Analytical methods for high speed impact behaviour of long rods

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Finally, I have to thank my parents, for being there during the good times and the bad, and for always believing in me and pushing me to do my best.
Resumo

Estudos sobre impactos de hastes longas a hiper- e altas velocidades têm sido um tópico de grande interesse desde a década de 60. Estes projéteis são considerados das ameaças mais letais devido ao nível de profundidade que podem atingir. O desenvolvimento de modelos analíticos que descrevam este tipo de impacto é particularmente relevante, pois estes proporcionam uma melhor compreensão dos fenômenos físicos em causa e apresentam-se como uma abordagem inicial ao problema. A investigação a este tipo de impacto já levou ao desenvolvimento de vários modelos unidimensionais capazes de prever a profundidade de um determinado impacto com êxito. Onde a investigação tem falhado é no desenvolvimento de modelos para descrever o impacto não ideal, isto é, impacto com ângulos, que é o tipo de impacto que acontece mais frequentemente em cenários fora do laboratório. Esta tese faz uma revisão dos modelos historicamente usados para descrever impacto normal e revê ainda alguns modelos propostos para impacto com ângulos. Os modelos de impacto normal são comparados a dados experimentais de forma a confirmar a sua validade, para serem posteriormente aplicados como parte dos modelos de impacto com ângulos. Estes também serão validados perante resultados experimentais envolvendo ângulos de inclinação e obliquidade – os seus méritos e limitações são discutidos.

Palavras-chave: modelos analíticos, impacto com ângulos, inclinação, obliquidade, hastes longas
Abstract

High speed and hypervelocity impact studies on long rods have been a subject of interest since the 1960's. They are considered one of the most lethal threats as a consequence of the very deep penetration depth that these projectiles reach. The development of analytical models is particularly relevant as they provide a comprehensive understanding of the physical phenomena and serve as an introductory approach to the problem of impact behavior. The research into this type of impact has produced several one-dimensional analytical models that have proven to be quite successful in predicting the penetration depth of a given ideal impact event. Where the research lacks is in models that describe non-ideal impact, that is, angled impact, which is the type of impact most likely to be encountered in real life scenarios. This work reviews the models that have historically been used to describe normal impact and some that have been proposed for angled impact. The normal impact models are then compared to experimental data and their validity confirmed, to subsequently be applied as part of the reviewed angled impact methods. The angled impact models are validated against experimental data regarding yawed impact onto oblique targets - their merits and limitations are discussed.

Keywords: analytical models, angled impact, yaw, obliquity, long rods
Contents

Acknowledgments ................................................................. iii
Resumo ........................................................................ v
Abstract ........................................................................ vii
List of Tables ........................................................................ xi
List of Figures ......................................................................... xiii
Nomenclature .......................................................................... xv
Glossary ............................................................................... 1

1 Introduction ........................................................................ 1
1.1 Problem Description ....................................................... 1
1.2 Relevance to the aerospace field ...................................... 1
1.3 Motivation ......................................................................... 4
1.4 Objectives ......................................................................... 4
1.5 Thesis Outline ................................................................. 5

2 Theoretical Background ...................................................... 7
2.1 Terminal Ballistics Overview .......................................... 7
  2.1.1 Main subfields of ballistics ...................................... 7
  2.1.2 Brief History of Terminal Ballistics ......................... 8
  2.1.3 Velocity Classification ........................................... 8
  2.1.4 Projectile Classification ........................................... 9
  2.1.5 Target Classification ............................................... 10
2.2 Long Rod Models .......................................................... 10
  2.2.1 Hydrodynamic Theory .......................................... 12
  2.2.2 Modified Hydrodynamic Theory ............................. 14
  2.2.3 Empirical approaches to the problem ...................... 15
2.3 Angled Impact ............................................................... 15
  2.3.1 Yawed impact ......................................................... 16
  2.3.2 Oblique Impact ....................................................... 18
  2.3.3 Oblique and Yawed Impact .................................... 19
  2.3.4 Horizontal yaw angle ............................................. 20
## List of Tables

2.1 Target classification .................................................. 10

3.1 Input parameters ...................................................... 28

4.1 Values for Validation 1 - Tungsten alloy rod onto steel target - L/D=10 ........................................ 36
4.2 Values for Validation 2 - Tungsten alloy rod onto steel target - L/D=10 ........................................ 38
4.3 Input values for yaw analytical model verification ................................................................. 41
4.4 Verification Results .................................................... 42
4.5 Material properties and dimensions from the Anderson et al. experimental data [26] .......... 42
4.6 Normal impact result comparison - Anderson et al. [26] vs. Tate's model .......................... 43
4.7 Result comparison with experimental data by Anderson et al. [26] .................................. 44
List of Figures

1.1 Flight 1549 on the Hudson River after emergency landing ........................................... 2
1.2 Impact test results on the Space Shuttle Atlantis wing panels to recreate the damage on
the Columbia Space Shuttle - Source: [3] ................................................................. 2

2.1 Classification of ballistic impact by velocity ................................................................. 9
2.2 Types of nose shapes - Adapted from [11] ................................................................. 9
2.3 Long rod in flight - Source: [13] ............................................................................. 11
2.4 Eroding Long Rod - Adapted from [9] ....................................................................... 12
2.5 Angled long rod impact ............................................................................................. 16
2.6 Triangular shaped craters caused by yawed long rod impact - Source: Bless et al. [24] .... 18
2.7 Ricochet effect on long rod oblique impact ................................................................. 19
2.8 Positive obliquity target and rod with positive yaw, zero yaw and negative yaw respectively 20

3.1 Crater profile for normal long rod impact. Naz and Lehr [31] ....................................... 24
3.2 Flowchart of the program methodology ..................................................................... 26
3.3 Graphical User Interface ............................................................................................ 27
3.4 Example of results for a single study analysis ............................................................ 29
3.5 Example of results for a yaw angle parametric analysis ............................................. 30

4.1 Validation 1 against Hohler and Stilp data - Tungsten alloy rods onto steel targets - L/D=10 36
4.2 Validation 2 against Hohler and Stilp data - Tungsten alloy rods onto steel targets - L/D=10 38
4.3 Modified Hydrodynamic Theory vs. Hohler and Stilp data for different L/D ratios- [20]
Tungsten alloy rods onto steel targets ................................................................. 40
4.4 Penetration phases - Adapted from [1] .................................................................... 40
4.5 Results for impacts on targets with -30° and +30° obliquity - Tungsten alloy rod on a RHA
steel target - V=1.66km/s L=126.4mm D=6.3mm ................................................... 46
4.6 Results for impacts on targets with -45° and +45° obliquity - Tungsten alloy rod on a RHA
steel target - V=1.66km/s L=126.4mm D=6.3mm ................................................... 47
4.7 Results for impacts on targets with -60° and +60° obliquity - Tungsten alloy rod on a RHA
steel target - V=1.66km/s L=126.4mm D=6.3mm ................................................... 48
4.8 ±3° yawed rods impacting a 60° target - \( \alpha_c \) dissimilarity ........................................... 49
Nomenclature

**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Yaw angle</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Vertical yaw angle</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Horizontal yaw angle</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>Critical yaw angle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Correction factor</td>
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<td>$\theta$</td>
<td>Oblique angle</td>
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<tr>
<td>$\theta_c$</td>
<td>Critical oblique angle</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Density ratio factor</td>
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<tr>
<td>$\rho_p$</td>
<td>Projectile density</td>
</tr>
<tr>
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<tr>
<td>$\phi$</td>
<td>Crater floor angle</td>
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**Roman symbols**

<table>
<thead>
<tr>
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<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Defined by equation 2.8</td>
</tr>
<tr>
<td>$BHN$</td>
<td>Brinell hardness number</td>
</tr>
<tr>
<td>$D$</td>
<td>Projectile diameter</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Crater diameter</td>
</tr>
<tr>
<td>$H_v$</td>
<td>Vickers hardness number</td>
</tr>
<tr>
<td>$k$</td>
<td>Crater ratio</td>
</tr>
<tr>
<td>$L$</td>
<td>Projectile length</td>
</tr>
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</table>
$P$  Penetration depth

$p$  Pressure

$R_t$  Target resistance factor

$S_p$  Projectile ultimate tensile strength

$S_t$  Target ultimate tensile strength

$U$  Penetration velocity

$V$  Projectile impact velocity

$Y_p$  Projectile strength factor

$Y_t$  Target strength factor
Chapter 1

Introduction

This thesis aims to examine some existing analytical methods of impact behavior, particularly those that apply in cases of hypervelocity impact. The models described will concern only to a particular type of projectile, long rods. The focus of this study is on both ideal and non-ideal impact on targets the subject of normal hypervelocity impact and subsequently the topic of angled impact will be broached. The aim is to study the analytical methods’ applicability and limitations, and to consider their use in regards to the aerospace field.

1.1 Problem Description

This work deals with the impact of eroding long rods into semi-infinite targets, which has been a subject of intense research since the 1960’s. These types of projectiles are metal cylinders with lengths that are a minimum of 10 times larger than their diameters. When they impact targets at high speeds, typically from 1000 m/s, they are eroding during the impact. These impacts are considered as one of the most lethal threats, as a consequence of the very deep penetration depth that the projectiles reach [1].

Throughout the years, analytical and empirical models have been proposed to describe these events. These efforts focus on reviewing one-dimensional penetration models that facilitate the complex physical phenomena and can be easily implemented. This study concerns models for both normal and angled impact, with the introduction of yaw and oblique angles to the impact events. These will be validated against experimental data from literature with the purpose of identifying each of their merits and faults.

1.2 Relevance to the aerospace field

In the aerospace field, the analysis of collisions on the structures of aircrafts and satellites, is of extreme importance. The damage they provoke can lead to the serious debilitation of the structural integrity of the impacted surfaces and may even result in an accident.

Although they are not the subject of these studies, aircrafts are prone to be affected by low and ordnance velocity impacts. Bird strikes are a serious threat to the aviation sector, having been the cause
of numerous accidents throughout the years. The problem gained widespread attention when in 2009, US Airways Flight 1549 was forced to an emergency landing on the Hudson River (Figure 1.1) after several birds were ingested into both engines. Ever since the first world war, the beginning of aerial combat, aircrafts are frequently also the target of warfare. Manufacturers have to take into consideration the amount of damage the aircraft can endure when struck by bullets and other ammunition.

In space, the threat of collisions is also a very concerning problem. Space debris from old satellites and rocket fragments, as well as natural debris such as asteroids and meteorites, often hit functioning space objects such as satellites and even the International Space Station. The infamous 2003 Columbia Space Shuttle disaster was caused by impact damage. A piece of foam debris from the external tank broke off and hit the leading edge of the left wing, originating in a breach in the Thermal Protection System which, in turn, resulted in the overheating and failure of the wing, causing the accident during reentry and killing all 7 crew members [2]. A recreation of that impact was performed in the investigation of the accident and the resulting damage is depicted in figure 1.2.

Figure 1.1: Flight 1549 on the Hudson River after emergency landing - Source: Andrew Gregory Lam Pak Ng

The mechanics of penetrating objects have been extensively studied throughout the years with a substantial emphasis on military applications. The usual shapes and sizes of projectiles with significance for military purposes, e.g. small diameter cylinders and spheres, are considerably different than space debris.

Figure 1.2: Impact test results on the Space Shuttle Atlantis wing panels to recreate the damage on the Columbia Space Shuttle - Source: [3]

debris fragments, which can have any number of irregular geometries and vary in size. Nevertheless, this research can provide concepts and models that can be expanded and modified to study the penetration of fragments in space.

High-speed debris fragments can be launched as a consequence of accidental spacecraft collisions, propulsion and electric system explosions or the intentional destruction of satellites. These fragmentation events, along with other debris sources, amount to the more than 10000 tracked pieces of the space debris population currently orbiting the earth, most of them in low earth orbits (LEO) [4]. These fragments present a concerning prospect of catastrophe, seeing that the velocities they travel at can reach 15km/s. Since most of the existing debris is situated in LEO, these potential collisions could lead to a significant loss in space infrastructure for military and commercial satellites orbiting LEO, and even cause damage on the ISS, risking human lives.

Analytical models concerning angled impact are of particular importance for these efforts since they present an initial approach to predict the damages and implications of an impact at hypervelocities. Fragmentation events resulting from explosions and collisions project debris that presents irregular rotation during flight and can hit a target at multiple different orientations. The studies presented in this work show that any angle presented at impact, whether related to the projectiles trajectory or to the targets obliquity in respects to the impending object, can significantly alter the consequences of the event, both in terms of penetration depth and crater size. Furthermore, knowing that any impact occurrence can lead to new fragmentation, thus further aggravating the space debris population problem, a better understanding of hypervelocity impacts in space is necessary to predict these episodes and prevent future accidents.

In 1986, Cour-Palais [5] gathered information on hypervelocity impact studies carried out in preparation for the Apollo lunar missions, to offer some insight into the potential damage a spacecraft might suffer. At the time the threats to those spacecrafts were mainly meteors and meteorites, but that concern has shifted to the orbiting debris. Some studies focused on the penetration depth of aluminium spheres onto aluminium targets at hypervelocities above 6km/s, since the average density of the debris fragments was determined to be around the same as the density of aluminium. These studies showed that the spheres would penetrate the target hydrodynamically, demonstrated by the raised lips on the target’s impacting surface and the crater was dependent on the projectile’s diameter and impact velocity. Further studies showed that the initial penetration would occur hydrodynamically and as the projectile slowed down, the strengths of the target’s materials now became significant to the process, influencing the penetration depth.

Cour-Palais [6] then expanded on his studies of spherical hypervelocity projectile impact, analysing if Whipple shields, which are the shields used to protect spacecraft from collisions, could prevent non-spherical projectiles including plates, discs, cylinders and rods from damaging the spacecrafts. The conclusions were that while these shields were effective in preventing damage from spherical projectiles, they most often did not prove to successfully protect against these non-spherical projectiles, whose shapes were more consistent with orbital debris fragments.

The hypervelocities of space debris fragment impacts is in the magnitude of $10^4 m/s$ and those ve-
velocities are very hard to achieve on earth, particularly in projectiles with significant mass, in the gram order of magnitude. Recreating these LEO velocities on earth is only possible when dealing with shaped charges, a type of eroding projectile along with long rods. The erosion of shaped charges into the targets is a very similar phenomenon to the one of long rods, the key difference being the non-constant velocity along the length of the jet resulting from the shaped charge.

1.3 Motivation

Due to the high intricacy and wide array of possible results of impact studies, particularly at high speeds, experimental studies require very complex testing infrastructure and are very costly. Due to the expensive nature of those tests, and the fact that they are mostly financed by various military organizations around the world, therefore not publicly available, other methods have to be developed in order to predict the behavior of structures under impact.

Numerical models are often used to predict impact behaviour, considering the increase in computer capabilities and their accessibility to a wider array of scientists, they constitute a very useful tool to determine these scenarios, often being the only form to analyse some impacts, in cases where no analytical methods have been developed and experimental studies are not a viable option.

Analytical models provide a simpler way of representing impact and are therefore important in cases where resources are not abundant. The development of analytical models also yields the advantage of a more comprehensive understanding of the physical phenomena at hand, highlighting which parameters are most important to these phenomena. The recent advances in numerical modeling are reducing the need of analytical simulations, since, when compared to experimental tests, these simulations present a greater error than their numerical counterparts. Still, it is important to mention their value as an introductory approach to the problem of impact behaviour, particularly when dealing with one dimensional models which are quick and easy to apply to a given event.

The choice between the use of the various different approaches will often fall on the resources available, as well as the level of preciseness desired. Most often the use of a complimentary approach is applied, with a combination of the three methods.

1.4 Objectives

This thesis’ intent is to review some simple models that describe hypervelocity impact events.

Outside of laboratory testing conditions, real impact events can present an array of distinct situations that influence the outcome. One of those situations is the presence of angles upon impact, both because of the presence of yaw or the obliquity of the target surfaces. Because not all possible situations can be recreated for experimental procedures, the objective of this thesis is to develop a tool that could calculate a wide number of different scenarios when applying the aforementioned simple analytical models. That way, it can serve as an initial approach to quickly determine the approximate depth a projectile might
reach, and this information can be used and improved upon with the employment of other tools, such as numerical simulations and experimental studies.

The intent is also to easily see the way the different parameter influence the results of the impact event, so this tool and its parametric studies capability should serve as a way to easily visualize this effects, so the user can learn which parameter to alter in order to obtain the desired results.

1.5 Thesis Outline

The content of this thesis is organized into five different chapters. The first chapter introduces the subject of the study, offers some background on its relevance to the aerospace field and the motivation behind the use of analytical models.

The second chapter presents a general overview on the topic of ballistics, containing a brief introduction into the development of the discipline throughout history as well as a basic classification of ballistic terminology. Then, specific analytical methods used historically to determine penetration depth for long rods and analytical methods to examine angled impact are presented.

In the third chapter, the methodology and implementation of the aforementioned analytical models are described. For this implementation a tool was developed that uses Microsoft Office Excel and Visual Basic for Applications language to automate the methods for the user. The chapter presents a description on how the program works in both of its operating modes and shows the user interface and mentioned the limitations that should be taken into account when using it.

The results of those efforts are presented in the fourth chapter and compared to experimental results obtained from literature. Parametric studies on the influence of the penetration depth by different conditions, first regarding the variation of the velocity parameter, then concerning the inflicting angles of target and projectile will be presented.

Finally the thesis finishes with a conclusion and perspectives on future work in the last section.
Chapter 2

Theoretical Background

This chapter will introduce main notions on the topic of Terminal Ballistics, offering some insights into the history of ballistic studies, defining impact and its intervening parties, projectile and target. The ways said impact can be characterized analytically will be reviewed by taking a closer look at how impact at hypervelocities can be described physically to see what the defining factors for the penetration process are, depending on the situation. Both normal impact and impact at an angle will be analysed, with focus on yaw and oblique angles of incidence.

The use of analytical models for the study of impact will depend heavily on the situation at hand. There is no catch-all method that one can apply whatever the situation might be. Additionally, the methods presented here focus only on a few aspects of the impact event, which in this case is penetration depth, crater size, critical angles, among others. Determining the circumstances of the impact and the aspects worth examining is the basis of the study.

The models proposed below seek to simplify very intricate three-dimensional problems, with many intervening factors. They focus on the most important physical aspects of the penetration process and result in one-dimensional models that represent the impact event and can be calculated without much difficulty.

2.1 Terminal Ballistics Overview

2.1.1 Main subfields of ballistics

In a general sense, ballistics is the study of the motion of projectiles, from the process of the initial acceleration of the projectile, to the trajectory of the projectile and finally it's impact on a physical target. Because it encompasses so many different aspects of physics, ballistics is usually divided into three main different subfields. These are Interior Ballistics, which deals with the factors that influence the motion of the projectile within the firearm; Exterior Ballistics, the studies of projectile flight trajectories and the factors that influence those motions; and finally, Terminal Ballistics, which handles the impact of the projectile onto a target and the effects it provokes both on the projectile and on the target [7].
2.1.2 Brief History of Terminal Ballistics

Since the beginning of civilization, conflict and war have been a constant presence in human history. The throwing of objects, projectiles, has always been a part of that history, from the days of stone and spear throwing to the present and has since then only gained more prevalence in war. The field of exterior ballistics, however, only began to be broached in the 17th century, when Isaac Newton and Galileo applied scientific principles to the problem. With the basis of Newton’s laws of motion, Galileo famously developed his theory of projectile motion, determining that projectiles would follow a parabolic trajectory when not subjected to friction. In the 18th century, mathematicians Benjamin Robins and Leonhard Euler started the subject of terminal ballistics, analyzing data for cannonballs and their penetration depth as a function of their velocity at impact [1]. In the 19th century, french engineer J. V. Poncelet conducted experiments and proposed his resistance law to the problem of projectile penetration. During this century, British military forces started running experiments to determine projectile capabilities and armor limitations, evaluating projectile nose shapes and composition, as well as target material effectiveness.

With the beginning of the 20th century and the start of World War I, terminal ballistic studies shifted to aerial combat, with the introduction of aircrafts as weapon carriers. Instead of damage through penetration depth, aerial bombardments were now the most effective method of structure destruction, given the deficiencies of targets at the time. After the war, focused ballistic laboratories were established to study new advancements. World War II then brought technologies like the shaped charge jets, guided missiles, rockets and the well known atomic bomb [8].

In recent years, several developments have been made in the methods to study terminal ballistics, both experimentally and in prediction methods, particularly in numerical methods. Advances in material properties measurement, improvements in highspeed camera technology have contributed to the garnering of more accurate results and the rise in computing capabilities has shown tremendous improvements in the processing of these complicated phenomenons and the prediction of penetration depth [9].

2.1.3 Velocity Classification

Impact is often classified by the velocity between its intervening parties, projectile and target. Because the range of impact velocities is so extensive, the physical phenomena can differ greatly, hence the importance to classify the events by velocity. Velocity often affects the variety of phenomena so deeply that almost any other consideration is overridden.

In the Ballistics field, velocity range is usually categorized in three ranges: low velocity range, which concerns velocities below 500 m/s; ordnance velocity range, involving velocities between 500 and 2000 m/s and, finally, the hypervelocity range, applicable to velocities above 2000 m/s [1], as depicted in Fig. 2.1.

Applying these categories to the impact field in a general setting, low velocity impact events are common in the automobile industry, with vehicle crashworthiness studies. It is also explored in the aviation industry, with emphasis on bird strikes in engines and in fuselages. Ordnance velocity impacts
are usually studied in conventional ballistics and blast impact events. Hypervelocity impact is usually reviewed in the aerospace field, with the impact of meteorites and space debris on satellites and other space objects, as well as impact on reentry vehicles [10].

2.1.4 Projectile Classification

Another way to categorize impacts is to do it in terms of the type of projectile. Different geometries of projectiles are going to instigate different responses by the impacted target. Projectile shapes are subcategorized by their main body shape and their nose shape.

The main body shape is usually one of three shapes: a sphere; a cylinder, like a rod or a bar; and some kind of irregular shape, which is the case of impacting fragments, birds, among others. These shapes can be hollow or solid, made of different materials, sometimes multiple materials in a single projectile. Different nose shapes can profoundly affect the impact behaviour of a projectile. There are several common types of nose shapes. The most common ones for long rods, according to [1], are depicted in Fig. 2.2.
2.1.5 Target Classification

Similar to the projectiles, targets are also classified by their shape, more specifically by their thickness. The classification used generally in Ballistic studies was introduced by Backman and Goldsmith [12] in 1978, and organizes targets into four categories represented in Table 2.1: thin targets, intermediate targets, thick targets and semi-infinite targets.

Thin targets are usually fully penetrated by a projectile. They serve the purpose of inflicting the most damage to the projectile. Intermediate and thick targets are targets where the projectile doesn’t reach the distal surfaces, but inflicts considerable damage to them, weakening the target. Both thin and intermediate/thick targets reflect real life situations for armor design when preventing impact damage. Semi-infinite targets are thick enough that the inflicting projectile does not influence the distal surfaces at all, since the penetration depth is substantially less than the targets thickness. Studies with semi-infinite targets are a rudimentary way to predict the thickness necessary to stop a projectile, though often adequate enough for design purposes [1].

<table>
<thead>
<tr>
<th>Target Classification</th>
<th>Definition</th>
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<tbody>
<tr>
<td>thin</td>
<td>usually fully perforated by projectiles</td>
</tr>
<tr>
<td>intermediate</td>
<td>enough to stop the projectile, considerable damage to the distal surfaces</td>
</tr>
<tr>
<td>thick</td>
<td>influence on the distal surface only after substantial penetration</td>
</tr>
<tr>
<td>semi-infinite</td>
<td>no influence on the distal surfaces</td>
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2.2 Long Rod Models

One of the areas of focus of ballistics is the impact of long rods on targets. A long rod is understood to be a rod with a length at least 10 times bigger than its diameter [1]. That is the case because of their ability to cause a great amount of damage, seeing that they can possess large amounts of kinetic energy and deposit that energy into a concentrated area. In certain situations, which this study will focus on, almost all of that kinetic energy is transformed into plastic deformation, originating a crater. When developing armor, the performance of the target is measured by the depth of the perforation, amongst other factors, so being able to predict depth of the crater formed by an impact event is of extreme importance, and the goal of numerous studies throughout the years.

Experimental studies on projectile impacts are usually done under set circumstances, in order to obtain comparable results. This means that the types of projectile- and target materials used in these studies tend to be the same, depending on the intended impact event. Given the case of high-speed long rod impact onto thick targets, which is what this work intends to study, these materials are usually
one of the following: tungsten or tungsten alloys for the projectiles and armoured steel for the targets.

In real world scenarios, long rod projectiles are usually incased by an outer shell, larger in diameter, known as a sabot. This allows for the inner projectile to be denser, and in turn heavier, because the projectile is still able to achieve very high muzzle velocities due to its larger gas surface area. This explains the use of tungsten and tungsten alloys, typically above 90% tungsten, which are much denser than the previously-used steel penetrators and can handle higher velocities and pressures. While in mid-flight, because of the drag effect, the projectile then sheds its outer shell, leaving just the long rod penetrator for the impact, as shown in Fig. 2.3.

![Figure 2.3: Long rod in flight - Source: [13]](image)

The targets used for these studies are made of armoured steel, specifically rolled homogeneous armoured (RHA) steel, on account of its historical use on common ballistic targets, i.e. tanks. RHA steel has standardized specifications, but is similar to AISI 4340. The use of these targets remains standard procedure for many ballistic tests despite the abandonment of RHA steel in most modern armoured vehicles in favour of composite materials.

Different projectile types require different analytical models, and for long rod models, the distinction between rigid long rods and eroding long rods has to be made. The difference between the two is the impact velocity. When colliding at low velocities, long rods behave as rigid bodies. At high velocities, the pressures involved are high enough to surmount the projectiles compressive strength, usually around 1.5 GPa, so it erodes and is consumed by the target as depicted in figure 2.4.

When considering eroding penetrators, the understanding is that it is referring either to shaped charge jets or eroding long rods. Shaped charged jets are formed when a hollow charge is filled with a metal liner, usually copper. When the explosive charge is detonated the liner collapses on itself and forms an extremely energetic long jet, that can strike a target and inflict a lot of damage [14]. Erod- ing long rods are made of tungsten heavy alloys or depleted uranium, dense materials that continually erode until the rod is consumed completely. Both long rods and shaped charged jets are considered to be amongst the most dangerous projectiles, because of their penetration capability [1]. Due to the similarities of projectile geometries, analytical models to describe them can be based on the same principles, such as the hydrodynamic theory, outlined in this section of the work.

This section will focus on introducing analytic methods that recreate the perforation of long rods
onto semi-infinite targets, to determine the penetration depth they achieve. Due to the complexity of high-velocity impact, almost all analytical models developed in this particular topic rely on some kind of empirical input, or require corrections to more closely represent previously observed patterns. When pertinent, these empirical and semi-empirical methods will also be presented. After introducing these penetration models to perpendicular impact, analytic methods to describe impact with yaw and oblique angles, and their influence to the penetration, are examined in the next section.

### 2.2.1 Hydrodynamic Theory

A simple way to describe perforation analytically is to employ one-dimensional models, such as the classic hydrodynamic theory of perfect fluids. The hydrodynamic theory can only be applied to impact in cases where materials behave like fluids, which occurs when the impact velocity is great enough to create pressures so high at the impact point, that the strengths of projectile and target materials are trivial in comparison. To achieve these pressures, velocities have to be higher than the critical hydrodynamic velocity. This hydrodynamic limit is dependent on the materials of both target and projectile, and can be as high as 5000m/s for ceramic materials and as low as 500m/s for soft projectiles hitting polymer materials [9].

The hydrodynamic theory was first applied to shaped charges [15]. It was then applied to metal projectiles and targets, in cases where pressures are high enough to disregard the strength of the target and projectile [16]. The hydrodynamic theory is used in cases of high speed impact, particularly of long
rods onto targets, because the initial surface impact and ultimate motion halt periods are trivial to the crater formation, and the period during which the rod is penetrating the target at a constant speed can be considered steady state. To apply this theory, one must also assume a semi-infinite target, to observe the whole depth of the crater to be within the targets thickness. The rate of erosion of the rod is assumed to be constant, so it can be described by steady hydrodynamic flow.

Applying a balance of linear momentum to a long rod impact event, and taking into consideration all the assumptions mentioned above, equating the pressures of both projectile and target at the stagnation point, Bernoulli's equation results in the following relation

$$\frac{1}{2} \rho_p (V - U)^2 = \frac{1}{2} \rho_t U^2$$

(2.1)

where $V$ is the velocity of the projectile at impact, $U$ the velocity of penetration and $\rho_p$ and $\rho_t$ the projectile and target densities, respectively. Relating the depth of the crater, $P$, and the time it takes for the projectile to erode , $\Delta t$

$$P = U \cdot \Delta t \quad \text{and} \quad \Delta t = \frac{L}{U - V}$$

where $L$ is the length of the projectile, one obtains

$$\frac{P}{L} = \frac{U}{U - V}$$

(2.2)

Substituting equation 2.1 into 2.2 one obtains the final relation of the hydrodynamic theory [17].

$$\frac{P}{L} = \sqrt{\frac{\rho_p}{\rho_t}}$$

(2.3)

This means that the penetration depth is not dependent on the projectile impact velocity, which is only the case for impacting velocities that are sufficiently high to generate pressures far above the target material yield strength. Assuming both projectile and target are of the same material, take steel for example, this formula would yield the result that the penetration depth is equal to the length of the rod, whatever the impact velocity might be. Though this may be the case for hypervelocities, where material strengths truly are negligible in regards to the aforementioned high pressures, for lower velocities it was observed that crater depth decreased with the reduction in velocity. It was also observed that the penetration depth was much higher than the one predicted by equation 2.3, in cases of penetration into very soft targets, such as lead [16]. In a further extensions of the hydrodynamic theory, the strengths of both the projectile and target were taken into account, considering the deceleration of the projectile along the perforation.
2.2.2 Modified Hydrodynamic Theory

Between 1966 and 1967 both Tate [15] and Alekseevskii [18] proposed a modified version of the hydrodynamic theory that took into account the strengths of projectile and target materials. These strengths need to be overcome in order to reach the stagnation point. In case of the rod this means that it would behave as a rigid body until the pressure was high enough to surmount its strength factor, \( Y_p \), related to its dynamic yield strength, and then flow hydrodynamically. The same applies to the target, only here the pressures have to be enough to also overcome the inertia of surrounding material, overcoming the target’s resistance to penetration, \( R_t \), and only then behaving hydrodynamically. This results in the following modified Bernoulli equation.

\[
\frac{1}{2} \rho_p (V - U)^2 + Y_p = \frac{1}{2} \rho_t U^2 + R_t \tag{2.4}
\]

In 1986 Tate [19] revisited his hydrodynamic theory model and proposed a relation of factors \( Y_p \) and \( R_t \) with the dynamic yield strength \( \sigma_{yp} \). The target’s resistance to penetration is a parameter dependent on the material’s dynamic yield strength and its Young elasticity modulus \( E \).

\[
Y_p = 1.7 \sigma_{yp} \tag{2.5}
\]

\[
R_t = \sigma_{yt} \left( \frac{2}{3} + \ln \left( \frac{0.57 E_t}{\sigma_{yt}} \right) \right) \tag{2.6}
\]

Solving equation 2.4 for \( U \) one obtains the following relation

\[
U = \frac{1}{1 - \mu^2} \left( V - \mu \sqrt{V^2 + A} \right) \tag{2.7}
\]

with

\[
\mu = \sqrt{\frac{\rho_t}{\rho_p}} \quad \text{and} \quad A = \frac{2(R_t - Y_p)(1 - \mu^2)}{\rho_t} \tag{2.8}
\]

Applying the relation 2.2 previously used on the classic hydrodynamic theory one more time results in the Alekseevskii Tate model for the modified hydrodynamic theory, now accounting for the the material strengths of both target and projectile.

\[
\frac{P}{L} = \frac{1}{\mu} \left( \frac{\mu \sqrt{V^2 + A} - V}{\mu V - \sqrt{V^2 + A}} \right) \tag{2.9}
\]

As mentioned, this model can only be applied in cases where the projectile erodes during penetration, which only happens at high velocities. This critical velocity [15], below which the penetration model no longer is valid, can be obtained by setting \( U = 0 \) in equation 2.4, which in cases where \( R_t > Y_p \) yields

\[
V_c = \sqrt{\frac{2(R_t - Y_p)}{\rho_p}} \tag{2.10}
\]
and in cases where \( Y_p > R_t \) yields

\[
V_c = \sqrt{\frac{2(Y_p - R_t)}{\rho_t}}
\]  

(2.11)

### 2.2.3 Empirical approaches to the problem

In 1992 Anderson et al. [20] gathered experimental results from a multiple researches across different decades in their penetration mechanics database. These results spanned a variety of different projectile and target materials, but the available data consisted mostly of tungsten alloy projectiles impacting steel targets. Most of the reported data was also concerning projectiles with length to diameter ratios of at least 10. Considering these shared properties of the data, the researchers performed multiple non-linear regressions to fit this data and arrived at an empirical relation that fitted most of the data reasonably well. This relation is the expression below, where \( S_t \) is the ultimate static tensile strength of the target.

\[
\frac{P}{L} = 1.121 \left( \frac{\rho_p}{\rho_t} \right)^{0.3704} \tanh \left( 0.0175 \left( \frac{L}{D} \right)^{-0.302} \left( \frac{\rho_p V^2}{S_t} \right)^{1.137} \right)
\]

(2.12)

with

\[
\frac{\rho_p V^2}{S_p} > 20
\]

(2.13)

This empirical relation applies in cases where the projectiles erodes, which Anderson et al. determined was when the condition 2.13, with \( S_p \) as the ultimate static tensile strength of the projectile, was met, otherwise the projectile penetrated the target as a rigid body.

### 2.3 Angled Impact

In ballistics, armor designers seek methods that hinder the projectile perforating capabilities, be it by way of strengthening the target’s materials, or by finding other ways to diminish penetration, inducing deceleration, erosion, fragmentation, and deflection to the projectile. These methods are known as defeat mechanisms [1].

The introduction of an angle to the impact of a long rod can drastically change the outcome of the event, leading to a decrease in penetration. Introducing yaw angles to incoming rods and altering a targets geometry in order to induce an oblique impact are ways to diminish penetration, therefore very useful in military armor design.

There are two angles that define a projectiles impact: the yaw angle and the oblique angle, \( \alpha \) and \( \theta \) respectively. The yaw angle describes the angle between the rod’s axis of revolution and its velocity. The obliquity of the impact is defined by the angle between the the rods velocity and the normal to the target plane, as depicted schematically in Fig. 2.5.

Between 2002 and 2003, Luttwak [21, 22] developed analytical methods to describe both normal yawed impact, and yawed impact at an oblique angle. Those methods will be introduced in this section and their validity will be discussed in subsequent chapters.
Presented here are, first, a discussion on normal impacts with yawed rods, then the introduction of oblique impacts and finally both aggregated for yawed oblique impact.

### 2.3.1 Yawed impact

As is the case in hydrodynamic theory, the methods described are based on the assumption that all of the kinetic energy that the long rod possesses is transformed into plastic deformation on the target, in the form of a crater. When the rod travels with a yaw angle and hits the target, that same assumption is true, so the crater volume has to stay consistent with a normal impact. Expecting the crater to widen because of the effect of the angle, the natural conclusion would be that the crater is shallower than one inflicted by normal impact, keeping the volume constant.

Contrasting with normal impact, in yawed impact, the practical length $L_\alpha$ of the colliding projectile is not $L$ anymore. It is, instead, related to the yaw angle $\alpha$ and the projectile’s diameter $D$,

$$L_\alpha = \frac{D}{\sin \alpha} \quad (2.14)$$

The penetration depth $P$ is therefore proportionally inverse to the sine of the impact angle $\alpha$. A factor $k$ that takes into account the widening of the crater also has to be introduced, which is the ratio by which the crater is wider than on normal impact.

$$\frac{P(\alpha)}{D/\sin \alpha} = \frac{kP(0)}{L} \quad (2.15)$$

$$P(\alpha) = \frac{kP(0)}{\frac{D}{\sin \alpha}} \quad (2.16)$$

with $k = D_h/D$ where $D_h$ is the diameter of the crater caused by the impact.

For small angles though, one can observe that equation 2.16 does not yield the right results, since a yawed impact crater is inherently shallower than a perpendicular impact crater, where the deepest
penetration occurs. For that reason \( P(\alpha) \) should always be smaller than \( P(0) \), so a critical angle must exist, where that condition is not met. To find that angle, one must equate both depths.

\[
1 = \frac{P(0)}{P(\alpha)} = \frac{\sin \alpha (\frac{L}{D})}{k}
\]

(2.17)

\[
\sin \alpha = \frac{kD}{L} \Rightarrow \alpha_c = \sin^{-1} \frac{kD}{L}
\]

(2.18)

In other literature, Bjerke et al. [23] described this angle by a different formula, instead with the difference of diameter of a yawed crater and the normal one.

\[
\alpha_c = \sin^{-1} \frac{kD - D}{2L}
\]

(2.19)

The physical meaning of this angle is that for angles below this critical angle, the rod tail passes through the hole formed initially by the head of the rod, for angles greater than \( \alpha_c \), the tail hits the edges of the hole, forming a larger crater and severely diminishing penetration depth.

Taking that into consideration, the denominator of equation 2.16 must be corrected to account for the critical angle. In order to do that, the term \( e^{\alpha/\alpha_c} \) is added, which will still yield the result \( P(0) \) for \( \alpha = 0 \), while behaving like \( L\sin(\alpha)/D \) for greater yaw angles. This results in the following equation:

\[
P(\alpha) = \frac{kP(0)}{\sin \alpha + ke^{\frac{-\alpha}{\alpha_c}}}
\]

(2.20)

To determine the parameter \( k \), one could turn to experimental data provided by Bjerke et al. in 1992, which resulted in an empirical relation for impact velocities between 1,2 and 4,7km/s [23]. To obtain \( k \), \( V \) has to be processed in km/s.

\[
k = \frac{D_h}{D} = 1.1524 + 0.3388V + 0.1286V^2
\]

(2.21)

This parameter can also be extrapolated by an approximation proposed by Luttwak [21], which takes into consideration the target material’s strength factor, \( Y_t \).

\[
k = \frac{D_h}{D} \approx \frac{V}{(1 + \sqrt{\frac{\rho_t}{\rho_p}})\sqrt{\frac{Y_t}{\rho_p}}}
\]

(2.22)

After determining this parameter, knowing the projectile’s dimensions and the velocity and angle at impact is all the necessary data needed to estimate the penetration depth of the impact event.

To increase the accuracy of this model, one has to take into consideration that the yaw angle will interfere in the shape of the crater formed. While at small yaws the crater will remain somewhat cylindrical, with the increase of the angle, the tail end of the rod will widen the crater, creating a triangular shape, as can be observed in the experimental data obtained by Bless et al. [24] and depicted in Figure 2.6.

Bless et al. [24] suggested that this shape of the crater, namely the crater floor, could be described
by a linear slope. The angle $\phi$ between this slope and the target’s surface can be determined

$$\tan \phi = \frac{k}{D} \sin \alpha$$

(2.23)

To optimize the analytical model for these triangular crater shapes, a shape factor of $(1 + \sin 2\alpha)$ is added to the numerator of equation 2.20. Furthermore, another parameter, $\gamma$, will need to be introduced to both the numerator and to the correction factor of $exp(-\frac{\alpha}{\alpha_c})$, as is the case with parameter $k$. This parameter is necessary because, for this model, the target’s strength factor $Y_t$ used to estimate $k$ in equation 2.22, is the static yield stress. To obtain results that more closely resemble experimental results, the constant $\gamma$ is used as a correction factor. With the addition of those correction factors and parameters, the final equation proposed by Luttwak [21] to describe yawed rod impact can be obtained.

$$P(\alpha) = \frac{\gamma k P(0)(1 + \sin 2\alpha)}{D \sin \alpha + \gamma k exp(-\frac{\alpha}{\alpha_c})}$$

(2.24)

### 2.3.2 Oblique Impact

In contrast to yawed impact, in oblique impact, the velocity vector is the same as the rod’s axis of revolution vector. The angled impact occurs due to the obliquity of the target, as previously depicted in Fig. 2.5. The angle $\theta$ is the angle between the velocity of the projectile and the normal to the target’s surface.

Following the same assumption of previous models, being that all kinetic energy is transformed into plastic work for the formation of the crater, this means that the crater formed by an unyawed rod on an oblique target is of same volume as one on a normal target. Therefore, the penetration depth of the rod along its velocity vector, often called the in-line penetration or line of sight penetration, will be the same as a normal unyawed impact, $P(0)$. To determine the penetration depth along the normal to the plane of the target, i.e. normal penetration, one simply has to multiply this depth with the cosine of the oblique angle.

$$P(\theta) = P(0) \cos \theta$$

(2.25)

By inflicting a target at an oblique angle, the effective length the projectile has to travel in order to fully perforate and come out on the other side is higher than if it would have inflicted normally. This is a strategy often used by armor manufacturers, as they project their vehicles and structures with oblique surfaces, the so called sloped armor. That is the reason many tanks throughout history have had multiple oblique surfaces, as they are often the target of deep penetrating projectiles.
When analyzing experimental results, one can observe however, that for higher oblique angles a ricochet effect is present. The rod only penetrates the target slightly, transforming only part of its kinetic into plastic deformation, gets deflected and continues to travel with a diminished velocity. This phenomenon is depicted in Figure 2.7.

![Figure 2.7: Ricochet effect on long rod oblique impact](image)

Ricochet can only be observed for high obliquity of the target. This means that an angle exists, the critical ricochet angle $\theta_c$, separating target perforation and ricochet of the rod. Tate [25] determined an analytical model based on his modified hydrodynamic theory, where he multiplied $Y_p$ with the eroding area at the tip of the rod, obtaining the perpendicular force to that area. This force is then multiplied by half the length of the rod and, obtaining the momentum around its center of gravity, which causes the rotation away from the target, leading to ricochet for angles according to the relation below.

$$
\tan^3 \theta_c > \frac{2 \rho_p V^2}{3 Y_p} \left( \frac{L + D}{LD} \right) \left( 1 + \sqrt{\frac{\rho_p}{\rho_t}} \right)
$$

(2.26)

### 2.3.3 Oblique and Yawed Impact

For yawed impact hitting an oblique target, the distinction between positive yaw and negative yaw and positive obliquity and negative obliquity needs to be made, since this difference will change the outcome of the impact considerably. The direction of the yaw is now important in regards to the target’s obliquity, because it can either make the angle at which it is impacting even more pronounced, effectively increasing its obliquity, or make the rod more perpendicular in regards to the target, as can be observed in Fig. 2.8.

In order to represent that analytically, an additional term that would decrease the penetration for positive yaw, and increase it for negative yaw is needed. The crater volume will be directly dependent on the face colliding with the target, which in this case of obliquity will increase proportionally to the term $\frac{\cos \theta}{\cos \theta + \alpha}$. By inserting that term into the denominator of equation 2.24 both the conditions are met.

The distinction of positive and negative angles also calls for the replacement of the variable $\alpha$ with its absolute value $|\alpha|$ in equation 2.24.

$$
P(\alpha) = \frac{\gamma k P(0)(1 + \sin (2|\alpha|))}{\frac{L}{2} \frac{\sin (|\alpha|) \cos \theta}{\cos (\theta + \alpha)}} + \gamma k exp \left( -\frac{|\alpha|}{\alpha_c} \right)
$$

(2.27)
2.3.4 Horizontal yaw angle

Until now, all the angled impact methods reviewed in this work have been regarding angles on the same plane, the plane of the surface of the page. As one can imagine, a projectile can also inflect a target at an off-plane angle, in fact, this is the case that most resembles real world yawed impact. When dealing with oblique targets, the yaw angle has to be separated into two orthogonal angle components, vertical yaw and horizontal yaw. Following traditional aerospace nomenclature, the pitch angle would be the vertical yaw angle, $\alpha_1$, in the vertical plane to the target, and the yaw angle would be the horizontal off-plane yaw, $\alpha_2$. The total yaw $\alpha$ is calculated applying the Pythagorean Theorem to the tangents of both yaw angles [26].

$$
\tan \alpha = \sqrt{\tan^2 \alpha_1 + \tan^2 \alpha_2}
$$

(2.28)

This is a contradiction to the way the vertical plane yaw angle has so far been referred to in this work, since according to this nomenclature it should have been referred to as pitch angle. In most of the literature that exists around angled impact, this distinction doesn’t have to be made, since it only comes into question when one introduces an oblique target, and because literature about oblique and yawed impact is very sparse, the angle is just generally referred as yaw. In addition, the simple analytical models described in the literature and summarized in this work, seek to transform these complicated events into one-dimensional models. The introduction of an off-plane angle would deem that very difficult, therefore, these studies will not dwell into the modeling of the effects of the horizontal yaw angle.
Chapter 3

Methodology and Implementation

This chapter of the work presents the chosen approach to solve hypervelocity impact events. The chapter starts with a review of the theoretical concepts discussed in the previous chapter highlighting the ones adopted for the implementation and continues with a discussion about the design of the application and the important criteria that should be met in order to maximise its purpose. Next, an overview of the developed computational tool is presented, its structure and user interface, along with an analysis regarding the fulfillment of the stated design goals and the program’s limitations. Finally, as a means of preparation for the validation against experimental data that will take place in the next chapter, the last section presents approaches for the treatment of material property data, indicating how information about a material’s hardness can be converted to the relevant parameters needed for the analytical models.

3.1 Methodology

This section highlights how the theoretical concepts discussed in chapter 2 are employed in practice.

To determine the penetration depth of a long rod impacting vertically onto a semi-infinite target, the results from the three methods mentioned previously are compared. The first by determining penetration depth using the basic hydrodynamic theory, employing the known expression 2.3, relating the length of the rod and the ratio of projectile and target densities to find $P$.

To calculate penetration depth using the Alekseevskii Tate modified hydrodynamic theory model, the formula 2.9 is used, obtaining $\mu$ and $A$ following the relations 2.8 The projectile’s and target’s strength factors, $Y_t$ and $R_t$ respectively, are determined following equations 2.5 and 2.6.

Lastly, Anderson et al.’s [20] empirical relation given by equation 2.12 will also be used to determine normal penetration depth.

When dealing with yawed rods, in order to calculate the penetration depth, the model developed by Luttwak [21] is used, which determines $P$ according to 2.24 and uses the expression 2.22 to calculate the parameter $k$ and expression 2.18 to determine the critical yaw angle $\alpha_c$. To emulate results obtained experimentally, both for $\alpha = 0^\circ$ from Bjerke et al. [23] and for $\alpha = 90^\circ$ from Yaziv et al. [27], Luttwak
suggests to take the parameter \( \gamma \) as approximately equal to 2/3.

Moving on to oblique impact, according to the assumptions of the analytical models, the penetration along the line of sight of the projectile will be equal to a normal impact in the same conditions. To calculate the penetration depth perpendicular to the impact surface, expression 2.25 is applied, which only multiplies the normal penetration by the cosine of the oblique angle.

In order to check if the desired oblique angle would result in penetration at all or if the rod would just bounce off due to the ricochet effect, it is important to verify if the angle is below the critical oblique angle, \( \theta_c \). An obliquity angle greater than the one determined by Tate's ricochet effect relation, given by expression 2.26, would result in the projectile being deflected away from the target.

In the presence of both oblique surfaces and yawed rods, relation 2.27 gives the final penetration depth for these events. This penetration is the penetration along the rod's trajectory, line of sight penetration. If one needs the penetration depth value perpendicular to the target's surface, one would again simply need to multiply it by the oblique angle's cosine.

These are all the elements needed to estimate both normal and angled long rod impact into semi-infinite targets using the simple one-dimensional models presented in this work. In the subsequent sections the requirements for the implementations are discussed, and the type of analyses the program intend to perform and what characteristics it should have are considered. The expressions presented above should be sufficient to be able to determine all the results for the set out analyses.

### 3.2 Design and Implementation

As previously stated in section 1.4, one of the objectives of this work was to create a widely available tool capable of calculating a large number of different scenarios of hypervelocity impact.

The program Microsoft Office Excel was chosen for the purposes of implementation of the previously described analytical models. Due to the simple nature of the chosen models, not much computing performance is required in order to obtain desired results.

Microsoft Excel also enables the use of its programming language Visual Basic for Applications, VBA, which allows the user to automate certain processes while still using the well known Excel graphic user interface. These processes can include the accessing of previously formulated tables, containing material properties, impact conditions and further relevant information to the problem, and the automatizing of the formulation of graphs depicting the results with varying input parameters, among other uses.

#### 3.2.1 Design Requirements

**Functional Requirements**

In the previous chapter, three different approaches were discussed to determine the penetration depth of normal hypervelocity impact events, \( P(0) \), the hydrodynamic theory, the modified hydrodynamic theory and an empirical relation. The analytical models differed on the inclusion of projectile strength and the
target's resistance to penetration and the empirical relation included the projectiles diameter to account for decreases in penetration for projectiles with greater $L/D$ ratios.

Basic hydrodynamic theory models are based on the assumption that the strengths of both projectile and target are negligible when compared to the pressures created by the impact. For this to be the case, the velocities involved have to be great enough to be within the hydrodynamic range. This range will vary with the materials involved in the event. Softer target materials will require a lower velocity than harder materials in order to be within the hydrodynamic range. This was studied by Anderson et al. [28] in 1999. Consequently, it would be of interest to compare the three methods and to assess their scope of application and their constraints. The goal is to analyse the results obtained from literature, and, with the assistance of the software tool, investigate which of the methods best describes the impact phenomenon, see where they converge with the experimental results and determine their faults.

Another aspect worth analysing is determining where projectile strength and target resistance become irrelevant and the penetration process occurs fully hydrodynamically, which will be apparent by the approximation of the results to the hydrodynamic limit set by the hydrodynamic theory. The empirical relation by Anderson et al. [20] includes a condition, given by the relation 2.13, that delimits where projectiles penetrate the targets as rigid bodies as opposed to eroding, so seeing where that transition happens depending on the impact conditions is also of interest.

As one can imagine, when analysing impact of projectiles, the dimensions of the projectile play a significant role in the outcome of the event. A larger projectile in volume will create a bigger crater and therefore greater damage than a smaller one impacting at the same speed. When the projectile is non-spherical, not only is the volume important to the crater size, but also the way it is distributed. In the case of long rod projectiles, this volume distribution is quantified by the L/D ratio. A number of efforts have been dedicated to determine the influence of this aspect ratio on penetration depth efficiency, defined as the depth of penetration normalized by the length of the projectile, $P/L$. These efforts, reviewed Anderson et al. [29] in 1996, are justified by evidence of loss of penetration efficiency for aspect ratios greater than 10. Rosenberg and Dekel [30] argued for a loss of efficiency of 15% for every 10 unit increment on the L/D ratio. With support from experimental and 2D simulations results, Rosenberg demonstrated that the normalized penetration depth decreased for higher aspect ratio rods. This loss of efficiency is, however, not predicted by either the hydrodynamic theory or Tate’s modified hydrodynamic theory. On the other hand, the empirical relation, 2.12 was proposed with this decreased penetration effect in mind, seeing that it includes the $L/D$ ratio of the projectile as a parameter. With that in mind, looking into the L/D effect is an effort that would merit further exploring.

In the sections 2.3.1-2.3.3, methods to describe angled impact penetration were presented, all of them dependent on the normal penetration depth $P(0)$. This means that the models describing angled impact are intrinsically tied to the model chosen to determine $P(0)$ and the results will vary depending on how this normal penetration was estimated. The choice for these efforts fell on the modified hydrodynamic theory, for reasons that will be discussed in the next chapter.

As discussed earlier in section 2.3, the yaw angle will have a significant influence on the crater size. The introduction of a yaw angle means that the rods velocity is not aligned with its center axis, and
consequently, the large amounts of kinetic energy that they possess are no longer deposited into the same concentrated area, as is the case for a normal impact. As shown in Figure 2.6, yawed impact provokes a triangular shape of the crater, which does not happen in normal impact, where the crater has a cylindrical shape, depicted below in Figure 3.1, consistent with the shape of the eroding rod. This difference in crater shape signifies a reduction in penetration depth. For this reason, it would be of relevance to analyse the penetration depth for a wide range of yaw angles.

![Figure 3.1: Crater profile for normal long rod impact. Naz and Lehr [31]](image)

The intent with the development of this tool was to easily see the way the different parameter influence the results of the impact event, so the program and its parametric studies capability should serve as a way to easily visualize these effects. By doing so, the user can learn which parameter to alter in order to obtain the desired results.

**Non-functional Requirements**

**Usability** - The goal is to create a user-friendly tool that would not need any sort of preface before launching.

**Extensibility** - This tool is developed knowing that even with the wide range of possibilities it offers, it could be insufficient for the needs of someone who might want to use it. A user may, for example, want to use an alternative method to determine the normal penetration $P(0)$. Or, one might want to alter the models for angled impact. With that in mind, a conscious decision was made to develop this tool in a way that is easy to personalize or iterate. This means that, for the most part, the functionalities should work independently from each other. All the expressions mentioned in section 3.1 were developed as independent Public Functions. These functions are then invoked in the separate modules, each one belonging to a different functionality. So if say, instead of using equation 2.18 to determine $\alpha_c$, a user intended to use the most commonly used equation 2.19 for the critical yaw, he would simply need to add another function where he defines that critical yaw and invoke that function in the Sub that does the calculations in each of the different modules.

**Robustness** - As with most physical models, the models described in section 2.2 can not be applied to all types of impact. Some restrictions have to be imposed in order to guarantee pertinent results.
Those restrictions are made in the form of geometry delimitation and value range determination and will follow the limitations that will be discussed in the upcoming section 3.2.3.

3.2.2 Structure of the program

The overall methodology of the program is depicted in figure 3.2. The diamond shapes represent decision points. The goal was to create a tool that could guide the user through the entire process without the prior understanding of the appropriate ranges and values. The program can, at the beginning, check to verify that the input parameters are within those ranges. Even if the values are not inside the intended ranges, the program is still able to extrapolate the results, though warning the user that the outcome might not be accurate. If the input value significantly exceeds the range of possibilities, such as negative values for projectile dimensions or velocity, the program will terminate the job and point the user to the issue.

Sheet 1 - User Interface

The first sheet of the Excel workbook, is the main user interface. This sheet, "Main", depicted in Figure 3.3, is where the user is asked for input values and is allowed to choose which operating mode he intends the program to run.

The program was developed, so that the user only has to specify the impact conditions, and then, with a single click, generate the results for an isolated impact event, or choose the option to look at parametric studies, selecting a varying parameter range to analyse the influence that variation has on the penetration depth.

On this main page the user is presented with two options to proceed. The first option is to find out the penetration depth of a single impact event, entering the exact variables associated to that event. Alternatively, the user can also look at the variation of the penetration depth along a range of values. This last option can be applied to two different parametric analyses: \( V \), \( \alpha \). These different analyses are very common for hypervelocity long rod impacts, since these are the parameters researchers have focused on for further studying, for the reasons discussed in section 3.2.1.

The input variables that are needed for the program to compute the results are listed in table 3.1, along with the unit they should be entered in, so that it yields fitting results. These units are typical when dealing with hypervelocity impacts. In contrast to the rest of the parameters, the user doesn’t have to manually insert the properties of both the projectile and target material.

Sheet 2 - Material database

The program includes an additional sheet, "Material Properties", where the user can access previously entered materials and their respective densities, yield and tensile strengths. In case the user wishes to analyse another material not yet present on the list, he can simply add its name along with the values of its properties below the existing ones. When inserting the input variables, the user is presented with a drop down list of all the available materials, chooses one for the projectile and one for the target, and
the program then accessed their respective properties. When inserting materials into the database, the user should have in mind that due to the restrictions imposed by the use of the empirical relation, that is, the obligation to restrict the analyses to steel targets, one should use only steels as targets, otherwise this relation will not be valid. It is also important to note that projectiles need to have a different density than the target in order for the modified hydrodynamic theory to compute results. These limitations will be further discussed in section 3.2.3.

**Sheet 3 - Single study results**

The third sheet in the workbook is the "Results" sheet. Here, the user can find the results and graphics corresponding to his single event calculation. At the top of the sheet, the program presents the user with the chosen input parameters, along with the densities, yield and tensile strengths of the materials.
Analytical methods of impact behavior

Impact Properties

Select Projectile Material: Tungsten alloy
Select Target Material: 4340 Steel

Projectile Dimensions

\[ L = 80 \text{ mm} \]
\[ D = 8 \text{ mm} \]

Velocity

\[ V = 1.50 \text{ km/s} \]

Yaw Angle

\[ \alpha = 10^\circ \]

Oblique Angle

\[ \theta = 0^\circ \]

Calculate and Draw Diagrams

Parametric Studies

Velocity Range:

\[ V_{\text{min}} = 1.00 \text{ km/s} \]
\[ V_{\text{max}} = 2.00 \text{ km/s} \]

Velocity Evaluation

Yaw Range:

\[ \alpha_{\text{min}} = 1^\circ \]
\[ \alpha_{\text{max}} = 90^\circ \]

Figure 3.3: Graphical User Interface
Table 3.1: Input parameters

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Projectile</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
<td>mm</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter</td>
<td>mm</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Density</td>
<td>$g/cm^3$</td>
</tr>
<tr>
<td>$\sigma_{yp}$</td>
<td>Dynamic Yield Strength</td>
<td>GPa</td>
</tr>
<tr>
<td>$S_p$</td>
<td>Static Tensile Strength</td>
<td>GPa</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_t$</td>
<td>Density</td>
<td>$g/cm^3$</td>
</tr>
<tr>
<td>$\sigma_{yt}$</td>
<td>Dynamic Yield Strength</td>
<td>GPa</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>Static Yield Strength</td>
<td>GPa</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Static Tensile Strength</td>
<td>GPa</td>
</tr>
<tr>
<td><strong>Impact Conditions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V$</td>
<td>Impact Velocity</td>
<td>km/s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Yaw Angle</td>
<td>deg</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Oblique Angle</td>
<td>deg</td>
</tr>
</tbody>
</table>

The user is presented with two sets of results together with their corresponding bar charts. The first are the results for the normal penetration depth $P(0)$. Here the user can find a comparison for this value calculated with the three different normal penetration methods discussed in sections 2.2.1 - 2.2.3, following the same order. Further below, the user finds the results regarding his angled impact analysis, with the values of $P(0)$, $P(\alpha)$, $P(\theta)$ and $P$, the final penetration depth along the projectile’s line of sight. For these calculations, $P(0)$ is generated using the modified hydrodynamic theory to determine $P(0)$. This is due to the fact that the modified version of the hydrodynamic theory garners credible results for a wider scope of situations in comparison to the basic version, as will be discussed in the next chapter. An example of these results is presented in Figure 3.4.

The choice to show the penetration depths in a bar chart pertains to the better visualizations of the different values in comparison to one another, and also due to the fact that depth is a physical measure of distance, so it seemed logical to show it a tangible way. From this perspective, the user can understand the effect of the different angles by seeing how much their presence reduces penetration in comparison to a normal event.

**Sheets 4 and 5 - Parametric Studies**

For the functionalities of parametric studies, the user can choose to analyse the influence of two different parameters on the penetration depth. These parameters are the projectile’s impact velocity $V$ and the yaw angle of the rod at impact $\alpha$.

In order to conduct a parametric analysis, the user must choose the range of values he wants to examine and set the remaining fixed variables to what he needs at the top of the “Main” sheet, like he would if he wanted to perform a single event analysis. See Figure 3.3 for an example of value inputs.

The program is set up to divide the desired range into 20 steps, that is say 20 equally spaced single values. Equation 3.1 shows the step calculation for the velocity parametric study, where $V_{\text{max}}$ is the top value and $V_{\text{min}}$ the bottom value of the user’s chosen range. The program then calculates the
penetration depth for each step value in a For Next loop until it reaches the range's top delimitation value.

\[ \text{step} = \frac{V_{\text{max}} - V_{\text{min}}}{20} \]  

(3.1)

If the user chooses to perform a velocity range analysis he will find those results in the fourth sheet. Here he is presented with the results for a velocity analysis for the normal penetration of the conditions he chose. At the top he can find the relevant input conditions he chose, and beneath that, the program generates a line chart with three different curves, corresponding to the three models for normal penetration discussed in the previous chapter. At the top of the chart is a dashed horizontal line, the curve for the basic hydrodynamic theory. The modified hydrodynamic curve is presented as a solid black line, and the Anderson et al. [20] relation is the grey line. The user is also presented with three values to left of the chart. These are: the value for the hydrodynamic limit, determined with the hydrodynamic theory; the velocity value where Tate’s model predicts penetration starts to occur; and finally the velocity value that delimits rigid body penetration according to Anderson et al. empirical relation.

The same velocity parametric analysis is then presented in terms of penetration efficiency, \( P/L \), further down below. This form of presenting the results is useful when comparing results for projectiles with different lengths and \( L/D \) ratios.

In case the user chooses the yaw angle parametric analysis, those results are generated and presented in the fifth and final sheet of the workbook. In terms of presentation, this sheet follows a similar structure as the two previous ones: at the top the user is presented with his input values and the chosen
materials’ properties and down below the program displays a line chart of the results of the analysis. Left of this chart are the penetration depth results for each step determined with the relation equivalent to 3.1 for yaw angles. This curve is determined using Luttwaks [21] oblique yawed penetration model with \( P(0) \) calculated with the modified hydrodynamic theory, consistent with the results in the single study analysis. An example of the results the program puts out in its parametric studies functionality is presented in Figure 3.5.

![Parametric Study (Yaw)](image)

Figure 3.5: Example of results for a yaw angle parametric analysis

### 3.2.3 Limitations

#### Physical limitations

The simplicity of the analytical models reviewed in this work entails the presence of some limitations that have to be taken into account in order to ensure pertinent results.

By definition, the long models described in sections 2.2 and 2.3, can only be applied to projectiles that qualify as long rods, meaning they have a length to diameter ratio \( L/D \) of at least 10. In shorter projectiles penetrations, this penetration process cannot be considered steady state, which is an assumption for these reviewed models.

The models developed to describe eroding rod penetration should inherently also only be applied when dealing with eroding penetrators, meaning the velocities of the projectiles at impact need to be sufficient to cause the projectile to erode. The transition velocity from rigid body penetration to eroding penetration will depend on the hardness of the materials involved. Tate [15] suggested the critical velocity for the validity of his model to be defined by equations 2.10 and 2.11. For Anderson et al.’s [20] empirical relation, the critical velocity is related to the rigidity condition 2.13, resulting in a critical velocity defined by

\[
V_c = \sqrt{\frac{20S_p}{\rho_p}}
\]  

(3.2)
Tate’s original modified hydrodynamic theory [15] included differential equations to describe projectile deceleration and erosion. As the objective of this work was to discuss simple analytical models, the need for numerical integration would contradict this idea, so for this reason, the simple solution to describe penetration depth was chosen. As a result, issues arise when one chooses materials with the same densities for projectile and target. These conditions result in a density ratio $\mu = 1$, which would yield parameter $A$ of equation 2.8 $A = 0$ and as a consequence lead to an indetermination, since equation 2.9 would be

$$P(\rho_p = \rho_t) = \frac{0}{0}$$

(3.3)

This means that in order to use this model, one can not set materials with the same densities for projectile and target.

Another limitation is imposed by the empirical relation of Anderson et al. [20], given by equation 2.12, since it was developed solely for steel targets and should not be applied to target densities different from $\rho_t = 7.85 g/cm^3$.

In regards to Luttwak’s [22] analytical method for angled impact, defined by equation 2.27, it requires the sum of the desired angles for the impact to be less than or equal to 0, since for $\theta + \alpha > 90$ the term $\cos (\theta + \alpha) < 0$ and yield a negative penetration depth, which does not make physical sense.

### Software limitations

One of the disadvantages in the use of Microsoft Excel operating with VBA, is that the user needs to enable the use of VBA macros to use the program. Office is set to disable all macros by default, so the user needs to manually allow them to run. This is due to the unsafe nature of the VBA language, which can access external files and delete or damage those. In order to mitigate this issue, the code developed for this work will be publicly available on Github, a web platform used to host open source software developed by its users.

One of the limitations of this program was discovered by accident. This program was at its early stages developed on a Macintosh operating system. After testing it on a Windows OS, an adverse effect was noticed, as the program was deformatted. Efforts were made to mitigate this effect by developing on a Windows and then testing it on a macbook running macOS High Sierra with Excel 16, which showed better results than development on macOS. The reason for this effect are unknown, but it might be caused by the difference in the definition of length and height of a cell.

### 3.3 Validation’s Methodology

The experimental data chosen for the validation of the analytical models did not provide information of projectile and target material properties in the specific parameters of table 3.1. Information on materials is instead commonly reported in terms of a material’s hardness, by providing the Brinell hardness number $BHN$ or the Vickers hardness number $HV$. 31
3.3.1 Normal impact validation

To perform validation on the normal impact models, data obtained by Hohler and Stilp [32] was chosen due to the high number of experimental data points. These can be found on Anderson et al. database [20].

A restriction imposed on the data selection, was the choice to disregard data on targets other than steel. Since most of Hohler and Stilp's data involved either steel or tungsten projectiles, only the data pertaining to tungsten or tungsten alloy projectiles is considered, since this simple version of Tate's formula would yield no results for steel on steel impact. These reasons for these were discussed in section 3.2.3.

The choice of Hohler and Stilp's data was mainly by virtue of their reporting on material hardness. A material with the same density can have widely different mechanical properties depending on treatment. Take AISI 4340 steel alloy for example, which depending on treatment - annealed, normalized or oil-quenched - will have yield strengths of 472MPa, 862MPa and 1620MPa respectively [33]. Seeing that the materials properties needed for the models can vary considerably, a material's hardness is a way to discern materials from each other. In Tate's 1986 extension of the modified hydrodynamic theory [19], a relation between a material's hardness and it's dynamic yield strength was proposed,

\[ \sigma_y = 4.2BHN \]  

(3.4)

where \( BHN \) is the Brinell hardness number, a form of categorization of a materials' hardness by subjecting it to Brinell hardness test, which indents the material with a steel ball indenter. This helps to determine parameters \( Y_p \) and \( R_t \) specific to Hohler and Stilp's data, which will yield more credible results than if one would have taken these values from other similar experiments.

The static tensile strength of the steel targets \( S_t \) needed to calculate the empirical formula proposed by Anderson et al. [20], equation 2.12, can also be obtained with the material's hardness, using the commonly known proportional relationship given by equation 3.5 [33].

\[ S = 3.45BHN \]  

(3.5)

3.3.2 Angled impact validation

For the angled impact analyses, Luttwak's [22] model will be compared to the experimental data from a recent (2013) study by Anderson et al. [26], which reported on experimental results for yawed rods onto oblique targets.

The article provides the following information about the projectiles and targets used: their materials, tungsten sinter alloy for the long rod and RHA steel for the target, and the projectile's dimensions. The article mentions the density of the tungsten alloy rods and points to another article by Rohr et al. [34] where more details can be found about this tungsten alloy, including the dynamic yield strength of the projectiles, \( \sigma_{yp} = 1,197GPa \), so parameter \( Y_p \) can be obtained using equation 2.5.
Regarding the target, the article mentions the RHA steel has a 300HV20 hardness. This is a form of categorization of a materials' hardness by their Vickers number, which means the material has a hardness of 300 kgf/mm$^2$ when subjected to a 20kgf load in a Vickers hardness test, which indents the material with a diamond pyramid indenter. Tate’s modified hydrodynamic theory uses the dynamic yield strength of the target to determine its resistance to penetration $R_t$, yet the model for angled impact calculates the parameter $k$ with the target’s static yield strength, so both need to be determined with the information about the hardness of the target. To determine the dynamic yield strength needed for Tate’s model of $P(0)$, one needs to convert this Vickers hardness number into the corresponding Brinell hardness number. According to the international standard ISO 18265:2013 [35], a Vickers hardness of 300 corresponds to a Brinell hardness number of 285, which applying equations 3.4 and 2.6 translates to a target resistance value of $R_t = 6.25 GPa$. To determine the static yield strength of this steel, Pavlina and Van Tyne [36] proposed equation 3.6,

$$Y = 90.7 + 2.876H_v$$ \hspace{1cm} (3.6)

This relation gives a static yield strength of $Y_t = 0.772 GPa$ for a hardness number of 300HV20. For the density of the target, the value $\rho_t = 7.85 g/cm^3$ will be used, as it was in several publications [28, 37] as the density of RHA steel, since this parameter is not specifically mentioned in the article.
Chapter 4

Results and Discussion

In this chapter results obtained with the analytical models reviewed earlier in this work are presented and discussed. In order to analyse results for angled impact, given that those models depend on the normal penetration $P(0)$, one starts by assessing how the normal impact methods perform against experimental results in an initial validation. A comparison between the different analytical methods is presented - the basic hydrodynamic theory, the modified hydrodynamic theory as well as the empirical method previously introduced - in order to determine which one is the most suitable for these efforts. After validation of the normal impact results, one may then be able to apply those methods in angled impact analyses in a comparison of experimental results from literature for yawed rods onto oblique targets to results obtained by the analytical model for that type of impact. These analyses and results are discussed and the merits and limitations of the different models are pointed out throughout the chapter.

4.1 Normal Impact

4.1.1 Hydrodynamic Theory vs. Modified Hydrodynamic Theory vs. Empirical

Validation of the methods reviewed in sections 2.2.1 - 2.2.3 will be performed against the extensive experimental data obtained by Hohler and Stilp [32, 38–40], published between 1977 and 1987, and later gathered by Anderson et al. [20] in their 1992 experimental results database. From all the available data, the choice of Hohler and Stilp's was due to the presence of information regarding the projectile's and target's hardness, as well as the focus on different aspect ratios for impacting projectiles. Their experiments involved projectiles with minimal yaw (all below 1°) impacting targets at 0° obliquity, which means they are adequate for normal impact analysis methods.

Hohler and Stilp analysed projectiles with aspect ratios of 1 to 32, but since the subject of this work is long rod behaviour, only $L/D$ ratios equal to or greater than 10, in accordance with the long rod definition, are considered.
Validation

To perform validation, two sets of data were chosen, both concerning tungsten alloy rods impacting steel semi-infinite targets. These can be found on Anderson et al. database [20], the first set in pages A-82 to A-84 and the second set in pages A-121 to A-124. These sets of data were chosen due to their high number of entries.

Validation 1 will be performed against data from $L/D = 10$ rods, with 17 data points for velocities between 0.74 and 3.71 km/s, the majority of those in the 1-2 km/s range. Projectile and target hardness have values of 294 and 388 respectively. Dimension and material properties’ values used for these tests, along with the resulting parameters determined from those values following the methodology described in section 3.1 and section 3.3, are summarized in table 4.1.

<table>
<thead>
<tr>
<th>$L$ (mm)</th>
<th>$D$ (mm)</th>
<th>$\rho_p$ (g/cm$^3$)</th>
<th>$BHN_p$</th>
<th>$\rho_t$ (g/cm$^3$)</th>
<th>$BHN_t$</th>
<th>$\sigma_{yp}$ (GPa)</th>
<th>$\sigma_{yt}$ (GPa)</th>
<th>$Y_p$ (GPa)</th>
<th>$R_t$ (GPa)</th>
<th>$\mu$ (km$^2$/s$^2$)</th>
<th>$S_t$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>2.8</td>
<td>17</td>
<td>294</td>
<td>7.85</td>
<td>388</td>
<td>1.23</td>
<td>1.63</td>
<td>2.10</td>
<td>8.01</td>
<td>0.68</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Applying all the reviewed methods for normal impact to this geometry and conditions, one obtains the results depicted in figure 4.1, set over the 17 data points of Hohler and Stilp [20].

![Figure 4.1: Validation 1 against Hohler and Stilp data](image)

The dashed straight line at the top is the result of the hydrodynamic theory using equation 2.3 as described in the methodology, which yields the same penetration depth, $P = 41.2\text{mm}$, regardless of impact velocity, since it is only dependent on the density ratio and the length of the projectile.

The solid black line is the curve of the modified hydrodynamic theory model, equation 2.9. It indicates that penetration only occurs for velocities above $V = 0.84\text{km/s}$, which suggests that for velocities below...
that, the pressure caused by the impact of the rod is not sufficient to surmount the target's resistance to penetration. The curve shows that penetration depth increases steadily up to velocities around 2 km/s, where it starts to asymptotically approach the hydrodynamic penetration depth limit.

The grey line curve is the prediction of the empirical relation by Anderson et al. [20], equation 2.12. The curve only starts at $V = 1 \text{km/s}$ because of the restriction imposed to exclude rigid projectile penetration, $\rho pV^2/S_p > 20$. This means that according to Anderson's relation, in these conditions, impacts below 1.1 km/s should be treated as rigid penetration, where the projectile doesn't erode. Even though it was not proposed for rigid penetration, Anderson's relation can still be applied to those values which results in the dotted grey line that meets the solid grey one. This solid grey curve shows a more drastic increase in penetration depth than the solid black line, and at $V = 2 \text{km/s}$ starts to taper off and approaches an asymptotic limit, $P = 41.8 \text{mm}$, that is higher than the hydrodynamic limit.

This representation incorporating all three analytical/empirical models for normal impact set on top of the experimental data allows one to examine which method shows the most conformity to that data. The results show that all three of the methods have their merits and limitations.

The first, the basic hydrodynamic theory, is essentially a means to determine maximum penetration, the hydrodynamic limit. For velocities greater than 2.3 km/s, experimental data start to deviate from and surpass the modified hydrodynamic curve and approach that theoretical hydrodynamic limit. As the impacting velocities get higher, the gain in penetration depth gets smaller. Looking at the last two experimental data points: a 32% increase in impact velocity (2.81 to 3.71 km/s) only yields a 12% gain in penetration depth, compared to the two values right below 2 km/s, where a 14% increase in velocity led to a 24% gain in penetration depth. This holds the claim that this hydrodynamic limit proposed by the basic hydrodynamic theory can be demonstrated experimentally, as that last data point only shows a slight (5%) deviation from that limit. For velocities lower than 2.3 km/s however, the basic hydrodynamic theory is not an accurate method to assess penetration depth, as data points below that value show considerable discrepancies to that penetration depth, as can be observed in the figure 4.1.

Moving on to the second curve, the one corresponding to the modified hydrodynamic theory, it exhibits the most correspondence to the data points among the three methods in the 0.9-1.7 km/s range, where the curve deviates less than 10% for all the values. At the same time, it is also noticeable that the experimental data approach the hydrodynamic limit much faster than in this prediction. Taking the values of the penultimate data point as an example, Tate's model would only reach this penetration depth of 38.9 mm for rods impacting at $V = 6 \text{km/s}$, more than doubling the velocity of $V = 2.81 \text{km/s}$ of this data point. The merits of this method are therefore restricted to lower velocities and not for the hypervelocities where penetration depth is close to the hydrodynamic limit.

The final curve, which corresponds to the empirical relation Anderson et al. [20] developed based on data from their database, most closely resembles the shape of the distribution of the data points for the full range of values. It exhibits a slow growth at the beginning, followed by a steady increase along a similar slope as the experimental data and finally a quick approach to the hydrodynamic limit, surpassing it by just 1.4%. The curve slightly underestimates penetration depth, with a minor right shift in regards to the data points. This good correspondence is justified by the fact that the empirical relation
was based on experimental data which included these data sets, so this small deviation is expected, as the researchers reported a correlation coefficient of 0.9838.

Validation 2 will be in regards to a second set of results from Hohler and Stilp’s data ([20] pages A-121 to A-124), these ones belonging to 58mm rods made from a harder tungsten alloy impacting softer steel targets at velocities between 0.5km/s and 2.1km/s (see dimensions and material properties in table 4.2). All three penetration methods are applied in the same manner as the first time, which leads to the following results depicted in figure 4.2.

Table 4.2: Values for Validation 2 - Tungsten alloy rod onto steel target - L/D=10

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>D (mm)</th>
<th>( \rho_p ) (g/cm(^3))</th>
<th>BH ( N_p )</th>
<th>( \rho_t ) (g/cm(^3))</th>
<th>BH ( N_t )</th>
<th>( \sigma_{yp} ) (GPa)</th>
<th>( \sigma_{yt} ) (GPa)</th>
<th>( Y_p ) (GPa)</th>
<th>( R_t ) (GPa)</th>
<th>( \mu )</th>
<th>( A ) (km/s(^2))</th>
<th>( S_t ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>5.8</td>
<td>17.6</td>
<td>406</td>
<td>7.85</td>
<td>255</td>
<td>1.71</td>
<td>1.07</td>
<td>2.90</td>
<td>5.71</td>
<td>0.67</td>
<td>0.40</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Figure 4.2: Validation 2 against Hohler and Stilp data - Tungsten alloy rods onto steel targets - L/D=10

Once again, the hydrodynamic theory, the straight dashed line, is apparent as the hydrodynamic limit at the top. Because the velocity values for this set of results are lower than for the first analysis, no data point is near that limit. The hydrodynamic velocity range is probably around 3.5km/s like in the first analysis.

Looking at the modified hydrodynamic theory curve, generated by equation 2.9, one can see that it predicts penetration occurring for velocities above 0.54km/s, significantly earlier than in the first analysis, as a result of the harder rods impacting softer targets, meaning the pressures needed to surmount the target’s resistance are lower.

The empirical relation 2.12 considers the penetration for velocities up to 1.27km/s to be rigid rod penetration. Since these rods are made of a harder material, they will have a higher tensile strength than...
softer ones so the vertical limit condition \( \rho_p V^2 / S_p > 20 \) will happen for higher velocities. Once again, the penetration limit of this curve exceeds the hydrodynamic limit established by the hydrodynamic theory.

In this validation the modified hydrodynamic theory shows a poorer correspondence to the experimental data than in the first validation. Up until velocities of 1.5km/s, the theory significantly overestimates penetration as the points seem to follow a sloped line. Only after 1.5km/s does the penetration depth of the data points come within 10% of the one predicted by this curve.

This poorer performance by the modified hydrodynamic theory could be justified by the strength of the rods, \( Y_p \), which is substantially higher than in the first validation. In 1996 [41] Rosenberg et al. argued that a rod's increasing strength resulted in a greater deceleration of the rod during penetration, meaning that it's propensity to penetrate more due to it's strength was contrasted by this deceleration, resulting in a more reduced penetration than one might expect. These researchers suggested another more realistic model to determine the actual yield of the rod.

In this case, the empirical relation is the curve with the best performance predicting penetration depth. With the exception of the first three data points, whose velocities are considered as rigid penetration velocities for this model, all the others points fall within a 5% deviation of this curve, showing excellent correspondence for the full range.

L/D effect

To examine the effect different \( L/D \) ratios have on the penetration depth, experiments Hohler and Stilp [32] performed with projectiles up to ratios of 32 are examined. In order to have comparable data from the different lengthed rods, rather than looking at penetration depth, alternatively, penetration efficiency, meaning \( P \) divided by the projectile's length, \( P/L \), is evaluated. Looking at data collected from projectiles with ratios of 10, 17.5, 22.5 and 32, all made from the same tungsten alloy impacting steel semi-infinite targets (see table 4.2 for material properties and mechanical parameters) and focusing on data points for velocities within the 1-2 km/s range, the analytical methods performance is assessed. The penetration efficiency according to the modified hydrodynamic theory for these conditions was plotted together with the experimental data of Hohler and Stilp for the different ratios in figure 4.3. According to this theory, penetration depth or efficiency is not influenced by a rod's \( L/D \) ratio, which results in only one curve for all four sets of experiments.

Before Hohler and Stilp published this data in 1987, the general understanding of researchers was that, in contrast to short projectiles, where penetration efficiency was proven to decrease with the increase in \( L/D \) ratio, for ratios greater than 10 (long rods) this tendency would stabilize and the values would fall on the same curve, regardless of ratio [1]. The experimental data presented in figure 4.3 demonstrated that this effect was still prevalent in long rods. Looking at the data points, it is noticeable that the ones with the higher penetration efficiency are the ones with the smallest ratio of 10. The values for higher ratioed rods all fall below those and the \( L/D = 32 \) rods clearly show the least penetration efficiency, losing around 50% of the efficiency.

Both Rosenberg and Dekel [30] and Anderson et al. [29] tried to explain this effect by performing simulations with rods of different ratios. They discovered that even in cases when they simulated the
events with zero strength rods, which would not decelerate during penetration, the effect was still present, so this deceleration was not the cause of this effect. They showed that for lower velocities, the initial non-steady entrance phase of the impact was responsible for the discrepancies in penetration efficiency, while for higher velocities this effect was less pronounced, but still present and caused by the terminal phase of penetration, which is also non steady. The different phases that influence the penetration depth of a rod are depicted in figure 4.4, which shows the pressure of the impact event at different times during the penetration.

The figure shows an initial transient phase with a rapid increase and decrease in pressure, which is
caused by a shock wave generated from the rod’s entrance into the target. The second phase characterized by a constant pressure, which is the steady state condition that the analytical models are based on. The third phase is another transient phase, which happens after the full erosion of the projectile [1]. Tate’s penetration model does not account for the initial and final transient phases, treating the whole process as steady state penetration.

These limitations spark some concern moving forward, as the validation of the angled impact model will be performed against experimental results for rods with ratios of 20. Still, as the focus of this study is on analytical models, and because the modified hydrodynamic theory is widely used to describe these types of impacts, the subsequent analyses will take this approach to calculate $P(0)$. The results particular to the conditions of the angled impact will need to be examined before to make sure the deviations are not substantial.

4.2 Angled Impact

4.2.1 Verification

To confirm that the computational implementation of these analytical models was executed correctly and the formulas were applied correctly, a verification is performed. Because it deals with analytical models, this verification is carried out against analytical results from literature.

In his efforts, Luttwak [21, 22] provided analytical results for the yawed rod analytical model of equation 2.24 (no oblique targets), so these are the results against which verification is performed. Luttwak indicated that for this model $P(0)$ could be determined using the expression

$$P(0) = L \sqrt{\frac{\rho_t}{\rho_p}} \left(1 - \frac{1 - \frac{\rho_pV^2}{Y_p}}{Y_p \sqrt{1 + \frac{\rho_p}{\rho_t}}} \right)$$

(4.1)

but the root does not yield a real results for velocities above 0.5km/s, regardless of the yield strength of the projectile (within the realm of possibilities). For this reason, the value $P(0) = 60mm$, a value that Luttwak stated as the result for $0^\circ$ yaw, is used. In addition, the exact values used as material properties for the tungsten rod and steel target are also unknown, so the values in table 4.3 are used for this verification.

<table>
<thead>
<tr>
<th>$V$ (km/s)</th>
<th>$L$ (mm)</th>
<th>$D$ (mm)</th>
<th>$\rho_p$ (g/cm$^3$)</th>
<th>$\rho_p$ (g/cm$^3$)</th>
<th>$Y_t$ (GPa)</th>
<th>$\alpha_c$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>80</td>
<td>8</td>
<td>19.25</td>
<td>7.85</td>
<td>0.65</td>
<td>11.2</td>
</tr>
</tbody>
</table>

These density values are the standard values for tungsten and steel [33], and Luttwak mentioned the value for the target’s yield strength and critical yaw angle, which he did not obtain with expression 2.18.
The results obtained for the implementation are in table 4.4.

<table>
<thead>
<tr>
<th>Yaw angle (°)</th>
<th>( P - [21] ) (mm)</th>
<th>( P - (2.24) ) (mm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>4°</td>
<td>57.1</td>
<td>57.1</td>
<td>0%</td>
</tr>
<tr>
<td>7.5°</td>
<td>52.3</td>
<td>52.3</td>
<td>0%</td>
</tr>
<tr>
<td>19°</td>
<td>38.7</td>
<td>38.6</td>
<td>0.3%</td>
</tr>
<tr>
<td>31°</td>
<td>30.2</td>
<td>30.2</td>
<td>0.7%</td>
</tr>
<tr>
<td>90°</td>
<td>8</td>
<td>8.4</td>
<td>5%</td>
</tr>
</tbody>
</table>

The results show that the implementation is in line with the results from literature. The small errors are presumably due to small differences in material properties.

### 4.2.2 Problem Description

Experimental data involving hypervelocity impacts with both yawed angles and oblique targets is very limited, at least publicly, as most efforts in this area focus on vertical impact. A recent (2013) study by Anderson et al. [26], reported on experimental results for yawed rods onto oblique targets. Those results are compared with one’s obtained by the analytical model proposed by Luttwak [22], equation 2.27.

The shared input parameters of all the impact tests are presented in table 4.5, while velocity, yaw angle and obliquity will depend on the individual events. \( Y_p \) and \( R_t \) were obtained following the methodology described in section 4.2.1.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_p = 17.68 , g/cm^3 )</td>
<td>( \rho_t = 7.85 , g/cm^3 )</td>
</tr>
<tr>
<td>( Y_p = 2.035 , GPa )</td>
<td>( Y_t = 0.772 , GPa )</td>
</tr>
<tr>
<td>( L = 126.4 , mm )</td>
<td>( R_t = 6.25 , GPa )</td>
</tr>
<tr>
<td>( D = 6.3 , mm )</td>
<td></td>
</tr>
</tbody>
</table>

As for the remaining parameters: projectile velocity at impact was approximately \( V = 1.65 \pm 0.03 km/s \); yaw angle varied between 0° and 90°; and targets were set at 30°, 45°, and 60° obliquity, both positive and negative. In the study, rods impacted the targets with vertical yaw (pitch), as well as horizontal yaw (yaw). The difference between those angles was discussed in the subsection 2.3.4. As discussed, the one-dimensional model proposed to describe angled impact does not account for horizontal yaw angles. To bypass this limitation, the total yaw, given by equation 2.28, was considered as the value for yaw in the calculations. Anderson’s study included a discussion on the effect of horizontal yaw on the impact results, with the conclusion being that these angles become negligible when greater pitch angles are present and when pitch angles are small their effect is more pronounced. Nevertheless, due to lack of a better option, total yaw will be used for these efforts. When considering negative obliquity targets, one can either introduce negative \( \theta \) angles, or, alternatively, set the yaw of those impacts as negative while...
keeping obliquity positive, which in practical terms is the same (any possible gravity effects are negligible in such high velocity impact events), and will yield the same results.

In the wake of the concerns that arose in section 4.1.1, due to the lower accuracy of Tate’s normal penetration model when applied to rods with a ratio larger than \( L/D = 10 \), and seeing that the rods in this experiment have a ratio of \( L/D = 20 \), Tate’s modified hydrodynamic theory was applied to these impact conditions and compared to the four baseline experiments for projectiles with minimal yaw (less than 1.2°) impacting targets with 0° obliquity. In table 4.6 is the comparison between their results and the one’s predicted by Tate’s model, equation 2.9.

<table>
<thead>
<tr>
<th>( V ) (km/s)</th>
<th>( P_{\text{exp}} [26] ) (mm)</th>
<th>( P_{\text{cal}} (2.9) ) (mm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.624</td>
<td>105.0</td>
<td>112.7</td>
<td>7.3%</td>
</tr>
<tr>
<td>1.748</td>
<td>118.8</td>
<td>120.6</td>
<td>1.5%</td>
</tr>
<tr>
<td>1.699</td>
<td>111.2</td>
<td>117.6</td>
<td>5.8%</td>
</tr>
<tr>
<td>1.556</td>
<td>91.4</td>
<td>107.9</td>
<td>18%</td>
</tr>
</tbody>
</table>

An example for the calculation of the first value is presented below.

\[
\mu = \sqrt{\frac{\rho_t}{\rho_p}} = 0.667
\]

\[
A = \frac{2(R_t - Y_p)(1 - \mu^2)}{\rho_t} = \frac{2(6.25 - 2.035)10^9(1 - 0.666^2)}{7.85 \times 10^3} \left( \frac{kg}{m \cdot s^2 \cdot kg} \right) = 0.597 \times 10^6 \frac{m^2}{s^2} = 0.597 \frac{km^2}{s^2}
\]

\[
P = \frac{L}{\mu} \left( \frac{\mu \sqrt{V^2 + A - V}}{\mu V - \sqrt{V^2 + A}} \right) = 126.4 \times 0.666 \times 10^3 \left( \frac{0.666 \sqrt{(1.624^2 + 0.597)10^6} - 1.624 \times 10^3}{0.666 \times 1.624 \times 10^3 - \sqrt{(1.624^2 + 0.597)10^6}} \right) \left( \frac{m \cdot s}{m \cdot m} \right)
\]

\[
P = 112.7 \times 10^{-3} \text{ m} = 112.7 \text{ mm}
\]

All the predictions are above the experimental values, which is to be expected given the implications of the L/D effect. Aside from the last set of values, whose rod had a yaw angle of 1.1°, the largest of all four, and could have influenced the reduced measured penetration depth compared to the theoretical one, all the other values have a reasonable error, so the validity of this normal penetration model is confirmed for these conditions and can be applied as \( P(0) \) to see how the angled impact model fares against the experimental data.

In an initial analysis, the data points with the least horizontal yaw from the Anderson et al. [26] experiments were selected and their individual impact velocities taken, to calculate the line-of-sight
LOS penetration of the rods onto the oblique targets with Luttwak’s proposed model [22], equation 2.27. The requirements were data points with less than 1° of horizontal yaw and at least three point for each target inclination, ideally with a good distribution of yaw angles. Because of the minimal horizontal yaw of these points, they should provide the best opportunity to match the analytical model.

As it happens, there were exactly three data points for each target inclination that filled those criteria, with the exception of one data point that impacted a 60° target at 6.5° total yaw, but had a horizontal yaw of 1.2°. Target obliquity, yaw angle, impact velocity, and final LOS penetration of those data points is presented in table 4.7, together with the results of the analytical model. The values represent a good distribution of yaw angles, as intended.

<table>
<thead>
<tr>
<th>θ (°)</th>
<th>α (°)</th>
<th>V (km/s)</th>
<th>P_{exp} [26] (mm)</th>
<th>P_{calc} (2.27) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>6.2</td>
<td>1.657</td>
<td>91.7</td>
<td>79.7</td>
</tr>
<tr>
<td></td>
<td>16.1</td>
<td>1.673</td>
<td>63.0</td>
<td>52.7</td>
</tr>
<tr>
<td></td>
<td>58.4</td>
<td>1.680</td>
<td>14.3</td>
<td>19.8</td>
</tr>
<tr>
<td>30</td>
<td>1.1</td>
<td>1.657</td>
<td>108.5</td>
<td>108.2</td>
</tr>
<tr>
<td></td>
<td>13.5</td>
<td>1.667</td>
<td>49.0</td>
<td>43.7</td>
</tr>
<tr>
<td></td>
<td>26.0</td>
<td>1.660</td>
<td>28.5</td>
<td>22.6</td>
</tr>
<tr>
<td>-45</td>
<td>5.7</td>
<td>1.686</td>
<td>100.1</td>
<td>86.6</td>
</tr>
<tr>
<td></td>
<td>10.5</td>
<td>1.647</td>
<td>92.3</td>
<td>67.4</td>
</tr>
<tr>
<td></td>
<td>24.8</td>
<td>1.669</td>
<td>48.8</td>
<td>48.1</td>
</tr>
<tr>
<td>45</td>
<td>2.1</td>
<td>1.654</td>
<td>100.1</td>
<td>99.9</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.672</td>
<td>61.2</td>
<td>60.5</td>
</tr>
<tr>
<td></td>
<td>26.9</td>
<td>1.656</td>
<td>16.7</td>
<td>15.0</td>
</tr>
<tr>
<td>-60</td>
<td>3.4</td>
<td>1.646</td>
<td>111.4</td>
<td>97.1</td>
</tr>
<tr>
<td></td>
<td>15.4</td>
<td>1.656</td>
<td>80.0</td>
<td>66.9</td>
</tr>
<tr>
<td></td>
<td>26.9</td>
<td>1.657</td>
<td>36.0</td>
<td>57.1</td>
</tr>
<tr>
<td>60</td>
<td>1.2</td>
<td>1.656</td>
<td>106.2</td>
<td>106.7</td>
</tr>
<tr>
<td></td>
<td>6.5</td>
<td>1.688</td>
<td>44</td>
<td>64.0</td>
</tr>
<tr>
<td></td>
<td>12.6</td>
<td>1.660</td>
<td>24.8</td>
<td>32.6</td>
</tr>
</tbody>
</table>

An example of the calculations for the first values in table 4.7 is presented below.

\[
P(0) = \frac{L}{\mu} \left( \frac{\mu\sqrt{V^2 + A} - V}{\mu V - \sqrt{V^2 + A}} \right) = 114.9 \text{ mm}
\]

\[
k = \frac{V}{(1 + \sqrt{\frac{m}{\rho_p}}) \sqrt{\frac{2\gamma}{\rho_t}} \sqrt{\frac{2\gamma}{\rho_t}}} = \frac{1.657 \times 10^3}{(1 + \sqrt{\frac{7.85}{17.68}}) \sqrt{\frac{2+0.772\times10^9}{7.85+10^3}}} \left( \frac{m}{s \ m^2 \ kg} \right) = 2.24
\]
$$a_c = \sin^{-1}\left( \frac{kD}{L} \right) = \sin^{-1}\left( \frac{2.24 \times 6.3}{126.4} \right) = 6.42^\circ$$

$$P(\alpha) = \frac{\gamma kP(0)(1 + \sin(2|\alpha|))}{L \sin(|\alpha| \cos \theta) + \gamma k \exp(-\frac{|\alpha|}{\alpha_c})}$$

$$= \frac{2/3 \times 2.24 \times 114.9 \times 10^{-3} \times (1 + \sin(2 \times 6.2))}{6.3 \times \sin(6.2) \cos(-30)} + 2/3 \times 2.24 \times \exp(-\frac{6.2}{6.42}) = 79.7 \times 10^{-3} \, m = 79.7 \, mm$$

One can easily tell that the results for the targets with negative obliquities (-30°, -45°, -60°) present greater deviations from the experimental results than the ones for positive obliquity targets. This finding suggests that Luttwak’s model, as presented, might not be applicable to negative oblique targets. The shortcomings of the analytical model, particularly in the cases of negative obliquities, warrant further investigation. In order to find out if the model is actually not suited to describe negative oblique impact (or negative yaw angles onto positive oblique targets), the curves from the parametric studies capability of the software tool are plotted.

### 4.3 Parametric Studies

Since the experimental data from Anderson et al. [26] included results for a wide range of yaw angles for each target obliquity, those results can be compared to the analytical model. As stated above, the rods had both vertical and horizontal yaw, so the total yaw values will be considered for the analysis. One can vary the yaw angle for equation 2.27 for each different target configuration and see the effect it has on the impact event.

As is apparent from table 4.7, the velocities of impact are not equal for all the impact events. To address this issue, the average velocity from the data points in each target configuration was applied to the model and the variation of penetration depth with the increasing yaw angles was plotted for the 6 different target configurations.

The first set of targets were set at ±30°. The average velocity for the impacts against targets at -30° was \(V_{-30} = 1.664\, km/s\) and the one for the 30° targets was \(V_{30} = 1.662\, km/s\). The results for these analyses are presented in figure 4.5. The curves follow the implementation of the analytical model for the average velocities mentioned here. The same handling was done for the results of the targets at -45°, 45°, -60° and 60°, whose average velocities were \(V_{-45} = 1.664\, km/s\), \(V_{45} = 1.662\, km/s\), \(V_{-60} = 1.660\, km/s\) and \(V_{60} = 1.661\, km/s\) respectively. Those results are presented in figures 4.6 and 4.7.

Figures 4.5(b), 4.6(b) and 4.7(b) suggest that Luttwak’s analytical model, equation 2.27, is consistent with experimental results. The results for the rods impacting the targets set at 45° obliquity are especially
Figure 4.5: Results for impacts on targets with -30° and +30° obliquity - Tungsten alloy rod on a RHA steel target - V=1.66km/s L=126.4mm D=6.3mm
Figure 4.6: Results for impacts on targets with -45° and +45° obliquity - Tungsten alloy rod on a RHA steel target - V=1.66km/s L=126.4mm D=6.3mm
Figure 4.7: Results for impacts on targets with -60° and +60° obliquity - Tungsten alloy rod on a RHA steel target - $V=1.66\text{km/s}$ $L=126.4\text{mm}$ $D=6.3\text{mm}$
encouraging, as most of the data points present a deviation smaller than 10% when compared to the model.

These figures also highlight the faults of this angled impact model, as the deviations for negative obliquities become very apparent. All the curves for the negative obliquity targets seem to follow the same trend. For yaw angles that are below the 20°-30° range, the curve underestimates penetration depth, while from there on out it does the contrary and overstates the penetration for these values. This trend is particularly noticeable in the results for the oblique targets set at −60°.

One explanation for this fact might have to do with the fact that the critical yaw angle is different for the two situations. When a rod with positive yaw impacts a target with positive obliquity, it is effectively increasing its obliquity, while a rod with negative yaw is impacting more vertically. Figure 4.8 is a representation of the scenario of two yawed rods with the same yaw angle, one positive, the other negative. If the tips of the rods are aligned at the center line of the crater, the tail of the positive yawed rod will hit the wall, while the negative yawed rod is just barely touching it. (This figure only serves as a means to show the different critical angles and is not scaled to reality). This means that a positive yawed rod hitting a positive oblique target will have a smaller critical yaw angle, \( \alpha_c \), than its negative counterpart.

![Figure 4.8: ±3° yawed rods impacting a 60° target - \( \alpha_c \) dissimilarity](image)

Since Luttwak [21] did not suggest another form to determine these different critical yaw angles, one can apply the same approach he did to find it for yawed penetration onto normal targets, correcting that critical yaw angle by applying the same method as in section 2.3.1, equating \( P(0) \) and \( P(\alpha) \) from equation 2.16 but with the obliquity term \( \frac{\cos \theta}{\cos \theta + \alpha} \) in the denominator.

\[
1 = \frac{P(0)}{P(\alpha)} = \frac{\sin \alpha \cos \theta (L_D)}{k \cos \alpha + \theta}
\]

\[
\frac{\sin \alpha}{\cos \alpha \cos \theta - \sin \alpha \sin \theta} = \frac{kD}{L \cos \theta}
\]

\[
\cot \alpha_c - 1 = \frac{L}{kD} + \tan \theta
\]

This gives a differing critical yaw angles for negative and positive obliquity targets, and roughly the same critical yaw for normal targets, as \( \tan(\alpha) \approx \sin(\alpha) \) for small angles.

Nevertheless, when taking this dissimilarity into consideration, the critical yaw angle is only respon-
sible for correcting the expression for small yaw angles that are below it. Therefore, the issues with the results for the negative oblique are not resolved in their entirety, so the model may need to be adapted for negative obliquities.
Chapter 5

Conclusions

This work reviewed one-dimensional analytical models to describe high speed and hypervelocity impacts of long rods. The goal was to provide insight into these particular cases of impact, as they have garnered a lot of attention throughout the years due to their highly destructive capabilities. Due to the magnitude of the velocities involved, experimental data is not easily generated and usually reserved for military applications, so the interest in credible analytical models is very high. With the development of computer hardware and software in the last decades, this interest has shifted to numerical simulations, as these are now widely available and provide good results. Nevertheless, analytical models offer valuable insight into the physical aspects that rule these events as researchers try to understand these highly complicated behaviours to better predict their outcome.

5.1 Achievements

The efforts focused on models to determine penetration depth into semi-infinite targets of normal impacts and subsequently those were applied on a model for angled impact, with the inclusion of yaw and oblique angles with the purpose of analysing how those angles influenced penetration depth.

In regards to the normal impact models, the well known model of the classic hydrodynamic theory was described, as this is the basis for most of the one-dimensional models that were further developed in this area of study. Another model that warranted attention in this work was the Alekseevskii-Tate model, also known as the modified hydrodynamic theory. An empirical approach was also presented, which was based on an extensive database of experimental data. Subsequently, the issue of angled impact was targeted, with a review of a proposed analytical model that included the terms for yaw and oblique angles and predicted the decrease in penetration depth caused by those angles.

A software tool was developed around these analytical methods, facilitating these analyses by calculating penetration depth for various input conditions chosen by the user. This software also provided comparisons between the different methods for normal impact and allowed for velocity range and yaw angle range analyses, allowing one to observe the penetration depth response with the varying parameters.
This tool was put into practice for the validation of the different models against experimental data obtained from literature. The models for normal impact were compared to two sets of results for tungsten alloy rods impacting steel targets. The first analysis showed a very good correspondence of the modified hydrodynamic theory and fairly good correspondence by the empirical relation. The classic hydrodynamic theory established itself as a viable option only for hypervelocity impacts above 3 km/s, as the projectile and target strength appeared to be considerable for velocities up until that point, and could therefore not be ignored.

In the second analysis, the modified hydrodynamic theory showed less promising results, which can be explained by the increased strength of the rods, showing that these models are highly susceptible to the material properties of projectile and target. Incorrect reporting of these properties is almost unavoidable, since these analyses generally ignore thermal effects, which will affect densities and strengths, particularly for very high impact velocities.

The software was then applied to angled impact analyses, where the impact model proposed for these purposes revealed itself to be a good choice to describe yawed impact onto targets with positive obliquity. The curves followed the same trends of the experimental data and showed good correspondence in terms of penetration depth values, particularly for the 45° targets. However, the model lacked in impacts with negative oblique targets. This was partially explained by the different definition of the critical yaw angle for negative inclination targets which should be different than it’s positive counterpart, but only for small angles, and the model showed issues with the whole range of values.

5.2 Future Work

The results for the angled impact analyses have shown that there is room for improvement in regards to negative oblique target impacts. The proposed analytical model is better suited for positive obliquities, and might need to be adapted to better fit experimental results for the full range of obliquities. The model also did not account for the effects of horizontal yaw on penetration depth, so it can also be improved in that regard.

The lack of experimental data on yawed rod impact onto oblique targets was a problem for the thorough analysis of the analytical model, as experiments with both yaw and oblique angled are very limited. The experimental data concerning angled impact focused only on a small range of impact velocities, a constant $L/D$ of 20 and the same projectiles and targets for all the experiments. More experimental data is needed in order to fully assess the influence of all these parameters have when dealing with angled impact.

Numerical simulations would also be welcomed for these types of impact, as these provide good results and are now widely available.
References


