Cubesat Structural and Thermal Analysis Methodology

ISTsat-1 Design

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Dedicated to my family who always believed in me, even when I myself didn't
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Resumo

O lançamento de um satélite promove condições extremas de pressão e temperatura, condições essas que necessitam de ser consideradas e avaliadas com o devido cuidado, por forma a minimizar o risco de problemas que daqui possam advir.

O presente trabalho procura assim garantir que uma dada configuração de um satélite em desenvolvimento é compatível com as condições referidas, com possíveis alterações ao cargo do analista no sentido de o garantir. Isto força a realização de uma detalhada análise térmica, onde as várias fontes de energia existentes são consideradas para avaliar as temperaturas extremas atingidas pelos vários componentes, e estrutural, onde são avaliados os efeitos das grandes acelerações sentidas pelo satélite, através de uma análise estática, assim como o comportamento dinâmico do satélite, através de uma análise modal, que servirá como base para prever o comportamento deste face às vibrações por ele sentidas aquando do lançamento, através de uma análise de vibrações aleatórias.

Surge a necessidade de realizar uma análise pormenorizada da estrutura e seus componentes que não pode ser garantida com simples cálculos à mão ou métodos analíticos, requerendo a utilização de processos numéricos patentes em software de engenharia assistida por computador. Os resultados finais das análises são assim acompanhados por uma explicação destas, juntamente com cuidados a ter na sua realização, desde o modelo, à malha e preparação da simulação, onde a maior parte do presente trabalho recai.

Palavras-chave: Análise térmica, Análise estática, Análise modal, Análise de vibrações aleatórias, Engenharia assistida por computador
Abstract

The launch of a satellite promotes extreme conditions of pressure and temperature which need to be considered and evaluated carefully, so as to minimize the risk of problems arising from it.

The present work therefore looks to guarantee that a given design configuration of a satellite under development is compatible with the referred conditions, with possible changes at the hands of the analyst in an attempt to ensure it. This forces a detailed thermal analysis, where the various energy sources are considered in order to appraise the extreme temperatures reached by the various components, and a structural one, for the evaluation of the effects of the great accelerations felt by the satellite, through a static analysis, as well as its dynamic behavior, through a modal analysis, serving as the basis to predict its reaction to the vibrations felt during the launch window, through a random vibrations analysis.

The need for a detailed analysis of the structure and its components arises, one that cannot be undertaken through simple hand calculations or analytical methods, requiring the use of numeric processes patented in CAE (Computer Assisted Engineering) software. The final results of these analyses are accompanied by their overview and extra considerations required for their use, from the model, to the mesh and to the preparation of the simulation, where most of the present work falls.

Keywords: Thermal analysis, Static analysis, Modal analysis, Random vibration analysis, Computer assisted engineering
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Nomenclature

Seeing as the document presents a multidisciplinary approach, within reasonable margins, the same symbols are often found to overlap. That being said, and in an attempt not to break with typical conventions, plenty of said symbols are presented as multiple references, with common sense dictating what to expect on a case by case basis.

Greek Letters

\( \alpha_R \) Absorptivity
\( \varepsilon \) Strain Tensor
\( \varepsilon_R \) Emissivity
\( \mu \) Statistical Mean Value
\( \nu \) Poisson's Ratio
\( \xi \) Harmonic Primary Variable Function in Modal Coordinates
\( \rho \) Density
\( \rho_R \) Reflectivity
\( \sigma \) Stress Tensor, Principle Stresses if subscripted with \( _{1,2,3} \) / Standard Deviation
\( \tau \) Shear Component of Stress
\( \tau_R \) Transmissivity
\( \phi \) Modal Shape
\( \omega \) Angular Frequency

Roman Symbols

\( a \) Albedo
\( A \) Area
\( B \) Heat Capacity Matrix
\( C \) Damping Matrix
$C_M$ Material Properties' Matrix

$E$ Young's Modulus

$E_R$ Radiated Energy

$E_b$ Energy Emitted by Black Body

$f$ Natural Frequency

$F$ Force Vector

$F_{i-j}$ View Factor between $i$ and $j$

$G_R$ Irradiated Energy

$h$ Element Length

$H$ Transfer Function

$I$ Current Intensity

$k$ Thermal Conduction Coefficient

$K$ Stiffness Matrix

$M$ Mass or Inertial Matrix

$N$ Shape Function

$p$ Distributed Internal Pressure

$P$ Pressure / Time Dependent Force Vector / Heat Load Vector

$Q$ Heat Flux

$R$ Residual

$R_c$ Autocorrelation Function

$S$ Power Spectral Density

$S_S$ Surface

$t$ Time

$T$ Temperature

$u$ Primary Variable according to $x$ direction

$U$ Electric Potential Difference

$u_h$ Primary Variable's Approximated Function
\[ v \quad \text{Velocity / Primary Variable according to } y \text{ direction} \]
\[ V \quad \text{Volume} \]
\[ w \quad \text{Weight Function / Primary Variable according to } z \text{ direction} \]
\[ x \quad \text{Harmonic Primary Variable Function} \]

**Subscripts**

0, 1, ... Referent to Nodal Values

\( i, j \) Referent to Components / Referent to Nodes

\( i, o \) Input, Output

\( x, y, z \) Directional Components

**Superscripts**

\( (e) \) Referent to Element

\( T \) Translated Matrix

**Differential Variables**

\( t \) Time

\( x, y, z \) Directional Components
Glossary

1U – 1 Unit CubeSat, same logic applying for all other versions (0.25U, 2U, 3U, 6U)

ADCS – Attitude Determination and Control System

ADS-B – Automatic Dependent Surveillance – Broadcast

ASD – Acceleration Spectral Density

CAE – Computer Assisted Engineering

Cal Poly – California Polytechnic State University

COM – Computer-on-Module

DSP – Digital Signal Processor

EPS – Electrical Power System

ESA – European Space Agency

FEM – Finite Element Method

FOS – Factor of Safety

FOSU – Ultimate Design Factor of Safety

FOSY – Yield Design Factor of Safety

FOV – Field of View

FYS – Fly Your Satellite

GS – Ground Station

ICBM – Intercontinental Ballistic Missile

ISS – International Space Station

IST – Instituto Superior Técnico

MOS – Margin of Safety

NASA – National Aeronautics and Space Administration
OBC – On-Board-Computer

OTS Parts – Off the Shelf Parts

P-POD – Poly Picosatellite Orbital Deployer

PCB – Printed Circuit Board

PL – Payload

PSD – Power Spectral Density

RBE1 – Rigid Body Element, Form 1, same logic applying to other forms (RBE2, RBE3)

RF – Radio Frequency

RMS – Root Mean Square

TT&C – Telemetry, Tracking and Control

UI – User Interface

UHF – Ultra High Frequency

VHF – Very High Frequency
Chapter 1

Introduction

1.1 Motivation

Space. The final frontier.

Many a science fiction media have used these sentences as a be all and end all motto of their more or less poignant story plot. As a way to provide a gripping opening line, or who knows, maybe a devilishly clever turn of phrase with a hidden meaning, it justifies its mainstay in the mainstream media. But perhaps the most ingenious way to look at it is that, no matter the context, there always seems to be an interpretation that makes it a rather true affirmation.

In what pertains to this document, and its writer, space, in its obvious interpretation in lieu of the context of this work as outer space, is that which contains all that is known and all that isn’t, an entity as old as time and as mysterious as its inception. This is where the fascination of Mankind with its place in the universe arises, its, as of yet, unknown fate and the fearsome realization that we are alone in this universe, or the equally frightening prospect that we may not be.

For there to be even a remote chance of reaching a conclusion to all the questions roaming the recesses of our minds, the same approach is required as that which is taken when faced with any other major doubt: investigation and dedication.

Herein lies the motivation for this work, just like with any other scientist, researcher, engineer, medic or any other, that came before or that shall come latter to finish what was started in prior generations, the sense of wonderment that comes from discovery, from the act of learning, the deeper understanding that all search for, that most human of desires, curiosity.

If you were to ask the writer for a less abstract answer one would certainly be harder to give, seeing as his main desire is just that, learning as much as possible in the short amount of time we have available to us. The opportunity of learning about one of the most fascinating environments there is, is a gift in and of itself, but the possibility of applying it for a greater purpose is even more so. Join that to his
innate interest for all things computational, mainly the modelling of real-life scenarios and their eventual prediction, and the only frontier that ends up mattering is that represented by time itself.

We live in a world filled to the brim with wondrous beauty, with mysteries galore and a plethora of puzzles to solve, yet all that bountifulness many a times blinds us to the fact that we are but a grain of sand. A grain of sand in a desert among deserts. We look up to the sky, but we don’t see the vastness, stuck as we are inside the prison we call our own minds, burdening us with the remembrance of our problems and our responsibilities, grasping at the chance to improve our status and material gains, looking for a shortcut towards becoming a slightly brighter speck of dust than the rest.

And yet, for as irrelevant as we may be, we have achieved much. We spend our time in daily commutes in highly impressive feats of engineering, the cars that once used to require an animal presence to slowly careen through the villages now burn rubber as they speed through the corners of the bustling streets in sprawling cities, the trains that used to require constant supervision of the coal supply for their steam engines have now given their place to ones that swiftly and quickly hover over their tracks powered by magnets, and the time where we looked up to the clouds and wondered how the birds managed to take it as their own has been left behind for one where the sight of a heavy frame carrying thousands of people soaring through the once restricted highway is not even worth a sigh of incredulity.

After countless years of arduous research, and the nail biting back and forth that it brings, a man-made object made its successful voyage beyond what any Human had ever seen, even if not for long, for once the march of progress is afoot, little can anyone do to stop it. And so, not too long after, it was Man’s time to leave its home and find its own place in the universe, a journey that many would say is still in its infancy.

But for as little time as has passed, today, space is less of an unknown factor and more of an unknown variable. While there are many fronts in which our cumulative knowledge lacks, there are more and more that have been taken by storm, and today, a launch into space, something previously considered an extremely daunting task, is a fairly common occurrence.

Time has showed us that anything is achievable, and yet people still question what is and isn’t possible, even when the answer is simple. Imagination. That is what drives our advances, and also what defines our limits. Since time immemorial, the great thinkers looked up and saw something no one else did. They imagined our capability to fly, to reach those unreachable heights, and they preached to anyone who would hear them that the sky was the limit. And we still do. But we now know that there is something beyond it, there is still room to improve.

Space. Indeed, the final frontier.
1.2 General Overview

The current subsection shall present all the required background on the project that may enable a better understanding of its inception, its importance, requirements and respective reasoning.

1.2.1 Satellites

A satellite is defined as an object that moves in an orbit [1] around a larger object than itself. This definition can be further enhanced by the classification of one as a natural satellite, whose orbit exists with no influence on behalf of Men, or as an artificial satellite, as is the case of the work at hand, where the design, production and launch of said object is fully dependent on Human intervention.

Satellites saw their first iterations, as most technologies do, motivated by conflict, as the United States and the Soviet Union raged a dangerous Cold War that saw little to no conflict. The advent of nuclear weaponry, and the means to deliver the payloads to any target from any point in the globe through the use of Intercontinental Ballistic Missiles (ICBMs) being developed by the major players involved, led to the establishment of a state of mutually assured destruction, where one false move could spark the start of what would certainly be an uncommonly short 3rd World War. With their military might rendered useless for the time being, the powers that be turned the conflict into a war of ideologies and propaganda, each hungrily clawing their way to space to prove their intellectual and technological superiority. And so began the Space Race, with the launch of the Soviet Union’s Sputnik 1 in 1957.

Eventually, the fires of war subsided, and the focus shifted towards research and quality of life improvements, with satellites assuming an invaluable role in our current knowledge base. Nowadays any number of different artificial satellites can be found orbiting around our planet, ranging from communication satellites to open communication channels between different points on Earth, to navigational satellites to aid in positioning any given receiver on the surface of the planet, Earth observation satellites for environment conditions monitoring, such as weather patterns or atmosphere quality, and a myriad of other satellites developed for the sole purpose of research in conditions too difficult or expensive to reproduce on Earth. Being some of the most known and prolific space entities in the world, as well as especially relevant when it comes to the kind of satellites to which this document pertains, the European Space Agency (ESA) and the National Aeronautical and Space Administration (NASA) have a vast history of successful missions of different kinds, which are made available online by ESA [2] and NASA [3], respectively.
1.2.2 Nanosats

As has been the typical path technology has adopted for the past couple of decades, so have satellites fallen prey to the latest tendencies of miniaturization. This has been helped by the side effects of the peace that has somewhat permeated most of the developed world. At the finish line of the last major conflicts, debts and damage incurred, both human and material, destabilized what was then an established monetary flow, leading to financial vacuums that needed to be padded from the nation’s internal revenue. This led to a change in priorities for many countries, with budget cuts in research and development areas in favor of infrastructure, defense systems and the creation of relief funds based on financial aid programs installed post war. Speculation arose that indicated a possible move towards privatization of non-essential companies, and even if most remained intact, their budgets certainly didn’t, and the space agencies suffered greatly from it, with increasing pressure to reduce in both costs and timelines certainly leading to many projects being scrapped.

In a time frame where the overall product development has shied away from military focus in a move towards a pro consumer market installation, the technology present in the resulting consumer level products has skyrocketed, leaving other areas, space exploration chief among them, severely lagging behind [4]. Nonetheless, this empowerment of the everyday citizen has manifested itself by way of development tools being readily available through low-powered, cheaper, more efficient and smaller, as its own name suggests, micro-electronics. With this groundwork in place, it was only a matter of time before satellites started making use of it, leading to a renaissance of sorts of the space age, with outer space being open for public use, albeit with very tight restrictions and regulations of course.

Long gone being the previous requirements of large structures and budgets to accommodate the heavy and burdensome hardware, satellites took to space in many different shapes and forms, sparking the need to categorize them to better keep track of all the man-made objects orbiting our planet. Mass being a universal variable common to all, regardless of configuration, it was chosen for the part, with the typical decimal prefix system at its base, as shown in Table 1.1.

<table>
<thead>
<tr>
<th>Main Classification</th>
<th>Sub Classification</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Satellites</td>
<td>-</td>
<td>&gt;1000 kg</td>
</tr>
<tr>
<td>Medium Satellites</td>
<td>-</td>
<td>500 to 1000 kg</td>
</tr>
<tr>
<td>Small Satellites</td>
<td>Minisatellites</td>
<td>100 to 500 kg</td>
</tr>
<tr>
<td></td>
<td>Microsatellites</td>
<td>10 to 100 kg</td>
</tr>
<tr>
<td></td>
<td>Nanosatellites</td>
<td>1 to 10 kg</td>
</tr>
<tr>
<td></td>
<td>Picosatellites</td>
<td>100g to 1 kg</td>
</tr>
<tr>
<td></td>
<td>Femtosatellites</td>
<td>10 to 100 g</td>
</tr>
<tr>
<td></td>
<td>Attosatellites</td>
<td>1 to 10 g</td>
</tr>
<tr>
<td></td>
<td>Zeptosatellites</td>
<td>0.1 to 1 g</td>
</tr>
</tbody>
</table>

Table 1.1 - Satellite Classification based on Mass
Do note that this classification is, by no means, a definitive one, and more of a guideline, with other aspects like size also playing a role. Case in point, some nanosatellites (nanosats) have been known to hold their mass value at just shy of the 1 kg lower bound for this class. At this point in time, almost 1000 nanosats have been launched, with the following 6 years expected to quadruple that value. A database of all known launched nanosats, together with mission details and status can be checked in [5].

### 1.2.3 CubeSats

Even though the cost of satellite development has plummeted, sending it to space is an entirely different endeavor, leading space agencies all around the globe to open the market for the launch of consumer developed artificial satellites. This poses a problem, nonetheless. The transportation and storage of such sensitive material is not as easy as any Earth-bound common product, requiring special structures to be designed to contain them, which would allow their presence as secondary payloads in a launcher vehicle. But the development of said structures on a case by case base would just drive the prices of satellite development back to unreasonable figures, which prompted an alternative approach. The creation of a group of standards, by which all consumer developed products should abide, allowed the mainstreaming of the launch procedures, resulting in faster development cycles and more prolific output.

One such standard, and unarguably the most used, as portrayed in Figure 1.1, is the type of nanosat aptly named CubeSat, based off of the overall shape of the most common sized element, developed in 1999 in a joint effort between the California Polytechnic State University (Cal Poly) and Stanford University [6]. Said standard defines dimensions, maximum mass boundaries, types of material to be used, venting areas, amongst other, leading to an unwavering envelope and satellites that only tend to differ drastically in their payload.

The established CubeSat element (previously referenced) can be considered an approximate cube with 10x10x10 cm and an upper mass budget of 1.3 kg, referred to as a 1U CubeSat, an example of which is presented in Figure 1.2. This cubic unit can be stacked in many ways (generally the most compact form, with a 3U assuming 10x10x30 cm dimensions in a vertical stack whilst a 6U assumes a 10x20x30 cm), allowing for multiple possible configurations, each generally referred to by the number of units they comprise (2U, 3U…). For the sake of full transparency, some exist that buckle the unregistered intuitive rule of keeping to integer values, with 1.5U CubeSats gaining particular traction. CubeSats with lower dimensions than that of the 1U have also been launched, with the smallest being a 0.25U. Also of note is the fact that most of the CubeSats are required to provide a well-defined minimum number of Separation Springs and Deployment Switches, for aiding in the projection of the satellite when released from the deployer and the activation of a countdown for activation of all electrical subsystems, respectively.
As previously stated, with the CubeSat becoming the de facto ruler of the launched nanosats, a structure capable of deploying these as secondary payloads, solo or multiple at once, also followed a pattern, with diversity between the interested parties coming down to a question of isolation performance and overall allowed configuration. The Poly Picosatellite Orbital Deployer (P-POD), seen in Figure 1.3, developed alongside the CubeSat in its first iteration, was, for many years, the most common platform, having been replaced in that regard by the newcomer NanoRacks when it comes to deployment from the International Space Station (ISS). These deployers have been designed and improved upon to ensure that mission critical objectives are met, even in case of malfunction, such as protecting the launcher vehicle, and the ISS itself in case of secondary deployment through their facilities, from any form of interference, be it mechanical, chemical or electromagnetic in nature.

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As one of the leading experts of all things space in Europe and Worldwide, ESA has obviously joined in on this technology taking the word by storm, providing their own take on the importance of it and their benefits for the agency itself, stating it as a "driver for miniaturization efforts", an "affordable means of demonstrating and testing technologies", a "means of delivering primary payloads", among others [7]. This patented interest justifies their investment in the development of multiple projects of this kind, in different markets of opportunity, which leads to the next subsection.

1.2.4 Fly Your Satellite

The task of diffusing knowledge and instilling in others an interest in its pursuit is vital to our development as a species and as individuals, making it its keepers’ duty to fulfill it. ESA’s standing as an authority in the realm of space makes it a target for such tasks, tasks that it takes rather seriously. The creation of ESA Education is living proof of it, being an entity solely dedicated to informing the public and promoting activities that foster interest in the field [8]. Beyond maintaining a dedicated platform for younger readers to learn and a healthy trickle of information being provided on a regular basis as to the status of their more meaningful work and discoveries, this branch of the European giant is responsible for putting together actual hands-on experiences to further prepare the next generations for what shall be required if space research and exploration is to proceed. These can be as simple as run of the mill demonstrations of physics concepts for the younger audience, or as complex as allowing and empowering teams made up of college students to devise, design, and build an actual satellite from scratch by offering expert guidance, state-of-the-art dedicated testing facilities, financial support for attending workshops and training scenarios and by sponsoring the launch itself [9].

![Figure 1.4 - Nanosats Deployed by Organization Type [5]](image)
This initiative has been dubbed *Fly Your Satellite! (FYS)* and is one of many similar programs offered by a number of entities with regards to the production and launch of CubeSats developed in college environments. In fact, a considerable amount of these projects have their origins in this area, left behind only by those presented by companies, while edging out Military developed CubeSats by a significant margin, patent in Figure 1.4.

As already stated, the definition of the standard of CubeSat brought with it a very robust set of ground rules to be followed for acceptance. Some of the most basic are enumerated here as a quick overview, some a subset to refer solely to what applies for the satellite at stake, mainly its **1U** configuration (refer to section 1.2.5), hopefully providing a clearer picture of the kind of requirements at play. The full list can be checked in [10], from which a bare bones drawing with the relevant dimensions patented has been lifted and posted on Appendix A for ease of use.

- The CubeSat structure shall comply with the defined dimensions, those being 100x100x135 mm, with protrusions being allowed up to 6.5 mm on the various faces, measured from the plane of the rails;
- Rails shall have a minimum width of 8.5 mm and a minimum of a 6.5x6.5 mm area on their ends to ensure stable contact with other CubeSats possibly stacked above or below it;
- The maximum mass of a **1U** CubeSat shall not surpass 1.3 kg;
- The CubeSat center of gravity of a **1U** CubeSat shall be within a 2 cm radius of the geometric one;
- Aluminum 7075, 6061, 5005 and/or 5052 shall be used for the entirety of the structure;
- All structural material that may enter into contact with the deployer or other CubeSats shall be hard anodized to prevent cold welding.

The CubeSat Design Specification document from **Cal Poly** [10], also defines a number of more general, electrical and operational requirements, including, but not limited to, restrictions on propulsion systems, hazardous materials, minimum venting area per volume, requirements regarding switches, battery protection and Radio Frequency (**RF**) transmissions. These are glossed over due to their irrelevance for most of the work presented.

Beyond these, there are also basic testing requirements referred, and even though no testing has been done on the satellite at the time of writing, knowledge of required tests and the levels they should adhere to are essential to defining what to look for and thrive to achieve at an analysis level. Many sources do discretize both test and analysis level qualification requirements and they shall be referenced at a later stage. It is important to refer nonetheless that these sources and values may differ between CubeSats and, most importantly, between initiatives, seeing as they are highly dependent on launch conditions and, therefore, might be specific to **FYS**.
1.2.5 ISTsat-1

The ISTsat-1 is the first CubeSat being developed in Portugal, being the result of the dedication of several students and teachers that make up the multidisciplinary team based out of Instituto Superior Técnico (IST).

As the first attempt at a fairly complex and sensitive product, the main goal is to provide the parties involved with firsthand experience on an actual engineering project, giving them ample opportunity to develop further capabilities in a variety of fields that may prove useful in future endeavors. Looking towards the future, it will also serve to establish the ground work for similar projects further down the line, through instrumentation and infrastructure investment and careful documentation for both acceptance and archival, not to mention expertise being personally passed down to the younger members of the team so that they may carry the torch for the ones that have yet to come.

The secondary goal is much more technically oriented, appealing to the research side, which, even though simple in nature, provides an interesting challenge from a scientific, technological and applications point of view.

The mostly in-house developed ISTsat-1 is a 1U CubeSat whose primary payload is an Automatic Dependent Surveillance – Broadcast (ADS-B) antenna and corresponding receiver with the mission being to detect and receive broadcasts from commercial aircrafts for tracking purposes. This type of technology has seen significant use in small satellites aimed at its demonstration, with the IST’s team attempting to build upon and improve it in the form of a wide Field of View (FOV), small form (patch) ADS-B antenna.

![Figure 1.5 - ADS-B Signal Reception "Store and Forward" [11]](image)

Seeing as the objective is to test the technology’s efficiency and efficacy in orbit, in a smaller form, and aboard a much smaller vessel than that which usually is used for this purpose, the data broadcast is not continuous, instead adopting a Store and Forward approach, where the information is stored and the Downlink only initiated as the satellite manages to achieve line of sight with the Ground Station(s),
one currently in set-up phase at IST’s Taguspark campus. This information is later expected to be analyzed and correlated with that attained through typical ground-based technology.

Assuming the data is enough for said correlation to be established, it should be possible to assess and characterize the *Cone of Silence*, volume of space where signals cannot be emitted to or received from due to the zenith null in the radiation pattern of a monopole antenna, and to measure performance parameters of the communication chain, particularly when it comes to *Probability of Target Acquisition*, *Probability of Detection* and *Probability of Identification*.

An extended presentation of the Mission Statement and a more detailed presentation of the technology in use can be perused in the team’s Mission Description Document [11].

Besides the payload, it is crucial that the overall configuration of the satellite be established at this point, considering its relevance towards the entire endeavor. With that in mind, the following images are provided, representing the structural design in Figure 1.6 and the subsystems in Figure 1.7.

![Figure 1.6 - Structural Components](image)
![Figure 1.7 – Subsystems’ Components](image)

The structure is comprised of four arms linking the two main frames, where the rails protrude to provide eight stand-offs for contact with either the deployer or other CubeSats. Connected to the arms are also four axes, that together with the spacers (not visible) are responsible for holding the printed circuit boards, housing the electrical components, in place.

The electrical subsystems are far more numerous and can be divided in the outer parts and the main stack. On the first group we have four solar panels on the sides of the satellite, with a fifth one resting on top of the Very High/Ultra High Frequency (VHF/UHF) antenna and this one locked to the upper section of the structure. On the bottom lies the primary payload, the *ADS-B* antenna. All these components are locked to the structure through the use of bolts. The stack contains five Printed Circuit Boards (PCBs), responsible for ensuring all the processing and power storage needs of the CubeSat. They communicate between each other through PC104 connectors, plastic based housings on the
upper face of each board where the copper pins protruding from the lower face of the upper board are inserted. Going from top to bottom, and according to [12] they are:

- Telemetry, Tracking and Control (TT&C) – Telemetry Controller. Responsible for maintaining a radio link with the Ground Station (GS). It handles modulation of digital to analog and demodulation of analog to digital data streams to and from the upper antenna, respectively;

- On-Board-Computer (OBC) – Main computer, brain of the system. Responsible for computations vital to the survival of the mission and its achievement. It handles typical housekeeping and attitude control functions;

- Electrical Power System (EPS) – Electrical Control, heart of the system. Responsible for power distribution as required. It holds the rechargeable batteries that store the surplus of energy;

- Computer-on-Module (COM) – Communications processor. Responsible for implementing the communications protocol stack. It handles intensive computational loads through data processing and handling;

- Payload (PL) – Payload antenna companion. Responsible for pre-processing the data obtained by the lower antenna. It handles decoding and filtering of mission related acquired data.

1.3 Objectives

In order for the satellite to be accepted into testing stages, and later on integration, all subsystems have to undergo a detailed analysis and testing campaign to improve the chances of critical mission success. This is the same for the Structural and Thermal departments, where worst case scenarios are evaluated and their negative effects mitigated by carefully selected, and documented, engineering solutions.

The first version of the developed work was intended to show the end results of the various analyses required for the all clear, together with a few extra What If scenarios to further expand the range of survivability insurance, questions like bolt-failure analysis, sensitivity analysis based on part dimensions, material properties, eccentric loading and some others.

This objective changed over time due to two major factors. The first being the late testing campaign prepared for the CubeSat not allowing a validation pipeline of analysis results to be developed, which considering the several problems faced by the miniaturization of said structures, many of which not even found in large scale satellites and therefore harder to predict and model, reduces the confidence in the values obtained. The second, and most important, was the unexpected lack of documentation focusing on actual procedures to take when approaching a finite element model problem, be it structural or thermal. Due to the high degree of experience required for reasonable results to be obtained in high level Computer Assisted Engineering (CAE) software packages, many pitfalls are typically encountered, and many more are not even recognized, possibly leading to less than ideal results. Assembly
connections, when to and which to use, Mesh definition, how to control it and ensure its convergence, the influence of Resonant Frequencies on Frequency Sweeps and Random Vibration analyses, assuring a proper Bolt Analysis, the list goes on, and whilst many are high level concepts that are far too complex for discussion at this level, many are there that could make it easier for newcomers to more easily get to grips with its use.

The objective at this stage becomes to present an intro of sorts to a proper CubeSat analysis by means of examples, an instructions manual if you will, perhaps motivating others to delve deeper into less obvious practices around a basic framework, whilst maintaining more frustrating aspects at bay.

1.4 State-of-the-art

The design of satellites within the CubeSat moniker follows a fairly tight standard, with little room for changes when it comes to the overall structure. This results in both similarity and flexibility, meaning that the plethora of satellites either in orbit, decommissioned or not, or in development stages find themselves in common ground when it comes to overlapping entities, whilst being vastly different with respects to the different specialized modules in use, according to the mission details. This perceived consistency allows for a high level of modularity to be achieved, so much so that many a company started specializing in the production of components for said satellites on the grounds of them being bought as Off the Shelf Parts (OTS Parts), ranging from solar panels, to antennas, and even the main structural frame itself. Enter Pumpkin Space Systems™, Clyde Space™, EnduroSat™, ISIS Space™, the list goes on, with these ones referenced specifically by name only comprising a relatively small sample off of the pool of suspects. This allows for the approach most teams end up taking, buying several of the required components according to budgetary and/or mission specific constraints from different companies, leading to the possibility of employing smaller teams, focused on mission specifics instead of attempting to reinvent the wheel. However, other reasons exist for foregoing this method and developing most of the satellite in-house, as is the case with the CubeSat to which the current work pertains to, mainly educational and financial related.

Regardless of these choices, the amount of work developed with this satellite framework in mind is fairly extensive, and within the academic world alone, the development has seen a staggering expansion for the last few years, with a variety of mission designs permeating this output. This ranges, in overall perspective, from well-defined systems implemented to run a particular function, such as signal detection and/or retransmission, to technological testing, such as new attitude control mechanisms, and even biological research, focused on data recovery. As a side note, currently, a particular interest has been installed in the development of clusters of CubeSats, all working within the same framework and towards the same goal.

The breadth of scenarios notwithstanding, the development campaign is universal to all teams involved in the FYS initiative. Therefore, plenty of documentation exists relating to the type of analyses presented in the current document, although their focus is often times less split due to separation of work among different team members, with structural and thermal designs typically individualized. The approach
taken, however, is very much the same, with similar software packages, assumptions and results expected among similar projects, that is, none that deviate from normal configurations or present abnormal energy requirements.

This development campaign is, nonetheless, in constant flux, with changes being made with every iteration of the project in an attempt to reduce the, still, statistically high likelihood of mission failure. The sources for this are innumerable, commonly involving critical parts, and registered cases exist of battery problems, communication issues, and even deployment missteps, with deeper scrutiny pointing fingers to lapses in software development, lack of testing rigor, or, more importantly for this case, simplified or even overlooked thermal analyses.

1.5 Thesis Outline

The current document is divided in several chapters, each focusing on different aspects of the work developed and with corresponding theoretical background accompanying it. This subsection serves as a lower level contents guide for quickly referencing a particular subject that may be discussed further on.

Chapter 1
The current section. It paints the backdrop for the satellite development project, requirements and mission included.

Chapter 2
A fairly detailed account of the basic mathematical procedures and logic behind the Finite Element Method (FEM), including discretization and matrix assembly.

Chapter 3
Dedicated to Structural Analysis. Discussion of idealization, mesh and simulation preparation, as well as results presentation. Included in this chapter are Static Analysis, Modal Analysis and Random Vibration Analysis.

Chapter 4
Dedicated to Thermal Analysis. Discussion of idealization, mesh and simulation preparation, as well as results presentation. Included in this chapter is a Transient Thermal Analysis.

Chapter 5
Conclusions presentation with future work to be developed.
Chapter 2

Finite Element Method Basics

2.1 Initial Notes

In the old days, the scientific community found its way to the answer of many a problem through careful mathematical approaches and high-level logical reasoning, leading to theoretical advancements that were only privy to a handful of scholars capable of following such abstract lines of thought. We lived an era of expansionary vision, in detriment of a practical one. This perspective started to change with the passage of time, with an increasing effort on the part of research parties to promote knowledge diffusion and abide to the common populace’s demands. Massive engineering and physics problems were tackled and resolved through analytical means, putting forth multi-disciplinary formulations that defined any number of behaviors within a random domain. Nonetheless, and even with visionary minds leading the charge of smart, innovative and productive processes, one thing still fell short, actual numerical output. Seeing as the capability for highly accurate calculation of the human mind is fairly limited, most engineering projects were developed on the basis of hand calculations, with sometimes overtly non-conservative simplifications to the analytical processes, leading to approximations with rather large error margins that could easily destroy an otherwise marketable product.

The advent of the so-called new technologies brought with it a drastic paradigm shift, leading to a vast number of processing units starting to permeate the world and all aspects of human life with it. Ideas that were, so far, only theoretical concepts could take shape and be implemented in highly modular systems that could be loaded with a particular set of instructions to be followed, instructions that included conditional and iterative behaviors and could be realized in a fraction of the time it would take any one person, or group of people for that matter, to finish. Anything that could be thought of as a process with specific and ordered instructions could potentially be fully automated, and the market for such products grew exponentially. The electronic world expanded, from highly advanced server farms to the smallest of chips embedded on a miniaturized camera started making rounds, bringing a highly improved research and development cycle with it.
One such improvement was the creation of program suites capable of solving a large number of calculations without requiring constant human supervision and control. Given a set of input data, that can be simplified through a graphical interface, no matter the amount of calculations required, as long as there are enough computational resources to foot the bill, the machines do the heavy lifting and come out with an output set to match. With this tool at our disposal, analytical solutions were no longer required, and numerical processes came to the forefront of the engineering efforts which, unlike many other areas, more exoteric ones, deal with actual tangible projects and require accurate solutions to a subset of problems, instead of all-encompassing ones.

The development of the ISTsat-1 is no exception, requiring a fairly accurate assessment of the various adverse conditions it may have to endure and its behavior as they occur. For this analysis, the Siemens NX™ 12.0 software package was used, chosen mostly due to its embedded space systems thermal module, which facilitates the thermal analysis through an easy to setup automatic simulation of the orbiting of the satellite.

Most of the technical documentation at the level of dissertations or articles that can be found seem to focus solely on the analysis of end results and their implications, and while that is still the case in this work, in an attempt to break with tradition, and hopefully provide a helpful guide to first timers and novices alike, a bigger focus is given to the intricacies behind the different types of analyses, although much of the knowledge presented is also applicable to others, with the respective caveats, of course.

Following the old adage garbage in, garbage out, that typically makes its rounds on most endeavors that require human input, a minimal knowledge of the background of the processes involved is required, lest the input be less than optimal and the results therefore unusable. This section serves that purpose, presenting a brief and consciously instinctual overview of the FEM in a hands-off approach, with little mention to any particular software suite.

As a disclaimer, the writer feels obligated to state that the content that follows is in no way, shape or form a complete description of the process, nor is it a source of deeper understanding seeing as, stated above, a less mathematically intensive approach is taken to improve comprehension. For such, a dedicated bibliography is recommended [13-18] to which the content of the present chapter can be traced back to.

2.2 Introduction

Mathematical representations of physical processes often times correspond to differential equations, that may vary in their nature, from partial to nonlinear, meaning that, while some are easily solvable through analytical methods, others require methods that are either too complex or offer no closed form solution. The answer to these problems is a numerical one, which may take many forms, from a traditional Finite Difference Method to the already referenced FEM, which is where this document’s focus lies.
The Finite Element Method is a numeric approach used to attain approximate solutions to boundary value problems, also known as field problems. These problems consist in the determination of functions defining a set of variables that satisfy a group of differential equations inside a known domain, whilst being in accordance with specific states at its boundaries. In simpler terms, a problem where the values our variables take at the boundaries of a given domain, often times a structure or volume, are known and the objective is to effectively categorize and quantify its distribution throughout it. This means that the formulation requires two elements:

- An integral statement referred to as the governing equation, expectedly representing the space and/or time dependent variation of particular quantities, such as stress, heat, electromagnetic fields, among others;
- Boundary or initial conditions, defined for static problems and time dependent ones respectively, representing the particular setup for the problem at hand, and therefore defining it. They represent, in layman’s terms, the data set that provide an envelope to the analysis, turning a potentially solvable but undefined problem into a fully defined one.

While the presentation methods of the concepts inherent to every step of the process are highly variable among the typically available literature, the hierarchical steps taken up to, and including, the processing of the result are rather straightforward, with a few caveats, and can be easily presented in a succinct and orderly fashion:

- Pre-Processing – The first, and arguably most important, step corresponds to that of the overall problem definition. At this point the domain is defined, the element types to use are chosen, leading to the discretization process, as well as the mesh connectivity and the appliance of the respective material properties, and the boundary conditions or initial values defined by the problem requirements are imparted on the model;
- Solution – The process running on the background of all analysis software, which takes the algebraic formulation of the governing equation and leads to the definition of the elementary matrices (through the use of appropriately chosen approximation functions) and their assembly for the overall global matrix equation, with which a particular solution for the primary variables are obtained. Other properties of interest can later be found with the resulting data;
- Post-Processing – A project of this kind tends to be followed by thorough documentation and several parties may be involved, so a methodic and clear approach for its presentation is advisable. Anything from maxima and minima, rates of change and model changes to more dynamic behaviors can and should, according to its relevance, be presented in any number of forms, from graphs and charts to images and videos, to better translate the intended information.

A few key points represent the logistics and benefits of the use of the FEM over similar methods:

- The difficulty inherent to the treatment of complex domains is subdued by a discretization process, leading to smaller, simpler finite subdomains, elements;
• The approximation functions to be defined can be so through the use of several simpler polynomials, seeing as a linear combination of them can result in any continuous function;

• The coefficients for the resulting equations can be obtained through the governing equations on an element by element basis.

The sections to follow go into more depth on some of the concepts referenced so far in a generalized form, mainly in what refers to the solution step, with most of them being presented, once more, in a more individualized note and through an example led approach, in later chapters, with the focus shifted mostly to the pre and post-processing. This seemingly unorthodox distribution is due to the fact that the pre-processing is mostly important from a software point of view, and requires experience and a basic knowledge of the inner workings of the solution phases to prepare properly, therefore, and, again, perhaps unexpectedly, at least for whom this is a first contact with the matter, it shall be briefly skipped, following the typical approach that presents the basics of the mathematical solution as a departure point.

2.3 Governing Equations

As is easily perceived from any undergraduate level physics related course, the physical world abides by specific sets of rules, some of which fairly known, and others posing an as of yet undeciphered riddle. Nonetheless, the existence of these so-called rules is fairly accepted in the scientific community, even under the immutable truth that its proof is unlikely to ever surface. These formulations can be as simple as the ever-increasing value of entropy in a given system, or the behavior of a mass-spring system, or as complex as the velocity profile of a fluid flow at any given point. Regardless, the insight gained by such discoveries tends to point towards the inevitable perspective that, given enough data, the immediate future is less of a variable per say and more of a result of circumstance, and whilst the fact that psychological patterns are somewhat uncharted at this point in time, given the lack of information, makes predictions on outcomes of a future predicated on emotional decisions all but impossible at this point, purely physical non-human dependent processes can and have been fairly accurately assessed, providing further credence to this ideology.

In the end, these kinds of equations can be aptly named governing equations, dictating the space-time variation of specific properties by us defined, from density, to velocity, displacement, temperature or any other relevant and dependent variable.

\[
\frac{d}{dx} \left( EA \frac{du}{dx} \right) + p = 0
\]

\[
\frac{\partial u}{\partial t} - k \nabla^2 u = 0
\]

\[
\rho \frac{Dv}{Dt} = -\nabla p + \rho g + \mu \nabla^2 v
\]
\[
\n\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)
\] (2.4)

The equations here presented are merely done so for illustrative purposes, warranting no further discussion than the point to come across, although the first two will prove to be essential in latter chapters of the current document. Nonetheless, for context, they shall be individually referenced, in the respective order:

- One dimensional Stress distribution on an axially solicited structure
- Multi-dimensional Heat distribution
- Navier-Stokes equations, specifying the behavior of Newtonian Incompressible Fluids
- Maxwell equations, dictating the behavior of Electromagnetic Fields

One of the commonalities easily singled out from all these examples is the nature of the equations themselves, them being Differential equations. These are defined as relations involving not only dependent and independent variables but also derivatives, or rates of change, of the independent variable at stake. In layman’s terms, and as a quick example, the time dependent temperature variation along a one-dimensional like rod of any given material, is not only dependent on the time passage but also the rate at which the temperature itself varies. Obviously, this points towards the fact that plenty of physical phenomena follow the same type of behavior, one that is, rule of thumb, not easily analyzed.

Within the concept of differential equations various types can be discretized, between ordinary or partial and scalar or vector in its nature, and while ordinary equations can, for the most part, be solved with a certain degree of ease, most partial differential equations become either unsolvable or require far too arduous a methodology to be methodically used for solving non trivial systems with possibly random configurations. A less orthodox, non-direct approach becomes necessary in order to obtain usable data.

\[
\sum_{i=1}^{n} u_i N_i
\] (2.5)

This approach presents an attempt to replace the exact result with an approximate one and the equation becomes the starting point for the method at work through the use of shape functions, \( N_i \), polynomial functions that approximate the end result between nodal points and which, by virtue of their nature, are more easily computed.

The approximated nature of this method means extra care has to be taken to ensure the induced error is within acceptable parameters, which is why the original differential equation is morphed into the integral form previously alluded to, equation (2.6), which allows this error, residual, to average to zero on the whole domain, instead of simply enforcing their inexistence at particular points in it. This formulation is also known as the Strong Form, or Method of Weighted Integrals.

\[
\int_{0}^{1} w R dx = 0
\] (2.6)
This weight parameter, \( w \), works in a similar fashion to the shape functions referenced above, with \( R \) corresponding to the residual function, all non-zero parcels of the governing equation in its differential nature with the independent variable taking the approximation form established in equation (2.5), and \( w \) to a trial function.

The weighted integral method is a variable system, with small differences leading to different thought processes and so on, while being a fairly straightforward example of what is possible to follow. Similarly, the community has documented a plethora of ways to obtain the referenced weight functions, each with their own incentives and faults. As an example, one of the most typical approaches is to use the trial functions as the weight functions themselves, basis for the so-called Ritz-Galerkin’s Method.

Seeing as the solution is obtained by numerical means, the equaling of the residual error to zero only impacts the integration points used in this mathematical computation problem, hence why the number of events/elements considered is typically directly related to the quality of the result.

\[ [M]\ddot{u} + [C]\dot{u} + [K]u = F \]  

(2.7)

Each of the matrices that make up this equation (2.7) are fairly known within the structural realm, but the relationship they sustain with respect to the primary variables, such as displacements, can be observed through the lens of a different problem, or better yet, in a somewhat abstract perspective:

- \( M \) – This is typically referenced to as Mass Matrix, or Inertial Matrix as it pertains more to its function within the set-up. Looking back to one of the most important equations in classical mechanics, that of the second law of Newton \( F = ma \) that defines the correlation between the mass of an object and its acceleration as a result of an imposed force, the term used to define this property inherent to the test subject is not mass, as is a typical misuse of the concept, but of inertial mass. In simple terms, this property defines the object’s capability to resist change as a function of time, typically velocity. Therefore, the inertial matrix incorporates all the

---

1 For additional information on how this result can be obtained verify [17]
information on the model under scrutiny that relates to its capacity of resisting the primary variable changes within a time frame, whether it be resisting deformation, temperature changes and so on.

- \( C \) – The Damping Matrix is very much what the name implies, a matrix that aggregates all the information pertaining to the damping of a system. To simulate this, it is typical in multiple applications for specific elements to be used to control great amplitude/directional changes, allowing for a minimum energy, stable, state to be achieved, within a certain margin, as fast as possible while avoiding too rigid a behavior so as to isolate it as much as required.

- \( K \) – The Stiffness Matrix is the most prominent of all the system dependent matrices. It relates, to an extent, to the roles played by the Inertial Matrix and the Damping Matrix in the sense that it is also responsible for taking into account the resistance offered by the model to deformation, the major difference being that it does not correlate to a time dimension, this becomes clearer in the next few paragraphs. In the end, many elements are simulated as having spring like behaviors, which dictate how changes in one part of the system affect others.

- \( F \) – The Force Vector is the simplest to understand, corresponding to a merger of all the external solicitations applied on the system. As this is purposefully generalized for all applications, the use of the term force is done so in a non-literal sense, seeing as it can also correspond to thermal power or electromagnetic field sources for instance.

The equation of motion of a generalized element serves therefore as an analogy for most any problem in engineering, from a typical structural application to the most complex electromagnetic conundrum. It is important to note though, that the end equation for any given problem might not present itself in the same shape as equation (2.7), often not accounting for all the terms in it. This variance is affected by both the nature of the problem and the type of analysis at hand, whether it be structural, thermal, fluid, electromagnetic, acoustic or other.

At its core, an analysis is only relevant if it takes into account the real-life conditions that it attempts to emulate and, in many cases, this dictates different problems within the same realm of an engineering class. For instance, the requirements for a given assessment may lie simply in a purely space-dependent approach where the time dimension is irrelevant, or the nature of the problem may translate into a time-dependent input, with a corresponding time-dependent output, all within the same type of analysis. This will be given a general rundown in the following paragraphs using a structural type analysis for aided understanding:

- Static Analysis – This is a type of analysis where the focus is on long term results. Subjecting a bound structure to a non-variable stress profile causes changes that eventually converge and stabilize. In other words, after an arbitrarily long time, the structure reaches a static state, a new equilibrium that takes into account its stiffness. The Inertial and Damping matrices have no effect seeing as time dependent changes are inconsequential, the requested output corresponds to the end result, not the process through which it is achieved.
• Dynamic Analysis – In opposition to the previous one, a Dynamic Analysis is time dependent. Making use of the same setup as the prior example, but with a stress profile that is highly variable in nature, it is simple to imagine that the inertia of the structure contributes to creating a scenario where reaching a static state, for no matter how small a time window, is impossible. At this point, both the Inertia and the Damping Matrix take a relevant role, becoming indispensable in achieving a reasonable output.

Beyond these two, a case can be made for an existence of a Quasi-Static Analysis, a sort of midway between them. This is characterized by an input stress profile that, while time-dependent, presents a small enough rate of change that allows for the structure to reach successive Static states, in essence allowing for it to be studied under Static Analysis conditions. Another way to look at this case can be from an energy perspective. If a Static case presents itself as purely channeling the internal energy of the system, and the Dynamic one as mostly affecting the kinetic energy, then a Quasi-Static case may be determined in such a case that most of the energy flow corresponds to internal energy, although, of course, this boundary is rather subjective and very much influenced by the accuracy required.

2.4 Domain Discretization

As previously stated, the results obtained from a computational method may present significant disparities when compared to the analytical ones. Reiterating what was discussed in the previous section, the residual can only be expected to reach zero at the points chosen for integration during the process, whilst the rest of the domain is interpolated from them as a result of the chosen trial functions. In other words, the more interpolation points, the bigger the accuracy, which serves as an argument in favor of the discretization process.

This promotes the use of meshing, a process that allows for the representation of a physical domain as a conjunction of several, smaller, simpler domains which shall be referred to as finite elements, each contributing with extra integration points and a set of approximation functions tailored to them.

In the end, the conclusion is fairly obvious, the more elements, the closer the end result becomes to the actual physical outcome, exemplified by Figure 2.2, the downside being a clear and unshakeable increase in complexity, time and hardware requirements. Many a practice, both from a programmer and a user perspective, result in some sort of mitigation of this drawback but the resulting increase in computational effort is unavoidable.

Even so, one cannot embark on an infinite process in search of a perfect solution, both unattainable and unverifiable. This is where the concept of convergence comes to light. As the size of the elements decreases, the computational errors tend to follow suit, and with each refining step the solution changes less, until it can be considered unchanged, the margin in use being mostly up to the analyst or the client. Some very important exceptions arise, but these shall be further discussed in later chapters.
Convergence is therefore a rather important part in computational analysis, and is required for results to be accepted, but while it may be a requirement, it isn’t a guarantee. This brings forth a couple of rules to the table, the explanation for which equation (2.8) is presented here, being a slightly different version of equation (2.5).

\[ u^{(e)} \approx \sum_{i=1}^{n} u_i^{(e)} N_i \]  

(2.8)

Obtaining a solution based off of equation (2.5) is exceedingly difficult considering its nature, that of its application to the entirety of the domain, which would therefore require a group of trial functions that extended throughout it. Using it in the form of equation (2.8) nonetheless, confers a higher degree of pattern retention, while also allowing for the definition of crucial points and trial functions on a per element base, hence the superscript \((e)\). The process becomes mainly locally focused instead of the global domain being the prioritized perspective. These shape functions can be defined in whatever way the user pleases, as long as it complies with the rules previously alluded to and now presented in extended form, but the most used correspond to polynomial formulations.

- Compatibility/Continuity – The interpolation functions must be continuous and belong to the differentiability class \( C^{n-1} \), which is to say that all its partial derivatives up to order \( n - 1 \) are continuous as well, where \( n \) is the highest order derivative present in the integral form of the element equations. This extends to the boundaries of the element where the functions also need to be able to ensure any type of previously defined conditions are met and compatible with the adjacent elements. This requirement of compatibility has its roots on the fact that the known world does not abide the existence of singularities, discontinuities in the domains of
physical phenomena, that is, the rate of change of any variable of interest follows a finite gradient.

• Completeness – The trial functions need to ensure that the integral form of the equations, as well as all partial derivatives up to and including the highest order derivatives \((n)\) can assume constant values, which, by the nature of the discretization, can be directly applied to the trial functions themselves. This can be ensured by imposing that the trial functions are complete, that is that they do not omit any lower term polynomial parcel. The constancy capability ensured by this condition allows for something like rigid body displacement to occur, in the case of a structural problem, or heat flow absence in the case of a thermal problem.

These are not the be all and end all of the approximation functions nature, serving only as a rule of thumb type of situation when choosing polynomial formulations for this step of the process. A generalized example follows, showing how these functions can be obtained for a one-dimensional problem, with concrete examples presented later down the line for direct application.

This uses both a global and a discretized means to generalize the shape function matrix, by defining the overall polynomial function over an element and following with its application to particular nodes in matrix form.

\[
\begin{align*}
    u_h(x) &= \sum_{j=1}^{n} a_{j-1} x^j + 1 = [1 \ x \ \ldots] \begin{bmatrix} a_0 \\ a_1 \\ \vdots \end{bmatrix} = \{\varphi\}[a] \text{ providing the overall polynomial} \\
    \begin{bmatrix} u_h(x_0) \\ u_h(x_1) \end{bmatrix} &= \begin{bmatrix} 1 & x_0 \\ 1 & x_1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = [C][a] \text{ providing the matrix form on a per node basis}
\end{align*}
\]

Considering the shape functions as defined in equation (2.8), it is possible to make use of these last formulations to come up with a matrix definition for said functions.

\[
\begin{align*}
    u_h(x) &= [N]\{u\} \text{ and } u_h(x) = \{\varphi\}[a] \text{ with } \{a\} = [C]^{-1}[u] \Rightarrow \\
    &\Rightarrow u_h(x) = \{\varphi\}[C]^{-1}[u] \Rightarrow [N] = \{\varphi\}[C]^{-1}
\end{align*}
\]

Rules notwithstanding, a visual presentation always improves comprehension, so a generalized example follows, starting with the simplest set-up and building on it for more complex topics, a single one-dimensional element with two nodes, one degree of freedom per node then.

\[\text{Figure 2.3 – Single One-Dimensional Element Two Node Example Solution Graph}\]
The above figure is a simple representation of a basic arbitrary result obtained for the referred element. On the left-hand side, the two linearly independent trial functions are presented, representing a base for any linear function to result from their sum, weighed by the primary variables’ values defined at the edges of the element, which can be seen on the graph on the right-hand side.

\[
    u_h(x) = \sum_{j=1}^{2} a_{j-1}x^{j+1} = (1 \quad x) \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \{\varphi\}[a] 
\]

\[
    \begin{bmatrix} u_h(0) \\ u_h(L) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & h \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = [C][a]
\]

Following the steps presented in the generalized method, the shape functions are obtained.

\[
    [N] = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & h \end{bmatrix}^{-1} = \left\{ 1 - \frac{x}{h}, \frac{x}{h} \right\}
\]

The convergence process, as previously stated, can be undertaken by increasing the number of elements in the model, in other words, reducing their size. This is what is known as \textit{h-refinement}. Nonetheless, improvements in the accuracy of the results can also be attained from the use of \textit{p-refinement}, a method that attempts to improve on the approximation function by increasing its polynomial order, achieved by the addition of internal nodes on the unchanged element. Building on the previous example, what follows is the same scenario where the two nodes give way to one extra, a single one-dimensional element with three nodes, one degree of freedom per each, resulting therefore in second order polynomials for shape functions.

\[
    u_h(x) = \sum_{j=1}^{3} a_{j-1}x^{j+1} = (1 \quad x \quad x^2) \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \{\varphi\}[a] 
\]

\[
    \begin{bmatrix} u_h(0) \\ u_h(h/2) \\ u_h(h) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & h/2 & (h/2)^2 \\ 1 & h & h^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = [C][a]
\]

Once again, following the steps presented in the generalized method, the shape functions are obtained.
\[ \begin{bmatrix} N \end{bmatrix} = \begin{bmatrix} 1 & x & x^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & h/2 & (h/2)^2 \\ 1 & h & h \end{bmatrix}^{-1} = \left\{ \left(1 - x/h\right) \left(1 - 2x/h\right), \frac{4x}{h} \left(1 - x/h\right), \frac{x}{h} \left(1 - 2x/h\right) \right\} \] (2.14)

The examples presented so far correspond to typical simple examples, serving purely for the purpose of presenting basic concepts in a more intuitive form, being that most practical examples tend to use far more complex types than a line element. However, the shown simplicity does not mean it isn’t applicable. A Two-Dimensional element is used for the image that follows, present to once again demonstrate the reasoning behind the method and its efficacy through visual cues, being that the calculations involved do not stretch far from what has been shown for prior examples, and the same goes for Three-Dimensional elements.

![Figure 2.5 – Single Two-Dimensional Element Three Node Example Solution Graph](image)

This process culminates in the final result, this section having showed how data is interpolated between nodal values that are previously calculated. The question of how these values come to be though still remains, which shall be tackled next.

### 2.5 Element Matrices and Global Assembly

The governing equations applied to any problem at hand, after several steps, result in the matrix equation (2.7), but so far only this end form and the interpolation functions have been grasped, the matrices required for the solution have yet to be discussed. In reference to the enumerated parcels, only those relating to the Stiffness matrix and Force vector will be mentioned, under the umbrella of a Static Analysis, the intent being, once again, to simplify as much as feasible without forsaking proper understanding of the concepts. This is achievable since the path leading to the assembly of the matrices of import is fairly prototypical, which is to say that the process to obtain the Coefficient and Force matrices can be replicated to an extent to obtain the Inertia and Damping ones as well.

\[ [K][u] = [F] \] (2.15)
The current section will therefore focus on the equation (2.15), the resulting matrix form for the weak formulation of the governing equation applied to a Static problem.

The structural problem involving the determination of the displacements at the edges of a bar can be solved with a fair degree of accuracy through the use of the one-dimensional stress distribution differential equation (2.1) already presented at the beginning of the current chapter, and copied here for ease of use.

\[
\frac{d}{dx}(EA \frac{du}{dx}) + p = 0
\]  \hspace{1cm} (2.1)

In this equation, the variables at play are the Young’s Modulus ($E$), explained in further detail in the beginning of the following chapter, the cross-sectional area of the bar ($A$), and the internal force distribution ($p$).

To reach the familiar matrix form of equation (2.15), this formulation must be applied as the residual on the integral equation (2.6), which is then solved through the use of the method of integration by parts, making use of equation (2.5) to approximate the primary variable. At this point, the requirements for the solution are reduced, the rigidity of the solution is alleviated, with the resulting form becoming laxer, taking therefore the name of Weak Form, in opposition to the Strong Form already discussed.

This solution brings into the spotlight the actual formulations behind the matrices, in this particular case with the weight functions taken as being the same as the approximation/trial ones, as per the Ritz-Galerkin’s Method previously alluded to.

\[
[K^{(e)}] \{u^{(e)}\} = \{F^{(e)}\} \quad \text{with} \quad K_{ij}^{(e)} = \int_{x_e}^{x_{e+1}} EN_i^{(e)} N_j^{(e)} dx
\]

\[
F_i^{(e)} = \int_{x_e}^{x_{e+1}} pN_i^{(e)} dx + p_1^{(e)} N_i^{(e)}(x_e) + p_2^{(e)} N_i^{(e)}(x_{e+1})
\]  \hspace{1cm} (2.16)

The shape functions, $N$, can at this point be chosen, following the rules presented in the previous section, to obtain the final matrices. Seeing as the case corresponds to a one-dimensional approximation of one single element, then equation (2.12) stands, resulting in the pretended result.

Note that the integrals presented before-hand are changed to a local coordinate system to accommodate the trial functions in said equation.
\[
N_1 = 1 - \frac{\bar{x}}{h_e} \quad \Rightarrow \quad N'_1 = -\frac{1}{h_e}
\]
\[
N_2 = \frac{\bar{x}}{h_e} \quad \Rightarrow \quad N'_2 = \frac{1}{h_e}
\]

\[
K_{ij} = \int_{0}^{h_e} EAN_i'N_j'd\bar{x} \quad \Rightarrow \quad K_{11} = K_{22} = \frac{EA}{h_e} \quad \text{and} \quad K_{12} = K_{21} = -\frac{EA}{h_e}
\]
\[
F_i = \int_{0}^{h_e} pN_i d\bar{x} + P_1N_i(x_e) + P_2N_i(x_{e+1}) \quad \Rightarrow \quad F_1 = \frac{ph_e}{2} + P_1 \quad \text{and} \quad F_2 = \frac{ph_e}{2} + P_2
\]

\[
\frac{EA}{h_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{ph_e}{2} + P_1 \\ \frac{ph_e}{2} + P_2 \end{bmatrix}
\]

Of note is the fact that, in the absence of constraints to the system, the stiffness matrix is singular, that is non-invertible, resulting in the impossibility of determining the actual individual displacements and only the overall length change. This comes about due to the fact that, without said constraints, the system is allowed to undergo rigid body motion.

As previously stated, in cases where a dynamic nature of the system is verified, inertia and dampening data is also required, yet these matrices can be built using the same principles here discussed.

A quick analysis of the formulations at the base of this section will easily denote an absence of the superscript \((e)\), referent to its elemental character, the exception being equation (2.16) seeing as it results from a non-individualized process before its application to a particular scenario. This is the case due to the fact that the examples in this section so far represent simple setups where a single element is enough to convey all required information, however, when that isn’t the case, extra steps need to be taken to force correct connectivity between different elements, that is, nodes that are common to different elements need to ensure that the primary variable value is mirrored between them, seeing as no discontinuities may arise from this discretization.

\[
\mathbf{u}_n^{(e)} = \mathbf{u}_1^{(e+1)}
\]

Figure 2.7 – Compatibility between Adjacent Elements
In a nutshell, the local coordinates used for systematically defining elements need to be converted into global coordinates that represent the entirety of the model. This change is a fairly simple one when it comes to the node coordinates, the only required steps being taking into account the displacement vector that represents the translation between the different coordinate systems. As a simple example, the local coordinate system of the two elements present in Figure 2.7 above defines both as having nodes at \( x = 0 \) and at \( x = h_e \). However, when passing them onto a global coordinate system, defining the first node of the first element as its origin, the nodes become present at \( x = 0 \) and \( x = h_e \) for the first element and \( x = h_e \) and \( x = 2h_e \) for the second, assuming both elements are equal in length.

The element matrices are different however, seeing as, unlike the nodal coordinates which are unique, the elements connecting different nodes can be of different natures or even be multiple in number. Fortunately, the workarounds are simple operations, only requiring that the matrix entries involved in this calculation be correctly chosen as to preserve the local to global coordinates transformation.

![Figure 2.8 – Four Spring-Element System with Applied Axial Forces](image)

The system represented in the above figure is, once again, one-dimensional in nature, with four nodal displacements represented by \( u_i \), and with the bolded lines connecting the end points of the springs of elements 2 and 3 being rigid. For this particular example, the spring constant, \( k \), is different between all elements.

\[
\begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} \\
K_{21} & K_{22} & K_{23} & K_{24} \\
K_{31} & K_{32} & K_{33} & K_{34} \\
K_{41} & K_{42} & K_{43} & K_{44}
\end{bmatrix}
\begin{bmatrix}
u_1 \\ u_2 \\ u_3 \\ u_4
\end{bmatrix} =
\begin{bmatrix}
F_1 \\ F_2 \\ F_3 \\ F_4
\end{bmatrix}
\] (2.21)

The matrix equation representing the full system is expected to fall into the pattern of equation (2.21), with the spring formulation summed up in equation (2.22).

\[
\begin{bmatrix}
k & -k \\
-k & k
\end{bmatrix}
\begin{bmatrix}
u_1 \\ u_2
\end{bmatrix} =
\begin{bmatrix}
f_1 \\ f_2
\end{bmatrix}
\] (2.22)

Furthermore, the correspondence between local and global coordinates is fairly easy to attain.

\[
u_1^{(1)} = u_1, \quad u_2^{(1)} = u_1^{(2)} = u_2, \quad u_3^{(2)} = u_2^{(3)} = u_1^{(4)} = u_3, \quad u_2^{(4)} = u_4
\] (2.23)
In the end, the effect of the stiffness of the springs and the applied forces are processed as a sum of their contributions towards the displacement of the attached nodal coordinates.

\[
\begin{bmatrix}
 k_1 & -k_1 & 0 & 0 \\
 -k_1 & k_1 + k_2 + k_3 & -k_2 - k_3 & 0 \\
 0 & -k_2 - k_3 & k_2 + k_3 + k_4 & -k_4 \\
 0 & 0 & -k_4 & k_4
\end{bmatrix}
\begin{bmatrix}
 u_1 \\
 u_2 \\
 u_3 \\
 u_4
\end{bmatrix} =
\begin{bmatrix}
 f_1 \\
 0 \\
 0 \\
 f_2
\end{bmatrix}
\] (2.24)

The zero values in the stiffness matrix warrant a closer look, with an analogy being made for the rows to represent the effect of the respective spring on the nodes corresponding to the respective column. As such, the entry (1,3), for instance, would correspond to the contribution of spring 1 toward the displacement of node 3, which can be easily predicted as being null at first glance, seeing as there is no physical interaction between them.

### 2.6 Boundary Conditions

The final part discussed in this chapter is that of the imposition of the boundary conditions. These correspond to known values taken by the system at particular points, mainly in the domain boundary, and come up in two different forms, essential and natural boundary conditions, distinction required when dealing with the Weak Form of the problem.

Essential boundary conditions are inputs that unequivocally define the primary variable value at particular geometries of the model. It can correspond to a null displacement at a fixed feature such as a wall mounted face or a known temperature at the surface of an object. As the name suggests, these types of conditions are required for an actual numerical result to be found, seeing as they allow the reduction of the stiffness matrix to a non-singular form, in opposition to beforehand, where rigid body motion was possible.

Natural boundary conditions on the other hand are part of the second member of the main equation, being sources of discontinuities of secondary variables, such as stress or heat flow, within the model or applied solicitations on the boundaries. In the case of a structural problem for instance, these involve external forces or distributed pressures or even concentrated forces at a given internal point. The lack of these last two however, implies the need for all but boundary nodal applied forces to be zero, equation (2.24), seeing as equilibrium dictates the sum of all applied forces at a given point to result in that same value.

The typical boundary value problem solved through the FEM requires both types of conditions to be applied, however, due to their nature, with essential ones relating to the exact variation and natural ones to the source of said variation, both types can't be applied to a single node.

All previous steps taken, the only thing remaining in the solution step is the actual solving of the equations for obtaining primary variables and their distribution on non-nodal coordinates as well as possible extra calculations for obtaining secondary variables when solicited.
2.7 Closing Remarks

An important note should be made aware before going forward. The matter of the use of the FEM is a highly complex one, not to mention ever evolving. This preludes the fact that the examples presented in this more general overview correspond to those of extreme simplicity, modelled around one or two-dimensional elements with a distinctively discrete-like nature.

This forced simplicity allows for an approach based on the strength of materials from the Mechanics of Materials or Applied Mechanics branches of the Engineering tree, where concepts like tension, compression, bending, torsion, can be somewhat decoupled and used on a case-by-case basis by extension of the element matrices to account for the required deformation source, which is viable and allows for optimal performance when evaluating one-dimensional and some simpler instances of two-dimensional elements. For additional information on this subject refer to [19] and [20].

These articulated systems, however, cannot represent the behavior of more complex, three-dimensional non-discrete systems, requiring a method based on the elasticity concepts of Solid Mechanics, the study of continua. The structure related concepts applied in this chapter, derived straight from the area of Mechanics of Materials from the previous paragraph, in fact connect to, or even stem from, Solid Mechanics, presenting themselves as simplifications that a more general focus on continuum allows, an example of this is the typical, simpler approach to torsion, where the cross sections are assumed to remain plane and undistorted, even if this is only true for circular shafts.

Nonetheless, even though these approximations are only exactly that, they are often precise enough to deal with the problem at hand, and this simplicity translates into the potential for easier mesh setup and, especially, vast improvements in computational speeds. As such, any part requiring discretization should be looked upon as potentially requiring only one or two dimensional meshes. The former type can often be represented by beam or bar elements, and the later by plates or shells, according to the problem requirements.

Of course, this is not always possible, and complex interactions can sometimes prevent this simplification, even in circumstances where the model itself is clearly skewed towards a lesser dimensional representation. This once again attest to the need for careful consideration of all possibilities with regards to any sort of analysis, with even the simplest having potentially unexpected pitfalls, which can only be circumvented through experience and validation, whether it is through real life testing or even through extra studies run on known or predictable results.
Chapter 3

Structural Analysis

3.1 Introduction and Chapter Organization

Man-made objects are fast taking the spotlight as the majority shareholders of this planet’s habitable space, surgically injecting themselves into every living being’s orbit at an often times alarming rate. The sheer volume of these is only eclipsed by the fact that comparatively few of them are structurally flawed, a tremendously positive outcome considering that the large number of, nowadays invaluable, habitation and vehicle related structures are constantly permeated by numerous human lives, turning any oversight or malfunction into a possible source of chaotic loss.

This attests to the quality, not only of the work but also of the methods and understanding developed throughout time, which allowed for the whole process from idea brainstorming to qualified prototypes to be increasingly shortened, whilst keeping production values and safety margins exceedingly high. Behind these improvements are systemic approaches, focused efforts on general problems as opposed to particular conundrums, leading to all-encompassing solutions, in other words, a one size fits all mentality that endeavors to allow any new problem to be tackled, regardless of specifics. This flexibility takes form in the software packages available for commercial use that have been designed with it in mind, allowing for most engineering problems to be solved under the same license, or new solutions to be applied for those that can’t.

The current chapter will therefore present a number of structural analyses to the aforementioned ISTSat-1, vital to path charting throughout its development and acceptance campaigns. Static, Modal and Random Vibration analyses will be the focus, including set-up, results and final conclusions, always taking into account that which is defined as required by the competent entities. Each of these shall also be accompanied by succinct, individual, expositions on their importance and objective, together with additional, smaller, simulation models and results to emphasize related concepts.

As previously stated, and although the typical approach for the kind of documentation here presented mirrors the iterative nature of the work itself, a bigger emphasis is given to the end results and lessons
learned from them than the actual means and presentation of every step of the long process towards an acceptable final state.

These will be preceded by a theoretical section, this time centered around structural concepts, particularly those defining the elasticity theory, in the context of the FEM, which should hopefully lend some help in grasping the basics of both how results are obtained and their validity from a qualitative perspective.

All of this section needs to take into consideration that the fact that the structure at stake needs to work mostly in a space environment requires several concessions or extra bits of knowledge that should orient the designers’ and analysts’ roles. It is therefore important to keep in mind that some design decisions may have derived from this, in particular from the work developed by the ECSS, or the European Cooperation for Space Standardization, which have, beyond defining the standards to be followed in any typical space related endeavor, drafted a number of handbooks to help guide the work under a typical workflow, some of which relating to the structural aspects [21-29]. Beyond this, a few theses within the same framework have been used as references, [30-32], here discussed seeing as their influence permeates this chapters, not just isolated passages.

### 3.2 Theory of Elasticity

The concept of elasticity is very much a universal one, defining a behavior that affects any known and, presumably also, unknown material. It reports to the fact that the application of a force on a structural element leads to a deformation which, depending on its nature, can be reversed if and when said force is removed from the system. Within certain limits, this configuration change can be predicted on the basis of a fairly linear relationship with the applied stress, being purely dependent on material properties. Considering its importance towards the intended simulations, a brief overview of these concepts and formulations is presented, following information that can be gleamed from a few, fairly known books [17], [20] and [34-35].

![Figure 3.1 – General State of Stress at an Arbitrary Point](image1.png)

![Figure 3.2 – General Strain Components (Normal Strain on the Left, Shearing Strain on the Right)](image2.png)
The first point of business is splitting the applied forces into two types, those that are distributed along the surface of the body, such as pressure, and aptly named surface forces, the general formulation of which can be seen in Figure 3.1, and those that are distributed throughout the volume of the body, again suggestively called body forces. For equilibrium to be achieved, it is required for both to equate to the inertial forces, bringing about the net force to zero on all directions, and as such defining the governing equations for the problem at hand, presented here as equation (3.1).

\[
\begin{align*}
\sigma_x \frac{\partial x}{\partial x} + \tau_{xy} \frac{\partial y}{\partial y} + \tau_{xz} \frac{\partial z}{\partial z} + F_x &= \rho \ddot{u} \\
\tau_{yx} \frac{\partial x}{\partial y} + \sigma_y \frac{\partial y}{\partial y} + \tau_{yz} \frac{\partial z}{\partial z} + F_y &= \rho \ddot{v} \\
\tau_{zx} \frac{\partial x}{\partial y} + \tau_{yz} \frac{\partial y}{\partial y} + \sigma_z \frac{\partial z}{\partial z} + F_z &= \rho \ddot{w}
\end{align*}
\]

The stress having been considered, the strain becomes the next point of focus, being a measure of the displacement normalized by the direction, similar to how the stress is a measure of the applied surface forces by unit of area.

The strain of any element can be easily separated, similar to stress, into several components, relating to the primary displacement variables \((u, v, w)\).

\[
\begin{align*}
\epsilon_x &= \frac{\partial u}{\partial x} \\
\epsilon_{xy} &= \frac{\partial u}{\partial y} \\
\epsilon_{xz} &= \frac{\partial u}{\partial z} \\
\epsilon_y &= \frac{\partial v}{\partial y} \\
\epsilon_{yz} &= \frac{\partial v}{\partial z} \\
\epsilon_z &= \frac{\partial w}{\partial z}
\end{align*}
\]

Do note that all the concepts regarding strain presented in this section are applicable seeing as the deformations are assumed to be infinitesimal in nature, which tracks with the fact that the analyses presented later on are taken as being linear.

Before moving on, however, an extra simplification can be made, deriving from moment considerations on the upper right diagram of Figure 3.2, that dictate that both the strain and stress matrices be symmetric. The effect of this on the stress matrix is simply defining shear stresses with same indexes, regardless of order, to be the same. The strain matrix nonetheless suffers a slight differentiation, with shear related components taking the form of a sum of these, and the new variables taking the notation \(\gamma\).

\[
\begin{align*}
\tau_{xy} = \tau_{yx} \\
\tau_{yz} = \tau_{zy} \\
\tau_{zx} = \tau_{xz}
\end{align*}
\]

\[
\begin{align*}
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\
\gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}
\end{align*}
\]

The constitutive formulation is at this point established, through Hooke’s Law, defining the relationship between the stress and strain components at an arbitrary point. Considering the matrix nature of both,
the material properties entity should be a 4th order tensor, however, due to the simplifications done in equation (3.3), the component matrices see their independent entries reduced to six, which presents an opportunity to present these in a vector form and allows for the tensor to suffer an order reduction.

\[ \sigma_{ij} = E_{ijkl} \varepsilon_{kl} \rightarrow \{\sigma\} = [C_m]\{\varepsilon\} \equiv \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} 1 - \nu & \nu & 0 & 0 & 0 & \nu \\ -\nu & 1 - \nu & 0 & 0 & 0 & 0 \\ \nu & -\nu & 1 - 2\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - \nu & 0 & 0 \\ \nu & 0 & 0 & 0 & 1 - \nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - \nu \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{pmatrix} \]  

\[ \text{with} \quad \nu = -\frac{\varepsilon_y}{\varepsilon_x} \]  

\[ E = \frac{\sigma_x}{\varepsilon_x} \]  

The number of independent entries in matrix \([C_m]\) is dictated by the type of material in use, whether it be anisotropic, orthotropic or isotropic, ranging from fully dependent to fully independent of direction respectively.

The case of isotropic materials is of particular interest seeing as most metals, in high usage in space applications, belong to this class. This type of material can be fully defined by the use of two constants only, resulting in a properties matrix with relatively few non-zero entries, which is presented in equation (3.5). These two constants are the Young’s Modulus (\(E\)), which relates same direction normal stress and strain components, and the *Poisson’s Ratio* (\(\nu\)), dictating the relation between normal strains in perpendicular directions.

\[ \{\sigma\} = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{pmatrix} 1 - \nu & \nu & 0 & 0 & 0 & \nu \\ -\nu & 1 - \nu & 0 & 0 & 0 & 0 \\ \nu & -\nu & 1 - 2\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - \nu & 0 & 0 \\ \nu & 0 & 0 & 0 & 1 - \nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - \nu \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{pmatrix} \]  

The case of orthotropic materials on the other hand, requires the Young’s Modulus and Poisson’s Ratio to be computed for each of the three major directions, resulting in six different constants to use for obtaining the properties matrix, the formulation will, however, be skipped for briefness’ sake.

At this point it is possible to define the differential operator matrix, easily obtained through analysis from equations (3.1) and (3.2) and immediately used to simplify both. The presentation of the transpose version of the matrix is simply for a smaller footprint.

\[ [T]^T = \begin{pmatrix} \partial/\partial x & 0 & 0 & \partial/\partial y & 0 & \partial/\partial z \\ 0 & \partial/\partial y & 0 & \partial/\partial x & 0 & \partial/\partial z \\ 0 & 0 & \partial/\partial z & 0 & \partial/\partial y & 0 \end{pmatrix} \rightarrow [T]^T\{\sigma\} + \{f\} = \rho(\ddot{u}) \]  

\[ \{\varepsilon\} = [T]\{u\} \]  

All relations having been established, a final solution with only the primary variable, displacements, as the dependent value is possible. Do note that the inertial factor is also present, indicating a dynamic problem. If this isn’t the case, the object’s initial momentum is unchanged, represented by a null right-hand side of equation (3.1), leading to a static problem.

\[ [T]^T[C_m][T]^T\{u\} - \rho(\ddot{u}) = -\{f\} \]  

Shifting focus towards a more **FEM** oriented approach, the use of the virtual displacement principle in its dynamic form, the Hamilton principle, presented in equation (3.8), allows for a quick route towards
defining all the required element matrices. The process can become quite extensive if all intermediate steps are presented and therefore only the final result is demonstrated, with the previously referred sources being an illuminating read for detailed understanding if at all necessary.

\[
\int_{V_e} \left( \sigma_{ij} \delta\epsilon_{ij} + \rho \ddot{u}_i \delta u_i \right) dV - \int_{V_e} f_i \delta u_i dV - \int_{S_e} t_i \delta u_i dS = 0
\]  

(3.8)

The last two integrals of the equation will not be treated at this point, being related to applied surface forces in the latter, and body forces in the former, being fairly easy to commit both to vector form. Either way, both contribute to the force vector, while the first integral can be subdivided into contributions to the stiffness matrix and the mass matrix respectively.

The shape functions can also be placed in matrix form in order to solve the weak formulation underneath, with the application of the differential operator matrix to it adding another fundamental matrix to the mix.

\[
[N] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots & N_n & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 & N_n & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \cdots & 0 & 0 & N_n \end{bmatrix} \quad \text{and} \quad [B] = [T][N] \quad (3.9)
\]

In the end, the joining of all concepts and matrix definitions presented in this section, applied to the principle of virtual displacements represented by equation (3.8) results in both required element matrices.

\[
[K^e] = \int_{V_e} [B^e]^T [C^e] [B^e] dV \quad [M^e] = \rho \int_{V_e} [N^e]^T [N^e] dV
\]  

(3.10)

### 3.3 Static Analysis

The current subsection deals with the possible effects derived from a constant, and fairly large, acceleration being applied to the satellite during its launch into orbit. References for this section include [34].

#### 3.3.1 Basic Notes

The elasticity theory as defined in section 3.2 is based on Hook’s Law, that is, it works under the assumption of linear behavior between stress and strain components of the material at stake. However, as is obvious, there is a limit to the amount of stress any object can withstand without long term consequences, that is, irreversibility becomes a reality, after which the regime becomes non-linear, before finally reaching the rupture point.
The end goal is therefore projecting a structure that manages to remain structurally sound when under typical usage conditions, that is, under worst case scenario occurrences, the object retains enough structural integrity to assure mission compliance. This requirement may be lax enough to only prohibit fracture, or as restrictive as forcing the material to remain in the elastic domain for the entirety of the predicted lifespan.

An understanding of this concept does lead to discovery of one basic, yet vital, problem. On a uniaxial tension problem for instance, the existence of a single stress component allows for direct comparison with yield and/or fracture values, ending in a simple Boolean logic result. The existence of multiple components of stress on an arbitrary three-dimensional object on the other hand sweeps the triviality off the table, and the need for a scaling standard becomes supremely important, enter the von Mises criterion.

This criterion is based on the distortion energy of any given material, that is, the energy associated with the variation in shape of an object. Underneath it lies the principle that the material foregoes yield as long as this distortion energy per unit volume on the deformed part remains below that of the same variable obtained at yield from a tensile-test specimen from the same material. Relating these concepts, it is feasible for a particular state of multi-component stress to be reduced to a single value, that of the so-called von Mises stress, for comparison with the yield strength.

This criterion’s usual form is presented with the principle stresses as their variables, eigenvalues from the stress tensor that correspond to the stress state along the coordinate axes defined by the eigenvectors. In general terms, they are the diagonal entries of the stress matrix in reference to a basis that maximizes normal stresses and minimizes shear ones, who reach zero. These stresses are defined as \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) in descending order of magnitude. The von Misses stress therefore results in the following equation.

\[
\sigma_v = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}
\]  

(3.11)
3.3.2 Singularities

One of the biggest problems that plague any structural problem solved through the FEM is that of singularities, often not understood, which is why this particular section delves a bit further into this concept, in hopes of providing some guidance, or at the very least some understanding of its nature, its origin and its impact on the end result.

The term singularity shows up in plenty of areas of study, from mathematics to natural sciences and even as far as technology, but even though its context may be taken from a large pool of influences, the overall idea tends to remain immutable in a sense. Singularity can be defined, perhaps ironically, as that which can’t be defined, the exception to the rule of a well-behaved function for instance, the point at which it tends to infinity.

The definition also applies in this context, mainly through the fact that for particular points in the model, where there are abrupt changes in direction of material, forming angular features, the strain and stress values, which are calculated based on displacement derivatives in order of general coordinates, see their values skyrocket due to a theoretically infinite value being reached, and the refinement of the mesh does not offer convergence, with each recurrent reduction in the element size being accompanied by an ever increasing value of the secondary variables, see Figure 3.5.

This is an example of the type of circumstance where the simulation does not represent the real world properly, for as much as that were the intention, any real structure cannot have true angles, with a small fillet or bevel showing up, and therefore bringing the strain and stress values back to a finite domain. This type of feature may be represented in the model itself, however, it increases the complexity of the meshing process and, in large structures with significant numbers of nodes, this may become impeditive.

Several methods exist to deal with these singularities, many of each attempt to eliminate them from the model itself, however, a common practice, and the one used in this work, is that of ignoring these results in detriment of values obtained in the adjacent nodes, which, if close enough to the singularity, represent the type of stress values expected to be found in case a fillet is simulated. An example of this kind follows.

![Figure 3.4 – Displacement of a Structure with Fillet (Left) and Without Fillet (Right)](image)
The values of stress taken from adjacent nodes to the singularity converge to similar values to those obtained with an analysis at the same structure with a fillet, which allows to show the relative validity of this compromise, where the model simplification by removal of these rounded features improves efficiency whilst not doing so at the expense of accuracy.

### 3.3.3 Analysis

The group of structural analyses presented in this chapter are guided by similar decisions, one of which is referent to the mesh in use, since, as previously stated, the converge is a rather important factor in the proper acceptance of the results. With this in mind, that shall be the first order of business when presenting these results, even though the analysis themselves are required for said conclusion, considering that the fact that all analyses are interconnected allows for the same mesh to be used, provided a significant change doesn’t occur in the conditions of the model. Of course, this only prevails in case due precautions are taken and the mesh is properly vetted.

Although an overall mesh refinement is often used to validate the finite element model, in most cases little to no refinement is required in the vast majority of the model, with typically critical areas being the ones surfacing as complex interfaces and/or unusual shapes. Therefore, to ensure correct results whilst avoiding computationally excessive calculations, the mesh was refined locally, mainly on holes and contact regions, to allow for singularity and stress concentrations to be identified and distinguished, leading to the values referent to the former case being ignored in detriment of the node values on adjacent elements.
This convergency study was made taking into account all three major directions, with each critical area being individually considered, an example of which is presented in Table 3.1 and Figures 3.6 and 3.7, showcasing the analysis to a hole with stress concentration, the refinement around this feature having been made through the use of a mesh control command defining the number of elements on its edge.

As can be seen, the variation between the use of 12 and 24 elements on the edge of the hole promotes a change in less than 2 percent of the stress value on a similarly placed node in all three meshes, leading to the conclusion that the mesh has converged in this particular area.

Similar considerations were made for other holes and contact areas where a stress concentration/singularity could not be easily distinguished until the numerical behavior could be clearly categorized as one or the other.

The convergency having been assured, the focus shifts to the enforcement of boundary conditions, both essential and natural, in different nodes. The structure’s lower standoffs are assumed to remain in contact with either the bottom of the P-POD or the upper standoffs of other satellites at all times, due

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Local Edge Elements</th>
<th>Maximum Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>6 Elements</td>
<td>0.0941</td>
</tr>
<tr>
<td>Vertical</td>
<td>12 Elements</td>
<td>0.0691</td>
</tr>
<tr>
<td>Position</td>
<td>24 Elements</td>
<td>0.0680</td>
</tr>
<tr>
<td>Final Error (%)</td>
<td>1.62%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 – Mesh Convergence Results
to compressive forces, which, for simplicity’s sake, is translated into a fixed constraint, while the outer surfaces of the rails are considered to slide along the rails of the pod, without perpendicular movement to them allowed, naturally simulated as a slider constraint.

As for the natural conditions, an overall acceleration was applied to the entirety of the satellite. Due to the small effect the possibility of extra CubeSats being placed atop the structure in case has, this when compared to the rather large accelerations to which it might be subjected during launch, those are dropped for the use of an overall acceleration of 19g’s, above the predicted maximum of 18.1g’s of the fastest launcher in consideration for the mission, as per reference [36].

The position and orientation in which the P-POD is to be placed within the launcher being an as of yet undecided factor, the analysis regarding directional loads are required to be ran in multiple configurations, in this particular case a repeat in each of the three main directions being the accepted approach.

The following groups of images provide an idea of the overall distribution of both displacement and stress throughout the model for each of these directional analyses. Of note is the fact that the stress images are presented with the colour gradient defined under a logarithmic scale, allowing for a better distinction of values along the structure in general, for the presence of outliers, such as singularities, could create too homogenous a colour pattern from a linear based scale.
The maximum displacement corresponds to 0.184 mm, a rather minute value, predictably reached by the EPS board for an acceleration directed on the Z direction. Besides this fact, the solar panels surrounding the structure can be seen to present even smaller displacements, which serves to predict a fairly comprehensible isolation of the overall envelope, important to ensure that there is no destructive interaction with other possibly present CubeSats or the pod itself.

As for the stress values, a Margin of Safety (MOS) is expected to exist so that, as the name implies, for unexpectedly worse conditions than those predicted for use in the analyses, a fair margin for error be guaranteed.

\[
MOS = \frac{\text{design allowable load}}{\text{design limit load} \times FOS} - 1 \tag{3.12}
\]

And seeing as the yield strength isn’t surpassed in the model, the load is assumed to vary linearly with the stress and the design allowable load and design limit load are replaced by design allowable stress and design limit stress respectively.

The design allowable stress corresponds to the yield or ultimate strength of a given material, depending on the values assumed for the Factor of Safety, see below.
The **design limit stress** corresponds to the highest obtained stress for a given material through the analysis at stake multiplied by the qualification factor (taken as 1.25 given the fact that it is a satellite analysis on the basis of global flight loads), the project factor (which given the final states of its development is taken as 1) and the model factor (which is taken as 1.1 considering the successful flight heritage and small variation between satellites of this type).

The **Factor of Safety (FOS)** corresponds to a Yield Design Factor of Safety (**FOSY**) or Ultimate Design Factor of Safety (**FOSU**) value, depending on whether the yield or the ultimate strength are taken as the **design allowable stress**, multiplied by a local design factor (taken as 1.2 in areas close to singularities and/or joints and as 1.1 elsewhere).

The various factors presented in the previous couple of paragraphs can be traced back to the reference documentation regarding Factors of Safety for Spaceflight Hardware [37].

This result of this calculation is presented for every main material in use in every direction, Aluminum (for the Structure and Upper Antenna), FR4 (for the **PCBs**, Solar panels and Lower Antenna) and Polyethylene (for the PC104 connectors), and the process taken is also repeated for the Random Vibration Analysis at a later section.

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Strength (MPa)</th>
<th>Limit Stress (MPa)</th>
<th>FOS</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>572</td>
<td>14</td>
<td>1.5</td>
<td>18.8</td>
</tr>
<tr>
<td>FR4</td>
<td>276</td>
<td>16</td>
<td>1.5</td>
<td>7.4</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>40</td>
<td>3</td>
<td>1.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

**Table 3.2 – Margin of Safety Calculation for the Static Analysis in the Z Direction**

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Strength (MPa)</th>
<th>Limit Stress (MPa)</th>
<th>FOS</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>572</td>
<td>19</td>
<td>1.5</td>
<td>13.6</td>
</tr>
<tr>
<td>FR4</td>
<td>276</td>
<td>8</td>
<td>1.5</td>
<td>15.7</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>40</td>
<td>1</td>
<td>1.5</td>
<td>18.4</td>
</tr>
</tbody>
</table>

**Table 3.3 – Margin of Safety Calculation for the Static Analysis in the Y Direction**

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Strength (MPa)</th>
<th>Limit Stress (MPa)</th>
<th>FOS</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>572</td>
<td>17</td>
<td>1.5</td>
<td>15.3</td>
</tr>
<tr>
<td>FR4</td>
<td>276</td>
<td>9</td>
<td>1.5</td>
<td>13.9</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>40</td>
<td>2</td>
<td>1.5</td>
<td>8.7</td>
</tr>
</tbody>
</table>

**Table 3.4 – Margin of Safety Calculation for the Static Analysis in the X Direction**
The absolute minimum expected being a positive value, the considerably high values obtained for the MOS for all materials and in all cases bring a considerable degree of confidence in the original structure’s design, ending the Static Analysis.

### 3.4 Modal Analysis

The current subsection deals with the determination of the system’s natural frequencies and corresponding modes of vibration, required for ensuring that typical long-term frequencies during the ascent do not put its integrity at risk. References for this section include [38-40].

#### 3.4.1 Basic Notes

The dynamic behavior of a system, as previously stated, comes into play when the results are of the time dependent variant. This can, in very general terms, be divided into two different scenarios, based off of the loads applied being a constant fixture or only the inducing factor, leading to forced vibrations, the former, or free vibrations, the latter.

A modal analysis worries about the case of free vibrations, the behavior of the structure when, after being forced into a vibrating state, it’s left to reach equilibrium. The set-up for this particular case is therefore an undamped and unforced system, as stipulated by equation (3.13).

\[
[M]\{\ddot{u}\} + [K]\{u\} = 0
\]  
(3.13)

The typical assumption for a problem of this kind is that of the primary variable function taking the form of a harmonic solution, which allows for a simple derivation and linearization process to lead to the familiar form of an eigenproblem, where \(\omega\) corresponds to the frequency assumed for the harmonic function and \(\phi\) to its amplitude.

\[
([K] - \omega^2[M])\{\phi\} = 0
\]  
(3.14)

Outside of the trivial solution corresponding to a zero displacement, the end result can be obtained by solving for the determinant of the matrix, leading to a number of different values, one for every degree of freedom in the system, each of which a so-called natural frequency, accompanied by the modal shapes.

\[
([K] - \omega_i^2[M])\{\phi_i\} = 0 \quad \rightarrow \quad f_i = \frac{\omega_i}{2\pi} \, \text{natural frequencies} \\
\{\phi_i\} \, \text{modal shapes}
\]  
(3.15)

The mode shapes are of particular interest seeing as they define the overall shape the structure takes when reacting to the corresponding natural frequencies, whilst owning the properties of orthogonality, which is to say that all are unique, completely distinct from all others, and it is not possible to obtain any one shape by linear combinations of the rest.
The modal analysis is extremely important to ensure that the structure remains far from any possibly long term applied load of frequency close to that of the fundamental one, first natural frequency, or a phenomenon known as resonance can kick in, whereas the natural damping of the structure reaches a minimum and the load provides an additive effect to the vibration of the structure, ultimately leading to failure.

![Figure 3.12 – First Modal Shapes of a Cantilever Beam](image)

The greatest contribution of the modal analysis, however, falls on the matter of the basis it provides for further dynamic analysis, both when it comes to informing the decisions of the analyst and reducing computational load, seeing as the deflected shape of a structure subjected to free or forced vibrations may be taken, once again, as a linear combination of the various modes.

An important note of contention is regarding the physical interpretation of the natural shapes, which cannot, as many may assume, be taken at face value. These only correspond to a relative displacement, giving an idea of the kind of shapes that should be expected, not the exact ones, the amplitude for which is dependent on both the natural frequencies and the applied loads. The result of this is that, for each natural frequency, as soon as one of the degrees of freedom’s displacement is obtained, all remaining $n - 1$ values are immediately defined.

Assuming that both stiffness and mass matrices are symmetric and real, and that the indexes of the mode shape vectors are the same, or else the result is null due to orthogonality, the diagonalization of the matrices leads to the following relations.

$$\{\phi_i\}^T[M]\{\phi_i\} = m_i \text{ ith generalized mass}$$

$$\{\phi_i\}^T[K]\{\phi_i\} = k_i \text{ ith generalized stiffness}$$

$$k_i = \omega_i^2 m_i \quad \rightarrow \quad \omega_i^2 = \frac{\{\phi_i\}^T[K]\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}}$$

The last equation presented in this formulation is known as the Rayleigh equation, and is often times used to roughly ballpark the natural frequencies of any given structure based on the element matrices at use. Even though the fine prints on how this is achieved is not discussed in this work, the equation itself presents the possibility to vaguely conclude that the natural frequencies of any object are lower for structures with less stiffness and more mass.
3.4.2 Typical Convergence and Rigid Body Motion

The matter of mesh refinement and the validity that comes with it has already been touched upon in the context of the Static analysis, however, it is retreaded at this point to point out the difference between the convergence of the natural frequencies and mode shapes obtained from a Modal analysis and the stress and strain components derived from the prior analysis due to their disparate natures.

Stress and strain, as presented in the theoretical background patent at the beginning of the current chapter, are known, in the guise of the FEM, as secondary variables, that is, after the primary variable, displacement, is calculated, the post-processing of the final data includes a step to use the main dependents to chart the distribution of extra physical properties, chief amongst which are the prior two.

A quick glance at the equations dictating the currently in discussion analysis, on the other hand, tells a different story, with the intended variables corresponding to the eigenvalues and eigenvectors taken directly from the governing equation.

The conclusion is that the convergence necessities for the Modal analysis are typically fairly shallow, in relative terms, although a detailed mesh analysis is not to be ruled out purely on the face of these allegations. This is, once again, demonstrated through a fairly basic example, where a simple metal sheet is clamped, for a Modal analysis, and solicited, for a Static one, as in Figure 3.13. The graph on the left presents the frequency and stress results obtained as a function of the number of elements of the model, while the one on the right graphs the error variation when compared to the previous iterations, starting at a common 10%. Do note that the stress evolution with mesh refinement takes a perceivable amount of iterations to reach an acceptably stable behavior, whilst the many natural modes taken for the example converge almost immediately, the exception being mode 3 that presents a peak early on, nonetheless dropping below the stress curve to join the rest of the modes with final relative errors within the 1% margin.

![Figure 3.13 – Convergency (Left) and Error (Right) of Mode Frequencies in Hz and Maximum von Mises Stress in MPa with Number of Elements](image)
This fast convergence can be justified by the fact that an accurate value for fundamental frequencies simply requires that the distribution of nodes throughout the mesh allow for a vague representation of the final shape of said mode, as per Figure 3.14, where even an unpresentably coarse mesh provides presentable values.

![Figure 3.14 – Fundamental Frequency with a Coarse (Left) and a Fine (Right) Mesh](image)

Also of note is the fact that the Modal analysis can be used as a rigid body check, seeing as any unconstrained element of the mesh will be flagged by a near zero value for the first few respective fundamental frequencies, corresponding to the unconstrained degrees of freedom that a given part(s) of the model is(are) allowed to change in the capacity of a rigid body movement.

### 3.4.3 Analysis

The Boundary Conditions applied for the analysis in case correspond to those that simulate a hard-mounted configuration, with the lower standoffs being fixed, as in the Static analysis, as well as the outer faces of the rails. This is achieved through the connection of all the nodes in question to an external one, Figure 3.15, to which a fixed constraint is applied. This connection is established through the use of one-dimensional elements known as *Rigid Body Elements* in their second form (*RBE2*), which allow for the connection of one node of independent degrees of freedom to multiple nodes whose degrees of freedom become dependent on the prior.

Although the modal analysis itself does not require this type of connected constraint, it is nonetheless a common practice for running Random Vibration analyses, difference being in that case the external node is not defined as a fixed feature but is instead the origin of the variable vibration load.
The Modal Analysis’ results are presented through both a table of a few of the lesser resonant frequencies and a group of images of corresponding modal behaviors to illustrate how the model, and therefore, within a certain range, the actual satellite resonates with a number of frequencies. Although for this Modal analysis only the first few are presented, all frequencies up to 2000Hz were calculated in anticipation of the requirements for the Random Vibration analysis to follow.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>192.41</td>
</tr>
<tr>
<td>2</td>
<td>295.09</td>
</tr>
<tr>
<td>3</td>
<td>328.27</td>
</tr>
<tr>
<td>4</td>
<td>331.06</td>
</tr>
<tr>
<td>5</td>
<td>374.97</td>
</tr>
<tr>
<td>6</td>
<td>390.51</td>
</tr>
<tr>
<td>7</td>
<td>450.73</td>
</tr>
<tr>
<td>8</td>
<td>475.37</td>
</tr>
</tbody>
</table>

Table 3.5 – First 8 Natural Frequencies of the Structure

Figure 3.16 – 1st and 2nd Mode Shapes
According to these results, the lowest frequency sits comfortably above the required 100Hz minimum resonant frequency required in hard-mounted configuration, as per [36]. Beyond this, a deeper look at the first few modes shows that the EPS is the main character, which is in line with the fact that this PCB is the heaviest of the stack, together with the fact that they are the elements that present a lesser stiffness due to the choice made for their mounting interface.

The Modal analysis also provided, as per the user request, a *modal effective mass distribution* output in the form of a table of values relating to each of the involved frequencies. The importance of this data becomes clearer with the requirements for a proper Random Vibration analysis, and can therefore be checked in the respective section.
3.5 Random Vibration Analysis

The current subsection deals with the effects of the non-deterministic vibration profile that the satellite is subjected to during the take-off phase, required for ensuring that peak stress values are unlikely to promote long term damage to the structure. References for this section include [39-41].

3.5.1 Basic Notes

The Random Vibration analysis is a subset of the Frequency Response analysis type, and therefore this last one shall be summarily touched upon before concepts more specific to the Random Vibration analysis are discussed.

The solution process for a Frequency Response analysis is rather similar to that of the Modal analysis already presented, the biggest departures being the damping and the fact that the vibration is forced, as opposed to the previously alluded to free vibration. At its root, instead of being allowed to respond naturally, the structure, or more particularly parts of the structure, are forcefully accelerated at a given frequency (or multiple). In the end, the equation at its base is the same, with the exception of the applied load, as well as the assumption that the primary variable’s change follows the shape of a harmonic function.

\[
[M]\ddot{x}(t) + [C]\dot{x}(t) + [K]x(t) = \{P(\omega)\}e^{i\omega t} \quad \text{with} \quad \{x\} = \{u(\omega)\}e^{i\omega t}
\] (3.17)

At this point a choice is made, represented in most, if not all software packages, as different solutions. The equation may be solved “as is” with the primary variable being derived twice and implemented in the equation of motion. This approach provides the most accurate result, although it is computationally much more expensive, and is aptly known as the Direct Frequency Response. On the other hand, the properties of the modal shapes may be used, resulting in the Modal Frequency Response. Through this method, the mode shapes are used to allow a change of coordinates, from physical ones, \(u\), to modal ones, \(\xi\).

\[
\{x\} = [\phi]\{\xi(\omega)\}e^{i\omega t}
\] (3.18)

The equality presented in equation (3.18) only applies when all eigenvectors have been calculated and applied, which is quite the tall order. Usually only a subset of the modes is used, making this formulation an approximation, albeit a practical one due to its decent results, assuming a minimal coverage of the modes, and its comparatively lightweight nature.

The derivation of this and subsequent insertion of the result in equation (3.17), together with the diagonalization of the matrices, results in a set of single degree of freedom equations, that can easily be solved for the modal coordinates and, therefore, to the overall displacement as a function of frequency.

\[
-\omega^2 m_i \xi_i(\omega) + i\omega c_i \xi_i(\omega) + k_i \xi_i(\omega) = p_i(\omega) \quad \Leftrightarrow \quad \xi_i(\omega) = \frac{p_i(\omega)}{-m_i \omega^2 + i c_i \omega + k_i}
\] (3.19)
It is important to note, however, that in case the damping matrix cannot be diagonalized, equation (3.19) can’t be reached and the solver falls back to a Direct Frequency Response analysis.

A Random Vibration, on the other hand and as the name suggests, is a type of non-deterministic vibration, as in, there is no way to describe it in an exact form as a function of time. This type of analysis is fairly recent in so far as structural requirements go, and is of particular interest to areas that fall under the purview of aerospace applications, where somewhat random phenomena such as flutter and violent pressure changes from atmosphere crossing need to be considered in some way.

Seeing as a deterministic definition for the input load is not possible, the probabilistic route is the most obvious solution, where well defined concepts like magnitude give way to those such as expected values and variances. Out of all the scenarios that may be defined under these conditions, a particular subset becomes exceedingly important for its relative ease of use, that of stationary processes. These follow a very particular set of rules, not to be covered here, with the exception of perhaps the most important for immediate comprehension, the fact that they are independent of time from a statistical standpoint, which is to say that the probability distribution is unchanged throughout the time dimension. In a broad sense, this can be evaluated through the concept of autocorrelation, or similarity between different points in time of a single random process, equation (3.20), with \( E \) being the expected value.

\[
R_c(\tau) = \lim_{T \to \infty} \left( \frac{1}{T} \int_0^T u(t)u(t-\tau)dt \right) \quad \text{or} \quad R_c(\tau) = E[u(t)u(t-\tau)] \tag{3.20}
\]

On the other hand, applying a Fourier Transform to the autocorrelation function, and taking the delay out of the equation and assuming a typically expected mean for a random process to be zero, the standard deviation, \( \sigma \), of the process comes to light as the Root Mean Square (RMS), the square root of the arithmetic mean of a power based function, in this case \( S(\omega) \), known as the Power Spectral Density (PSD), the power of a signal distributed over frequency, equation (3.21). This concept is a bit of a misnomer, seeing as the part of the acronym that defines the P, power, is not necessarily related to the concept of energy as a function of time, but to the squaring of a signal type function, meaning that concepts like velocity or displacement can be used. For instance, one of the most typical inputs, and the one used in this work, is based on acceleration, Acceleration Spectral Density (ASD), with units of \( g^2/Hz \).

\[
R_c(0) = E[|u(t)|^2] \quad \rightarrow \quad \sigma^2 = \frac{1}{2\pi} \int_0^\infty S(\omega)d\omega \tag{3.21}
\]

This is when the results obtained from the Frequency Response analysis come to bear. The primary variable’s changes as a result of a given variable load having been charted as a function of frequency, a transfer function can be modelled for the whole spectrum of interest, \( H(\omega) \). Also, any random vibration can, at this point, be defined statistically in the form of a PSD function, and it can be proved that the output follows this pattern as well, whilst being defined by equation (3.22).

\[
S_y(\omega) = |H(\omega)|^2S_i(\omega) \tag{3.22}
\]
The output is then obtained, and it can be analyzed in the form of the Power Spectral Density Function for the primary variable or, more likely considering the requirements, in the form of the RMS based on said function, allowing for components of stress and strain to be computed. The Root Mean Square, as stated prior, corresponds to the standard deviation of a statistical function, and therefore stands to reason that it doesn’t encompass the worst case scenario values, only a fraction of the whole probability field, which can be estimated by following the assumption that the type of distribution at play is a normal one. Equation (3.23) presents the overall equation dictating a normal distribution, but in the case in question it is important to remember that the mean value \( \mu \) has already been taken as zero.

\[
f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3.23}
\]

![Figure 3.20 – Normal Distribution](image)

In the end, and as should be known from basic statistics, to ensure that possible results aren’t being ignored, ones to which material properties should be reviewed against, the value obtained for any RMS variable should be further multiplied by 3, therefore ensuring that the bases are covered for 99.7% of the time.

### 3.5.2 Modal Effective Mass Fraction and Unit Consistency

The fact that a Modal Frequency Response analysis is at the basis of all this discussion, with its results being dependent, as already stated, on the modes chosen for the approximation, brings forth the question regarding the number and particulars of those chosen for it. The possibility exists of using every single natural mode of the mesh at stake but only in theoretical terms, for in practice, the frequency range for any decently refined mesh is far too wide and results in an unfeasible number of modes to be considered. This assumedly complicated query may be resolved through the use of the Modal Effective Mass, a construct that correlates any given natural frequency with the mass of the model whose displacement it promotes, discretized in each of the six degrees of freedom, or more particularly, the Modal Effective Mass Fraction, which, as the name implies, provides a percentage based approach. Seeing as each natural mode correlates to different parts of the model, it is predictable that their total sum for each direction reaches unity. This is the fulcrum of the simplification and therefore the typical path to take involves assuring that most, not all, of the mass is involved in the solution,
particularly 80 to 90%, or use of the entire range of frequencies of interest if it resolves into a smaller set.

This information is usually an optional one, with settings required to be changed for it to be part of the provided output, in which case it tends to appear not on the graphical interface of the software, at least not directly, but on the result files, where it can still be easily accessed. The sheet metal example used in the Modal analysis section is once again the basis for the present one, where the effect of the main modes, as in those with the biggest mass contribution for the particular direction at stake, is verified.

Table 3.6 – Modal Effective Mass Fraction with Frequency

<table>
<thead>
<tr>
<th>Frequency</th>
<th>T3–Z Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>3207.54 Hz</td>
<td>3.797869E-25</td>
</tr>
<tr>
<td>3575.34 Hz</td>
<td>5.115994E-01</td>
</tr>
<tr>
<td>7306.09 Hz</td>
<td>5.116203E-29</td>
</tr>
<tr>
<td>7838.78 Hz</td>
<td>1.025826E-28</td>
</tr>
<tr>
<td>8632.41 Hz</td>
<td>2.385070E-01</td>
</tr>
<tr>
<td>9601.85 Hz</td>
<td>4.286219E-27</td>
</tr>
<tr>
<td>10256.72 Hz</td>
<td>4.425273E-29</td>
</tr>
<tr>
<td>14137.21 Hz</td>
<td>8.312528E-02</td>
</tr>
<tr>
<td>14189.06 Hz</td>
<td>1.559134E-21</td>
</tr>
</tbody>
</table>

Immediately a comparison between Figure 3.21 and Table 3.6 leads to a correlation being established between the higher values for the Modal Effective Mass Fraction and the peaks found on the maximum displacement graph, and the same kind of reasoning applies to other types of variables as well, with the most coveted overall maximum stress results showing up due to one of, or combination thereof, said principal modes. Important to note is the fact that with the nine modes presented, around 83% of the model, in mass terms, is affected, with a negligible different existing if all but the three most significant ones are ignored, the result from which is nigh on indistinguishable in the displacement front whilst saving considerably in the computational power department.

As for the case of the Random Vibration analysis itself, it follows very closely to that of the Frequency Response one, although something that warrants further attention is the consistency of units. Considering the number of exoteric concepts that become vital for a proper setup and later post-processing of the results, together with some possible metaphorical walls that restrict access under the hood of the software, sometimes ensuring that the simulation is valid becomes less than straightforward, particularly when it comes to the units required for the input load. The example of a frequency dependent enforced acceleration comes to mind, seeing as it becomes the focus of the actual analysis, although the end result being in RMS doesn’t exactly help. A good practice to dismiss this uncertainty, however, is to request the Acceleration Spectral Density function from the node(s) where the original acceleration
is enforced, meaning that, in case units are consistent, the value of this function should mirror that of the input ASD.

The graph presented in Figure 3.22 provides enough proof that the input is properly setup seeing as the blue line, corresponding to the acceleration of the driving node, correctly follows the predicted path. The red line is presented purely for illustrative purposes, showing the acceleration profile of a randomly selected node, where the same three peaks previously alluded to can still be found. The checks done, the von Mises Stress distribution presented in Figure 3.23 is validated, at least from an analysis standpoint.

### 3.5.3 Analysis

In the case of the Random Vibration Analysis, and with respect to the boundary conditions, an external node was created, connected by 1D elements to all the nodes required, as discussed in the Modal section.

The load to be applied serves as an orientation and extra multiplicative factor, seeing as the PSD input is presented in g’s as the units for acceleration. In this case, instead of a typical load, an acceleration is enforced on the referenced external node, with the magnitude of 9.81 m/s² and applied in the negative axis direction according to the simulation’s main axis.

The Random Vibration analysis, as previously discussed, requires a Frequency Response to be run beforehand, forcing a deck of excitation frequencies to be chosen. Since the structure naturally deforms to larger degrees closer to the natural frequencies obtained from a modal analysis, the choice ends up falling on these, to an extent, with a couple of extra frequencies around each of them also applied to ensure that the behavior close to resonance is also taken into account.

Seeing as some of the most significant parts of the CubeSat, mainly the structure itself, have no resonant frequencies below the 2000Hz threshold defined for the Random Vibration analysis profile, the typical 80 to 90% of the effective mass model being excited per direction can’t be reached, being instead exchanged for the use of all frequencies within the range of interest.
To avoid unrealistic deformations, a structural damping is applied to the entirety of the model, using a fairly typical 3% value.

Information pertaining to the Random Vibration itself is also vital for the setup, in this case, the qualification requirements defined as PSD inputs, for which Table 3.7 is used, taken from [42]. It is also presented in graphical form in Figure 3.24.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>PSD (g^2/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.057 (g^2/Hz)</td>
</tr>
<tr>
<td>20-153</td>
<td>0 (dB/oct)</td>
</tr>
<tr>
<td>153</td>
<td>0.057 (g^2/Hz)</td>
</tr>
<tr>
<td>153-190</td>
<td>+7.67 (dB/oct)</td>
</tr>
<tr>
<td>190</td>
<td>0.099 (g^2/Hz)</td>
</tr>
<tr>
<td>190-250</td>
<td>0 (dB/oct)</td>
</tr>
<tr>
<td>250-750</td>
<td>-1.61 (dB/oct)</td>
</tr>
<tr>
<td>750</td>
<td>0.055 (g^2/Hz)</td>
</tr>
<tr>
<td>750-2000</td>
<td>-3.43 (dB/oct)</td>
</tr>
<tr>
<td>2000</td>
<td>0.018 (g^2/Hz)</td>
</tr>
<tr>
<td>OA (Grms)</td>
<td>9.47</td>
</tr>
</tbody>
</table>

Table 3.7 – PSD Input per Frequency Band

As was the case with the Static analysis presented prior, the Random Vibration analysis is run in all three major directions, to ensure that the satellite can withstand the typical vibrations felt during launch, regardless of its position relative to the launcher vehicle itself. Again, following the pattern presented on the Static analysis, images of the end results are shown for illustrative purposes, later followed by a closer look at the highest registered values of stress for each material at stake and their respective Factor and Margin of Safety calculations.
The statistical considerations presented prior are now taken into account, with values obtained as the Root Mean Squares of the von Mises Stresses interpreted as corresponding to a standard deviation, only accounting for the values that are predictable to be limiting around 64% of the time. To increase this up to 99%, and with it ensuring a high likelihood of the attained values not being surpassed in a real-world scenario, the values taken as the limit loads for the Margin of Safety calculations are that of 3\(\sigma\). This is the most typical approach, as stated, however there are misconceptions around the fact that this procedure is acceptable for all scenarios. Cases such as those of Shock analysis are plenty of times extreme to the point of requiring higher probability levels to be assured for qualification purposes, with values given by 4 or even 5 times the standard deviation to be the norm.
Table 3.8 – Margin of Safety Calculation for the Random Vibration Analysis in the Z Direction

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Strength (MPa)</th>
<th>Limit Stress (MPa)</th>
<th>FOS</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>572</td>
<td>42</td>
<td>1.5</td>
<td>5.6</td>
</tr>
<tr>
<td>FR4</td>
<td>276</td>
<td>39</td>
<td>1.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>40</td>
<td>6</td>
<td>1.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 3.9 – Margin of Safety Calculation for the Random Vibration Analysis in the Y Direction

<table>
<thead>
<tr>
<th>Material</th>
<th>Ultimate Strength (MPa)</th>
<th>Limit Stress (MPa)</th>
<th>FOS</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>572</td>
<td>48</td>
<td>1.5</td>
<td>4.7</td>
</tr>
<tr>
<td>FR4</td>
<td>276</td>
<td>24</td>
<td>1.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>40</td>
<td>6</td>
<td>1.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 3.10 – Margin of Safety Calculation for the Random Vibration Analysis in the X Direction

The Margins of Safety are all positive, and giving a wide berth to the null value that could be cause for concern. This being the case, no changes are deemed necessary and the design from a structural point of view is considered acceptable in view of the requirements for the current development phase.
Chapter 4

Thermal Analysis

4.1 Introduction and Chapter Organization

The presentation provided at the start of the last chapter, focusing on matters of structural nature, can easily be extended to ring true for thermal considerations as well. Similar to the safety requirements arising with the ever increasing complexity of physical structures, the prevalence of growing power hungry technologies bring to the table an inordinate amount of energy, and its’ density distribution, especially throughout a litany of small form factored gadget and gizmos, promotes a serious need for thermal control, lest the induced temperatures and heat flows end up negatively affecting their performance, in the best case that is. Space based applications are no exception, with extreme thermal conditions routinely being a cause for contention, involving cases of high thermal loads due to solar radiation all the way to low temperatures derived from the overall cold space environment.

The designers and/or analysts responsible for keeping these problems from arising have at their disposal two different types of thermal control, active and passive, which are differentiated by the presence or absence, respectively, of external systems that make use of extra energy beyond the originally present in the system. However, the relatively small size of the satellite under development, aided by the also comparatively small complexity and power output of its systems, lead to an active control being too cumbersome, not to mention expensive, so most of the efforts in this regard are directed into finding passive systems for this task.

The current chapter presents all the required data and reasoning behind the Transient Heat analysis that is its focus, with an entire chapter dedicated to it due to both a different nature when compared to the structural ones and also its significantly higher degree of difficulty. Once again, the results and setup are the focus, encapsulated by the concepts of heat transfer laid out in the following subsection.

As a counterpart to the several documents that together make up the ECSS structural handbook, the equivalent documentation regarding the thermal aspects are also a significant part of the research process [43-58], together with a few other documents, from theses to articles [59-65].
4.2 Heat Transmission

The transmission of energy in its thermal form can be differentiated into different mechanisms, the predominance of each of which is dependent on the environment and application at stake. These modes are enumerated as follows.

• Conduction – Heat flow within a stationary medium, where a temperature gradient leads to kinetic energy transfer between particles in said medium in a naturally stabilizing fashion.

• Convection – Heat flow between a moving fluid in relation to a stationary surface at differing temperatures, with the random particle movement (diffusion) and the bulk movement (advection) of the fluid being the main contributors.

• Radiation – Energy emitted in the form of electromagnetic waves due to a non-zero temperature.

The condition of space application relegated to the currently in discussion project eliminates the need for convection to be considered, seeing as the relative vacuum denies the existence of a fluid-based transmission mode.

Any number of publications exist that relate these concepts in a multitude of ways, from simple explanations to in-depth discussions on their applicability. Beyond the several references presented in the introduction, a typical bibliographical mention for this body of work falls on [66], from which most of the information presented in this section was sampled. This book in particular comes highly recommended for anyone searching for extra information on the topic, especially seeing as the theoretical presentation to follow is rather lightweight, a necessity seeing as, just as was the case with Random Vibrations, the concepts of heat transfer are too wide for the scope of this text, especially considering the transient nature of the orbital simulations.

4.2.1 Conduction

As stated, conduction governs the flow of energy within a stationary element, whether it be solid, liquid or gaseous, with said energy being passed down by particle collisions from areas with high kinetic energy concentrations, higher temperatures, to those with lower. This is arguably the simplest of the heat transmission modes, from a general point of view, and its extension shows precisely that, with only a couple of concepts required for minimally acceptable understanding. Equation (4.1) presents the basic formulation for conduction, taken from Fourier’s Law.

\[
Q = -\left(k_x A_x \frac{\partial T}{\partial x} + k_y A_y \frac{\partial T}{\partial y} + k_z A_z \frac{\partial T}{\partial z}\right)
\]  

(4.1)

The heat transfer rate, \(Q\), is, as expected, proportional to the negative temperature gradient on a direction basis, as discussed, with the cross-sectional area, \(A\), and the thermal conductivity, \(k\), a material property, as coefficients.
This equation is simple enough for application on a single part to predict its temperature distribution, however, the case of different parts in contact becomes a whole different problem.

A common analogy made with respect to this problem is that of the thermal resistance, where the matter of heat transfer is equated to that of a resistance in an electrical circuit. An example of two objects in contact with a one-dimensional heat conduction transfer setup can be approximated by equation (4.2).

\[ Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{Total}}} \quad \text{with} \quad R_{\text{Total}} = \frac{L_1}{k_1 A_1} + R_{\text{Contact}} + \frac{L_2}{k_2 A_2} \]  

(4.2)

The important point to take from this equation is the presence of the term \( R_{\text{Contact}} \), that corresponds to a resistance responsible for simulating the intricacies of heat flow through the interface between the two objects. This is a highly sensible and highly volatile variable, a little more of which is discussed in latter sections.

### 4.2.2 Radiation

Radiation is the only heat transfer method that does not require matter to be present, seeing as the energy is transferred in the form of electromagnetic waves, and, therefore, able to propagate in the vacuum of space. In rather general terms, any object receives and emits radiation in a permanent fashion, which goes to say that a thermal system may become stationary, but never static. Although extremely varied in its sources and target objects, the quantitative impact of this process is fairly easy to account for, at least in a simplified form, which most of the times tends to be enough for actionable intel to be obtained.

The flow of energy transferred through radiation is essentially a play on weighted components, with all the steps from emission, to absorption and back to emission being a chain of non-perfect processes where the efficiency is dictated by what are called optical properties of the material. The departure for this analysis centers on the concept of a black body, a theoretical entity, physically impossible, that absorbs all radiation that reaches it and emits the maximum amount of energy possible by unit area, dictated by the *Stefan-Boltzmann’s Law*, a simplified version of which is presented in equation (4.3).

\[ E_b = \sigma T^4 \quad \text{where} \quad \sigma = 5.670 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \]  

(4.3)

This defining a perfect object with regards to radiation processes, both the emission and absorption of a real entity is treated as a fraction of this total, impact of which is safeguarded by the application of coefficients that dictate these properties. Therefore, in a somewhat intuitive form, the emissivity, \( \epsilon_R \), dictates the relation between the amount of energy emitted by the object to which the coefficient is applied, radiated energy \( (E_R) \), and a black body, whereas the absorptivity, \( \alpha_R \), reflectivity, \( \rho_R \), and transmissivity, \( \tau_R \), dictate similar relations between the amount of energy absorbed, reflected or transmitted, respectively, and the total incident energy on a body, irradiated energy \( (G_R) \).

\[ E_R = \epsilon_R E_b \quad , \quad G_{R\text{abs}} = \alpha_R G_R \quad , \quad G_{R\text{ref}} = \rho_R G_R \quad , \quad G_{R\text{trs}} = \tau_R G_R \]  

(4.4)
These coefficients, so called optical properties, as previously referred, follow a particular set of rules, dictated by concepts of conservation of energy, mainly that the sum of those related to the irradiated energy, incident radiation, needs to return unity, and due to the opaque nature of plenty of materials, the value of transmissivity is often neglected. Also of note is the fact that the simple equations presented hide a crucial, yet often overlooked aspect, which is their dependency on direction and frequency, although they are not of particular interest for this work.

The emitted and incident radiation having been referred, there is still the matter of how these two properties relate to each other, which, perhaps predictably, is mostly dependent on geometric properties, with the effects of non-vacuum like interstitial mediums in between or light bending considerations not taken into account. This leads to the definition of View Factors, percentage-based values that relate the radiation emitted by a given surface and the one intercepted by another, as per equation (4.5).

\[ F_{i-j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j \] (4.5)

The overall process discussed, all that is left is to define all the radiative components that affect a change in the thermal environment of the satellite in development, quickly enumerated as follows.

Solar radiation is pretty self-explanatory, accounting for the direct transfer between the sun and the satellite, equation (4.6).

\[ Q_{solar} = \alpha_{satellite} \varepsilon_{sun} F_{sun-satellite} A_{sun} E_b \] (4.6)

In this case, the emissivity of the sun is considered to be unity, in other words, it is assumed to behave as a black body, a typical approximation considering the fact that a significant number of celestial corpses present an emissivity coefficient that approaches it.

This radiation extends to the Earth as well, where a significant fraction is reflected back and captured by the satellite, meaning the last equation may be considered, substituting the satellite for the Earth and the absorptivity for the reflectivity, which in turn can be considered as the emitted radiation towards the satellite, equation (4.7).

\[ Q_{albedo} = \alpha_{satellite} \varepsilon_{Earth} F_{Earth-satellite} a F_{sun-Earth} A_{sun} E_b \] (4.7)

The a component corresponds to the albedo, serving a role much like the reflectivity, only that it represents a non-wavelength dependent value that defines the reflection of the entire solar spectrum on the Earth as a whole, accounting for the presence of atmosphere and its unpredictability, as well as typical optical properties of materials on the ground, therefore being mostly dependent on latitude.

Besides the reflected radiation, Earth also emits its own, on the infrared spectrum, which, given its closeness to the satellite when relativized by that of the sun, also presents a significant contribution. This follows much the same idea as that of the direct solar radiation, equation (4.8).

\[ Q_{Earth} = \alpha_{satellite} \varepsilon_{Earth} F_{Earth-satellite} A_{Earth} E_b \] (4.8)
In this case, just like with the radiation originating on the sun, the Earth is assumed to behave like a black body. Also of note is the fact that the fairly stationary thermal equilibrium verified on Earth leads credence to the fact that the energy emitted by the Earth as infrared radiation mirrors that of the incident radiation coming from the space environment, which mostly boils down to that of the sun, due to its energy output.

Finally, the satellite itself also radiates to space, regardless of the target of said radiation, equation (4.9).

\[ Q_{\text{satellite}} = A_{\text{satellite}} \varepsilon_{R_{\text{satellite}}} E_b \]  

(4.9)

This sums up all the contributions that radiation-based heat transfer provides from the space environment, to or from the exterior of the system. Beyond this, however, the radiation couplings within the CubeSat itself, equation (4.10), and the heat generated due to thermal bleed from electronic components, equation (4.11) must be accounted for.

\[ Q_j = \alpha_j \sum_i F_{i-j} A_i \varepsilon R E_b \]  

(4.10)

\[ Q_{\text{joule}} = UI \]  

(4.11)

The last equation derives from Joule’s Law, relating the potential and current intensity through a given component to the power expelled as heat by that very same component.

### 4.2.3 Transient Heat Equation for FEM

The discussed concepts of conduction and radiation are applied to a finite element model, according to both the specifications of the project and the discretion of the analyst, their contributions to each node being patent in the transient heat equation used for the solution of the problem, equation (4.12).

\[ [B][\dot{T}] + [K][T] + [R][T]^4 = [P] \]  

(4.12)

In this equation, the matrices \([K]\) and \([R]\) can easily be taken as representing the conduction and radiation respectively, with the vector \([P]\) the heat loads. The bigger difference between a static and a transient heat problem lies in the presence of the matrix \([B]\), known as the heat capacity matrix, which works analogously to that of the damping matrix in structurally oriented problems, defined by grouping the heat capacity values for the different parts. This material specific property defines the amount of energy required to enact a unit change in temperature, and is often one of the most important reasoning factors, from a thermal perspective, behind the requirement of cross-checking density values assigned to parts of the model with the actual part’s expected mass, seeing as a significant discrepancy may drastically affect the results.
4.3 Thermal Conduction Between Parts

Considering the importance and relative complexity of the conduction of heat between different parts, an extra section devoted to it is presented here, with the main objective being a presentation of the sensitivity of the thermal energy distribution on how the interfaces are modelled, seeing as, according to equation (4.2), this behavior can be summed up in a resistance value, or, as most commonly used in this work, its inverse, from here on out referred to as Conductive Transfer Coefficient.

Any material, no matter the treatment it receives, possesses a degree of roughness to it, a perfectly smooth surface being, by definition, unattainable. As such, only a small fraction of interacting surfaces are actually in contact (typically less than 2%), which provides the reasoning behind the difficulty inherent to this problem, with contact thermal resistances being exceedingly dependent on geometrical, thermal and mechanical properties, among which the actual geometry of the contact points, the thickness of the gap, the interstitial material/fluid, the registered pressure values, thermal conductivities, elasticity modulus and others. This leads to a need for simplification, however, even this reduction of complexity is generally difficult enough to model as it is, resulting in cumbersome expressions with large numbers of parameters, many of which impossible to determine at a design and analysis phase.

As an attempt to simulate this variation whilst maintaining enough simplicity for the analyses at stake, a conscious conduction coefficient variation based on pressure is ultimately used, with higher stress values promoting higher conduction coefficients and vice-versa. The impact of this parameter can be gleamed from a simple example, presented through Figures 4.1, 4.2 and 4.3, using two blocks of the same material in contact with each other, an enforced temperature value at one extremity of the assembly and varying values for the Conductive Transfer Coefficient being used.

The presented results purport to the state of the system after a set time is past, and the graphs in Figures 4.2 and 4.3 clearly demonstrate a significant change in the temperature value at the free end of the simulated model as a function of the conditions defined for the interfacing surfaces. The obvious conclusion arises that, in the case of a non-finite conduction parameter, the corresponding resistance would take a null value and its effect would be non-existent, which added to the fact that both parts in the model are assumed to be made of the same material, leads to the predicted behavior being indistinguishable from the use of a single part.
Figure 4.2 – Temperature Variation with Length as a Function of Conductive Transfer Coefficient

Figure 4.3 – Temperature Variation at End Surface of Model as a Function of Conductive Transfer Coefficient

4.4 Analysis

The overall process for acceptance on the structural side is simple enough to present, generally boiling down to an applied force or enforced acceleration as input and a limiting factor to which the end results are to be compared as requirements. The case of the thermal analysis is a lot more involved, from individual temperature ranges for each part when it comes to requirements, and everything from orbit definition and conduction and radiation contacts to heat loads applied based on mission profile input wise. This is data that needs to be part of the analysis definition, to both inform and orient it, which is why a good amount of this section is dedicated to it.
4.4.1 Requirements

Due to a lack of information on the part of the hardware providers, true operational and non-operational temperature ranges are hard to ascertain for any of the involved parts, with the exception of the battery, the most critical item to be assessed. Nonetheless, for completeness’ sake, a historically based and piecewise created table is here presented with what is taken as being a fairly accurate estimation of the typical numbers expected and used as basis for the project at hand, see Table 4.1. The operational and non-operational temperature ranges correspond, respectively, to those that need to be maintained during the operation phases and those that need to be maintained for all times, including storing and satellite deployment.

<table>
<thead>
<tr>
<th>Parts</th>
<th>Operational Range (°C)</th>
<th>Non-Operational Range (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Up to 700</td>
<td>Up to 700</td>
</tr>
<tr>
<td>Solar Panels</td>
<td>-55 to 150</td>
<td>-65 to 160</td>
</tr>
<tr>
<td>ADS-B Antenna</td>
<td>-50 to 105</td>
<td>-60 to 115</td>
</tr>
<tr>
<td>UHF Antenna</td>
<td>-20 to 60</td>
<td>-50 to 85</td>
</tr>
<tr>
<td>TTC</td>
<td>-40 to 70</td>
<td>-65 to 105</td>
</tr>
<tr>
<td>OBC</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
</tr>
<tr>
<td>EPS</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
</tr>
<tr>
<td>COM</td>
<td>-30 to 80</td>
<td>-65 to 105</td>
</tr>
<tr>
<td>PL</td>
<td>-40 to 85</td>
<td>-65 to 105</td>
</tr>
<tr>
<td>Battery</td>
<td>-10 to 50</td>
<td>-20 to 60</td>
</tr>
</tbody>
</table>

Table 4.1 – Temperature Requirements

It is worth noting that in the cases where the non-operational temperature range couldn’t be reliably obtained, Solar Panels and the ADS-B Antenna in particular, the non-operational range for analysis comparison was defined as a conservative +/-10 °C. Additionally, temperature ranges for each individual printed circuit board correspond to the limiting elements’ respective range, mainly microcontrollers, microprocessors and DSPs.

4.4.2 Simulation Components

The present section serves to describe all the heat-based components considered in the simulation, comprising of the Orbital Heat and Radiation modules, Radiation and Conduction Couplings and Heat Loads applied.

Seeing as a steady state analysis doesn’t provide actionable data, considering that a satellite never remains in the same place long enough for it to reach this state, such a simulation is foregone in substitution of a transient analysis, defined with a convergence constraint based on cyclic orbit, meaning
that it considers the solution as converged when the differences between orbits, from a temperature point of view, are less than 0.1ºC. Additionally, and as latter presented, the time dependent nature of the thermal loads leads to an impossibility, in many cases, to reach such convergence, at which point a full day is used as an average, corresponding to, approximately, 16 orbits, which generally allow for the minimum and maximum overall temperatures to be obtained.

To take into account all possible scenarios of a typical mission, leading to temperature ranges that faithfully represent all probable outcomes, different analyses were ran, according to different mission profiles, the differences amongst which are presented in due course.

**Orbital Heat and Radiation Modules**

The Orbital Heat Module allows for model elements to be chosen so that view factors may be calculated with respect to the sun and the earth, and therefore the solar, albedo and Earth’s infrared radiation contributions absorbed by the satellite. These values are calculated based on the definition of a particular orbit, together with the satellite’s attitude. An example of a considered orbit, taken directly from the software in use, is hereby presented.

![Figure 4.4 – Satellite Orbit Representation](image)

Besides this, it is necessary to also consider the radiation emitted from the satellite to the exterior environment, task which is taken up by the Radiation module. This simulation object works in tandem with the Orbital Heat one, defined above, by calculating view factors and allowing for calculations of radiation exchange with the outside to be carried on.

The attitude of the satellite can be represented through some additional data given as a part of the input for the Orbital Heat Module, mainly the rotational velocity vector and the amplitude. This functionality is taken advantage of by simulating a value of 5 degrees per second with the axis of rotation being the normal of the upper Solar Panel and applied at its center point. Additionally, the **ADS-B** antenna is defined as pointing towards Nadir at all times. The rotational velocity value presented was assured to be attainable by the Attitude Determination and Control System (**ADCS**) department.
Beyond this, a tumbling case is also defined, based on the Nominal parameters to be described below, and with a rotational axis defined by the diagonal of the CubeSat and a rotational speed of 30 degrees per second. Furthermore, and to improve the randomness of the rotation, the upper Solar Panel is directed towards the Sun at all times.

**Radiative and Conductive Couplings**

A more precise approach has to be taken when considering the inside of the satellite, requiring manual establishment of radiation and conduction between different parts of the assembly model. This is the part that requires the most experience from an analysis point of view, seeing as defining every single existing radiation and conduction pairing is not only time consuming, impedingly so, but may also drastically increase computational time. Fortunately, the model in case is fairly small and the amount of conduction and radiation couplings possible is substantially smaller than in any typical large scale engineering project, and therefore a more encompassing approach may be taken, although many of the possible radiative couplings are ignored due to the clearly small view factor between them, as well as their relatively small dimensions.

The radiation couplings are therefore mostly defined between each adjacent printed circuit board and the upper and lower ones with the **UHF** and **ADS-B** antennas respectively. Additionally, the radiation between the back plate of the solar panels and each of the **PCBs**, together with the batteries, is also accounted for.

From a software point of view, the radiation couplings have, as inputs, solely the pairs of surfaces between which the radiation should be considered, the rest being automatically defined on the basis of the previously presented optical properties, defining the fraction of radiation absorbed and emitted from both parts of the radiative pairs.

The conductive couplings on the other hand are defined between every interacting surface from different parts in contact, allowing for heat to be transferred from any part and to every part of the assembled model. Unfortunately, as presented, the thermal contact resistance, or its counterpart, the conductive transfer coefficient, is a highly variable parameter, and very difficult to truly estimate, hence the different values eventually used for the same materials in contact as a function of predicted pressure.

Many simulations were run, and different results checked against predicted outcomes and temperature readings from satellites already in flight in an attempt to validate the choice of coefficients, having historically used and experimentally obtained results as the basis for a typical range of values. As such, those presented here were obtained after careful comparison with a number of different sources, scientific papers and books alike [59] and [67-68], and the aforementioned temperature readings made available to the team.
<table>
<thead>
<tr>
<th>Interfacing Materials</th>
<th>Loose ($W/m^2\cdot K$)</th>
<th>Normal ($W/m^2\cdot K$)</th>
<th>Tight ($W/m^2\cdot K$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum-Aluminum</td>
<td>20</td>
<td>150</td>
<td>800</td>
</tr>
<tr>
<td>Copper-Copper</td>
<td>30</td>
<td>300</td>
<td>1000</td>
</tr>
<tr>
<td>Aluminum-Copper</td>
<td>30</td>
<td>300</td>
<td>1000</td>
</tr>
<tr>
<td>Copper-Polyethylene</td>
<td>-</td>
<td>5</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.2 – Interface Conductive Coefficients

As a rule of thumb, the values under the **Tight** column are used when the interfacing surfaces are pressed against each other by the effect of fasteners, such as bolts. An example of this is the contact between the frames and the solar panels. Under the **Loose** column the values are used in interfacing surfaces that do not suffer from normal pressure between them, such as the contact between the printed circuit boards and the axes. The values under the **Normal** moniker are used elsewhere, mainly in the contacting interfaces between PCBs and spacers.

**Heat Loads**

The Heat Load Module allows for a specific value of power to be applied to any model element. In this particular case, it is used to represent the energy dissipated by the printed circuit boards due to the Joule effect that translates into energy lost by heat. Due to the difficulty inherent to the modelling of all the different components within each of the PCBs, defining heat sources within their plane can become complicated, not to mention the fact that not all information regarding said boards’ organization was privy to the analyst before the corresponding simulations and, as such, the heat loads are defined as uniform throughout the allotted area.

**4.4.3 Mission Profiles**

To ensure that all scenarios are accounted for, four different profiles are established and simulated: Cold, Nominal, Hot and Tumbling cases. The Cold and Hot cases represent the absolute extremes of the breadth of situations that the satellite might face, and therefore, the non-operational temperature ranges at least have to be respected. On the other hand, the Nominal case is presented as an overall likely scenario that should be expected to represent the behavior during the vast majority of the time. The Tumbling case, based on the Nominal parameters, provides assurance that the satellite maintains operability, from a thermal standpoint, in case issues arise in the ADCS or before it is employed.

The differences between these profiles exist mainly in the definition of the orbit, through the Orbital Heat module, and the Heat Loads applied, all the rest remaining the same. These variations are presented in the following subsections.
Orbital Heat Module

The main difference between the various cases when it comes to the Orbital Heat module is the average sun flux that reaches the satellite. Those values are presented in Table 4.3, together with the rest of the orbital parameters in use.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Cold Case</th>
<th>Nominal Case</th>
<th>Hot Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun Flux ( W/m^2 )</td>
<td>1323.6</td>
<td>1357.4</td>
<td>1411.6</td>
</tr>
<tr>
<td>Minimum Altitude ( km )</td>
<td></td>
<td>401</td>
<td></td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.000432</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbit Inclination ( ^\circ )</td>
<td>51.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argument of Periapsis ( ^\circ )</td>
<td>264.38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.3 – Orbital Parameters Divided by Cases*

Besides this, and to account for even harsher possible conditions, the values of Albedo and Earth's Infrared (IR) radiation are assumed to suffer small variations, across the different scenarios, around the average values of 0.3 and 237 \( W/m^2 \) respectively, as in Table 4.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Cold Case</th>
<th>Nominal Case</th>
<th>Hot Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albedo</td>
<td>0.285</td>
<td>0.3</td>
<td>0.315</td>
</tr>
<tr>
<td>Earth's IR ( W/m^2 )</td>
<td>227</td>
<td>237</td>
<td>247</td>
</tr>
</tbody>
</table>

*Table 4.4 – Albedo and IR Variation*

Heat Loads

As for the case of heat loads, the different conditions considered involve variations in heat dissipation on the part of the printed circuit boards, with the values applied for each case presented in table form Table 4.5. Duty-cycle based variations in applied heat loads, presented in the table as two values in the same entry, are discussed and explained bellow.

<table>
<thead>
<tr>
<th>Case</th>
<th>TTC (mW)</th>
<th>OBC (mW)</th>
<th>EPS (mW)</th>
<th>COM (mW)</th>
<th>PL (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold</td>
<td>52</td>
<td>28</td>
<td>175</td>
<td>28</td>
<td>6</td>
</tr>
<tr>
<td>Nominal</td>
<td>800/52</td>
<td>40</td>
<td>680</td>
<td>120/28</td>
<td>360/6</td>
</tr>
<tr>
<td>Hot</td>
<td>1000/65</td>
<td>180</td>
<td>770</td>
<td>150/35</td>
<td>450/8</td>
</tr>
</tbody>
</table>

*Table 4.5 – Heat Loads Applied to each PCB on a per Case Basis*

- TTC – In the Nominal case, the load applied to the board is modelled as dissipating 800 mW for 10 minutes followed by 4 hours and 38 minutes of 52 mW dissipation. This cycle is repeated, leading to 5 transmission periods per day. In the Hot case, the dissipation and time of transmission is increased, to
1000 $mW$ and 15 minutes respectively, followed by the remaining 4 hours and 33 minutes with 65 $mW$ dissipation.

- **COM** – In the Nominal case, the load applied to the board is modelled as dissipating 120 $mW$ for 4 hours followed by 28 $mW$ of dissipation for the remaining duration of the day, 20 hours. In the Hot case the power and duration of the peak dissipation are increased to 150 $mW$ and 8 hours respectively, followed by the remaining 16 hours with 35 $mW$ dissipation.

- **PL** – The duty cycle of the Payload follows closely to that of the COM board, with 4 hours in the Nominal or 8 hours in the Hot cases of peak dissipation and the rest of the day at lower levels, 20 or 16 hours accordingly. The dissipation values for this case are given as 360 $mW$ followed by 6 $mW$ in the Nominal case and 450 $mW$ followed by 8 $mW$ in the Hot one.

Beyond the dissipation applied to the different boards based on their use, the cold case also requires the use of battery heaters to maintain their temperature within operating ranges, simulated by the application of a heat load of 0.4 $W$, aided be the heater functionality of the software, allowing for cut-in and cut-off temperatures to be defined, in this case 1 and 6.5 °C respectively.

All of the cases also have an applied thermal load of 40 $mW$ on the upper antenna to simulate its dissipation.

### 4.4.4 Results

The present section serves as a place to provide the end results of each of the stipulated analyses in the format of a table with operational, non-operational and minimum and maximum values for each of the components. The accompanying images serve purely for the purpose of exemplification of the result formats through visual cues. The columns corresponding to the minimum and maximum with uncertainties accounted for are obtained by adding -10 or 10 °C respectively to the minimum and maximum obtained directly from the software, a conservative measure to account for the absence of an extensive battery of sensitivity analyses.

![Figure 4.5 – External View of Results](image1.png) ![Figure 4.6 – Internal View of Results](image2.png)
### Cold Case

<table>
<thead>
<tr>
<th>Parts</th>
<th>Operational Range (°C)</th>
<th>Non-operational Range (°C)</th>
<th>Minimum (°C)</th>
<th>Maximum (°C)</th>
<th>Minimum with Uncertainty (°C)</th>
<th>Maximum with uncertainty (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>&lt;700</td>
<td>&lt;700</td>
<td>-31.7</td>
<td>3.8</td>
<td>-41.7</td>
<td>13.8</td>
</tr>
<tr>
<td>Solar Panels</td>
<td>-55 to 150</td>
<td>-65 to 160</td>
<td>-34.9</td>
<td>8.4</td>
<td>-44.9</td>
<td>18.4</td>
</tr>
<tr>
<td>ADS-B Antenna</td>
<td>50 to 105</td>
<td>-60 to 115</td>
<td>-27.4</td>
<td>-1.2</td>
<td>-37.4</td>
<td>8.8</td>
</tr>
<tr>
<td>UHF Antenna</td>
<td>-20 to 60</td>
<td>-50 to 85</td>
<td>-30.1</td>
<td>2.9</td>
<td>-40.1</td>
<td>12.9</td>
</tr>
<tr>
<td>TTC</td>
<td>-40 to 70</td>
<td>-65 to 105</td>
<td>-12.3</td>
<td>-5.8</td>
<td>-22.3</td>
<td>4.2</td>
</tr>
<tr>
<td>OBC</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
<td>-8.4</td>
<td>-6.0</td>
<td>-18.4</td>
<td>4.0</td>
</tr>
<tr>
<td>EPS</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
<td>-4.4</td>
<td>-3.4</td>
<td>-14.4</td>
<td>6.6</td>
</tr>
<tr>
<td>COM</td>
<td>-30 to 80</td>
<td>-65 to 105</td>
<td>-7.7</td>
<td>-5.9</td>
<td>-17.7</td>
<td>4.1</td>
</tr>
<tr>
<td>PL</td>
<td>-40 to 85</td>
<td>-65 to 105</td>
<td>-12.6</td>
<td>-6.9</td>
<td>-22.6</td>
<td>3.1</td>
</tr>
<tr>
<td>Battery</td>
<td>-10 to 50</td>
<td>-20 to 60</td>
<td>0.4</td>
<td>2.2</td>
<td>-9.6</td>
<td>12.2</td>
</tr>
</tbody>
</table>

*Table 4.6 – Temperature Values for Cold Case*

### Nominal Case

<table>
<thead>
<tr>
<th>Parts</th>
<th>Operational Range (°C)</th>
<th>Non-operational Range (°C)</th>
<th>Minimum (°C)</th>
<th>Maximum (°C)</th>
<th>Minimum with Uncertainty (°C)</th>
<th>Maximum with uncertainty (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>&lt;700</td>
<td>&lt;700</td>
<td>-30.8</td>
<td>18.5</td>
<td>-40.8</td>
<td>28.5</td>
</tr>
<tr>
<td>Solar Panels</td>
<td>-55 to 150</td>
<td>-65 to 160</td>
<td>-34.7</td>
<td>13.0</td>
<td>-44.7</td>
<td>23.0</td>
</tr>
<tr>
<td>ADS-B Antenna</td>
<td>50 to 105</td>
<td>-60 to 115</td>
<td>-26.9</td>
<td>6.9</td>
<td>-36.9</td>
<td>16.9</td>
</tr>
<tr>
<td>UHF Antenna</td>
<td>-20 to 60</td>
<td>-50 to 85</td>
<td>-27.8</td>
<td>5.8</td>
<td>-37.8</td>
<td>15.8</td>
</tr>
<tr>
<td>TTC</td>
<td>-40 to 70</td>
<td>-65 to 105</td>
<td>-5.9</td>
<td>2.0</td>
<td>-15.9</td>
<td>12.0</td>
</tr>
<tr>
<td>OBC</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
<td>2.2</td>
<td>6.4</td>
<td>-7.8</td>
<td>16.4</td>
</tr>
<tr>
<td>EPS</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
<td>15.0</td>
<td>19.2</td>
<td>5.0</td>
<td>29.2</td>
</tr>
<tr>
<td>COM</td>
<td>-30 to 80</td>
<td>-65 to 105</td>
<td>5.6</td>
<td>13.3</td>
<td>-4.4</td>
<td>23.3</td>
</tr>
<tr>
<td>PL</td>
<td>-40 to 85</td>
<td>-65 to 105</td>
<td>-4.7</td>
<td>1.2</td>
<td>-14.7</td>
<td>11.2</td>
</tr>
<tr>
<td>Battery</td>
<td>-10 to 50</td>
<td>-20 to 60</td>
<td>14.7</td>
<td>18.9</td>
<td>4.7</td>
<td>28.9</td>
</tr>
</tbody>
</table>

*Table 4.7 – Temperature Values for Nominal Case*
### Hot Case

<table>
<thead>
<tr>
<th>Parts</th>
<th>Operational Range (°C)</th>
<th>Non-operational Range (°C)</th>
<th>Minimum with Uncertainty (°C)</th>
<th>Maximum with Uncertainty (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>&lt;700</td>
<td>&lt;700</td>
<td>-26.9</td>
<td>30.8</td>
</tr>
<tr>
<td>Solar Panels</td>
<td>-55 to 150</td>
<td>-65 to 160</td>
<td>-30.0</td>
<td>20.0</td>
</tr>
<tr>
<td>ADS-B Antenna</td>
<td>-50 to 105</td>
<td>-60 to 115</td>
<td>-22.3</td>
<td>9.3</td>
</tr>
<tr>
<td>UHF Antenna</td>
<td>-20 to 60</td>
<td>-50 to 85</td>
<td>-26.1</td>
<td>14.2</td>
</tr>
<tr>
<td>TTC</td>
<td>-40 to 70</td>
<td>-65 to 105</td>
<td>0.7</td>
<td>19.1</td>
</tr>
<tr>
<td>OBC</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
<td>14.6</td>
<td>21.9</td>
</tr>
<tr>
<td>EPS</td>
<td>-40 to 85</td>
<td>-55 to 105</td>
<td>24.9</td>
<td>31.5</td>
</tr>
<tr>
<td>COM</td>
<td>-30 to 80</td>
<td>-65 to 105</td>
<td>14.9</td>
<td>24.9</td>
</tr>
<tr>
<td>PL</td>
<td>-40 to 85</td>
<td>-65 to 105</td>
<td>3.7</td>
<td>20.1</td>
</tr>
<tr>
<td>Battery</td>
<td>-10 to 50</td>
<td>-20 to 60</td>
<td>26.0</td>
<td>31.1</td>
</tr>
</tbody>
</table>

*Table 4.8 – Temperature Values for Hot Case*

### Tumbling Case

<table>
<thead>
<tr>
<th>Parts</th>
<th>Operational Range (°C)</th>
<th>Non-operational Range (°C)</th>
<th>Minimum with Uncertainty (°C)</th>
<th>Maximum with Uncertainty (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>&lt;700</td>
<td>&lt;700</td>
<td>-30.5</td>
<td>19.0</td>
</tr>
<tr>
<td>Solar Panels</td>
<td>-55 to 150</td>
<td>-65 to 160</td>
<td>-33.7</td>
<td>14.3</td>
</tr>
<tr>
<td>ADS-B Antenna</td>
<td>-50 to 105</td>
<td>-60 to 115</td>
<td>-26.3</td>
<td>8.5</td>
</tr>
<tr>
<td>UHF Antenna</td>
<td>-20 to 60</td>
<td>-50 to 85</td>
<td>-27.2</td>
<td>7.1</td>
</tr>
<tr>
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*Table 4.9 – Temperature Values for Tumbling Case*
An overview of the results returns the conclusion that the temperature values obtained for most of the parts are within the operational range, even with the uncertainties accounted for, and all without exception are within the respective non-operational ranges. The UHF Antenna is the only one where the operational range is not respected. Nonetheless, the operational range defined for this particular part is done so in respect to the processors responsible for the deployment action, the melting of the lines, which takes a few seconds at most. The temperature variation of the printed circuit board in the antenna, as in Figure 4.7, however, makes it clear that most of the time the antenna remains well within the required operational range for the deployment, after which the processors become passive components and do not require the given operational range to maintain the antenna functioning properly.

Figure 4.7 – UHF Antenna Temperature Variation in Nominal Case

Heat Paths

The temperature distribution obtained from the different mission profiles seems to validate the end design of the satellite, however, especially due to the variability of the results with small material property changes, the heat flow throughout the model becomes a valuable tool, both as a possible source of data for specifically targeted changes and as a sanity check, by providing an idea of the direction of flow and areas of energy accumulation that may require change should the test campaign prove unfavorable towards the design.

The heat circulates essentially in the direction of the structure followed by the solar panels, whether it has its origin in the solar panels themselves, due to incident solar radiation, or in the various PCBs due to energy dissipation as a result of the Joule effect from their functioning.

On the outside, the energy tends to accumulate in the solar panels in direct view of the sun, being conducted towards the structural frames afterwards and eventually redirected to the rest of the solar panels.
On the inside, the energy responsible for heating the printed circuit boards circulates to the axes and, subsequently, the structural frames. During this process, some of the flow is absorbed by the spacers, allowing for more energy to be retained on the inside. Besides the conduction towards the axes, the energy also travels towards the components in direct contact with the PCBs, mainly the battery packs.

On the tail end of the process, with most of the heat having been redirected to the solar panels and/or structural elements, the radiation process emits a fraction of said energy to the exterior.

Figure 4.8 – Conduction Flow in the Frames (Left) and in the Axes (Right)
Chapter 5

Conclusion

5.1 Final Remarks

The process required for validation of a CubeSat is rather lengthy, and detailed within the expected margins. It does not require as much work as a fully-fledged typically sized satellite, mostly due to questions such as survivability, long term mission requirements or existing risks of debris reaching the ground in case of re-entry, not to mention the obvious question of dimensions, but even still, plenty of often faced issues are common to both sides of this vast spectrum. Regardless of size, the matters of structural integrity, thermal balance, energy storage, electronic hardware, software suites, everything needs to be accounted for and designed with a variable mission statement and/or environment in mind, seeing as changes are a common occurrence in this line of work, and a certain flexibility is therefore not only appreciated but vital. A simple discussion and application of concepts is insufficient, a research and development pipeline is a necessity, and besides software based analysis, actual real-life scenarios must be simulated and a true hardware testing campaign completed, more so than in any typical, run-of-the-mill engineering application, for the risks to human lives and other possible vital material losses represent too heavy a variable to be left to chance.

The results provided in this body of work should prove that, from an analysis point of view, the design of the satellite is sufficiently robust to tackle the predicted structural and thermal environment variability, especially after some of the changes made to the original designs, not presented here. The choices regarding spacers’ materials and optical properties chosen for the structure, through the use of a particular finishing, in this case, black-anodization, were enough to clear all requirements and reduce the likelihood of any particular issues arising, whilst leaving the groundwork developed for further improvement after the testing campaign, in case the results differ significantly from those predicted by the software in use and patent in the previous chapters.

Before the enunciation of work to be developed in the close future, the following small section is dedicated to the problems encountered and developed work not presented in this document.
5.2 Difficulties and Work not Presented

The matter of FEM is a fairly complex one, the level of which is far from having been represented in the presented outline, however, some particular problems arose that should perhaps be discussed.

Unlike other software packages, the documentation provided with the Siemens NX™ is not exactly forthcoming, and the typical sources for help with regards to this kind of package are diminished in both number and quality. The end result was a lot of time spent with attempts to understand the underlying systems for a minimum productivity seeing as, although simple problems such as Static and Modal analyses are fairly easy to setup properly, even without guidance, the user interface (UI) becomes less intuitive for problems of greater complexity, the Random Vibration analysis being particularly opaque. This lack of information further accentuates the issues surrounding error codes provided by the software console, with often times inscrutable handles. The matter of mesh control is also of particular relevance, whilst providing better control than most others, memory and mesh quality problems arise with unfathomable frequency, tending to run computationally heavy modules for long periods of time before outputting errors in situations where similar meshes in disparate software packages output reasonable results in a fraction of the time.

The amount and variation within the different analyses required led to a less in-depth discussion of each, in order to present most of the required results, but still some remained outside of these pages due, mostly, to space constrictions. It is therefore important to refer that beyond the work presented, buckling, frequency sweep and bolt analyses were run to ensure the stability of the satellite under a number of other possible scenarios were assuredly safeguarded. These would account for matters such as those of extra CubeSats being present on the launcher and stacked above the one presently in development, resulting in extra compressive effects, or the likelihood of the selected bolts supporting the loads in question, together with what if scenarios involving failure of several sets of bolts and their impact on the mission.

5.3 Future Work

The current point of the development of the satellite is already quite far in the timeline, and therefore little work remains to be developed in the area of discussion. It all boils down to the already mentioned testing campaign, due to be initiated in the present year, and whose final results should be cross-checked with both requirements from the launch responsible entities and the results obtained from the analyses presented in the design documentation, already provided to the competent authorities. A similar result pattern should serve to validate the results obtained from the analysis campaign, whilst a disparate one should serve as the basis for a new iteration of the work presented, with especially sensible parameters changed in an attempt to approach the end results obtained from both campaigns.
Bibliography


Appendix A

Technical Documentation
Figure A.1 - CubeSat 1U Specification [10]

Notes:
- CubeSat coordinate system located in the geometric center of the CubeSat.
- No external components other than the rails shall touch the inside of the P-POD.
- All 1U CubeSats must have two 121 separation springs and at least one 111 deployment switch on the -Z face. See drawing DDS-13-006, Deployment Switch and Separation Spring Location, for more details.
### Parts' General Properties

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**Table A.1 – Model Parts’ General Properties**

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**Table A.2 – Model Parts’ Structural Analysis Particular Properties**

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**Table A.3 – Model Parts’ Thermal Analysis Particular Properties**