Model predictive control for optimal supply chain management

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Abstract Currently, the main challenge in supply chain management consists on the coordination of efforts and the fulfilment of goals of the various actors composing the chain. On the one hand, suppliers, manufacturers, and retailers are interested in providing customers the right product, in the right amount, at the right time, for the right price, and at the right place. On the other hand, transport providers want to efficiently allocate goods to their resources and minimise the number of movements. To solve the trade-of between (i) on-time delivery, and (ii) efficient transportation management, this work presents a novel dynamic approach for real-time supply chain management integrating transportation operations, based on a model predictive control framework. Focusing on the discrete time case, a method for developing linear, time-invariant, state-space representations for supply chains that are both controllable and observable is outlined. The performance of the proposed methodology is illustrated resorting to a case study based on a real-world scenario. The results demonstrate that the devised controller is able to (1) deal with multi-products and multi-transports, (2) manage stocks, (3) monitor WIP, and (4) schedule manufacturing and transportation operations in an autonomous and integrated way, while respecting predefined time-windows.

Keywords MPC, Multi-Product, Multi-Transport, Supply Chain Management

1. Introduction

Industry and trade are the driving forces behind economical growth and the improvement of living standards [1]. Over the last decades, the increasing consumption and globalization have created the need for more transportation, strengthening the internal competition of this sector [2]. In turn, in order to respond to the growing demands in terms of product customisation, price and service levels from the customer side, companies are urged to lower their costs, while still maintaining high quality standards, which in addition to global warming and other environmental concerns have pressured even more distributors, and transportation service providers in general, towards reducing movement costs [2, 3]. This strong competition between goods owners (i.e. suppliers, manufacturers, and retailers) and transportation providers have created a permanent state of tension between the two sectors, increasing the demand for higher levels of efficiency, quality of service, timeliness, and responsiveness across supply chains [2].

Various scientific communities have devoted attention to the management and optimization of operations in supply chains. Undoubtedly, operations research methods are the most widely used when modelling those systems. Nevertheless, over the past years, control theory has been attracting the attention from the scientific community as a powerful method to analyse and design supply chains from a dynamical system point of view [4, 5]. However, even though transportation management is an integral part of supply chain management, these topics have either been studied independently from each other, or integrated for strategic and tactical purposes only.

This document sets forth a new approach for real-time supply chain management based on the model predictive control framework. The proposed methodology is based on a flow perspective and focuses on the discrete time case. It integrates ideas from operations research and control theory, resulting in interpretable, tractable and flexible dynamic models. The outlined modelling framework produces linear, time-invariant, state-space supply chain representations that are both controllable and observable. The presented approach was initially based on the work of Nabais et al. [6], and evolved into an extension of the works of Perea-Lopez et al. [7] and Braun et al. [8]. While their work focused on coordinating production and inventory activities across the network, this work generalises and integrates both the perspectives of the goods owners, and the transportation actors, enabling the proposed managing tool to be employed by any supply chain member, regardless of its role. That is, on the one hand suppliers, manufacturers and retailers are able to (1) deal with multi-products, (2) monitor and manage stocks, (3) schedule production activities, (4) monitor WIP, and (5) define reception and dispatch time-windows. On the other hand, transportation providers can (1) monitor different transportation types, (2) deal with costs associated with the different resources' capacities, and (3) monitor the location and state (i.e. with or without cargo) of the transportation resources composing the fleet.

2. Modelling supply chains

To quickly respond to demand changes, supply chains (SCs) usually work in a pull system, meaning that a given node reacts to a replenishing order placed by its succeeding node by either producing and/or replenishing it, or by transferring that order to upstream nodes if it is not possible to fulfil that order [7], producing a cascade effect. Figure 1 illustrates the basic flows and mechanisms present in such systems.

In SCs, actors (e.g. suppliers, manufacturers, retailers, transporters) must cooperate to move commodities from the point of origin to the point of consumption in order to meet customers' requirements. However, different SC members

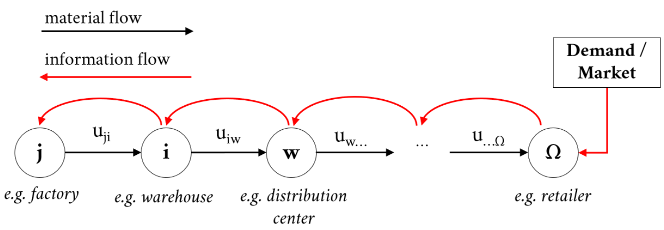


Figure 1. Supply chain dynamics — definition of nodes, arcs, and flows of material and information.

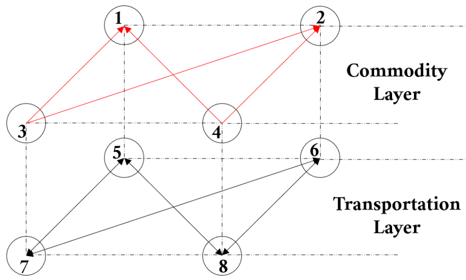


Figure 2. Fundamental layers in a SC. The movement of a commodity from node 3 to node 1, implies the synchronized flow of material from nodes 3 and 7 to nodes 1 and 5, respectively.

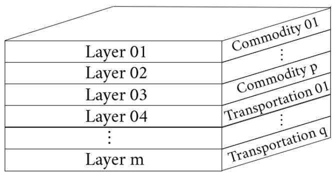


Figure 3. Generic supply chain stack of p commodities and q transportation networks.

may have different goals. For instance, suppliers usually do not work exclusively for one particular manufacturer, and retailers often do not depend on one single manufacturer. This distinction is particularly accentuated when considering transportation as a service. In that case, the primordial goal of transport providers is not the delivery of goods to customers on time, but rather to efficiently allocate goods to their transportation resources.

Thus, from a modelling perspective a supply chain is a network of independent entities or nodes, connected by links (or arcs) representing the flows of material and information. In turn, these flows affect the contents of each node. Therefore, the dynamics of a SC can be modelled resorting to mass balances at each node, where may exist different types of material resources: (1) goods, i.e. raw materials, WIP, and finished products, and (2) transportation vehicles. Throughout the document, raw materials and/or finished products will be referred to as *goods* or *commodities* interchangeably, and transportation vehicles as *transport agents* (or simply *agents*) or *vehicles* interchangeably.

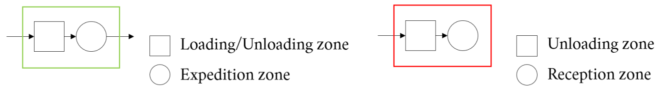
The movement of material resources across the network creates two different and independent material flows: (1) the flow of commodities, and (2) the flow of transport agents. Furthermore, one can distinguish two different groups of node contents (i.e. inventories): (1) inventory of commodities (e.g. stocked raw materials and/or products awaiting to be shipped), and (2) inventory of transport agents (e.g. free, parked vehicles awaiting for shipment assignment). Hence, one can discriminate two fundamental layers: (1) the commodity layer, and (2) the transportation layer. Each layer consists of (possibly different) independent networks across which material is allowed to flow. Thus, the SC dynamics is given as a superposition of these two fundamental layers, in which the movement of a commodity is interpreted as a synchronised, superimposed flow of material in both the commodity and the transportation layers, as shown in Figure 2.

Generally, however, each commodity poses particular transportation requirements with respect to its weight, physical state, heating, packaging, etc. The combination of these characteristics specifies a certain commodity type. Analogously, a transportation type may be characterised by different features as well, such as speed, loading capacity, transportation cost, authorised areas of operation, etc. Consequently, a generic supply chain is given as a collection of stacked networks (or layers) of different commodity and transportation types. Figure 3 illustrates a generic supply chain consisting on stacked commodity and transportation networks.

From an operations management point of view, there are a variety of operations one can find in any SC. It is therefore expected that nodes with different functions within a SC should present different mechanisms of mapping inputs onto outputs. Taking a holistic view of the SC, two fundamental types of node can be discerned. Namely, *source nodes*, and *sink nodes*. Sink nodes are defined as the most down-stream members of the network, whereas the rest are referred to as source nodes. The difference in nomenclature is important because their internal mechanisms are different. Source nodes receive (and/or transform) commodities that will be dispatched and flow through the network until they arrive at the sink nodes, which form the last echelon in a supply chain, usually representing the retailing level. Therefore, sink nodes receive and stock commodities. Since these two types of node are fundamentally different, the names given to their internal zones should also differ.

Source nodes: composed of a *loading/unloading zone*, followed by an *expedition zone*. The loading/unloading zone refers to the areas where commodities and transportation agents are stored awaiting for assignment, either because cargo has arrived or because it is waiting to be shipped. Once an order is placed, if the required resources are available, assignment takes place and cargo is loaded. Then, the loaded transport goes into the expedition zone, from where it can take different paths to the down-stream nodes. Figure 4(a) presents a schematic representation of such nodes. Source nodes may be further divided into transformation nodes.

Sink nodes: are composed of an *unloading zone*, followed by a *reception zone*. The unloading zone is the area where commodities and transportation agents are decoupled. Transport agents must stop and wait for further instructions, while commodities proceed to the reception zone, a restricted area where commodities are stocked for the final customer to pick them up. Figure 4(b) presents a schematic representation of such nodes.



(a) Source node. (b) Sink node.

FIGURE 4. Schematic representation of source and sink nodes.

Transformation nodes: are a special type of source node that represent a production or assembly process, where the output is given as a combination of the inputs. Figure 5 presents a schematic representation of such nodes.

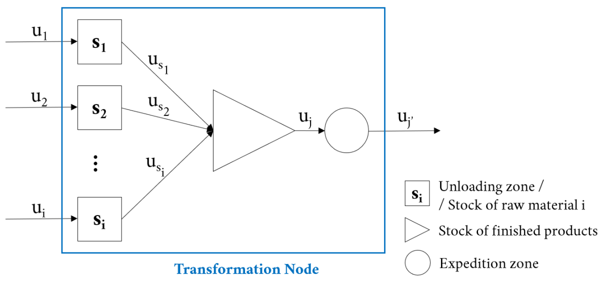


FIGURE 5. Transformation nodes.

Transshipment nodes: may not represent precise geographical locations, but rather virtual ones, in order to give information about: (1) the amount and whereabouts of in-transit commodities, and (2) the amount, whereabouts and status (i.e. if agents are *occupied* moving cargo, or *free* going towards a new location) of moving transport agents. An illustration of such nodes can be found in Figure 7.

In turn, one can describe the transportation process by a sequence of five actions: (1) wait assignment; (2) reallocate, if needed; (3) load cargo; (4) move cargo, (5) unload goods. Figure 6 presents a summary of the transportation process.

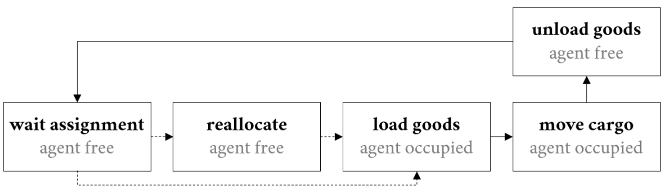


FIGURE 6. Transportation process.

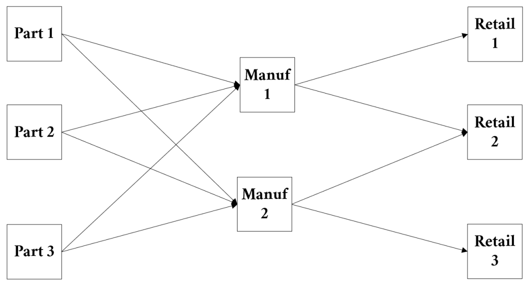
Finally, Figure 7 presents a sample SC network and the respective final model employing the presented modelling approach.

2.1. Notation

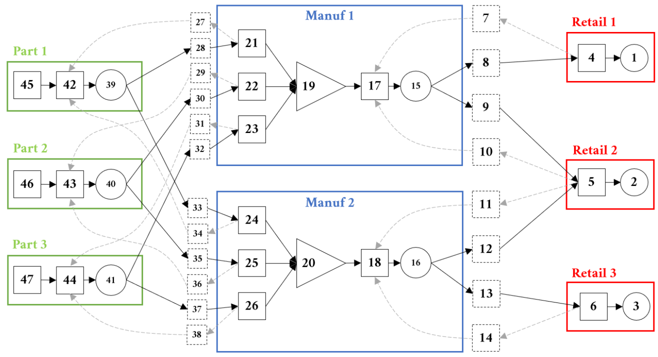
As previously stated, the dynamics of a SC may be modelled resorting to mass balances at each node. Consider Figure 8 as a starting point.

Generically, the inventory of a given resource m at node i can be computed as follows:

$$x_{i_m}(k+1) = x_{i_m}(k) + \Delta x_{i_m}(k) \tag{1}$$



(a) Sample network. (Based on Willems [9].)



(b) Final model.

FIGURE 7. Supply chain modelling.

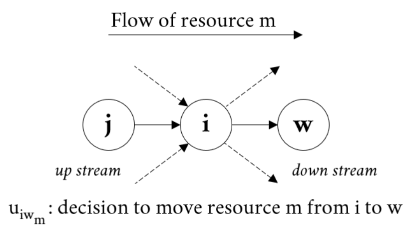


FIGURE 8. Transformation nodes.

$$= x_{i_m}(k) + \sum_j u_{ji_m}(k) - \sum_w u_{iw_m}(k) + d_{i_m}(k), \tag{2}$$

where x_{i_m} is the inventory level of resource m at node i; Δx_{i_m} is the variation of inventory level of resource m at node i; u_{ji_m} is the incoming stream of a given resource m to be processed at node i coming from node j; u_{iw_m} is the outgoing stream of a given resource m from node i to be processed at node w; d_{i_m} represents an exogenous input into node i regarding resource m (e.g. acquisition of new transport agents, unavailability of a given agent due to maintenance, reception/shipment of commodities from/to suppliers/customers outside the standard supply chain, end-customer acquisition of finished goods); and k is the discrete-time base period, also referred to as "sampling time instant", which depends on the dynamic characteristics of the network, i.e. is dependent on the application.

Transformation nodes belong to a special type of node which require more manipulations. These nodes define their outputs as a combination of the inputs, i.e. whenever the necessary raw materials required to produce one unit of finished product are in place, they are simultaneously processed and transformed into the final product. This transformation takes place whenever material flows from the stock of raw materials nodes into the stock of finished products one. Thus, the stream of raw materials works as a "transformation switch".

From a mathematical point of view, the stock of transformed (or finished) products can be modelled as follows:

$$x(k+1) = x(k) + Q_B\ C_G \sum_i u_{s_i} - u_j, \tag{3}$$

where Q_B is the batch quantity, and C_G is the compatibility gain, defined as $C_G = \frac{1}{N}$, where N is the total amount of different raw materials required to produce one unit of finished product.

In turn, the inventory of raw materials can be defined as follows:

$$s_i(k+1) = s_i(k) - Q_B\ M_i\ u_{s_i} - u_i, \tag{4}$$

where Q_B is the same batch quantity, and M_i is the necessary quantity of raw material i to produce a transformed product.

Denoting by n the number of nodes, n_m the number of resources, n_u the number of links between adjacent nodes, and n_z the number of output nodes, one can represent the model by making use of a state-space representation as follows,

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) \ + \ \mathbf{B}\mathbf{u}(k) \ + \ \mathbf{B}_d\mathbf{d}(k), \\ \mathbf{y}(k) &= \mathbf{C}_y\mathbf{x}(k), \\ \mathbf{z}(k) &= \mathbf{C}_z\mathbf{x}(k), \end{aligned} \tag{5}$$

where

$$\mathbf{x}(k) = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{n_m}]^T \in \mathbb{R}^{[n \times n_m] \times 1}, \tag{6}$$

$$\mathbf{u}(k) = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_{n_m}]^T \in \mathbb{R}^{[n_u \times n_m] \times 1}, \tag{7}$$

$$\mathbf{d}(k) = [\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_{n_m}]^T \in \mathbb{R}^{[n \times n_m] \times 1}, \tag{8}$$

$$\mathbf{y}(k) = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_{n_m}]^T \in \mathbb{R}^{[n \times n_m] \times 1}, \tag{9}$$

$$\mathbf{z}(k) = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_{n_m}]^T \in \mathbb{R}^{n_z \times 1}. \tag{10}$$

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_d$, $\mathbf{C}_y$, and $\mathbf{C}_z$ are matrices of appropriate size given as follows,

$$\mathbf{A} = diag(\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_{n_m}), \tag{11}$$

$$\mathbf{B} = diag(\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_{n_m}), \tag{12}$$

$$\mathbf{B}_d = diag(\mathbf{B}_{d_1}, \mathbf{B}_{d_2}, \cdots, \mathbf{B}_{d_{n_m}}), \tag{13}$$

$$\mathbf{C}_y = \mathbf{I}, \tag{14}$$

and $\mathbf{C}_z$ is defined by the user.

2.2. Performance indexes

Since one may be interested in gaining a panoramic and integrated overview of the supply chain as whole, two performance indexes are proposed. Namely, *Service rate*, and *Fleet usage rate*.

Service rate: service (or fill) rate measures the number of units filled as a percentage of the total ordered, and can be defined as follows,

$$SR = \left[1 - \frac{max(D) - N}{max(D)}\right] \times 100, \quad \text{for } D > 0, \tag{15}$$

where, D is the demand of a given product type, and N is the number filled units of a given product type.

Fleet usage rate: fleet usage rate measures the number of transportation resources that are in use as a percentage of the total transportation resources, and can be defined as follows,

$$FUR = \left[1 - \frac{A - O(k)}{A}\right] \times 100, \quad \text{for } A > 0, \tag{16}$$

where, A is the existing transportation resources (or agents), and O the number of occupied agents at time k.

3. Controlling supply chains

Model Predictive Control (MPC) has become an important framework for controlling complex, dynamic systems. Over the last decades, MPC has proven to be successful in the process industry [8, 10], and its growing popularity on SCM applications is rooted in the relative ease with which it can be understood, and its ability to handle constraints [8]. Figure 9 presents the basic features and mechanisms of a model predictive control.

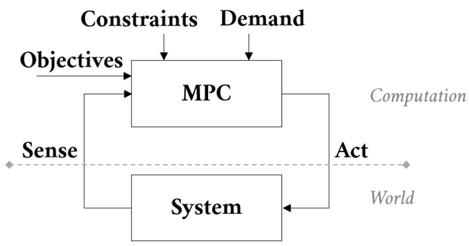


Figure 9. Basic features and mechanisms of MPC. (Based on Nabais et al. [6].)

MPC is a control strategy that produces a sequence of control actions based on the predicted behaviour of the system. The control actions are chosen by repeatedly minimising (or maximising) a performance index at each time-step. At each sampling time it predicts a control sequence into the future over some horizon (prediction horizon, H_p) but implements only the first one, resulting in what is called the *receding horizon window*. Any MPC problem considers three main components: a prediction model, a performance index (or cost function) and a set of constraints.

Regarding the cost function, the problem at hand is formulated as a modified standard reference tracking problem, and is defined as follows,

$$J = \sum_{i=1}^{Hp} [\mathbf{r}(k+i) \ - \ \tilde{\mathbf{z}}(k+i)]^2_{\mathbf{Q}} \ + \ [\mathbf{u}(k+i-1)]^2_{\mathbf{R}}, \tag{17}$$

where $\mathbf{u}$ is the collection of predicted decision variables that minimise J, and $\mathbf{Q}$ and $\mathbf{R}$ are weighing parameters of appropriate dimensions.

Table 1 presents the variables' mapping from the MPC into the supply chain domain.

In the previous section the concept of stacked resource layers was presented, in which the movement of a particular commodity was interpreted as a synchronised, superimposed flow of material in both the commodity's layer, and its

Table 1
Variables mapping from MPC to supply chain domains.

MPC	Supply Chain
references $\mathbf{r}$	inventory level targets per node
predicted outputs $\tilde{\mathbf{z}}$	future inventory levels per node (individual and/or aggregate levels)
inputs $\mathbf{u}$	"nodes' connections, or links", decision to move/allocate resource
weighing matrix $\mathbf{Q}$	cost of not achieving a prescribed stock level
weighing matrix $\mathbf{R}$	transportation costs

respective transport agent's one. From the control perspective, the flow of material is the decision variable to be optimized. Therefore, for the controller to be able to produce a synchronised, superimposed flow of material the following constraint should be imposed. Let $u_{ij_{m_P}}$ and $u_{ij_{m_T}}$ represent the decisions to move a product, P, and a transportation type, T, respectively. Then, the superimposed flow of material (i.e. products, plus transportation) can be written as follows,

$$\sum_{m_P} u_{ij_{m_P}}(k) \leq \lambda u_{ij_{m_T}}(k), \tag{18}$$

where $\lambda > 0$, and represents the maximum load capacity.

In turn, transformation nodes also require some constraints. Recall that in the previous section it was stated that whenever the necessary raw materials required to produce one unit of finished product are in place, they are then *simultaneously* processed and transformed into the final product. The transformation takes place when the material flows from the stock of raw materials nodes into the stock of finished products node, making the stream of raw materials work as a "transformation switch". To do so, control actions, u_{s_i} (see Figure 5) must assume only two values, either 0 or 1. Further, to accomplish the "switch-like" mechanism all control actions must assume the same value (either 0 or 1) at the same time. To translate this constraint into a formal mathematical way, consider the following.

Let $\mathcal{S}$ be the set of all inputs to a given stock of finished products node. Let $u_{s_i}(k)$ denote the i$^{\text{th}}$ element in $\mathcal{S}$, and $u_{s_i'}(k)$ the i$^{\text{th}}$ element in $\mathcal{S} \setminus \{u_{s_i}(k)\}$, at any given time-instant k. Thus, two elements, $u_{s_i}(k)$ and $u_{s_i'}(k)$, are said to be equal *iff* the following condition holds,

$$|u_{s_i}(k) - u_{s_i'}(k)| \leq 0. \tag{19}$$

Forcing all elements in $\mathcal{S}$ to simultaneously assume the same value, then, would imply a set of combinatorial conditions of the form of equation (19). Since the number of necessary combinations are mathematically expressed by C_2^N, where N denotes the total number of elements in $\mathcal{S}$, the following expression will be used as a shorthand to represent them,

$$|u_{s_i}(k) - u_{s_i'}(k)|_{C_2^N} \leq 0. \tag{20}$$

Conversely, to prevent transformation nodes of producing two commodities, m and n, in parallel, one can define the following constraint,

$$u_{s_{i_m}}(k) + u_{s_{i_n}}(k) \leq 0. \tag{21}$$

where $u_{s_{i_m}}$ and $u_{s_{i_n}}$ are the flows of raw material i of commodities m and n, respectively. Generalising such a constraint would imply a set of conditions of the form of equation (21). Namely, the number of necessary conditions is equal to the number of commodities that must be produced in sequence minus one. For brevity's sake, the following expression will be used as a shorthand to represent them,

$$\left[u_{s_{i_m}}(k) + u_{s_{i_n}}(k)\right]\Big|_{\forall\ m,n} \leq 0. \tag{22}$$

To guarantee that a given resource is at a given node at the time of pulling, one could define the following constraint:

$$\sum_i u_{ij_m}(k) \leq x_{i_m}(k). \tag{23}$$

However, since different nodes may have different processing mechanisms, their processing-times may also vary. From a modelling perspective, the processing-time can be considered as a pure time-delay. Thus, one can rewrite equation (23) as follows,

$$\sum_i u_{ij_m}(k) \leq x_{i_m}(k - \tau_i), \tag{24}$$

where τ_i is the time-delay produced by the processing-time of node i.

Equation (24) makes explicit use of past information, namely the past inventories $x_{i_m}(k - \tau_i)$, which means the controller must have sufficient internal memory to save the necessary information. The most efficient way of encapsulating the required information is to make a dynamical model that updates constraints each time the controller is updated with the system's state. Figure 10 illustrates the control scheme.

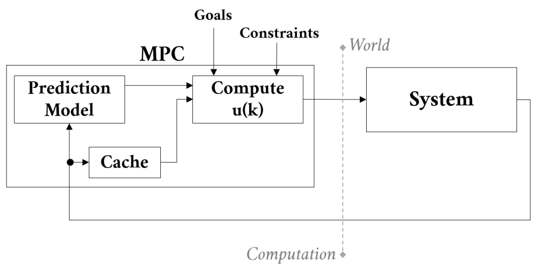


Figure 10. Control scheme.

Expanding equation (24) yields,

$$\sum_i u_{ij_m}(k) \leq x_{i_m}(k - \tau_i),$$
$$\vdots$$
$$\sum_i u_{ij_m}(k+1) \leq x_{i_m}(k - \tau_i + 1),$$
$$\vdots$$
$$\sum_i u_{ij_m}(k + \tau_i) \leq x_{i_m}(k).$$

Noticing that as k increases, $x_{i_m}(k)$ "moves upwards", it is possible to build a state-space model for a system of n nodes and n_m commodities, as follows,

$$\mathbf{t}(k+1) = \mathbf{A}_\tau \mathbf{t}(k) + \mathbf{B}_\tau \Delta\mathbf{y}(k) + \mathbf{\Omega}\mathbf{B}_{u_\tau}\mathbf{u}(k-1), \quad (25)$$

$$\mathbf{T}(k) = \mathbf{\Omega}_\tau^T \mathbf{t}(k), \quad (26)$$

where

$$\mathbf{A}_\tau = diag\left(\mathbf{A}_{\tau_1},\ \mathbf{A}_{\tau_2},\ \cdots,\ \mathbf{A}_{\tau_{n_m}}\right) \in \mathbb{R}^{[n\times n_m(\tau_i+1)]\times[n\times n_m(\tau_i+1)]}, \quad (27)$$

$$\mathbf{B}_\tau = diag\left(\mathbf{B}_{\tau_1},\ \mathbf{B}_{\tau_2},\ \cdots,\ \mathbf{B}_{n_m}\right) \in \mathbb{R}^{[n\times n_m(\tau_i+1)]\times n_m}, \quad (28)$$

$$\mathbf{\Omega} = diag\left(\mathbf{\Omega}_1,\ \mathbf{\Omega}_2,\ \cdots,\ \mathbf{\Omega}_{n_m}\right) \in \mathbb{R}^{[n\times n_m(\tau_i+1)]\times n_m}, \quad (29)$$

$$\mathbf{B}_{u_\tau} = f\left(\mathbf{B}_u\right)^{1} \in \mathbb{R}^{n_m\times n_m}, \quad (30)$$

$$\mathbf{A}_{\tau_{\mathbf{n_m}}} = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n(\tau_i+1)\times n(\tau_i+1)}, \quad (31)$$

$$\mathbf{B}_{\tau_{\mathbf{n_m}}} = [0,\ \cdots,\ 0,\ 1]^T \in \mathbb{R}^{n(\tau_i+1)\times 1}, \quad (32)$$

$$\mathbf{\Omega}_{\mathbf{n_m}} = [1,\ 0,\ \cdots,\ 0]^T \in \mathbb{R}^{n(\tau_i+1)\times 1}, \quad (33)$$

$$\mathbf{t}(k) = \left[\mathbf{x}_1(k-\tau), \cdots,\ \mathbf{x}_{n_m}(k-\tau)\right]^T \in \mathbb{R}^{[n\times n_m(\tau_i+1)]\times 1}, \quad (34)$$

$$\mathbf{x}_{n_m}(k-\tau) = \left[x_{1_m}(k-1), \cdots,\ x_{i_m}(k-\tau_i)\right]^T \in \mathbb{R}^{n(\tau_i+1)\times 1}, \quad (35)$$

$$\Delta\mathbf{y}(k) = \mathbf{y}(k) - \mathbf{y}(k-1) \in \mathbb{R}^{[n\times n_m(\tau_i+1)]\times 1}, \quad (36)$$

Note that at any given time the controller must be updated with $\mathbf{t}(k)$, $\Delta\ \mathbf{y}\ (k)$, and $\mathbf{u}(k-1)$. The last two are trivial, since they depend only on already computed variables. On the other hand, $\mathbf{t}(k)$ must be updated at every time-step using the following relationship,

$$\mathbf{t}(k) = \mathbf{A}_\tau \mathbf{t}(k-1) + \mathbf{B}_\tau \Delta\mathbf{y}(k). \quad (37)$$

The MPC optimisation problem can then be written as follows:

$$\min_{\tilde{u}} J = \sum_{i=1}^{Hp} \left[\mathbf{r}(k+i)\ -\ \tilde{\mathbf{z}}(k+i)\right]^2_{\mathbf{Q}}\ +\ \left[\mathbf{u}(k+i-1)\right]^2_{\mathbf{R}}, \quad (38)$$

$$s.t. \quad \mathbf{x}(k) \geq 0, \quad (39)$$

$$\underline{\mathbf{z}}\ \leq \mathbf{z}(k)\ \leq\ \overline{\mathbf{z}}, \quad (40)$$

$$\underline{\mathbf{u}}\ \leq \mathbf{u}(k)\ \leq\ \overline{\mathbf{u}}, \quad (41)$$

$$\sum_i u_{ij_m}(k) \leq x_{i_m}(k-\tau_i), \quad (42)$$

$$\sum_{m_P} u_{ij_{m_P}}(k) \leq \lambda\ u_{ij_{m_T}}(k), \quad (43)$$

$$|u_{s_i}(k) - u_{s_i'}(k)|_{C_2^N} \leq 0 \quad (44)$$

$$\left[u_{s_{i_m}}(k) + u_{s_{i_n}}(k)\right]\Big|_{\forall\ m,n} \leq 0. \quad (45)$$

[1] $\mathbf{B}_{u_\tau}$ is given as a function of the control matrix, $\mathbf{B}_u$, of the system to be controlled. More accurately, $f(\cdot)$ was meant to represent two operations: 1) substitute every positive element by zero, and 2) delete null rows.

Constraints (39) – (45) impose the network's structural features. Equation (39) assures all states are positive at every time-step. Equation (40) imposes the minimum and maximum nodes' capacities, denoted by $\underline{\mathbf{z}}$ and $\overline{\mathbf{z}}$, respectively. Equation (41) limits the material flow between nodes, imposing minimum and maximum admissible values as well, denoted by $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$, respectively. Equation (42) guarantees the pulled resource is available at the node at the time of pulling, where τ represents the time node i requires to process resource m. Equation (43) assures the maximum transport loading capacity, λ, is respected. Equation (44) assures the raw materials flow in the right quantity, and at the right time into the transformation nodes. Equation (45) assures the raw materials of different commodities flow in the right quantity, and at the right time into the transformation nodes, forcing the controller to schedule manufacturing activities. It should however be stressed that only constraints (42) – (45) depend on the network's configuration.

4. Simulation experiment

To validate the proposed approach a case-study based on a real-world supply chain was devised. The following paragraphs focus on a comprehensive description of the problem, followed by the presentation of the proposed model and its computational implementation, as well as the assumptions made. The section ends with the exhibition of the achieved results.

4.1. Problem description

The supply chain under study is presented in Figure 11 and is based on a data set made publicly available by Willems [9] comprising 38 real-world supply chains which have been implemented in practice by either company analysts or consultants. The chosen network is a three-echelon vertical integrated chain dedicated to the production of three types of product (P1, P2, and P3). To produce each product-type, manufacturing sites (Manuf 1, and Manuf 2) require three different raw materials (RM1, RM2, and RM3) which are then mixed in various proportions to produce the finished goods, which must be delivered to three different retailers (Retail 1, Retail 2, and Retail 3), each one requiring specific daily amounts of finished products. Table 2 presents the total lead time (in hours) for each commodity. Tables 3 and 4 present the bill of materials and the average daily demand of each retailer, respectively.

Differences in hardware, available space, and product-mix require different production schemes for each manufacturing site. While Manuf 1 produces P1 and P2, Manuf 2 focuses on producing P2 and P3. Each product type requires different processing times, resulting in heterogeneous production rates and throughput times. Namely, a batch of 500 units of P1 and a batch of 300 units of P3 take 1 hour, and after that time commodities of such types can readily be stored. However, products of type P2 require one extra hour before storage. A batch of 200 units of P2 is accomplished in 1 hour, thus the processing time of each batch equals 2 hours. Moreover, both manufacturing sites work in a flow shop scheme, meaning that

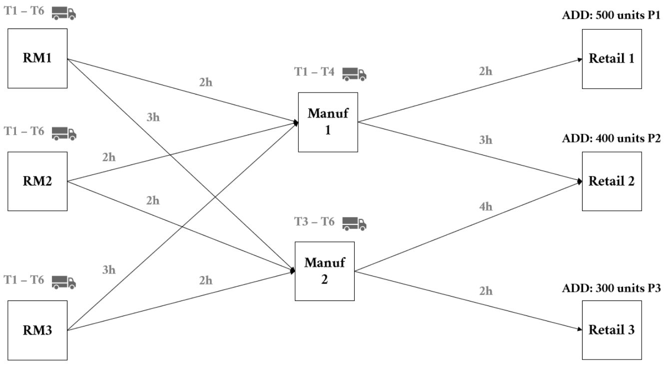


Figure 11. Supply chain configuration. (Based on Willems [9].)

Table 2
Total lead time, in hours.

	Retail 1	Retail 2 via Manuf 1	Retail 2 via Manuf 2	Retail 3
RM1	2 + 2	2 + 3	3 + 4	3 + 2
RM2	2 + 2	2 + 3	2 + 4	2 + 2
RM3	3 + 2	3 + 3	2 + 4	2 + 2
P1	5	—	—	—
P2	—	6	6	—
P3	—	—	—	5

Table 3
Bill of materials — units of raw material per unit of finished product.

	RM1	RM2	RM3
P1	2	1	1
P2	1	1	1
P3	1	1	2

Table 4
Average daily demand.

	P1 (units)	P2 (units)	P3 (units)
Retail 1	500	—	—
Retail 2	—	400	—
Retail 3	—	—	300

it is not possible to produce two product types in parallel. Table 5 presents a summary of manufacturing information.

Table 5
Manufacturing information.

	Production rate			Processing time		
	P1 (units/h)	P2 (units/h)	P3 (units/h)	P1 (h/batch[1])	P2 (h/batch[1])	P3 (h/batch[1])
Manuf 1	500	200	—	1	2	—
Manuf 2	—	200	300	—	2	1

[1] One batch equals the amount of produced products in one hour.

In terms of raw material supply, it is assumed that they are always and immediately available to be shipped whenever necessary.

Regarding transportation, each commodity can be moved by two different modes, where different transportation types differ only on load capacity. Table 6 presents the specifications of each transportation type, as well as the type of commodity each mode can be assigned to. In turn, the average trip duration is shown in Figure 11 and Table 2.

Table 6
Maximum load capacity by transportation mode and commodity type.

	RM1 (units)	RM2 (units)	RM3 (units)	P1 (units)	P2 (units)	P3 (units)
T1	250	250	250	250	—	—
T2	500	500	500	500	—	—
T3	100	100	100	—	100	—
T4	200	200	200	—	200	—
T5	100	100	100	—	—	100
T6	200	200	200	—	—	200

Finally, each supply chain member defines a working-day as a 12 hour period (from 8h to 20h), meaning that there is no processing, nor transportation, of commodities outside this time window. Moreover, the demand nodes (Retail 1, Retail 2, Retail 3) have specific time windows of cargo reception. Namely, these members are able to receive goods only at two particular times per day: from 8h to 10h, or from 16h to 19h.

The problem to be solved consists on monitoring and control (1) the transportation of goods from source to demand nodes, and (2) the position and status of the available transport agents over time, while assuring that the delivery is made on time.

4.2. Implementation and initial set up

To solve the case described in the previous section, the model presented in Figure 7 was implemented.

Two simulation tests are performed (*Simulation 1* and *Simulation 2*). The difference between simulations consists in the weights attributed to the different transportation modes, as will be discussed below. For clear illustrations, each simulation assumes the SC to be empty of commodities at starting time. Transportation wise, Figure 11 presents the fleet disposition at starting time.

Both simulations represent a 48h period, in which each time-instant represents 1 hour. The main objective is the same in both tests: to deliver the right amount of finished goods (500 units of P1, 400 units of P2, and 300 units of P3), at the right place, and at the right time, while minimising transportation and inventory costs. Namely, the desired delivery time is at 18h (of the first day). However, due to processing times in upstream sites, it is still acceptable to receive the goods at the next window of opportunity, that is the following pre-established reception time. It is further assumed that the maximum capacity of each node is much larger than the amounts of commodities being transported. Besides, at each time-step it can be transported as many commodities as necessary, the only limitation being the availability of transportation.

To solve this problem, the MPC controller is set to have a prediction horizon, H_p, equal to 20 time-instants. That is, at each time-step the controller plans the current control action considering the next 20 hours. Moreover, matrix $\mathbf{Q}$ is set to be equal in both simulations and is defined as follows $\mathbf{Q} = diag(1000, 100, \cdots, 100, 1, \cdots, 1)$.

As previously stated, the difference between simulations consists in the weights attributed to different transportation modes. Consequently, matrix $\mathbf{R} = diag(\rho_1, \;\cdots,\; \rho_i)$ is set to be different in each case. In Simulation 1 the movement of transport agents with greater loading capacity is considered to be more *costly* than the rest. Namely, T1, T3, and T5 are set to have a weight, ρ_i, equal to 1.5, whereas a $\rho_i = 3.5 \times 10^4$ was set for T2, T4, and T6. In turn, in Simulation 2 all transportation types were set to have the same weight, yielding $\mathbf{R}_{ii} = 1.5$. Table 7 presents a summary of the MPC parameter specifications for both simulations.

Table 7
MPC parameter specifications.

	H_p	$\mathbf{Q}$	ρ	
			T1, T3, T5	**T2, T4, T6**
Simulation 1	20	$diag(1000, 100, \cdots, 100, 1, \cdots, 1)$	1.5	3.5×10^4
Simulation 2	20	$diag(1000, 100, \cdots, 100, 1, \cdots, 1)$	1.5	1.5

4.3. Results

Figure 12 presents the inflow of commodities into the different demand nodes (Retail 1, Retail 2, and Retail 3), whereas Figure 13 presents the transformation of raw materials into finished products. As can be seen, the demand is met, while respecting the pre-established reception time-windows. It is interesting to take note of the delay on the shipment of P3. To better understand this, recall that a transport agent takes 2h to move from Manuf 2 to Retail 3 (see Figure 11). Inspecting Figure 13(b) it is clear P3 is stored at 17h (i.e. it enters node 17, see Figure 7(b)). Considering that goods would yet need to be loaded (i.e. pass through node 15) — which would require 1h more — and the travelling between sites takes 2h, the order would arrive at Retail 2 at 20h, which would violate the reception time-windows previously defined. Therefore, the controller postponed its shipment to the next available time-slot.

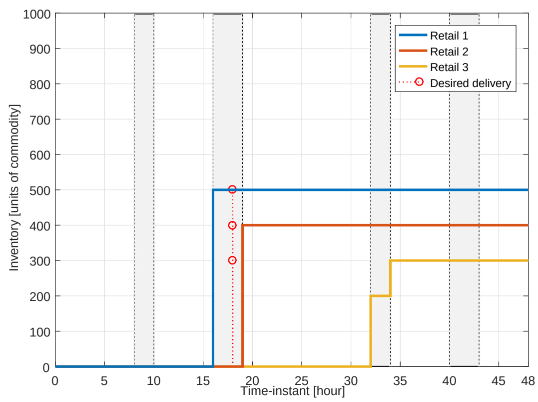
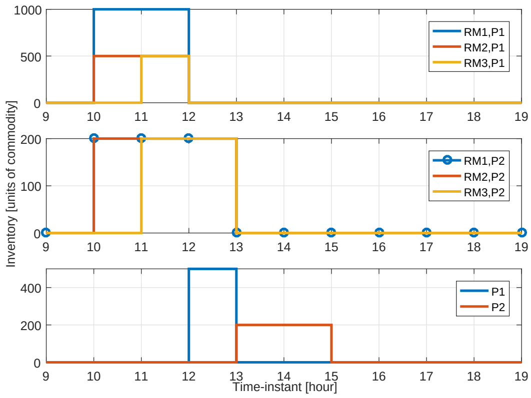
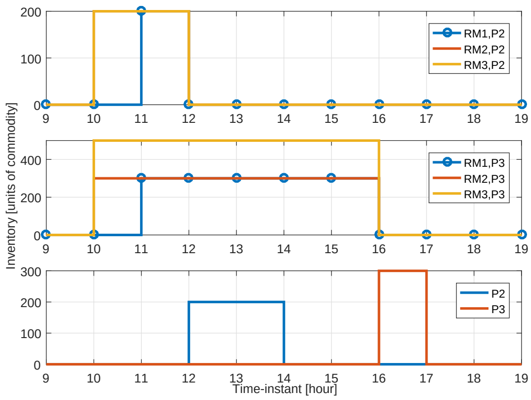


Figure 12. Flow of commodities into demand nodes (Retail 1, Retail 2, and Retail 3). Reception periods are represented as grey area.

Figure 13 clearly shows the production scheduling. Two points should be noted. First, the restriction that manufacturing sites work in a flow shop, i.e. it is not possible to have parallel production, is satisfied. Second, the choice of which product to produce first was a decision left entirely to the controller, which had to decide what would be the best sequence of decisions in order to meet the demand.



(a) Manuf 1.



(b) Manuf 2.

Figure 13. Transformation of raw materials into finished products. (Nodes 19 — 26, see Figure 7(b).)

Transportation wise, one is interested in monitoring the position and status of the available transport agents over time. To illustrate how the proposed approach is able to provide the required information, Figure 14 presents a holistic view of the transportation resources over time. At any given moment, it is possible to see how many agents of each type are free, i.e. awaiting task assignment. To have a more detailed information about how and where are these resources being used, one has to resort to a representation such as the one shown in Figure 15, where one can have a much deeper insight into the flow of transportation occurring across a specific area (in this case the manufacturing sites). Note as Figures 13 and 15 complement each other, giving a detailed account of what happened in manufacturing sites, and showing the impact of transportation on all supply chain activities. In this respect, note the production of P2 and P3 (Figure 13(b)) was postponed by a delivery delay of 100 units of raw material RM3 that only arrived at 16h at Manuf 2. Additionally, one can make use other ways of monitoring the fleet, as shown in Figure 16.

Namely, one can monitor what is happening in a single node, regarding a particular transportation type as presented in Figure 16(a), or take a more holistic take and monitor the occupied agents across the network, to know how many are waiting assignment, how many are loading, and how many are already in-transit, as show in Figure 16(b).

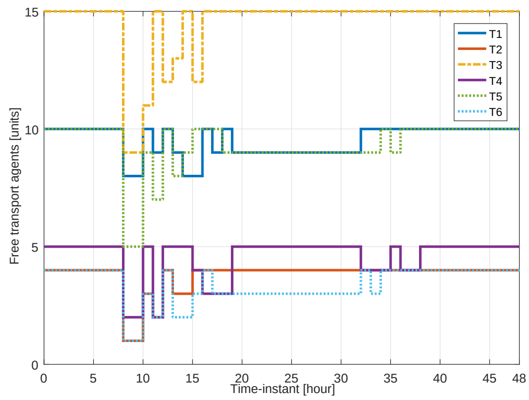
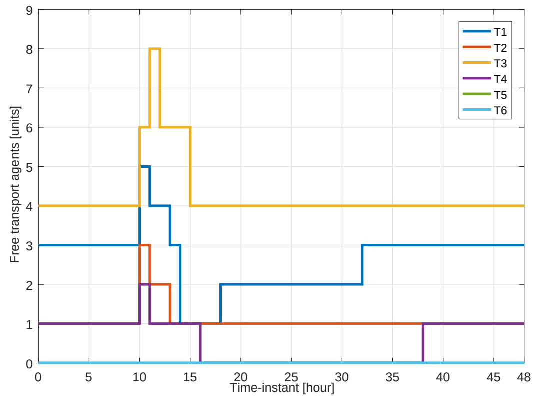
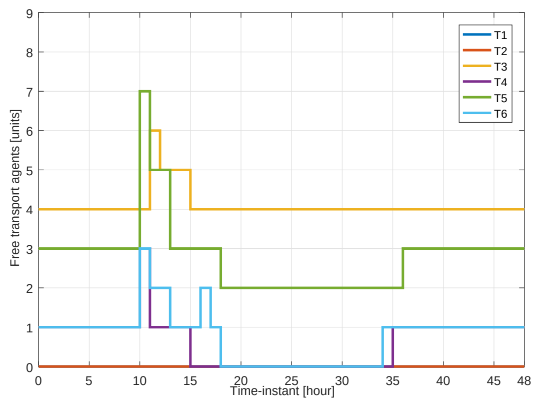


Figure 14. Aggregate number of transport agents awaiting task assignment, over time.



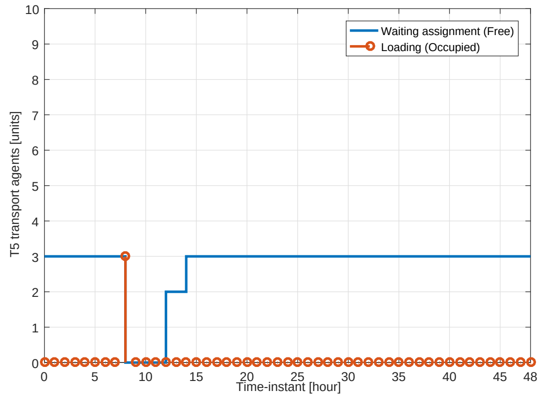
(a) Manuf 1.



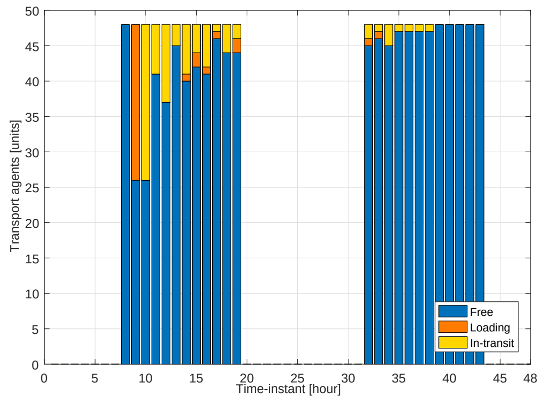
(b) Manuf 2.

Figure 15. Flow of transport agents across manufacturing sites. (Nodes 17 — 18, and 21 — 26, see Figure 7(b).)

Notwithstanding, it is yet possible to monitor how many trips were made, and if they consist on either shipments, or repositioning trips. As can be seen in Table 8, all transport



(a) Agents of type T5 in RM3. (Nodes 47 – 41, see Figure 7(b)).



(b) Fleet status overview.

Figure 16. Additional fleet monitoring tools.

agents got back to their initial position, which extends the insight of Figure 14.

Table 8
Total performed trips in both simulations.

	Simulation 1						Simulation 2					
	T1	T2	T3	T4	T5	T6	T1	T2	T3	T4	T5	T6
With cargo	8	7	14	10	12	10	4	9	12	11	14	8
Without cargo	8	7	14	10	12	10	4	9	12	11	14	8

As can be seen, the controller tended to assign more transport agents with bigger loading capacity in Simulation 2. More, it is clear that whenever there is a possibility to choose between two modes, the controller chose the cheapest one more often. This is specially visible when comparing T1 and T2. In Simulation 1, the assignment of T1 occurred twice more often than in Simulation 2, which enabled to save 2 of the more expensive T2 shipments.

Furthermore, one may be interested on gaining a panoramic and integrated overview of the supply chain as whole. Figure 17 presents a SC overview, integrating both the perspective of goods owners and transportation providers, where it is clear fleet-usage decreases over time as commodities are moved from up- to down-stream nodes.

Finally, Table 9 presents information regarding computational performance. As can be seen, the maximum iteration time is bellow 3 minutes (i.e. each hour is simulated in less

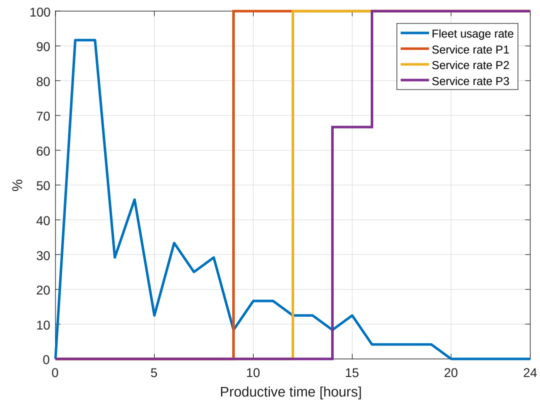


Figure 17. SC overview, integrating both the perspective of goods owners and transportation providers.

than 3 minutes), which in a supply chain management context is fast enough to be considered *real-time*.

Table 9
Computational performance.

	Simulation 1	Simulation 2
min It. Time [min]	1.65	1.59
max It. Time [min]	2.10	2.32
avg It. Time [min]	1.72	1.69
Total Sim. Time [h]	1.37	1.35

5. Conclusion and future work

To answer to the increasing demand for higher levels of efficiency, quality of service, timeliness, and responsiveness across SCs, a new dynamic modelling approach for real-time supply chain management integrating transportation operations, based on an MPC framework was proposed. The devised notation was used to develop a centralised, constrained MPC scheme, where the variables' mapping from the MPC framework to the SC domain was accomplished by representing inventories as states, and flows of material as control actions. To achieve the desired system behaviour, a set of constraints was defined. The MPC problem was formulated as a quadratic programming problem, in which desired inventory levels must be achieved. The proposed modelling framework was shown to result in linear, time-invariant, state-space representations of supply chains that are both controllable and observable.

Results have validated the proposed approach and shown the proposed approach is able to integrate manufacturing and transportation operations so that: (1) pre-defined reception time-windows are respected; (2) moving costs are minimised; (3) transportation reallocation policies are respected; and (4) path-planning for on-time delivery is accomplished. Regarding inventory and manufacturing activities alone, the devised approach is able to effectively and efficiently manage production (i.e. to determine the amount of goods to produce, where to produce them, and in which sequence), resulting in optimal inventory levels, i.e. the right amount of commodities was stored at the right time, and at the right place only to assure manufacturing activities could take place, enabling supply chain members to reduce their inventory levels to zero.

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