CubeSat Structural and Thermal Analysis Methodology ISTSat-1 Design

Miguel Bruno Veludo Guedes
miguel.v.guedes@tecnico.ulisboa.pt

Instituto Superior Técnico, Universidade de Lisboa, Portugal

June 2019

Abstract

The launch of a satellite promotes extreme conditions of pressure and temperature which need to be considered and evaluated carefully, so as to minimize the risk of problems arising from it. The present work therefore looks to guarantee that a given design configuration of a satellite under development is compatible with the referred conditions, with possible changes at the hands of the analyst in an attempt to ensure it. This forces a detailed thermal analysis, where the various energy sources are considered in order to appraise the extreme temperatures reached by the various components, and a structural one, for the evaluation of the effects of the great accelerations felt by the satellite, through a static analysis, as well as its dynamic behavior, through a modal analysis, serving as the basis to predict its reaction to the vibrations felt during the launch window, through a random vibrations analysis. The need for a detailed analysis of the structure and its components arises, one that cannot be undertaken through simple hand calculations or analytical methods, requiring the use of numeric processes patented in CAE (Computer Assisted Engineering) software. The focus of the work at hand lies therefore in the presentation of results obtained from said methods.
Keywords: Static Analysis, Modal Analysis, Random Vibration Analysis, Transient Heat Transfer Analysis, Computer Assisted Engineering

1. Introduction

Space. The Final Frontier.

Many a science fiction media have used these sentences as a be all and end all moto of their more or less poignant story plot. As a way to provide a griping opening line, or who knows, maybe a devilishly clever turn of phrase with a hidden meaning, it justifies its mainstay in the mainstream media. But perhaps the most ingenious way to look at it is that, no matter the context, there always seems to be an interpretation that makes it a rather true affirmation.

In what pertains to this document, and its writer, space, in its obvious interpretation in lieu of the context of this work as *outer space*, is that which contains all that is known and all that isn't, an entity as old as time and as mysterious as its inception. This is where the fascination of Mankind with its place in the universe arises, its, as of yet, unknown fate and the fearsome realization that we are alone in this universe, or the equally frightening prospect that we may not be.

It is at this point that the matter of space exploration becomes an important part of the discussion, leading to the appearance of satellites, defined as objects that move in an orbit around a larger object than themselves. These run the gamut of sizes, with the larger ones taking precedence in the early stages of the field, however, as has been the typical path technology has adopted for the past couple of decades, so have satellites fallen prey to the latest tendencies of miniaturization, together with an opening of the marketplace for input from outside sources. The large structure and budgets required to accommodate the heavy and burdensome hardware gave way to smaller teams, with less expensive endeavors, and, ultimately, to small form-factor projects.

Even still, though the cost of satellite development has plummeted, sending it to space is an entirely different endeavor, leading space agencies all around the globe to provide launch services to consumer developed artificial satellites. But ensuring that any satellite could be launched would not be cost-effective, while its development costs are lower, the launch itself would need to be considered on a case by case basis, driving the prices back up again. The need for a standard arises, and the solution presents itself in the form of CubeSats.

Developed in 1999 in a joint effort between the

California Polytechnic State University and Stanford University, the CubeSat standard [1] defines dimensions, maximum mass boundaries, types of material to be used, venting areas, among others, leading to an unwavering envelope and satellites that only tend to differ drastically in their payload.

The potential these satellites provide from a learning perspective lead to the appearance of projects oriented towards universities, giving the opportunity to students of gaining hands-on experience with the development and validation processes of this type of hardware, chief amongst which is the *Fly Your Satellite!* (FYS) initiative, governed by the European Space Agency, where the context of the current work lies, the development of the ISTSat-1.

The ISTsat-1 is the first CubeSat being developed in Portugal, being the result of the dedication of several students and teachers that make up the multidisciplinary team based out of Instituto Superior Tcnico (IST). The primary payload carried is an Automatic Dependent Surveillance Broadcast (ADS-B) antenna and corresponding receiver, with the mission being to detect and receive broadcasts from commercial aircrafts for tracking purposes, a technology which has seen significant use in small satellites.

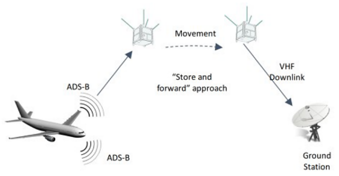

Figure 1: ADS-B Signal Reception [2]

The data obtained through this antenna is later expected to be analyzed and correlated with that attained through typical ground-based technology, and together with it, assess and characterize the *Cone of Silence*, volume of space where signals cannot be emitted to or received from due to the zenith null in the radiation pattern of a monopole antenna, and to measure performance parameters of the communication chain, particularly when it comes to *Probability of Target Acquisition, Probability of Detection and Probability of Identification.*

With the modularity that comes with the CubeSat standard, several companies have started to create parts for sale with this type of application in mind, everything from actual structures, to solar panels, batteries, antennas, among others, allowing for the teams to dedicate themselves to the primary mission and the required payload, ranging from well-defined systems implemented to run a particular function, such as signal detection and/or retransmission, to technological testing, such as new attitude control mechanisms, and even biological research, focused on data recovery.

The breadth of scenarios notwithstanding, the development campaign is universal to all teams involved in the FYS initiative, with changes being made with every iteration of the project in an attempt to reduce the, still, statistically high likelihood of mission failure. In the end, for acceptance of the satellite to later testing stages, a number of structural and thermal analyses need to be considered, and these studies are the main focus of the present document.

2. Finite Element Method

The development of an engineering project in the current days requires more than simple hand calculations, incapable of dealing with complex domains, enforced conditions and the need for accurate results for safety factor calculations, ensuring that the adverse conditions in place during the lifetime of said project do not affect its capability of seeing the main objective through. A numerical approach is therefore unavoidable, with several software packages available for the effect, the Siemens NX™, being the one used in the current work.

Following the old adage *garbage in, garbage out*, that typically makes its rounds on most endeavors that require human input, a minimal knowledge of the background of the processes involved is required, lest this input be less than optimal and the results therefore unusable. This section serves that purpose, presenting a brief and consciously instinctual overview of the Finite Element Method (FEM).

This method is a numeric approach used to attain approximate solutions to boundary value problems, also known as *field problems*, described in Reddy [3]. These problems consist in the determination of functions defining a set of variables that satisfy a group of differential equations inside a known domain, whilst being in accordance with specific states at its boundaries. Generally speaking, the method resorts to two elements, Governing Equations and Boundary Conditions.

2.1. Governing Equations and Boundary Conditions

The physical world abides by specific sets of rules, with everything from the behavior of a mass-spring system to the velocity profile of a fluid flow being dictated by Governing Equations, typically differential in nature.

Within the concept of differential equations various types can be distinguished, between ordinary or partial and scalar or vector in its dimension, and while ordinary equations can, for the most part, be solved with a certain degree of ease, most partial differential equations become either unsolvable or require far too arduous a methodology to be methodically used for solving non trivial systems with possibly random configurations. This emphasizes the need for an approximate solution, obtained through the use of Shape functions, N_i, polynomial functions that approximate the end result between nodal points and which, by virtue of their nature, are more easily computed.

$$u \quad \sum_{(i=1)}^{n} u_i \; N_i \tag{1}$$

The approximated nature of this method means extra care has to be taken to ensure the induced error is within acceptable parameters, which is why the original differential equation is morphed into the integral form, allowing this error, residual, to average to zero on the whole domain, instead of simply enforcing their inexistence at particular points in it. This formulation is also known as the Strong Form, or Method of Weighted Integrals.

$$\int_0^1 w \; R \; dx = 0 \tag{2}$$

Equation (2) can be solved through the use of the integration by parts method, at which point the requirements for the solution are reduced, the rigidity of the solution is alleviated, with the resulting form becoming laxer, taking therefore the name of Weak Form, whose manipulation into functionals dependant on different order time derivatives results in a matrix form reminiscent of the equation of motion of a damped mass spring system.

$$[M]f\ddot{u}g + [C]f\dot{u}g + [K]fug = fFg \tag{3}$$

The result of the majority of finite element problems can be summed up in this equation, with u defining the primary variable and each of the matrices presented representing a behavioral tendency of the system, briefly expanded upon below.

M - Mass or Inertia Matrix, defines the object's capability to resist change in its momentum as a function of time;

C - Damping Matrix, defines the damping of the system, its capability to flush kinetic energy as a way to reach stability;

K - Stiffness Matrix, defines the object's capability to resist direct changes to its primary variable, without time dependency, as opposed to the Mass Matrix;

F - Force Vector, defines the external solicitations imposed on the system.

The equation in matrix form having been defined, the Boundary Conditions become the last input set required before the mathematical solving process can begin in earnest. These conditions define the particular state of the system, in opposition to its behavior, like the Governing equation does, and when dealing with the Weak Form of the problem these can be categorized as either Essential or Natural.

Essential Boundary Conditions - Inputs that unequivocally define the primary variable value at particular points in the model geometry;

Natural Boundary Conditions - Sources of discontinuities of secondary variables within the model, such as stress or heat flow, or applied solicitations on the boundaries.

The overall equation already discussed, equation (3), is not necessarily used while considering all the matrices at stake. For instance, if a non-time varying solution is required then a simplification can be made, and this brings to light another possible distinction.

Static Analysis - The focus is on long term results with an initial transient response that eventually converges and stabilizes. In other words, after an arbitrarily long time, a static state is achieved, the result of interest. The Inertial and Damping matrices have no effect seeing as time dependent changes are inconsequential.

Dynamic Analysis - Contrastingly, the transient does not converge, usually due to non-static Natural Conditions applied. The time dimension becomes an inherent part of the solution, and therefore the Inertial matrix takes a relevant role, becoming indispensable in achieving a reasonable output, with the Damping one being optional.

The solution presented in this subsection refers to an overall process. The complexity of the equations, domains, applied conditions and problems in general, however, promote the need for a more practical approach, leading to the concept of *meshing*, a discretization of the domain in simpler elements as a way of both simplifying computational approach and improving solution accuracy by establishing more nodal points where the results are directly calculated.

2.2. Shape Functions and Element Matrices

The fact that a domain discretization is used for solving purposes means that, for each finite element in the mesh, individual shape functions and element matrices are required. Fortunately, due to the use of local coordinate systems and the constancy of material properties and overall formulations, the path to defining these properties follows a pattern.

The Shape functions, as previously alluded to, correspond to functions defined on a per element basis, chosen by the user, whose sum, weighted by the nodal values, approximate the distribution of the primary variable along the corresponding element. The standard approach taken by most software packages sees Shape functions defined as polynomials, helpful for a computational solver.

$$
\begin{gathered}
u_h(x) = \{1 \quad x \quad :::\} \begin{Bmatrix} a_0 \\ a_1 \\ ::: \end{Bmatrix} = f\,'gfag \\
\begin{Bmatrix} u_h(x_0) \\ u_h(x_1) \\ ::: \end{Bmatrix} = \begin{bmatrix} 1 & x_0 & ::: \\ 1 & x_1 & ::: \\ ::: & ::: & ::: \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ ::: \end{Bmatrix} = [C]fag \\
u_h(x) = f\,'g[C]^{\;1}fug \;) \;\; [N] = f\,'g[C]^{\;1}
\end{gathered}
\quad (4)
$$

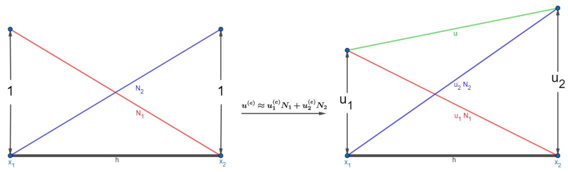

Figure 2: Two-Node Elements's Shape Functions

Equation (4) and Figure 2 represent the overall formulation for determining one-dimensional elements' Shape functions and a visual representation for illustrative purposes, respectively.

The Weak Form is obtained from the use of the Governing equation as a residual in the Strong Form and by approximating the primary variable through the Shape functions. As such, at this point, and assuming the Weight functions and the Shape functions to be the same, a typical approach, the element matrices can be obtained. Once again purely for illustrative purposes, an example formulation is presented, that of a bar under pressure, within the margins of a static problem, therefore allowing the Inertial and Damping matrices to be ignored, although they can be built using the same principles.

$$
\begin{gathered}
[K]fug = fFg \\
K_{ij} = \int E \; A \; N_i' N_j' dx \\
F_i = \int p \; N_i dx + P_1 \; N_i(x_e) + P_2 \; N_i(x_{e+1})
\end{gathered}
\quad (5)
$$

Of note is the fact that, in the absence of constraints to the system, the Stiffness matrix is singular, that is non-invertible, resulting in the impossibility of determining the actual individual displacements and only the overall length change. This comes about due to the fact that, without said constraints, the system is allowed to undergo rigid body motion.

These equations refer to a particular element, which means it provides a result for the case of a simple element representing the entirety of the domain. However, as previously alluded to, the main objective of using this process is to allow for the discretization of the domain, and with it the solving of far more complex systems. For this to work though, the element matrices need to be assembled in order to obtain a single matrix equation that can be solved for the entire model, so extra steps need to be taken to force correct connectivity between different elements, that is, nodes that are common to different elements need to ensure that the primary variable value is mirrored between them, seeing as no discontinuities may arise from the meshing. Also, unlike the nodal coordinates which are unique, the elements connecting different nodes can be of different natures or even be multiple in number, meaning multiple elements may impact the same entry in the assembled matrices.

Fortunately, the workarounds are simple operations, only requiring that the matrix entries involved in this calculation be correctly chosen as to preserve the local to global coordinates transformation, and ensuring they are processed as a sum of their contributions towards the displacement of the attached nodal coordinates.

2.3. Closing Remarks

The matter of the use of the Finite Element Method is a highly complex one, not to mention ever evolving. This preludes the fact that the examples generally presented for academic purposes correspond to those of extreme simplicity, modelled around one or two-dimensional elements with a distinctively discrete-like nature.

Nonetheless, even though these approximations are only exactly that, they are often precise enough to deal with the problem at hand, and this simplicity translates into the potential for easier mesh setup and, especially, vast improvements in computational speeds. As such, any part requiring discretization should be looked upon as potentially requiring only one or two dimensional meshes. The former type can often be represented by beam or bar elements, and the later by plates or shells, according to the problem requirements.

Of course, this is not always possible, and complex interactions can sometimes prevent this simplification, even in circumstances where the model itself is clearly skewed towards a lesser dimensional representation. This once again attest to the need for careful consideration of all possibilities with regards to any sort of analysis, with even the simplest having potentially unexpected pitfalls, which can only be circumvented through experience and validation, whether it is through real life testing or even through extra studies run on known or predictable results.

3. Structural Analysis

The fact that relatively few engineering projects in their final stages present drastic structural flaws relates to the use of software packages capable of presenting accurate results with which informed decisions can be made regarding materials to use or structural changes needed to ensure a sound structure. This section deals in precisely those types of analyses.

3.1. Static Analysis

The overall objective of a structural analysis being the verification of whether a structure can withstand a particular scenario, it is usual for the validation to hinge on how close to failure it comes. This depends mostly on the concept of stress, analyzed in Timoshenko [4]. Assuming small displacements, the strain can be calculated through the application of a differential matrix to the displacements themselves. On the other hand, the stress values result from *Hook's Law*, defining a material properties' matrix that relates strain to stress, equation (6).

$$f \quad g = [C] f \quad g \qquad (6)$$

This value of stress, however, is direction dependent, and therefore not particularly helpful in determining if the structure is in the imminence of failure. For this, the typical approach is the use of the von Mises criterion, with energy considerations at its source, that takes the stress tensor and provides a single value to be compared with the yield/ultimate tensile strength as required.

As for the analysis itself, with the objective of simulating the effects of the acceleration imposed on the structure during lift-off, a 19Gs acceleration is enforced in each direction of the satellite, as different load cases, required due to the fact that the orientation of said CubeSat is currently unknown. The Boundary conditions, on the other hand, are defined as fixed constraints on the bottom standoffs and slider constraints on the side frames, simulating the environment inside the launch pod. An example of the displacement and stress results are presented in Figure 3.

Regarding the stress values, a Margin of Safety (MOS) is expected to exist so that, as the name implies, for unexpectedly worse conditions than those predicted for use in the analyses, a fair margin for error is guaranteed, equation (7).

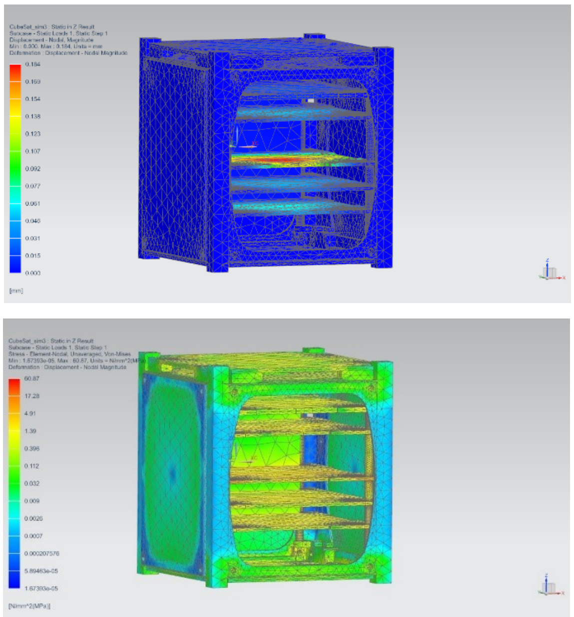

Figure 3: Displacement (Above) and Stress in Logarithmic Scale (Below) of Static Analysis

$$MOS = \frac{\text{design allowable stress}}{\text{design limit stress} \quad FOS} \quad 1 \qquad (7)$$

The FOS in equation (7) corresponds to a Factor of Safety, resulting from the multiplication of multiple coefficients accounting for unquantifiable variabilities, everything from model detail to project maturity, beyond the normal 1.25 value, typical in space related projects of this magnitude.

The maximum values obtained from all three directional analysis are presented for every material in use, Table 1. Due to the similarity of most of the components when it comes to material properties, a small amount of entries are registered, with Aluminum corresponding to the entirety of the structural frame, FR4 to the solar panels, antennas and printed circuit boards and the Polyethylene to the connectors between these last ones. Do note that A.S. and L.S. correspond to allowable and limit stress respectively.

Material	A.S. (MPa)	L.S. (MPa)	MOS
Aluminum	572	19	13.6
FR4	276	16	7.4
Polyethylene	40	3	5.5

Table 1: Margin of Safety Calculation for the Static Analysis

The margin of safety being positive indicates that the structure should support the launch acceleration.

3.2. Modal Analysis

The dynamic behavior of a system comes into play when the results are of the time dependent variant, as discussed in Rao [5]. This can, in very general terms, be divided in two different scenarios, based off of the loads applied being a constant fixture or only the inducing factor, leading to forced vibrations, the former, or free vibrations, the latter. The Modal analysis deals with the case of free vibrations, dictating the behavior of the structure when, after being forced into a vibrating state, it is left to reach equilibrium.

The typical assumption for a problem of this kind is that of the primary variable function taking the form of a harmonic solution, which allows for a simple derivation and linearization process to lead to the familiar form of an eigenproblem, where ω corresponds to the angular frequency assumed for the harmonic function and ϕ to its amplitude, present in equation (8).

$$([K] - \omega^2[M])\{\phi\} = 0 \tag{8}$$

Outside of the trivial solution corresponding to a zero displacement, the end result can be obtained by solving for the determinant of the matrix, leading to a number of different values, one for every degree of freedom in the system, each of which a so-called *natural frequency*, f_i, accompanied by the *modal shapes*, ϕ_i.

The modal analysis is extremely important to ensure that the structure remains far from any possibly long term applied load of frequency close to that of the fundamental one, first natural frequency, or a phenomenon known as resonance can kick in, whereas the natural damping of the structure reaches a minimum and the load provides an additive effect to the vibration of the structure, ultimately leading to failure.

The greatest contribution of the modal analysis, however, falls on the matter of the basis it provides for further dynamic analysis, both when it comes to informing the decisions of the analyst and reducing computational load, seeing as the deflected shape of a structure subjected to free or forced vibrations may be taken as a linear combination of the various modes.

The Modal analysis can also be used as a rigid body check, seeing as any unconstrained element of the mesh will be flagged by a near zero value for the first few respective fundamental frequencies, corresponding to the unconstrained degrees of freedom that a given part(s) of the model is(are) allowed to change in the capacity of a rigid body movement.

Regarding the analysis itself, just as in Aboobakar [6], the Boundary conditions defined correspond to those of a Hard-Mounted configuration, that is, besides the fixed constraint applied to the bottom stand-offs, the slider constraint defined in the static case is also changed to a fixed one. No Natural conditions are required given the free vibrational scenario.

The results of the Modal analysis are presented in the form of images representing the first couple modes of vibration, Figure 4, and a table presenting some of the lower calculated natural frequencies, Table 2.

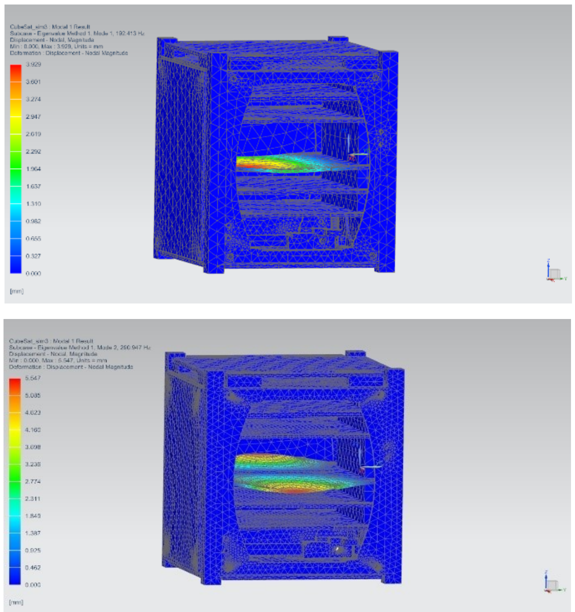

Figure 4: First Two Natural Mode Shapes

Mode	Frequency (Hz)
1	192.41
2	295.09
3	328.27
4	331.06
5	374.97
6	390.51
7	450.73
8	475.37

Table 2: First Eight Natural Frequencies of the Satellite

According to these results, the lowest frequency sits comfortably above the required 100Hz minimum resonant frequency required in hard-mounted configuration, meaning that resonance is unlikely to occur.

The Modal analysis also provides, per user request, a *modal effective mass fraction* output, in the form of a table of values, that correlates any given natural frequency with the mass of the model whose displacement it promotes, discretized in each of the six degrees of freedom.

3.3. Random Vibration Analysis

The Random Vibration analysis is a subset of the Frequency Response analysis type, which in turn is rather similar to that of the Modal analysis already presented, the biggest departures being the damping and the fact that the vibration is forced, in other words, instead of being allowed to respond naturally, the structure, or more particularly parts of the structure, are forcefully accelerated at a given frequency (or multiple). This means that the originally presented Weak formulation in its matrix form, equation (3), is the one in use, with all element matrices considered, and, like in the Modal analysis, the primary variables change follows the shape of a harmonic function.

At this point a choice is made, represented in most, if not all software packages, as different solutions, see Siemens NX [7]. The equation may be solved as is with the primary variable being derived twice and implemented in the equation of motion. This approach provides the most accurate result, although it is computationally much more expensive, and is aptly known as the *Direct Frequency Response*. On the other hand, the properties of the modal shapes may be used, resulting in the *Modal Frequency Response*. This is a much less expensive solution, which justifies the advantage of running a Modal analysis prior, as stated, but at the cost of some error. Full accuracy can only be reached if all modes are used for the approximation, which isn't viable seeing as they would number equal to the degrees of freedom of the mesh.

Unlike most dynamic problems, a Random Vibration, as the name suggests, is a type of non-deterministic vibration, as in, there is no way to describe it in an exact form as a function of time, leaving only a statistical approach. This type of analysis is fairly recent in so far as structural requirements go, and is of particular interest to areas that fall under the purview of aerospace applications, where somewhat random phenomena such as flutter and violent pressure changes from atmosphere crossing need to be considered in some way.

The probabilistic nature of this vibration may present itself in multiple ways, but the most important for its ease of use, and therefore how Random Vibrations are defined, is through the concept of stationary processes. These are independent of time from a statistical standpoint, which is to say that the probability distribution is unchanged throughout the time dimension. In a broad sense, this can be evaluated through the concept of autocorrelation, or similarity between different points in time of a single random process, equation (9)

$$R \quad = \lim_{T \; ! \; 1} \left(\frac{1}{T}\int_0^T u(t)u(t \qquad)dt\right) \tag{9}$$

Applying a Fourier Transform to the autocorrelation function, and taking the delay out of the equation while assuming a typically expected mean for a random process to be zero, the standard deviation, , of the process comes to light as the Root Mean Square (RMS), the square root of the arithmetic mean of a power based function, in this case $S(!)$, known as the Power Spectral Density (PSD), the power of a signal distributed over frequency.

$$R(0) = \quad ^2 = \frac{1}{2}\int_0^1 S(!)d! \tag{10}$$

This is when the results obtained from the Frequency Response analysis come to bear. The primary variables changes as a result of a given variable load having been charted as a function of frequency, a transfer function can be modelled for the whole spectrum of interest, $H(!)$. Also, any random vibration can, at this point, be defined statistically in the form of a PSD function, and it can be proved that the output follows this pattern as well, being defined by equation (11).

$$S_o(!) = jH(!)j^2 S_i(!) \tag{11}$$

The output is then obtained, and it can be analyzed in the form of the Power Spectral Density Function for the primary variable or, more likely considering the requirements, in the form of the RMS based on said function, allowing for components of stress and strain to be computed. The Root Mean Square, as stated prior, corresponds to the standard deviation of a statistical function, and therefore stands to reason that it doesn't encompass the worst case scenario values, only a fraction of the whole probability field, which can be estimated by following the assumption that the type of distribution at play is a normal one.

In the end, and as should be known from basic statistics, the standard deviation only accounts for around 68.3% of the probability field, therefore, to ensure that possible results arent being ignored, ones to which material properties should be reviewed against, the value obtained for any RMS variable should be further multiplied by 3, with it ensuring that the bases are covered for 99.7% of the time.

The analysis requires two specific inputs that differ from the previous examples, as defined in the current section, a deck of frequencies to use as the forced vibrations and the input PSD function. The modal effective mass fraction indicates that less than the typically required 80 to 90% of the model's mass is envolved, within the frequency range of consideration (20 to 2000Hz), meaning that all the natural modes within said range are used. Beyond it, the PSD function, Figure 5, is presented in $g^2{=}Hz$, so the Boundary condition defined as fixed in the

Modal analysis is now enforced with a 9.81Gs acceleration, ensuring unit cohesion.

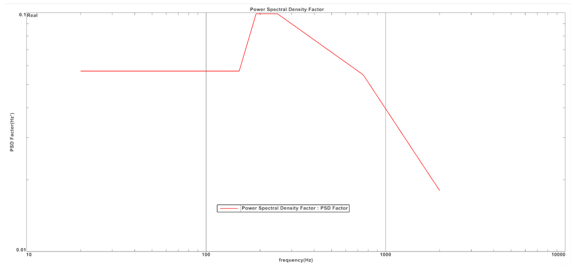

Figure 5: PSD Input in a Log-Log Scale

Otherwise, the results are presented in the same format as for the Static analysis already presented, with images for illustrative purposes, Figure 6, and a table with the stress maxima and corresponding Margins of Safety for the worst cases taken from the three directional studies, Table 3.

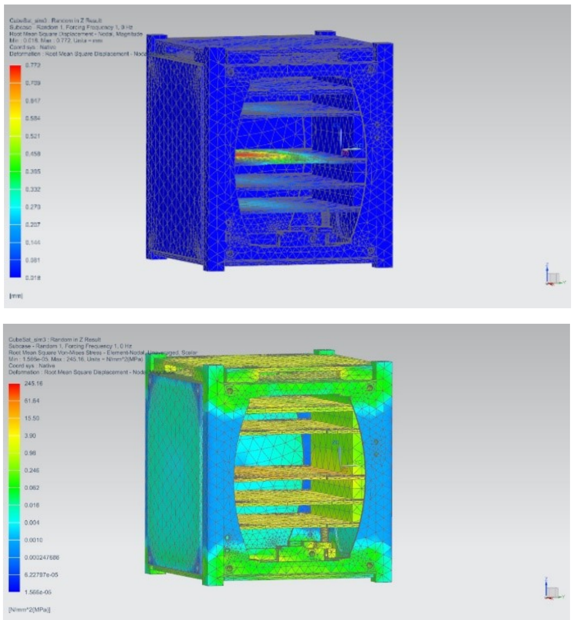

Figure 6: Displacement (Above) and Stress in Logarithmic Scale (Below) of Random Vibration Analysis

Material	A.S. (MPa)	L.S. (MPa)	MOS
Aluminum	572	48	4.7
FR4	276	39	2.4
Polyethylene	40	6	2.2

Table 3: Margin of Safety Calculation for the Random Vibration Analysis

Once again, the structure appears stable, considering the margins of safety are positive.

4. Thermal Analysis

The thermal profile of a project is often times overlooked, the focus being placed on the structural aspects, but its importance is paramount to a viable product, especially with the push for miniaturization, leading to a dense grid of electronic components that may source a substantial temperature increase due to power output. This section deals with the kind of analyses required for dealing with these issues.

4.1. Theoretical Background

The transmission of energy in its thermal form can be differentiated into different mechanisms, from Incropera [8], conduction, radiation and convection, although the latter is of no consequence for this particular type of satellite and is therefore ignored.

Conduction translates into heat flow within a stationary medium, where a temperature gradient leads to kinetic energy transfer between particles in said medium in a naturally stabilizing fashion, whose basic formulation, presented in equation (12), can be taken from *Fourier's Law*.

$$Q = -(k_x A_x \frac{\partial T}{\partial x} + k_y A_y \frac{\partial T}{\partial y} + k_z A_z \frac{\partial T}{\partial z}) \quad (12)$$

Radiation, on the other hand, corresponds to the transfer of energy emitted in the form of electromagnetic waves due to a non-zero temperature, not requiring matter to be present.

The flow of energy transferred through radiation is essentially a play on weighted components, with all the steps from emission, to absorption and back to emission being a chain of non-perfect processes where the efficiency is dictated by what are called optical properties of the material. The departure for this analysis centers on the concept of a black body, a theoretical entity, physically impossible, that absorbs all radiation that reaches it and emits the maximum amount of energy possible by unit area, dictated by the Stefan-Boltzmann's Law, a simplified version of which is presented in equation (13).

$$E_b = \sigma T^4 \quad \text{where} \quad \sigma = 5.670 \times 10^{-8} \quad (13)$$

This defining a perfect object with regards to radiation processes, both the emission and absorption of a real entity is treated as a fraction of this total, impact of which is safeguarded by the application of coefficients that dictate these fractions, known as optical properties. Furthermore, the emitted and incident radiations are mostly dependent on geometric properties, defined through the use of a View Factor.

The discussed concepts of conduction and radiation are applied to a finite element model, according to both the specifications of the project and the discretion of the analyst, resulting in equation (14).

$$[B]f\dot{T}g + [K]fTg + [R]fTg^4 = fPg \tag{14}$$

In this equation, the matrices [K] and [R] can easily be taken as representing the conduction and radiation respectively, with the vector fPg the heat loads, whilst the matrix [B], known as the heat capacity matrix, works analogously to that of the damping matrix in structurally oriented problems.

4.2. Transient Heat Transfer Analysis

The setup for the thermal analysis is vastly more complex than that required for the structural related ones, with all radiation sources and both radiation and conduction interfaces requiring individual attention, as presented in Chandrashekar [9].

Seeing as a steady state analysis doesn't provide actionable data, considering that a satellite never remains in the same place long enough for it to reach this state, such a simulation is foregone in substitution of a transient analysis, as presented through equation (14) defined with a convergence constraint based on cyclic orbit, meaning that it considers the solution as converged when the differences between orbits, from a temperature point of view, are less than 0.1C. The time dependent nature of some of the inputs, however, impede this convergence to be reached, and therefore a safety net of 16 orbits is established as the maximum.

The different external radiation sources, that is both into and out of the satellite, are accounted for by the use of the Orbital Heat and Radiation environment modules, responsible for calculating View Factors and applying them to solve for direct solar radiation, solar radiation reflected by the Earth, through albedo, Earth's infrared radiation and radiation emitted by the CubeSat, all of which as functions of position within orbit.

Within the satellite, the radiation and conduction between different components are defined individually, with the radiation between parts clearly not in view, or whose View Factor is predictably small, ignored in favor of those with obvious impact, such as in-between adjacent printed circuit boards, and the parts in contact having a particularly important setup process, with the definition of a coefficient limiting the heat flow through the conducting interface.

The thermal output of the electronic components are also of relative importance, being defined as distributed heat loads with variable output.

The methods having been touched upon, it is important to note that the variability of the mission conditions promotes the use of multiple mission profiles. These are chosen in order to predict the thermal behavior of the satellite for particular environment configurations. The Hot and Cold cases are self-explanatory, through which the worst case scenarios may be analyzed and changes to the systems be made, if required, the Nominal case should provide an idea of the baseline for the mission, the expected thermal distribution, and the Tumbling case allows for the study of the scenario where the satellite's attitude is not being controlled, an expected occurrence at the beginning of the mission, before the Attitude Determination and Control System kicks in, and an unexpected one afterwards.

The differences between these profiles fall to variations in particular parameters, mainly the orbit, influencing the sun flux, Earth's albedo and infrared radiation output, and different duty cycles for the various electronic components. Of particular interest for the simulation is the presence of heaters on the battery packs, connected to a sensor and endowed with a cut-off function, defining a narrow range of temperatures between which they are activated, a feature available on the software package. The battery heaters are nonetheless only expected to be relevant when considering the Cold case.

Unlike the structural considerations already discussed, the actual limitations to be imposed on the results are also more involved, mirroring the setup, with each different component presenting operational and non-operational temperatures, dictating, respectively, as the name suggests, the range between which the component needs to be for functions not to be impacted, and the range in which the component may be kept overall.

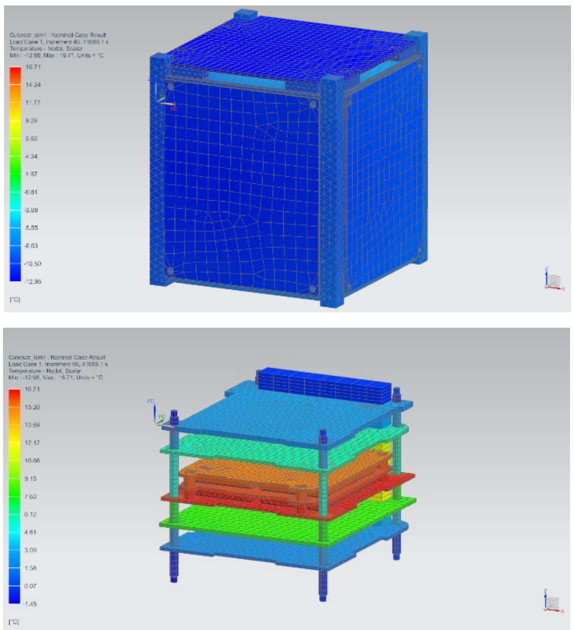

Figure 7: External (Above) and Internal (Below) Temperature distribution

The image presented prior, Figure 7, shows the typical output format through which the temperature distribution may be studied, and the table that follows, Table 4, provides actual values for the minima and maxima of temperature obtained for each of the components across all mission profiles. These are presented with a -/+10 C margin due to the sensitivity of the analysis, and the background color dictates the existence of these values within the operational range, green, the non-operational range only, yellow, or outside of both, red.

Part	Min. (C)	Max. (C)
Structure	-41.7	40.8
Solar Panels	-44.9	30
ADS-B Antenna	-37.4	19.3
UHF Antenna	-40.1	24.2
TTC	-22.3	29.1
OBC	-18.4	31.9
EPS	-14.4	41.5
COM	-17.7	34.9
PL	-22.6	30.1
Battery	-9.6	41.1

Table 4: Minimum and Maximum Temperature Values

An overview of the results returns the conclusion that the temperature values obtained for most of the parts are within the operational range, even with the uncertainties accounted for, and all without exception are within the respective non-operational ranges. The UHF Antenna is the only one where the operational range is not respected. Nonetheless, the operational range defined for this particular part is done so in respect to the processors responsible for the deployment action, the melting of the lines, which takes a few seconds at most. As such, and considering that in all mission profiles the antenna goes well within the operational range for considerable stretches of time, this is considered as a non critical issue and the satellite is taken as test ready.

5. Conclusions

The results provided in this body of work should prove that, from an analysis point of view, the design of the satellite is sufficiently robust to tackle the predicted structural and thermal environment variability. The choices, not presented here, regarding spacers materials and optical properties chosen for the structure, through the use of a particular finishing, in this case, black-anodization, were enough to clear all requirements and reduce the likelihood of any particular issues arising, whilst leaving the groundwork developed for further improvement after the testing campaign, in case the results differ significantly from those predicted by the software in use.

With that in mind, and regarding future work, the current point of the development of the satellite is already quite far in the timeline, and therefore little work remains to be developed in the area of discussion. It all boils down to the already mentioned testing campaign, due to be initiated in the present year, and whose final results should be cross-checked with both requirements from the launch responsible entities and the results obtained from the analyses presented in the design documentation, already provided to the competent authorities. A similar result pattern should serve to validate the results obtained from the analysis campaign, whilst a disparate one should serve as the basis for a new iteration, with especially sensitive parameters changed in an attempt to approach the end results obtained from both campaigns.

Acknowledgements

The author would like to thank professor Filipa Moleiro, thesis supervisor, for the constant help and availability.
Nuno Andrada, co-worker and original structural department leader.
The ISTSat-1 team, João Monteiro in particular in his capacity as team manager.
Other friends and family in general, unrelated to the project, for their constant support.
All referenced parties were vital to the finishing of the project.

References

[1] California Polytechnic State University. *CubeSat Design Specification.* Revision 13, 2015.

[2] Instituto Superior Tcnico. *ISTsat-1 Mission Description Document.* Revision B, 2017.

[3] J. N. Reddy. *An Introduction to the Finite Element Method.* McGraw-Hill, 2^{nd} Edition.

[4] S. Timoshenko and J. N. Goodier. *Theory of Elasticity.* McGraw-Hill, 2^{nd} Edition.

[5] S. S. Rao. *Mechanical Vibrations.* Pearson Education, 5^{th} Edition.

[6] S. F. C. Aboobakar. *Dynamic and Thermal Models for ECOSat III.* 2016.

[7] Siemens NX. *Advanced Dynamic Analysis Users Guide.* NX Nastran Help Library, 2014.

[8] T. L. Bergman F. P. Incropera, D. P. DeWitt and A. S. Lavine. *Fundamentals of Heat and Mass Transfer.* Wiley, 6^{th} Edition.

[9] C. Chandrashekar. *Thermal Analysis and Control of MIST CubeSat.* 2017.