Sensor-Based Cooperative Control of Multiple Autonomous Marine Vehicles

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Abstract

This work focuses the control of multiple autonomous marine vehicles acting in cooperation, using so-called cooperative control systems. With this objective in mind, we study individual vehicle control algorithms that serve as a basis for the later study of cooperative control techniques in which vehicles exchange relevant information via acoustic communication networks. We present an innovative method that allows a group of vehicles to perform cooperative path following with respect to a path defined in a moving reference frame. The introduction of a coordination controller, on an outer loop, guarantees convergence to the desired formation which, in the context of the present work, corresponds to an equally spaced distribution over the path. Other formations can be achieved by simply adjusting a path parameter. To reproduce the scenario of an actual mission to the full, the robustness of the solution against ocean currents is analyzed and, a control law with current compensation is provided. Simulations (in *MATLAB*), considering the MEDUSA class vehicle model, are provided to validate the efficacy of the proposed control systems.oe

Keywords: Autonomous marine vehicle motion control, Cooperative control, Path following

1. Introduction

From an early age, the human being showed interest in exploring the ocean environment. In recent years technological progress has led to major developments in the area of autonomous marine vehicles, which are the most viable option in the study of the oceans and its living or non-living resources. Enormous efforts have been conducted over the last decades to develop more sophisticated and robust control systems for single vehicle motion control. More recently, more challenging problems in the area of cooperative motion control have been receiving worldwide attention.

An example of a project in this area is GREX [2006-2009] [3], entitled *Coordination and Control of cooperating heterogeneous unmanned systems in uncertain environments*. Both theoretical an practical issues were addressed in the scope of the project. One of the main goals was to create a conceptual framework and middleware to coordinate a group of diverse, heterogeneous physical objects working in cooperation to achieve a defined practical goal in an optimized manner. CO^3-AUVs [2009-2012] [11], entitled *Cooperative Cognitive Control for AUVs*, aimed at the development, implementation and test of advanced cognitive systems for coordination and cooperation between multiple AUVs. At the time, the state of the art was dominated by single AUVs limited to open-sea preplanned trajectories. During the project the cooperation between AUVs and humans was also addressed, as one of the research topics was the use of AUVs to perform functions as companion and support platforms during scientific and commercial dives. In the scope of MORPH [2012-2016] [5], entitled *Marine Robotic System of Self-Organizing, Logically Linked Physical Nodes*, also addressed the problem of cooperative control. Groups of AUVs were required to operate in areas with low visibility, and unknown obstacles, where a single vehicle would have very limited capabilities. During the project a group of vehicles worked on a vertical cliff in cooperation, working as a large virtual vehicle. More recently, the WiMUST [2015-2018] [4], entitled *Widely Scalable Underwater Sonar Technology*, aimed at the development of advanced cooperative navigation and control systems for groups of autonomous, as a mean to fully automate geotechnical surveys at sea. It is against the above background of ideas, namely in what regards the objectives set forth in the scope of the WiMUST project, that in this paper we address the problem of cooperative motion control with a view to enable groups of marine vehicles perform increasingly demanding scientific and commercial missions. To this end we study an interesting problem that is an extension of PF. In the new set-up, a group of vehicles are requested to converge to and follow a desired path while maintaining a desired formation along the path, the latter undergoing translational motions in response of the motion of the target to be tracked. See for example [1] and [2] for an introduction to this problem, therein referred to as the the problem of circular formation control for cooperative target tracking. In the results obtained in T. Oliveira [8] the author used the parallel transport frame to derive a control law to ensure that an unmanned aerial vehicle tracks a target on ground. It must be stressed that in [2] the formation control is done based on a exosystem that generates relative reference positions for the vehicles to track. This paper introduces a method with similarities with the moving path following methodology in [8], but: i) it is very easy to implement using inner-outer structure and, ii) allows cooperation between the trackers and moving target.

The results of the work exposed in [10] and [6] on basic vehicle modeling and path following are used as stepping stones towards the final goal of deriving efficient cooperative path following algorithms, including an algorithm for cooperative target tracking.

2. Vehicle Model

Before obtaining the vehicle model, it is common practice to define two reference frames and the notation used. The *Inertial Reference Frame*, $\{U\}$ is composed by the axes $\{x_U,\, y_U,\, x_U\}$ and the *Body Reference Frame*, $\{B\}$ is composed by the axes $\{x_B,\, y_B,\, x_B\}$. The frame $\{U\}$ can be placed on any place on Earth, the frame $\{B\}$ is, for simplicity, fixed to the vehicle's center of mass, which means that its axis coincide with the principal axes of inertia. We use the nomenclature defined by the SNAME for treating motion of a submerged body through a fluid to represent the vehicle's position and orientation. Regarding this work, the vehicle will operate in the xy plane. This assumption reduces the number of Degrees of Freedom (DOF) to three, since $\phi = 0$, $\theta = 0$ and $z = 0$. Let $\mathbf{p} = [x, y]^T$ be the position vector, ψ the heading angle and $\boldsymbol{v} = [u, v]^T$ the velocity vector. With this notation the kinematic equations are given by

$$\dot{\mathbf{p}} = \mathrm{R}(\psi)\boldsymbol{v}. \tag{1}$$

We can also apply these conditions to the vehicle's dynamics, neglecting roll, pitch, and heave motion. In this situation, the dynamic equations involving u, v, and r are described by

$$\begin{aligned} m_u\dot{u} - m_v vr + d_u u &= \tau_u \\ m_v\dot{v} + m_u ur + d_v v &= 0 \\ m_r\dot{r} - m_{uv} uv + d_r r &= \tau_r, \end{aligned} \tag{2}$$

where τ_u represents the forward thruster force, τ_r represents the thruster torque around the z-axis, and

$$\begin{aligned} m_u &= m - X_{\dot{u}} \\ m_v &= m - Y_{\dot{v}} \\ m_z &= I_z - N_{\dot{r}} \\ m_{uv} &= m_u - m_v \end{aligned} \qquad \begin{aligned} d_u &= -X_u - X_{u|u|}|u| \\ d_v &= -Y_v - Y_{v|v|}|v| \\ d_r &= -N_r - N_{r|r|}|r| \end{aligned} \tag{3}$$

where m_u, m_v, m_z and m_{uv} represent mass and hydrodynamic added mass and d_u, d_v and d_r are hydrodynamic damping effects.

3. Motion Control

The Path Following (PF) problem can be briefly described as that of affording a vehicle the capability to follow a specified spatial path, without explicit temporal constraints. Inspired in the work in P. Maurya et al. [7] we adopt an inner-outer loop control structure. At the inner loop, the objective is to design a heading controller for the heading angle to track a reference ψ_{ref}. By examining (3), we conclude that the model is not linear, meaning that it is not possible to apply linear control designs tools to obtain the heading controller. We therefore linearize the model about the vehicle's operation conditions, defined by $v \approx 0$ and $r \approx 0$. This assumption allows us to rewrite some parcels from (2) which, in turn, allows to obtain a linear model for the vehicle. Let $\boldsymbol{x} = [\psi, r, v]^T$ be the system's state, $y = \psi$ be the system's output and $u = \tau_r$ the system's input. With this notation we define the linear model as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{N_r}{m_r} & \frac{m_{uv}}{m_r} \\ 0 & -\frac{m_u}{m_v} & \frac{Y_v}{m_v} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ \frac{1}{m_r} \\ 0 \end{bmatrix} u \\ \mathrm{y} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}. \end{aligned} \tag{4}$$

We adopt the state variable model presented in (4) to design the heading controller. We start by projecting a regulator that stabilizes the state variables at the origin, using the Linear Quadratic Regulator (LQR). Since the objective is to design a heading controller that makes the vehicle's heading track a desired reference, we will later modify the feedback scheme to obtain a servomechanism that allows tracking heading references. Let $\tilde{\psi} = \psi - \psi_{ref}$ be the heading error that we want to make converge to zero. Let $J = \int_0^{+\infty} (\boldsymbol{x}^T Q \boldsymbol{x} + u^T R u) dt$ be the quadratic cost function to be minimized, where $\boldsymbol{\eta} = [\frac{1}{\psi_{\max}}, \frac{1}{r_{\max}}, \frac{1}{v_{\max}}]$, $Q = \boldsymbol{\eta}^T \boldsymbol{\eta}$ and $R = \frac{1}{\tau_{\max}}$. The matrices Q and R are defined so that we normalize all variables according to the maximum values of their absolute values. If the pair (A, B) is controllable and the pair (A, C) is observable, the solution for the LQR problem exists, is unique, and is given by $u = -K\boldsymbol{x}$, with $K = R^{-1}B^T P$, where P

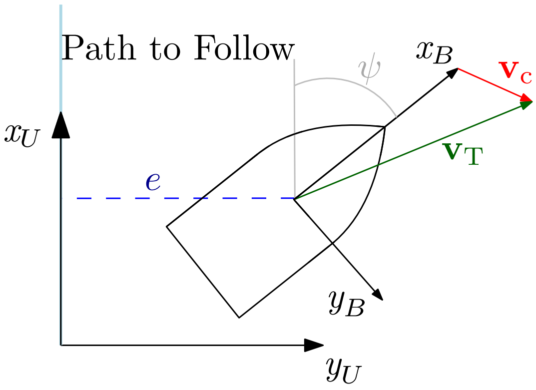

Figure 1: P. Maurya concepts and nomenclature

is the only positive-definite solution of the algebraic Riccati equation. From the above regulator structure, a servomechanism is obtained by introducing the heading error and computing u_d as

$$u_d = -\, k_p\tilde{\psi} \,-\, k_d\dot{\psi}, \tag{5}$$

where $u_d = \tau_r$ represents the differential mode of the thrusters and k_p, k_d represent the proportional and derivative gain, respectively.

In PF we must take into account that the vehicle is required to follow a path with a specific speed profile without explicit temporal restrictions. By going back to (2) and to (3), we can neglect the vehicle's accelerations over the x-axis of the Body Frame. Assuming that there is no velocity in sway and after some manipulations, we obtain an equation that connects the desired speed u^* with the forward thrusthers' force τ_u, given by

$$\tau_u = u^*\big(- X_u - X_{u|u|}|u^*|\big). \tag{6}$$

3.1. Path Following Guidance Law

The outer loop, often times called guidance loop, will provide a heading reference for the inner loop, based on the desired path and the vehicle's position. We consider a path defined by successive waypoints and that a straight line connects two consecutive waypoints. Let $e = -\sin{(x - x_k)} + \cos{(y - y_k)}$ be the cross-track error (the distance between the vehicle and its projection in the path), where x_k and y_k are the coordinates of the last waypoint which the vehicle has already passed. The objective is to generate the heading reference so as to guarantee convergence of e to the origin. We base our work in the results obtained in P. Maurya et al. [7]. We start by considering a straight path aligned with the x-axis of the Inertial Frame. The concepts used by the P. Maurya's guidance law are captured in figure 1. The ocean current velocity vector is represented by $\mathbf{v}_\mathrm{c}$ and the total velocity vector by $\mathbf{v}_\mathrm{T}$. By assuming that the side-slip angle is zero and the absence of ocean current component in the inertial y-axis v_{cy}, we get a simplified case, and we can write

$$\dot{e} = u\sin{(\psi)} = uU. \tag{7}$$

By making the observation that the variable U should be free to manipulate and by setting $U = -k_1 e$ it is trivial to show that we can ensure convergence of the cross-track error to the origin. However, the possible existence of a bias v_{cy} motivates the introduction of an additional integral term of the cross-track error to the virtual input U, that can be rewritten as

$$U = \frac{1}{u}\big(- k_1 e \,-\, k_2 \int_0^t e d\tau\big). \tag{8}$$

By replacing the previous equation on the dynamics of e, (7), we get a equation that characterizes a second-order system and, the gains k_1 and k_2 should be chosen to obtain the desired natural frequency and damping factor. Taking into consideration the error dynamics, (7), that the variable U should be free to be manipulated, and the intended expression for U, it becomes clear that the desired angle must be of the form $\psi_d = \sin^{-1}\big(\mathrm{sat}(U)\big)$. Thus, the virtual control law can be written as

$$\psi_d = \sin^{-1}\Big(\mathrm{sat}\big(- \frac{k_1}{u}e - \frac{k_2}{u}\int_0^t e d\tau\big)\Big), \tag{9}$$

where $\mathrm{sat}\big(\,\cdot\,\big)$ is the saturation function limited to the range $[-1, 1]$. As mentioned in [7] it is possible to show using Lyapunov-based analysis, that the non-linear control law yields convergence of the e to zero if the actual vehicle heading equals the desired heading reference ψ_d. A path is defined as concatenations of segments of straight lines and circumferences. The P. Maurya's guidance law was designed for straight paths. For a curved path, with constant curvature, the guidance law guarantees convergence of e to the origin if the circumference radii is not too small.

3.2. Ocean Current Estimation

There is no sensor capable of providing directly the inertial velocity of the water. This can only be done indirectly. For example, at the surface, the water velocity can be obtained using a Doppler velocity log (DVL) and GPS. We base our work on the results obtained in Oliveira et al. [9]. The problem consists in that of estimating the ocean current based on the measurements $\mathbf{p}_m$ and $\mathbf{v}_m$of the vehicle's position and velocity with respect to the water, respectively. Considering the existence of ocean currents, the kinematic equations can now be written as $\dot{\boldsymbol{p}} = \mathrm{R}(\psi)\boldsymbol{v} + \mathbf{v}_\mathrm{c}$. The process model

$\mathcal{M}_{pv}$ is given by

$$\mathcal{M}_{pv} = \begin{cases} \dot{\mathbf{p}} = \mathbf{v}_m + \mathbf{v}_c \\ \dot{\mathbf{v}}_c = 0 \\ \mathbf{p}_m = \mathbf{p} \end{cases} . \tag{10}$$

Given the above process model it is possible to develop a complementary filter with the realization

$$\mathcal{F} = \begin{cases} \dot{\hat{\mathbf{p}}} = k_3(\mathbf{p}_m - \hat{\mathbf{p}}) + \mathbf{v}_m + \mathbf{v}_c \\ \dot{\hat{\mathbf{v}}}_c = k_4(\mathbf{p}_m - \hat{\mathbf{p}}) \end{cases} , \tag{11}$$

where k_3 and k_4 are gains, and $\hat{\mathbf{p}}$ and $\hat{\mathbf{v}}_c$ represent the position estimates and current velocity estimates, respectively. In [9], the authors showed that for $k_3, k_4 > 0$ the above filter is asymptotically stable.

4. Multiple Vehicle Motion Control

The key to achieve synchronization is to correctly parametrize each path, so that if the synchronization parameters of the vehicles achieve consensus, then the vehicles are in the desired formation. We start with the analysis of a case considering a formation required to maneuver side-by-side along parallel lines. In this scenario the synchronization parameter γ used is the distance travelled in the path by the projection of each vehicle S, that is $\gamma_i = S_i$, where the subscript i denotes an arbitrary vehicle in the formation. The coordination error $\gamma_{i,j}$ is defined as the difference between the synchronization parameters, that is

$$\gamma_{i,j} = \gamma_i - \gamma_j, \tag{12}$$

The vehicle's speed can be represented using this parameter, that is

$$\frac{\partial \gamma_i}{\partial t} = v_r, \tag{13}$$

where v_r is the desired speed for the formation. Once we achieve coordination between vehicles, meaning $\gamma_{i,j} = 0$ we can make the speed of all vehicles equal to v_r.

For segments of circumferences, the along-path length cannot be used as a parameter. An example is a scenario containing two circular paths with different radii, where two vehicles are required to maintain the radial alignment. Since the two circumferences have different radii, the perimeters will differ. This means that when the vehicles are synchronized they will travel different distances, thus the need to define a different synchronization parameter. The goal is to drive $\theta_{i,j}$ to zero, where $\theta_{i,j}$ is the "angular" distance between the vehicles. We can do an equivalent process by normalizing the the length of the circumference to 2π. To do so, we divide the along-path length by the circumference radius, R to obtain

$$\gamma_i = \frac{S_i}{R_i}, \tag{14}$$

where S denotes the arc length given by $S_i = R_i\theta_i$, where θ is the central arc of the angle expressed in radians. Thus, we get a good measure of the coordination error, defined in (12). By making $\partial\gamma_i/\partial t = v_r$ we obtain the same speed profile in all γ_i coordinates. In fact recalling the derivative of composition function yields

$$\frac{\partial \gamma_i}{\partial t} = \frac{\partial \gamma_i}{\partial S_i}\frac{\partial S_i}{\partial t} = v_r. \tag{15}$$

Thus, we conclude that the inertial speeds scale naturally and proportionally as

$$\frac{\partial S_i}{\partial t} = R_i v_r, \tag{16}$$

as desired.

For situations where the formation is not characterized by the vehicles navigating side by side, the path parametrization is adjusted, by performing a shift in the coordination parameter for curvature paths and by adding an offset for straight paths. This adjustment must ensure that the coordination error $\gamma_{i,j}$ is zero when the vehicles are in the desired formation.

We use a graph to represent the inter-vehicle communication network, where the vehicles are represented as nodes and the communication links are represented as edges. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represent an undirected graph with the node set $\mathcal{V} = \{1, 2, \cdots, N\}$, where N is the number of vehicles involved, and with the edge set $\mathcal{E} = \{\{(v_i, v_j) \in \mathcal{V} \times \mathcal{V}\}\}$ where the edge (v_i, v_j) exists only if vehicle i communicates with vehicle j. We define a square matrix, $\mathrm{A} = \left(a_{ij} \in \Re^{N\times N}\right)$, whose elements indicate whether two vehicles are adjacent or not. By resorting to graph theory, it is possible to define the degree matrix D for the graph $\mathcal{G}$, indicating the number of communication links associated with every vehicle. From A and D we can define the normalized Laplacian matrix $\mathrm{L_D}$ as,

$$\mathrm{L_D} = \mathrm{D}^{-1}\big(\mathrm{D} - \mathrm{A}\big). \tag{17}$$

The matrix $\mathrm{L_D}$ from (17), will be used later to design a coordination controller.

Considering the above graph representation, it remains to define a coordination controller that, in thew presence of continuous communications, ensures convergence of the vehicles to the desired formation. The key idea is to add a correction term to the reference speed of each vehicle. Thus, the speed reference provided to the speed law in (6) is equal to the sum of the reference speed for the vehicle with the speed correction, that is,

$$\boldsymbol{v}_d = \boldsymbol{v}_r + \boldsymbol{v}_c, \tag{18}$$

where $\boldsymbol{v}_d$ is the desired speed. The correction term should be bounded to prevent the desired speed from reaching negative values as well as high values. Inspired by the work in F. Vanni [12] we introduce a control law defined by

$$\boldsymbol{v}_c = -k_e \tanh\big(L_D \boldsymbol{\gamma}\big), \tag{19}$$

where $k_e > 0$ is the bound for the correction term. The desired speed for each vehicle is given by

$$v_{d_i} = v_{r_i} - k_e \tanh\big(\gamma_i - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \gamma_j\big) \tag{20}$$

where $\mathcal{N}_i$ denotes the vehicles in the neighborhood of vehicle i. The use of the hyperbolic tangent ensures the boundedness of the correction term. The proof that the speed v_d converges asymptotically to v_r is made in the mentioned work by assuming that the graph is connected. This, in turn, implies that the Laplacian L_D observes some conditions that are required in the proof of convergence. Full details are available in [6].

5. Target Encircling

We now introduce two different control structures that allow a set of agents to perform an encircling maneuver around a moving target. We start with the analysis of a control structure based on the results in L. Arranz [2], that uses the principles of trajectory tracking. Later we design an innovative control law that solves the PF problem on a local reference frame and later the local guidance law is transferred to the Inertial Frame.

5.1. Trajectory Tracking

In the original design proposed in [2], the authors proposed a decentralized control structure that allows each vehicle to take its independent decisions. The strategy relies, tightly, in the generation, by an exosystem, of coordinated relative positions that are tracked over time by the vehicles, so that the vehicles converge to a encircling trajectory around the target. Nevertheless, the exosystem is a complex design that requires a model dynamics and a circular control law to create a virtual vehicle whose motion converge to a circular trajectory around the origin. The headings of the virtual vehicles are exchange through the communication network to ensure synchronization of the virtual vehicles (using a potential function that introduces a repulsion force in the virtual vehicles). Considering the circular path intended, the reference positions can be generated without the need of such complex design. We resort to trigonometry to generate the reference positions, that are initialized over the circular trajectory, without the need to converge to the circular trajectory. Consider a circumference with radius R centered at the origin, its coordinates are given by

$$\big[x, y\big] = R\big[\cos(\theta), \sin(\theta)\big]. \tag{21}$$

To obtain the desired speed profile, the angle θ should evolve in time according to

$$\theta = \omega t \tag{22}$$

where ω is the angular reference speed for the formation. Yet to be defined is a mechanism that replaces the potential function and equally spaces the vehicles along the circumference. When synchronized, the reference positions for two consecutive vehicles have a "angular" distance of $\frac{2\pi}{N}$. To achieve this spacing, we resort to the phase shifting of the trigonometric function, i.e., we introduce an offset in the trigonometric function, that depends on the vehicle number. The reference relative position can thus be defined as

$$\begin{aligned}\mathbf{p}_i^* = R\Big[&\cos\big(\omega t + 2(i-1)\frac{2\pi}{N}\big),\\ &\sin\big(\omega t + 2(i-1)\frac{2\pi}{N}\big)\Big]\end{aligned} \tag{23}$$

where $\mathbf{p}^*$ is the relative position reference vector. The previous law replaces the exosystem in the original control design. By tracking the reference positions, the vehicles converge to the desired formation and perform the desired maneuver along the moving path.

However, the reference positions are being generate independently inside the exosystem without taking in consideration the vehicle's status, i.e., its capability of tracking its reference. Because of this, in the presence of a malfunction that prevents the vehicle from tracking its reference, no compensation is provided when generating the reference positions. Trying to overcome this phenomenon, we performed a series of simulations using the original exosystem, but using the real vehicles' heading inside the potential function. Apart from a small difference in the initial behavior, the steady-state results are approximately the same. However, it must be stressed that this experiments are not legitimized by a solid theoretic analysis, and its efficacy is not guaranteed in mission applications.

5.2. Cooperative Moving-Path Following

Throughout this section we will design an innovative control law for target encircling purposes, whereby a group of vehicles is required to converge to a closed path and follow it at a desired speed, adopting a desired geometric configuration with respect to the path, while the center of the latter undergoes motions to pursue a target. We address the

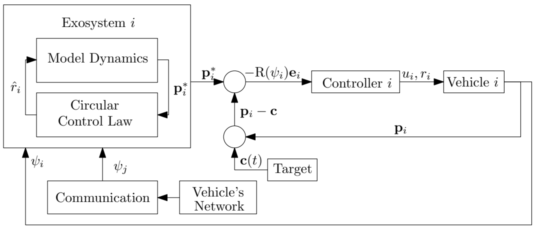

Figure 2: Control design using a decentralized exosystem with vehicle's heading feedback ψ_i to the exosystem

scenario where the geometric path has both linear and angular velocities. At the target's center we fix a frame named $\{P\}$. The position of the origin of $\{P\}$ is coincident with the target's center, is represented by ${}^U\mathbf{p}_{\mathrm{o_p}}$. Defined in $\{P\}$ is the path intended for the vehicles, denoted $\mathcal{P}$. It is essential to note that $\mathcal{P}$ is rigidly attached to $\{P\}$, which means that $\mathcal{P}$ is rigidly attached to $\{P\}$. The linear and angular velocities of $\mathcal{P}$ are represented as $\mathbf{v}_P$ and ω_P, respectively. The setup for the moving path following (MPF) is represented in figure 3. As

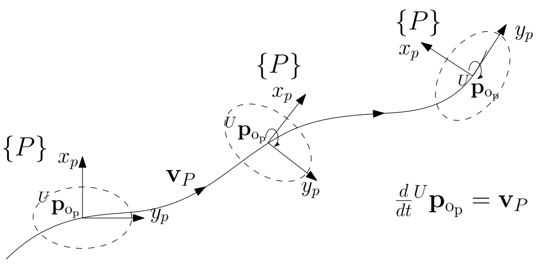

Figure 3: Moving-path following set-up considering a rotating path

a first step, we focus on the frame $\{P\}$ and solve the PF problem in $\{P\}$, that is, in local coordinates. The solution will then be transferred to the inertial frame. We formulate the problem considering a single vehicle. Let ${}^P\mathbf{p}$ denote the position of the vehicle in $\{P\}$. The objective is to derive a local guidance law for the vehicle's velocity in $\{P\}$, ${}^P\mathbf{v}$ of the form ${}^P\mathbf{v} = {}^P\mathbf{v}\big({}^P\mathbf{p}, \mathcal{P}\big)$, meaning that the vehicle computes an instantaneous velocity vector in the local coordinates that is a function of the position of the vehicle in $\{P\}$, the geometry of $\mathcal{P}$ and the desired speed profile along $\mathcal{P}$. For the local guidance law we adopted the P. Maurya's guidance law, presented in (9). The inertial position if a vehicle can be expressed as the sum of the position of the origin of $\{P\}$, ${}^U\mathbf{p}_{\mathrm{o_p}}$, with the position of the vehicle in $\{P\}$, ${}^P\mathbf{p}$, properly multiplied by the rotation matrix from $\{P\}$ to $\{U\}$, that is

$$ {}^U\mathbf{p} = {}^P\mathbf{p}_{\mathrm{o_p}} + {}^U_P\mathrm{R} \cdot {}^P\mathbf{p} \tag{24} $$

By computing the derivative of both sides of equation (24) we obtain the equation for the global guidance law given by

$$ {}^U\mathbf{v} = \frac{d}{dt}{}^U\mathbf{p} = \mathbf{v}_P + {}^U_P\dot{\mathrm{R}} \cdot {}^P\mathbf{p} + {}^U_P\mathrm{R} \cdot {}^P\mathbf{v} \tag{25} $$

The derivative of the rotation matrix can be simplified using some properties of the skew-symmetric matrix and the cross product of a skew-symmetric matrix. Thus, equation (25) is rewritten as

$$ {}^U\mathbf{v} = \mathbf{v}_P + {}^U_P\mathrm{R} \cdot \omega_P \times {}^P\mathbf{p} + {}^U_P\mathrm{R} \cdot {}^P\mathbf{v} \tag{26} $$

In (26) is defined the global guidance law so steer the vehicles to a moving path. In mission there exist often external disturbances such as ocean currents. Thus, and considering that the currents cause a misalignment between the heading angle and the course angle, in (26) we subtract the values of the estimations,$\hat{\mathbf{v}}_{\mathrm{c}}$, provided by (11) to compensate the existence of ocean currents. Taking into consideration that the initial distance between the vehicles and the target is corrected by action of the local guidance law that generates ${}^P\mathbf{v}$ and that this distance may reach high values we introduce a virtual target. This virtual target is initialized at the mean initial position of the vehicles. Fixed on the virtual target's position is the frame $\{VP\}$. In frame $\{VP\}$ we define a copy of $\mathcal{P}$ denoted $\mathcal{VP}$. Instead of formulating the problem in $\{P\}$ we redone the derivation considering that the vehicles are require to follow $\mathcal{VP}$. To avoid redundant derivation we will take the global guidance law from (26), and rewrite it to the situation where the path following task is formulated in $\{VP\}$,

$$ {}^U\mathbf{v} = \mathbf{v}_{VP} + {}^U_{VP}\mathrm{R} \cdot \omega_{VP} \times {}^{VP}\mathbf{p} + {}^U_{VP}\mathrm{R} \cdot {}^{VP}\mathbf{v} \tag{27} $$

The guidance law from (27) guarantee the convergence of the vehicles to $\mathcal{VP}$. In order to make the vehicles converge to $\mathcal{P}$ we will move the virtual target so that $\{VP\}$ converges to $\{P\}$. We design a feedback control law that adds a correction term to the virtual path reference frame velocity given by

$$ \mathbf{v}_{\mathrm{VP}} = \mathbf{v}_{\mathrm{P}} + \mathbf{v}_{\mathrm{corr}} \tag{28} $$

where $\mathbf{v}_{\mathrm{VP}}$ is the velocity of the origin of $\{VP\}$, $\mathbf{v}_{\mathrm{P}}$ is the velocity of the origin of $\{V\}$ and $\mathbf{v}_{\mathrm{corr}}$ is the correction term given by

$$ \mathbf{v}_{\mathrm{corr}} = K \tanh\left(\mathbf{p}_{\mathrm{o}}^{P} - \mathbf{p}_{\mathrm{o}}^{VP}\right) \tag{29} $$

where K is the gain of the correction term.

The controller from (29) directly synchronizes the positions of $\{VP\}$ and $\{P\}$, indirectly synchronizing its velocities. $\{VP\}$ and $\{P\}$ are initialized with the same angular orientation and $\omega_{VP} = \omega_P$.

The use of $\{VP\}$ is not mandatory, and the problem could be formulated directly on the path reference frame, without the need of the controller from

(29). Nevertheless, its use allows a smoother approach to the path. The vehicles are performing the desired maneuver while converging to the target.

At the implementation level each vehicle must perform the following steps:

1. Compute its position ${}^{U}\mathbf{p}$ in $\{U\}$
2. Compute its position ${}^{VP}\mathbf{p} = {}^{VP}\mathbf{p} - \mathbf{p}_{o}^{VP}$ in $\{VP\}$
3. Compute the local kinematic guidance law ${}^{VP}\mathbf{v} = {}^{VP}\mathbf{v}({}^{VP}\mathbf{p}, \mathcal{P})$
4. Transfer the local guidance law to the *Inertial Reference Frame*

To achieve synchronism among the vehicles performing the moving-path following, the coordination controller from (19) is added to the control algorithms so that the local guidance laws, ${}^{VP}\mathbf{v}$, is adjusted so that the vehicles converge to the desired formation. The absolute value of the local velocity vector is now equal to the formation nominal speed added the correction term. The orientation of the local velocity vector is given by the P. Maurya guidance law.

In this mission type, the target corresponds to the center of reflected beams. The target speed is equal to the velocity of the surface vehicles carrying the emitting devices. In situations where the vehicles are not capable of performing the desired maneuver, the target velocity should be decreased so that the vehicles have a higher chance of performing the maneuver. The parameter used to evaluate the vehicle's capability to perform the maneuver is the cross-track error. Its sign only indicates the relative position of the vehicle with respect to the path. Thus, it has no interest in this application. We use the absolute value of the cross-track error to act over the target velocity. The communication between the surface and the underwater formation is limited. Thus, the communication between formations is carried out by one vehicle of each formation. Besides its role in the maneuver it has the role of communicating with the other formation. At the surface vehicle, an auxiliary algorithm compares the cross-track error of each vehicle and if the maximum value is inside a the actuation region it communicates with the other formation so as to decrease the velocity of the vehicles carrying the emitting devices. A lower bound in the velocity is imposed to prevent the target velocity from being to low. Once all cross-track errors are outside the actuation zone the target velocitty is reset to the nominal speed.

All the control mechanisms necessary to steer a set of vehicles towards a moving path are now defined. The coordination controller ensures the coordination between vehicles by adjusting the vehicles' velocities.

6. Conclusions

A controller for cooperative moving path following was presented, by decoupling the overall objective into smaller control tasks. The communication network through which the vehicles exchange relevant information is considered in the coordination controller to adjusts the vehicles' speed according to its neighbors state. The simulation results look very promising and illustrate the effectiveness of the proposed control strategy. Future work include the implementation of the developed software in the robots to further test the performance of the control architecture.

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