for the aforementioned analytical models for different input conditions defined by the user. This way, one can for example introduce the impact conditions of an experimental study and obtain the theoretical results given by the analytical model to later compare them and validate the models.

The three different approaches discussed for the penetration depth of normal hypervelocity impact

events, $P(0)$ - the hydrodynamic theory, the modified hydrodynamic theory and the empirical relation - differ on the inclusion of projectile strength and the target's resistance to penetration and the empirical relation included the projectiles diameter to account for decreases in penetration for projectiles with greater L/D ratios.

Some aspects that are relevant to analyse in these types of analyses are the point where projectile strength and target resistance become irrelevant and the penetration process occurs fully hydrodynamically, which will be apparent by the approximation of the results to the hydrodynamic limit set by the hydrodynamic theory. Since Anderson's et al. [3] includes a condition that delimits where projectiles penetrate the targets as rigid bodies as opposed to eroding, analysing where that transition happens depending on the impact condition is also of interest for this work. Consequently, the software should compare the three methods and to assess their scope of application and their constraints

In regards to the angled impact analyses, the introduction of a yaw angle means that the rods velocity is not aligned with its center axis, and consequently, the large amounts of kinetic energy that they possess are no longer deposited into the same concentrated area, as is the case for a normal impact, which leads to a widening of the craters. This difference in crater shape signifies a reduction in penetration depth, so the goal is to analyse the penetration depth for a wide range of yaw angles.

3.2. Limitations

The simplicity of the analytical models reviewed in this work entails the presence of some limitations that have to be taken into account in order to ensure pertinent results.

By definition, the long models described in the previous section can only be applied to projectiles that qualify as long rods, meaning they have a length to diameter ratio L/D of at least 10.

The models developed to describe eroding rod penetration should inherently also only be applied when dealing with eroding penetrators, meaning the velocities of the projectiles at impact need to be sufficient to cause the projectile to erode.

Tate's original modified hydrodynamic theory [16] included differential equations to describe projectile deceleration and erosion. As the objective of this work was to discuss simple analytical models, the need for numerical integration would contradict this idea, so for this reason, the simple solution to describe penetration depth was chosen. As a result, equal density materials result in a density ratio $\mu = 1$, which would yield to $A = 0$ in equation (6) and as a consequence lead to an indetermination, since equation (7) would be

$$P(\rho_p = \rho_t) = \frac{0}{0} \tag{14}$$

Another limitation is imposed by the empirical relation of Anderson et al. [3], given by equation (10), since it was developed solely for steel targets and should not be applied to target densities different from $\rho_t = 7.85 g/cm^3$.

In regards to Luttwak's [11] analytical method for angled impact, defined by equation (13), it requires the sum of the desired angles for the impact to be less than or equal to 0, since for $\theta + \alpha > 90$ the term $\cos(\theta + \alpha) < 0$ and yield a negative penetration depth, which does not make physical sense.

3.3. Validation's Methodology

The input parameters one needs to introduce in the software tool are presented in table 1, along with the units those parameters should be in. These units are in multiples of the SI units because these are the typical magnitudes for these types of impacts and some empirical relations require these units to compute the correct results.

Table 1: Input parameters and their units

	Projectile	
L	Length	mm
D	Diameter	mm
ρ_p	Density	g/cm^3
σ_{yp}	Dynamic Yield Strength	GPa
S_p	Static Tensile Strength	GPa
	Target	
ρ_t	Density	g/cm^3
σ_{yt}	Dynamic Yield Strength	GPa
Y_t	Static Yield Strength	GPa
S_t	Static Tensile Strength	GPa
	Impact Conditions	
V	Impact Velocity	km/s
α	Yaw Angle	deg
θ	Oblique Angle	deg

The experimental data chosen for the validation of the analytical models did not provide information of projectile and target material properties in these specific parameters. Information on materials is instead commonly reported in terms of a material's hardness, by providing the Brinell hardness number BHN or the Vickers hardness number HV.

In order to compute a material's dynamic yield strength, Tate [17] provided a relation to its Brinell hardness number:

$$\sigma_y = 4.2BHN \tag{15}$$

The static yield strength of steels can be obtained by using a linear relation, dependent on Vickers hardness number, proposed by Pavlina and Van Tyne [12].

$$Y = 90.7 + 2.876HV \tag{16}$$

The static tensile strength of steels can be determined using the commonly known proportional relationship given by equation (17).

$$S = 3.45BHN \tag{17}$$

If a steel's hardness is reported on another hardness scale and there is the need to convert into the corresponding Brinell or Vickers hardness number, one should follow the standardized conversion tables of ISO 18265:2013 [9].

4. Results

In order to analyse results for angled impact, given that those models depend on the normal penetration $P(0)$, one starts by assessing how the normal impact methods perform against experimental results in an initial validation. A comparison between the different analytical methods is presented - the basic hydrodynamic theory, the modified hydrodynamic theory as well as the empirical method previously introduced - in order to determine which one is the most suitable for these efforts. After validation of the normal impact results, those methods can then be applied in angled impact analyses in a comparison of experimental results from literature for yawed rods onto oblique targets to results obtained by the analytical model for that type of impact.

4.1. Normal impact

To perform validation, two sets of data were chosen from the extensive experimental data obtained by Hohler and Stilp [8], both concerning tungsten allow rods impacting steel semi-infinite targets. These can be found on Anderson et al. database [3]. These sets of data were chosen due to their high number of entries.

Validation 1 was performed against data from $L/D = 10$ rods, with 17 data points for velocities between 0.74 and 3.71 km/s, the majority of those in the 1-2 km/s range. Projectile and target hardness have values of 294 and 388 respectively. Validation 2 was performed against data belonging to 58mm rods made from a harder tungsten alloy impacting softer steel targets at velocities between 0.5km/s and 2.1km/s. Dimension and material properties' values used for these tests, along with the resulting parameters needed for the analytical models determined from those values, are presented in table 2.

Table 2: Validation parameters

	Validation 1	Validation 2
L (mm)	28	58
D (mm)	2.8	5.8
ρ_p (g/cm^3)	17	17.6
BHN_p	294	406
ρ_t (g/cm^3)	7.85	7.85
BHN_t	388	255
σ_{yp} (GPa)	1.23	1.71
σ_{yt} (GPa)	1.63	1.07
Y_p (GPa)	2.10	2.90
R_t (GPa)	8.01	5.71
μ	0.68	0.67
A (km^2/s^2)	0.81	0.40
S_t (GPa)	1.34	0.88

Applying all the reviewed methods for normal impact to these geometries and conditions, the results are depicted in figures 2 and 3, set over the data points of Hohler and Stilp [3].

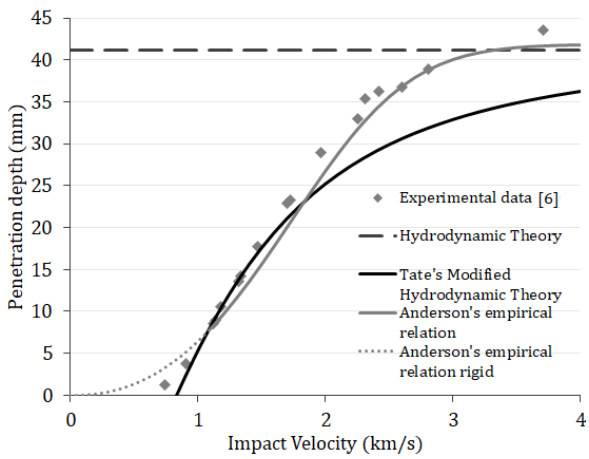

Figure 2: Validation 1

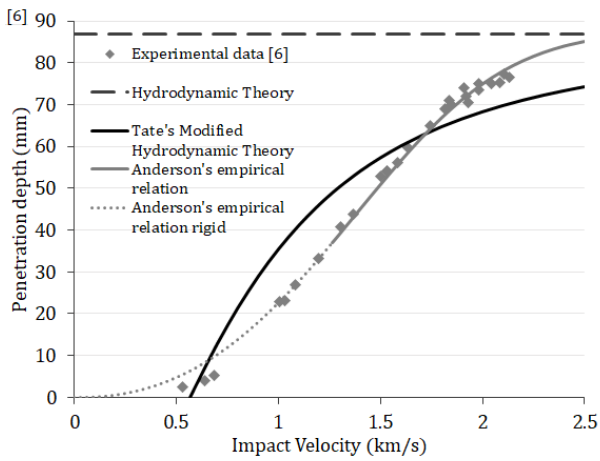

Figure 3: Validation 2

The dashed straight lines at the top are the results of the hydrodynamic theory using equation (2), which yields the same penetration depth, $P = 41.2mm$ for validation 1 and $P = 86.8mm$ for validation 2, regardless of impact velocity, since it is only dependent on the density ratio and the length of the projectiles. Since the density ratios are very similar for the two sets fo materials, the difference in penetration depth is due to the much longer rods of validation 2.

The solid black line are the curves of the modified hydrodynamic theory model, equation (7). It indicates that penetration only occurs for velocities above $V = 0.84km/s$ for validation 1 and $V = 0.57km/s$ for validation 2, which suggests that for velocities below that, the pressure caused by the impact of the rod is not sufficient to surmount the target's resistance to penetration. The curve shows that penetration depth increases steadily at first and as the velocities increase the curves start to asymptotically approach the hydrodynamic penetration depth limit. The curve for validation 2 predicts penetration at a lower velocity than the one corresponding to validation 1 because of the difference in projectile and target hardnesses, as a result of the harder rods of validation 2 impacting softer targets, meaning the pressures needed to surmount the target's resistance are lower.

The grey line curves are the predictions of the empirical relation by Anderson et al. [3], equation (8). These curves don't start registering penetration depth at $P = 0$ because of the restriction imposed to exclude rigid projectile penetration, $\rho_p V^2/S_p > 20$. This means that according to Anderson's relation, in these conditions, impacts with velocities below those along the grey lines should be treated as rigid penetration, where the projectile doesn't erode. Even though it was not proposed for rigid penetration, Anderson's relation can still be applied to those values which results in the dotted grey lines that meet the solid grey ones. These solid grey curves show a more drastic increase in penetration depth than the solid black lines, and also start to taper off and approach an asymptotic limit, that is higher than the hydrodynamic limit.

These representations incorporating all three analytical/empirical models for normal impact set on top of the experimental data allow for a comprehensive analysis into each of methods' merits and limitations.

The first, the basic hydrodynamic theory, is essentially a means to determine maximum penetration, the hydrodynamic limit. In both cases, as velocities increase past 2 km/s, experimental data starts to deviate from and surpass the modified hydrodynamic curve and approach that theoretical hydrodynamic limit. As the impacting velocities get higher, the gain in penetration depth gets smaller. In the first analysis, looking at the last two experimental data points: a 32% increase in impact velocity (2.81 to 3.71 km/s) only yields a 12% gain in penetration depth, compared to the two values right below 2 km/s, where a 14% increase in velocity led to a 24% gain in penetration depth. This holds the claim that this hydrodynamic limit proposed by the basic hydrodynamic theory can be demonstrated experimentally, as that last data point only shows a slight (5%) deviation from that limit.

For velocities lower than 2 km/s however, the basic hydrodynamic theory is not an accurate method to assess penetration depth, as data points below that value show considerable discrepancies to that penetration depth, as can be observed in the figures.

Moving on to the second curves, the ones corresponding to the modified hydrodynamic theory, they exhibit some correspondence to the data points, particularly in the first analysis. This correspondence is particularly apparent for the results in the 0.9-1.7 km/s range, where the curve deviates less than 10% for all the values. At the same time, it is also noticeable that the experimental data approach the hydrodynamic limit much faster than in this curve.

In the second validation the modified hydrodynamic theory has poorer correspondence with the experimental data. Up until velocities of 1.5km/s, the theory significantly overestimates penetration as the points seem to follow a sloped line. Only after 1.5km/s does the penetration depth of the data points come within 10% of the one predicted by this curve.

This poorer performance by the modified hydrodynamic theory could be justified by the strength of the rods, Y_p, which is substantially higher than in the first validation. In 1996 [14] Rosenberg et al. argued that a rod's increasing strength resulted in a greater deceleration of the rod during penetration, meaning that it's propensity to penetrate more due to it's strength was contrasted by this deceleration, resulting in a more reduced penetration than one might expect.

The final curves, which correspond to the empirical relation by Anderson et al. [3] developed based on data from their database, are the ones that show the best correspondence with the data points for the full range of values. It exhibits a slow growth at the beginning, followed by a steady increase along a similar slope as the experimental data and finally a quick approach to the hydrodynamic limit, surpassing it slightly (only noticeable in figure 2, as it happens for velocities greater than 2.5 km/s fro figure 3). This good correspondence is justified by the fact that the empirical relation was based on experimental data which included these data sets, so

this small deviation is expected, as the researchers reported a correlation coefficient of 0.9838.

4.1.1 L/D effect

To examine the effect different L/D ratios have on the penetration depth, experiments Hohler and Stilp [8] performed with projectiles up to ratios of 32 can be examined. In order to have comparable data from the different lengthed rods, rather than looking at penetration depth, alternatively, penetration efficiency, meaning P divided by the projectile's length, P/L, can be evaluated. Looking at data collected from projectiles with ratios of 10, 17.5, 22.5 and 32, all made from the same tungsten alloy impacting steel semi-infinite targets and focusing on data points for velocities within the 1-2 km/s range, the analytical methods performance will be assessed.

The penetration efficiency according to the modified hydrodynamic theory for these conditions was plotted together with the experimental data of Hohler and Stilp for the different ratios in figure 4. According to this theory, penetration depth or efficiency is not influenced by a rod's L/D ratio, so only one curve for all four sets of experiments.

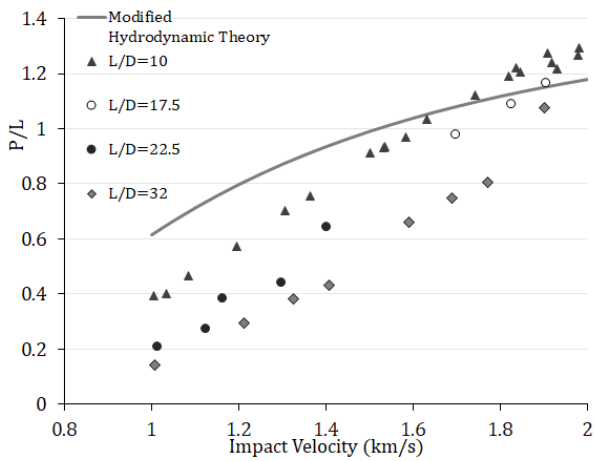

Figure 4: Validation 2

Before Hohler and Stilp published this data in 1987, the general understanding of researchers was that in that, in contrast to short projectiles, where penetration efficiency was proven to decrease with the increase in L/D ratio, for ratios greater than 10 (long rods) this tendency would stabilize and the values would fall on the same curve, regardless of ratio [15]. The experimental data presented in figure 4 demonstrated that this effect was still prevalent in long rods. Looking at the data points, it is clear that the ones with the higher penetration efficiency are the ones with the smallest ratio of 10. The values for higher ratioed rods all fall below those and the $L/D = 32$ rods clearly show the least penetration efficiency, losing around 50% of the efficiency.

Both Rosenberg and Dekel [13] and Anderson et al. [4] tried to explain this effect by performing simulations with rods of different ratios. They showed that for lower velocities, the initial non-steady entrance phase of the impact was responsible for the discrepancies in penetration efficiency, while for higher velocities this effect was less pronounced, but still present and caused by the terminal phase of penetration, which is also non steady. Tate's penetration model does not account for the initial and final transient phases, treating the whole process as steady state penetration.

4.2. Angled impact

Experimental data involving hypervelocity impacts with both yawed angles and oblique targets is very limited, at least publicly, as most efforts in this area focus on vertical impact. A recent (2013) study by Anderson et al. [2], reported on experimental results for yawed rods onto oblique targets. Using the developed software tool, one can obtain the results predicted by the analytical model proposed by Luttwak [11], equation (13), and compare them to the experimental data. The shared input parameters of the impact tests are presented in table 3, while velocity, yaw angle and obliquity will depend on the individual events. Y_p and R_t were obtained following equations (4) and (5).

Table 3: Material properties and dimensions from the Anderson et al. experimental data [2]

Projectile	**Target**
$\rho_p = 17.68\ g/cm^3$	$\rho_t = 7.85\ g/cm^3$
$Y_p = 2.035$ GPa	$Y_t = 0.772$ GPa
$L = 126{,}4$ mm	$R_t = 6.25$ GPa
$D = 6.3$ mm	

As for the remaining parameters: projectile velocity at impact was approximately $V = 1.65 \pm 0.03 km/s$; yaw angle varied between 0° and 90°; and targets were set at 30°, 45°, and 60° obliquity, both positive and negative.

In the study, rods impacted the targets with vertical yaw (pitch), as well as horizontal yaw (yaw). The one-dimensional model proposed to describe angled impact does not account for horizontal yaw angles. To bypass this limitation, the total yaw, given by $\tan\alpha = \sqrt{\tan^2\alpha_1 + \tan^2\alpha_2}$ was considered as the value for yaw in the calculations. Anderson's study included a discussion on the effect of horizontal yaw on the impact results, with the conclusion being that these angles become negligible when greater pitch angles are present and when pitch angles are small their effect is more pro-

nounced. Nevertheless, due to lack of a better option, total yaw will be used for these efforts.

4.2.1 Initial validation

In the wake of the concerns that arose with the lower accuracy of Tate's normal penetration model when applied to rods with a ratio larger than $L/D = 10$, and seeing that the rods in this experiment have a ratio of $L/D = 20$, Tate's modified hydrodynamic theory was applied to these impact conditions and compared to the four baseline experiments for projectiles with minimal yaw (less than 1.2°) impacting targets with 0° obliquity. In table 4 is the comparison between their results and the one's predicted by Tate's model.

Table 4: Normal impact result comparison - Anderson et al. [2] vs. Tate's model

V (km/s)	P_{exp} (mm)	P_{cal} (mm)	Error
1.624	105.0	112.7	7.3%
1.748	118.8	120.6	1.5%
1.699	111.2	117.6	5.8%
1.556	91.4	107.9	18%

All the predictions are above the experimental values, which is to be expected given the implications of the L/D effect. Aside from the last set of values, whose rod had a yaw angle of 1.1°, the largest of all four, and could have influenced the reduced measured penetration depth compared to the theoretical one, all the other values have a reasonable error, so this model for normal penetration can be applied as $P(0)$ in the angled impact studies.

4.2.2 Angled impact analyses

The experimental data from Anderson et al. [2] included results for a wide range of yaw angles for each target obliquity, and can therefore be plotted against the results predicted by the analytical model of equation (13) implemented in the software tool. This comparison will indicate if the model predicts oblique yawed impact accurately.

The velocities of impact are not consistent for all the impact events of the experimental data, so they would not fall on the same line in the analytical model. To address this issue, the average velocity from all the data points in each target configuration was applied to the model and the variation of penetration depth with the increasing yaw angles was plotted for the 6 different target configurations. The results of these analyses are presented in figures 5 to 10.

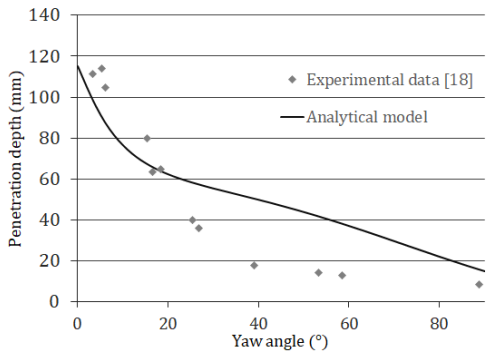

Figure 5: Results for impacts on θ=-60° targets

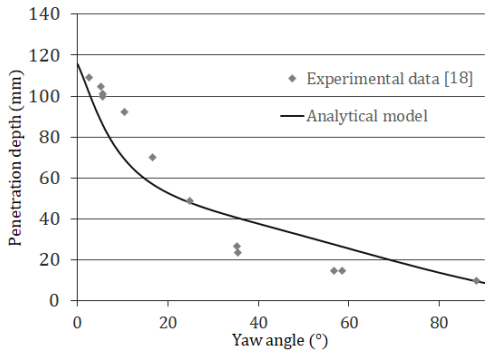

Figure 6: Results for impacts on θ=-45° targets

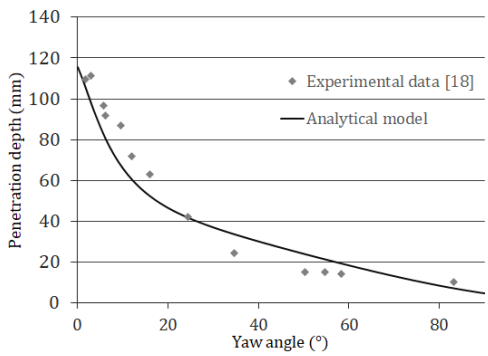

Figure 7: Results for impacts on θ=-30° targets

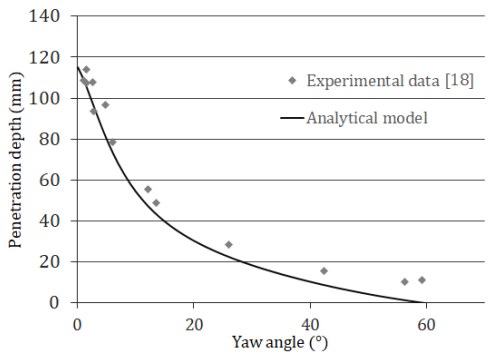

Figure 8: Results for impacts on θ=30° targets

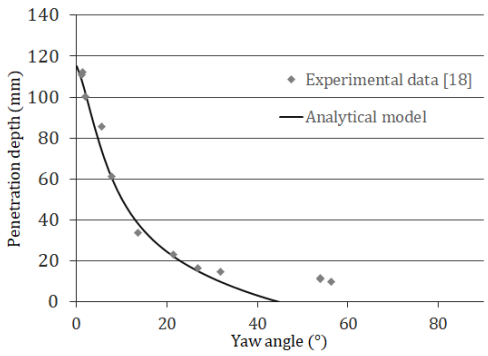

Figure 9: Results for impacts on θ=45° targets

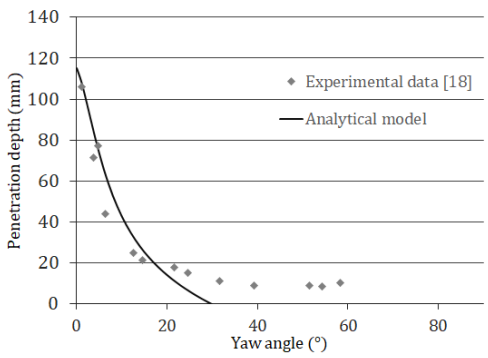

Figure 10: Results for impacts on θ=60° targets

Figures 8, 9 and 10 suggest that Luttwak's analytical model is consistent with experimental results. The results for the rods impacting the targets set at 45° obliquity are especially encouraging, as most of the data points present a deviation smaller than 10% when compared to the model.

On the other hand, the figures that show the results for the targets with negative obliquities (-30°, -45°, -60°), present greater deviations from the experimental results. All the curves for these negative obliquity targets seem to follow the same trend. For yaw angles that are below the 20°-30° range, the curve underestimates penetration depth, while from there on out it does the contrary and overstates the penetration for these values. This trend is particularly noticeable in the results for the oblique targets set at $-60°$.

These findings suggest that Luttwak's model might not be applicable to negative oblique targets. These shortcomings of the analytical model, particularly in the cases of negative obliquities, warrant further investigation.

One explanation for this fact might have to do with the fact that the critical yaw angle is different for the two situations. When a rod with positive yaw impacts a target with positive obliquity, it is effectively increasing its obliquity, while a rod with negative yaw is impacting more vertically. The critical yaw angle, which represents the angle where the tail of the rod hits the crater wall, will be smaller for a positive yawed rod hitting a positive oblique target, than for its negative counterpart.

Nevertheless, when taking this dissimilarity into consideration, the critical yaw angle is only responsible for correcting the expression for small yaw angles that are below it. Therefore, the issues with the results for the negative oblique are not resolved in their entirety, so the model may need to be adapted for negative obliquities.

5. Conclusions

This work reviewed one-dimensional analytical models to describe high speed and hypervelocity impacts of long rods. Analytical models offer valuable insight into the physical aspects that rule these events as researchers try to understand these highly complicated behaviours to better predict their outcome.

5.1. Achievements

The efforts focused on models to determine penetration depth into semi-infinite targets of normal impacts and subsequently those were applied on a model for angled impact, with the inclusion of yaw and oblique angles with the purpose of analysing how those angles influenced penetration depth.

In regards to the normal impact models, the well known model of the classic hydrodynamic theory was described, as this is the basis for most of the one-dimensional models that were further developed in this area of study. Another model that warranted attention in this work was the Alekseevskii-Tate model, also known as the modified hydrodynamic theory. An empirical approach was also presented, which was based on an extensive database of experimental data. Subsequently, the issue of angled impact was targeted, with a review of a proposed analytical model that included the terms for yaw and oblique angles and predicted the decrease in penetration depth caused by those angles.

A software tool was developed around these analytical methods, facilitating these analyses by calculating penetration depth for various input conditions chosen by the user.

This tool was put into practice for the validation of the different models against experimental data obtained from literature. The models for normal impact were compared to two sets of results for tungsten alloy rods impacting steel targets. These analysis showed a very good correspondence of the empirical relation and fairly good correspondence bymodified hydrodynamic theory, particularly for the first set of results. The classic hydrodynamic theory established itself as a viable option only for hypervelocity impacts above 3 km/s, as the projectile and target strength appeared to be considerable

for velocities up until that point, and could therefore not be ignored.

The software was then applied to angled impact analyses, where the impact model proposed for these purposes revealed itself to be a good choice to describe yawed impact onto targets with positive obliquity. However, the model lacked in impacts with negative oblique targets. This was partially explained by the different definition of the critical yaw angle for negative inclination targets which should be different than it's positive counterpart.

5.2. Future Work

The results for the angled impact analyses have shown that there is room for improvement in regards to negative oblique target impacts as the model might need to be adapted to better fit experimental results for the full range of obliquities. The model also did not account for the effects of horizontal yaw on penetration depth, so efforts can be made to improve it in that regard.

The lack of experimental data on yawed rod impact onto oblique targets was a problem for the thorough analysis of the analytical model, as experiments with both yaw and oblique angled are very limited. The experimental data concerning angled impact focused only on a small range of impact velocities, a constant L/D of 20 and the same projectiles and targets for all the experiments. More experimental data is needed in order to fully assess the influence of all these parameters have when dealing with angled impact.

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