Pumps as turbines (PATs): Series and parallel connections

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Abstract—As a result, from the high energy consumption of the water supply sector, several improvements have arisen for the sake of minimizing this consumption and maximizing the energy efficiency. The use of the pump operating as a turbine (PAT) is one example. It has demonstrated to be a viable solution for energy recovery and pressure control in water systems, especially in remote areas where there is no access to the electrical grid. This is accomplished by means of a generator, whose operation in this specific application has been overlooked. Therefore, this dissertation is intended to bridge this gap between the hydraulic and electric system when using PATs. This was achieved through the assessment of the electrical stability of the association of multiple PATs connected to self excited induction generators (SEIG) in water distribution systems.

In order to accomplish this, a dynamic model of a SEIG was developed. Subsequently, a model of a PAT was modelled from its characteristic curves. Both models were validated after their implementation and together they formed a generating system. When two generating systems were connected in series, it was perceived how an electrical variation in one generating system highly affected the subsequent system where the perturbance was not imposed. However, it is important to note that this conclusion applies only for this specific PAT and SEIG. If this application would be evaluated in a high-power system, the implications observed in the PAT where no variation was imposed would most likely turn out to be insignificant.

I. Introduction

Water supply systems have a significant environmental and energetic impact due to the large amount of energy consumed. According to the Internation Energy Agency [3], the energy consumption of the water sector worldwide in the form of electricity corresponded to 4% of the total global electricity consumption in 2014. Of this part of electricity consumed for water, around 40% was estimated to be used to extract water, 25% for wastewater treatment and 20% for water distribution. As a result from the large energy consumption in the entire water sector, society has become aware that improving the management of these systems is a crucial step towards a more sustainable and economical use of water resources. In particular, solutions to minimize energy consumption and to maximize the energy efficiency have to be implemented.

The application of a pump operating as a turbine (PAT) has been proven in Ramos and Borga [4] to be an alternative and sustainable solution for a better management of these water systems. This solution has drawn a lot of attention since it combines water saving with power generation. In detail, the PAT is able to recover energy where it is available in excess, instead of allowing its dissipation, and when coupled to a generator this energy can be converted into electrical energy. Therefore, this is a reliable solution in the case of rural and remote areas, where the absence of the electrical grid could be compensated with power generation by means of a PAT and a generator, in locations with natural or artificial falls of water. Even though the efficiency of a PAT is usually lower than that of conventional turbines, using PATs has still demonstrated to be a favourable alternative to the use of turbines for energy recovery systems. A primary advantage is the potential cost savings. As pumps are mass produced and there are less manufacturers of turbines, considering the use of a pump operating as a turbine shows to be less expensive than obtaining a specifically designed hydraulic turbine. In addition, pumps are available in a wide range of heads and flows and in a large number of standard sizes. Also, spare parts (such as seals and bearings) are easily available, allowing for easier maintenance.

To complement the on going advancement regarding the use of PATs, this dissertation was proposed. Its aim is to assess the reliability of associating multiple groups of PATs, each connected to a generator, in order to increase the amount of energy recovered. This purpose demonstrates to be pertinent given that it is more cost-effective to invest in several low power pumps rather than in a single high power pump. Depending on the hydraulic system, the connections between the PATs can be made in parallel or in series. Pumps connected in parallel are suitable for a hydraulic system characterized by high flows, as in the case of sewage treatment. On the other hand, pumps connected in series are appropriate for a hydraulic system characterized by a high head, for example in waterfalls. Furthermore, the associations of PATs could prove to be advantageous if acting in one group of PAT coupled to a generator does not affect the other groups significantly. If this is proven, a fault occurring in one group does not compromise the others groups such that they are still able to supply their respective low-power consumers. The evaluation of this effect consists of the goal of this thesis. Only by assessing this interaction between generating groups can a conclusion be drawn regarding the reliability of these associations.

In order to achieve this goal, a model of the SEIG and a model of the PAT had to be implemented. This development and the results from the final assessment of the association of PATs are presented here.

II. Self excited induction generator - a review

Not only Capelo [1], but also Williams et al. [5] have identified the squirrel cage induction generator as the most appropriate electrical machine to take into account for energy recovery in WDS. Simplicity, robustness, reliability and low cost are the main reasons pointed out for this choice.
For the induction machine to run as a generator, a source of reactive power is required. In case the machine is connected to the grid it can draw excitation current from it. But, in the case where the machine is operating isolated from the grid, an alternative source has to be considered. For this it is common to use capacitors as, when connected to the stator terminals, they provide the magnetizing current necessary to establish the air-gap magnetic field. As this dissertation directs to the use of PATs for energy recovery in remote areas, the grid cannot be taken for granted, so the main focus here will be the stand alone operation. Figure 2 demonstrates the complete generating system from which the SEIG is a part of. The power flow represented in this figure can be interpreted. As the prime mover transfers mechanical power to the shaft, the machines decelerate and the SEIG will supply the load with reactive power when the torque developed by the prime mover equals the torque requested by the mechanical load, which is the generator, plus the losses. In this research the prime mover starts as a DC motor to validate the model of the SEIG and, afterwards, it is changed to the PAT.

Bearing this in mind, the model of the SEIG is developed next.

III. DYNAMIC MODEL OF THE SEIG

The dynamics of the induction machine is characterised by differential equations with time-varying inductances due to the continuous change in the position of the rotor with respect to stator. This implies that when calculations are taken place in the usual abc reference frame, being this a stationary coordinate system, a large set of complex equations have to be derived needing a greater effort to accomplish the dynamic analysis. In order to lower this complexity a change to a rotating reference frame is considered, accomplished by the direct-quadrature-zero (dq0) transformation. To be more precise, in the case of balanced three-phase circuits, the application of the dq0 transformation reduces the three AC quantities to two DC quantities. As a result, the time dependency on these three-phase quantities is eliminated and the analysis becomes much simpler.

To start with, the three-phase stator and rotor voltage equations are as follows:

\[
v_{abc, s} = R_s i_{abc, s} + \frac{d\Psi_{abc, s}}{dt} (3) \\
v_{abc, r} = 0 = R_r i_{abc, r} + \frac{d\Psi_{abc, r}}{dt} (4)
\]

where the voltages \((v_{abc})^T = [v_a \ v_b \ v_c]\) are zero since the rotor windings have their terminals short-circuited and \(R_s\) and \(R_r\) are diagonal matrices, each with equal nonzero elements. These three-phase voltages can be transformed into two-phase voltages by considering the direct and quadrature axes in a rotating reference frame. In fact, any direct and quadrature variable can be obtained using the Park’s transformation, \(P_\gamma\), as follows:

\[
f_{dq0} = P_\gamma f_{abc} (5)
\]

where

\[
f_{dq0} = \begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix}
\]

with \(f_d\) being the direct component, \(f_q\) the quadrature component and \(f_0\) the zero component. Here \((f_{abc})^T = [f_a \ f_b \ f_c]\) can represent any variable such as voltage, current or flux. As for the Park’s matrix, it is defined by:

\[
P_\gamma = \frac{2}{3} \begin{bmatrix} \cos\gamma & \cos(\gamma - 120^\circ) & \cos(\gamma + 120^\circ) \\ \sin\gamma & \sin(\gamma - 120^\circ) & \sin(\gamma + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} (7)
\]

where \(\gamma\) can either be the phase difference between the d-axis and the stator phase a-axis or the angle between the d-axis and the rotor phase a-axis, depending on the variable we refer to. For this reference frame it is admitted that the q-axis lags...
the d-axis by 90° as it can be visualized next in figure 2 and suggested by analysing matrix 7.

\( f_0 \) (eq. 5) is used to make the d-q model suitable for unbalanced three-phase systems and also to allow performing the inverse of Park’s transformation. Under balanced three-phase conditions this component is null. Therefore, it will not be considered any longer.

At this point, it will be assumed that the d-q reference frame is rotating at synchronous angular speed, \( \omega_s \). Therefore when seen from the stator’s perspective, the d-q axis is seen to be rotating at synchronous angular speed, \( \omega_s \), since this component is null. Therefore, it will not be considered any longer.

At the same time when seen from the rotor’s perspective the d-q reference frame is seen to be rotating at an angular speed \( \omega_r \). This implies that the angular displacement of the d-axis with respect to the stator geometry position will be \( \theta = \omega_s t \) at any instant. This is confirmed in figure 2 as the angle between the angle displacement \( \theta \) and the rotor axis is seen to be rotating at synchronous angular speed, \( \omega_s \), as it can be visualized next in figure 2 and equation 9.

By inspecting equations 10 to 13 it can be perceived that the system is simplified if either \( \theta \) or \( \beta \) are assumed to be zero. For this reason, the d-q reference frame has been chosen to be stationary. In detail, as the d-q axis stopped rotating, the angle displacement \( \theta \) is 0, such that the d-axis is aligned with the stator phase-a axis. Following this consideration, the direct and quadrature components of the flux linkages of the stator and rotor are obtained from equations 10 to 13 as:

\[
\begin{align*}
\Psi_{ds} &= \int (v_{ds} - R_s i_{ds}) dt \\
\Psi_{qs} &= \int (v_{qs} - R_s i_{qs}) dt \\
\Psi_{dr} &= \int (0 - R_r i_{dr} - \omega_r \Psi_{qr}) dt \\
\Psi_{qr} &= \int (0 - R_r i_{qr} - \omega_r \Psi_{dr}) dt
\end{align*}
\]

Furthermore, the d-q components of the flux linkages of the stator and the rotor can also be determined from the d-q components of the current of the stator and rotor:

\[
\begin{align*}
\Psi_{ds} &= L_s i_{ds} + L_m i_{dr} \\
\Psi_{qs} &= L_s i_{qs} + L_m i_{qr} \\
\Psi_{dr} &= L_r i_{dr} + L_m i_{ds} \\
\Psi_{qr} &= L_r i_{qr} + L_m i_{qs}
\end{align*}
\]

with \( L_s = \lambda_s + L_m \) and \( L_r = \lambda_r + L_m \). Here \( \lambda_s \) and \( \lambda_r \) are the stator and rotor self inductances, respectively, and \( L_m \) is the magnetizing inductance. Taking into account equations 18 to 21, the direct and quadrature currents of the stator and the rotor are computed as follows:

\[
\begin{align*}
i_{ds} &= \frac{\Psi_{ds} - L_m i_{dr}}{L_s} \\
i_{qs} &= \frac{\Psi_{qs} - L_m i_{qr}}{L_s} \\
i_{dr} &= \frac{\Psi_{dr} - L_m i_{ds}}{L_r} \\
i_{qr} &= \frac{\Psi_{qr} - L_m i_{qs}}{L_r}
\end{align*}
\]

Regarding the magnetizing inductance, \( L_m \), in [2] it was verified that the model of the SEIG was more accurate when the influence of the magnetization level, \( E_f \), in this parameter was accounted for, with the remainder of the electrical parameters fixed. This variation of \( L_m \) with the magnetization level
A. Model of the DC motor

The DC motor was modelled from its torque equation, $T_{mec}$, and from the relation between the back emf $e_a$ and the armature terminal voltage $u_a$, as follows:

$$ T_{mec} = K_a \phi_f i_a $$

$$ u_a = e_a + R_a i_a + L_a \frac{di_a}{dt} $$

with

$$ e_a = K_a \phi_f \omega_m $$

where $R_a$ is the armature winding resistance, $i_a$ is the armature current and $K_a$ is a constant which is determined by the configuration of the winding.

B. Test results

The transient and steady state response obtained by the model of the SEIG was validated with experimental tests where either the load or the capacitor bank was connected or disconnected. A brief demonstration of this validation is presented here.

Figure 4a shows the process of self-excitation of the induction machine. From this figure it can be seen the experimental and simulated stator voltage building-up when the capacitor bank was connected to the SEIG. Contrarily, figure 4b demonstrated the loss of excitation when the capacitor bank was disconnected from the SEIG. In both experimental and simulated results, the stator voltage decreased as the machine demagnetized through the resistances.
Therefore, according to the Faraday’s law of induction, a decrease in the speed causes the decrease of the frequency of the stator voltages and, consequently, so does the magnitude of these voltages. Subsequently, when the load was disconnected, the stator voltage was seen to increase. Now this can be understood from the inverse logic.

Fig. 5: Terminal voltage of the SEIG while (a) connecting and (b) disconnecting the load, for a phase resistance of $R_L = 600\Omega$ and for $C = 35\mu F$.

Analysing figures 4a to 5b, it is perceived how the simulated voltage varies accordingly to the experimental one. In addition to this, their magnitudes were verified to be practically the same, as well as the time taken to converge in each transient.

V. MODEL OF THE PAT

The PAT to be modelled was the one used in the experimental work described in Capelo [1]. The characteristic curves of the PAT, namely the Q-H curves and the Q-$\eta_{PAT}$ curves, were obtained in these experiments and they were the starting point to this modelling.

In the experimental set-up assembled in [1], the pressure was the hydraulic parameter which was imposed to the system. For the sake of simplicity, it was admitted that the pressure imposed to the hydraulic system equals the differential pressure between the inlet and outlet of the PAT, $P$. This relates with the head drop $H$ as follows: $P = \rho g H$. Given this, the characteristic curves obtained in [1] had to be adapted so as to be defined as a function of the head $H$ instead of the flow rate $Q$.

The original Q-H curves were defined as:

$$H = \alpha^2 A + \alpha BQ + CQ^2$$  \hspace{1cm} (34)

where $\alpha$ relates the any speed $N$ and a chosen reference speed $N_{ref}$: $\alpha = \frac{N}{N_{ref}}$. During this implementation, the operating points studied in [1] were tested as this reference speed in order to determine which would be more suitable for the interpolation of the curves. In the end, the $N_{ref}$ and corresponding $A$, $B$ and $C$ parameters for which the mode of the errors was the lowest possible was: $A = 3.6644$, $B = -694.45$, $C = 314560$ and $N_{ref} = 1050$ rpm.

Therefore, by inverting equation 34 one obtains the H-Q curves:

$$Q = -\frac{\alpha B \pm \sqrt{(\alpha B)^2 - 4C(\alpha^2 A - H)}}{2C}$$  \hspace{1cm} (35)

These H-Q curves are represented in figure 6 for the operating points studied in [1].

Fig. 6: Relationship between the head and flow rate for different rotational speeds considered for the implementation of the model.

Regarding the efficiency of the PAT, it was defined by several curves which were obtained from experimental data provided by the hydraulics laboratory. These curves are marked with dots in figure 7. When all the adjacent dots are connected, a tendency between them is established and defined by the three-dimensional surface in the figure. This surface enables the estimation of the efficiency of the PAT even for points which were not on these curves marked with the dots. As for the efficiency of the PAT for points outside this surface, it can be obtained from the similarity laws as follows:

$$\eta_i = \eta_{ref} \frac{H_i}{H_{ref}} \left( \frac{N_i}{N_{ref}} \right)^2$$  \hspace{1cm} (36)

Fig. 7: Efficiency of the PAT as a function of the head, for different rotational speeds.
As soon as the head $H$ and flow rate $Q$ are defined, the hydraulic power $P_{hyd}$, which is produced as the water moves through the pump, can be determined:

$$P_{hyd} = QP = \rho gQH \quad (37)$$

In the end, the model of the PAT returns the torque caused by the movement of the pump impeller, which is the mechanical torque, $T_{mec}$, and this is given by:

$$T_{mec} = T_{hyd} \eta_{PAT} \quad (38)$$

$$T_{hyd} = \frac{P_{hyd}}{\omega_m} \quad (39)$$

where $T_{hyd}$ is the hydraulic torque and $\omega_m$ is the mechanical speed.

When the model of the PAT was complete, it was put together with the model of the SEIG just like in figure 1. Afterwards, the model of this generating system was validated with the experimental results from [1] since those tests proceeded with the PAT driving the SEIG which was modelled previously.

VI. SERIES CONNECTION

A series connection between two PATs has two principles that must be fulfilled. First, the flow leaving the outlet port of one PAT has to be exactly the same flow which enters the inlet port of the second PAT: $Q_1 = Q_2$, where $Q_1$ is the flow rate from PAT1 and $Q_2$ is the flow rate in PAT2. In addition, the summation of the pressure drop associated to each of the two PATs has to be equal to the total pressure which is imposed to the system at any moment: $P_{total} = P_1 + P_2$, where $P_1$ corresponds to the pressure drop in PAT1 and $P_2$ to the pressure drop in PAT2. The model of the series connection between the two PATs must have this in account.

Figure 8 presents the schematic representation of the model of the series connection of the two PATs. At first glance it can be noticed that it is composed of two generating systems as the ones represented in figure 1. Then the connection itself can be analysed. From figure 8 it can be perceived that the pressure drop in PAT1, $P_1$, is defined as the difference between the total pressure imposed to the system, $P_{total}$, and the pressure drop in PAT2, $P_2$. The pressure drop in PAT2 is determined from the flow which is discharged by PAT1 through its Q-H curves, given that $Q_1 = Q_2$. In turn, the flow in PAT1, $Q_1$, is obtained from the pressure drop in PAT1, $P_1$, through its H-Q curves, and this whole cycle repeats. In each iteration the cycle repeats numerous times until the system converges to specific hydraulic parameters $P_1$, $P_2$ and $Q$. During the transient, each iteration results in different hydraulic parameters as the rotational speed is changing. Nonetheless, as initially intended this model is implemented such that these hydraulic parameters are adjusted in such a way that the flow remains the same in the two PATs, $Q_1 = Q_2$, and the total pressure drop is still divided by the pressure drop in PAT1 and PAT2: $P_{total} = P_1 + P_2$.

The rated power of the SEIG used in the experimental tests is 550 W, corresponding to a nominal current of 1.6 A. Examining the results from [1] it can be perceived how the hydraulic power delivered to the PAT did not exceed 500 W in any of the tests. Given this, the SEIG could never produce its rated power, nor did it reach its rated current. In order to increase the hydraulic power resulting from the simulations one could increase the pressure imposed to the system. However, this would result in a flow rate which would be higher than the maximum verified in the laboratory. So not to exceed the limits of this PAT, its model was adapted to create a fictitious PAT which would be able to deliver more power to the shaft. For this, the basic characteristics of the original PAT were intended to be the same. That is, the way the head varies according to changes in flow rate would remain the same as in the original PAT. Only now a higher range for the head would be admitted. This was accomplished by a vertical shift in the Q-H curve. Instead of defining these curves with equation 35, for the series connection they were defined by $H = 3\alpha A + \alpha BQ + CQ^2$.

Each simulation starts by imposing a fixed pressure to the system. In the beginning each generator was connected to a capacitor bank with equal capacitance $C$ per phase, as well as a resistive load again with equal $R$ per phase. As initially $C$ and $R_L$ are equal for both groups PAT+SEIG and as the two PATs are equal (i.e. same characteristic curves), in the beginning the system always converged to the same operating point in each of the two groups. In this case of the series connection specifically, the system evolved in a way that when steady state was reached the pressure imposed to the system was equally divided between the two PATs. After the system stabilized, a perturbation was imposed to one subsystem, so that an analysis on how the complete generating system reacts to the disturbance could be cleared out. To be more specific, these perturbances were the variation in the capacitance value $C_2$ or a variation in the resistive load $R_{L2}$. The choice of the subsystem in which the variation is applied is irrelevant given that both subsystems are equal.

Each plot and each notation which will be clarified next have to be considered presuming the complete generating already reached an equilibrium in the beginning, when all the hydraulic and electrical components were the same. That is, the variables...
resultant from the simulated tests are to be shown over time but only for a specific timespan, only during the transient when the parameters \( C \) or \( R_L \) were changed. The first part regarding the transient in which the SEIG excites and the two PATs converge to the same flow rate is not shown here.

The analysis of the test results is divided into two categories: one is the effect of increasing \( C \) or \( R_L \) and the second is the effect of decreasing \( C \) or \( R_L \). The results are grouped in this way since, as demonstrated in the dissertation, increasing \( C_2 \) or \( R_{L2} \) has a similar effect on the other subsystem, PAT1+SEIG1, and the same applies for decreasing \( C_2 \) or \( R_{L2} \). In this current work, only one case from each group is presented.

For simplicity, the subscripts \( A \) and \( B \) were adopted during the following explanation to refer to the operating point before and after the transient respectively.

A. Decreasing \( R_L \)

In order to demonstrate the consequences that decreasing the value of the load \( R_L \) of one PAT has on the generating system a specific test will be shown. In the case which will be presented.

- The rotational speed \( N_2 \) is fixed. Figure 9b, where the rotational speed \( N_2 \) increases as the capacitance value \( C_2 \) remains fixed. Figure 9b, where the rotational speed \( N_2 \) is plotted during the transient, confirms this change in the speed.

Here it can be seen that given this variation in the electrical circuit, it reacted by increasing the speed from \( N_{A2} \) to \( N_{B2} \). As the SEIG2 and the PAT2 are mechanically coupled, there is a high dependency between the electrical and hydraulic systems. So the hydraulic system will suffer repercussions from this electrical variation in its mechanical speed. As it was verified previously, a change in the rotational speed causes a shift in the Q-H curve of the PAT. More specifically, in this case for PAT2 as the speed increases it is expected that the curve corresponding to the operating point \( B \) will be above the one of the operating point \( A \). This can be observed in figure 10a.

As the two groups PAT+SEIG are connected, it is expected that a perturbation imposed to one group will also cause a change in the operation of the second group. More specifically, as in this case the two PATs are connected in series, the change in flow and head of PAT2 imposes a change in flow and head on PAT1, since it must be guaranteed that the flow is the same for the two PATs and that the sum of the pressure drop remains unchanged. For this same reason, the changes verified in PAT1 also influence the operation of PAT2. So basically, the two PATs keep interacting with each other until the whole generating system converges.

From figure 9b it can be noticed that as the load \( R_{L2} \) decreases, the speed \( N_2 \) increased. Considering the Q-H curves for PAT2 (fig. 10a), as \( N_2 \) increases, the flow \( Q_2 \) tends to decrease. This will affect PAT1 given that in series connection the two PATs share the same flow rate. So looking at the Q-H curves for PAT1 in figure 10b, it can be perceived that a drop in the flow rate \( Q_1 \) leads to a decrease of the head of PAT1, \( H_1 \). Now, this hydraulic adjustment in PAT1 will also have consequences in PAT2. As the head in PAT1 decreased, the head in PAT2, \( H_2 \), will be seen to increase, given that the total head is maintained fixed. At the same time that \( H_2 \) increases, the flow rate \( Q_2 \) continues to decrease, and this whole process repeats. This evolution of the operation of the two PATs is perceived in figure 10, where the progression of the operation of each PAT is represented from point \( A \) to point \( B \) including intermediate points. This transient response from point \( A \) to point \( B \) can be underdamped, critically damped and overdamped, as it can be seen ahead when the hydraulic parameters are represented over time.

![Fig. 9: (a) Generic representation of the change of the curve relating the minimum capacitance required to excite the SEIG2 with the mechanical speed \( N_2 \), when the value \( R_{L2} \) is decreased. (b) Evolution of the mechanical speed \( N_2 \) over time, during a decrease of 20% of \( R_{L2} \).](image)

![Fig. 10: Characteristic curves of (a) PAT2 and (b) PAT1, before (A) and after (B) the decrease of \( R_{L2} \), and intermediate points of the operation of each PAT (in red).](image)
25%. At the same time, as the head $H_2$ increased by 14% and the flow rate $Q_2$ decreased by 13%, the hydraulic power transferred to PAT2, $P_{hyd2}$, converted to its initial value. This means that requesting more power from PAT2 (by decreasing the resistance) leads to a similar power in PAT2, in a different speed, but reduces the power of PAT1.

Moreover, the rotational speed in PAT1, $N_1$, can be seen to be decreasing during the transient, given that the Q-H curve corresponding to point B for PAT1 is seen to be below the curve corresponding to point A. Again this could have been foreseen through the dynamics of the system: \[ \frac{d\omega}{dt} = T_{PAT1} - T_{SEIG1} - T_{losses}. \] During the transient, the flow rate and the head in PAT1 are seen to decrease, meaning that the hydraulic power which is being transferred to PAT1 is decreasing. Thus, the group PAT1+SEIG1 is seen to decelerate. Given that $P_{hyd1}$ decreases by 25%, the active power supplied to the load $R_{L1}$ also decreases by 25%.

Under these circumstances, it is important to highlight that the variation in the hydraulic power is shown to be much greater for PAT1 than in PAT2, even though the perturbation was applied to SEIG2. This was anticipated has in PAT2 $H_2$ was seen to increase while $Q_2$ was seen to decrease, and for PAT1 $H_1$ and $Q_1$ both were seen to decrease. As a consequence, SEIG1 supplies the load $R_{L1}$ with a much lower active power than the power delivered by SEIG2 to the load $R_{L2}$. Before the perturbation, each SEIG was supplying the load with $P_s = 423.6W$. When $R_{L2}$ decreased by 20%, the active power that SEIG2 delivered to $R_{L2}$ decreased to $P_{s2} = 392.5W$, while the active power that SEIG1 delivered to $R_{L1}$ already decreased to $P_{s1} = 315.3W$. This brings upon an interesting conclusion. Even though the load was changed in the second group PAT2+SEIG2, the biggest impact is seen to be felt in the first group PAT1+SEIG1.

\[ N_2 = C_2 = 27.6\mu F, \] while the remainder of the components remained unchanged.

As the capacitance value $C_2$ increased it could have been envisioned that the mechanical speed $N_2$ would decrease according to figure 12a, given that the load $R_{L2}$ remained fixed. Figure 12b where the rotational speed is plotted over time, during the transient, confirms the drop in speed of PAT2.

As it was already perceived, when the two PATs are connected in series this implies a dependency between them. When the two groups PAT+SEIG are connected to equal loads and capacitor banks, their outputs are the same. However, when a variation is imposed to a group the other group will also be affected. Here it can be verified that, starting from the same conditions in the two groups, when the capacitance value $C_2$ increased the system evolved in such a way that the mechanical speed $N_2$ decreased. Following this, the reaction of the second subsystem PAT1+SEIG1 to this variation in the group PAT2+SEIG2 has to be analyzed.

Since PAT2 is mechanically coupled to SEIG2, the rotational speed of PAT2 decreased. In chapter ?? it was concluded how the rotational speed of the PAT changes its operation by a shift in the Q-H curves. Here, as the speed decreased from $N_A$ to $N_B$, the Q-H curve corresponding to the situation after the transient, $N_B$, is bellow the curve corresponding to the initial conditions, $N_A$. Observing the Q-H curves of each PAT before and after the transient in figure 13, the evolution of the operation of each one can be explained. Starting from point A, a decrease in the rotational speed causes an increase in the flow $Q_2$. As the two PATs are connected in series they share the same flow rate. So, as $Q_2$ increases, so does the flow in PAT1, $Q_1$. Following the behaviour of the Q-H curve in PAT1, as its flow $Q_1$ increases, so does the head $H_1$, as it can be observed in figure 13b. Since the sum of the heads in the two PATs must be constant for this specific connection, the increase in $H_1$ causes a decrease in the head of PAT2, $H_2$. With the speed in PAT2 gradually decreasing, the flow rate $Q_2$ continues to increase and this whole cycle repeats until the speed $N_2$ converges to $N_B$, which is dictated by $C_B$.

The variation of the flow and head in each PAT is more
explicit in figure 14, where these hydraulic parameters are described over time, during the variation of the capacitance $C_2$. In figure 14a it can be seen how the flow rate is shared by the two PATs. Again as the speed $N_2$ decreased, PAT2 imposes the flow rate to increase, in this case from 6.69 l/s to 6.95 l/s, an equivalent of 4%. Accordingly PAT1 reacted by increasing the head $H_1$ from 25.56 mwc to 26.81 mwc, an equivalent of 5%. Given this, PAT1 imposed the head $H_2$ to decrease by the same proportion (−5%), from 25.56 mwc to 24.31 mwc. Thus, the changes in flow and head for PAT2 cancel out again, leading to the hydraulic power transferred to PAT2, $P_{hyd2}$, to converge to the same value as before the perturbation. Contrarily, as the changes in flow and head for PAT1 are both positive, this led to an increase in the hydraulic power of PAT1 of 9%. This increase in $P_{hyd1}$ caused the rotational speed of PAT1, $N_1$, to increase over time. In addition, as PAT1 can now deliver more power to SEIG1, the active power delivered to the load $R_{L1}$ also increased by 9%.

As for the parallel connection, there is only one constraint. In this case the pressure drop in each PAT, $P_1$ and $P_2$ must equal the pressure which is imposed to the complete generating system, $P_{total}$. Regarding the flow rate, its summation is not necessarily constant since the pressure is the hydraulic parameter which is imposed to the system. This way, the sum of the flow rate of the two PATs changes as $Q_1$ and $Q_2$ change individually. Due to this, in this case a change in the flow rate of one PAT does not affect the other PAT.
The results demonstrated that the model of the PAT can be validated with experimental results. This modelling enabled the analysis of the parallel connection, joint efforts between the hydraulic and electric department are suggested in the future to implement a more accurate model of the PAT.

VIII. CONCLUSION

As originally proposed, in the course of this dissertation a model of the SEIG coupled to the PAT was developed and validated with experimental results. This modelling enabled a subsequent analysis of the series and parallel connection between PATs, from which conclusions were drawn.

Regarding the d-q model of the SEIG, it was validated with the experimental results, for both transient and steady-state operation. The simulation of this model accurately reproduced the voltage build up for different capacitance values. In addition, it was verified both in the simulation and in the experimental tests how the reactive power \( Q_s \), supplied by the capacitor bank, and the mechanical power \( P_{mech} \), supplied by the prime mover have an impact on the power supplied to the load \( P_s \). Besides, as this operation was analysed for an off-grid scenario, the voltage and frequency are not defined by the electric grid. Instead, these two parameters vary with the rotor speed, the load and the capacitance of the capacitor bank. This lack of voltage and frequency regulation presents a challenge for off-grid applications.

As for the model of the PAT, it was implemented to enable the simulation of the operation of the PAT driving the SEIG. The results demonstrated that the model of the PAT can replicate the real behavior of the PAT. It was verified that the SEIG was able to supply the load with more active power as the pressure and flow rate were raised.

Subsequently, after the model of the PAT coupled to the SEIG was complete, a series connection between two generating groups composed of a PAT and a SEIG was assessed. For this particular PAT used in these tests, this connection demonstrated to be unsteady. More specifically, it was verified how a disturbance in the resistive load or in the capacitance value of one generating group turned out to highly impact the second group connected in series, while having a lower impact on its own group. This effect of the variation of an electrical parameter was also seen to have different consequences depending if the variation was imposed to the load or to the capacitance. However, this conclusion can only be guaranteed to this specific generating system. The generating group which was indirectly affected by the variations of the electric parameters of the other group exhibited large fluctuations of the hydraulic parameters. This was verified since this specific PAT demonstrated to be quite sensitive to pressure or flow variations. Since disturbances applied to a generating group highly affected the operation of the second group, its performance became compromised. Nonetheless, too overcome this it is proposed for future work the conduction of tests with the series connection in a high-power hydraulic system, where this effect will most likely be mitigate.

The parallel connection of the PAT and SEIG systems was not performed due to the simplifications admitted for the implementation of the model of the PAT. To enable the analysis of the parallel connection, joint efforts between the hydraulic and electric department are suggested in the future to implement a more accurate model of the PAT.

REFERENCES