# Photovoltaic System with Multi-Terminal Power Electronic Converter 

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Abstract- In this article a multi-terminal converter that allows the connection of a photovoltaic (PV) generation system to the Low Voltage (LV) grid is proposed and sized. More specifically, a 4 -arm matrix converter will be used, allowing the connection of 3 photovoltaic panels to the threephase electrical grid.

The objective of the article is to control independently the PV panels, ensuring that all guarantee Maximum Power Point Tracking (MPPT), even if when total or partial shading of one or more panels occurs. To do this, a sliding mode controller will be used, in order to get a quick response of the system in different operating conditions.

The currents injected in the grid will be controlled, as well to minimize the impact of the connection of the PV system in the grid, allowing for minimum harmonic content and reactive power control.

The proposed system will be tested in a simulation environment for several operating scenarios: normal operation, partial converter failure, or irradiance variation in the PVs.

Keywords- Multiterminal PV system, 4 arm matrix converter, MPPT, unitary power factor

## I. Introduction

With the growing need to reduce the use of fossil fuels as an energy source, comes the opportunity to explore the sources of renewable energy, including photovoltaic energy.

Portugal is a country with a high incidence of solar radiation when compared to most European countries. For example, in the south of the country, in Faro it is possible to generate $1718 \mathrm{kWh} / \mathrm{kWp}$ of electricity with photovoltaic (PV) panels. The reasons to invest in this type of energy are, therefore, evident. [1]

Despite this high potential for exploitation and the growing investment in renewable energies, the production of electricity through solar energy remains very low. For example, between January and August 2018, only $1.50 \%$ corresponded to PV sources, even though $55.3 \%$ of the energy produced in Portugal was generated from renewable sources. [2]

The high cost associated with constructing and developing panels, coupled with reduced efficiency and low annual power utilization, provide some of the reasoning that explains the low market penetration of this source, Therefore, it's necessary to focus on improving efficiency of the PVs. [3]

Since PVs panels produce DC voltage and DC current,
and the low voltage distribution grid is AC , it is necessary to use a DC-AC converter, as a matrix converter.

Matrix converters have many advantages over Voltage Source Converter based topologies such as rectifier-inverter back-to-back association. One of these advantages is the fact of not having intermediate DC link with energy storage elements, which usually have a reduced lifetime. Also, the lack of energy storage elements (except for a small input filter) allows a reduction in cost and size. This converter has other advantages like reduce harmonic content, adjustable power factor, and bi-directional power flow [4] [5].

However, it has a couple of disadvantages. The inexistence of storage elements makes it sensitive to disturbances in the AC network, for example, 5th and 7th harmonic voltage [6] and it needs more semiconductors than equivalent converters.

Besides the advantages presented by the matrix converters, using a converter with multiple outputs and having a independent MPPT system for each PV panel, allows not only a optimization of the MPPT system, but also reduces the number of necessary converters., thus decreasing the overall volume of the system.

Another great advantage of this proposed system is to permit independent control of all PVs without using a boost converter or a transformer. [7] [8]

## II. Proposed Topology

The proposed PV system consists of a multi-terminal converter, with a connection of several panels to the same converter, that allows for independent control of each panel, without needing additional boost converters, reducing the number of semiconductors and increasing the efficiency of the system.

## A. Current source inverter

The DC-AC inverter contains 12 bi-directional switches, $\left.S_{k j}, k \in\{1,2,3,4\}, j \in\{1,2,3\}\right)$. Each switch has 2 possible values $S_{k j}=1$, when closed ( ON ) or $S_{k j}=0$ when open (OFF). The state of switches can be represented by the matrix (1): [9]

$$
S=\left[\begin{array}{lll}
\mathrm{S}_{11} & \mathrm{~S}_{12} & \mathrm{~S}_{13}  \tag{1}\\
\mathrm{~S}_{21} & \mathrm{~S}_{22} & \mathrm{~S}_{23} \\
\mathrm{~S}_{31} & \mathrm{~S}_{32} & \mathrm{~S}_{33} \\
\mathrm{~S}_{41} & \mathrm{~S}_{42} & \mathrm{~S}_{43}
\end{array}\right]
$$



Fig. 1 General scheme of the proposed system

The number of switches combination is restricted to 81 because the short circuit in AC voltages has to be avoided, and the continuity of Dc currents has to be guaranteed. As a result, the instantaneous sum of each line of $S$ must be 1 $\left(\sum_{j=1}^{3} S_{k j}=1, k \in\{1,2,3,4\}\right)$.

The relationship between the converter input ( $v_{a}, v_{b}, v_{c}$ ) and output $\left(v_{A}, v_{B}, v_{C}, v_{D}\right)$ voltages and currents is given by (2). [9]

$$
\left[\begin{array}{l}
v_{A}  \tag{2}\\
v_{B} \\
v_{C} \\
v_{D}
\end{array}\right]=[S]\left[\begin{array}{l}
v_{a} \\
v_{b} \\
v_{c}
\end{array}\right],\left[\begin{array}{l}
i_{a} \\
i_{b} \\
i_{c}
\end{array}\right]=\left[S^{T}\right]\left[\begin{array}{l}
i_{A} \\
i_{B} \\
i_{C} \\
i_{D}
\end{array}\right]
$$

Through these relationships, the 81 combinations of the switches are obtained (Tab 1). These combinations represent the value of voltage and current in each time instant.

Tab 1 - Combination of switching states of semiconductors

| $\begin{aligned} & \text { \#. } \\ & \text { ت゙ } \end{aligned}$ | $S_{11}$ | $S_{12}$ | $S_{13}$ | $S_{21}$ | $S_{22}$ | $S_{23}$ | $\mathrm{S}_{31}$ | $S_{32}$ | $S_{33}$ | $S_{41}$ | $S_{42}$ | $S_{43}$ | $V_{A}$ | $V_{B}$ | $V_{C}$ | $V_{N}$ | $V_{A N}$ | VBN | $V_{C N}$ | $I_{a}$ | $I_{b}$ | $I_{c}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{2}$ | $\mathrm{V}_{2}$ | V, | $\mathrm{V}_{2}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\text {AB }}$ | 0 | 0 | $\mathrm{I}_{A}$ | $-\mathrm{I}_{\text {A }}$ | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-V_{A B}$ | 0 | 0 | - $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{I}_{\mathrm{A}}$ | 0 |
| 6 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{c}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $V_{B C}$ | 0 | 0 | 0 | $\mathrm{I}_{\mathrm{A}}$ | - $\mathrm{I}_{\mathrm{A}}$ |
| 7 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $-\mathrm{V}_{\mathrm{BC}}$ | 0 | 0 | 0 | -IA | $\mathrm{I}_{\mathrm{A}}$ |
| 8 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\text {CA }}$ | 0 | 0 | - $\mathrm{I}_{\mathrm{A}}$ | 0 | $\mathrm{I}_{\mathrm{A}}$ |
| 9 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{3}$ | $V_{c}$ | $\mathrm{V}_{\mathrm{c}}$ | $V_{c}$ | - $\mathrm{V}_{C A}$ | 0 | 0 | IA | 0 | - ${ }_{\text {A }}$ |
| 10 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | $\mathrm{V}_{\mathrm{AB}}$ | 0 | $\mathrm{I}_{\mathrm{B}}$ | $-\mathrm{I}_{\mathrm{B}}$ | 0 |
| 11 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{2}$ | $\mathrm{V}_{3}$ | 0 | $-V_{A B}$ | 0 | - $\mathrm{I}_{\mathrm{B}}$ | IB | 0 |
| 12 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\text {c }}$ | 0 | $\mathrm{V}_{B C}$ | 0 | 0 | $\mathrm{I}_{\mathrm{B}}$ | - $\mathrm{I}_{\mathrm{B}}$ |
| 13 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | $-V_{B C}$ | 0 | 0 | $-\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 14 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{c}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | 0 | $\mathrm{V}_{\text {CA }}$ | 0 | - $\mathrm{IB}_{B}$ | 0 | $\mathrm{I}_{\mathrm{B}}$ |
| 15 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | - $\mathrm{V}_{\mathrm{CA}}$ | 0 | $\mathrm{I}_{\mathrm{B}}$ | 0 | - $\mathrm{I}_{\mathrm{B}}$ |
| 16 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | 0 | $\mathrm{V}_{\text {AB }}$ | $\mathrm{I}_{\mathrm{C}}$ | - $\mathrm{I}_{\mathrm{C}}$ | 0 |
| 17 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | 0 | 0 | $-\mathrm{V}_{\text {AB }}$ | - $\mathrm{IC}_{C}$ | $\mathrm{I}_{C}$ | 0 |
| 18 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | 0 | $\mathrm{V}_{B C}$ | 0 | IC | -IC |
| 19 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{h}}$ | $\mathrm{V}_{\text {c }}$ | $\mathrm{V}_{\mathrm{h}}$ | 0 | 0 | $-V_{B C}$ | 0 | - $\mathrm{I}_{\mathrm{C}}$ | IC |
| 20 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | 0 | 0 | $\mathrm{V}_{\mathrm{CA}}$ | - $\mathrm{IC}_{\text {c }}$ | 0 | IC |
| 21 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\text {c }}$ | 0 | 0 | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{C}}$ | 0 | -IC |
| 22 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\text {AB }}$ | $\mathrm{V}_{\text {AB }}$ | 0 | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{C}}+\mathrm{I}_{\mathrm{N}}$ | 0 |
| 23 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-V_{A B}$ | $-V_{A B}$ | 0 | $\mathrm{IC}_{\mathrm{c}+\mathrm{IN}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ | 0 |
| 24 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $V_{B C}$ | $V_{B C}$ | 0 | 0 | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ | ${\mathrm{IC}+\mathrm{I}_{\mathrm{N}} \text { }}_{\text {d }}$ |
| 25 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $V_{c}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $-V_{B C}$ | $-V_{B C}$ | 0 | 0 | $\mathrm{I}_{\mathrm{C}+\mathrm{I}_{\mathrm{N}}}$ | $\mathrm{I}_{A}+\mathrm{I}_{B}$ |
| 26 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\text {c }}$ | $\mathrm{V}_{\text {c }}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\text {CA }}$ | $\mathrm{V}_{\text {CA }}$ | 0 | $\mathrm{I}_{\mathrm{c}+\mathrm{I}_{N}}$ | 0 | $\mathrm{I}_{A}+\mathrm{I}_{B}$ |
| 27 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{3}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\text {c }}$ | - $\mathrm{V}_{C A}$ | $-V_{C A}$ | 0 | $\mathrm{I}_{A}+\mathrm{I}_{\mathrm{B}}$ | 0 | $\mathrm{I}_{\mathrm{C}+\mathrm{I}_{\mathrm{N}}}$ |
| 28 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | $\mathrm{V}_{\text {AB }}$ | $\mathrm{V}_{A B}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\Delta}+\mathrm{I}_{N}$ | 0 |
| 29 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{3}$ | 0 | $-V_{A B}$ | $-V_{A B}$ | $\mathrm{I}_{A}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | 0 |
| 30 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | $V_{B C}$ | $\mathrm{V}_{\mathrm{BC}}$ | 0 | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | $+\mathrm{IN}^{\text {N}}$ |


| 31 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | $-\mathrm{V}_{\mathrm{BC}}$ | $-V_{B C}$ | 0 | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | 0 | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{IN}$ | 0 | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ |
| 33 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | - $\mathrm{V}_{\mathrm{CA}}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | 0 | $\mathrm{I}_{\mathrm{A}+}+\mathrm{IN}$ |
| 34 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\text {AB }}$ | 0 | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{C}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | 0 |
| 35 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{h}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{3}$ | $-V_{A B}$ | 0 | $-\mathrm{V}_{\text {AB }}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{A}+\mathrm{I}_{C}$ | 0 |
| 36 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{BC}}$ | 0 | $\mathrm{V}_{\mathrm{BC}}$ | 0 | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{C}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ |
| 37 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | - $\mathrm{V}_{\mathrm{BC}}$ | 0 | - $\mathrm{V}_{\text {BC }}$ | 0 | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{C}}$ |
| 38 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{CA}}$ | 0 | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | 0 | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{C}$ |
| 39 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | - $\mathrm{V}_{\mathrm{CA}}$ | 0 | - $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{C}}$ | 0 | $\mathrm{I}_{\mathrm{B}}+\mathrm{IN}_{N}$ |
| 40 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{3}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{h}}$ | $\mathrm{V}_{\text {AB }}$ | $\mathrm{V}_{\text {AB }}$ | $\mathrm{V}_{\text {AB }}$ | - $\mathrm{IN}_{N}$ | IN | 0 |
| 41 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | - $\mathrm{V}_{\text {AB }}$ | $-\mathrm{V}_{A B}$ | $-\mathrm{V}_{A B}$ | IN | - $\mathrm{IN}_{\mathrm{N}}$ | 0 |
| 42 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\text {BC }}$ | $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{BC}}$ | 0 | $-\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{N}}$ |
| 43 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\text {c }}$ | $\mathrm{V}_{\sim}$ | $\mathrm{V}_{\text {c }}$ | $\mathrm{V}_{\mathrm{h}}$ | - $\mathrm{V}_{\mathrm{BC}}$ | - $\mathrm{VBC}_{\text {B }}$ | $-V_{B C}$ | 0 | $\mathrm{I}_{\mathrm{N}}$ | $-\mathrm{I}_{\mathrm{N}}$ |
| 44 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{N}}$ | 0 | - $\mathrm{IN}_{\mathrm{N}}$ |
| 45 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $-V_{C A}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $-\mathrm{I}_{\mathrm{N}}$ | 0 | $\mathrm{I}_{\mathrm{N}}$ |
| 46 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $-\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ | $\mathrm{IN}_{\mathrm{N}}$ | IC |
| 47 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-\mathrm{V}_{\mathrm{AB}}$ | $-V_{A B}$ | $\mathrm{V}_{\text {CA }}$ | $\mathrm{IN}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ | IC |
| 48 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{BC}}$ | $-\mathrm{V}_{\mathrm{CA}}$ | IC | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ | $\mathrm{IN}_{\mathrm{N}}$ |
| 49 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $-\mathrm{V}_{\mathrm{BC}}$ | $-V_{B C}$ | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}$ |
| 50 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $-\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{N}}$ | IC | $\mathrm{I}_{A}+\mathrm{I}_{B}$ |
| 51 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $-\mathrm{V}_{C A}$ | $-V_{C A}$ | $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\Delta}+\mathrm{I}_{B}$ | IC | $\mathrm{I}_{\mathrm{N}}$ |
| 52 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $-V_{B C}$ | $\mathrm{V}_{\text {AB }}$ | $\mathrm{I}_{A}+\mathrm{I}_{C}$ | $\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 53 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $-\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 54 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{BC}}$ | $-V_{C A}$ | $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{4}+\mathrm{I}_{C}$ | $\mathrm{IN}_{\mathrm{N}}$ |
| 55 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $-\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $-\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\mathrm{B}}$ | IN | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{C}}$ |
| 56 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $-V_{A B}$ | $\mathrm{V}_{\mathrm{CA}}$ | IN | IB | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{C}}$ |
| 57 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{BC}}$ | $-\mathrm{V}_{C A}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{C}$ | IB | $\mathrm{I}_{\mathrm{N}}$ |
| 58 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $-V_{B C}$ | $-\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\text {A }}$ | $\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ |
| 59 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-\mathrm{V}_{A B}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\text {A }}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ |
| 60 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{BC}}$ | $-V_{C A}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\Delta}$ | $\mathrm{I}_{\mathrm{N}}$ |
| 61 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $-\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}$ |
| 62 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $-\mathrm{V}_{\text {AB }}$ | $-\mathrm{V}_{\mathrm{AB}}$ | IN | $\mathrm{IB}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\text {A }}$ |
| 63 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | - $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{BC}}$ | IA | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{C}$ | IN |
| 64 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | 0 | $-\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{C}}$ |
| 65 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{AB}}$ | 0 | $-V_{B C}$ | $\mathrm{I}_{A}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | IC |
| 66 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{V}_{\mathrm{BC}}$ | 0 | $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{C}+} \mathrm{I}_{\mathrm{N}}$ |
| 67 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | Va | 0 | $\mathrm{V}_{\mathrm{CA}}$ | - $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 68 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\text {AB }}$ | $-\mathrm{V}_{\mathrm{BC}}$ | 0 | $\mathrm{I}_{A}$ | $\mathrm{I}_{\mathrm{C}+} \mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 69 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $-\mathrm{V}_{\mathrm{CA}}$ | 0 | $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{N}$ |
| 70 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-\mathrm{V}_{\mathrm{AB}}$ | 0 | $\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{I}_{\mathrm{C}}$ |
| 71 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | $\mathrm{V}_{\mathrm{AB}}$ | - $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{N}}$ | IC |
| 72 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{3}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{B C}$ | $-\mathrm{V}_{C A}$ | 0 | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{I}_{\mathrm{C}+}+\mathrm{IN}$ |
| 73 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $-\mathrm{V}_{\text {AB }}$ | $\mathrm{V}_{\mathrm{CA}}$ | 0 | $\mathrm{IC}_{\mathrm{C}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 74 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | 0 | $-\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{C}$ | $\mathrm{I}_{4}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{B}}$ |
| 75 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{B C}$ | 0 | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{C}$ | $\mathrm{I}_{\Delta}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{N}$ |
| 76 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{CA}}$ | 0 | $-\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{A}}$ |
| 77 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{b}}$ | - $\mathrm{V}_{\mathrm{BC}}$ | $\mathrm{V}_{\mathrm{AB}}$ | 0 | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{C}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}$ |
| 78 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | $-\mathrm{V}_{\mathrm{CA}}$ | $V_{B C}$ | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{N}}$ |
| 79 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{CA}}$ | $-\mathrm{V}_{\mathrm{AB}}$ | 0 | $\mathrm{I}_{\mathrm{C}+\mathrm{IN}}$ | IB | IA |
| 80 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{b}}$ | $-\mathrm{V}_{B C}$ | 0 | $\mathrm{V}_{\mathrm{AB}}$ | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{N}}$ | $\mathrm{I}_{\mathrm{A}}$ |
| 81 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | $\mathrm{V}_{\mathrm{c}}$ | $\mathrm{V}_{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{a}}$ | $\mathrm{V}_{\mathrm{c}}$ | 0 | $\mathrm{V}_{B C}$ | $-\mathrm{V}_{\mathrm{CA}}$ | $\mathrm{I}_{C}$ | IB | $\mathrm{I}_{A}+\mathrm{I}_{\mathrm{N}}$ |

## B. Model of the PV arrays



Fig. 2 Curve V-I


Fig. 3 General scheme of the conversion system

## C. State space model of the proposed system

The proposed PV system is based on a 4-arm matrix converter, with 3 PV panels connected in its output, as can be seen in Fig. 3.

Considering Fig. 3 and knowing that the state variables will be the currents in the coils ( $i_{L A}, i_{L B}, i_{L C}$ and, $i_{L N}$ ), it is possible to obtain the equations of the DC currents dynamics.

Applying Kirchhoff's voltage law to the converter of Fig. 3, results (3)

$$
\left\{\begin{array}{l}
v_{A N}=L \frac{d i_{l A}}{d t}+R i_{L A}-R i_{L N}-L \frac{d i_{L N}}{d t}  \tag{3}\\
v_{B N}=L \frac{d i_{L B}}{d t}+R i_{L B}-R i_{L N}-L \frac{d i_{L N}}{d t} \\
v_{C N}=L \frac{d i_{L C}}{d t}+R i_{L C}-R i_{L N}-L \frac{d i_{L N}}{d t}
\end{array}\right.
$$

After simplification of (3), results in (4)

$$
\left\{\begin{array}{l}
\frac{d i_{I A}}{d t}=\frac{1}{4 L}\left(3 v_{A N} V_{B N}-V_{C N}-3 R i_{L A}+R i_{L N}+R i_{L B}+R i_{L C}\right)  \tag{4}\\
\frac{d i_{L B}}{d t}=\frac{1}{4 L}\left(-V_{A N}+3 v_{B N}-V_{C N}+R i_{L A}+R i_{L N}+R i_{L C}-3 R i_{L B}\right) \\
\frac{d i_{L C}}{d t}=\frac{1}{4 L}\left(-v_{A N}-V_{B N}+3 v_{C N}+R i_{L N}+R i_{L A}+R i_{L B}-3 R i_{L C}\right)
\end{array}\right.
$$

Due to the linear dependence of currents, it is still possible to obtain equations (5):

$$
\begin{gather*}
v_{A N}+v_{B N}+v_{C N}=0 \\
v_{A N}=-v_{B N}-v_{C N}  \tag{5}\\
v_{B N}=-v_{A N}-v_{C N} \\
v_{C N}=v_{A N}-v_{C N}
\end{gather*}
$$

Using (5) in (4) results in the state model (6):

$$
\left\{\begin{array}{l}
\frac{d i_{L A}}{d t}=\frac{1}{L} v_{A N}+\frac{1}{4 L}\left(-3 R i_{L A}+R i_{L N}+R i_{L B}+R i_{L C}\right)  \tag{6}\\
\frac{d i_{L B}}{d t}=\frac{1}{L} v_{B N}+\frac{1}{4 L}\left(+R i_{L A}+R i_{L N}+R i_{L C}-3 R i_{L B}\right) \\
\frac{d i_{L C}}{d t}=\frac{1}{L} v_{C N}+\frac{1}{4 L}\left(R i_{L N}+R i_{L A}+R i_{L B}-3 R i_{L C}\right)
\end{array}\right.
$$

Analyzing (6) it is possible to conclude that variation of the current in the coils depends linearly on only one variable, that will simplify when controlling the currents in the PVs.

To develop the state model of the entire PV conversion system, it is also necessary to consider the dynamics of the AC side. For this, it is assumed that:

- The converter is connected to a symmetrical and balanced three-phase system, whose voltages are defined by $v_{a}, v_{b}$ and $v_{c}$;
- The input filter is a three-phase association of $L_{f}, C_{f}$ and a parallel damping resistance $\left(R_{s}\right)$;
- The components of the input filter have the same values in all the phases, ie $C_{a}=C_{b}=C_{c}=C_{f}$ and $L_{a}=L_{b}=L_{c}=L_{f}$;
- The state variables will be the capacitor voltages ( $v_{a b}, v_{b c}$ and $v_{c a}$ ), and the current in the input coils ( $\left.i_{L a}, i_{L b}, i_{L c}\right)$;

However, the state model present in (7) can be represented using only linearly independent variables (8), as $i_{L a}=-i_{L b}-i_{L c}$ and $v_{a b}=-v_{b c}-v_{c a}$

$$
\begin{align*}
& {\left[\begin{array}{l}
\frac{d i_{L a}}{d t} \\
\frac{d i_{L b}}{d t} \\
\frac{d i_{L c}}{d t} \\
\frac{d v_{a b}}{d t} \\
\frac{d v_{b c}}{d t} \\
\frac{d v_{c a}}{d t}
\end{array}\right]=\left[\begin{array}{cccccc}
0 & 0 & 0 & 0 & \frac{1}{3 L_{f}} & \frac{2}{3 L_{f}} \\
0 & 0 & 0 & 0 & -\frac{2}{3 L_{f}} & -\frac{1}{3 L_{f}} \\
0 & 0 & 0 & 0 & \frac{1}{3 L_{f}} & -\frac{1}{3 L_{f}} \\
\frac{1}{3 C_{f}} & -\frac{1}{3 C_{f}} & 0 & -\frac{1}{3 C_{f}} & 0 & 0 \\
\frac{1}{3 C_{f}} & \frac{2}{3 C_{f}} & 0 & 0 & -\frac{1}{3 C_{f}} & 0 \\
-\frac{2}{3 C_{f}} & -\frac{1}{3 C_{f}} & 0 & 0 & 0 & -\frac{1}{3 C_{f}}
\end{array}\right]\left[\begin{array}{c}
i_{L a} \\
i_{L b} \\
i_{L c} \\
v_{a b} \\
v_{b c} \\
v_{c a}
\end{array}\right]+\left[\begin{array}{ccc}
\frac{1}{L} & 0 & 0 \\
0 & \frac{1}{L} & 0 \\
0 & 0 & \frac{1}{L} \\
\frac{1}{3 C_{f}} & -\frac{1}{3 C_{f}} & 0 \\
\frac{1}{3 C_{f}} & \frac{2}{3 C_{f}} & 0 \\
-\frac{2}{3 C_{f}} & -\frac{1}{3 C_{f}} & 0
\end{array}\right]\left[\begin{array}{l}
v_{a} \\
v_{b} \\
v_{c}
\end{array}\right]}  \tag{7}\\
& +\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
-\frac{1}{3 C_{f}} & \frac{1}{3 C_{f}} & 0 \\
-\frac{1}{3 C_{f}} & -\frac{2}{3 C_{f}} & 0 \\
\frac{2}{3 C_{f}} & \frac{1}{3 C_{f}} & 0
\end{array}\right]\left[\begin{array}{l}
i_{\text {mat }} \\
i_{\text {mat } B} \\
i_{\text {mat } C}
\end{array}\right] \\
& {\left[\begin{array}{l}
\frac{d i l_{a}}{d t} \\
\frac{d i l_{b}}{d t} \\
\frac{d v_{b c}}{d t} \\
\frac{d v_{c a}}{d t}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 0 & \frac{1}{3 L_{f}} & \frac{2}{3 L_{f}} \\
0 & 0 & -\frac{2}{3 L_{f}} & -\frac{1}{3 L_{f}} \\
\frac{1}{3 C_{f}} & \frac{2}{3 C_{f}} & -\frac{1}{3 C_{f}} & 0 \\
-\frac{2}{3 C_{f}} & -\frac{1}{3 C_{f}} & 0 & -\frac{1}{3 C_{f}}
\end{array}\right]\left[\begin{array}{c}
i_{l a} \\
i_{l b} \\
v_{b c} \\
v_{c a}
\end{array}\right]+\left[\begin{array}{cc}
\frac{1}{L} & 0 \\
0 & \frac{1}{L} \\
\frac{1}{3 C_{f}} & \frac{2}{3 C_{f}} \\
-\frac{2}{3 C_{f}} & -\frac{1}{3 C_{f}}
\end{array}\right]\left[\begin{array}{cc}
v_{a} \\
v_{b}
\end{array}\right]+\left[\begin{array}{cc}
0 & 0 \\
0 & 0 \\
-\frac{1}{3 C_{f}} & -\frac{2}{3 C_{f}} \\
\frac{2}{3 \mathrm{C}_{\mathrm{f}}} & \frac{1}{3 C_{f}}
\end{array}\right]\left[\begin{array}{l}
i_{\text {mata }} \\
i_{m_{\text {matB }}}
\end{array}\right]} \tag{8}
\end{align*}
$$

## III. CONTROL OF THE SYSTEM

In order to assure the correct operation of the system it is necessary to guarantee an almost unitary power factor in the connection to the grid, ie the currents injected in the grid should have a sinusoidal waveform and be nearly in phase with the voltages, and also to ensure the MPPT mechanism, ie, the PVs panels always work at the maximum available power.

## A. MPPT Control

The MPPT is an algorithm that optimizes the operation of the PVs panels, guaranteeing that they generate the maximum possible power under certain conditions of both temperature and irradiation. [9]

In this case, it decided to perform a MPPT system, taking into account the value of the power derivative in order to the current $\frac{d P}{d I}$, where the maximum power point occurs when $\frac{d P}{d I}=0$. Knowing that the power in each PV is $P=V I$, deriving it in order to $I$ and knowing that it should be zero, results in (9) :

$$
\begin{equation*}
\frac{d P}{d I}=0 \Leftrightarrow V+\frac{d V}{d I} I=0 \Leftrightarrow V=-\frac{d V}{d I} I \tag{9}
\end{equation*}
$$

Due to the small variation of voltage and current the power derivative can be (10) :

$$
\begin{equation*}
\frac{d P}{d I}=V(t)+\frac{V-V(t-\Delta t)}{I-I(t-\Delta t)} I \tag{10}
\end{equation*}
$$

Since the goal of this controller is to guarantee the operation of the PV at its maximum power, it is intended that $\frac{d P}{d \boldsymbol{I}}=\mathbf{0}$. Then:

- when $\frac{d P}{d I}<\mathbf{0}$, (right side of Fig. 4), the value of the derivative should increase, that is, the value of the current should be decreased;
- when $\frac{d P}{d I}>\mathbf{0}$, (left side of Fig. 4), the value of the derivative should decrease. Therefore, the value of $I$ should be increased;

In order to simplify the analysis of the derivative signal, a 3-level control signal (SP) is generated:

- $\mathrm{SP}=0$ if $\frac{d P}{d l}<0$;
- $\mathrm{SP}=1$ if $\frac{d P}{d I}=0$;
- $\mathrm{SP}=0$ if $\frac{d P}{d I}>0$;


To control the current, a particular vector (combination of switches) is chosen. The vector is chosen taking in account the voltage applied to the converter terminals.

As the grid voltages are sinusoidal, the phase-to-phase voltages to be applied to the converter DC side is dependent on the time instant. Consequently, it is necessary to divide the grid period of the phase-to-phase voltage in 6 zones, as shown in Fig. 5.

Fig. 4 P-I power curve


Fig. 5. Division of phase-to-phase voltage in zones

After that, the voltage is calculated at the terminals of the PV, to obtain the derivative of $I_{L}$ (6)

If it is desired to increase $\mathrm{i}_{\mathrm{L}}$, then $\frac{d i L}{d t}>0$, otherwise if it is desired to decrease $\mathrm{i}_{\mathrm{L}}$, then $\frac{d i L}{d t}<0$, so analyzing Fig. 6 , it is concluded that the $\frac{d p}{d i}$ will be similar to $\frac{d i L}{d t}$, that is, if $\frac{d i L A}{d t}>0$ so $\frac{d p}{d i}>0$.

In conclusion, the current control is performed as follows:

- If $\frac{d p}{d i}>0$, the value of the derivative must decrease, so it will be necessary to increase the value of I. Then, taking in account the zone where the input voltage is (Fig. 5) the maximum value of the voltage should be applied;
- If $\frac{d p}{d i}<0$ the value of the derivative must increase, so it will be necessary to decrease the value of $I$. Then, taking in account the zone where the input voltage is (Fig. 5) the minimum value of the voltage should be applied;
- If $\frac{d p}{d i} \approx 0$, an intermediate value of the voltage should be applied

To simplify this control 3-level hysteresis comparators are used to generate control signals (SP, one for each PV) for the MPPT controllers.

- $S e \frac{d p}{d i}>0 \rightarrow S P=2$
- $S e \frac{d p}{d i}=0 \rightarrow S P=1$
- $S e \frac{d p}{d i}<0 \rightarrow S P=0$

Following this logic, the voltage vector which is more
suitable to control the currents in the coils is chosen. For example, if the voltage zone is zone $4(\mathrm{SV}=4)$ and $\mathrm{SP}=$ $(1,1,0)$, a voltage vector has to be chosen to guarantee an intermediate voltage value ( $V_{B C} /-V_{B C}$ ) in the first and second panels, and a minimum value in the third panel, that is, $-V_{C A}$ or $V_{A B}$. Analyzing Tab 1 we conclude that two possible vectors are 48 and 49.

However, the aim of this work is to control the current in each PV, so applying Kirchhoff's current law to the converter of Fig. 3 results in (11):

$$
\left\{\begin{array}{l}
i_{P v 1}=-i_{A}=i_{L A}  \tag{11}\\
i_{P v 2}=-i_{A}-i_{B}=i_{L A}+i_{L B} \\
i_{P v 3}=-i_{A}-i_{B}-i_{C}=i_{L A}+i_{L B}+i_{L C}
\end{array}\right.
$$

Using (11) in (6), results in (12) that allows the control of currents in each PV.

$$
\left\{\begin{array}{c}
\frac{d i_{p V 1}}{d t}=\frac{1}{L} V_{A N}+\frac{1}{4 L}\left(-3 R i_{L A}+R i_{L N}+R i_{L B}+R i_{L C}\right)  \tag{12}\\
\frac{d i_{p V 2}}{d t}=\frac{1}{L}\left(V_{B N}+V_{A N}\right)+\frac{1}{4 L}\left(-2 R i_{L A}+2 R i_{L N}+2 R i_{L C}-2 R i_{L B}\right) \\
\frac{d i_{p V 3}}{d t}=\frac{1}{L}\left(V_{B N}+V_{A N}+V_{C N}\right)+\frac{1}{4 L}\left(3 R i_{L N}-R i_{L A}-R i_{L B}-R i_{L C}\right)
\end{array}\right.
$$

As the PV currents depend linearly on the coil currents, controlling the coil currents will result in the control of the PV currents.

## B. AC current control

To ensure correct operation of the converter, in addition to ensuring the control of the output currents, it must be ensured that the input currents are sinusoidal and are in phase with the mains voltages.

For this condition to be guaranteed, it is necessary to
control the input power factor in the connection to the grid, so that it is approximately unitary, ie, $\cos (\varphi)=1$.

It is known that the active power and reactive power injected in the grid are given by (13):

$$
\begin{align*}
& P_{3 f}=3 \cdot V_{\text {aef }} \cdot I_{\text {aef }} \cdot \cos (\varphi)  \tag{13}\\
& Q_{3 f}=3 \cdot V_{\text {aef }} \cdot I_{\text {aef }} \cdot \operatorname{sen}(\varphi)
\end{align*}
$$

As $\cos (\varphi)=1$ and $\operatorname{sen}(\varphi)=0$, it results (14):

$$
\begin{gather*}
P_{3 f}=3 . V_{a e f} \cdot I_{a e f}  \tag{14}\\
Q_{3 f}=0
\end{gather*}
$$

To guarantee that the input currents are sinusoidal and in phase with the voltages, the reactive power will have to be zero.

To perform this control, the Blondel-Park transformation (15) will be used: [5]

$$
\left[P_{B}\right]=\left[\begin{array}{ccc}
\cos \theta & \operatorname{sen} \theta & 0  \tag{15}\\
-\operatorname{sen} \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{array}\right]
$$

This matrix allows the transformation from $\alpha \beta$ [10] components to dq components, as shown in (16) [5]:

$$
\left[\begin{array}{l}
v_{d}  \tag{16}\\
v_{q} \\
v_{0}
\end{array}\right]=\left[P_{B}\right]\left[\begin{array}{l}
v_{\alpha} \\
v_{\beta} \\
v_{0}
\end{array}\right]
$$

Since $\theta$ is the transformation angle, in the new coordinate system, the voltages will be given by (17)

$$
\left\{\begin{array}{l}
v_{d}=V \cos (\omega t-\theta)  \tag{17}\\
v_{q}=V \operatorname{sen}(\omega t-\theta) \\
v_{0}=0
\end{array}\right.
$$

Considering as reference the grid voltage $v_{a}$ and synchronizing the d-axis with $v_{a}$, the grid voltages are obtained in the new coordinate system (18) [11].

$$
\left\{\begin{array}{c}
v_{d}=\sqrt{3} V  \tag{18}\\
v_{q}=0
\end{array}\right.
$$

In dq coordinates, the active and reactive power are given by (19)

$$
\begin{align*}
& P_{d q}=v_{d} i_{d}+v_{q} i_{q}  \tag{19}\\
& Q_{d q}=i_{d} i_{q}-v_{q} i_{d}
\end{align*}
$$

Using (18) in (19), it is possible to conclude that the reactive power depends only on the $q$ component of the input current (20)

$$
\begin{equation*}
Q_{d q}=v_{d} i_{q} \tag{20}
\end{equation*}
$$

If $Q_{d q}=0$, then the power factor is unitary, and $i_{q}$ must be zero.

In order to ensure that the current $i_{q}$ is close to zero, a sliding mode controller is used. The error of $i_{q}$ is defined, taking in account the reference value $i_{\text {ref }}$ (21). In this case,
$i_{\text {qref }}=0$ to guarantee nearly unitary power factor. [12]

$$
\begin{equation*}
e_{I q}=i_{q}-i_{q}^{r e f} \tag{21}
\end{equation*}
$$

As the current values are very low, in order to amplify the error, it is multiplied by a gain $k\left(e_{I q}=k\left(i_{q}-i_{q}{ }^{\text {ref }}\right)\right)$.

After, a hysteresis comparator is used to obtain a new control signal, wherein:

- If $e_{i q}<0, \mathrm{SQ}=0$, then $i_{q}<i_{q r e f}$ and the value of $i_{q}$ must increase;
- If $e_{i q}>0, \mathrm{SQ}=1$, then $i_{q}>i_{q r e f}$ and the value of $i_{q}$ must decrease;
This control is performed by choosing one of the 2 vectors previously selected when controlling the output currents of the converter, ie between the 2 chosen vectors the current $i_{q}$ is calculated and then the vector is chosen depending on the value of $i_{q}$.


## IV. RESULTS

The converter of Fig. 3 and the current control methods described in Section III were implemented through Simulink in Matlab. To check the correct operation of the system, it was simulated in different scenarios.

## A. Scenario 1: Irradiation variation

In this scenario a radiance step input is applied to each of the panels in order to simulate a sudden variation of the radiance value in the PV .

In this specific case the following variations were chosen:

- PV1: $G_{P V I}=400 \mathrm{~W} / \mathrm{m}^{2}$ to $G_{P V I}=450 \mathrm{~W} / \mathrm{m}^{2}$;
- PV2: $G_{P V 2}=700 \mathrm{~W} / \mathrm{m}^{2}$ to $G_{P V 2}=600 \mathrm{~W} / \mathrm{m}^{2}$;
- PV3: $G_{P V 3}=800 \mathrm{~W} / \mathrm{m}^{2}$ to $G_{P V 3}=900 \mathrm{~W} / \mathrm{m}^{2}$;

The results obtained, Fig. 7 are according to expected ones, because at the moment in which the variation of G occurs, it is possible to verify that the current $I_{p v}$ is controlled instantaneously, to a new value.


Fig. 7. variation of $i_{p v}$, in step

## B. Scenario 2: Partial fault of the converter

In this scenario it is intended to simulate the partial fault of the converter. To simulate this, in the fourth arm of the converter a function is defined that allows to select if the chosen vector is applied to the semiconductor (if the auxiliary function is 1 ), or if the semiconductors in that arm
are turned OFF (if the auxiliary function is 0 ), thus simulating the partial fault of the converter.

As can be seen in Fig. 8, the system continues to work correctly despite the failure, because the PV panel connected to the arm 4 ceases to produce current, but the remaining panels continue to operate at maximum power.


Fig. 8 Variation of the currents $i_{p v 1}, i_{p v 2}$ and $i_{p v 3}$ in the PVs in the event of a fault in the arm 4 of the converter

Another result to take into account is the variation of the current at the output of the converter, ie the currents that go through coils. In this case, it is intended that the current in the arm in which the failure occur (arm 4) is zero and that the sum of the current in the remaining arms is zero. From the analysis of Fig. 9 the results are consistent with those expected.


Fig. 9 Variation of the currents at the output of the inverter, in the event of a partial fault

In the case of currents injected in the grid (Fig. 10), although after the fault there is more distortion, the sinusoidal shape is maintained and the currents remain in phase with the voltage.


Fig. 10 Variation of $v_{a b c} / 200$ and $i_{a b c}$, for partial fault of the converter

## V. Conclusion

This project developed a control system that guarantees MPPT mechanism in each PV and unitary FP in the connection to the grid. In order to decrease the harmonic content present in the currents, input and output filters were sized.

The system was tested for various operating scenarios. The results obtained were as expected, ie, the current injected into the grid is nearly sinusoidal and out of phase with the voltage. In the DC side, the PV system always operates at its maximum power, and even when the value of the irradiance changes the current value in the PVs is immediately adjusted to the requested value.

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