Program Synthesis with Constraint Solving for the OutSystems Language

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Resumo

Síntese de programas é o problema de gerar automaticamente implementações concretas de programas a partir de especificações de alto nível que definem a intenção do utilizador. OutSystems é uma plataforma low-code para desenvolvimento rápido de aplicações e integração simples com sistemas já existentes, com recurso à programação visual ou textual. Neste trabalho, focamo-nos no lado da programação textual. Abordamos o problema de sintetizar expressões OutSystems em um ambiente onde as especificações são fornecidas na forma de exemplos de entrada-saída, com foco em expressões que manipulam dados dos tipos inteiro e texto. Fazemos um estudo do estado da arte em síntese de programas e implementamos dois sintetizadores baseados em componentes e numa arquitetura de síntese inductiva guiada a oráculos. Ambos os sintetizadores aplicam uma mistura de satisfação de restrições com procura enumerativa básica, diferindo um do outro na quantidade de trabalho que colocam na fase de satisfação de restrições. Ambos os sintetizadores são aferidos e comparados com o SyPet num conjunto de problemas do mundo real fornecidos pela OutSystems, demonstrando resultados interessantes para programas até tamanho 4.

Palavras-chave: Síntese de Programas, Síntese Baseada em Componentes, Satisfação de Restrições, Satisfação Módulo Teorias
Abstract

Program synthesis is the problem of automatically generating concrete program implementations from high-level specifications that define user intent. OutSystems is a low-code platform for rapid application development, and easy integration with existing systems, featuring both visual and textual programming. In this work, we focus on the textual programming side. We tackle the problem of synthesizing OutSystems expressions in a setting where the specifications are given in the form of input-output examples, focusing on expressions that manipulate the integer and text datatypes. We survey the state of the art in program synthesis, and implement two component-based synthesizers based on an oracle-guided inductive synthesis architecture. Both synthesizers employ a mixture of constraint solving with basic enumerative search, differing from each other on the amount of work they put on the constraint solving phase. Both synthesizers are benchmarked and compared to SyPet on a set of real world problems provided by OutSystems, showing promising results for expressions of up to 4 lines in their correspondent static single assignment form.

Keywords: Program Synthesis, Programming by Examples, Component-Based Synthesis, Constraint Solving, Satisfiability Modulo Theories
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Chapter 1

Introduction

Program synthesis is the problem of automatically generating program implementations from high-level specifications. Amir Pnuelli, former computer scientist and Turing Award winner, described it as “one of the most central problems in the theory of programming” [26], and it has been portrayed as one of the holy grails of computer science [15, 34]. It is easy to understand why: if only we could tell the computer what you want and let it figure out how to do it, the task of programming would be much easier. However, program synthesis is a very hard problem. Programming is a task that is hard for humans and, given its generality, there is no reason to believe it should be any easier for computers. Computers lack algorithmic insight and domain expertise. Hence, the challenge is actually twofold: we need to find out both how to tackle the intractability of the program space, and how to accurately capture user intent [15].

It is important to keep in mind that program synthesis is not a panacea for solving problems in computer programming. For example, we might be interested in other properties besides functional correctness, such as efficiency or succinctness of the generated program. Also, in the general case, it is impossible for program synthesis to eliminate all sources of bugs. Particularly, it cannot solve problems originating from bad specifications that arise from a poor understanding of the problem domain.

Indeed, writing specifications is a delicate process. This might be better understood with a simple, yet non-trivial, motivating example.

1.1 Motivating Example: Sorting

In order to exemplify how the interaction between the user and the computer (from now on referred to as the “synthesizer”) may occur, let us suppose we are interested in developing a sorting procedure, sort, for lists of integers:

\[
\begin{align*}
\text{sort: } & (xs: \text{List Int}) \rightarrow (xs': \text{List Int}) \\
\text{sort}([]) &= [] \\
\text{sort}(x::xs) &= \text{insert}(x, \text{sort}(xs))
\end{align*}
\]

\[
\begin{align*}
\text{insert: } & (x: \text{Int}, xs: \text{List Int}) \rightarrow (xs': \text{List Int})
\end{align*}
\]
The function \texttt{sort} takes a list of integers as input and returns it sorted in ascending order (\texttt{[]} represents the empty list, and \texttt{y::ys} means that element \texttt{y} is prepended to the list \texttt{ys}). One approach to implement \texttt{sort} is to resort to an auxiliary function, \texttt{insert}, taking an element \texttt{x} and a list \texttt{x}s as inputs, and returning a new list \texttt{x}s'. Assuming that \texttt{x}s is sorted, \texttt{insert} guarantees that \texttt{x}s' is also sorted by placing \texttt{x} in the \textit{right place}. It is easy to see, by induction, that \texttt{sort} is correctly defined.

Nevertheless, it would be preferable if we could just give the synthesizer a specification of what it means for a list to be sorted and let it figure out the implementation. For example, the synthesizer could support using type signatures and predicates as specifications. We could also hint the structure of the implementation to the synthesizer by giving a specification for another function, \texttt{insert}, which could be useful for implementing \texttt{sort}:

\begin{verbatim}
isSorted: List Int \rightarrow Bool
isSorted([]) = True
isSorted([x]) = True
isSorted(x::y::ys) = x <= y and isSorted(y::ys)

sort: (xs: List Int) \rightarrow (xs': List Int)
sortSpec: isSorted(xs')

insert: (x: Int, xs: List Int) \rightarrow (xs': List Int)
insertSpec: isSorted(xs) \Rightarrow isSorted(xs') and sameContents(xs', x::xs)
\end{verbatim}

However, there is a problem with this specification, since there exist many \textit{unwanted} programs that satisfy it. The program that ignores its input and simply outputs the empty list is one such example. The problem is that the specification does not model our intent precisely. It should be clear that the output \texttt{x}s' should be in some way related to the input \texttt{x}s, but the current specification does not capture that relation. We must require that \texttt{x}s' has exactly the same contents as those of \texttt{x}s, meaning that every element of \texttt{x}s should occur in \texttt{x}s' the exact same number of times. We could express that requirement as a binary predicate \texttt{sameContents} over lists (here the implementation is omitted) and add it to the specification:

\begin{verbatim}
sort: (xs: List Int) \rightarrow (xs': List Int)
sortSpec: isSorted(xs') and sameContents(xs, xs')

insert: (x: Int, xs: List Int) \rightarrow (xs': List Int)
insertSpec: isSorted(xs) \Rightarrow isSorted(xs') and sameContents(xs', x::xs)
\end{verbatim}
From this example we can infer that writing specifications is prone to oversimplification by omitting details that might seem obvious at first. What might be less clear is that the specification as it is might still not be precise enough. There are countless other properties that we might want our sorting procedure to satisfy, such as, for example, stability, complexity and/or adaptability.

### 1.2 OutSystems and Programming by examples (PBE)

This work was done in the context of an internship at OutSystems.\(^1\) OutSystems is also the name of a low-code platform that features visual application development, easy integration with existing systems, and the possibility for the user to add their own code when needed (see Figures 1.1 and 1.2).\(^2\) OutSystems was developed with the goal of cutting down web and mobile app development time from several months to a few weeks. Recently, the OutSystems.AI team was created to research and implement more degrees of automation into the platform. In this work, we focus on synthesizing data transformation expressions in the OutSystems language.\(^3\).

**Example 1.1** Suppose that, given a text representing a person’s name, we are interested in extracting the first name and prepend it with a prefix. For example, given the text “John Michael Doe” and the prefix “Dr. ” we would like to obtain “Dr. John”. The following expression satisfies this specification.

\[
\text{prog(name, prefix)} = \text{Concat(prefix, \\
\quad \text{Substr(name, 0, \\
\quad \quad \text{Index(name, " ", 0)))}}
\]

We are interested in building a synthesizer of OutSystems expressions that is performant and easy to use. This implies that the synthesis process finishes in a matter of seconds (instead of hours or days),

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\(^1\)https://www.outsystems.com

\(^2\)https://www.outsystems.com/platform/

\(^3\)https://success.outsystems.com/Documentation/11/Reference/OutSystems_Language/Logic/Expressions
and the synthesizer has a “push-button”-style interface that does not force the user to acquire new skills. The paradigm in program synthesis that best fits this scenario is called programming by examples (PBE). In PBE, the synthesizer should be able to synthesize correct programs merely from a (small) set of input-output examples. For instance, one possible input-output example for the program from example 1.1 is, for example, (<“John Michael Doe”, “Dr. “>, “Dr. John”). By correct we mean that the program captures the user’s intent.

The most notable success in the area of program synthesis is Microsoft’s FlashFill [12] tool. FlashFill employs PBE technology and is currently integrated in Microsoft Excel. FlashFill is able to synthesize programs that manipulate strings very fast (typically under 10 seconds). However, the programs are written in a special-purpose domain-specific language (DSL) that does not suit our needs. In particular, it is not obvious how one could map the FlashFill DSL to OutSystems expressions. Another notable PBE synthesizer is SyPet [9], which has been applied to the synthesis of Java programs.

1.3 Contributions

In this work, we survey the state of the art in program synthesis, and implement two component-based PBE synthesizers based on an oracle-guided inductive synthesis (OGIS) architecture. Both synthesizers employ a mixture of constraint solving with basic enumerative search, differing in the amount of work they put in the constraint solving phase, for which they make use of a satisfiability modulo theories (SMT) solver. The components that we use correspond to a small subset of expressions of the OutSystems API that manipulate integers and text. However, the synthesizers are not specific to OutSystems expressions.
and could be instantiated in other domains. We benchmark both synthesizers and compare them to SyPet [9].

1.4 Document Structure

Chapter 2 introduces some basic definitions and examples that will be useful later on. Chapter 3 surveys related work and provides a brief introduction to the most common techniques in program synthesis, not only in PBE, but in general. Chapter 4 details the implementation of our contributions. Namely, we implement two synthesizers for OutSystems expressions based on constraint solving, a concept that we introduce in Chapter 3. In Chapter 5 we evaluate and compare the synthesizers presented in Chapter 4. We compare both synthesizers to each other and to SyPet. Chapter 6 concludes the work and suggests future research directions.
Chapter 2

Preliminaries

This chapter introduces some definitions and examples that will be useful later on. It might be practical to just skim this chapter at first reading, and then come back as needed.

2.1 Terminology and Basic Definitions

This section covers some essential terminology that is used in the document.

Program synthesis is fundamentally a search problem, hence, for it to be well-defined there must be some search space that contains the objects we are searching for. In our case, these objects are programs, so we have a program space.

Definition 1 (Program Space) A program space is the set of all programs in a given programming language.

Programming languages are usually defined by means of a grammar. For our purposes, we are interested in a particular kind of grammar, called a context-free grammar (CFG).

Definition 2 (Context-Free Grammar) A context-free grammar (CFG) is a tuple \((V, \Sigma, R, S)\), where:

- \(V\) is a finite set denoting the set of non-terminal symbols.
- \(\Sigma\) is a finite set denoting the set of terminal symbols.
- \(R\) is a finite relation from \(V\) to \((V \cup \Sigma)^*\), where * denotes the Kleene star operation.
- \(S\) is a non-terminal symbol, corresponding to the start symbol of the grammar.

Definition 5 in Section 2.2 provides an example of a CFG.

In our context, we are working in a PBE setting, where the user explains their intent by means of a set of input-output examples.

Definition 3 (Input-Output Example) An input-output example is a pair \((x, y)\) where \(x\) is a list of \(n\) inputs \(<x_0, x_1, \ldots, x_n>\), and \(y\) is the output.
We say that a program \( f \) satisfies an input-output example \((x, y)\) if \( f(x) = y \). We say that a program satisfies a set of input-output examples if it satisfies each example in that set.

### 2.2 Satisfiability Modulo Theories

Many problems in the real world can be modeled in the form of logical formulas. Thus, it is of great interest to have access to efficient off-the-shelf logic engines, usually called solvers, for these formulas. The satisfiability problem is the problem of checking if a given boolean logical formula has a solution. Satisfiability modulo theories (SMT) solvers extend this problem and check the satisfiability of first-order logic formulas with symbols and operations drawn from theories, such as the theory of uninterpreted functions, the theory of strings, or the theory of linear integer arithmetic. SMT solvers have seen a multitude of applications, particularly in problems from artificial intelligence and formal methods, such as program synthesis, or verification. This section gives a short introduction to SMT, and is based on the chapter about SMT of the Handbook of Satisfiability [6].

#### 2.2.1 Syntax of SMT Formulas

Here we define the language of well-formed SMT formulas. Formulas are composed of symbols and logical connectives over those symbols.

**Definition 4 (Signature)** A signature \( \Sigma = \Sigma^F \cup \Sigma^P \) is a set of \((\Sigma-)\)symbols. \( \Sigma^F \) is the set of function symbols, and \( \Sigma^P \) is the set of predicate symbols. Each symbol has an associated arity. A zero-arity symbol \( x \) is called a constant symbol if \( x \in \Sigma^F \), and is called a propositional symbol if \( x \in \Sigma^P \).

**Definition 5 (Terms and Formulas)** A \( (\Sigma-)\)term \( t \) is an expression of the form:

\[
 t ::= c \mid f(t_1, \ldots, t_n) \mid \text{ite}(\phi, t_0, t_1)
\]

where \( c \in \Sigma^F \) with arity 0, \( f \in \Sigma^F \) with arity \( n > 0 \), and \( \phi \) is a formula. A \( (\Sigma-)\)formula \( \phi \) is an expression of the form:

\[
 \phi ::= A \mid p(t_1, \ldots, t_n) \mid t_0 = t_1 \mid \perp \mid T \mid \neg \phi \mid \phi_0 \rightarrow \phi_1 \mid \phi_0 \leftrightarrow \phi_1 \mid \phi_0 \land \phi_1 \mid \phi_0 \lor \phi_1 \mid (\exists x. \phi_0) \mid (\forall x. \phi_0)
\]

where \( A \in \Sigma^P \) with arity 0, and \( p \in \Sigma^P \) with arity \( n > 0 \).

#### 2.2.2 Semantics of SMT Formulas

In this section we explore how SMT formulas are given meaning.

**Definition 6 (Model)** Given a signature \( \Sigma \), a \( (\Sigma-)\)model \( A \) for \( \Sigma \) is a tuple \( (A, (\_)^A) \) where:

1. \( A \), called the universe of the model, is a non-empty set;
2. \((\_)^A\) is a function with domain \(\Sigma\), mapping:
   - each constant symbol \(a \in \Sigma^F\) to an element \(a^A \in A\);
   - each function symbol \(f \in \Sigma^F\) with arity \(n > 0\) to a total function \(f^A : A^n \rightarrow A\);
   - each propositional symbol \(B \in \Sigma^P\) to an element \(B^A \in \{\text{true}, \text{false}\}\);
   - and each predicate symbol \(p \in \Sigma^P\) with arity \(n > 0\) to a total predicate \(p^A : A^n \rightarrow \{\text{true}, \text{false}\}\).

**Definition 7 (Interpretation)** Given a model \(A = (A, (\_)^A)\) for a signature \(\Sigma\), an interpretation for \(A\) is a function, also called \((\_)^A\), mapping each \(\Sigma\)-term \(t\) to an element \(t^A \in A\) and each \(\Sigma\)-formula \(\phi\) to an element \(\phi^A \in \{\text{true}, \text{false}\}\), in the following manner:

1. \(f(t_1, \ldots, t_n)^A\) is mapped to \(f^A(t_1^A, \ldots, t_n^A)\);
2. \(p(t_1, \ldots, t_n)^A\) is mapped to \(p^A(t_1^A, \ldots, t_n^A)\);
3. \(\text{ite}(\phi, t_1, t_2)^A\) is equal to \(t_1^A\) if \(\phi^A\) is \text{true}, and equal to \(t_2^A\) otherwise;
4. \(\bot^A\) is mapped to \(\text{false}\);
5. \(\top^A\) is mapped to \(\text{true}\);
6. \((t_1 = t_2)^A\) is mapped to \(\text{true}\) if \(t_1^A\) is equal to \(t_2^A\), and is mapped to \(\text{false}\) otherwise.
7. \(\Sigma\)-symbols are mapped according to the mapping of the model just as before.

**Definition 8 (Satisfiability)** Given a model \(A = (A, (\_)^A)\) for a signature \(\Sigma\), the model \(A\) is said to satisfy a \(\Sigma\)-formula \(\phi\) if and only if \(\phi^A\) is \text{true}. The formula \(\phi\) is said to be satisfiable.

**Definition 9 (Theory)** Given a signature \(\Sigma\), a (\(\Sigma\)-)theory \(T\) for \(\Sigma\) is a non-empty, and possibly infinite, set of models for \(\Sigma\).

**Definition 10 (\(T\)-Satisfiability)** Given a signature \(\Sigma\) and a \(\Sigma\)-theory \(T\), a \(\Sigma\)-formula \(\phi\) is said to be \(T\)-satisfiable if and only if (at least) one of the models of \(T\) satisfies \(\phi\).

**Definition 11 (SMT Problem)** Given a signature \(\Sigma\) and a \(\Sigma\)-theory \(T\), the SMT problem is the problem of determining the \(T\)-satisfiability of \(\Sigma\)-formulas.
Chapter 3

Background

This chapter discusses the fundamentals of the field of program synthesis and gives a broad overview of the state of the art, with a focus on constraint solving and programming by examples (PBE). Section 3.1 discusses several ways to describe intent to the synthesizer. Section 3.2 discusses techniques to search the program space for the intended program. Gulwani et al. presented a broader overview of the field as of 2017 [15].

3.1 Specifications

The first part of solving a program synthesis problem is figuring out how the user will communicate their intention to the synthesizer. An intention is communicated by a specification, and may be given in many different ways, including: logical specifications [16], type signatures [11, 23, 27]; syntax-guided methods [2] such as sketches [34], or components [8–10, 32]; inductive specifications such as input-output examples [11, 14, 22], demonstrations [20], or program traces [19]; or even other programs [35]. The kind of specification should be chosen according to the particular use case and to the background of the user, and it might dictate the type of techniques used to solve the problem (see Section 3.2).

3.1.1 Logical Specifications

Logical specifications are the canonical way of introducing specifications. In the sorting example from the introduction (1.1) we already saw an example of this where the specifications were given as logical pre/post-conditions over the inputs/outputs of the program. In that case, the specifications were written as predicates in the host programming language. Logical specifications may also be given as loop invariants or general assertions in the code, in order to give more hints to the synthesizer. Complete logical specifications are often difficult to write, or expensive to synthesize from. They usually require a great deal of knowledge about the domain of operation, and are typically not suitable for non-technical users.
3.1.2 Syntactic Specifications

A specification can be seen as a constraint over the space of all possible programs. We have already seen one type of specification, logical specifications, which are a kind of semantic specifications, meaning that they constrain the program space over behavior. Syntactic specifications are a method of specifying intent in program synthesis where a semantic specification is complemented with some form of syntactic constraints on the shape that the desired program can take.

Syntactic specifications are typically provided in the form of a CFG [2], or with sketches [34]. These restrictions provide structure to the set of candidate programs, possibly resulting in more efficient search procedures. They can also be used for the purpose of performance optimizations, e.g., by limiting the search space to implementations that only use a limited amount of lines of code. The learned programs also tend to be more readable and explainable.

Sketching

The idea of sketching is to provide skeletons of the programs we want to synthesize, called sketches, leaving missing details, called holes, for the synthesizer to fill. The synthesizer is then directed by the high-level structure of the skeleton while taking care of finding the low-level details according to user-specified assertions. Sketching is an accessible form of program synthesis, as it does not require learning new specification languages and formalizations, allowing the users to use the programming model with which they are already familiarized. This approach was introduced in the SKETCH system by Solar-Lezama [34], which allowed the synthesis of imperative programs in a C-like language.

Component-Based Synthesis

In component-based synthesis [8–10, 18, 32] we are interested in finding a loop-free program made out of a combination of fundamental building blocks called components. These components could be, for example, methods in a library application programming interface (API) [9, 32], and the way in which they can be combined forms the syntactic specification for the programs we want to find. They may also be supplemented by additional constraints in the form of logical formulas [10].

Syntax-Guided Synthesis

The problem of program synthesis with syntactic specifications was generalized and formalized in the work on syntax-guided synthesis (SyGuS) [2]. SyGuS is a community effort with the objective of "formulating the core computational problem common to many recent tools for program synthesis in a canonical and logical manner". The input to this problem consists of a background theory, that defines the language, a semantic correctness specification defined by a logical formula in that theory, and a syntactic specification in the form of a CFG. This effort has helped to create a common format for the definition of program synthesis problems and a growing repository of benchmarks. It has also led to the creation of the SyGuS-Comp annual competition.

\[\text{https://sygus.org}\]
3.1.3 Inductive Synthesis

Inductive synthesis is an instance of the program synthesis problem where the constraints are under-specified. Sometimes the domain we want to model is complex enough that a complete specification could be as hard to produce as the program itself, or might not even exist. In other cases, we might want the synthesizer to be as easy and intuitive to use as possible, e.g., for users coming from non-technical backgrounds.

Programming by examples (PBE)

PBE is an instance of inductive synthesis where the specification is given by input-output examples that the desired program must satisfy. Explicitly giving examples can be preferred due to their ease of use, especially by non-programmers, when compared to more technical kinds of specification, such as logical formulas. The examples may be either positive, i.e., an example that the desired program must satisfy, or negative, i.e., an example that the desired should not satisfy. More generally, given some (implicit) input-output example, we may include asserting properties of the output instead of specifying it completely [28]. This can be helpful if it is impractical or impossible to write the output concretely, e.g., if it is infinite.

Programming by demonstration (PBD)

In programming by demonstration (PBD) the user does not write a specification per se; instead the synthesizer is given a sequence of transformation steps (a demonstration) on concrete inputs, and uses them to infer the intended program. The program must be general enough to be used with different inputs. PBD can be seen as a refinement of PBE that considers an entire execution trace (i.e., step-by-step instructions of the program behavior on a given input) instead of a single input-output example. It depicts how to achieve the corresponding output instead of just specifying what it should be.

Though the concept of PBD is easy to understand, the task of the user can be tedious and time-consuming. Therefore, the synthesizer must be able to infer the intended program from a small set of user demonstrations. Ideally, it would also be able to interact effectively and receive feedback from the user. However, the concept might also be interesting when applied to non-interactive contexts, such as reverse engineering.

Lau et al. applied PBD to the text-editing domain by implementing SMARTedit, a system that induces repetitive text-editing programs from as few as one or two examples. The system resembles familiar keystroke-based macro interfaces, but it generalizes to a more robust program that is likely to work in more situations [20]. They have also presented a language-neutral framework and an implementation of a system that learns procedural programs from just 5.1 traces on average [19].

Ambiguity

In inductive synthesis the specifications are inherently ambiguous. Therefore, the intended program should not only satisfy the specifications, but also generalize, effectively trying to figure out the user's
intention. Typically, two approaches have been used to solve this problem. The first approach works by ranking the set of programs consistent with the examples according to their likelihood of being the desired program. This ranking function should allow for the efficient identification of the top-ranked program without having to perform costly enumeration. There have been manual approaches to create such functions, but it is a time-consuming process that requires a lot of domain expertise. It is also a fragile approach because it depends too much on the underlying DSL. Recently, more automated approaches have been proposed [29, 33], usually relying on machine learning techniques. However, as is usual with machine learning techniques, they require large labeled training datasets.

The second approach is called active learning, and (usually) relies on interaction between the user and the synthesizer. Typically, this happens by asking the user for (a small set of) additional input-output examples. Another idea, introduced by Jha et al. [18], works by finding a distinguishing input, an input on which two candidate programs differ, and query the user what the expected output should be for the intended program. Other types of active learning exist such as rephrasing the program in natural language, or accepting negative examples [11].

3.1.4 Programs

A program can also be used as a specification, and the job of the synthesizer is then to find another program with the same semantics. We might also be interested in programs that behave the same way as the original one, but, for example, are more efficient, or shorter.

Typically, the specification programs are not made to be efficient, but to be easy to read, or to prove correct. They may also appear naturally as specifications in certain use cases, such as superoptimization [25], deobfuscation [18], and synthesis of program inverses [35].

3.2 Search Techniques

The second part of solving a program synthesis problem is deciding which search technique to apply in order to find the intended program. First, we want to ensure that the program satisfies the semantic and syntactic specifications. Second, we want to leverage the specifications and the knowledge we have from the problem domain in order to guide the search. Common search techniques are enumerative search [3, 25] (Section 3.2.1), stochastic search [31, 33] (Section 3.2.2), and constraint solving [8–10] (Section 3.2.3). Modern synthesizers usually apply a combination of those, enabling us to consider frameworks and techniques for structuring their construction, such as counterexample-guided inductive synthesis (CEGIS) [34], CEGIS(7) [1], and OGIS [17] (Section 3.2.4).

3.2.1 Enumerative Search

In the context of program synthesis, enumerative search consists of enumerating programs by working the intrinsic structure of the program space in order to guide the search. The programs can be ordered
using many different program metrics, the simplest one being program size, and pruned by means of
semantic equivalence checks with respect to the specification. Perhaps surprisingly, synthesizers based
on enumerative search have been some of the most effective to synthesize short programs in complex
program spaces. A reason why is that the search can be precisely tailored for the domain at hand,
encoding domain-specific heuristics and case-by-case scenarios that result in highly effective pruning
strategies.

In their overview of the field of program synthesis [15], Gulwani et al. describe some enumerative
search algorithms for finding programs in program spaces defined by a CFG, which we describe next.

Top-Down Tree Search

Algorithm 3.1: Enumerative Top-Down Tree Search. Adapted from Gulwani et al.’s overview [15].

\begin{algorithm}
\textbf{input} : A specification $\phi$ and a CFG $G$
\textbf{output} : A program $p$ in the grammar $G$ that satisfies $\phi$
\begin{algorithmic}
\State $P \leftarrow \text{Queue}()$
\State $P' \leftarrow \{S\}$
\While {$P \neq \emptyset$}
\State $p \leftarrow \text{popFirst}(P)$
\If {$p$ satisfies $\phi$}
\State \textbf{return} $p$
\EndIf
\For {$a \in \text{nonTerminals}(p)$}
\For {$b \in \{b | (a, b) \in R\}$}
\If NOT subsumed($p[a \rightarrow b]$,$P'$)
\State $P \leftarrow P \cup \{p[a \rightarrow b]\}$
\State $P' \leftarrow P' \cup \{p[a \rightarrow b]\}$
\EndIf
\EndFor
\EndFor
\EndWhile
\end{algorithmic}
\end{algorithm}

The first enumerative strategy is the top-down tree search algorithm (Algorithm 3.1). It takes as
input a CFG $G = (V, \Sigma, R, S)$ and a specification $\phi$, and works by exploring the derivations of $G$ in a
best-first top-down fashion. The algorithm stores the current programs in a priority queue, $P$, and stores
all the programs found so far in the set $P'$. Both $P$ and $P'$ are initialized with the partial program that
Corresponds to the start symbol $S$ of $G$. The algorithm runs until it finds a program $p$ that matches the
specification $\phi$, or until there are no more programs waiting in the queue (meaning that the algorithm
fails). At every iteration, we take the program $p$ with the highest priority from the queue and check
whether it satisfies $\phi$. If yes, we return $p$. Otherwise, the algorithm finds new (possibly partial) programs
by applying the production rules of the grammar to $p$. The program space is pruned in the next step by
ignoring programs that are semantically equivalent (with respect to $\phi$) to programs already considered
in the past (i.e., subsumed within $P'$).

Bottom-Up Tree Search

The bottom-up tree search algorithm (Algorithm 3.2) is dual to the top-down tree search algorithm. It
also takes a CFG $G = (V, \Sigma, R, S)$ and a specification $\phi$, and works by exploring the derivations of
Algorithm 3.2: Enumerative Bottom-Up Tree Search. Adapted from Gulwani et al.'s overview [15].

input: A specification $\phi$ and a CFG $G$
output: A program $p$ in the grammar $G$ that satisfies $\phi$

begin
    $P \leftarrow \emptyset$
    for progSize $= 1, 2, \ldots$ do
        $P' \leftarrow \text{enumerateExprs}(G, E, \text{progSize})$
        for $p \in P'$ do
            if $p$ satisfies $\phi$ then
                return $p$
            if not subsumed($p, P$) then
                $P \leftarrow P \cup \{p\}$

the grammar in a bottom-up dynamic programming fashion. This strategy has the advantage over the top-down search that (in general) only complete programs may be evaluated for semantic equivalence. The algorithm maintains a set of equivalent expressions, first considering the programs corresponding to leaves of the syntax tree of the grammar $G$, and then composing them in order to build expressions of increasing complexity, essentially applying the rules of the grammar in the opposite direction.

Bidirectional Tree Search

We can see that a top-down tree search starts from a set of input states, while a bottom-up tree search starts from a set of output states. In both approaches the size of the search space grows exponentially with the size of the programs. The bidirectional tree search algorithm tries to attenuate this problem by combining the previous two approaches, starting from both a set of input states and a set of output states. It maintains both sets, evolving in the same way as the previous two algorithms, and stops when it finds a state that belongs to both sets in a sort of meet-in-the-middle approach.

3.2.2 Stochastic Search

Stochastic search is an approach to program synthesis where the synthesizer uses probabilistic reasoning to learn a program conditioned by the specification (i.e., the specification induces a probability distribution over the program space).

Typical stochastic synthesis approaches include, for example: genetic programming [37], where a population of programs is repeatedly evolved by application of biological principles (such as natural selection) while optimizing for a given fitness function (e.g., the number of input-output examples that are satisfied); neural networks that learn how to reproduce the intended behavior, or that learn actually interpretable programs [24]; or learning a distribution (a guiding function) over the components of the underlying DSL in order to guide a weighted enumerative search in the direction of a program that is most likely to meet the desired specification [4, 21].
Sampling the Search Space

In this section we describe the stochastic synthesizer used by Alur et al. in their syntax-guided synthesis paper [2]. Their synthesizer learns from examples and is adapted from work on superoptimization of loop-free binary programs [31]. Their algorithm uses the Metropolis-Hastings procedure to sample expressions that are more likely to meet the specification. They define a score function, Score, that measures the extent which a given program is consistent with the specification. Then they perform a probabilistic walk over the search space while maximizing this score function. The algorithm works by first picking a program \( p \) of fixed size \( n \) uniformly at random. They then pick a node from its parse tree uniformly at random, and consider the subprogram rooted at that node. They then substitute it with another subprogram of same size and type, chosen uniformly at random, obtaining a new program \( p' \). The probability of discarding \( p \) for \( p' \) is given by the formula \( \min(1, \frac{\text{Score}(p')}{\text{Score}(p)}) \). It remains to say how to pick the value of \( n \). Typically we do not know the size of the desired program from the start. They tackle this by running multiple concurrent searches for different values of \( n \). The first search has \( n \) fixed at value 1, and at each iteration they switch to the search with size \( \max(1, n \pm 1) \) with some small probability (default is 0.01).

3.2.3 Constraint Solving

Another approach to program synthesis is to reduce the problem to constraint solving by the use of off-the-shelf automated constraint solvers [8–10, 18, 32, 34] (typically SAT or SMT solvers). The idea is to encode the specification in a logical constraint whose solution corresponds to the desired program. Gulwani et al. [15] illustrate this in a simple way with an example which we show here. This example also serves as a short introduction to the SMT-LIB language [5], a standard language for describing SMT problems. Suppose our programs are composed of operations over two bitvectors of length 8:

\[
\begin{align*}
\text{program } P &::= \text{plus}(E, E) \mid \text{mul}(E, E) \mid \text{shl}(E, C) \mid \text{shr}(E, C) \\
\text{expression } E &::= x \mid C \\
\text{constant } C &::= 00000000_2 \mid 00000001_2 \mid \ldots \mid 11111111_2
\end{align*}
\]

We consider an expression to be either an input variable \( x \), or an 8-bit constant. A program consists of additions and multiplications between expressions, or of shift left/right operations over an expression by a constant. We can declare the type of bitvectors of length 8 in SMT-LIB as:

1. \( \text{(define-sort Bit8 () (_ BitVec 8))} \)

To encode the grammar of well-formed programs we first need to introduce the constant symbols \( hP \), \( hE0 \), \( hE1 \), \( c0 \) and \( c1 \), as well as the function symbol \( \text{prog} \):

1. \( \text{(declare-const hP Int)} \)
2. \( \text{(declare-const hE0 Bool)} \)
(declare-const hE1 Bool)
(declare-const c0 Bit8)
(declare-const c1 Bit8)

(define-fun prog ((x Bit8)) Bit8
  (let ((left (ite hE0 c0 x))
        (right (ite hE1 c1 x)))
    (ite (= hP 0) (bvadd left right)
         (ite (= hP 1) (bvmul left right)
              (ite (= hP 2) (bvshl left c1)
                           (bvlshr left c1))))))

The symbol hP encodes the choice of which language construct to pick, while the symbols hE0, hE1 encode the choice of whether left or right are assigned the values of constants (c0, c1) or the value of the input (x). An assignment to these constant symbols corresponds to a valid program in our grammar, if we ensure that hP is in a valid range. We can use assert to introduce new clauses that must be satisfied:

(assert (>= hP 0))
(assert (< hP 4))

Finally, we can also use assert to encode a semantic specification. For example, suppose we are interested in a bitvector program P that, for all input x, its output is always positive, i.e., ∀x. P(x) ≥ 0. In SMT-LIB that can be written as:

(assert (forall ((x Bit8)) (bvsge (prog x) #x00)))

This example shows an end-to-end constraint solving approach to program synthesis. However, encoding the problem this way can sometimes be non-trivial, or time-consuming. This led to the appearance of the concept of solver-aided programming, where programming languages are enlarged with high-level constructs that give the user access to synthesis without having to deal with the constraint solvers directly. For example, Gulwani et al. describe the SKETCH system as a “compiler [that] relies on a SAT solver to materialize some language constructs”. ROSETTE [36] is a framework for developing solver-aided programming languages embedded in Racket that provides constructs not only for synthesis, but also for verification, debugging and angelic execution.

### 3.2.4 Oracle-Guided Inductive Synthesis

**Oracle-guided inductive synthesis (OGIS)** is an approach to program synthesis where the synthesizer is split into two components: the learner and the oracle. The two components communicate in an iterative query/response cycle, as shown in Figure 3.1. The learner implements the search strategy to find the program and is parameterized by some form of semantic and/or syntactic specifications (see 3.1).
The usefulness of the oracle is defined by the type of queries it can handle and the properties of its responses. The characteristics of these components are typically imposed by the application.

![Oracle-guided inductive synthesis](image)

Figure 3.1: Oracle-guided inductive synthesis. Adapted from Jha and Seshia [17].

Typical queries and response types are some of the following [17]:

- **Membership queries**, where given an input-output example \( x \) the oracle responds with the answer to whether \( x \) is positive or not.

- **Positive (resp. negative) witness queries**, where the oracle responds with a positive (resp. negative) input-output example, if it can find any, or \( \perp \) otherwise.

- **Counterexample queries**, where given a candidate program \( p \) the oracle responds with a positive input-output counterexample that \( p \) does not satisfy, if it can find any, or \( \perp \) otherwise.

- **Correctness queries**, where given a candidate program \( p \) the oracle responds with the answer to whether \( p \) is correct or not. If it is not, the oracle responds with a positive input-output counterexample.

- **Verification queries**, where given program \( p \) and specification \( \phi \) the oracle responds with the answer to whether \( p \) satisfies \( \phi \) or not, or \( \perp \) if it cannot find the answer.

- **Distinguishing input queries**, where given program \( p \) and a set \( X \) of input-output examples that \( p \) satisfies, the oracle responds with a new counterexample \( x \) to \( p \) such that another program \( p' \) exists that satisfies both \( x \) and all the other examples in \( X \).

An OGIS system responding to counterexample queries corresponds to the CEGIS system, introduced by Solar-Lezama [34] in the context of the SKETCH synthesizer. Correctness oracles are more powerful than counterexample oracles because they are guaranteed to return a counterexample if the program is not correct, where the counterexample oracles might not.

The concept of OGIS was introduced by Jha et al. [18] as a generalization of CEGIS when they applied this idea to a PBE synthesizer based on distinguishing inputs in order to deobfuscate malware and to generate bit-manipulating programs. Jha et al. further developed this idea by presenting a new theoretical framework for inductive synthesis [17].

In general, the higher the capabilities of the oracles, the more expensive they are to run. Distinguishing oracles are (typically) not as strong as counterexample or correctness oracles because the returned
counterexample is not necessarily positive. To understand why they might be effective tools we can turn to the Bounded Observation Hypothesis [34], which asserts that “an implementation that works correctly for the common case and for all the different corner cases is likely to work correctly for all inputs.”

In a setting where the synthesizer is allowed to interact with the user, we could see the users take the role of the oracles. However, the interesting cases are the ones where the ratio between the amount of work the users are given and the information given to the synthesizer is minimized. A system that frequently queries the users for correctness checks would probably feel very cumbersome. On the other hand, a system that queries for membership or positiveness checks might be more realistic, as usually the user has an idea of what sort of examples fit their desired model.
Chapter 4

Synthesis

This chapter describes the problem (Section 4.1) and the approaches that were studied in order to solve it. Both approaches (Sections 4.2 and 4.4) are adaptations of a component-based constraint solving approach previously applied to the synthesis of bitvector programs [13, 18]. Section 4.3 explains how the components that we use are encoded in SMT.

4.1 Problem Description

We are working in the context of the OutSystems platform. OutSystems is a low-code platform that features visual application development, easy integration with existing systems, and the possibility to add own code when needed. To that effect, the kind of programs we are interested in are expressions in the OutSystems language.¹

We can think of OutSystems expressions as a simple functional language of operands and operators that one may combine in order to create pure, stateless, and loopless programs. This means that OutSystems expressions do not have side-effects, like printing to the screen, or writing to a database, and do not allow variable declarations, or (for/while) loops.² They do, however, have conditional expressions in the form of “if” statements. The library of builtin expressions includes functions that manipulate builtin data types such a text strings, numbers, or dates.³,⁴

In this work we are mainly interested in synthesizing expressions that manipulate text strings, like concatenation, substring slicing or whitespace trimming. The reason for this focus is that text manipulation is a tedious and error prone task, and it’s often easier to provide examples than it is to describe them in terms of programming language primitives. As some of these operations involve indexing, we are also interested in synthesizing simple arithmetic expressions involving addition and subtraction. Therefore, the data types we are working with are text strings and integers. In particular, we are not dealing neither with booleans, nor conditional expressions. Table 4.1 describes the builtin expressions that our synthe-

²Strictly speaking, it is possible to have expressions that produce side effects, but these are not the focus of this work.
⁴https://success.outsystems.com/Documentation/10/Reference/OutSystems_Language/Logic/Built-in_Functions
sized programs can be composed of, i.e., the components of our DSL. These expressions were chosen because they are some of the most frequently found in OutSystems applications, and because their semantics are easily translatable to SMT.

**Example 4.1** Suppose that, given a text representing an email, we are interested in extracting its domain part. For example, given the text “john.doe@outsystems.com”, we would like to obtain “outsystems.com”. The following expression satisfies the specification.

\[
\text{prog}(\text{email}) = \text{Substr} (\text{email}, \text{Add}(\text{Index} (\text{email}, "@", 0), 1), \text{Length} (\text{email}))
\]

**Example 4.2** Suppose that, given a text representing a person’s name, we are interested in extracting the first name and prepend it with a prefix. For example, given the text “John Michael Doe” and the prefix “Dr. ” we would like to obtain “Dr. John”. The following expression satisfies the specification.

\[
\text{prog} (\text{name}, \text{prefix}) = \text{Concat} (\text{prefix}, \text{Substr} (\text{name}, 0, \text{Index} (\text{name}, " ", 0)))
\]

**Example 4.3** Suppose now that, given a text representing a person’s name, we are interested in extracting not the first but the second name. For example, given the name “John Michael Doe” we would like to obtain “Michael”. The following expression satisfies the specification.

\[
\text{prog} (\text{name}) = \text{Substr} (\text{name}, \\
\text{Index} (\text{name}, " ", 0), \\
\text{Index} (\text{name}, " ", \text{Index} (\text{name}, " ", 0) + 1) - \text{Index} (\text{name}, " ", 0))
\]

In our context, we are working in a PBE setting, so we are interested in synthesizing an OutSystems expression from a set of input-output examples. For example, \{("John Michael Doe", "Michael")\}
4.2 Setwise Encoding

Given that OutSystems expressions are composed of self-contained pure functions, this synthesis problem fits nicely in the component-based synthesis paradigm (Section 3.1.2). Therefore, assume we are given a library of base components \( F \) that the synthesizer can use in order to compose the programs. These components will be built-in functions drawn from the OutSystems library, or combinations of them. Each component can take a finite number of inputs and return exactly one output. More formally, a component \( f \in F \) is specified by a first-order formula \( \phi_f \) that relates its input parameters \( P_f \) to its return value \( r_f \) (see Section 4.3).

We can see from the previous examples that OutSystems expressions can also include constant literals, like " ", or 0. These could been given as input, but we would like them to be computed automatically by the synthesizer. Ignoring well-typedness, an OutSystems expression is a tree-like program whose syntax can be succinctly described using a CFG:

\[
S ::= f(S, \ldots, S) \mid x \mid c
\]

where \( f \in F, x \in I, \) and \( c \in C, \) and where \( I \) is the set of inputs of the program, and \( C \) is the set of constant literals in the OutSystems language.

It is useful to reason about OutSystems expressions in another representation, called static single assignment (SSA) form. A program in SSA form is a line program where every variable is assigned exactly once and defined before it is used. For example, the program from Example 4.2 could be written in SSA form as shown in Figure 4.1. The body of a program in this format can be described succinctly with the following CFG:

\[
S ::= ID = c \mid ID = f(x_1, \ldots, x_n) \mid S; S
\]

where \( ID \) stands for an identifier in the OutSystems language. The non-terminal \( S \) represents a line in the program. A line is an assignment of a variable to a constant literal \( c \) or to the return value of a
component \( f \) on inputs \( x_1, \ldots, x_n \). As long as the program is well-typed, an input to a component can be one of the inputs of the program, or a variable defined in a preceding line. Thus, the general structure of a program in SSA form is a sequence of assignments.

The approach described in this section is based on Jha et al.'s program encoding [18]. The idea is to encode the program space in a formula. The formula is then constrained further in order to encode only those programs that satisfy the input-output examples. A solution to the formula can then be decoded back yielding a program that satisfies the set of examples.

The synthesizer (Figure 4.2) follows the OGIS model, described in Section 3.2.4. It has a learner part and an oracle part, which we will refer to as the enumerator and the solver, respectively.

The enumerator receives the set of input-output examples as input, and is parameterized by the library of components. The enumerator is responsible for drawing a subset of components from the library. The components are drawn by trying all sets of combinations (with replacement) in order of increasing size. It then passes these components to the solver, along with the input-output examples, and queries whether there exists a program composed only of those components that satisfies the examples. There is the additional restriction that the program must use each of the components in the query exactly once.

The solver works by encoding the query into an SMT formula, and uses an automated SMT solver to check for satisfiability. The SMT solver might or might not be able to solve the formula. If the formula is satisfiable, then the solver responds to the enumerator with SAT and a solution (called a model) to that formula. If not, the solver returns UNSAT or UNKNOWN, depending on whether the formula is unsatisfiable, or the SMT solver could not, for some reason, verify its satisfiability, respectively.

The procedure keeps going in a loop until the enumerator receives SAT from the solver. The enumerator then decodes the model into an actual program, which is then returned to the user.

### 4.2.1 Program Formula

Consider the program from Figure 4.1 as a running example in order to understand how we can construct a formula whose model can be decoded into a program that satisfies the examples. This is a small, non-trivial program, which uses non-input constant variables (" ", "0"), and not every component has the same return type.
prog(name, prefix):
c0 = " "
c1 = 4
c2 = 0
r1 = Concat(prefix, name)
r2 = Index(r1, c0, c1)
r3 = Substr(r1, r2, c1)

c0 = " "
c1 = 0
r1 = Index(name, c0, c1)
r2 = Substr(name, c1, r1)
r3 = Concat(r2, prefix)

Figure 4.3: Two other well-formed programs using the components Index, Substr, and Concat.

In order to encode the space of valid programs, the solver has to decide (1) how many constant variables to create and which values to assign them, (2) in which order the components appear in the program, and (3) which actual values to pass to the formal parameters of each component.

For instance, Figure 4.3 shows two other valid programs using the components Index, Substr, and Concat. The program on the left satisfies the sole input-output example of Example 4.2, although it does not generalize. It does so by switching the order of the components, and using one more variable than program 4.1. The program on the right, however, does not satisfy the example because the values passed to Concat are reversed.

In order to encode the program we need variables in the formula to model several entities: (1) the input variables to the program; (2) the constant variables; (3) the formal parameters of each component; (4) the return variables of all components; (5) the output variable of the program; and (6) the connections between the variables, that specify which actual parameters are passed to the formal parameters of each component. Thus, we have a set $I$ of input variables, a set $C$ of constant variables, a set $P$ of the formal parameters of all components, a set $R$ of the return variables of all components, and a variable $o$, the output variable of the program. We will denote the formal parameter variables and return variable of component $f$ by $P_f$ and $r_f$, respectively. Also, we will use $F'$ to refer to the components available to the solver (in this case, Index, Substr, and Concat) — recall that $F$ is used to denote the library of all components). For program 4.1 we have $I = \{name, prefix\}$, $C = \{c_0, c_1\}$, and $R = \{r_1, r_2, r_3\}$. We also have $P = P_{Index} \cup P_{Substr} \cup P_{Concat}$, with $P_{Index} = \{p_{11}, p_{12}, p_{13}\}$, $P_{Substr} = \{p_{21}, p_{22}, p_{23}\}$, and $P_{Concat} = \{p_{31}, p_{32}\}$.

Well-Formedness Constraint

To encode the connections, we require a set $L$ of integer-valued location variables $l_x$ for each variable $x \in I \cup C \cup P \cup R \cup \{o\}$. Intuitively, if $x$ is the return variable of component $f$, then $l_x$ is the line number where $f$ appears in the program. If $x$ is a formal parameter of some component, then $l_x$ is the line number where the actual parameter is defined. In practice, each variable in $I$ is assigned a line number from 1 to $|I|$ sequentially, variables in $C$ are assigned a number from $|I| + 1$ to $|I| + |C|$, and the output variable $\{o\}$ is assigned the line number $|I| + |C| + |R|$ (the last line). The locations of variables in $R$ range from $|I| + |C| + 1$ to $|I| + |C| + |R|$. The location of each formal parameter $x \in P$ ranges from 1 up to the location of its corresponding component. For program 4.1 we would have $l_{name} = 1$, $l_{prefix} = 2$, $l_{c_0} = 3$, $l_{c_1} = 4$, and $l_{o} = 7$. The range constraints are $4 \leq l_x \leq 7$ for $x \in R$, $1 \leq l_x < l_{Index}$ for
\( x \in P_{\text{Index}}, 1 \leq l_x < l_{\text{Substr}} \) for \( x \in P_{\text{Substr}} \), and \( 1 \leq l_x < l_{\text{Concat}} \) for \( x \in P_{\text{Concat}} \). In general, we can capture these constraints with the following formula \( \psi_{\text{range}} \):

\[
\psi_{\text{range}}(I, C, P, R) = \bigwedge_{f \in F'} (|I| + |C| + 1 \leq l_{r_f} \leq |I| + |C| + |R|) \land \bigwedge_{f \in F'} \bigwedge_{p \in F_f} (1 \leq l_p < l_{r_f})
\]

The locations of the variables \( x \in I \cup C \cup \{a\} \) are known as soon as we decide how many constant variables the program will have at its disposal. The objective is then to find an assignment to the locations of the variables \( x \in P \cup R \). These give us all the information we need to decode back the program. For program 4.1 we have \( l_{r_1} = 5, l_{r_2} = 6, l_{r_3} = 7; l_{p_{11}} = 1, l_{p_{12}} = 3, l_{p_{13}} = 4; l_{p_{21}} = 1, l_{p_{22}} = 4, l_{p_{23}} = 5 \); and \( l_{p_{31}} = 2, l_{p_{32}} = 6 \). Because the program has two inputs, we need to subtract two to the location variables to get the corresponding “line numbers”. This means, for example, that \( \text{Index}, \text{Substr}, \) and \( \text{Concat} \) appear on lines 3, 4 and 5, respectively, and so on.

We need a few more constraints in order to encode the space of well-formed programs. First, no two components should have the same location. Thus, we have \( l_{r_1} \neq l_{r_2} \neq l_{r_3} \). In the general case, these constraints are captured by the following formula \( \psi_{\text{rloc}} \):

\[
\psi_{\text{rloc}}(R) = \bigwedge_{x,y \in R, x \neq y} (l_x \neq l_y)
\]

Second, the program must be well-typed, so the location of each formal parameter \( x \in P \) should differ from the location of any \( y \in I \cup C \cup R \) whose type does not match with \( x \). In the same vein, only components which return value has the same type as the output may appear in the last line.\(^5\) These constraints are given by the following formula \( \psi_{\text{tloc}} \):

\[
\psi_{\text{tloc}}(I, o, C, P, R) = \bigwedge_{p \in P} \bigwedge_{x \in I \cup C \cup R} (l_p \neq l_x) \land \bigwedge_{r \in R} (l_r \neq l_o)
\]

Combining formulas \( \psi_{\text{range}}, \psi_{\text{rloc}}, \) and \( \psi_{\text{tloc}} \) we get the full program well-formedness constraint \( \psi_{\text{wfp}} \):

\[
\psi_{\text{wfp}}(I, o, C, P, R) = \psi_{\text{range}}(I, C, P, R) \land \psi_{\text{rloc}}(R) \land \psi_{\text{tloc}}(I, o, C, P, R)
\]

**Functional Constraint**

Formula \( \psi_{\text{wfp}} \), which we arrived to in the last section, encodes the space of all *syntactically* well-formed programs. However, in no way does it constrain the programs to have the correct *semantics*. In particular, it (1) does not relate the return values to their corresponding components; (2) does not ensure that variables share the same value if they share the same location; nor (3) does not ensure that the program satisfies the input-output example. For example, for program 4.1 we need to ensure (1) that the value of \( r_1 \) is actually equal to \( \text{Index(name, c0, c1)} \); (2) that \( p_{21} \), the first formal parameter of \( \text{Substr} \), and \( \text{name} \) share the same value; and (3) that \( r_3 \) equals \( o \), the output variable.

\(^5\)Strictly speaking, the OutSystems language has implicit type conversions, but we ignore those in the scope of this work.
Constraint (1) are encoded with the following formula $\psi_{\text{spec}}$:

$$\psi_{\text{spec}}(P, R) = \bigwedge_{f \in F'} \phi_f(P_f, r_f)$$

Recall that $\phi_f$ denotes the specification of component $f$, which relates its formal parameters to its return value. Constraint (2) refers to the dataflow properties of the program. For example, we need to have either the constraint $l_{p_{21}} = l_{\text{prefix}} \implies p_{21} = \text{prefix}$ (because we do not know beforehand that $p_{21} \neq \text{prefix}$), or the constraint $l_{r_2} = l_o \implies r_2 = o$ (because we do not know which component is going to be on the last line), but we do not want the constraint $l_{p_{21}} = l_{r_1} \implies p_{21} = r_1$ (because $p_{21}$ and $r_1$ have different types). In general, these properties are encoded in the following formula $\psi_{\text{flow}}$:

$$\psi_{\text{flow}}(I, C, P, R) = \bigwedge_{p \in P} \bigwedge_{x \in I \cup C \cup R} \left( l_p = l_x \implies p = x \right) \land \bigwedge_{r \in R} \left( l_r = l_o \implies r = o \right)$$

We would like to ensure that every component given by the enumerator is effectively used in the generated program, meaning that their correspondent return value should be either the actual parameter of some other component, or the final output of the program. This makes sense because of the way that the enumerator draws components from the library (combinations with replacement in order of increasing size), as every subset of $F'$ would have already been passed to the solver and deemed insufficient in order to build a satisfying program. For instance, the return value of $\text{Concat}$ could be either the output of the program, or one of the actual parameters of the same type of $\text{Index}$ or $\text{Substr}$. Thus, we would have $l_{r_3} = l_o \lor l_{r_3} = l_{p_{11}} \lor l_{r_3} = l_{p_{12}} \lor l_{r_3} = l_{p_{21}}$. In general, this is encoded in the following formula $\psi_{\text{out}}$:[6]

$$\psi_{\text{out}}(o, P, R) = \bigwedge_{f \in F} \bigvee_{p \in P - F_f} \left( l_{r_f} = l_p \lor l_{r_f} = l_o \right)$$

Formula $\psi_{\text{flow}}$ along with formula $\psi_{\text{out}}$ guarantee that the generated program has the correct output, thus ensuring that constraint (3) is satisfied. Moreover, we would also like to ensure that no program input is ignored, significantly cutting down the search space, which is guaranteed by formula $\psi_{\text{in}}$ (similar to $\psi_{\text{out}}$):

$$\psi_{\text{in}}(I, P) = \bigwedge_{i \in I} \bigvee_{p \in P} (l_i = l_p)$$

The functional constraint $\psi_{\text{prog}}$ is obtained by adding to $\psi_{\text{wfp}}$ the formulas from this section, wrapping the formal parameter and return value variables $x \in P \cup R$ under an existential quantifier:

$$\psi_{\text{prog}}(I, o, C) = \exists P, R. \left( \psi_{\text{wfp}}(I, o, C, P, R) \land \psi_{\text{spec}}(P, R) \land \psi_{\text{flow}}(I, C, P, R) \land \psi_{\text{out}}(o, P, R) \land \psi_{\text{in}}(I, P) \right)$$

---

[6]: The equality $l_{r_f} = l_o$ is only included in the disjunction if type($r_f$) = type($o$).
Table 4.2: Some symbols constrained by the theory of strings with linear integer arithmetic.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + y$</td>
<td>Function symbol. The terms $x$ and $y$ are integers. Its value, an integer, is the addition of $x$ and $y$.</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Function symbol. The terms $x$ and $y$ are integers. Its value, an integer, is the result of subtracting $y$ from $x$.</td>
</tr>
<tr>
<td>$++$</td>
<td>Function symbol. The terms $x$ and $y$ are strings. Its value, a string, is the string concatenation of $x$ and $y$.</td>
</tr>
<tr>
<td>$\text{IndexOf}(s,t,i)$</td>
<td>Function symbol. The terms $s$ and $t$ are strings, and $i$ is an integer. Its value, an integer, is the first occurrence of $t$ in $s$ after the index $i$, or $-1$ if $t$ is not in $s$.</td>
</tr>
<tr>
<td>$\text{Length}(s)$</td>
<td>Function symbol. The term $s$ is a string. Its value, an integer, is the length (number of characters) of $s$.</td>
</tr>
<tr>
<td>$\text{Substr}(s,i,j)$</td>
<td>The term $s$ is a string, and the terms $i$ and $j$ are integers. Its value, a string, is the substring of $s$ starting at index $i$ with length $j$ (or from $i$ until the end of $s$ if the length of $s$ is less than $i+j$).</td>
</tr>
</tbody>
</table>

Full Constraint

The formula $\psi_{\text{prog}}(I,o)$ encodes a well-formed program that satisfies the input-output example $(I,o)$. We can get a formula $\Psi$ that works over all provided input-output examples $(I,o) \in E$ with a simple conjunction over $E$ like so:

$$\Psi = \exists L, C. \bigwedge_{(I,o) \in E} \psi_{\text{prog}}(I,o,C)$$

In essence, formula $\Psi$ encodes the different runs of the program over all the provided input-output examples. A model of this formula corresponds to a program that uses only the components provided by the enumerator, and satisfies all input-output examples. The variables $x \in L \cup C$ retain their values across all runs, and are the only information we need in order to extract the program.

4.3 SMT Interlude

At this point it might be nice to sit back and understand how the components of our language (see Table 4.1) are encoded in SMT. However, it should be noted that the encoding is independent of the particular components used, and works as long as their semantics can be fully encoded in SMT.

In section 2.2 we introduced the concepts of SMT necessary for the purpose of this thesis. One in particular, was the concept of theory (Definition 9). Theories constrain the interpretation given to certain symbols. Because we are dealing with text and integers, we make use of the SMT theory of strings with linear integer arithmetic. This theory allows one to reason about operations such as integer addition, string length, or substring extraction. The symbols we are interested in are represented in Table 4.2.

The semantics of the components $\text{Concat}$, $\text{IndexOf}$, $\text{Length}$, $\text{Substr}$, $\text{Add}$, and $\text{Sub}$ are directly captured by their SMT counterparts $++$, $\text{IndexOf}$, $\text{Length}$, $\text{Substr}$, $+$, and $-$, respectively. Concretely, their
specifications are as follows:

\[
\phi_{\text{Concat}}(x, y, r) = (x + y = r)
\]

\[
\phi_{\text{IndexOf}}(s, t, i, r) = (\text{IndexOf}(s, t, i) = r)
\]

\[
\phi_{\text{Length}}(s, r) = (\text{Length}(s) = r)
\]

\[
\phi_{\text{Substr}}(s, i, j, r) = (\text{Substr}(s, i, j) = r)
\]

\[
\phi_{\text{Add}}(x, y, r) = (x + y = r)
\]

\[
\phi_{\text{Sub}}(x, y, r) = (x - y = r)
\]

4.4 Whole Encoding

In the previous approach the workload was split between the enumerator and the solver: the enumerator would exhaustively search for all combinations (with replacement) of the library of components in order of increasing size and pass them one by one to the solver. In turn, the solver would then verify if a program existed that satisfied the input-output examples using each of the given components exactly once. However, the number of combinations of a given size grows exponentially. One wonders if it is possible to free the enumerator by pushing as much workload as possible to the solver.

In the next approach we reduce the enumerator to query the solver with a single number \(n\) (plus the input-output examples). Instead of drawing components from the library and passing them to the solver, the enumerator now queries if there is any program of exactly \(n\) components that satisfies the examples. Thus, the solver is now parameterized by the library of components \(F\). This process is repeated for increasing values of \(n\) until a program is found.

4.4.1 Program Formula

In this section, we adapt the location-based encoding from the previous section in order to find a program with the exact number of allowed components, \(n\). Just as before, we will have a set \(I\) of input variables, an output variable \(o\), a set \(C\) of constant variables, a set \(L\) of location variables, a set \(R\) of return variables, and a set \(P\) of formal parameter variables, with some changes, as we will see next.

In this scenario we do not know how many times each component will be used in the synthesized program. This means we must have some way to model the choices of which components get picked and which do not. For this reason we introduce a new set of integer-valued variables \(A = \{a_1, a_2, \ldots, a_n\}\), which we will refer to as the activation variables. The activation variables have values in the set \(\{1, 2, \ldots, |F|\}\), with the interpretation that if we have \(a_i = k\), then component \(f_k\) will be the \(i\)-th component in the program (the one appearing in line \(|I| + |C| + i\)). For the same reason, this time we do not assign return variables to each component. Instead, since we know the number \(n\) of components we are aiming for, we have one return variable \(r_i\) associated with each \(a_i\), for \(i = 1, \ldots, n\). These components will have their locations fixed beforehand: variable \(r_i \in R\) will be assigned to location \(l_{r_i} = |I| + |C| + i\).
\[ \text{prog}(i_0, i_1): \]
\[ c_0 = ? \]
\[ c_1 = ? \]
\[ r_1 = f_{a_1}(p_{11}, p_{12}, p_{13}) \]
\[ r_2 = f_{a_2}(p_{21}, p_{22}, p_{23}) \]
\[ r_3 = f_{a_2}(p_{31}, p_{32}, p_{33}) \]

Figure 4.4: Symbolic representation of a program with two inputs, \( i_0, i_1 \in I \), two constant variables \( c_0, c_1 \in C \), three return value variables \( r_i \in R \), for \( i = 1, \ldots, n \), and nine formal parameter variables \( p_{ij} \in P \), for \( i = 1, \ldots, n \), \( j = 1, \ldots, m \), with \( n = m = 3 \). The question marks '?' are values to be found by the solver.

Example 4.4 Consider the program in Figure 4.1. Assuming that component \( \text{Concat} \) corresponds to \( k = 1 \), \( \text{Index} \) to \( k = 2 \), and \( \text{Substr} \) to \( k = 3 \), then we would have \( a_1 = 2 \), \( a_2 = 3 \), and \( a_3 = 1 \). We would also have \( l_{r_1} = 5 \), \( l_{r_2} = 6 \), and \( l_{r_3} = 7 \).

Since the values of the activation variables \( x \in A \) will only be known after solving the formula, we do not know apriori the concrete types of the return variables \( x \in R \). We circumvent the problem by augmenting the type of each return variable to be the union of all possible return types. In our case, this implies that all return variables are either of type \text{Text}, or of type \text{Integer}.

The same happens for the types of the formal parameter variables \( x \in P \). Moreover, we do not know exactly how many formal parameter variables to create because we do not know the arities of the components that will be picked for each line. We know, however, the maximum arity \( m \) of all components. Hence, there is an upper bound on the number of formal parameter variables we might need, namely \( n \ast m \). Thus, we introduce variables \( p_{ij} \in P \), for \( i = 1, 2, \ldots, n \), and \( j = 1, 2, \ldots, m \), where each \( p_{ij} \) is the \( j \)-th parameter of \( f_{a_i} \), the component activated by \( a_i \). We also augment their type to be the union of all possible formal parameter types, plus a \text{Null} type, inhabited by a single value, \text{null}, to indicate the absence of value. A variable \( p_{ij} \in P \) will be \text{null} if and only if the arity of component \( f_{a_i} \) is less than \( j \), and consequently will be ignored by \( f_{a_i} \) if that is the case. Finally, we perform the same \text{Null} type augmentation to the location variables \( x \in L \), because it makes no sense in having a location number if a variable is null.

Example 4.5 Consider program 4.1 and its corresponding symbolic skeleton from Figure 4.4. We have three components: \( \text{Concat} \), \( \text{Index} \), and \( \text{Substr} \). Components \( \text{Index} \) and \( \text{Substr} \) have arity 3, which is the largest among the three components. Thus, we have \( n = m = 3 \). This means we have nine variables \( p_{ij} \in P \). Variables \( p_{1j} \), with \( j = 1, \ldots, 3 \) are the formal parameters of \( f_{a_1} \), which is component \( \text{Concat} \). However, \( \text{Concat} \) only takes two parameters. These will be \( p_{11} \) and \( p_{12} \), meaning that \( p_{13} \) will take value \text{null}.

Well-Formedness Constraint

As in the setwise encoding (Section 4.2), each input variable \( x \in I \) is assigned a distinct location \( l_x \in L \) from 1 to \( |I| \), and each constant variable \( x \in C \) is assigned a distinct location from \( |I| + 1 \) to \( |I| + |C| \). As said in the previous section, this time we do the same for each return variable \( x \in R \), assigning to
each a location from \(|I| + |C| + 1\) to \(|I| + |C| + |R|\). The upper bound on the location (if non-null) of each parameter variable \(p_{ij} \in P\) is now statically known to be \(|I| + |C| + i\), so now our well-formedness constraint \(\psi_{wfp}\) is simply:

\[
\psi_{wfp}(L, C, I, P) = \bigwedge_{i=1,\ldots,n} \bigwedge_{j=1,\ldots,m} (l_{p_{ij}} \neq \text{null} \implies 1 \leq l_{p_{ij}} < |I| + |C| + i)
\]

Compared to the well-formedness constraint of the setwise encoding, this constraint is arguably simpler due to the fact that the lines of the return variables are fixed apriori, and because we are not doing any type checking.

**Example 4.6** Consider the symbolic skeleton from Figure 4.4. We have:

\[
\begin{align*}
l_{i_0} &= 1 & l_{i_1} &= 2 \\
l_{c_0} &= 3 & l_{c_1} &= 4 \\
l_{r_1} &= 5 & l_{r_2} &= 6 & l_{r_3} &= 7
\end{align*}
\]

Also, the well-formedness constraint is the conjunction of the following:

\[
\begin{align*}
l_{p_{11}} \neq \text{null} &\implies 1 \leq l_{p_{11}} < 5 \\
l_{p_{12}} \neq \text{null} &\implies 1 \leq l_{p_{12}} < 5 \\
l_{p_{13}} \neq \text{null} &\implies 1 \leq l_{p_{13}} < 5 \\
l_{p_{21}} \neq \text{null} &\implies 1 \leq l_{p_{21}} < 6 \\
l_{p_{23}} \neq \text{null} &\implies 1 \leq l_{p_{23}} < 6 \\
l_{p_{22}} \neq \text{null} &\implies 1 \leq l_{p_{22}} < 6 \\
l_{p_{31}} \neq \text{null} &\implies 1 \leq l_{p_{31}} < 7 \\
l_{p_{32}} \neq \text{null} &\implies 1 \leq l_{p_{32}} < 7 \\
l_{p_{33}} \neq \text{null} &\implies 1 \leq l_{p_{33}} < 7
\end{align*}
\]

**Functional Constraint**

Similarly to the last encoding, we have the following concerns regarding the semantics of the program: (1) relating return variables to the values of their components and their parameters; (2) value sharing between variables with the same location; and (3) effectively mapping the inputs \(x \in I\) to the output \(o\).

Constraint \(\psi_{spec}\), which guarantees constraint (1), should be similar to the one from section 4.2.1, but we will have to pay attention to some special points. First, we have to make sure that, for every component, we relate the correct number of formal parameters to the return value.\(^7\)

\[
\psi_{pnum}(A, P, R) = \bigwedge_{i=1,\ldots,n} \bigwedge_{k=1,\ldots,|F|} (a_i = k \implies \phi_{f_k}(p_{i1}, p_{i2}, \ldots, p_{ijk}, r_i))
\]

In the constraint shown above, \(j_k\) is used as shorthand for the arity of component \(f_k\). Second, we should ensure that formal parameter variables that are in excess have value \(\text{null}\):

\[
\psi_{null}(A, P) = \bigwedge_{i=1,\ldots,n} \bigwedge_{k=1,\ldots,|F|} (a_i = k \implies \bigwedge_{j=j_k+1,\ldots,m} p_{ij} = \text{null})
\]

Finally, we need to constrain the formal parameters to have the correct type, because the specifications\(^7\) Recall that we may have more formal parameter variables than we may need because we do not know apriori which component goes on which line.
\( \phi \) of the components are typed. An example might, perhaps, be the best way to show how this constraint, which we will call \( \psi_{\text{type}} \), can be materialized.

**Example 4.7** Consider the symbolic skeleton from Figure 4.4, and suppose that we are working with components \( f_1 = \text{Concat} \), \( f_2 = \text{Index} \), and \( f_3 = \text{Substr} \). We will use the predicates \( \text{isstring} \) and \( \text{isint} \) to mean that a variable must be of type \( \text{Text} \), or \( \text{Integer} \), respectively. In this concrete case, the following constraint is needed to ensure that the type signatures of these components are satisfied:

\[
\psi_{\text{type}}(A, P) = \bigwedge_{i=1, \ldots, n} (a_i = 1 \implies (\text{isstring}(p_{i1}) \land \text{isstring}(p_{i2}))) \\
\land \bigwedge_{i=1, \ldots, n} (a_i = 2 \implies (\text{isstring}(p_{i1}) \land \text{isstring}(p_{i2}) \land \text{isint}(p_{i3}))) \\
\land \bigwedge_{i=1, \ldots, n} (a_i = 3 \implies (\text{isstring}(p_{i1}) \land \text{isint}(p_{i2}) \land \text{isint}(p_{i3})))
\]

Constraint \( \psi_{\text{spec}} \) is then just the conjunction of the three previous constraints:

\[
\psi_{\text{spec}}(A, P, R) = \psi_{\text{pnum}}(A, P, R) \land \psi_{\text{null}}(A, P) \land \psi_{\text{type}}(A, P)
\]

The dataflow properties of constraint (2) are encoded in a similar fashion as in Section 4.2.1, with the added constraint that every formal parameter variable with \( \text{null} \) value must also have a \( \text{null} \) location:

\[
\psi_{\text{flow}}(C, I, P, R) = \bigwedge_{i=1, \ldots, n} \bigwedge_{j=1, \ldots, m, x \in I \cup C \cup R} (l_{p_{ij}} = l_x \implies p_{ij} = x) \\
\land \bigwedge_{i=1, \ldots, n} \bigwedge_{j=1, \ldots, m} (l_p = \text{null} \implies p = \text{null})
\]

Again, we need to ensure that every return variable is used (as an actual parameter or as the output of the program). This property is encoded as follows:

\[
\psi_{\text{out}}(o, P, R) = \bigwedge_{k=1, \ldots, n-1} \bigvee_{i=k+1, \ldots, n} \bigvee_{j=1, \ldots, m} (l_{r_k} = l_{p_{ij}}) \land (r_n = o)
\]

Formula \( \psi_{\text{out}} \), along with \( \psi_{\text{flow}} \), ensures that constraint (3) is satisfied. We also need to ensure that every input variable appears as an actual parameter of some component, a property encoded by the following formula:

\[
\psi_{\text{in}}(I, P) = \bigwedge_{x \in I} \bigvee_{i=1, \ldots, n} \bigvee_{j=1, \ldots, m} (l_x = l_{p_{ij}})
\]

The functional constraint is obtained by conjoining the previous constraints with the well-formedness constraint \( \psi_{\text{wfp}} \), again wrapping the formal parameter and return value variables \( x \in P \cup R \) under an
The functional constraint $\psi_{prog}$ encodes the space of well-formed programs that satisfy a specific input-output example $(I, o)$. To obtain a formula that ranges over all input-output examples, we just make a conjunction over the set $E$:

$$\Psi = \exists A, L, C. \bigwedge_{(I, o) \in E} \psi_{prog}(A, L, C, I, o)$$

A model of this formula is an assignment to the variables $x \in A \cup L \cup C$, which can then be used to reconstruct a program that satisfies the input-output examples $E$ using only $n$ components.
Chapter 5

Experimental Results

In this chapter, we evaluate and compare the synthesizers presented in Chapter 4. The synthesizers are compared in terms of their running time and number of problems solved. A description of the benchmarks is provided in Section 5.1. Section 5.2 presents the experimental evaluation of the experiments and a comparison between the setwise synthesizer (Section 4.2) and the whole synthesizer (Section 4.4).

5.1 Benchmark Description

A set of 285,522 expressions were provided by OutSystems. We conducted an analysis to determine which builtin functions and combinations of functions were the most common in that set. We picked 51 expressions containing only functions from Table 4.1, with sizes (number of components, i.e., the number of lines of the program in SSA form minus the number of constants in the expression) ranging from 1 to 7. The size distribution of these 51 expressions is shown in Figure 5.1. Figure 5.2 shows the number of expressions in which each component occurs.

The hardness of a benchmark may depend on the size of the solution, the number of input-output examples, and the library of components. Typically, the higher the size of the intended solution and the

![Figure 5.1: Number of expressions per size, out of 51 expressions. For example, there are 31 expressions of size 2, and 16 expressions of size 1. Most expressions have size between 2 and 4.](image)
number of components in the library, the harder it is to synthesize a program.

We obtained a set of 3 input-output examples for each of these 51 expressions. In order to do that, we developed an *interpreter* for OutSystems expressions, and manually created a set of 3 different inputs for each expression. Then we interpreted the expressions over their set of inputs in order to obtain the corresponding outputs. The inputs were created in order to try to eliminate as much ambiguity as possible, i.e., seeking to make the set of expressions matching the examples as small as possible. A first approach was tried where we encoded the expressions in SMT in such a way that a solution to the formulas yielded valid input-output examples. Although the approach worked, it was ultimately too slow, and the automatically generated examples were not natural. As a result, we resorted to the manual approach.

The SMT solver used to solve the formulas generated in the synthesis process was Microsoft’s Z3 solver [7] (version 4.8.5). Each benchmark was monitored by the *runsolver* tool [30], typically used in SAT\(^1\), MaxSAT\(^2\) and PB\(^3\) competitions, and restricted to a wall-clock time limit of 600 seconds and a memory limit of 16 GB. We ran the experiments in an computer with Intel(R) Xeon(R) CPU E5-2620 v4 @ 2.10GHz processors running Ubuntu 16.04.5 LTS.

### 5.2 Evaluation

We are interested in evaluating the impact of the number and quality of the input-output examples on the performance of the synthesizer in terms of runtime and program quality. In this section, we only study the impact of the number of input-output examples.

We present the results for both synthesizers described in Chapter 4 with the following configurations. For each instance with 3 input-output examples we ran both synthesizers using 1, 2, or 3 of the input-output examples. We ran the setwise synthesizer (Section 4.2) configured to synthesize programs with a maximum of one integer and one string constants (we refer to these configurations as S-e1:c1, S-e2:c1, and S-e3:c1, for 1, 2, and 3 input-output examples, respectively), and also with a maximum of two integer and two string constants (configurations S-e1:c2, S-e2:c2, and S-e3:c2). We ran the whole synthesizer

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\(^1\)http://satcompetition.org/  
\(^2\)http://www.maxsat.udl.cat  
\(^3\)http://www.cril.univ-artois.fr/PB16/
(Section 4.4) configured to synthesize programs with a maximum of one constant. The synthesizer can choose whether this constant is an integer or a string (configurations W-e1:c1, W-e2:c1, and W-e3:c1, for 1, 2, and 3 input-output examples, respectively). Similarly, we also run the whole synthesizer configured to use a maximum of four constants (configurations W-e1:c4, W-e2:c4, and W-e3:c4).

In addition, we also instantiated SyPet [9], a PBE component-based synthesizer for Java programs, with our library of components. SyPet employs a search technique that is guided by the type signatures of Java methods, together with constraint-solving techniques. However, SyPet does not support guessing the value of constants. With this in mind, we set up two configurations for SyPet. For the first configuration (SyPet-All), we introduced a new 0-arity component for each constant in a pool of predefined constants.\(^4\) For the second configuration (SyPet-User), we looked at each particular instance, and introduced new components only for the constants that appear in the expected solution. This mimics a setting where the constants are declared by the user, instead of being guessed by the synthesizer. Both SyPet-User and SyPet-All were configured for using the three input-output examples of each benchmark, and may use as many constants as they need (out of the available ones). Table 5.1 shows a comparison between the different configurations in terms of the number of instances solved and running wall-clock time (mean and median for solved instances). An instance is considered solved if the synthesized program satisfies the input-output examples (but might or might not be what the user intended). Matching the expected solution means that the synthesizer outputs a program that, not only satisfies the input-output examples, but also captures the original intent and generalizes to more examples (this is checked by manual inspection). Figures 5.3 and 5.4 show the number of solved instances per size of the expected solution for the configurations S-e1:c1, S-e2:c1 and S-e3:c1, and for configurations S-e1:c4, S-e2:c4 and S-e3:c4, respectively. Figures 5.5 and 5.6 show the same for configurations W-e1:c1, W-e2:c1 and W-e3:c1, and for configurations W-e1:c4, W-e2:c4 and W-e3:c4, respectively. Figures 5.7 and 5.8 show the number of solved instances matching the expected solution per size of the expected solution for configurations S-e1:c1, S-e2:c1 and S-e3:c1, and for configurations S-e1:c4, S-e2:c4 and S-e3:c4, respectively. Figures 5.5 and 5.10 show the same for configurations W-e1:c1, W-e2:c1 and W-e3:c1, and for configurations W-e1:c4, W-e2:c4 and W-e3:c4, respectively. Figures 5.11 and 5.12 show a comparison between all configurations on three input-output examples.

5.3 Discussion

The sizes of both encodings grow linearly with the number of input-output examples. Indeed, we can verify that the hardness of the problem increases with the number of examples (Figures 5.3, 5.4, 5.6 and 5.6). This effect becomes more significant as we increase the size of the expected solution. On the other hand, the number of programs matching the expected solution should increase with the number of input-output examples, which we can also verify from the results in Figures 5.7, 5.8, 5.10 and 5.10.

\(^4\)The pool of constants, 43 in total, is the following: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 16, 20, 30, 40, 50, 60, 70, 80, 90, 100, ".", ", .","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":","":"}\n
\[\text{\textcopyright Created on "}, \text{\textcopyright updated on "}, \text{\textcopyright At "}.\text{These were chosen in order to roughly match those found in the expected solutions. The expected solutions employ a total of 34 constants from this pool.}
Table 5.1: Comparison between the different configurations by number of instances solved and running wall-clock time for solved instances (not necessarily matching the expected solution).

<table>
<thead>
<tr>
<th>Configuration</th>
<th># Instances solved</th>
<th># Instances solved (matching expected solution)</th>
<th>Mean Time (s)</th>
<th>Median Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-e1:c1</td>
<td>48</td>
<td>3</td>
<td>4.62</td>
<td>0.48</td>
</tr>
<tr>
<td>S-e1:c2</td>
<td>49</td>
<td>6</td>
<td>0.67</td>
<td>0.29</td>
</tr>
<tr>
<td>S-e2:c1</td>
<td>39</td>
<td>21</td>
<td>51.28</td>
<td>3.21</td>
</tr>
<tr>
<td>S-e2:c2</td>
<td>42</td>
<td>13</td>
<td>46.30</td>
<td>1.12</td>
</tr>
<tr>
<td>S-e3:c1</td>
<td>34</td>
<td>23</td>
<td>51.38</td>
<td>4.47</td>
</tr>
<tr>
<td>S-e3:c2</td>
<td>34</td>
<td>19</td>
<td>39.97</td>
<td>1.34</td>
</tr>
<tr>
<td>W-e1:c1</td>
<td>47</td>
<td>2</td>
<td>6.76</td>
<td>0.65</td>
</tr>
<tr>
<td>W-e1:c4</td>
<td>50</td>
<td>4</td>
<td>2.69</td>
<td>0.09</td>
</tr>
<tr>
<td>W-e2:c1</td>
<td>13</td>
<td>4</td>
<td>11.14</td>
<td>2.24</td>
</tr>
<tr>
<td>W-e2:c4</td>
<td>17</td>
<td>7</td>
<td>50.37</td>
<td>0.15</td>
</tr>
<tr>
<td>W-e3:c1</td>
<td>9</td>
<td>5</td>
<td>21.45</td>
<td>7.48</td>
</tr>
<tr>
<td>W-e3:c4</td>
<td>10</td>
<td>5</td>
<td>0.27</td>
<td>0.17</td>
</tr>
<tr>
<td>SyPet-All</td>
<td>11</td>
<td>6</td>
<td>31.27</td>
<td>3.00</td>
</tr>
<tr>
<td>SyPet-User</td>
<td>32</td>
<td>30</td>
<td>52.75</td>
<td>5.00</td>
</tr>
</tbody>
</table>

In Figures 5.11 and 5.12 we can see that SyPet-All has similar results to the whole synthesizer. The large number of components makes the search space intractable. Besides, it cannot take advantage of its type-directed algorithm because we are dealing with only two types (integers and strings). This last point also applies to SyPet-User, but the advantage given by the user-provided constants proves crucial for its fairly good results, which are actually better than the ones obtained for the setwise synthesizer (Table 5.1). However, recall that the setwise synthesizer is solving a harder problem because it computes the constants itself.

Many expected solutions require more than one constant, with some requiring more than one constant of each type. This could lead one to think that that is the origin of the poor performance of the whole encoding. However, we can see that increasing the number of available constants leads to even worse results in both synthesizers (on these instances). The configurations with more constants available lead to more instances solved and in less time, but also lead to less intended solutions. It could also be speculated from the start that the whole synthesizer would perform worse than the setwise synthesizer. Typically, one would expect that solving one large constraint is more expensive than solving multiple smaller constraints.\(^5\)

The running times for the solved benchmarks are reasonably fast, allowing for tolerable interaction times with the user. However, synthesizing programs with 4 or more lines seems to be out of reach for these configurations (Figure 5.12).

\(^5\)Note that the SMT problem with the theory of strings and linear arithmetic is undecidable.
Figure 5.3: Number of solved instances per size of the expected solution for the setwise synthesizer with one, two, and three examples, one integer constant, and one string constant (S-e1:c1, S-e2:c1, S-e3:c1, respectively).

Figure 5.4: Number of solved instances per size of the expected solution for the setwise synthesizer with one, two, and three examples, two integer constants, and two string constants (S-e1:c2, S-e2:c2, S-e3:c2, respectively).
Figure 5.5: Number of solved instances per size of the expected solution for the whole synthesizer with one, two, and three examples, and one constant (W-e1:c1, W-e2:c1, W-e3:c1, respectively).

Figure 5.6: Number of solved instances per size of the expected solution for the whole synthesizer with one, two, and three examples, and four constants (W-e1:c4, W-e2:c4, W-e3:c4, respectively).
Figure 5.7: Number of solved instances matching the expected solution per size of the expected solution for the setwise synthesizer with one, two, and three examples, one integer constant, and one string constant (S-e1:c1, S-e2:c1, S-e3:c1, respectively).

Figure 5.8: Number of solved instances matching the expected solution per size of the expected solution for the setwise synthesizer with one, two, and three examples, two integer constants, and two string constants (S-e1:c2, S-e2:c2, S-e3:c2, respectively).
Figure 5.9: Number of solved instances matching the expected solution per size of the expected solution for the whole synthesizer with one, two, and three examples, and one constant (W-e1:c1, W-e2:c1, W-e3:c1, respectively).

Figure 5.10: Number of solved instances matching the expected solution per size of the expected solution for the whole synthesizer with one, two, and three examples, and four constants (W-e1:c4, W-e2:c4, W-e3:c4, respectively).
Figure 5.11: Number of solved instances per size of the expected solution for the setwise and whole synthesizers with three examples, and both SyPet configurations.

Figure 5.12: Number of solved instances matching the expected solution per size of the expected solution for the setwise and whole synthesizers with three examples, and both SyPet configurations.
In this work, we tackle the problem of synthesizing OutSystems expressions from examples, focusing on expressions that manipulate integers and text. OutSystems sells solutions to help building enterprise-grade applications swiftly and with a great degree of automation. Thus, we were interested in an “push-button”-style approach that would be performant, and could generalize from a small number of examples. We surveyed the state of the art in program synthesis, and implemented two component-based PBE synthesizers – setwise and whole. Both synthesizers employ a mixture of constraint solving with basic enumerative search, differing on the amount of work they put on the constraint solving phase. We benchmarked both synthesizers and compared them to SyPet [9] with two different configurations: one where all constants needed to solve the instances are given as new components, and another that simulates user-provided constants by only allowing the constants needed for each particular instance.

The setwise synthesizer can consistently synthesize programs up to four lines that satisfy the given input-output examples, but only manages to match the user intent on programs with size up to three lines. Still, it manages to be competitive with SyPet, even when we configured the latter for a scenario simulating user-provided constants. However, the enumerator of the setwise synthesizer is very simple, and it would be interesting to see how more elaborate approaches, such as stochastic search, would fare. On the other hand, the whole synthesizer fails to produce good results on both fronts in all but the most trivial instances (programs with one line). However, additional variations and refinements of the encodings should be tested. It would also be interesting to test configurations with user-provided inputs. Another interesting scenario, probably involving extensions to the synthesizers includes trying to figure out from the input-output examples which constants the program might use. Moreover, it would be interesting to see how our synthesizers compare to a synthesizer implemented using the PROSE framework.¹

Both approaches use encodings which size scales linearly with the number of input-output examples. While not ideal, it is not unacceptable given that we are interested in offloading as much work as possible from the user, and so the approach should generalize with a small number of examples. Given the experimental results, tending towards approaches more reliant on general-purpose constraint

¹https://microsoft.github.io/prose/
solving might not be the best way to tackle this kind of problems. On the other hand, it would be interesting to explore how solvers specialized for synthesis, or approaches with a tighter integration with the underlying solver might fare. In fact, some approaches in the literature apply this idea [1, 10]. Moreover, it should be explored whether this approach scales when a larger library of components is used, especially components that do not have a direct encoding in SMT (and, in turn, could be more difficult, or even impossible to synthesize). It would be particularly interesting to support if-then-else expressions, along with frequently used predicates over Integers and Texts.
Acronyms

API  application programming interface 10

CEGIS  counterexample-guided inductive synthesis 12, 17

CFG  context-free grammar 6, 10, 13, 14, 21

DSL  domain-specific language 4, 12, 14, 20

OGIS  oracle-guided inductive synthesis xiii, 4, 12, 16, 17, 22

PBD  programming by demonstration 11

PBE  programming by examples 4–6, 11, 17, 20, 34, 41

SMT  satisfiability modulo theories ix, x, 7, 8, 15, 19, 20, 22, 26, 33, 35, 42

SSA  static single assignment xiii, 21, 22, 32

SyGuS  syntax-guided synthesis 10
Bibliography


