Maximum Power Point Tracking in Photovoltaic Systems
with Partial PV Shadowing

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Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.
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Resumo

Com a assinatura do Acordo de Paris de 2016 a população mundial admitiu que as alterações climáticas se tornaram um problema global e que medidas devem ser tomadas de maneira a limitar o aumento da temperatura média mundial a menos de 2 °C acima de níveis pré-industriais. Sendo o recurso renovável mais promissor, a energia solar tem sido alvo de muita atenção e, ao mesmo tempo, os seus custos de produção e instalação têem descido todos os anos. Uma maneira de reduzir ainda mais os custos é agregando múltiplos painéis fotovoltaicos em série e usando apenas um controlador para encontrar o ponto de maior potência do sistema. Contudo, um dos maiores problemas que esta tecnologia enfrenta são as condições de sombreamento parcial, quando diferentes painéis estão sujeitos a níveis diferentes de irradiação (devido a uma nuvem ou a sujidades cobrindo parcialmente alguns dos painéis).

Esta tese visa resolver o problema causado pelas condições de sombreamento parcial criando um controlador MPPT (aplicado no conversor boost do sistema) que seja capaz de encontrar o ponto de potência máxima global da característica não-côncava potência-tensão do sistema baseado no algoritmo de condutância incremental. O controlador é criado usando um algoritmo de backstepping e testado num sistema com três painéis em série. É também aplicado a três painéis individuais com os seus conversores ligados tanto em série como em paralelo, de maneira a testar a flexibilidade e robustez do controlador. Em todas as situações o controlador exibe uma eficiência acima de 99%.

Esta tese usa também o algoritmo de backstepping para desenhar o controlador do inversor que conecta o sistema à rede elétrica e testa a sua resposta contra o geralmente usado controlador PI. O controlador baseado em backstepping demonstra ser mais rápido e mais agressivo que o controlador PI, sendo então preferível em termos de rapidez em sistemas pouco sensíveis a variações extremas. Por outro lado, recomenda-se a utilização do controlador PI em sistemas mais sensíveis a rápidas variações de potência.

Palavras-chave: Energia solar, Condições de sombreamento parcial, MPPT, Algoritmo backstepping, Condutância incremental
Abstract

With the signing of the Paris Agreement of 2016 the world population admitted that climate change has become a global issue and that measures need to be taken in order to limit the increase of global average temperature to below 2 °C above pre-industrial levels. Being the most promising renewable resource, solar power has entered the spotlight with production and installation costs decreasing every year. One way to further reduce costs is to aggregate multiple PV panels in a string and having just one controller finding the maximum power point of the system. However, one of the biggest challenges this technology faces is partial shading conditions, when different panels have different irradiation levels (because of a passing cloud or dirt covering some of the panels).

This thesis aims to address the problem of partial shading conditions by designing a MPPT controller (applied to the boost converter of the system) able to find the global maximum power point of the non-concave power-voltage characteristic based on the incremental conductance algorithm. The controller is designed using a backstepping algorithm and tested in a system with three PV panels in a string. It is also applied to three individual panels with their different converters connected both in series and in parallel, in order to test the controller's flexibility and robustness. In all tested situations the controller was able to exhibit an efficiency above 99%.

This thesis also uses the backstepping algorithm to design the controller of the grid-connected inverter and tests its response against the commonly used PI controller. The backstepping controller proves to be faster and more aggressive than the PI controller, thus being preferable in terms of speed in systems not very sensitive to extreme variations. On the other hand, it is recommended the use of the smoother PI controller in systems more sensitive to quick power variations.

Keywords: Solar power, Partial shading conditions, MPPT, Backstepping algorithm, Incremental conductance
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List of Acronyms

AC  Alternate Current

CCM  Continuous Conduction Mode

CF  Command Filter

DC  Direct Current

MPP  Maximum Power Point

MPPT  Maximum Power Point Tracking

NOCT  Nominal Operating Cell Temperature

pu  per-unit system

PV  Photovoltaic

PWM  Pulse-Width Modulation

STC  Standard Test Conditions

THD  Total Harmonic Distortion
List of Symbols

**Greek symbols**

- $\eta_c$ MPPT efficiency
- $\eta_i$ Inverter efficiency
- $\eta_t$ MPPT efficiency when comparing to maximum possible generation
- $\gamma$ Boost converter control signal
- $\gamma_i$ Inverter control signal
- $\mu_{Isc}$ Short circuit current temperature coefficient
- $\mu_{Voc}$ Open circuit voltage temperature coefficient
- $\omega_f$ Command filter natural frequency
- $\omega_g$ Grid frequency
- $\omega_n$ Inverter controller natural frequency
- $\phi_g$ Phase difference between the inverter output current and grid voltage
- $\phi_M$ Inverter controller phase margin
- $\theta_a$ Atmospheric temperature
- $\theta_c$ Cell temperature
- $\xi$ Inverter controller damping coefficient
- $\xi_f$ Command filter damping coefficient

**Roman symbols**

- $C_1$ Boost converter input capacitance
- $C_2$ Boost converter output capacitance
- $C_D$ Solar cell dynamic model capacitance
- $C_{DC}$ Inverter input capacitance
$e_{IL}$ Boost converter inductor current error

$e_{inv}$ Inverter inductor current error

$e_{IU}$ Integral of the inverter input voltage error

$e_I$ Integral of the boost converter input voltage error

$e_{VC}$ Boost converter input voltage error

$e_{VDC}$ Inverter input voltage error

$e_{VPV}$ Boost converter voltage reference error

$f_{PWM}$ Inverter switching frequency

$G$ Irradiance level on a PV panel

$GI$ Average value of inverter control signal

$I_L$ Boost converter inductor current

$I_o$ Boost converter output current

$I_C$ Inverter capacitor current

$I_{DC}$ Inverter input current

$I_I$ Inverter input current

$I_{LI}$ Inverter inductor current

$I_{MPP}$ Maximum power point current

$I_O$ Diode saturation current

$I_{pan}$ PV panel current output

$I_{PV}$ PV system current output

$I_{sc}$ Short circuit current of a PV panel

$I_s$ Current generated by a PV cell

$K$ Boltzmann constant

$KI$ Integral gain of the inverter controller

$KP$ Proportional gain of the inverter controller

$k_v$ Boost converter voltage reference controller gain

$k_{IU}$ Inverter integration gain

$k_I$ Boost converter integration gain
\( k_U \)  Inverter input voltage controller gain
\( k_{vc} \)  Boost converter input voltage controller gain
\( L \)  Boost converter inductor self-inductance coefficient
\( L_I \)  Inverter inductor self-inductance coefficient
\( P_{g} \)  Active power delivered to the grid
\( P_I \)  Inverter input active power
\( P_{MPP} \)  Maximum power point power
\( P_{pan} \)  PV panel power output
\( P_{PV} \)  PV system power output
\( R_s \)  Series resistance
\( R_{sh} \)  Shunt resistance
\( S \)  Semiconductor switch (MOSFET)
\( S_g \)  Apparent power delivered to the grid
\( T \)  Boost converter switching period
\( T_c \)  Absolute cell temperature
\( T_d \)  Average delay of inverter controller
\( T_i \)  Inverter switching period
\( T_p \)  Time constant of the pole of the inverter controller
\( T_z \)  Time constant of the zero of the inverter controller
\( V_C \)  Boost converter input capacitance voltage
\( V_o \)  Boost converter output voltage
\( V_g \)  Grid voltage
\( V_I \)  Inverter output voltage
\( V_{LI} \)  Inverter inductor voltage
\( V_{Ly} \)  Lyapunov candidate function
\( V_L \)  Boost converter inductor voltage
\( V_{MPP} \)  Maximum power point voltage
\( V_{oc} \)  Open circuit voltage of a PV panel
$V_{pan}$ PV panel voltage output

$V_{PV}$ PV system voltage output

**Subscripts**

av Average value

ref Reference condition

RMS Root mean square value

**Superscripts**

r STC conditions
Chapter 1

Introduction

On December 12th 2015, in Le Bourget, France, 196 state parties adopted by consensus the Paris Agreement. It aims to strengthen the global response to the threat of climate change by keeping the increase in global average temperature to well below 2°C above pre-industrial levels (limiting the increase to 1.5°C would substantially reduce the risks and effects of climate change), increasing the ability to adapt to the adverse impacts of climate change and stimulating climate resilience and low greenhouse gas emissions development. [1]

As of July 2018, 195 UNFCCC (United Nations Framework Convention on Climate Change) members have signed the agreement, and 179 have become party to it [2]. The EU (European Union) was the first major economy to submit its intended contribution to the new agreement in March 2015 and sets three key minimum targets for the year 2030 [3]:

- cut greenhouse gas emissions in EU territory by 40% (from 1990 levels), with a long-term objective of cutting emissions by 80-95% by 2050;

- boost the share of renewables to 27% of EU energy consumption;

- 27% improvement in energy efficiency. This target will be reviewed in 2020 having in mind a 30% target.

The share of gross inland energy consumption in the European Union coming from renewable energy sources in 2016 was 13.2% [4] and Figure 1.1 shows that solar energy and wind power are the fastest growing technologies in Europe. This means that, in order to achieved the Paris Agreement commitment, the EU will likely be investing more in those promising fields.

Furthermore, solar energy is the fastest growing technology worldwide, as shown in Figure 1.2. This is driven by continuous technology cost reductions and unprecedented market dynamics in China [5]. Also, it is currently the only renewable energy technology capable of being installed in common households, making them very appealing to the public.
Solar energy is the radiant light and heat from the Sun that is harnessed using a number of different technologies, such as Photovoltaic (PV) panels and solar thermal collectors. PV panels take advantage of the photoelectric effect to produce electrical current when exposed to radiation and usually have a global efficiency of around 16-18%, with more expensive products presenting efficiencies of 21-22% [6].

PV, or solar, modules are simple strings of PV cells, the basic blocks for photovoltaic power generation. The most common PV cells are made from silicon crystals but pure silicon is an insulator because there are no free electrons/holes to carry charge. It is only after introducing impurities to the silicon crystal (doping) that it forms a p-n junction and turns into a semiconductor, positive ("p" side, free holes) on one side and negative ("n" side, free electrons) on the other. If a given electron in the negative side of the cell is struck with a photon with the right frequency of radiation it will be “freed” and flow away from...
the p-n junction before being recombined. The cell will generate a current flowing in one direction - from the n side to the p side - through the load attached to the system, thus delivering electric power. Figure 1.3 depicts these interactions.

A diode is a simple p-n junction like the one being discussed. If kept enclosed, the silicon crystal will have a current ($I_{\text{dark}}$) dependent on the voltage applied to its terminals, as per Shockley’s diode equation [7]. But it will act as previously described if exposed to radiation with photoelectric capabilities, generating a current with the opposite direction ($I_{\text{pe}}$), proportional to incoming irradiance. Figure 1.4a shows the model of this ideal cell and reveals that the output current of the cell will be $I_{\text{cell}} = I_{\text{pe}} + I_{\text{dark}}$, resulting in the current-voltage (IV) curve depicted in bold in Figure 1.4b, where the difference between $I_{\text{dark}}$ and $I_{\text{cell}}$ is the photoelectric generated current $I_{\text{pe}}$ (note that $I_{\text{dark}}$ is negative because it is being measured in the opposite direction of a diode current and the y-axis is inverted to show the usual depiction of a diode current).

Single PV cells have low power outputs (maximum achieved in [8] was 12 W), so it makes sense to aggregate groups of them inside PV modules. They are set-up in a string so that all of them are subject to approximately the same current (Figure 1.5 shows the most common layout of a PV module).

PV panels are usually more than 1 module connected in series and/or in parallel in order to increase the power output of the final product. For higher power demands, PV panels are arranged in strings,
with strings being connected in parallel to form arrays.

Given that a PV panel is made up of photovoltaic cells, its current-voltage and power-voltage (PV) curves will be similar to a photovoltaic cell’s. From Figure 1.4b, one can then construct the PV curve of an ideal solar panel, portrayed in Figure 1.6. This Figure shows that the Maximum Power Point (MPP), i.e., the operating point that maximizes the power generated by the system, can be found by deriving the PV curve and finding $V_{pan}$ such that $\frac{dP_{pan}}{dV_{pan}} = 0$. This voltage will be the MPP Voltage ($V_{MPP}$) and the corresponding output current will be the MPP Current ($I_{MPP}$).

### 1.2 Partial Shading Condition

A control algorithm that can track and maintain the system at the MPP is called a Maximum Power Point Tracking (MPPT) controller. Every MPPT algorithm in the literature assumes a strictly concave PV curve, i.e., there is only one local maximum point, the MPP. This is true for a single solar panel or even an array of solar panels in the same environment conditions of irradiance and temperature. However, different panels in an solar array sometimes have different conditions and a single controller for the entire array (instead of one for each PV panel) would help decrease the cost of the system.

It is improbable that two similar solar panels have different cell temperatures while in the same
environment but partial shading (shading that affects all solar panel differently, such as a passing cloud) poses a problem for MPPTs that control more than one panel because the PV curve stops being strictly concave, which means there are more than one local maxima.

Figure 1.7 shows the IV and PV curves of three panels with different irradiance levels ($G_2 = \frac{2}{3}G_1$ and $G_3 = \frac{1}{3}G_1$) when connected in series (summing the individual voltages of the three panels). Since they have different irradiances, the three panels would output different currents, if isolated. Since they are in series they create the staircase IV curve depicted. The resulting multiple local maxima make it difficult for a single controller to track the Global Maximum Power Point (GMPP) since it can become trapped in a lower local maximum.

These situations are common since they only require something that shades the PV panels unevenly but the resulting PV curves are unpredictable, making them hard to deal with using only simple maximizing algorithms.

1.3 Motivation

Since the discovery of the existence of the partial shading condition (PSC), research on the topic has rapidly spread in the literature. Currently, there are several types of methods with different priorities such as cost, accuracy, efficiency, implementation complexity, etc.. They can be divided in 6 categories [10]:

- Meta-heuristic methods (the most commonly used is the particle swarm optimization (PSO)), where the PV curve is sampled with "particles" in low and high voltages in search of the GMPP;
- Fuzzy logic based methods, where the PV curve is scanned for the location of the GMPP and then fuzzy logic is implemented to find the real GMPP;
- Numerical and mathematical application methods. These are not commonly used because they either are not able to find the GMPP for all conditions or the proposed algorithm is complex and requires an advanced microcontroller;
• Modified conventional methods, where the PV curve of the array is sampled at specific voltages and then the conventional methods are called as subroutines to search the local maxima around those specific points. When all local maxima are known the system returns to the biggest one, the GMPP;

• Hardware solutions. These rely only on changing how the PV panels are set up and their connections to the MPPT;

• Other methods.

The main obstacle concerning most of the more successful methods is that they need to sample or scan the whole PV curve. This means that while sampling/scanning the PV curve the array will output less power, at some points a lot less than the GMPP, which can cause considerable power loss in large-scale PV systems.

As mentioned in section 1.2, the PSC effects can be generally circumvented by installing one MPPT for every panel, cutting down the difficulties of finding the GMPP. This, however, raises the cost for the whole system. Hence, a compromise must be reached: in order to avoid drops in efficiency, more MPPTs are required which makes the system more expensive.

1.4 Objective

The main purpose of this thesis is to implement an incremental conductance algorithm that takes advantage of the fact that all local maxima of the non-concave power-voltage curve of a partially shaded PV system are approximately spaced by a fixed voltage, and thus a fixed number of points can be used to reliably track the GMPP. The obtained controller, called three-state single MPPT search algorithm, is designed using a backstepping algorithm [11] and applied to the boost converter connected to a string of three PV panels. The MPPT search algorithm is also used in a system with three individual PV panels with their different converters connected both in series and in parallel to test its flexibility.

This project also designs the inverter controller using the backstepping algorithm and compares it to the more commonly employed PI controller in order to see which one deals best with the fast changing conditions of the proposed three-state single MPPT search algorithm.

To simulate the model and the design of the proposed controllers the numerical computing environment MATLAB® will be used.

1.5 Thesis Outline

This dissertation is divided in five different parts:

• Chapter 2 begins by describing the mathematical model of a PV panel used in this project, which is then implemented using the parameters found in Appendix A. Then, the most common MPPT algorithms found in the literature are described, outlining the pros and cons of each one;
• Chapter 3 introduces the backstepping based controller of the boost converter as well as how to calculate the converter's parameters;

• Chapter 4 presents the three different tested architectures of this thesis: the three-state single MPPT search algorithm and the algorithm that allows for the search of the GMPP; the series connected converters, and the alteration that they have to undergo in order to avoid being disconnected from the system; and the parallel connected converters;

• Chapter 5 presents the grid-connected inverter and how to simplify its design in order to simplify its simulation, as well as how to calculate its parameters. Then the inverter controller is designed using the backstepping algorithm together with a linear PI controller;

• Finally, Chapter 6 shows the designed controllers evaluated under different conditions to demonstrate their correct operation and compare them against one another. In Chapter 7 some final conclusions are made about the performed work and obtained results and some further developments not contemplated in this dissertation are suggested.
Chapter 2

Solar Panel Module

2.1 Mathematical Model

In order to study different strategies for MPPT there is a need to simulate a real photovoltaic module. This simulation model needs to take into consideration not only the load but also the environment conditions in which the module is, as real solar panels are affected by both irradiance and temperature (as shown in section 2.3).

There are three models typically used to represent PV cells [12]:

- Ideal single diode model (three unknown parameters)
- Practical single diode model (five unknown parameters)
- Two diode model (seven unknown parameters)

These three models use transcendent equations, making them solvable only through iterations. Since they have an increasing number of unknown variables, they will be increasingly complex to solve.

The ideal single diode model is the simplest of the models but it does not provide accurate IV and PV curve characteristics of real solar cells. Concurrently, the two diode model presents simulation results closer to real photovoltaic cells when compared with the practical single diode model but it requires more complex equations and two more unknown parameters, taking a bigger number of iterations to solve [12].

For the purposes of this thesis, the practical single diode model was considered more appropriate for it combines accuracy with some level of simplicity, making it the best candidate for simulations.

In this model (shown in Figure 2.1), the current source $I_S$ represents the electrical current generated by the PV cell, when exposed to an irradiance $G$ and at temperature $\theta_c$, while the diode represents the p-n junction of the cell. The resistance $R_s$ represents the voltage drop and internal conduction losses of the system while the resistance $R_{sh}$ represents the leakage currents in the cell.
Remembering Shockley’s diode equation,

\[ I_D = I_O \left[ \exp \left( \frac{V_D}{mV_T} \right) - 1 \right] = I_O \left[ \exp \left( \frac{V + R_s I}{mV_T} \right) - 1 \right], \]

(2.1)

using Ohm’s law in resistance \( R_{sh} \)

\[ I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{V + R_s I}{R_{sh}}, \]

(2.2)

and applying Kirchhoff’s current law to the model gets

\[ I = I_s - I_D - I_{sh} = I_s - I_O \left[ \exp \left( \frac{V + R_s I}{mV_T} \right) - 1 \right] - \frac{V + R_s I}{R_{sh}}, \]

(2.3)

where

- \( I_O \) - diode saturation current [A];
- \( m \) - ideality factor of the diode;
- \( V_T \) - thermal voltage equivalent [V];

\[ V_T = \frac{KT_c}{q} \]

- \( K \): Boltzmann constant \((K = 1.38 \times 10^{-23} [\text{J/K}])\);
- \( T_c \): absolute cell temperature [K];
- \( q \): electron charge \((q = 1.6 \times 10^{-19} [\text{C}])\);

- \( V \) - PV cell output voltage [V];
- \( I \) - PV cell output current [A].

By analysing (2.3), the model’s five unknown parameters are \( I_s, I_O, m, R_s \) and \( R_{sh} \), which demands five independent equations. Three of those equations can come from three different operating points, with values given by the manufacturers at Standard Test Conditions (STC)\(^1\): short-circuit \((V = 0, I = I_{sc})\), open-circuit \((V = V_{oc}, I = 0)\) and MPP \((V = V_{MPP}, I = I_{MPP})\).

\(^1\)Standard Test Conditions: \( G^r = 1000 \text{ W/m}^2; \theta_c^r = 25 \text{ °C}.\) Referenced by the superscript \(^r\).
The short-circuit current is given by

$$I_{sc}^r = I_s^r - I_O^r \left[ \exp \left( \frac{R_s I_{sc}^r}{mV_T} \right) - 1 \right] - \frac{R_s}{R_{sh}} I_{sc}^r,$$  \hspace{1cm} (2.5)

the open-circuit operating point is characterized with

$$I_s^r - I_O^r \left[ \exp \left( \frac{V_{oc}^r}{mV_T} \right) - 1 \right] - \frac{V_{oc}^r + R_s}{R_{sh}} = 0,$$  \hspace{1cm} (2.6)

and finally, the MPP current is given by

$$I_{MPP}^r = I_s^r - I_O^r \left[ \exp \left( \frac{V_{MPP}^r + R_s I_{MPP}^r}{mV_T} \right) - 1 \right] - \frac{V_{MPP}^r + R_s I_{MPP}^r}{R_{sh}}.$$  \hspace{1cm} (2.7)

Assuming, for simplification,

$$\begin{cases} \exp \left( \frac{V + R_s I}{mV_T} \right) \gg 1, \\ \exp \left( \frac{V_{oc}}{mV_T} \right) \gg \exp \left( \frac{R_s I_{sc}^r}{mV_T} \right), \end{cases}$$  \hspace{1cm} (2.8)

(2.5), (2.6) and (2.7) can be rewritten. Equation (2.6) becomes

$$I_s^r = I_O^r \exp \left( \frac{V_{oc}^r}{mV_T} \right) + \frac{V_{oc}^r + R_s}{R_{sh}},$$  \hspace{1cm} (2.9)

applying (2.9) to (2.5) yields

$$I_{sc}^r = I_O^r \exp \left( \frac{V_{oc}^r}{mV_T} \right) + \frac{V_{oc}^r + R_s}{R_{sh}} - \frac{R_s}{R_{sh}} I_{sc}^r$$  \hspace{1cm} (2.10)

$$\Leftrightarrow I_O^r = \left( I_{sc}^r - \frac{V_{oc}^r}{R_{sh}} - \frac{R_s}{R_{sh}} I_{sc}^r \right) \exp \left( -\frac{V_{oc}^r}{mV_T} \right),$$

and applying (2.9) and (2.10) to (2.7) gives

$$I_{MPP}^r = I_{sc}^r + \frac{R_s I_{sc}^r - V_{MPP}^r - R_s I_{MPP}^r}{R_{sh}} + \left( \frac{V_{oc}^r - R_s I_{sc}^r}{R_{sh}} - I_{sc}^r \right) \exp \left( \frac{V_{MPP}^r + R_s I_{MPP}^r - V_{oc}^r}{mV_T} \right).$$  \hspace{1cm} (2.11)

After these three equations, two additional equations are still required in order to solve the five parameter system. One of them can be obtained from the knowledge that the derivative of the power at MPP is 0,

$$\frac{dP}{dV} \bigg|_{V = V_{MPP}^r, I = I_{MPP}^r} = \frac{d(VI)}{dV} \bigg|_{V = V_{MPP}^r, I = I_{MPP}^r} = I_{MPP}^r + V_{MPP}^r \frac{dI}{dV} \bigg|_{V = V_{MPP}^r, I = I_{MPP}^r} = 0,$$  \hspace{1cm} (2.12)

and the last equation is achieved from the empirical notion that the shunt resistance obeys the following relation [13],

$$\frac{dI}{dV} \bigg|_{I = I_{sc}^r} = -\frac{1}{R_{sh}}.$$  \hspace{1cm} (2.13)
To solve (2.12) and (2.13) it is necessary to derive the output current of the cell (equation (2.3)) with respect to the output voltage of the cell. However, (2.3) is transcendent and thus a non-linear approach is required [14].

Since

\[ I = f(I, V), \]

\[
\frac{dI}{dV} = \frac{\partial f(I, V)}{\partial V} - \frac{\partial f(I, V)}{\partial I}.
\]

(2.14)

Applying (2.14) to (2.12) yields

\[
I_{MP} + V_{MP} - \left(\frac{R_s I_{sc} - V_{oc}}{1 + \frac{R_s}{R_{sh}}} \right) \exp \left(\frac{R_s I_{sc} - V_{oc}}{mV_T R_{sh}}\right) = 0,
\]

(2.15)

and applying (2.14) to (2.13) gets

\[
- \left(\frac{R_s I_{sc} - V_{oc}}{1 + \frac{R_s}{R_{sh}}} \right) \exp \left(\frac{R_s I_{sc} - V_{oc}}{mV_T R_{sh}}\right) = - \frac{1}{R_{sh}}.
\]

(2.16)

(2.11), (2.15) and (2.16) make a system of three equations that can be used to compute the three unknowns \( m, R_s \) and \( R_{sh} \) since all the other variables are given by PV panel catalogs. The remaining two parameters can be computed using (2.9) and (2.10).

2.1.1 Influence of Irradiance and Temperature

The practical single diode model assumes that the parameters \( m, R_s \) and \( R_{sh} \) are independent from the environment conditions in which the PV cell is in. Unfortunately this is not true and thus, as stated before, the model is imperfect. However, since the variation of these parameters with the atmospheric conditions is small compared to the variation of \( I_{sc} \) and \( V_{oc} \), assuming them to be constant simplifies the model without great loss of accuracy.

The short circuit current and the open circuit voltage of the PV cell have, respectively, a linear and a logarithmic dependency with the irradiance and both of them have a linear dependency with the temperature, hence the following relationships can be established:

\[
I_{sc}(G, \theta_c) = \frac{G}{G_T} [I_{sc}^r + \mu I_{sc}^r (\theta_c - \theta_c^r)]
\]

\[
V_{oc}(G, \theta_c) = V_{oc}^r + \mu V_{oc}^r (\theta_c - \theta_c^r) + mV_T \ln \left(\frac{G}{G_T}\right),
\]

(2.17)

where
• $\mu_{Isc}$ - short circuit current temperature coefficient [A/°C];

• $\mu_{ Voc}$ - open circuit voltage temperature coefficient [V/°C].

These relations can be proven both experimentally and theoretically [14].

### 2.1.2 PV Cell Temperature

Equations (2.4) and (2.17) use the PV cell temperature and yet the temperature usually available in a solar panel’s data sheet is atmospheric temperature ($\theta_a$) and the Nominal Operating Cell Temperature (NOCT)$^2$. In order to calculate the PV cell temperature (in °C) the following temperature model can be used,

$$\theta_c = \theta_a + \frac{(NOCT)G - 20}{800}.$$

### 2.2 Implementation of the Mathematical Model

Currently, the most widely used PV module technology is mono-crystalline silicon [15], with an average peak power$^3$ of 270 W$p$, making it a good candidate for study. The main electrical data for the chosen PV module can be found in Table 2.1 (datasheet available in Appendix A).

Table 2.1: PV module main electrical data.

<table>
<thead>
<tr>
<th>$P_{MPP}$ [W$p$]</th>
<th>$V_{MPP}$ [V]</th>
<th>$I_{MPP}$ [A]</th>
<th>$V_{oc}$ [V]</th>
<th>$I_{sc}$ [A]</th>
<th>NOCT [°C]</th>
<th>$\mu_{Voc}$ [V/°C]</th>
<th>$\mu_{Isc}$ [A/°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>270</td>
<td>31.1</td>
<td>8.67</td>
<td>38.2</td>
<td>9.19</td>
<td>45±2</td>
<td>-118.4</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Since the equations to compute $m$, $R_s$ and $R_{sh}$ are transcendent some initial predictions are required. These predictions$^4$, as well as the final values of the parameters, can be seen in Table 2.2.

Table 2.2: Initial predictions and final values for the model’s parameters at STC.

<table>
<thead>
<tr>
<th>$m_j$</th>
<th>$R_{sj}$ [Ω]</th>
<th>$R_{shj}$ [Ω]</th>
<th>$m$</th>
<th>$R_s$ [Ω]</th>
<th>$R_{sh}$ [Ω]</th>
<th>$I_D$ [nA]</th>
<th>$I_s$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.2</td>
<td>7000</td>
<td>66.9</td>
<td>0.247</td>
<td>7328</td>
<td>2.12</td>
<td>9.19</td>
</tr>
</tbody>
</table>

Finally, with the model complete, the IV and PV-curves can be drawn. By assigning a vector of values to the diode voltage, $V_D$,

$$V_D = V + R_s I,$$  

$^2$G = 800 W/m$^2$; $\theta_a$ = 20 °C.

$^3$Peak Power ($P_{p}$ [W$p$]): a PV module’s maximum power output at STC.

$^4$Referenced by the subscript $j$. 

13
in (2.3), which causes it to be explicit and the values of the output current can be easily calculated. The same applies to the output voltage: by manipulating (2.19) and using the value vectors of $V_D$ and $I$, $V$ is computed. With these vectors the characteristic curves of the PV panel can be drawn, as seen in Figure 2.2.

![Figure 2.2: IV and PV curves at STC.](image)

### 2.3 Maximum Power Point Tracking

All PV installations require MPPT controllers because the $V_{MPP}$ is dependent on environment conditions such as cell temperature and irradiance, as can be seen in Figure 2.3. Every time these conditions change (atmospheric temperature and solar irradiance vary throughout the course of a single day) the controller must react in order to keep the system at the MPP to maximize energy production.

MPPT controllers can be classified under two categories: conventional methods, like Perturb and Observe (P&O) and Incremental Conductance algorithms; and advanced methods, such as fuzzy logic and neural networks [16].

#### 2.3.1 P&O Algorithm

The P&O algorithm is widely employed in practice, due to its low-cost, simplicity and ease of implementation. It measures the solar panel power output, changes the applied voltage by a fixed step size in a given direction and measures it again. If the power output increased then it infers that the step it took is in the right direction and repeats the process. It will keep this direction until the measured power output is lower than the last measurement (it has passed the peak of the PV curve), which means it has gone too far and needs to search in the other direction. The flow chart algorithm is shown in Figure 2.4.

This algorithm will oscillate around the MPP leading to efficiency loss, a problem that can be attenuated by having a variable perturbation size that gets smaller towards the MPP. However, under rapidly changing atmospheric conditions this algorithm starts diverging from the MPP and since the step size was reduced the controller will be slow to find the new MPP. [18]
2.3.2 Incremental Conductance Algorithm

This method is based on the fact that the slope of the PV array power curve (Figure 1.6) is zero at the MPP, positive on the left of the MPP and negative on the right,

\[
\begin{align*}
\frac{dP_{\text{pan}}}{dV_{\text{pan}}} &= 0, \quad V_{\text{pan}} = V_{\text{MPP}} \\
\frac{dP_{\text{pan}}}{dV_{\text{pan}}} &> 0, \quad V_{\text{pan}} < V_{\text{MPP}} \\
\frac{dP_{\text{pan}}}{dV_{\text{pan}}} &< 0, \quad V_{\text{pan}} > V_{\text{MPP}}
\end{align*}
\]  

(2.20)

Since

\[
\frac{dP_{\text{pan}}}{dV_{\text{pan}}} = I_{\text{pan}} + V_{\text{pan}} \frac{dI_{\text{pan}}}{dV_{\text{pan}}} \approx I_{\text{pan}} + V_{\text{pan}} \frac{\Delta I_{\text{pan}}}{\Delta V_{\text{pan}}},
\]

(2.21)

(2.20) can be rewritten as
The MPP can thus be tracked by comparing the instantaneous conductance \( (I/V) \) to the incremental conductance \( (\Delta I/\Delta V) \), as shown in the flowchart in Figure 2.5.

\[
\begin{align*}
\Delta I_{\text{pan}}/\Delta V_{\text{pan}} &= -I_{\text{pan}}/V_{\text{pan}}, \quad V_{\text{pan}} = V_{\text{MPP}} \\
\Delta I_{\text{pan}}/\Delta V_{\text{pan}} &> -I_{\text{pan}}/V_{\text{pan}}, \quad V_{\text{pan}} < V_{\text{MPP}} \\
\Delta I_{\text{pan}}/\Delta V_{\text{pan}} &< -I_{\text{pan}}/V_{\text{pan}}, \quad V_{\text{pan}} > V_{\text{MPP}}
\end{align*}
\] 

\[ (2.22) \]

In this case, the step size is adaptive, it will be proportional to the derivative. Once the MPP is
reached, the operation of the PV array is maintained at this point unless a change in current is noted, indicating a change in atmospheric conditions and the MPP. The algorithm decrements or increments the applied voltage to track the new MPP. [19]

### 2.3.3 Fuzzy Logic

This method has received attention of a number of researchers in the area of power electronics. The fuzzy logic control is somewhat easy to implement because it does not need the mathematical model of a system. Since it gives robust performance, the interest in practical application of fuzzy logic is growing rapidly. [20]

It generally consists of three stages: fuzzification, rule base table lookup and defuzzification. During fuzzification, numerical input variables are converted into linguistic variables such as: NB (negative big), NS (negative small), ZE (zero), PS (positive small), and PB (positive big). These variables are based on a membership function, a technique that relies more on experience rather than knowledge, making the rules for their definition "fuzzy" too. The inputs to a MPPT fuzzy logic controller are usually an error $E$ and a change in error $\Delta E$ with the user having the flexibility of choosing how to compute $E$ and $\Delta E$.

One common method [21] is

$$
E(n) = \frac{P(n) - P(n-1)}{V(n) - V(n-1)} \\
\Delta E(n) = E(n) - E(n-1)
$$

(2.23)

Once $E$ and $\Delta E$ are calculated and converted to the linguistic variables, the fuzzy logic controller output, which is typically a change in duty ratio $\Delta D$ of the power converter, can be looked up in a rule base table such as Table 2.3 [22]. The linguistic variables assigned to the output for the different combinations of $E$ and $\Delta E$ are based on the power converter being used and also on the knowledge of the user.

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\Delta E$</th>
<th>NB</th>
<th>NS</th>
<th>ZE</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>ZE</td>
<td>ZE</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
</tr>
<tr>
<td>NS</td>
<td>ZE</td>
<td>ZE</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>ZE</td>
<td>NS</td>
<td>ZE</td>
<td>ZE</td>
<td>ZE</td>
<td>PS</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>PS</td>
<td>PS</td>
<td>PS</td>
<td>ZE</td>
<td>ZE</td>
<td>ZE</td>
</tr>
<tr>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>ZE</td>
<td>ZE</td>
</tr>
</tbody>
</table>

Table 2.3: Example of a fuzzy rule base table.

In the defuzzification stage, the fuzzy logic controller output is converted from a linguistic variable to a numerical variable still using the same membership function mentioned previously [22]. This provides an analogue signal that will control the power converter to the MPP.

MPPT fuzzy logic controllers have been shown to perform well under varying atmospheric conditions
and have the advantages of working with imprecise inputs, not needing an accurate mathematical model and handling nonlinearity [23]. However, their effectiveness depends a lot on the knowledge of the user or control engineer in choosing the right error computation and coming up with the rule base table, which may change from one PV system to another as mentioned before.

### 2.3.4 Neural Network

Artificial neural networks (ANN) are computing systems inspired by biological neural networks of real animal brains [24]. These systems "learn" to perform tasks by considering examples fed by a training set, generally without being programmed with any task-specific rules. Neural networks commonly have three layers: input, hidden and output layers, as shown in Figure 2.6.

![Figure 2.6: Example of a neural network (based on Figure 2 in [25]).](image)

Each node (represented by a circle in Figure 2.6) has connections to other nodes, like neurons in a brain. The objective of an ANN is to adapt the weights given to each connection in order to minimize a loss function $L$ defined by the user. A very simple example would be [26]

$$L = \sum_{i=1}^{n} |y'_i - y_i|. \quad (2.24)$$

L1 loss function is the sum of absolute errors between the output of the system ($y'_i$, its prediction of reality) and the real value of that variable ($y_i$), where $i$ represents a single output and $n$ represents the number of outputs in the network. Each connection has their weight changed until it has minimized the loss function. The robustness of the model increases along with the decrease of the value of the loss function. More complex loss functions are generally used for improved results.

In PV systems, the inputs can be PV array parameters like the short-circuit current ($I_{sc}$) and open-circuit voltage ($V_{oc}$), atmospheric data like irradiance and temperature, or any combination of these. The output is usually one or several reference signals like the duty cycle used to drive the power converter. The efficiency of this method is highly dependent on the algorithms used by the hidden layer and how well the neural network has been trained. To accurately identify the MPP, the weights of each connection have to be carefully determined through a training process, meaning the PV array is tested over months
or years and the patterns between the input(s) and output(s) of the neural network are recorded. Since most PV arrays have different characteristics, a neural network has to be specifically trained for the PV array with which it will be used. The characteristics of a PV array also change with time, implying that the neural network has to be periodically trained to guarantee accurate MPPT. [27]
Chapter 3

Backstepping Control for MPPT of PV Panels

In control theory, backstepping is a recursive procedure that breaks down a design problem for the full system into a sequence of design problems of lower order subsystems [11]. The subsystems radiate out of a known stable system, with the designer creating controllers to stabilize each new outer subsystem ending with the last external controller [28]. This is a useful method because it allows the designer to treat a complex problem as simpler control problems, each encompassing the one before.

As stated before in section 1.1, the lowest order subsystem or, in other words, the main control objective of this controller, is to find and stabilize the voltage around the MPP, so voltage control is required.

Since the nominal single-phase voltage level in residential areas in Portugal is 230 V [29], in order to connect a PV system of three panels (each with an output voltage of 30-40 V) to the main grid the output voltage of the system must be increased. Series connection of panels and boost converters, as shown in Figure 3.1, can be used for that purpose. With the output voltage constant, the converter can be controlled to maintain an input reference voltage.

![Figure 3.1: Boost converter.](image)
3.1 Converter Parameters

The voltage increase in the boost converter is done by controlling the ON/OFF position of switch S (implemented by a transistor), which controls the current in the inductor ($I_L$), which, in turn, controls the voltage drop at capacitor $C_1$. This means that the MPPT controller can be implemented here: by controlling the ON/OFF position of switch S, the controller is able to control the operating point of the PV system.

In Figure 3.1, $I_{PV}$ and $V_{PV}$ represent, respectively, the output current and output voltage of the PV system that is connected to the converter. This means that, for an array of $N_p$ parallel strings of panels with $N_s$ panels in series per string, $V_{PV} = N_s V_{pan}$ and $I_{PV} = N_p I_{pan}$, assuming the same environment conditions of irradiance and temperature for all panels.

The inductor self-inductance coefficient ($L$) of the converter can be computed using [30]:

$$L = \frac{V_o}{4\Delta I_L}, \quad (3.1)$$

where

- $T$ - switching period [s];
- $\Delta$ - allowed ripple of the corresponding variable.

However, the input capacitance ($C_1$) is not so easy to find in the literature, so its calculation must be derived. By definition, capacitance is the ratio of the change in the electric charge of a system to the corresponding change in electric potential applied to the system, so

$$C_1 = \frac{\Delta Q_1}{\Delta V_C}. \quad (3.2)$$

This electric charge change ($\Delta Q_1$) is due to the nature of the converter. The switching of the transistor causes the inductor current to have a triangular waveform (as depicted in Figure 3.5, section 3.4) and since $I_{PV}$ is constant (assuming the PV system to be an ideal current source) the capacitor current will also have a triangular waveform.

At steady-state and in CCM, the average value of the capacitor current is zero ($I_{C1_{av}} = 0$) and $\Delta I_{C1} = \Delta I_L$, as shown in Figure 3.2. This Figure also shows that the maximum electric charge change can be calculated by integrating the capacitor current when it is positive, which at steady-state and in CCM is half the switching period.

Then, $C_1$ can be calculated by

$$C_1 = \frac{\Delta Q_1}{\Delta V_C} = \frac{\int_0^{T/2} i_{C1}(t)dt}{\Delta V_C} = \frac{1}{2} \frac{\Delta I_L}{\Delta V_C} = \frac{T \Delta I_L}{8 \Delta V_C}. \quad (3.3)$$

Because of the high discrepancy between one panel’s output voltage and the necessary input voltage of the inverter (discussed in more detail in section 5.1), the boost converter has to have a very high duty cycle ($\approx 1$), leading to a significant drop in the efficiency of the converter [31]. To counteract this,
converter should work at variable frequency which reduces the output voltage ripple and the inductor current ripple, ultimately improving efficiency [32].

Although both equations shown previously assume a fixed duty-cycle and fixed switching frequency, they remain good approximations to the desired converter parameters. The output capacitance \(C_2\) of the converter is the input capacitance \(C_{DC}\) of the inverter to which the converter is connected, and thus its calculation is shown in Chapter 5.

3.2 Lyapunov Calculation of the MPP Voltage

As stated before, the main control objective is to find and stabilize the voltage around the MPP. The Incremental Conductance algorithm discussed in section 2.3.2 states that

\[
\frac{dP_{PV}}{dV_{PV}} = 0. \tag{3.4}
\]

Since \(P_{PV} = V_{PV}I_{PV}\), (3.4) can be rewritten as

\[
\frac{dP_{PV}}{dV_{PV}} = \frac{dI_{PV} \Delta V_{PV}}{dV_{PV}} \approx -I_{PV} \frac{\Delta I_{PV}}{\Delta V_{PV}}. \tag{3.5}
\]

This linear approximation can be made by assuming that the sampling frequency of the current and voltage is much higher than the switching frequency of the power converter.

(3.6) can be rewritten as

\[
V_{MPP} = V_{PV} = -\frac{dI_{PV}}{dV_{PV}} \approx -I_{PV} \frac{\Delta V_{PV}}{\Delta I_{PV}} = -I_{PV}(k) \frac{V_{PV}(k) - V_{PV}(k - 1)}{I_{PV}(k) - I_{PV}(k - 1)}, \tag{3.7}
\]

where \(k\) represents a discrete sample. With this, a controller could be designed to enforce \(V_{PV}^{ref} = \ldots\)
However, it would be very aggressive, e.g., for a given $\Delta V_{PV}$ if $\Delta I_{PV}$ approaches zero then the computation of $V_{MPP}$ can return too high of a result, making the system hard to stabilize. This also means that the system is very sensible to noise, the slightest error in the measurements will not give the true MPP.

As a result, instead of enforcing $V_{PV_{ref}} = V_{MPP}$, the controller can work to minimize a voltage error function such as

$$e_{V_{PV}} = V_{MPP} - V_{PV_{ref}}.$$  \hfill (3.8)

Lyapunov’s second method for stability \cite{34} guarantees stability in a Lyapunov candidate function $V_{Ly}(x)$ such that

$$\begin{cases}
V_{Ly}(x) = 0, & x = 0 \\
V_{Ly}(x) > 0, & x > 0 \\
\frac{dV_{Ly}(x)}{dt} \leq 0, & \forall x \neq 0
\end{cases}$$  \hfill (3.9)

Creating a Lyapunov candidate function like

$$V_{Ly}(e_{V_{PV}}) = \frac{e_{V_{PV}}^2}{2},$$  \hfill (3.10)

and applying the method to it yields

$$e_{V_{PV}} \frac{de_{V_{PV}}}{dt} \leq 0, \quad \forall e_{V_{PV}} \neq 0$$

$$\Rightarrow \frac{de_{V_{PV}}}{dt} = -k_v e_{V_{PV}}, \quad k_v > 0.$$  \hfill (3.11)

By using (3.7) and (3.8) in the last equation of (3.11) and assuming $\frac{dV_{MPP}}{dt} = 0$ (at constant environment conditions $V_{MPP}$ is constant),

$$\frac{dV_{MPP}}{dt} - \frac{dV_{PV_{ref}}}{dt} = -k_v (V_{MPP} - V_{PV_{ref}})$$

$$\Rightarrow \frac{dV_{PV_{ref}}}{dt} = -k_v \left( I_{PV} \frac{dV_{PV}}{dI_{PV}} + V_{PV_{ref}} \right).$$  \hfill (3.12)

As stated before $P_{PV} = V_{PV} I_{PV}$, therefore

$$\frac{dP_{PV}}{dI_{PV}} = I_{PV} \frac{dV_{PV}}{dI_{PV}} + V_{PV}.$$  \hfill (3.13)

Assuming that the system is able to follow the reference closely means that $V_{PV} \approx V_{PV_{ref}}$. Then, (3.13) can be applied in (3.12), yielding
\[
\frac{dV_{PV\text{ref}}}{dt} = -k_v \frac{dP_{PV}}{dI_{PV}}
\]

\[
\Rightarrow V_{PV\text{ref}} = \int_0^t -k_v \frac{dP_{PV}}{dI_{PV}} \, dt + V_{PV\text{ref}}(0).
\]

This result makes intuitive sense when regarding Figure 3.3. If \( \frac{dP_{PV}}{dI_{PV}} > 0 \), then the system is on the right side of the curve and \( V_{PV\text{ref}} \) will decrease towards the MPP. Concurrently, if \( \frac{dP_{PV}}{dI_{PV}} < 0 \), then the system is on the left side of the curve and \( V_{PV\text{ref}} \) will increase. Also, the integral action dampens the controller response to new measurements, facilitating stabilization. Furthermore, the system can now be fine-tuned to be more or less aggressive by changing the gain value of \( k_v \).

\[\text{Figure 3.3: Characteristic curves of a PV panel.}\]

3.3 Backstepping Control of the Reference Current

With the voltage reference controller established, there needs to be a controller that keeps \( V_{PV} = V_{PV\text{ref}} \). From analysing the circuit in Figure 3.1 come the following equations:

\[ V_{PV} = V_C, \quad (3.15) \]

\[ \frac{dV_C}{dt} = \frac{I_{PV} - I_L}{C_1}, \quad (3.16) \]

meaning \( V_{PV} \) can be controlled by controlling the inductor current \( I_L \).

Defining a second error function

\[ e_{V_C} = V_{C\text{ref}} - V_C, \quad (3.17) \]

a Lyapunov candidate function
\[ V_{Ly}(e_{Vc}) = \frac{e_{Vc}^2}{2}, \quad (3.18) \]

and applying Lyapunov’s 2nd method of stability to it gives

\[ \frac{d e_{Vc}}{dt} = -k_{ve} e_{Vc}, \quad k_{ve} > 0. \quad (3.19) \]

Rewriting (3.19) with (3.17) and then (3.16) gets

\[ \Rightarrow \frac{d V_{Cref}}{dt} - \frac{d V_{C}}{dt} = -k_{ve} e_{Vc} \]

\[ \Leftrightarrow \frac{d V_{Cref}}{dt} - \frac{I_{PV} - I_L}{C_1} = -k_{ve} e_{Vc} \]

\[ \Rightarrow I_{Lref} = I_L = I_{PV} - C_1 \left( k_{ve} e_{Vc} + \frac{d V_{Cref}}{dt} \right), \quad (3.20) \]

assuming that the inductor current closely follows the reference.

### 3.3.1 Integral Action Backstepping Control of the Reference Current

As stated before, (3.14) assumes \( V_C = V_{Cref} \) and thus requires \( e_{Vc} = 0 \). However, a steady-state error may appear. To prevent this steady-state error, an integral control action is added by defining an error function \( e_I \) like

\[ e_I = \int_0^t e_{Vc} dt = 0, \quad (3.21) \]

and a Lyapunov candidate function

\[ V_{Ly}(e_I, e_{Vc}) = k_I e_I^2 + \frac{e_{Vc}^2}{2}, \quad (3.22) \]

and apply Lyapunov’s second method for stability

\[ k_{fI} \frac{d e_I}{dt} + e_{Vc} \frac{d e_{Vc}}{dt} = -k_{ve} e_{Vc}, \quad \left\{ \begin{array}{l} k_{ve} > 0 \\ k_I > 0 \end{array} \right. \quad (3.23) \]

Through mathematical manipulation and using (3.16), (3.17) and (3.21), (3.23) can be rewritten:

\[ \Rightarrow k_{fI} e_{Vc} + e_{Vc} \left( \frac{d V_{Cref}}{dt} - \frac{d V_C}{dt} \right) = -k_{ve} e_{Vc} \]

\[ \Leftrightarrow k_{fI} e_I + \frac{d V_{Cref}}{dt} - \frac{I_{PV} - I_L}{C_1} = -k_{ve} e_{Vc} \]

\[ \Rightarrow I_{Lref} = I_L = I_{PV} - C_1 \left( k_I e_I + k_{ve} e_{Vc} + \frac{d V_{Cref}}{dt} \right). \quad (3.24) \]

Comparing (3.24) to (3.20) shows that only the integral term was added, helping the system follow
the voltage reference. This is the Lyapunov equivalent of a PI controller, since the control action depends on the error $e_{V_C}$ and on the integral $e_I$ of that error.

### 3.4 Hysteresis Control of the Reference Current

(3.20) gives the reference value of current that the inductor should have but the only input that can be controlled in the system is the ON/OFF position of the transistor of the boost converter. Therefore, a final controller is necessary to convert the reference inductor current $I_{Lref}$ into an ON/OFF signal. As mentioned in section 3.1, the converter needs to work at variable frequency to counteract the effects of the high duty cycle. Hysteresis controllers are able to do this by relying on a different input than frequency: deviation from the reference.

From the circuit depicted in Figure 3.1, the voltage drop in the inductor ($V_L$) is given by

$$V_L = V_C - \gamma V_o,$$  \hspace{1cm} (3.25)

where $\gamma$ represents the ON/OFF position of the switch. If ON, $\gamma = 0$, there is a short circuit and $V_L = V_C$. If OFF, $\gamma = 1$, the diode conducts and $V_L = V_C - V_o$. The inductor voltage drop can also be given by the derivative of current in the inductor,

$$V_L = L \frac{dI_L}{dt}.$$  \hspace{1cm} (3.26)

Joining the two equations gives

$$\frac{dI_L}{dt} = \frac{V_C - \gamma V_o}{L},$$  \hspace{1cm} (3.27)

and this equation shows how the inductor current can be affected by the switch position. If $\gamma = 0$ then the derivative of the current will be positive (since both $V_C$ and $L$ are positive), and the current will increase. If $\gamma = 1$ then the derivative of the current is negative (since the circuit is a boost converter, $V_o$ is always bigger than $V_C$) and the current will decrease.

By creating an error function such as

$$e_{I_L} = I_{Lref} - I_L, \quad -\Delta I_L/2 < e_{I_L} < \Delta I_L/2,$$  \hspace{1cm} (3.28)

where $\Delta I_L$ is the allowed maximum variation for the error, a simple algorithm can be devised to keep the error between the allowed bounds (also depicted in Figure 3.4) and thus make the inductor current follow the reference calculated by the controller in (3.24):

$$\begin{cases} 
\text{If } e_{I_L} > \Delta I_L/2 \Rightarrow I_L \text{ needs to increase } \Rightarrow dI_L/dt > 0 \Rightarrow \gamma = 0 \\
\text{If } e_{I_L} < -\Delta I_L/2 \Rightarrow I_L \text{ needs to decrease } \Rightarrow dI_L/dt < 0 \Rightarrow \gamma = 1
\end{cases}.$$  \hspace{1cm} (3.29)
The resulting $I_L$ current waveform can be seen in Figure 3.5. In it, $i_a$ represents $I_L$, $i_a^*$ represents $I_{Lref}$, $\Delta i_a$ represents $\Delta I_L/2$ and $q$ represents $\gamma$.

And so, the full controller will be able to keep $V_{PV} = V_{PV_{ref}}$, a neighbourhood where $\frac{dP_{PV}}{dV_{PV}} \approx 0$, which means the main control objective is fulfilled and the backstepping procedure ends.

### 3.5 Recursive Backstepping Control of the MPP Voltage and Gain Estimation

#### 3.5.1 MPP Recursive Backstepping Control Law

First there is the need to obtain an equation for the control signal $\gamma$. Creating a Lyapunov candidate function such as

$$V_{Ly}(e_I, e_{Vc}, e_{IL}) = k_I \frac{e_I^2}{2} + \frac{e_{Vc}^2}{2} + \frac{e_{IL}^2}{2}, \quad (3.30)$$

and enforcing its time derivative to be negative yields (with $k_{vc}, k_{IL}, k_I > 0$),
\[ \Rightarrow k_I e_I \frac{dI}{dt} + e_I C_{ref} \frac{dV_{ref}}{dt} + e_L \frac{dI}{dt} = -k_v e_c^2 - k_I e_c^2 L. \]

\[ \Rightarrow k_I e_I e_V + e_V \left( \frac{dV_{ref}}{dt} - \frac{dV_C}{dt} \right) + e_L \left( \frac{dI_{ref}}{dt} - \frac{dI_L}{dt} \right) dt = -k_v e_c^2 - k_I e_c^2 L. \]

\[ \Rightarrow k_I e_I e_V + e_V \left( \frac{dV_{ref}}{dt} - \frac{dV_C}{dt} \right) + e_L \left( \frac{dI_{ref}}{dt} - \frac{dI_L}{dt} \right) dt = -k_v e_c^2 - k_I e_c^2 L. \]

\[ \Rightarrow k_I e_I e_V + e_V \left( \frac{dV_{ref}}{dt} - \frac{dV_C}{dt} \right) + e_L \left( \frac{dI_{ref}}{dt} - \frac{dI_L}{dt} \right) dt = -k_v e_c^2 - k_I e_c^2 L. \]

The last equation of (3.24) can be rewritten with the addition of \(-e_{IL}/C_1\) in each side as

\[ \frac{dV_{ref}}{dt} - \frac{I_{PV} - (I_{ref} - e_{IL})}{C_1} = -k_I e_I - k_v e_V - \frac{e_{IL}}{C_1}. \] (3.32)

and applying this to (3.31) gives

\[ \Rightarrow k_I e_I e_V + e_V \left( \frac{dV_{ref}}{dt} - \frac{dV_C}{dt} \right) + e_L \left( \frac{dI_{ref}}{dt} - \frac{dI_L}{dt} \right) dt = -k_v e_c^2 - k_I e_c^2 L. \]

\[ \Rightarrow k_I e_I e_V + e_V \left( \frac{dV_{ref}}{dt} - \frac{dV_C}{dt} \right) + e_L \left( \frac{dI_{ref}}{dt} - \frac{dI_L}{dt} \right) dt = -k_v e_c^2 - k_I e_c^2 L. \]

Using the last equation of (3.24) again and (3.27),

\[ \Rightarrow \frac{dI_{ref}}{dt} - \frac{dI_L}{dt} = \frac{e_V}{C_1} - k_I e_{IL} \]

\[ \Rightarrow \frac{d}{dt} \left[ \frac{I_{PV} - I_L}{C_1} \left( k_I e_I + k_v e_V + \frac{dV_{ref}}{dt} \right) \right] - \frac{V_C - \gamma V_o}{L} = \frac{e_V}{C_1} - k_I e_{IL}. \] (3.34)

Rearranging (3.32) gets

\[ \frac{dV_{ref}}{dt} - \frac{I_{PV} - I_L}{C_1} = -k_I e_I - k_v e_V - \frac{e_{IL}}{C_1}. \] (3.35)

and applying (3.16) followed by (3.17) results in

\[ \frac{dV_V}{dt} = -k_I e_I - k_v e_V - \frac{e_{IL}}{C_1}. \] (3.36)

Replacing this result in the last equation of (3.34) gives the final equation,

\[ \frac{dI_{PV}}{dt} - C_1 \left[ k_I e_V + k_v \left( -k_I e_I - k_v e_V - \frac{e_{IL}}{C_1} \right) + \frac{d^2V_{ref}}{dt^2} \right] - \frac{V_C - \gamma V_o}{L} = \frac{e_V}{C_1} - k_I e_{IL}. \] (3.37)
which, rearranged, calculates the value of $\gamma$ that minimizes $e_I$, $e_{V_C}$, and $e_{I_L}$.

$$\gamma = \frac{V_C}{V_o} - \frac{L}{V_o} \left( \frac{dI_{PV}}{dt} - C_1 \left[ k_I e_{V_C} + k_{vc} \left( -k_I e_I - k_{vc} e_{V_C} - \frac{e_{I_L}}{C_1} \right) + \frac{d^2 V_{Cref}}{dt^2} \right] - \frac{e_{V_C}}{C_1} + k_{I_L} e_{I_L} \right)$$

(3.38)

### 3.5.2 Gain Estimation

To determine $k_I$, $k_{I_L}$ and $k_{vc}$ one can use the first derivative of (3.16) and using (3.27) results in

$$\frac{d^2 V_C}{dt^2} = \frac{1}{C_1} \left( \frac{dI_{PV}}{dt} - \frac{V_C}{L} \right) - \frac{C_1}{V_o} \left[ k_I e_{V_C} + k_{vc} \left( -k_I e_I - k_{vc} e_{V_C} - \frac{e_{I_L}}{C_1} \right) + \frac{d^2 V_{Cref}}{dt^2} \right] - \frac{e_{V_C}}{C_1} + k_{I_L} e_{I_L} \right)$$

(3.39)

Replacing $\gamma$ in the last equation,

$$\frac{d^2 V_C}{dt^2} = \frac{1}{C_1} \left( \frac{dI_{PV}}{dt} - \frac{V_C}{L} \right) - \frac{C_1}{V_o} \left[ k_I e_{V_C} + k_{vc} \left( -k_I e_I - k_{vc} e_{V_C} - \frac{e_{I_L}}{C_1} \right) + \frac{d^2 V_{Cref}}{dt^2} \right] - \frac{e_{V_C}}{C_1} + k_{I_L} e_{I_L} \right)$$

(3.40)

and using (3.36),

$$\frac{d^2 V_C}{dt^2} = \frac{1}{C_1} \left( \frac{dI_{PV}}{dt} - \frac{V_C}{L} \right) - \frac{C_1}{V_o} \left[ k_I e_{V_C} + k_{vc} \left( -k_I e_I - k_{vc} e_{V_C} - \frac{e_{I_L}}{C_1} \right) + \frac{d^2 V_{Cref}}{dt^2} \right] - \frac{e_{V_C}}{C_1} + k_{I_L} e_{I_L} \right)$$

(3.41)

Rearranging this last equation,

$$\Leftrightarrow \frac{d^2 V_C}{dt^2} = \frac{1}{C_1} \left[ C_1 \left( k_I e_{V_C} + k_{vc} \frac{dV_{Cref}}{dt} + \frac{d^2 V_{Cref}}{dt^2} \right) + \frac{e_{V_C}}{C_1} - k_{I_L} e_{I_L} \right]$$

$$\Leftrightarrow \frac{d^2 V_C}{dt^2} = k_I e_{V_C} + k_{vc} \frac{dV_{Cref}}{dt} + \frac{d^2 V_{Cref}}{dt^2} - \frac{e_{V_C}}{C_1} + \frac{k_{I_L} e_{I_L}}{C_1}$$

$$\Leftrightarrow - \frac{d^2 e_{V_C}}{dt^2} = k_I e_{V_C} + k_{vc} \frac{dV_{Cref}}{dt} + \frac{e_{V_C}}{C_1} \frac{k_{I_L} e_{I_L}}{C_1}$$

(3.42)

Rearranging (3.36),

$$e_{I_L} = -C_1 \left( k_I e_I + k_{vc} e_{V_C} + \frac{dV_{Cref}}{dt} \right).$$

(3.43)
and applying it to (3.42),

\[
\begin{align*}
\L \left\{ \frac{df(t)}{dt} \right\} &= sL \{ f(t) \} - f(0) = sF(s), \\
\Rightarrow \quad -\frac{d^2 e_{vc}}{dt^2} &= k_I e_{vc} + k_{vc} \frac{de_{vc}}{dt} + \frac{e_{vc}}{C_I^2} + k_{IL} k_I e_I + k_{IL} k_{vc} e_{vc} + k_L \frac{de_{vc}}{dt} \\
\Rightarrow \quad \frac{d^3 e_{vc}}{dt^3} + (k_{vc} + k_{IL}) \frac{d^2 e_{vc}}{dt^2} + \left( k_I + k_{IL} k_{vc} + \frac{1}{C_I^2} \right) \frac{de_{vc}}{dt} + k_{IL} k_I e_I &= 0 \\
\Rightarrow \quad \frac{d^3 e_{vc}}{dt^3} + (k_{vc} + k_{IL}) \frac{d^2 e_{vc}}{dt^2} + \left( k_I + k_{IL} k_{vc} + \frac{1}{C_I^2} \right) \frac{de_{vc}}{dt} + k_{IL} k_I e_V &= 0.
\end{align*}
\tag{3.44}
\]

This is a third order system, in which the error tends asymptotically to zero (assuming \( k_{vc}, k_{IL}, k_I > 0 \)). Comparing it to

\[
a_e^3 T_c^3 s^3 + a_e^3 T_c^2 s^2 + a_e^2 T_c s + 1 = 0,
\tag{3.45}
\]

results in the following equations:

\[
\begin{align*}
\frac{1}{(k_{IL} k_I)} &= a_e^3 T_c^3 \\
(k_{vc} + k_{IL})/(k_{IL} k_I) &= a_e^3 T_c^2 \\
(C_I^2 k_I + C_I^2 k_{IL} k_{vc} + 1)/(k_{IL} k_I) &= a_e^2 T_c.
\end{align*}
\tag{3.46}
\]

By defining \( a_e \) and \( T_c \), this three equation system can be used to calculated the three gains that minimize the error functions \((e_{vc}, e_{IL} \text{ and } e_I)\). Parameter \( T_c \) can be tuned to make the system more aggressive and faster or more gentle and slower, while parameter \( a_e \) is usually \( 2 \leq a_e \leq 4 \) [30].

### 3.6 Command Filtering Backstepping

In section 3.2, the system’s sensibility to noise was not addressed since the computation of \( V_{PV,ref} \) is still reliant on the derivative of measurement inputs. The controller stability is guaranteed but the existence of measurement noise may slow it, causing it to be less efficient.

In Laplace transform, the derivative in time of a given function \( f(t) \) is represented by

\[
\L \left\{ \frac{df(t)}{dt} \right\} = sL \{ f(t) \} - f(0) = sF(s),
\tag{3.47}
\]

with \( F(s) \) being the Laplace transform of \( f(t) \) and assuming an initial condition \( f(0) = 0 \). Unfortunately, as Figure 3.6 shows, transfer function \( TF = s \) has high gain at higher frequencies, meaning it will strengthen high frequency noise.

This calculation can instead be made in differentiation by filtering using a command filter [36] as shown in Figure 3.7, where \( \xi_f \) and \( \omega_f \) represent, respectively, the damping coefficient and the fixed natural frequency of the filter.

Figure 3.7 shows that the signal \( \dot{x} \) is obtained using integral action instead of a differential method,
Figure 3.6: Bode plot of open loop transfer function $TF = s$. 

Figure 3.7: Command Filter (CF) block diagram.

which attenuates the high frequency disturbances [37]. The CF depicted has the following state-space representation:

$$
\begin{bmatrix}
\dot{x}_f_1 \\
\dot{x}_f_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\omega_f^2 & -2\xi_f\omega_f
\end{bmatrix}
\begin{bmatrix}
x_f_1 \\
x_f_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
-\omega_f^2
\end{bmatrix} \alpha, \quad (3.48)
$$

where $\alpha$ is the filter input signal and $x_{f_1}$ and $x_{f_2}$ are filter states. Since

$$
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{f_1} \\
x_{f_2}
\end{bmatrix}, \quad (3.49)
$$

the CF transfer function is given by

$$
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
\omega_f^2 \\
s^2 + 2\xi_f\omega_f + \omega_f^2
\end{bmatrix}
\begin{bmatrix}
1 \\
s
\end{bmatrix} \alpha, \quad (3.50)
$$

where the first output is the input signal filtered through a simple bandpass second-order filter, and the second output is the derivative of the first output. The CF should be implemented with a finite natural
frequency not necessarily much greater than that of the natural frequency of the input \[38\].

And so, the transfer function used to calculate the filtered derivative of an input signal \(\alpha\) is

\[
TF = \frac{\omega_f^2 s}{s^2 + 2\xi_f \omega_f + \omega_f^2},
\]

(3.51)

which has the frequency response depicted in Figure 3.8 (\(\xi_f = \sqrt{2}/2\) and \(\omega_f = 2\pi 1000\) rad/s, to be far from the 50 Hz grid frequency and well below the converter switching frequency). It shows filtering at high and low frequencies except for the selected natural frequency, which was the objective.

Since the derivative in the voltage reference controller can be rewritten as

\[
\frac{dP_{PV}}{dI_{PV}} = \frac{dP_{PV}}{dI_{PV}}
\]

(3.52)

it can be performed by dividing the filtered derivative of \(P_{PV}\) by the filtered derivative of \(I_{PV}\), hence minimizing problems related to measurement noise.

### 3.7 Proposed MPPT Backstepping Controller

The whole block diagram for the proposed controller can be seen in Figure 3.9. The two saturation blocks work to relax the controller when the system is far away from the reference value, helping to curb an initial aggressive response.
This result is applied in the MPPT controllers of Chapter 4.
Chapter 4

MPPT Controllers for Partial PV Shading

To find the global maximum power point of the non-concave power-voltage characteristic, this thesis studies three different solutions for dealing with partial PV shading, shown in Figure 4.1:

- One MPPT controller for three PV panels connected in series (Figure 4.1a);
- Three MPPT controllers, one for each PV panel, connected in series (Figure 4.1b);
- Three MPPT controllers, one for each PV panel, connected in parallel (Figure 4.1c).

Each one grants a specific advantage:

- a controller that could track the MPP of three series connected PV panels would reduce the cost of a multipanel system, making it more economically viable for private households;
- one controller for each PV panel of the system is more suitable for situations with heavy partial shading since it allows the system to track the maximum power output of every single panel;
  - series connection presents the convenience of not having connections between all the controllers and the inverter, reducing the amount of cable required in the system, and thus also reducing the system cost;
  - parallel connection permits all three controllers to be independent from each other, unlike the series connection that has to have the same current passing through all panels.

4.1 Three-state Single MPPT Search Algorithm

Since the power-voltage curve in a series of differently shaded solar panels is non-concave (it has more than one local maximum, as seen in section 1.2), the controller will have to search around some predefined points in order to find all the local maxima and then work out which is the GMPP. The controller
Figure 4.1: Studied architectural solutions.

(a) One MPPT, panels connected in series.

(b) Three MPPT, series connection.

(c) Three MPPT, parallel connection.
must be designed to be reliable (able to find every available local maximum) and fast (the controller does not search unnecessary points if possible).

Figure 2.3a of section 2.3 shows that the different levels of irradiance in each panel affect more its output current than its output voltage. This suggests that if every panel is considered alone its MPP should always be around the same voltage which in turn indicates that the local maxima should be approximately evenly spaced. Indeed, the literature agrees that, in partial shading conditions, the voltages of local maxima are about $0.8V_{oc}$ apart [39], meaning that the system can just search at

$$V_n = 0.8nV_{oc}, \quad n = 1, 2, ..., N, \tag{4.1}$$

with $N$ being the number of panels in the PV system string and also the number of points around which the controller will search for the GMPP, which in the scope of this thesis is three. Figure 4.2 displays the same PV and IV curve depicted in section 1.2 but with the MPP voltages labelled and it shows that, although the voltages are not perfectly evenly spaced, the approximation mentioned before is valid.

![Figure 4.2: Example of the locations of local maxima.](image)

This thesis proposes an algorithm divided into three states of operation: Setup, SteadyState and NewSearch. As the names imply, Setup operates when the controller is first turned on, either because the PV system was just installed or because the controller was turned off for maintenance reasons; NewSearch is called when a change in power output suggests that the GMPP may have been changed to another local maxima and thus the controller needs to find it; SteadyState is the idle state, it executes just the backstepping controller in order to keep the system at the GMPP.

### 4.1.1 Setup State

The Setup state starts with the reference voltage at its lowest possible value ($V_{PVref} = 0.8V_{oc}$). It will wait for the backstepping controller to get close to the local maximum and measure the power output in that point saving it as the GMPP ($P_{mem}$) and the voltage at which it occurs ($V_{mem}$). Then, it increases
the reference voltage, waits for the controller and measures the power output again: if it is bigger than $P_{mem}$, then it becomes the new maximum and its power and voltage are saved. It will repeat this process until the next increase in the voltage reference leaves the system outside the PV system power curve ($V_{PVref} > NV_{oc}$) and at that point the system has measured all local maxima.

The system now has to return to the GMPP. It could just make $V_{PVref} = V_{mem}$ and arrive directly at the GMPP but for strings with more than two panels the system may have to drastically change its output voltage in just a step change. Also, the power output change can also be drastic, which can cause some disturbances in the grid. In order to avoid this as best as possible, the algorithm will make step changes of $0.8V_{oc}$ each until it is close to the global maximum. When it finally arrives near the GMPP, it waits for the system to stabilize before measuring the power output one final time and saving it as the true maximum, after which $SteadyState$ will begin. The flowchart for this algorithm can be seen in Figure 4.3.

![Flowchart for Setup state of the three-state single MPPT search algorithm.](image)

**Figure 4.3: Flowchart for Setup state of the three-state single MPPT search algorithm.**

### 4.1.2 NewSearch State

After $Setup$, the system will remain in $SteadyState$ indefinitely, with the backstepping controller keeping the system at MPP and measuring the power output continuously. The algorithm compares this power with $P_{mem}$ and will call $NewSearch$ if the difference between them crosses a certain threshold (e.g., $|P_{PV} - P_{mem}| > 0.1P_{mem}$).

Once called, $NewSearch$ starts a searching algorithm similar to the one in $Setup$: it will measure the power output in the same voltage points and then return a step at a time to the biggest local maximum. It has, however, a slightly more complex search process due to the fact that unlike in $Setup$ the initial voltage may not be $V_{PV} \approx 0.8V_{oc}$.

If the initial voltage is the lowest possible then $NewSearch$ will work as $Setup$, increasing the voltage
until all maxima are checked, and then work backwards towards the GMPP. Alternatively, if the initial voltage is the highest possible it will have the opposite order of search, first decreasing the voltage and then increasing it towards the GMPP.

Finally, if the initial voltage is neither the highest nor the lowest then $V_{PV} \approx 1.6V_{oc}$. The algorithm will need to look first in one direction (e.g., lowering the voltage) and then turn back to search the other direction, as can be seen in Figure 4.4, where $V_n = 0.8nV_{oc}$.

$$V_1 \leftrightarrow V_2 \leftrightarrow V_3$$

Figure 4.4: Example of NewSearch algorithm.

A flag (dir) can be created to indicate the initial direction of the search, thus reducing the complexity of the algorithm since the first two cases follow direct opposite patterns of search. As for the third case, if after the first iteration $V_{PV} = V_1$, then it can be concluded that the initial voltage was $V_{PV} = V_2$, meaning that the next step should be $V_{PV} = V_3$. After this adjustment, this case follows the pattern established in the first case.

The flowchart for the NewSearch algorithm can be seen in Figure 4.5.

4.1.3 Solar Panel Dynamic Model

Given that this solution works by changing the operating point of the PV system (in some situations, rather drastically), the solar panel static model (described in Chapter 2) is not enough to characterize the system and, as a result, the dynamic model of the solar cell must be used. The simplified dynamic model is defined by the parallel circuit of a photocurrent source, a diode and an equivalent capacitor [40]. This capacitor is small, around $1 - 4 \mu F$ [41] for a given solar panel.

Also, in order to allow differently shaded panels to work in series, they must have a bypass diode ($D_B$) placed in parallel since each panel will output a different current. With no bypass diode, the system would have a low output current, the current of the more shaded panel. Combining it with the 5-parameter model already studied in Chapter 2 results in the model depicted in Figure 4.6.

4.2 Three MPPT Controllers, Series Connection

After implementing the MPPT search algorithm described in Chapter 3 to three series connected boost converters, one comes across a problem: all three converters have the same output current and the sum of their output voltages equals the input voltage of the inverter (which is constant) since they are
Figure 4.5: Flowchart for NewSearch state of the three-state single MPPT search algorithm.
connected in series. This means that, in partial shading conditions, the converters connected to the more heavily shaded panels (which output less power) will have smaller output voltages.

In more extreme situations, if one panel is too poorly irradiated compared with the other two, its converter output voltage may fall beyond the output voltage of the mentioned panel and then the boost converter stops working, no longer being able to deliver the panel’s produced power to the system. Even if this panel stops being shaded the boost converter has no way of increasing the output voltage from zero, meaning it will remain unable to deliver the produced power. Concluding, a simple boost converter is not enough for this solution to work.

The proposed converter can work as a Buck converter and as a Boost converter based on the situation, shown in Figure 4.7. If $S_1$ is kept OFF, the converter will work as a Buck and if $S_2$ is kept ON it will work as a Boost converter.

These two switches can be easily coordinated using a comparator that outputs $c = 1$ if $V_o > V_{oc}^{r}$ and $c = 0$ if $V_o < V_{oc}^{r}$. Switch $S_1$ control signal can be done by multiplying the control signal of the MPPT controller ($\gamma$) by the comparator’s result (comparable to a logic AND gate). The control signal of $S_2$ must be done using a logic OR gate that can be implemented by summing the comparator’s output with the control signal and limiting the result between 0 and 1. This coordination will yield the following results:

$$\begin{align*}
V_o > V_{oc}^{r} & \Rightarrow c = 1 \Rightarrow S_1 = \gamma || S_2 = 1 \\
V_o < V_{oc}^{r} & \Rightarrow c = 0 \Rightarrow S_1 = 0 || S_2 = \gamma
\end{align*}$$

which is the desired result. This way, panels will not be in danger of being forcefully disconnected when
heavily shaded with no way of reconnecting.

This solution also requires the use of the output capacitor \( C_2 \) in each converter (in addition to the inverter capacitor \( C_{DC} \)) because of the backstepping controller of the inverter. This capacitor can be calculated with [30]

\[
C_2 = \frac{V_o T}{R_o \Delta V_o},
\]

with \( R_o \) being the theoretical load of the converter and which can be calculated as \( R_o = \frac{(V_o/N_s)^2}{P_{MPP}} \), with \( N_s \) being the number of converters in series.

### 4.3 Three MPPT Controllers, Parallel Connection

This is the simplest architecture since it does not require different control algorithms (like in section 4.1) or different converters (like in section 4.2). Each controller works independently from each other, reducing the complexity of the system. It may, however, be the most expensive solution: it needs as many controllers (and boost converters) as there are solar panels and, because of the parallel connection, the cables that connect the panels to the inverter will need to be bigger to allow for higher currents, the more panels in the system, the bigger the cables.

Nonetheless, it may be the most efficient solution, given that each MPPT controller tracks the MPP of just one panel and that it uses a boost converter, a converter known for its high efficiency [42].
Chapter 5

Inverter Control

5.1 Inverter Types

The power inverter is an equipment that converts Direct Current (DC) to Alternate Current (AC) [43]. It is responsible for injecting the power produced by the PV array into the main electrical grid, with the requirement being that

\[ V_{DC} > \sqrt{2}V_{AC_{RMS}}, \]  

(5.1)

where

- \( V_{DC} \) - input voltage of the inverter (output voltage of the boost converter);
- \( V_{AC_{RMS}} \) - output RMS voltage of the inverter.

The voltage type inverter can be operated as both the voltage source and the current source when viewed from the AC side, only by changing the control scheme of the inverter. Assuming that the voltage level of the grid is mostly constant and independent from the inverter [44], a self-commutated voltage source inverter with a current control scheme is needed. Consequently, a controller that keeps the input voltage constant while injecting produced power into the grid must be designed (in other words, an input voltage controller).

5.2 Inverter Model

Since this thesis focuses on an array of only three PV panels of peak power \( P_P = 270W \), the power output of the system won’t be bigger than 1 kW, meaning a single-phase inverter (depicted in Figure 5.1) would be sufficient. Also, the DC/AC converter used is the full-bridge, H4 type, as this project aims at implementing an inverter with high efficiency and reliability and low cost [45].

In order to reduce the complexity of the system being tested in this thesis, the inverter seen in Figure 5.1 can be modelled without semiconductors.
Figure 5.1: Single-phase full-bridge inverter.

Starting with the capacitor’s voltage equation and applying Kirchhoff’s current law gets

\[
V_{DC} = \frac{1}{C_{DC}} \int_0^t I_C dt
\Rightarrow V_{DC} = \frac{1}{C_{DC}} \int_0^t (I_{DC} - I_I) dt.
\]

(5.2)

Next, applying Kirchhoff’s voltage law to the inductor’s current equation leads to

\[
I_{LI} = \frac{1}{L_I} \int_0^t V_{LI} dt
\Rightarrow i_{LI}(t) = \frac{1}{L_I} \int_0^t (V_I - v_g(t)) dt.
\]

(5.3)

Analysing Figure 5.1 shows that when switches \(S_1\) and \(S_4\) are ON while \(S_2\) and \(S_3\) are OFF \(V_I = V_{DC}\). In opposition, when \(S_2, S_3\) are ON with \(S_1, S_4\) OFF, \(V_I = -V_{DC}\). Furthermore, with \(S_1, S_2\) ON and \(S_3, S_4\) OFF (or vice versa), \(V_I = 0\).

Like the connection between the output voltage \(V_I\) and the input voltage \(V_{DC}\), Figure 5.1 also shows a similar connection between the input current \(I_I\) and the output inductor current \(I_{LI}\). Hence, a variable \(\gamma_i\) can be created in order to allow the following equations:

\[
\begin{align*}
I_I &= \gamma_i I_{LI} \\
V_I &= \gamma_i V_{DC}
\end{align*}
\]

\[
\begin{cases}
1, & S_1, S_4 = \text{ON} \\
0, & S_1, S_2 \mid S_3, S_4 = \text{ON} \\
-1, & S_2, S_3 = \text{ON}
\end{cases}
\]

(5.4)

where \(\gamma_i\) is a control signal responsible for turning the inverter’s switches ON and OFF. The resulting inverter outputs are shown in Figure 5.2 in per-unit system (pu).

The switching behaviour depicted is called 3-level Pulse-Width Modulation (PWM). It has an acceptably low Total Harmonic Distortion (THD) and while lower THD are possible they usually require different inverter architectures in order to allow for a higher number of \(V_I\) voltage levels [30].

As a result, (5.2) and (5.3) can be rewritten as
With these equations, the inverter model can be simulated without the use of semiconductors (depicted in Figure 5.3), highly lowering the complexity of the calculations and thus reducing simulation time.

**5.2.1 Inverter Parameters**

The inverter’s parameters can be calculated by the following equations [30]:

\[
\frac{V_{DC}}{4\Delta I_{LI} f_{PWM}} \leq L_I \leq \frac{\sqrt{V_{DC}^2/2 - V_{gRMS}^2}}{\omega_g I_{LIRMS}},
\]

(5.7)

\[
C_{DC} = \frac{I_{DC}}{\omega_g \Delta V_{DC}},
\]

(5.8)
where $f_{PWM}$ represents the switching frequency [Hz] and $\omega_g$ represents the grid frequency [rad/s].

### 5.3 Non-linear Backstepping Control

Since the MPPT controller was designed non-linearly using a backstepping algorithm, this thesis also proposes a non-linear controller for the inverter. The design procedure used is similar to the one applied in Chapter 3 [11].

Since the control objective of this controller is to have $V_{DC} = V_{DCref}$, the first step is to create an error function like

$$
e_{V_{DC}} = V_{DCref} - V_{DC}.$$  \(5.9\)

Then, a Lyapunov candidate function like

$$V_{Ly}(e_{V_{DC}}) = \frac{e_{V_{DC}}^2}{2},$$  \(5.10\)

must be created. Applying Lyapunov’s second method of stability to it, using (5.5) and (5.9) and assuming $dV_{DCref}/dt = 0$ (since it is a DC voltage) yields

$$\Rightarrow \frac{dV_{DC}}{dt} = -k_U e_{V_{DC}}, \quad k_U > 0$$

$$\Leftrightarrow \frac{dV_{DCref}}{dt} - \gamma I_{LI} \frac{I_{LI}}{C_{DC}} = -k_U e_{V_{DC}}.$$  \(5.11\)

Unfortunately, $\gamma$ cannot be dynamic in this calculation since its computation comes after the calculation of the reference current and is thus subject to a delay. Then, $\gamma$ should be approximated by its average value,

$$G_I = \int_0^T \gamma_i(t)dt.$$  \(5.12\)

This value can be calculated by analysing the power flow in the inverter,

$$P_g = \eta_i P_I,$$  \(5.13\)

where $P_g$ represents the output power to the grid, $P_I$ the input power and $\eta_i$ represents the inverter efficiency. This equation can be further developed into the following equations (assuming power factor $f_p = 1$):

$$\Rightarrow V_{GRMS} I_{LI\text{RMS}} f_p = \eta_i V_{DC} I_I$$

$$\Leftrightarrow I_I = I_{LI\text{RMS}} \frac{V_{GRMS}}{\eta_i V_{DC}} \Rightarrow G_I = \frac{V_{GRMS}}{\eta_i V_{DC}}.$$  \(5.14\)
Using the average value of \( \gamma_i \) means that (5.11) does not output the instantaneous value of the reference current, but its root mean square (RMS) value,

\[
I_{LIrefRMS} = \frac{I_{DC} - C_{DC}k_U e_{V_{DC}}}{G_I},
\]

assuming \( i_L \) follows the reference closely.

Furthermore, as shown in section 3.3.1, additional integral action can be added directly to the reference inductor current calculation, like

\[
\Rightarrow I_{LIrefRMS} = \frac{I_{DC} - C_{DC}(k_U e_{V_{DC}} + k_{IU} e_{IU})}{G_I},
\]

where \( e_{IU} \) is defined as \( e_{IU} = \int e_{V_{DC}} \, dt \) and \( k_{IU} > 0 \), in order to help the system decrease the DC voltage offset. \( k_U \) and \( k_{IU} \) can be calculated using a similar process to the one described in section 3.5.

Since the active power delivered by the solar system to the electrical grid is calculated by

\[
P_g = |S_g| \cos(\phi_g),
\]

where \( S_g \) is the apparent power delivered to the grid and \( \phi_g \) is the phase difference between the inverter output current and the grid voltage, minimizing \( \phi_g \) is a priority in order to maximize efficiency and make the approximation in (5.14) valid.

Near unity power factor inverters are made by applying a phase-locked loop (PLL) to the output voltage. This block outputs the frequency and phase of the voltage, which can then be applied to the reference RMS value of the inductor current to get the instantaneous reference inductor current \( (i_{LIref}(t)) \).

Finally, an hysteresis controller, like the one described in section 3.4, can be applied to the error function

\[
e_{inv}(t) = i_{LIref}(t) - i_{LI}(t),
\]

in order to output the control variable \( \gamma_i \) needed to control the inverter.

Since \( \gamma_i \) is non-binary, as opposed to the control signal in section 3.4 which had only two possible values (0 and 1), two hysteresis loops are required to output this controller’s three possible values (-1, 0 and 1). These two partial loops can be added together, each with binary outputs (-0.5 and 0.5). They have the following algorithms:

\[
\begin{align*}
\text{If } e_{inv} > \Delta I_{LI}/2 \Rightarrow I_{LI} \text{ needs to increase} \Rightarrow dI_{LI}/dt > 0 \Rightarrow \gamma_1 &= 0.5 \\
\text{If } e_{inv} < -\Delta I_{LI}/2 \Rightarrow I_{LI} \text{ needs to decrease} \Rightarrow dI_{LI}/dt < 0 \Rightarrow \gamma_1 &= -0.5
\end{align*}
\]

and...

47
\[
\begin{align*}
\text{If } e_{\text{inv}} > \Delta I_{LI}/4 \Rightarrow I_{LI} \text{ needs to increase } & \Rightarrow dI_{LI}/dt > 0 \Rightarrow \gamma_2 = 0.5 \\
\text{If } e_{\text{inv}} < -\Delta I_{LI}/4 \Rightarrow I_{LI} \text{ needs to decrease } & \Rightarrow dI_{LI}/dt < 0 \Rightarrow \gamma_2 = -0.5.
\end{align*}
\] (5.20)

The resulting sum of these two control signals \((\gamma_1 + \gamma_2 = \gamma_i)\) will then follow the following algorithm (also depicted in Figure 5.4):

\[
\begin{align*}
\text{If } e_{\text{inv}} > \Delta I_{LI}/2 \Rightarrow \gamma_i = 1 \Rightarrow dI_{LI}/dt > 0 \\
\text{If } e_{\text{inv}} < -\Delta I_{LI}/2 \Rightarrow \gamma_i = -1 \Rightarrow dI_{LI}/dt < 0 \\
\text{If } -\Delta I_{LI}/2 < e_{\text{inv}} < \Delta I_{LI}/2 \Rightarrow \gamma_i = 0 \Rightarrow \text{signal}(dI_{LI}/dt(t)) = \text{signal}(-v_g(t)).
\end{align*}
\] (5.21)

![Figure 5.4: Single phase inverter.](image)

The complete backstepping inverter controller can be seen in Figure 5.5.

### 5.4 Linear PI Control of the Inverter DC Input Voltage

PID controllers are one of the most used control algorithms in the world because of their known good transient responses and their simple tuning mechanisms [46]. They are also very adaptable to all kinds of different environments and flexible to almost all situations. This thesis studies this control method in order to evaluate the differences with the previously described non-linear backstepping controller.

In every power converter controller, control outputs are defined at the beginning of each period, and can only be changed in the next period. That means that if, after defining the control output for that period, there is a change in the controller’s controlled variable that would incite a change in the controller output, there will be a randomly variable time delay \(t_d\) until the controller can act on it. Usually, \(0 < t_d < T_i\) (\(T_i\) being the controller’s period) and it is dependent on when the change occurred. [30]

When switching times are very small compared to the reactive elements time constants (and assuming small disturbances) it is possible to consider the average value of the random variable \(t_d\) to
The transfer function of this time delay can be obtained using the delay property of the Laplace transform:

\[ f(t) \xrightarrow{\mathcal{L}} F(s); f(t - T_d) \xrightarrow{\mathcal{L}} F(s)e^{-sT_d} \]  

In direct converters in CCM (and for small changes), the transfer function of the time delay can be given by

\[ TF = K_D e^{-sT_d}. \]  

This transfer function can be expressed more conveniently considering the Taylor series of the exponential and neglecting high order terms, valid for \( \omega T_d \leq \sqrt{2}/2 \):

\[ K_D e^{-sT_d} = \frac{K_D}{e^{sT_d}} = \frac{K_D}{1 + sT_d + \frac{s^2}{2!}T_d^2 + \ldots + \frac{s^n}{n!}T_d^n + \ldots} \approx \frac{K_D}{1 + sT_d}. \]  

Using (5.5), the voltage controller block diagram can be drawn (Figure 5.6).

As seen in Figure 5.6, the system already has an origin pole and thus one could consider that
the system would only require a proportional compensator. This, however, would not guarantee a null steady-state offset nor insensitivity to $V_{DC}$ disturbances, meaning a PI compensator really is required. Concurrently, cancelling the origin pole with a zero in the controller could result in a slow system, highly oscillatory or even unstable, because of the imprecision of where the pole and the placed zero are.

Then, considering a PI controller with

$$C(s) = \frac{(1 + sT_s)}{sT_p}, \quad (5.27)$$

the open-loop transfer function (assuming $I_{PV} = 0$) is

$$I_L(s)\Big|_{I_{PV}=0} = \frac{(1 + sT_s)}{sT_p}\frac{K_{D\alpha_v}}{(1 + sT_d)}(-G_i)\frac{1}{sC_{DC}}I_{Lref}(s) - \frac{1}{sC_{DC}}V_{DC}. \quad (5.28)$$

From this, the closed-loop transfer function is

$$I_L(s)\Big|_{I_{PV}=0} = \frac{(1 + sT_s)I_{Lref}(s) - sT_p(1 + sT_d)V_{DC}}{s^3C_{DC}T_dT_{e} + s^2\frac{C_{DC}T_p}{(-\alpha_v)K_D\alpha_v} + s\frac{C_{DC}T_p}{(-\alpha_v)K_D\alpha_v} + sT_z + 1}. \quad (5.29)$$

Applying the $b_k^2 = 2b_{k-1}b_{k+1}$ criteria to the third order denominator polynomial yields the values for $T_z$ and $T_p$ and, by extension, the values for $K_p$ and $K_I$:

$$\begin{align*}
T_z &= 4T_d \\
T_p &= -\frac{8T_D^2K_DG_i\alpha_v}{C_{DC}} \\
\Rightarrow \quad K_p &= \frac{1}{T_p} = -\frac{C_{DC}}{8T_D^2G_iK_D\alpha_v} \\
K_I &= \frac{1}{T_p} = -\frac{C_{DC}}{2T_DG_iK_D\alpha_v}.
\end{align*} \quad (5.30)$$

Substituting these results in the denominator polynomial gets

$$I_L(s)I_{Lref}(s) = \frac{1 + 4T_d s}{8T_D^2s^3 + 8T_D^2s^2 + 4T_ds + 1} = \frac{1 + 4T_d s}{(1 + 2T_d s)(4T_D^2s^2 + 2T_ds + 1)}, \quad (5.31)$$

which can be shown [30] to have an overshoot of 43.3%. This result can be optimized by using $b_k^2 = ab_{k-1}b_{k+1}$ (in the previous case $a = 2$) resulting in the following equations:

$$\begin{align*}
T_z &= a^2T_d \\
T_p &= -\frac{a^2T_D^2K_DG_i\alpha_v}{C_{DC}} \\
\Rightarrow \quad K_p &= \frac{1}{T_p} = -\frac{C_{DC}}{a^2T_D^2G_iK_D\alpha_v} \\
K_I &= \frac{1}{T_p} = -\frac{C_{DC}}{aT_DG_iK_D\alpha_v}.
\end{align*} \quad (5.32)$$

Now, the controller’s natural frequency is $\omega_n = 1/(aT_d)$ and the phase margin is given by $\phi_M = \arcsin((a^2 - 1)/(a^2 + 1))$ which means that the open-loop transfer function has unit gain at $\omega = \omega_n$. 

![Figure 5.6: Block diagram of voltage control with internal current control.](image-url)
frequency at which the phase margin is maximum.

\(a\) can be calculated in order to get a specific phase margin \(\phi_M\) or damping factor \(\xi\) by considering the denominator of the transfer function (5.31),

\[
a^3T_d^3s^3 + a^3T_d^2s^2 + a^2T_ds + 1 = (1 + aT_d s)(a^2T_d^2s^2 + aT_ds + 1),
\]

(5.33)

with poles in \(\omega_n = 1/(aT_d)\) and \(\omega_n[-(a - 1)/2 \pm \sqrt{[(a - 1)/2]^2 - 1}] = -\xi\omega_n \pm j\omega_n\sqrt{\xi^2 - 1}\), with \(\xi = (a - 1)/2\).

Usually, \(2 \leq a \leq 4\) is in order to have \(36^\circ \leq \phi_M \leq 62^\circ\) with damping factor \(\xi \geq 0.5\). Given the desired \(\xi = \sqrt{2}/2\), \(a\) can be calculated by

\[a = 2\xi + 1 \approx 2.7,\]

(5.34)

yielding a phase margin \(\phi_M \approx 49^\circ\).

Then, a PLL can be implemented to have near-unity power factor and an hysteresis controller to command the inverter switches, just like in section 5.3. Figure 5.7 shows the complete PI controller.

These inverter controllers (linear PI and non-linear backstepping) are simulated and compared against one another in Chapter 6.
Chapter 6

Results

6.1 Parameter Calculation

Table 6.1 shows the Boost converter parameters while Table 6.2 shows the single-phase inverter parameters used in this thesis.

Table 6.1: Parameters used in boost converter.

<table>
<thead>
<tr>
<th>$L$ [mH]</th>
<th>$C_D$ [$\mu$F]</th>
<th>$C_1$ [mF]</th>
<th>$C_2$ [$\mu$F]</th>
<th>$k_v$</th>
<th>$k_{vc}$</th>
<th>$k_L$</th>
<th>$\Delta I_L$ [A]</th>
<th>$\omega_f$ [rad/s]</th>
<th>$\xi_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.7</td>
<td>3.5</td>
<td>1.98</td>
<td>681$^1$</td>
<td>5</td>
<td>1725</td>
<td>1.1e6</td>
<td>1.32</td>
<td>628</td>
<td>0.707</td>
</tr>
</tbody>
</table>

Table 6.2: Parameters used in single-phase inverter.

<table>
<thead>
<tr>
<th>$L_1$ [mH]</th>
<th>$C_{DC}$ [mF]</th>
<th>$k_U$</th>
<th>$k_{1U}$</th>
<th>$K_P$</th>
<th>$K_I$</th>
<th>$\Delta I_{L1}$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>20.9</td>
<td>15.4</td>
<td>638</td>
<td>-4.62</td>
<td>-0.67</td>
<td>0.44</td>
</tr>
</tbody>
</table>

6.2 Three-state Single MPPT Search Algorithm - Maximum Power Point Tracking

In order to test the tracking ability of the three-state single MPPT search algorithm the following case was designed:

- Stage 1 - the system is at steady-state with all three panels at STC ($G_A = G_B = G_C = 1000$ W/m$^2$);
- Stage 2 - a shadow appears in one of the panels ($G_A = G_B = 3G_C = 1000$ W/m$^2$);
- Stage 3 - the shadow becomes more pronounced and shades another panel ($G_A = 3G_B = 5G_C = 1000$ W/m$^2$).

$^1$ Applicable only for the series connected controllers.
As Figure 6.1 shows, this situation demands that the system searches all local maxima and tests the ability to find and stay on the GMPP, regardless of its initial operating point (with $V_n = 0.8nV_{oc}$). Figure 6.2 depicts the boost converter power output in response to these changes and Table 6.3 shows the measured converter power output at each step, the theoretical maximum values and the measured efficiency at the DC side ($\eta_c$).

Figure 6.1: Power-voltage characteristic at each different stage of the growing shadow situation.

Figure 6.2: Three-state single MPPT search algorithm response to the growing shadow situation.

Figure 6.2, together with the information in Table 6.3, shows that the system is indeed able to track the GMPP with very little error. Since the boost converter was designed almost ideally the drop in efficiency is mainly due to the MPPT.

It also shows the tracking process: the system begins at stage 1 and, after experiencing a 10%
output power change, saves the preliminary power point (in this case $P_{\text{mem}} = 650 \, W$ and $V_{\text{mem}} = V_3$) and checks the other two points ($V_2$ and $V_1$). Since none is better than the saved power point, the system will return to $V_3$ (at time $t = 7.4 \, s$) after passing through $V_2$.

For demonstration purposes the change in environment conditions between the stages is very dramatic, such a drop in power output in almost no time is not common in a real environment. Because of this, the algorithm is slower than the demonstrated change and when the system returns to $V_3$, it is no longer at the GMPP. Therefore, the system starts another search process (after saving the now stabilized power point of $V_3$) and checks again $V_2$ and $V_1$. $V_2$ is now the GMPP and thus the system stabilizes there (at time $t = 9 \, s$).

The change between stage 2 and 3 is the worst case scenario because it requires the biggest number of voltage changes: starts at $V_2$, checks $V_1$, passes through $V_2$ to get to $V_3$ and returns to $V_2$ ($t = 13 \, s$). Because of the fast changing environment conditions, it will execute these changes twice in order to find the new GMPP at $V_1$.

### 6.3 Three MPPT Controllers

Both controller solutions that use one MPPT per panel were also submitted to the situation described in section 6.2 in order to see if both were compatible with the proposed backstepping algorithm of Chapter 3. Figure 6.3 depicts the system response of both the parallel connection and the series connection, while Table 6.3 shows the measured power output at each step, the theoretical maximum values and the measured efficiency at the DC side.

#### Table 6.3: Measured output power and efficiency with three-state single MPPT search algorithm.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Measured power [W]</th>
<th>Theoretical maximum power [W]</th>
<th>$\eta_c$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_A = G_B = G_C$</td>
<td>724</td>
<td>727</td>
<td>99.6</td>
</tr>
<tr>
<td>$G_A = G_B = 3G_C$</td>
<td>483</td>
<td>484.6</td>
<td>99.7</td>
</tr>
<tr>
<td>$G_A = 3G_B = 5G_C$</td>
<td>241</td>
<td>242.2</td>
<td>99.5</td>
</tr>
</tbody>
</table>

Figure 6.3, with the results from Table 6.4, demonstrates that the backstepping algorithm is also successful in single panel controllers, either connected in parallel or in series.
Figure 6.3: Response of one MPPT per PV panel solutions to the growing shadow situation.

The theoretical maximums presented in Table 6.3 are lower than the ones presented in Table 6.4 because in the first case there is a trade-off, the system needs to pick the operating point that maximizes the power produced by the whole system, it is not possible to maximize the power produced by each panel, while in the second case the production of all three panels is maximized, leading to higher power outputs.

### 6.4 Three-state Single MPPT Search Algorithm - Worst Case Situation

The efficiency of the three-state single MPPT search algorithm can then be calculated using the theoretical maximums of Table 6.4, so that a comparison can be drawn regarding total efficiency ($\eta_t$), with the results shown in Table 6.5.

**Table 6.5: Measured output power and total efficiency with three-state single MPPT search algorithm.**

<table>
<thead>
<tr>
<th>Situation</th>
<th>Measured power [W]</th>
<th>Theoretical maximum power [W]</th>
<th>$\eta_t$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_A = G_B = G_C$</td>
<td>724</td>
<td>727</td>
<td>99.6</td>
</tr>
<tr>
<td>$G_A = G_B = 3G_C$</td>
<td>483</td>
<td>571.1</td>
<td>84.6</td>
</tr>
<tr>
<td>$G_A = 3G_B = 5G_C$</td>
<td>241</td>
<td>380.5</td>
<td>63.7</td>
</tr>
</tbody>
</table>

These results are intuitive: when choosing a GMPP, the three-state single MPPT search algorithm dismisses the other local maxima and neglects some power produced by the photovoltaic panels in order to maximize the total power produced. The effect is more pronounced when the various local maxima have similar values, as is the case in stage 3. This indicates that the total efficiency is lowest when all local maxima have equal values and, thus, this setup has a lowest possible theoretical efficiency.

Assuming environment conditions where

$$P_1 = P_2 = P_3,$$  \hspace{1cm} (6.1)
with $P_n$ being the different local maxima values (and $V_n$, $I_n$ the voltage and current of the same local maxima). Assuming that every individual panel would have its own MPP at the same voltage as the others if measured independently ($V_{MPP_A} \approx V_{MPP_B} \approx V_{MPP_C} = V_{MPP}$) and assuming that the local maxima are separated by $V_{oc}$ (with $V_{MPP} \approx 0.8V_{oc} \Rightarrow V_{oc} \approx 1.25V_{MPP}$), (6.1) can be manipulated as

\[ \iff V_1I_1 = V_2I_2 = V_3I_3 \]
\[ \iff V_{MPP}I_1 = (V_{oc} + V_{MPP})I_2 = (2V_{oc} + V_{MPP})I_3 \]
\[ \iff V_{MPP}I_1 = 2.25V_{MPP}I_2 = 3.5V_{MPP}I_3 \]
\[ \iff I_1 = 2.25I_2 = 3.5I_3 \]
\[ \iff I_1 = 2.25(wI_1) = 3.5(zI_1) \]
\[ \iff w = \frac{1}{2.25} \quad \text{and} \quad z = \frac{1}{3.5} \]

with $w$ and $z$ being, respectively, the ratio between $I_2$ and $I_1$ and $I_3$ and $I_1$.

It can be shown experimentally that the different local maximum currents correspond to the different MPP currents of each individual panel: $I_1 \approx I_{MPP_A}$, $I_2 \approx I_{MPP_B}$, and $I_3 \approx I_{MPP_C}$ and thus

\[ P_A + P_B + P_C = V_{MPP_A}I_{MPP_A} + V_{MPP_B}I_{MPP_B} + V_{MPP_C}I_{MPP_C} \]
\[ = V_{MPP_A}I_{MPP_A} + V_{MPP_A}\frac{I_{MPP_A}}{w} + V_{MPP_A}\frac{I_{MPP_A}}{z} \]
\[ = P_A(1 + 1/w + 1/z) \]
\[ \approx 1.73P_A, \]

which corresponds to the maximum theoretical power the three panels can output given these environment conditions. Since it can be shown experimentally that $P_A \approx P_1$, at the worst possible environment conditions when the three-state single MPPPT search algorithm chooses $P_1$ as the GMPP it will have a theoretical total efficiency of

\[ \eta_t = \frac{P_1}{P_A + P_B + P_C} \times 100\% \approx \frac{P_A}{1.73P_A} \times 100\% = 57.8\%. \]

Since the output current in each panel is proportional to its irradiance (as per equation (2.17)), the conditions that create this result are $G_A = 2.25G_B = 3.5G_C$. Using $G_A = 1000 \text{ W/m}^2$, the system will have a PV curve depicted in Figure 6.4, with the efficiency of the system response to these conditions calculated in Table 6.6.

Figure 6.4 shows that, although the local maxima values are not exactly equal the assumptions and approximations made are valid for an approximated result ($P_1 = 242.2 \text{ W}$, $P_2 = 245 \text{ W}$ and $P_3 = 245.3 \text{ W}$).

This demonstrates that this architectural solution of PV panels (one controller for three panels con-
Table 6.6: Measured output power and total efficiency with three-state single MPPT search algorithm at worst case situation.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Measured power [W]</th>
<th>Theoretical maximum power [W]</th>
<th>ηt [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_A = 2.25G_B = 3.5G_C$</td>
<td>243</td>
<td>431.2</td>
<td>56.4</td>
</tr>
</tbody>
</table>

Connected in series) never has a total efficiency below 50% when compared with the architectures that use one MPPT per panel, no matter the environment conditions. This effect decreases when adding more panels to the string.

### 6.5 Inverter Controllers

Both inverter controllers discussed in this thesis (linear PI and non-linear backstepping) were tested in the environment conditions depicted in Figure 6.5. These conditions are slower and less pronounced than the ones simulated in sections 6.2 and 6.3 but they are closer to real situations. The power output of the inverter (the power injected into the grid) when using each controller can be seen in Figure 6.6, while the input voltage can be seen in Figure 6.7 and the output (inductor) current is shown in Figure 6.8.

![Irradiance variation in time](image)
Figure 6.6: Inverter output power ($P_g$) controlled by PI and backstepping.

(a) PI controller.  
(b) Backstepping controller.

Figure 6.7: Inverter input voltage ($V_o$) controlled by PI and backstepping.

(a) PI controller.  
(b) Backstepping controller.

Figure 6.6 shows that the backstepping controller is more aggressive than the PI, it follows very closely the power output at the DC side (Figure 6.2). This can be explained by analysing Figure 6.7: the backstepping controller is faster than the PI controller at keeping the DC voltage constant, while the PI controller allows the voltage to fluctuate when the MPPT is searching for the new GMPP. Figure 6.8 also demonstrates the aggressiveness of the backstepping controller against the PI controller, with the PI outputting a much smoother current waveform.

Figure 6.9 and Figure 6.10 show the inverter output power (using PI and backstepping controllers) for series and parallel connected converters, respectively. They demonstrate that both controllers will have similar responses when there is no fast change in power output (unlike during the search process of the three-state single MPPT search algorithm). However, the PI controller shows again a smoother response than the backstepping controller.
Figure 6.8: Inverter output current ($I_{LI}$) controlled by PI and backstepping.

Figure 6.9: Inverter output power ($P_g$) for series connected converters.
(a) PI controller.
(b) Backstepping controller.

Figure 6.10: Inverter output power \( (P_g) \) for parallel connected converters.
Chapter 7

Conclusions

This dissertation presents a different approach to the MPPT controller from what is usually found in literature. A backstepping controller is proposed based in incremental conductance and is designed in a way that minimizes three distinct variables: voltage error between the reference value and the instantaneous output voltage of the PV system; the integral of the voltage error; the current error between the reference value and the instantaneous inductor current. Also proposed was an original method to calculated the gains $k_I$, $k_{I_L}$ and $k_{vc}$ of the backstepping algorithm with integral action. Since the obtained efficiency was above 99% the main objective was achieved, demonstrating the stability and reliability of the backstepping algorithm and the gain estimation.

The backstepping controller is implemented both in solutions that possess one controller per PV panel (series and parallel connected) and in the single MPPT search algorithm with all three panels connected in series. The latter requires the design of an additional algorithm, called three-state single MPPT search algorithm, to be able to search the local maxima of the non-concave power-voltage curve of a PV system subject to partial shading conditions. The three-state single MPPT search algorithm takes advantage of the fact that the local maxima are approximately spaced by $0.8V_{oc}$. In both cases, the controller proves to be able to track the GMPP in all tested conditions, displaying its flexibility to different conditions and architectures.

The three-state single MPPT search algorithm presented the expected results. It showed around 99.6% efficiency in the three different situations it was tested in ($G_A = G_B = G_C = 1000$ W/m$^2$, $G_A = G_B = 3G_C = 1000$ W/m$^2$ and $G_A = 3G_B = 5G_C = 1000$ W/m$^2$), with power outputs of 724 W, 483 W and 241 W.

The series and parallel connected MPPT controllers presented around the same efficiency (around 99.7% and 99.4%, respectively) but with higher overall power outputs (725 W, 570 W and 379.5 W for the series connection and 724 W, 568 W and 377.5 W for the parallel connection). This was also the expected result given the fact that a single controller connected to three solar panels will have some individual panel power loss in order to maximize the power production of the whole system, unlike three independent MPPT controllers. These results also demonstrate the feasibility of using the converter proposed in section 4.2 for the series connection. Although these associations require a boost converter
(or a boost-buck for the series connection) in each panel, this is not a big disadvantage, since each panel has a low power rating (270 W) and converters for these power ranges and respective microprocessor controllers are inexpensive compared to the cost of the PV panel itself. The series association of MPPT controlled panels could also alleviate the need and present trend of very high static gain boost converters.

This thesis compares the power output of the three-state single MPPT search algorithm against the sum of the power output of the three solar panels in the same conditions and concludes that, as the power-voltage local maxima of the three-state single MPPT search algorithm become closer in value to each other, the total efficiency of the system goes down. It then discusses the existence of a lowest possible total efficiency when all local maxima are equal and, through simple calculations, demonstrates that this efficiency exists (57.8%) and that it happens when \( G_A = 2.25G_B = 3.5G_C \). Simulating with \( G_A = 1000 \text{ W/m}^2 \), the system displays a total efficiency of 56.4%, showing that this setup will never have a total efficiency lower than 50% when compared with the one MPPT per panel alternatives, no matter the environment conditions.

One of the drawbacks of the nonlinear backstepping control design (based on Lyapunov’s second method for stability) is the presence of time derivatives. These can lead to instability if not properly managed and thus a command filtering approach is used to ensure the operation of the MPPT controller.

Once the MPPT controller is designed, this project focuses on applying the backstepping algorithm to the inverter controller and comparing it to the more commonly employed PI controller. This thesis tests both controllers to see which achieves a better performance when controlling the previously mentioned systems.

In the three controller solutions both PI and backstepping controllers proved to be fast and stable, with the PI controller displaying a more steady control effort (the AC input currents) at the cost of some overshoots in the controlled output (the DC capacitor voltage). In the three-state single MPPT search algorithm solution, the PI controller showed a bigger fluctuation in the DC voltage when the system is searching for a new maximum, while the backstepping controller is faster to react, at the cost of AC currents with higher ripple.

In summation, the backstepping controller, both in series or parallel connection or coupled to the three-state single MPPT search algorithm, proves to be a reliable option when designing a MPPT controller, to be employed either with a single panel or for a string of PV panels. It is robust and since most panels have similar characteristics it can be applied to control almost any PV string, with very high efficiency and fast response. It can also be applied to the inverter controller although in the three-state single MPPT search algorithm case the searching process is very dynamic, combined with the aggressiveness of the backstepping controller results in fast changes in output power which can be hard for the grid or the system connected load to deal with. So, this thesis recommends the use of more smooth controllers like the PI controller. It is slower than the backstepping controller and thus is able to output a slower, and smoother, changing current. The series association of MPPT controlled boost converters is able to extract the maximum power in shading conditions. Therefore, if shading conditions are frequent the main recommendation of this thesis is the use of PV panels series connection of individual panel
MPPTs. Since each panel has a low power rating, this distributed solution of boost converters and respective MPPT controllers is inexpensive and may alleviate the need of very high static gain centralized boost converters.

7.1 Future Work

- The boost converters in this thesis have been designed with relatively high capacitances in order to allow the correct operation of the MPPT. These capacitors are expensive and eventually short lived which means they could increase the cost of the system. Therefore, this system should be designed using smaller capacitors, perhaps adjusting the controller gains or testing some other techniques like the non-linear adaptive backstepping. This technique allows for the controller to tune itself for different power levels by adjusting its gains according to the output power error of the system.

- The search algorithm used in the three-state single MPPT search algorithm waits a given amount of time between steps. This allows the system to stabilize and give an accurate measurement of the power point of that location. However, when the algorithm is getting close to the local maximum the incremental conductance decreases, so instead of waiting a fixed time the algorithm could measure the incremental conductance and decide to wait for the system to stabilize or jump to the next step.

- Another approach can be tried for the three-state single MPPT search algorithm, searching for the maximum power of each PV panel fitted with its own boost converter, instead of using only a single boost. This would decrease the cost of the boost converter in each PV panel while providing central control.

- This thesis also adopts an approximated model for the inverter and employs ideal boost converters. In order to better cement the feasibility of this method this project should be done with more realistic parameters and maybe even a prototype could be created to test the designed controllers.
Bibliography


Appendix A

PV Datasheet

Figure A.1: Canadian Solar - CS6K - 270 | 275 | 280 M PV Module datasheet.