Optimal Meter Placement under High Load and High DER Volatility in Low Observability Distribution Networks

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Abstract

Recently, due to the process of decarbonization of the energy sector, distribution networks have faced a massive integration of distributed energy resources (DER), whose injection increases net load volatility. To better understand how to act under such increased volatility, an efficient monitoring of the network and an improved accuracy of state estimation (SE) is required. However, due to economic constraints, distribution networks are characterized for having few meters for their large extension. The main objective of this thesis is to provide economically viable meter placement solutions that efficiently mitigate the impact of the volatility associated with loads and DER on SE of distribution networks. For this reason, it is presented a mixed integer linear programming (MILP) approach that improves SE accuracy by providing meter placement solutions that take into account uncertainty as expressed by multiple load and DER profile scenarios, this way mitigating the impact of net load volatility. Solution results are illustrated and discussed for different case studies carried out over a radial nine bus test feeder.

Keywords: Distribution networks, limited observability, load and DER volatility, MILP, optimal meter placement.

I. Introduction

Recently, the European Union (EU) has been implementing a continuing set of policies that intends to drastically decrease greenhouse gas emissions by around 80-95% until 2050 [1, 2, 3]. To fulfill this objective, there has been a continuous integration of DER in the low-voltage and medium-voltage distribution networks. This integration is increasingly dissipating the well-established and one-sided relation between consumer and producer. Furthermore, this paradigm shift will lead to recurring excessive voltage variations and bottlenecks. Therefore, to deal with these changes, more investment in protection and controlling the system is required. Accordingly, Distribution System Operators (DSO) need to promote active system management to fully maintain the reliability of supply and quality of service for all consumers [2, 4], which is only possible if a more efficient monitoring of the network is established [2]. However, the volatile nature of loads and the economic constraints that come from the large extension of distribution networks compromise the efficient monitoring of the network. With this new reality in mind, there is the need for a growing focus of meter placement research towards distribution networks, specifically considering volatile loads and DER, low observability and a restricted budget.

This thesis addresses the meter placement problem in distribution networks under low observability, mitigating the impact of load and DER volatility in meter placement solutions. The objective is to attain maximum accuracy in SE by providing a meter configuration that not only comprises the allocation of a limited number of high accuracy meters but also prevents a significant increase in SE errors. For this purpose, the original MILP formulation by Chen et al. [5] was extended so as to deal with the uncertainty expressed by multiple scenarios of different load and DER profiles.

II. State of the Art

Schweppes and Wildes [6] introduced the concepts associated with the state representation of a system in power systems through static-state estimation. Static-state estimation enabled the identification of the operating point of the system, taking into account the uncertainties associated with the measurements available. With this breakthrough, the state of the system could be more accurately determined, no longer relying on load flows that did not take into account the uncertainties of measurements. However, although SE may have enabled a more accurate path for obtaining knowledge of the
system it has also lead to new problems that need attention such as bad data detection and identification, observability and meter placement. One of the main thematics in which Schweppe and Wildes also focused in their paper [6] was the meter placement problem. This new problem was identified to be essential for the performance of SE and can be defined as the choice of types of meters, location and associated accuracy in a network. In fact, it was concluded that for a good choice of meter placement, the state estimator converged rapidly and that for a bad choice of meter placement convergence was found to be slow or even nonexistent. These conclusions can only emphasize the importance that the meter placement problem has on SE and consequently on power systems.

The meter placement problem was initially mainly studied on transmission systems accompanied by the research that was being carried out in SE. The main assessment requirements for a good meter placement were summarized by Baran et al. [7] and involved cost requirements, accuracy requirements, reliability requirements and bad data processing requirements. On the other hand, several works also focused on observability. Monticelli and Wu [8] and Krumpholz, Clements, and Davis [9] developed pioneer works in observability of networks through numerical and topological approaches, respectively. In their papers, they strived not only on identifying which parts of the network were observable and, consequently, where it was possible to perform SE but also on how to connect them through meter placement.

More recently, with a growing focus of approaches towards meter placement in distribution networks, new approaches were proposed. Singh, Pal and Vinter [10] aimed to decrease the relative errors in the voltage and angle estimates at all buses of the network, which was derived to be equivalent to reducing the error ellipse generated by the error covariance matrix. Chen et al. [5] used a circuit representation model based on the gain matrix to represent SE errors and a disjunctive model to exactly relax and convert the meter placement problem from a mixed integer nonlinear programming (MINLP) problem into a MILP problem. Furthermore, some papers focused on addressing the meter placement problem in active distribution networks [11, 12]. These approaches relied on the use of heuristics to obtain the desired meter placement solution, specifically a genetic algorithm and a binary particle swarm optimization method.

III. Proposed Method

III.1. State Estimator

The state estimator considered in this approach is the weighted least squares based on an AC network model. Hence, the relation between the state variables and the measurements can be defined as

$$z = h(x) + e$$

(1)

where \( z \) corresponds to the measurement vector (\( m \)-vector); \( x \) is the true state vector (\( n \)-vector); \( h(\cdot) \) is the measurement function, which is a nonlinear vector function that relates measurements to states (\( m \)-vector) and \( e \) is the measurement error vector (\( m \)-vector) \([13, 14]\). The measurement error considered has zero mean, and the errors are assumed independent. From these properties, the covariance matrix of the measurement error is formulated as a diagonal matrix, where each diagonal element is the variance associated with each measurement allocated.

$$Cov(e) = E[ee^T] = R_z = diag\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2\}$$

(2)

Furthermore, once measurement errors are considered random variables, the same assumption can be extended to the measurements \( z \). These variables can also be represented according to a Gaussian distribution with a covariance matrix identical to (2), but with mean \( h(x) \) [15]. Therefore, to obtain the maximum likelihood estimate, the following objective function, \( J(x) \), is minimized

$$\min_{x} [z - h(x)]^T R_z^{-1} [z - h(x)]$$

(3)

The first-order optimal condition of (3) is given by

$$g(x) = \frac{\partial J(x)}{\partial x} = 0 \iff -H^T(x) R_z^{-1} [z - h(x)] = 0$$

(4)

where \( g(x) \) is the gradient of \( J(x) \) and \( H(x) \) is the measurement Jacobian

$$H(x) = \frac{\partial h(x)}{\partial x}$$

(5)

The gradient obtained in (4) is a nonlinear function and, consequently, in order to obtain a solution it is necessary to use an iterative method. For that purpose, Newton’s method is used to solve \( g(x) = 0 \). After applying the Taylor expansion to \( g(x) \),

$$g(x) = g(x^k) + G(x)(x - x^k) + \cdots = 0$$

(6)

where \( G(x) \) is referred to as the gain matrix. This Hessian matrix of the objective function can be expressed as

$$G(x) = \frac{\partial^2 J(x)}{\partial x^2} = H^T(x) R_z^{-1} H(x)$$

(7)

after ignoring the second derivative terms. Finally, after disregarding higher order terms of (6) and using (4), an iterative solution can be found

$$G(x) \Delta x^k = H^T(x) R_z^{-1} [z - h(x)]$$

$$\Delta x^k = x^{k+1} - x^k$$

(8)

to solve SE [13, 14, 15].
III.2. Circuit Representation of the Gain Matrix

The gain matrix is a key element in meter placement. It reflects both the type, accuracy and location of the measurements placed in a system. In addition, according to Schewppe and Wildes [6] its inverse matrix

$$R_k = G^{-1}$$

(9)
can be interpreted as a measure of accuracy of the state estimate

$$R_k = E\{(x - \bar{x})(x - \bar{x})^T\}$$

(10)

Accordingly, \(R_k = \langle \epsilon_{ij} \rangle_{n \times n}\) can be referred to as the covariance matrix of the state estimate, as it provides information about the accuracy which can be achieved at each bus with the available measurements.

Furthermore, the gain matrix is a symmetric sparse matrix. Consequently, its structure is similar to the admittance matrix \(Y\), which is characterized for its use to describe networks in power systems. Chen et al. [5] used this fact to represent SE errors through a circuit representation model of the gain matrix

$$G = \sum_{k=1}^{N_b} M_k y_k M_k^T$$

(11)

where \(M_k\) is a vector that represents the location of each admittance \(y_k\), and \(N_b\) is the number of branches. The reasoning used to obtain the admittances from matrix \(Y\) is also used for the gain matrix

$$\begin{align*}
 y_{ij}^{SA} &= \sum_{j=1}^{N_b} G_{ij} \\
 y_{ij}^{BA} &= -G_{ij}
\end{align*}$$

(12)

where \(y_{ij}^{SA}\) is the shunt admittance of bus \(i\) and \(y_{ij}^{BA}\) is the branch admittance between buses \(i\) and \(j\).

The only remaining issue to complete the circuit representation is to know how to compute the elements of the gain matrix. For that purpose, it can be used equation (7) but in a summation form expressed as

$$G_{ij} = \sum_{k=1}^{m} \frac{h_{ki} h_{kj}}{\sigma_k^2}$$

(13)

where \(k\) is the measurement index; \(i, j\) are the indices of each element of the gain matrix and \(h_{ki}\) and \(h_{kj}\) are the individual elements of \(H\), i.e. represent partial derivatives with respect to only one state variable, as in

$$h_{ki} = \frac{\partial h_{kj}}{\partial x_i}$$

(14)

By combining equations (12) and (13) and considering the contribution of a single measurement \(k\), the admittances of the gain matrix can be computed through

$$\begin{align*}
 y_{k,i}^{SA} &= \frac{h_{ki}}{\sigma_k^2} \sum_{j=1}^{n} h_{kj} \\
 y_{k,i}^{BA} &= -\frac{h_{ki} h_{ij}}{\sigma_k^2}
\end{align*}$$

(15)

In this thesis, it is assumed that the state vector \(x\) is composed by the set of bus voltage magnitudes of the network

$$x = [V_1, V_2, ..., V_n]^T$$

(16)

Furthermore, it is assumed that network branches follow the general two-port \(\pi\) model with negligible shunt admittances, as in Fig. 1

![Figure 1: Two-port \(\pi\) model network branch with negligible shunt admittances.](image)

where \(g_{ij}\) and \(b_{ij}\) are, respectively, the conductance and the susceptance of the network branch. Based on this assumption, the available measurements used in this thesis are listed in Table 1.

<table>
<thead>
<tr>
<th>Type of measurement</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_i)</td>
<td>(V_i \sum_{j \in \mathbb{B}<em>i} V_j (g</em>{Y,ij} \cos \theta_{ij} + b_{Y,ij} \sin \theta_{ij}))</td>
</tr>
<tr>
<td>(Q_i)</td>
<td>(V_i \sum_{j \in \mathbb{B}<em>i} V_j (g</em>{Y,ij} \sin \theta_{ij} - b_{Y,ij} \cos \theta_{ij}))</td>
</tr>
<tr>
<td>(I_{ij})</td>
<td>(\sqrt{\left(g_{ij}^2 + b_{ij}^2\right) \left(V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}\right)})</td>
</tr>
<tr>
<td>(V_i)</td>
<td>(V_i)</td>
</tr>
</tbody>
</table>

where \(\theta_{ij} = \theta_i - \theta_j\); \(P_i\) and \(Q_i\) are the real and reactive power injection at bus \(i\); \(I_{ij}\) is the line current flow magnitude from bus \(i\) to bus \(j\); \(V_i\) is the voltage magnitude at bus \(i\); \(G_Y\) and \(B_Y\) are in this context, respectively, the real and imaginary matrices of the admittance matrix \(Y\) and \(\mathbb{N}_b\) is the set of all consecutive buses to bus \(i\) [13].

From Table 1, it is possible to derive the expressions that determine the elements of the measurement matrix \(H\). It is important to note that when considering a distribution network, \(\theta_{ij} \approx 0\). This simplification can be used to obtain the admittances of the measurements intended for allocation. However, although \(\theta_{ij} \approx 0\) may still be applicable for other measurements, when they are not being allocated as admittances, it is adopted the formulation presented in Table 1. The reasoning is to obtain the
most precise solution whenever full detail is possible, specifically when using either real and reactive power injection measurements.

When considering a voltage magnitude meter at bus \( i \), \( h_k = x_i = V_i \). Based on that and according to equation (14)

\[
\begin{align*}
  h_{ki} &= \frac{\partial h_i}{\partial V_i} = \frac{\partial V_i}{\partial V_i} = 1 \\
  h_{kj} &= \frac{\partial h_i}{\partial V_j} = \frac{\partial V_i}{\partial V_j} = 0
\end{align*}
\]

(17)

Following that, from (15) comes that

\[
\begin{align*}
  y_{k,i}^A &= \frac{1}{\pi_k} \\
  y_{k,ij}^A &= 0
\end{align*}
\]

(18)

As a result, when a meter of this type is allocated at bus \( i \), the only resulting admittance is a shunt admittance, i.e. an admittance which connects bus \( i \) to the ground. It is also important to point out that its value only depends on the accuracy of the meter, through \( \sigma_k^2 \). Furthermore, the associated position of the admittance is represented by \( M_k \), which is a zero vector, except on the index of the bus where the meter is placed, which has the value 1.

On the other hand, when considering a current magnitude meter between two consecutive buses \( i \) and \( j \) and a non-consecutive bus \( l \) comes that

\[
\begin{align*}
  h_{ki} &= \frac{\partial h_i}{\partial V_i} = \sqrt{g_{ij}^2 + b_{ij}^2} \\
  h_{kj} &= \frac{\partial h_j}{\partial V_i} = -\sqrt{g_{ij}^2 + b_{ij}^2} \\
  h_{kl} &= \frac{\partial h_l}{\partial V_i} = 0
\end{align*}
\]

and consequently

\[
\begin{align*}
  y_{k,i}^A &= 0 \\
  y_{k,ij}^A &= \frac{g_{ij}^2 + b_{ij}^2}{\sigma_k^2} \\
  y_{k,il}^A &= y_{k,jl}^A = 0
\end{align*}
\]

(19)

(20)

In this case, the resulting admittance is not a shunt admittance but instead a branch admittance. Moreover, its value does not only depend on the accuracy of the measurement placed but also on the values of the parameters of the corresponding branch. The associated position of the admittance comes from \( M_k \), which is a zero vector except on the indices of the buses that connect the branch where the meter is placed. In this vector, one of these indices has the value 1 and the other the value -1.

It is also worth mentioning the elements of \( H \) related with the real power injections

\[
\begin{align*}
  h_{ki} &= \frac{\partial P_i}{\partial V_i} = 2V_i G_{Y,ii} \\
  &+ \sum_{j \in E_i} V_j(G_{Y,ij} \cos \theta_{ij} + B_{Y,ij} \sin \theta_{ij}) \\
  h_{kj} &= \frac{\partial P_j}{\partial V_i} = V_i(G_{Y,ij} \cos \theta_{ij} + B_{Y,ij} \sin \theta_{ij}) \\
  h_{kl} &= \frac{\partial P_l}{\partial V_i} = 0
\end{align*}
\]

(21)

and reactive power injections

\[
\begin{align*}
  h_{ki} &= \frac{\partial Q_i}{\partial V_i} = -2V_i B_{Y,ii} \\
  &+ \sum_{j \in E_i} V_j(G_{Y,ij} \sin \theta_{ij} - B_{Y,ij} \cos \theta_{ij}) \\
  h_{kj} &= \frac{\partial Q_j}{\partial V_i} = V_i(G_{Y,ij} \sin \theta_{ij} - B_{Y,ij} \cos \theta_{ij}) \\
  h_{kl} &= \frac{\partial Q_l}{\partial V_i} = 0
\end{align*}
\]

(22)

as they comprise the set of pseudo-measurements available in this study.

III.3. MILP Formulation

The objective of this optimal meter placement method is to find the optimal location for meters in order to minimize the SE standard error at each bus. This information can be understood to be in the diagonal elements of the error covariance matrix of the state estimate. Consequently, the performance index, which shall be referred to as error index, can be expressed as a linear combination of the diagonal elements of \( R_k \). However, this matrix only refers to the SE accuracy obtained under one load level. If one is to consider multiple load levels and wishes to get a compromise meter solution that best fits all scenarios considered, then it is necessary to consider all the corresponding \( R_k \) matrices. Accordingly, the error index can be formulated as

\[
\min \sum_{i=1}^{N_o} \varepsilon_{ii}
\]

(23)

where \( N_o \) corresponds to the number of scenarios considered.

The elements of each error covariance matrix of the state estimate can be obtained through (9). However, this operation is nonlinear and threatens the formulation of the problem in a MILP format.

The solution to this issue comes from decomposing the gain matrix into two parts

\[
G = G_0 + \sum_{k=1}^{N_{\text{cand}}} M_k b_k y_k M_k^T
\]

(24)

The first part, \( G_0 \), corresponds to the gain matrix obtained from the initial configuration of measurements already allocated in the network. The second represents a summation of gain matrices. Each gain matrix is computed based on the allocation, in the network, of only one of the total amount of meters that can be allocated, \( N_{\text{cand}} \). This second part is similarly formulated to equation (13). The sole difference is the introduction of the decision variable \( b_k \). This decision variable translates the allocation or not of meter \( k \). Since the meters that are to be allocated are voltage magnitude meters and current magnitude meters, the decision vector \( b \) can be separated into two

\[
b = [b_{\text{vol}} \quad b_{\text{cur}}]^T
\]

(25)
These vectors $b^{vol}$ and $b^{cur}$ store the decision of allocation, $b_k = 1$, or not, $b_k = 0$ for each meter. In the case of voltage meters at each bus, while in the case of current meters at each branch. From the decomposition of the gain matrix it is possible to combine both equations (9) and (24) into one

$$G_0 R_b + \sum_{k=1}^{N_{sc}} M_k b_k y_k M_k^T R_{\hat{z}} = I$$

(26)

At this point, equation (26) is nonlinear, and it is still not possible to formulate the optimal meter placement problem as a MILP. However, it is possible to define an instrumental vector $z_k$ to deal with the nonlinear nature of the equation

$$z_k = b_k y_k M_k^T R_{\hat{z}} \quad k = 1, 2, \ldots, m$$

(27)

The purpose of this vector is to select the nonlinear part of the equation in order to apply the disjunctive model proposed by Bahiense et al. [16]. According to this model, the multiplication of decision variables in equation (27) disappears, and this equation is replaced by a linear relaxation

$$-L(1 - b_k) \mathbf{1} \leq z_k - y_k M_k^T R_{\hat{z}} \leq L(1 - b_k) \mathbf{1}$$

(28)

where $L$ is a large number and $\mathbf{1} = [1 \ 1 \ \ldots \ 1]$. Nevertheless, this inequality does not ensure that when $b_k = 0$, the resulting gain matrix associated with measurement $k$ is a zero matrix. For that purpose, another inequality is needed

$$-L b_k \mathbf{1} \leq z_k \leq L b_k \mathbf{1}$$

(29)

The joint effort of both inequalities (28) and (29) leads to the intended result without any nonlinearities involved. When there is no allocation $b_k = 0$, and consequently $z_k = \mathbf{0}$. On the contrary, when there is allocation $b_k = 1$ and $z_k = y_k M_k^T R_{\hat{z}}$.

This formulation also takes into account the maximum amount of meters intended to employ. For that purpose, two new constraints can be added to the problem

$$\sum_{i=1}^{n} b_i^{vol} \leq N_{vol}^{new}, \sum_{i=1}^{N_b} b_i^{cur} \leq N_{cur}^{new}$$

(30)

where $N_{vol}^{new}$ and $N_{cur}^{new}$ are, respectively, the maximum amount of voltage and current magnitude meters that can be allocated.

Additionally, a cost can be associated with each meter allocation

$$C^T b \leq c_{total}$$

(31)

where $C$ is a vector with the costs of allocating each meter and $c_{total}$ is the maximum amount to spend in the allocation of meters.

A new constraint is also introduced such that it is possible to define the initial position of the meters intended to allocate

$$A^T b = a_{total}$$

(32)

where $A$ is a zero vector with ones in the positions where the meters are to be allocated. Accordingly, $a_{total}$ is the result of the sum of the elements of $A$.

The final MILP formulation can be expressed as

$$\min \sum_{i=1}^{n} \varepsilon_{ii}$$

s.t. $G_0 R_{\hat{z}} + \sum_{k=1}^{N_{sc}} M_k b_k y_k M_k^T R_{\hat{z}} = I$

$$-L(1 - b_k) \mathbf{1} \leq z_k - y_k M_k^T R_{\hat{z}} \leq L(1 - b_k) \mathbf{1}$$

$$-L b_k \mathbf{1} \leq z_k \leq L b_k \mathbf{1}$$

$$\sum_{i=1}^{n} b_i^{vol} \leq N_{vol}^{new}, \sum_{i=1}^{N_b} b_i^{cur} \leq N_{cur}^{new}$$

$$C^T b \leq c_{total}$$

$$A^T b = a_{total}$$

(33)

The final decision variables are the elements of all matrices $R_{\hat{z}}$, the elements of the instrumental vectors $z_k$ and the elements of $b$.

This formulation allows obtaining meter placement solutions that take into account not only one scenario but $N_s$, extending the MILP formulation presented by Chen et al. [5]. By allowing the use of $N_s$ scenarios instead of one, it is now possible to obtain a compromise meter placement solution that is capable of dealing with load and DER volatility. This solution is defined as a compromise solution, as it may not be the optimal meter solution for every single or even any scenario individually, but instead for the whole set of possible scenarios considered.

IV. Case Studies and Tests Performed

In this thesis, it is used a radial 9 bus test feeder. Its line model follows the same structure as presented in Figure 1 and the branch admittance is $0.01 + j0.01$ p.u.. It is also considered that all feeder buses, except the first, have real and reactive power injections. The measurements associated with these power injections are considered to be pseudo-measurements with 50% standard deviation. Furthermore, it is already placed a current magnitude meter between buses 1 and 2 with 1% standard deviation, as in Figure 2.
The situations under which this test feeder is tested are the following. In the first situation, two distinct scenarios that differ in the presence of DER are used. The first scenario describes a scenario without the presence of DER, in which all buses with loads have injected power equal to \(-0.22 - j0.022\) p.u.. On the contrary, the second scenario describes a situation with the presence of DER, where the sole difference is the injected power at the fifth bus, which is equal to \(0.88 + j0.088\) p.u..

Each of the following situations are composed of twenty four scenarios and intend to reflect an evolution in the integration of DER. The second situation represents a situation where there is still low DER integration, and consequently, there is low load and DER volatility. As such, only one bus 5 has DER and the values for the real power injections are obtained using two uniform distributions. The uniform distribution used for obtaining the values for the fifth bus has bounds between 0.1 and 1 p.u., while the uniform distribution used for the remaining buses has its bounds between \(-1\) and \(-1\) p.u.. The reactive power values are assumed to be 10% of the values obtained for the real power. The third situation assumes that all buses, except the first, can have allocated DER. Consequently, the volatility associated with this situation is higher than in the previous. The uniform distribution for obtaining the real power injections at each node follows the uniform distribution with bounds between \(-1\) and \(-1\) p.u.. The intention was that the uniform distribution used in this situation had the most similar interval possible to the previous case. In the same way, the reactive power values are assumed to be 10% of the values obtained for the real power. The fourth and final situation compresses ninety six individual meter readings of real and reactive power injections from a single day for each bus into twenty four scenarios just as in the previous two situations. In this situation only bus 5 has DER. For this bus, the reactive power is assumed to be 10% of the corresponding active power.

The tests performed comprise the allocation of one voltage magnitude meter and one current magnitude meter, both with 1% standard error. Furthermore, the allocation cost at any bus or branch is considered to be the same. These meters are allocated according to the solution obtained from the MILP formulation (33). In the first test, three optimal meter configurations are to be obtained and compared under the first situation. An optimal meter configuration considering the scenario without DER, an optimal meter configuration considering the scenario with DER and an optimal meter configuration considering both scenarios with and without DER. This last optimal meter configuration is the compromise meter solution of this situation. Each of these configurations is tested under each individual scenario to understand how they perform in relation to the optimal meter configuration for the scenario at hand. This analysis is carried out using both the results obtained for the error index and the SE accuracy at each bus. The following tests follow a similar reasoning to the last test, but this time for the second, third and fourth situations. First, the optimal configurations for each individual scenario are obtained and then, their error index is compared with the error index obtained for the compromise solution. In these cases the comparison is done in regard to the compromise meter solution as it represents the optimal configuration that is obtained when considering the whole set of scenarios of each situation.

V. Results and Discussion

V.1. Two Distinct Scenarios with Generated Data

Regarding the scenario that describes a situation without the presence of DER, the meter placement solution is presented in Figure 3.

Figure 3: Optimal meter configuration considering the scenario without DER.

The error index obtained for this meter configuration is 0.0009 p.u..

On the other hand, in regard to the scenario where there is the presence of DER, the meter placement solution is presented in Figure 4.

Figure 4: Optimal meter configuration considering the scenario with DER.

In this case, the error index obtained for this meter configuration is 0.0018 p.u.. The optimal meter configuration that takes into account both scenarios with and without DER is presented in Figure 5.
Figure 5: Optimal meter configuration considering both scenarios with and without DER.

The optimality that was previously ensured for the meter configurations that dealt with one scenario only is now also ensured by the compromise meter solution, as presented in Figure 6.

Figure 6: Contour plot of the error index considering both scenarios with and without DER.

In fact, according to Figure 6, the best possible configuration that considers both scenarios has a voltage meter at bus 7 and a current meter at branch 5. This branch corresponds to the branch between buses 5 and 6. The result obtained sustains the fact that the compromise meter solution remains as its counterparts a globally optimal solution.

After obtaining the three optimal meter configurations, each configuration was assessed under each scenario individually. The resulting error indices are presented in Table 2.

Table 2: Error Index Comparison between each Configuration

<table>
<thead>
<tr>
<th>Configurations</th>
<th>Scenario Error Index (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without DER</td>
</tr>
<tr>
<td>Without DER</td>
<td>0.0009</td>
</tr>
<tr>
<td>With DER</td>
<td>0.0015</td>
</tr>
<tr>
<td>Compromise</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

As anticipated and according to Table 2, the configuration that has the lowest error index for a determined scenario is the configuration specifically designed for that scenario. Furthermore, in both scenarios, the remaining configurations present a worse error index and, consequently, a worse SE accuracy. However, the results obtained with the compromise meter solution stand out in each scenario, specifically when comparing to the other configuration that was not designed for the scenario considered. In fact, the increase in the error index is significantly lower when using the compromise meter solution rather than when using the configuration that was not designed for that scenario. This is an important result, as this is verified not only on the scenario without DER but also on the scenario with DER. Thus, these results support the use of a compromise meter solution when dealing with load and DER volatility as it can be further verified through Figure 7.

Figure 7: Comparison between error index variations obtained for different optimal meter solutions. Error variations are referred to the error index found for the optimal meter configuration of each scenario.

According to the results presented in Figure 7, the error index variation of a meter configuration that was not designed according to the scenario at hand is significantly higher than when using a compromise meter solution.

Further analysis on the effectiveness of the choice of the compromise solution can be obtained through the comparison of the SE standard error at each bus. This comparison for the scenario without DER is presented in Figure 8.
From Figure 8, some conclusions can be drawn both according to the comparison of performance between meter configurations and according to the performance of each meter configuration individually. When comparing performances between meter configurations, it is noticeable that on average, the configuration that was designed for this scenario has the best SE accuracy. On the other hand, the configuration designed for the scenario with DER has, on average, the worst SE accuracy. The SE accuracy of the compromise solution tends to stay in between the two other configurations, as intended. It is also important to notice that these conclusions are for the average and not for the individual SE of each bus. In fact, in some cases such as when considering bus 4, the configuration for the scenario with DER has a better SE accuracy than the configuration without DER. This happens because in the configuration with DER there is a voltage magnitude meter placed at bus 4 and the meters used are allocated until bus 6.

The comparison of the SE standard error at each bus considering the scenario with DER is presented next in Figure 9.

From Figure 9, it is possible to understand the reason behind the less effective performance of the configuration with DER in the previous scenario, based on the performance of the configuration without DER in this scenario. In fact, the configuration without DER has a significantly worse SE accuracy at the beginning of the feeder, as it does not take into account the effect of the presence of DER. On the other hand, as intended, the compromise solution still has a reasonable performance.

V.2. Twenty Four Distinct scenarios with Generated Data

This situation represents a situation with low integration of DER and consequently with low load and DER volatility. The compromise meter solution obtained is presented in Figure 10.

Considering that each individual scenario has an optimal meter configuration, four distinct configurations were found. One of these configurations is the same as the compromise solution and agglomerates the largest amount of scenarios. The distribution of the individual optimal meter configurations according to their error variation regarding the compromise solution is presented in Figure 11.

In this situation, it is shown that the impact of
preferring the use of a compromise meter solution rather than an individual optimal solution is small. The reason for this low increase in the error index is that in this situation, there is low load and DER volatility at each bus. This low volatility, in turn, translates into a situation where there is low diversity in the choice of optimal meter configurations for each scenario and the impact of choosing distinct individual optimal configurations is not significant.

The previous situation can be further extended considering that all buses, except the first, can have DER. The immediate result is a substantial increase in load and DER volatility of the network. The compromise solution for the set of these scenarios is presented in Figure 12.

![Figure 12: Compromise optimal meter configuration. The set of scenarios considered is from the third situation.](image)

From the considered scenarios, eleven distinct configurations were found. One of these configurations is the compromise meter configuration, which, in this case, does not agglomerate the largest amount of scenarios. The distribution of the individual optimal meter configurations according to their error index variation is presented in Figure 13.

![Figure 13: Distribution of the individual optimal meter configurations, in percentage, according to their error index variation. Error variations are referred to the error index found for the compromise meter solution. The set of scenarios considered is from the third situation.](image)

It is important to point out that the increased volatility in this situation had a severe impact on the increase of the error index of the individual meter configurations, specifically when comparing with the previous situation. This shows why is it not enough to optimally choose the optimal meter configuration for one scenario and apply to the rest, but instead it is necessary to take into account the set of scenarios in study such that the effect of load volatility on SE accuracy is mitigated.

V.3. Twenty Four Distinct Scenarios with Real Data

This situation differs from the others because of the use of real data. However, it is similar to the first situation presented with generated data for twenty four scenarios. According to the provided data, the compromise solution for this situation is presented in Figure 14.

![Figure 14: Compromise optimal meter configuration. The set of scenarios considered is from the fourth situation.](image)

Considering all scenarios, two distinct optimal meter configurations were found. Similarly to the situation with generated data, the majority of scenarios chose an optimal meter configuration that coincided with the compromise solution leading to a null error variation. The remaining did not choose an optimal meter configuration that surpasses a 10% variation. These results are shown in Figure 15.

![Figure 15: Distribution of the individual optimal meter configurations, in percentage, according to their error index variation. Error variations are referred to the error index found for the compromise meter solution. The set of scenarios considered is from the fourth situation.](image)
The main reason for the similarity between this situation and the situation with generated data is the low load and DER volatility, as it leads to a smaller number of possible optimal meter configurations, whose differences do not significantly increase the error index.

VI. Conclusions

This thesis proposes a MILP approach that is able to mitigate the impact of load and DER volatility on SE of distribution networks. This approach improves SE accuracy by providing globally optimal meter placement solutions that take into account uncertainty as expressed by multiple load and DER profile scenarios, this way mitigating the impact of net load volatility.

The results obtained show that, if one is to consider only one scenario for obtaining the meter placement solution, when faced with a distinct scenario, the SE standard error can become significant. Furthermore, the results also show that a compromise meter solution can be found to efficiently deal with scenario variations, mitigating the effects on SE accuracy of the intrinsic volatility of distribution networks loads and DER. Accordingly, the use of a compromise meter solution will become growingly important over time in enhancing SE accuracy, as the reality of distribution networks may change from the second or fourth situation to a situation similar to the third, where there is high load and DER volatility.

References


