Development of an Unsteady BET Model for Forward Flapping Flight: Performance Comparison of Different MAV Configurations

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To my friends and family, who were there for me all along, encouraging me to tackle every challenge that this work continuously threw in my direction. And to my ingenuity, which allowed me to embrace this problem with the impression that solving bird flight would be a “piece of cake”. May I remain this idiotic forever!
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Resumo

Neste trabalho desenvolveu-se um modelo aerodinâmico de asas em batimento em voo nivelado, partindo de dois modelos já existentes para casos de voo pairado e recorrendo à Teoria dos Elementos de Pás, bem como a conceitos de aerodinâmica não-estacionária. Este modelo foi depois transformado numa ferramenta de fácil manuseio que permite agilizar o processo de estudo e otimização destas asas, e serviu de base para a comparação do desempenho de uma asa em batimento com soluções de voo mais tradicionais como a asa fixa e rotativa. A comparação também foi feita de forma puramente analítica, através de manipulação de expressões de potência, por forma a que houvesse vários métodos para a avaliação.

Os resultados analíticos mostram que, em algumas situações, a asa em batimento tem uma melhor performance. No entanto, em todos os casos estudados com a ferramenta aqui desenvolvida, a asa em batimento consome mais energia que ambas as outras opções. Porém, dado que as asas aqui estudadas apenas foram sujeitas a uma otimização rudimentar e que muitos casos diferentes não foram testados, não é possível retirar a conclusão de que esta configuração não se possa afigurar como vantajosa em certas situações. Só se pode afirmar que, em casos muito específicos, a asa em batimento mostrou ter uma pior performance que as outras configurações.

Abstract

This work set out to develop an aerodynamic model describing flapping wing forward flight, starting from two existing models for hovering flapping flight and resorting to the Blade Element Theory, as well as unsteady aerodynamics concepts. The developed model was later fitted into an user-friendly tool that allows for the optimization process of a flapping wing to be dramatically sped up, and served as the basis for a performance comparison between the flapping wing and the more traditional flight solutions like the fixed and the rotary wing. The comparison was also done in a purely analytical way, through the manipulation of power equations, so as to provide various methods to compare the different configurations.

The analytical results show that, in some situations, the flapping wing has the edge in terms of power consumption. However, the results obtained by the tool show that in every case studied here the other configurations consume less power. Nevertheless, since the flapping wings that were put to the test were only optimized in a rudimentary way, and given that several cases remained unstudied, it is not possible to conclude that there are no situations where the flapping wing is advantageous. It can only be said that, in very specific cases, this configuration showed to perform worse than its counterparts.

Keywords: Flapping Wing, Comparison, Blade Element Theory, Power Consumption, Unsteady Aerodynamics
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Nomenclature

**Greek symbols**
- \( \alpha \) Angle of attack.
- \( \beta \) Added mass coefficient.
- \( \Delta \) Lift fluctuation factor.
- \( \eta \) Efficiency.
- \( \Gamma \) Circulation.
- \( \gamma \) Flapping angle.
- \( \kappa \) Induced power correction factor.
- \( \lambda \) Inflow ratio.
- \( \mu \) Advance ratio.
- \( \Omega \) Angular velocity.
- \( \omega \) Dimensionless angular velocity.
- \( \Phi \) Wagner function.
- \( \phi \) Induced angle of attack.
- \( \Psi \) Azimuth angle.
- \( \rho \) Density.
- \( \sigma \) Rotor solidity.
- \( \tau \) Dimensionless time.
- \( \theta \) Pitch angle.

**Roman symbols**
- \( \mathcal{A} \) Aspect Ratio.
- \( T \) Period.
- \( A \) Area.
- \( a \) Acceleration.
- \( b \) Wingspan.
- \( C \) Coefficient.
- \( c \) Chord length.
- \( D \) Drag.
- \( e \) Oswald’s efficiency factor.
- \( F \) Force.
\( f \)  
Frequency.

\( G \)  
Rotational force coefficient.

\( g \)  
Gravitational acceleration.

\( K \)  
Profile power correction factor for helicopters.

\( k \)  
Reduced frequency.

\( L \)  
Lift.

\( m \)  
Mass.

\( N \)  
Number of.

\( P \)  
Power.

\( R \)  
Radius.

\( r \)  
Nondimensional radius.

\( Re \)  
Reynold's number.

\( T \)  
Thrust.

\( t \)  
Time.

\( V \)  
Velocity.

\( x, y, z \)  
Cartesian distances.

**Subscripts**

0  
Profile component.

\( \perp \)  
Perpendicular component.

\( \Gamma \)  
Circulatory component.

\( \infty \)  
Free-stream condition.

\( \parallel \)  
Parallel component.

\( A \)  
Added mass force component.

\( \text{av} \)  
Average value.

\( b \)  
Blade.

\( c \)  
Climb component.

\( \text{crit} \)  
Critical value.

\( \text{cycle} \)  
Quantity per cycle.

\( D \)  
Drag, 3D.

\( d \)  
Drag, 2D.

\( f \)  
Rotation axis.

\( \text{fix} \)  
Fixed wing.

\( \text{flap} \)  
Flapping wing.

\( H \)  
Heaving component.

\( h \)  
Helicopter.

\( i \)  
Induced component.

\( L \)  
Lift, 3D.

\( l \)  
Lift, 2D.

\( M \)  
Magnus force component.
\textbf{\textit{M}} \quad \text{Moment, 3D.}

\textbf{\textit{m}} \quad \text{Moment, 2D.}

\textbf{\textit{max}} \quad \text{Maximum value.}

\textbf{\textit{min}} \quad \text{Minimum value.}

\textbf{\textit{p}} \quad \text{Parasitic component.}

\textbf{\textit{prop}} \quad \text{Propeller.}

\textbf{\textit{ref}} \quad \text{Reference point.}

\textbf{\textit{rot}} \quad \text{Rotational component.}

\textbf{\textit{s}} \quad \text{Incidence relative to the stream.}

\textbf{\textit{T}} \quad \text{Thrust, 3D.}

\textbf{\textit{t}} \quad \text{Thrust, 2D.}

\textbf{\textit{tip}} \quad \text{Wing or blade tip.}

\textbf{\textit{vert}} \quad \text{Vertical component.}

\textbf{\textit{w}} \quad \text{Wing.}

\textbf{\textit{\eta, \xi, \zeta}} \quad \text{Local frame of reference components.}

\textbf{\textit{x, y, z}} \quad \text{Inertial frame of reference components.}

\textbf{\textit{T, R, P}} \quad \text{Local frame of reference components.}
# Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>2D</td>
<td>Two dimensions.</td>
</tr>
<tr>
<td>3D</td>
<td>Three dimensions.</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of attack.</td>
</tr>
<tr>
<td>BET</td>
<td>Blade Element Theory.</td>
</tr>
<tr>
<td>CFD</td>
<td>Computer Fluid Dynamics.</td>
</tr>
<tr>
<td>FWMAV</td>
<td>Flapping Wing Micro Air Vehicle.</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface.</td>
</tr>
<tr>
<td>LEV</td>
<td>Leading Edge Vortex.</td>
</tr>
<tr>
<td>LE</td>
<td>Leading Edge.</td>
</tr>
<tr>
<td>MAV</td>
<td>Micro Air Vehicle.</td>
</tr>
<tr>
<td>SSL</td>
<td>Standard Sea Level.</td>
</tr>
<tr>
<td>TE</td>
<td>Trailing Edge.</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle.</td>
</tr>
<tr>
<td>UBET</td>
<td>Unsteady Blade Element Theory.</td>
</tr>
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</table>
Chapter 1

Introduction

1.1 Motivation

In their quest to achieve the goal of affordable and efficient flight, engineers have spent the better part of the last century trying to improve the flight solutions already proven to work, namely fixed-wings and, to a smaller extent, rotary wings. However, these solutions can only improve so much, and the search for the next big breakthrough in the aerospace field is on.

Through countless years of evolution, Nature has reached the state of near perfection when it comes to flying animals. Birds and insects are capable of extraordinary feats such as long-range flight, high-thrust propulsion and efficient hovering, some of which going as far as to combine several of these capabilities. All of this is due to wings that evolved to be as efficient and lightweight as possible, while remaining manoeuvrable enough to give flyers a wide range of movements. In short, there is a lot of knowledge that can be taken from Nature, but so far this objective has proven elusive for engineers due to the difficulty in studying the aerodynamic and mechanical behaviour of these animals. Computational models describing flapping flight already exist, but they are either too costly or too specific; a simpler, more flexible and more affordable model that maintains the accuracy of the more sophisticated methods could then help in the quest to understand flapping flight.

The scientific community and the industry are faced with the objective of technology miniaturization. There is a need for more efficient aircraft, and now is the time to analyse the pros and cons of flapping flight and, if proven to work in a real-life implementation, to push for its widespread adoption. However, the tools for such a detailed study are not yet readily available, and therefore this work sets out to develop a model that quickly predicts both aerodynamic loads and power consumption of a flapping wing vehicle. Birds have cracked the code of efficient flight, so why can’t we?

1.2 Topic Overview

Technology miniaturization is generally one of the main goals in any field of research, and the Unmanned Aerial Vehicle (UAV) field is no exception. In order to build Micro Air Vehicles (MAVs), the most
efficient propulsion strategies need to be found. With the demand for new solutions on the rise, considerable efforts have been made to assess the question of the efficiency of different flying strategies, hence the increase in interest on the topic of flapping flight in the past years.

1.2.1 Configurations Comparison

The subject of performance comparison between different propulsion strategies has seen, like previously stated, several developments during the recent past. Liu and Moschetta [1] tried to compare the hovering power expenditure of two different MAV configurations, rotary and flapping wings, having reached the conclusion that, based on a purely theoretical approach, there is not an appreciable difference between the designs. Pesavento and Wang [2], on the other hand, set out to find if there was a flapping wing configuration that could prove to be more efficient than its fixed wing counterpart. Through the use of CFD and optimization of the flapping wing, they showed that it was possible to get a configuration that consumed 27% less power than an identical fixed wing. The topic of rotating vs. flapping wings was again explored by Zheng et al. [3], this time using CFD, and the conclusion was that for small Reynolds numbers ($Re < 100$), flapping wings have a lift-to-power ratio twice as large as the rotating configuration. More recently, flapping wings were again compared to fixed wings by Sachs [4], and the analysis was made solely from a mathematical standpoint. The study focused particularly on two parameters, propeller efficiency for fixed wings and induced drag for flapping wings, with the overall efficiency being dependent on both factors. The conclusion was that, for low induced drag factors, flapping wing vehicles show a higher efficiency, so they achieve a correspondingly higher level of overall performance, therefore requiring fixed wing vehicles to have a high propeller efficiency in order to yield a comparable flight performance.

The conclusion, after careful analysis of all the mentioned studies, is that the flapping configuration likely has the potential to help achieve the goal of technology miniaturization, particularly for devices intended to hover. However, the topic of forward flight remains very much unexplored due to the difficulty of the analysis and to the wide range of factors that play a role in the flight of a flapping wing vehicle.

1.2.2 Flapping Flight Models

Several attempts have been made to describe the forces that arise from this wing movement and to establish a model that reliably predicts them. Perhaps the most famous one, Dickinson et al. [5], sets out to describe the forces created by an insect wing in a hovering state. It does so by assembling a *Drosophila melanogaster* (fruit fly) mechanical wing, measuring the forces it generates during a flapping period and then trying to explain the measured values through several different mechanisms: delayed stall, rotational circulation and wake capture. After isolating the measured values into their translational and rotational components, the authors come to the conclusion that even though delayed stall plays a big role in the aerodynamic performance of the insect, a very important contribution comes from rotational forces generated by the wing's rotation and from the wake capture when it changes direction (end of the upstroke and the downstroke). This study is followed by another one, Dickinson and Sane [6], done by
practically the same investigators, where the focus is mainly to describe the behaviour of the rotational forces and to incorporate them in a revised model of flapping flight. After studying the phenomenon of varying force generation with different centres of rotation and rotational speeds, a model was developed that accurately described the translational forces and that better allowed for the isolation of the wake capture force, paving the way for future studies to better analyse and predict it.

Almost at the same time, Walker [7] tackled a similar problem when trying to find the importance of rotational lift for flapping flight. In doing so, he created a very detailed model of the forces that arise from a hovering wing and paved the way for what later would become one of the modified models of the present thesis. In his work, Walker found evidence that rotational forces are indissociable from the translational component of the forces, and could explain the generated forces using this idea. The model developed for the study was an unsteady one, and after comparing it to both a quasi-steady model and CFD measurements of the same wing, the conclusion was that the Unsteady Blade Element Theory (UBET) model was a viable option to predict how freely flying animals move, hence the choice to study this model in detail for this work. Another study aimed at creating an UBET based model was the one by Truong et al. [8], that set out to recreate the values measured by Dickinson et al. [5] through the modification of a normal Blade Element Theory (BET) model. After dividing the forces into different components, namely translational, rotational and apparent mass, the work describes the model’s implementation, validation and subsequent application to the case of a beetle. With their validation, the authors showed that the model was able to achieve values very similar to those measured in the original article, suggesting that their model was accurate enough to be used as a standalone method in different cases, therefore justifying its use for the case of the hovering beetle. Again, due to the potential showed by this method, this was also chosen as a building block of the analysis later done in this report.

1.2.3 CFD Analysis of Flapping Flight

Even though all of the mentioned studies apply to the case of hovering vehicles and animals, the same search focused on a BET model for forward flapping flight yields no results. Still, the existence of so many hovering models suggests that an UBET model could be modified to accommodate for the differences that flapping forward flight presents. This model could then be validated by values obtained through a more sophisticated CFD analysis, something that was done in studies like the one by Choi et al. [9]. This paper tried to find the most effective flapping motions and how that influenced the overall performance of the flapping wing vehicle, resorting to a CFD simulation of a NACA 0012 plunging and rotating. Through an optimization routine, the study finds the cases where the unsteady aerodynamic mechanisms efficiently maximize lift and thrust generation, for the cases of forward flight, hovering and high-thrust flight, providing important information for the validation of this work in terms of force measurements and optimized flapping motion. Another study on the subject is LinLin et al. [10], which, even though not directly used in the validation of this work, provided a valuable insight into the forces generated by flapping wings in forward flight. In this paper the authors explain how they modelled a pigeon-sized bird and its flapping motion, from takeoff to steady flight, to achieve a self-propelled bird
model. They then analysed, using a CFD package combining the immersed boundary and the volume of fluid methods, how changing parameters like the center of rotation of the wing influenced the overall flight performance of the bird. The conclusion was that slight changes to the centres of rotation and oscillation allow the bird to manoeuvre easily and climb without a significant increase in power consumption. Even though this analysis focuses on a case with a \( Re \approx 10^4 \), a value significantly higher than the other studies here mentioned, it still provided precious information on the topic of lift and thrust generation.

1.3 Objectives

This work aims to develop an aerodynamic model for forward flapping flight, based on the Unsteady Blade Element Theory and through modifications to existing flapping wing hovering models. This development will enable an accurate comparison between the performance of a flapping-wing Micro Air Vehicle (FWMAV) and similar devices that use the more traditional approaches to flight like fixed and rotary wings. This comparison will have various levels of complexity, with the UBET model being only the last step of an increasingly sophisticated process that starts in a purely analytical way.

1.4 Thesis Outline

In the present section, chapter 1, there is a brief introduction of the theme and a detailed investigation of the work done in recent years on the field of FWMAVs is presented. In chapter 2, the theoretical background of the thesis is explained. This mainly consists of power consumption concepts and calculation methods for different flight configurations, a brief explanation of the Blade Element Theory and a detailed description of the workings of two hovering flapping wing models, which form the basis for the development of the forward flight models later on. Then, in chapter 3, the challenge of analytically comparing the three flight solutions is tackled. This is done by manipulating the power equations of each configuration, resulting in curves that are plotted as a function of speed and conceal the information of where exactly each solution is preferable.

Chapter 4 is dedicated to another part of this work, which is creating the aforementioned forward flight flapping wing model. It starts by implementing two existing hovering models, then modifying each one for the forward case, and finally combining the best aspects of each one to form one definitive forward flight BET model, while also spending some time on its validation. Finally, the joint model is turned into a tool that calculates the performance of a flapping wing with a given set of user-defined values, while also being able to vary the parameters for optimization purposes. This functionality is later used in chapter 5 to analyse how every parameter of a flapping wing affects its overall performance, allowing some conclusions to be inferred in terms of optimization and leads to the generation of two different and relatively optimized wings that are put to the test through a head-to-head comparison with one existing flying wing and a typical small rotary wing.
Chapter 2

Background

Before tackling the issue of forward flapping wing flight, several concepts have to be understood. First power consumption in fixed, rotary and flapping wings will be discussed, followed by an introduction to Blade Element Theory. Finally, two UBET models for hovering flapping flight, which will be the basis for the work developed in the more advanced stages of this work, will be described.

2.1 Power Consumption

In terms of the power consumption, all of the analysis are done under the assumption that there is no fuselage, as that is a variable not directly related to the performance of each configuration. It is also important to explain that the analysis is made in terms of the power itself and not in terms of power coefficients, due to the fact that the objective is to compare different configurations that have completely distinct ways to nondimensionalise its power value, possibly leading to a wrong perception of which vehicle would be more or less efficient.

2.1.1 Fixed Wing

The case of power consumption for the fixed wing configuration in steady flight is relatively simple to evaluate. According to Sachs [4], assuming that there is a propeller powering the contraption it is possible to estimate its power expenditure through the formula

\[ P_{fix} = \frac{F_x V_\infty}{\eta_{prop}}, \]  

(2.1)

where \( F_x \) is the drag force that the body has to overcome, \( V_\infty \) is the free stream velocity and \( \eta_{prop} \) is the propeller’s efficiency. Since the drag can be more conveniently written in terms of the drag coefficient, the expression then becomes

\[ P_{fix} = \frac{\rho}{2 \eta_{prop}} A_w V_\infty^3 C_D, \]  

(2.2)
where \( \rho \) is the fluid's density, \( A_w \) is the total area of the wing and \( C_D \) is the wing's drag coefficient. In order to compare the performance of this configuration with the others, some other parameters must be introduced. Making use of the relation \( C_D = C_{D0} + C_{Di} \), which separates the drag coefficient into profile and induced drag, and noting that the induced drag can be further expanded with the relation \( C_{Di} = C_T^2 / (\pi Re) \), one can rewrite Equation 2.1 into

\[
P_{fix} = \rho \frac{A_w V_\infty^3}{2n_{prop}} \left( C_{D0} + \frac{C_T^2}{\pi Re} \right).
\]

(2.3)

A more comprehensive review of the physical mechanisms behind each type of drag is offered in subsection 2.1.2. Nonetheless, it is worth noting that, in this form, the power equation for fixed wing aircraft shows the separation between different types of power. While profile power is related to the \( C_{D0} \) term, induced power is related to \( C_{Di} \).

### 2.1.2 Rotary Wing

The case of rotary wings requires some more considerations. Due to the uneven movement of the blade's points and to the complex induced velocity, it is not possible to apply a simple formula like in the fixed wing case. The method used in this work follows Leishman's [11] approach, so the first consideration to make is that the power consumption of a helicopter in forward flight can be broken down into

\[
P_h = P_i + P_0 + P_p + P_c,
\]

(2.4)

where \( P_i \) is the induced power, \( P_0 \) is the profile power, \( P_p \) is the fuselage's parasitic power and \( P_c \) is the climbing power. Since the focus of this work is on the forward flight case, with no climbing velocity and assuming no fuselage, the terms \( P_p \) and \( P_c \) will be dropped from now on.

**Induced Power**

This component of power owes its existence to lift generated vorticity shed into the blade's wake as it moves through the fluid. In its simplest form is defined as

\[
P_i = \kappa T v_i,
\]

(2.5)

where \( \kappa \) is an empirical correction factor to account for several aerodynamic phenomena, especially those related to tip losses and nonuniform flow, \( T \) is the thrust created by the rotating blades and \( v_i \) is the induced flow speed due to the wing's rotation. This is an expression that can still be expanded, however. Noting that

\[
T = \rho A_b (\Omega R_b)^2 C_T
\]

(2.6)
and

\[ v_i = \lambda_i \Omega R_b, \tag{2.7} \]

where \( \rho \) is the fluid density, \( A_h \) is the rotor disc area \( (A_h = \pi R_b^2) \), \( R_b \) being the blade’s radius and \( \Omega R_b \) is the tip speed of the blades (referred to, from now on, as \( V_{tip} \)), \( C_T \) is the thrust coefficient of the helicopter and \( \lambda_i \) is the induced inflow ratio. Since this last quantity can be still expanded with the relation

\[ \lambda_i = \frac{C_T}{2 \sqrt{\mu^2 + \lambda^2}}, \tag{2.8} \]

where \( \lambda \) is the inflow ratio, parameter that is described in detail in section 2.1.2, and \( \mu \) is the advance ratio, a quantity that becomes extremely relevant later on, defined as the ratio between the free stream speed \( V_\infty \) and the blade’s tip speed \( V_{tip} \), or

\[ \mu = \frac{V_\infty}{V_{tip}}. \tag{2.9} \]

With all this in mind, Equation 2.5 can be transformed into

\[ P_i = \rho A_h V_{tip}^3 \frac{\kappa C_T^2}{2 \sqrt{\mu^2 + \lambda^2}}. \tag{2.10} \]

The end formula, Equation 2.10, albeit appearing in an unusual form, is written in such a way as to simplify the analysis further down the road and to highlight how every basic parameter affects the overall power consumption.

**Profile power**

Profile power, or the power required to overcome both the pressure drag (associated with the form factor of the object) and the skin friction drag (which can be laminar or turbulent, greatly impacting overall performance), can be written as

\[ P_0 = \rho A_h V_{tip}^3 \frac{\sigma C_{D0}}{8} \left(1 + K \mu^2\right), \tag{2.11} \]

where \( C_{D0} \) is the blade’s profile drag coefficient and \( K \) is yet another empirical correction factor to adjust the formula. At this point there is also the introduction of the parameter \( \sigma \), which is the rotor solidity, defined as the ratio between the total area of the blades and that of a filled circle with the same radius

\[ \sigma = \frac{A_b}{A_h} = \frac{N_b c_b R_b}{\pi R_b^2} = \frac{N_b c_b}{\pi R_b}, \tag{2.12} \]

where \( N_b \) is the number of blades and \( c_b \) is the blade chord length, assumed constant. Note that the chord might vary along the blade, complicating the formula, but for the sake of simplicity its length is
assumed to remain constant along the blade. There is also the need to introduce another parameter, the aspect ratio of the blade ($A_b = \frac{R_b}{c_b}$) to further simplify Equation 2.12. This yields

$$\sigma = \frac{N_b}{\pi R_b}.$$  \hfill (2.13)

**Total Power**

Inserting Equation 2.10 and Equation 2.11 into Equation 2.4, the expression describing helicopter power consumption used throughout this work is obtained

$$P_h = \rho A_h V_{tip}^3 \left[ \frac{\sigma C_D 0}{8} \left( 1 + K \mu^2 \right) + \frac{k C_T^2}{2 \sqrt{\mu^2 + \lambda^2}} \right].$$  \hfill (2.14)

**Inflow Ratio**

This quantity is essential for the calculations that are going to be performed, therefore requiring an in-depth perspective on how to calculate it. The inflow ratio is then defined as

$$\lambda = \mu \tan(\alpha) + \frac{C_T}{2 \sqrt{\mu^2 + \lambda^2}}.$$  \hfill (2.15)

Since analytical solutions to Equation 2.15 are really hard to find, a numerical process is normally employed to find the solutions to this equation. According to Leishman [11], the most efficient way to calculate the inflow ratio is through the Newton-Raphson method, with the iteration scheme being

$$\lambda_{n+1} = \lambda_n - \frac{f(\lambda)}{f'(\lambda)}.$$  \hfill (2.16)

Rearranging Equation 2.15 to find $f(\lambda)$ and subsequently $f'(\lambda)$

$$f(\lambda) = \lambda - \mu \tan(\alpha) - \frac{C_T}{2 \sqrt{\mu^2 + \lambda^2}} = 0,$$  \hfill (2.17)

$$f'(\lambda) = 1 + \frac{C_T}{2} (\mu^2 + \lambda^2)^{-3/2} \lambda.$$  \hfill (2.18)

The only thing missing is the starting value, which, in the majority of the cases, can be assumed to equal the hover value $\lambda = \sqrt{C_T/2}$. The process converges after just 3 or 4 iterations, making this a very effective way to calculate the inflow ratio and therefore the method followed in this work.

**2.1.3 Flapping Wing**

The problem of power calculation in the case of flapping wings is a rather complex one. It is no longer described by a simple, straightforward expression like in the fixed wing problem because now the unsteady effects must be taken into account, as well as the power required to move and rotate the wings. Several approaches are possible, but here the one developed by Sachs [4] is adopted. In his 2016 paper, Sachs resorts to a rather unusual, yet accurate, method to calculate the power consumption.
of flapping wings. He considers that the power obeys the same equation as in the case of fixed wings (Equation 2.2), the only difference being how the induced drag is defined. It is now

$$C_{Di} = \frac{C_d^2}{\pi Re_{flap}}.$$  \hfill (2.19)

The difference is the substitution of the Oswald coefficient $e$ by $e_{flap}$, which can be thought of as a modified Oswald coefficient that includes all of the flapping induced variations to the behaviour of the device. Its formula is

$$e_{flap} = \frac{\gamma_{av} e}{1 + \Delta},$$  \hfill (2.20)

where $\gamma_{av}$ is the average flapping angle, calculated through the expression

$$\gamma_{av} = \frac{\sin(\gamma_{max})}{\gamma_{max}}.$$  \hfill (2.21)

Note that $\gamma_{max}$ is the maximum flapping angle, which varies between 0° and 90°, and $\Delta$ is the dimensionless factor that quantifies the changes in the value of the lift vector during the course of the flapping cycle, given by

$$\Delta = \frac{1}{T_{flap}} \int_0^{T_{flap}} \frac{C_L(t) - C_{L,av}}{C_{L,av}} \, dt.$$  \hfill (2.22)

where $T_{flap}$ is the flapping cycle period, $C_L(t)$ is the instantaneous lift coefficient and $C_{L,av}$ is the average lift coefficient value. The power expression, using this approach, is calculated in a very similar fashion to the fixed wing. It is given by

$$P_{flap} = \frac{1}{2} \rho A_w V^3 \left( C_{D0} + \frac{C_{L,vert}^2}{\pi Re_{flap}} \right),$$  \hfill (2.23)

where $C_{L,vert} = C_L \gamma_{av}$ is the vertical component of the force being generated by the wings, and the velocity $V$ is defined through its relation with the lift coefficient in the following manner

$$mg = \frac{1}{2} \rho A_w V^2 C_{L,vert} \iff V = \sqrt{\frac{2mg}{\rho A_w C_{L,vert}}}.$$  \hfill (2.24)

Since normally $V_{\infty}$ is defined as being

$$V_{\infty} = \sqrt{\frac{2mg}{\rho A_w C_L}},$$  \hfill (2.25)

One can now rewrite the velocity expression to showcase its relation with the flapping angle, $V = \ldots$
Expanding Equation 2.23 with these relations, the total power is obtained

$$P_{\text{flap}} = \frac{1}{2} \rho A_w V_{\infty}^{3} \gamma_{av}^{-\gamma/\mu} \left( C_{D0} + \frac{C_{L,\text{vert}}^{2}}{\pi R e_{\text{flap}}} \right), \tag{2.27}$$

which is the expression that will be used in the first stages of this work for calculating the power consumption of a flapping wing contraption.

### 2.2 Blade Element Theory

An analysis based on the Blade Element Theory is a simple and effective tool to find the total forces acting on a wing. The procedure assumes that the wing is formed by several adjacent 2D sections, then it determines the forces being applied on each of the thin strips and finally it calculates the total forces by integrating these values along the entirety of the wingspan. Even though this tool is mostly used in helicopter design, its simplicity and the fact that the flapping motion shares some similarities with that of the rotary wing turn it into a great option to study flapping flight.

![Figure 2.1: Top and side view, respectively, of the reference scheme and wing environment used throughout this work (adapted from [11]).](image)

This approach does have some limitations, however. For example, it considers that there is no interaction between sections, effectively removing the option of a 3D solution. When studying rotary wings this problem is somewhat mitigated by including components like the tip-loss factor, but in the case of flapping wings there is not a widely accepted and common way to incorporate 3D effects into the analysis. Furthermore, it is dependent on factors like the lift and drag coefficient, which in turn need to be calculated by some other analysis and are oftentimes only available for a limited range of airfoils and a few different unsteady conditions.

Figure 2.1 depicts a generic blade’s environment in a BET analysis. The scheme for a flapping wing is really similar, with the main difference being the orientation of $\Omega$, the angular velocity (see Figure 2.2). It is worth noting that, since this work focuses only on the topic of forward flight, this will be the only mode explained in detail.
2.2.1 Forward Flight

As previously stated, a BET model is very useful when studying rotary wing blades. While the hovering state is the flying mode most usually studied with this method, it is possible to also model forward flight, even though this means adding several layers of complexity to the problem. Two very important concepts that are now needed are the azimuth angle of the blade and also the flapping angle, both depicted in Figure 2.2.

![Figure 2.2: Side and top view of the blade, showing the flapping angle $\gamma$ and the azimuth angle $\psi$, respectively, as well as the relevant velocity components (adapted from [11] and [12]).](image)

Comparing this case to the previous one, some things become apparent; the flow is no longer symmetrical, a reverse flow zone appears, compressibility effects become more relevant, there is the possibility of stall at the tip of the advancing side of the blade and several unsteady effects occur, but the effect of these issues can be diminished with some simplifying assumptions, as shown next. The procedure to determine blade forces begins by determining the different velocities at each element, which (in their nondimensional form) are written as follows:

\[
\frac{V_T}{\Omega R} = r + \mu \sin(\psi), \tag{2.28}
\]

\[
\frac{V_P}{\Omega R} = \lambda + \frac{r \dot{\gamma}}{\Omega} + \mu \gamma \cos(\psi), \tag{2.29}
\]

\[
\frac{V_R}{\Omega R} = \mu \cos(\psi), \tag{2.30}
\]

where $r = \nu/\nu_{\infty}$ is the dimensionless distance between the element and the rotor axis and $\dot{\gamma}$ is the flapping rate. Knowing the velocities, it is now possible to calculate the induced angle of attack $\phi$, depicted in Figure 2.1, which is equal to

\[
\phi = \arctan\left(\frac{V_P}{V_T}\right), \tag{2.31}
\]

which in turn renders out the aerodynamic angle of attack (AoA) as

\[
\alpha = \theta - \phi. \tag{2.32}
\]
This value is then used to evaluate both $C_l$ and $C_d$, and having these values one can calculate the incremental lift and drag in each section, at a given azimuthal position

$$dL = \frac{1}{2} \rho V^2_\infty c C_l \, dy \, d\psi,$$

$$dD = \frac{1}{2} \rho V^2_\infty c C_d \, dy \, d\psi.$$

(2.33)

(2.34)

where $c$ is the chord’s length. Using Figure 2.1 as a reference, solving for the vertical and horizontal forces acting on the blade renders out

$$dF_z = dL \cos(\phi) - dD \sin(\phi),$$

$$dF_x = dL \sin(\phi) + dD \cos(\phi).$$

(2.35)

(2.36)

Finally, to obtain the total values along the wing and during an entire rotation, one must integrate these values along the blade’s length and also during a complete rotation, resulting in

$$F_z = \frac{1}{2\pi} \int_0^R \int_0^{2\pi} dF_z,$$

$$= \frac{\rho}{4\pi} \int_0^R \int_0^{2\pi} V^2_\infty c (C_l \cos(\phi) - C_d \sin(\phi)) \, d\psi \, dy,$$

$$F_x = \frac{1}{2\pi} \int_0^R \int_0^{2\pi} dF_x,$$

$$= \frac{\rho}{4\pi} \int_0^R \int_0^{2\pi} V^2_\infty c (C_l \sin(\phi) + C_d \cos(\phi)) \, d\psi \, dy.$$

(2.37)

(2.38)

2.3 Hovering Flapping Wing Models

One of the main challenges in the aeronautical field is, arguably, to totally understand the mechanisms that allow birds and insects to fly so efficiently as they do. Some of these processes are already partially understood, like the clap-and-fling mechanism [13], the Leading Edge Vortex (LEV) [14], delayed stall and wake capture [5], therefore the great challenge resides in how to model them in an efficient way that does not require countless hours of processing time. Here a collection of the most promising models that describe hovering flight is presented.

2.3.1 Truong’s model

Based on the 1999 findings by Dickinson et al. [5] regarding force generation in an insect mimicking robot in a hovering state, Truong and his team developed a paper detailing an UBET based model that could approximate the average forces measured by Dickinson with an estimated error of only 5.7% [8]. Their method relies on dividing the wing in several different elements and then calculating the forces created by each section, exactly as one would do in a normal BET analysis. The difference, however, is that generated forces are divided into three components (translational, rotational and added mass), later explained in-depth. The results obtained by the model are shown in Figure 2.3, and analysing
them it becomes clear that the model can accurately describe the overall aerodynamic behaviour of the wing. The model does show some limitations, however, as one can tell looking at the peaks in force generation. Even though the model follows the overall trend of force generation, it does not reach the maximum measured drag and lift forces, and both drag peaks are shifted to the right. It is, nonetheless, a very good approximation since this is a model that can run instantly, compared to the several hours the same CFD analysis would take.

Figure 2.3: Results obtained by Truong's model, with and without rotational forces, compared to the results obtained by Dickinson. To the left, the horizontal forces, to the right the vertical ones [8].

Model Overview

First, some general aspects of the work have to be addressed. The movement of the wings, be it the translation or the rotation, was digitized from the reference graph shown in Figure 2.4.

Figure 2.4: Time history of the translational (green) and rotational (purple) velocities of the wing [5].

With these values it is possible to determine the lift and drag coefficients, again through a relationship that Dickinson developed. Based on measurements of translational force in rapidly accelerated wings from rest to a constant tip velocity of $0.25 \text{ m s}^{-1}$, a model for $C_l$ and $C_d$ was developed, whose equations are

$$C_l = 0.225 + 1.58 \sin(2.13 \alpha - 7.20),$$

$$C_d = 1.92 - 1.55 \cos(2.04 \alpha - 9.82).$$

(2.39)

(2.40)
The reference plane and general force environment used for the whole analysis is shown in Figure 2.5. It is actually very similar to Leishman’s scheme, with the difference that here there is the inclusion of a local reference frame that accompanies the movement of the wing, denoted by the letters $\xi$, $\eta$ and $\zeta$.

![Figure 2.5: Truong's wing reference scheme and terminology [8].](image)

The translational velocity $V_T$ is given by

$$V_T = \dot{\psi}r, \quad (2.41)$$

where $\dot{\psi}$ is the angular translational velocity and $r$ is the distance between the origin and the element. As for the vertical velocity $V_P$, despite being included in the formulas derived next, it is always approximated as being zero in the hovering case due to the great difference in magnitude to the translational velocity. Subsequently $V$, which is defined as $V = \sqrt{V_T^2 + V_P^2}$, equals $V_T$.

This model, as explained before, separates the generated forces into the three separate components. Therefore a comprehensive analysis of each component and the method to calculate each one is presented next.

**Translational Force**

Component of the force that would be created by a fixed wing flying at the same speed, in translation. To calculate it, one starts with the forces acting perpendicular and parallel to the local stream, defined as

$$dL = \frac{1}{2} \rho V^2 c C_l \, dr, \quad (2.42)$$

$$dD = \frac{1}{2} \rho V^2 c C_d \, dr. \quad (2.43)$$

The forces are then calculated in the inertial reference frame shown in Figure 2.5, through a combination
of momentum and blade element theory, resulting in the equations

\[
F_{T,\zeta} = \frac{1}{2} \rho \int_{0}^{R} c \left( C_l V_T - C_d V_P \right) V \, dr,
\]

(2.44)

\[
F_{T,\eta} = \frac{1}{2} \rho \int_{0}^{R} c \left( C_l V_P + C_d V_T \right) V \, dr.
\]

(2.45)

In practice, the decision to ignore the induced velocity results in a null induced AoA, meaning that the lift equals the whole vertical component of the force, while drag is the whole horizontal part. That notion renders part of Equations 2.45 and 2.44 useless, but the complete version of the equations is shown here because it will be of use later on, when adapting the model to forward flight.

**Rotational Force**

Part of the force that comes from the circulation around the wing. This is a mechanism similar to the Magnus force [5], where the wing’s own rotation drags the air around into the boundary layer, effectively serving as a circulation source. That circulation then creates a velocity differential between the sides of the wing, and where the net flow velocity is increased a low pressure zone is formed. The pressure gradient created is responsible for the rotational force, in the direction from the low pressure to the high pressure one. Rotational force is given by the expression

\[
dF_{\text{rot}} = \rho V_T d\Gamma_{\text{rot}},
\]

(2.46)

where \(d\Gamma_{\text{rot}}\) is the wing section’s rotational circulation, defined as

\[
d\Gamma_{\text{rot}} = G \dot{\theta} c^2 \, dr,
\]

(2.47)

where \(G\) is the rotational force coefficient, a parameter that is a function of the dimensionless rotational velocity \(\omega\), defined as

\[
\omega = \frac{\dot{\theta} c_{av}}{V_T}.
\]

(2.48)

where \(\dot{\theta}\) is the pitch rate and \(c_{av}\) is the average chord length. This is the truly unsteady part of the model, because as \(\omega\) varies, different \(G\) values will arise. For values of \(\omega\) between 0.166 and 0.374, the work by Dickinson and Sane [6] is used. If the value does not fit into these limits, however, another method is used, namely the equation

\[
G = \pi \left( 0.75 - x_f \right).
\]

(2.49)

where \(x_f\) is the dimensionless distance from the leading edge to the rotation center. Having performed all these calculations, one is finally in the position to be able to calculate the rotational force. Divided
into its local vertical and horizontal parts, it becomes

\[ F_{\text{rot},\zeta} = \int_0^R dF_{\text{rot}} \cos(\theta) , \]  
\[ F_{\text{rot},\eta} = \int_0^R dF_{\text{rot}} \sin(\theta) . \]  

### Added Mass Force

Force component that appears as a reaction to pushing and rotating the air around the wing during the stroke movement. As the wing exerts a force on the air to accelerate it, the surrounding air acts on the wing in the opposite direction, thus creating the force that here is referred to as added mass force. Calculating this value in a BET model is complicated because it requires making an assumption regarding the mass of air that is affected by the wing’s translational and rotational movement. The authors chose to follow the most common approach, where every section of length \( c \) is considered to affect a cylinder of fluid of the same diameter around itself. In practice this is the same as considering that the whole wing is constantly affecting a cylinder of air around it, that at every point has a diameter equal to the chord. Its expression is

\[ dF_A = \frac{\pi}{4} \rho c^2 a_\perp dr , \]  

where \( a_\perp \) is the component of the acceleration normal to the wing. Projecting this force into the same reference frame as before results in

\[ F_{A,\zeta} = \int_0^R dF_A \cos(\theta) , \]  
\[ F_{A,\eta} = \int_0^R dF_A \sin(\theta) . \]  

### Total Forces

Finally, to obtain the total vertical and horizontal forces in the \( xyz \) reference frame, the following sum and transformation is necessary

\[ F_z = F_{T,\zeta} + F_{\text{rot},\zeta} + F_{A,\zeta} , \]  
\[ F_y = (F_{T,\eta} + F_{\text{rot},\eta} + F_{A,\eta}) \cos(\psi) , \]  

which corresponds to the instantaneous forces acting on the wing, both in the vertical and horizontal direction, completing the calculations.

### Inertial Forces

One aspect which is not directly related to the work later developed in this thesis is the inertial forces acting on the wing. The model considers the wing movement to be in 3D, and, as a by-product of the algebra it performs, it deduces the equations that describe the inertial loads that the wings must
withstand. Even though the model does not later employ the inertial forces, it is an important aspect of a wing analysis and is therefore shown here. Starting with the accelerations of a generic particle in the x, y and z directions

\[\ddot{x} = -\left[r\ddot{\psi} + x_m\dot{\theta}\sin(\theta) + x_m\dot{\theta}^2 \cos(\theta) + x_m\ddot{\psi}\cos(\theta)\right] \sin(\psi)\]
\[\ddot{y} = +\left[r\ddot{\psi} + x_m\dot{\theta}\sin(\theta) + x_m\dot{\theta}^2 \cos(\theta) + x_m\ddot{\psi}\cos(\theta)\right] \cos(\psi)\]
\[\ddot{z} = +\left[\dot{\theta}^2 \sin(\theta) - \ddot{\theta} \cos(\theta)\right] x_m .\]  

where \(x_m\) is the distance from the axis to the point under analysis. To obtain the inertial forces one must then integrate the acceleration values of every particle on the wing, both along its depth, span and chord, resulting in

\[F_{Ix} = \int_0^R \int_0^{LE} \int_0^h \rho_w \ddot{x} dh dc dr,\]  
\[F_{Iy} = \int_0^R \int_0^{LE} \int_0^h \rho_w \ddot{y} dh dc dr,\]  
\[F_{Iz} = \int_0^R \int_0^{LE} \int_0^h \rho_w \ddot{z} dh dc dr,\]  

where \(\rho_w\) is the density of the wing and \(h\) is its thickness, while LE and TE are the leading and trailing edge of the wing, respectively.

### 2.3.2 Walker’s model

In his 2002 paper [7], Walker focuses on the importance of rotational lift for hovering flapping flight. His work appears in the wake of Dickinson’s study [5], like several articles of the same period on the topic of flapping flight. The main objective was to find just how essential rotational lift is for the hovering aerodynamics of insects and birds, and if there was any truth to the thesis put forward by Dickinson that the force peaks were in some way caused by the interaction between the wing and its previously shed wake. In other words, were the forces being generated by something not yet thought about, like the Magnus force, or was it just another example of the already known mechanisms at the time, namely the circulatory-and-attached-vortex force?

The results of this work, compared to those measured by Dickinson, are depicted in Figure 2.6. They show that the hypothesis of being the Magnus force the responsible for the peaks observed at the change of stroke direction is likely wrong, since the inclusion of this force makes the model overshoot its prediction of the values. Not including the Magnus force, however, leads to a model that predicts the forces fairly well, similarly to what Truong was able to achieve.
Figure 2.6: Forces obtained by Walker with his model (red) compared to the values measured by Dickinson (blue). For the lift, a model including the Magnus force (green) was also included [7].

Model Overview

To test the hypothesis he proposed, Walker developed an UBET model that aimed to recreate the *Drosophila* robot tested by Dickinson, to determine the origin of the forces being generated. It models the theoretical force as

\[
F = \rho V_\infty (\Gamma_T + \Gamma_H + \Gamma_{\text{rot}} + \Gamma_M),
\]

where \( \rho \) is the fluid density, \( V_\infty \) is the free stream velocity and \( \Gamma_T, \Gamma_H, \Gamma_{\text{rot}}, \Gamma_M \) are the translational, heaving, rotational and Magnus circulation, respectively. All of these components of circulation arise from the respective kinematic changes that the wing undergoes, namely translation, heaving and rotation, that in turn generate a difference in velocities leading to an induced AoA that differs from the geometrical angle of attack. The Magnus force, however, is different because it is independent of this induced angle of attack. The author argues that the forces Dickinson calls “Magnus-like”, are, in reality, just a result of the change in the angle of attack and therefore he follows through with the analysis defining \( \Gamma_T, \Gamma_H, \Gamma_{\text{rot}} \) as the circulatory-and-attached-vortex force while \( \Gamma_M \) is considered to be the Magnus force in its stricter sense.

The model itself relies on the scheme shown in Figure 2.7, where the parameters \( \alpha, \theta \) and \( \alpha_s \), later used in the calculations, can be seen. The parameter \( \alpha \) is defined as being the angle between the chord and the vertical, \( \theta = \pi/2 - \alpha \) is the angle between the chord and the horizontal (please note that, having \( \theta, \alpha \) becomes redundant, but it is still used to simplify the calculations) and \( \alpha_s \) is the incidence angle, or the angle between the chord and the resulting speed generated by the movements of the wing, namely translation and rotation. Not only that, it can also be seen how the different dimensionless distances in the wing are referred to: \( x_f \) is the distance between the leading edge and the center of rotation of the wing, while \( x_{\text{ref}} \) quantifies the distance between the leading edge and the reference point of contact with the incident flow.
The next step of the algorithm is to define the velocities needed for the calculations. In this case, these are the normal and parallel velocities to the wing, referred to as $V_\perp$ and $V_\parallel$, respectively. They are defined as

$$V_\perp = V_T \cos(\alpha) + (x_{ref} - x_f) c \dot{\alpha},$$

$$V_\parallel = V_T \sin(\alpha),$$

where $V_T$ is the translational velocity of the element and $\dot{\alpha}$ is the rate of rotation of the profile. The total velocity $V$ is then defined as $V = \sqrt{V_\perp^2 + V_\parallel^2}$. From these values the angle of incidence can also be evaluated, which is defined as

$$\alpha_s = \arctan \left( \frac{V_\perp}{V_\parallel} \right).$$

Another quantity that is important to define is the Wagner function, $\Phi(\tau)$. This factor is the element in the analysis that accounts for the interaction between the wing and the shed vortex at the beginning of every stroke, and owes its existence to the fact that circulation does not establish itself immediately around the wing as it rotates or moves around [15, p.157]. This coefficient is more pronounced the bigger the rate of change in the wing’s movement. In the words of the author, this is the element that effectively turns the model into an unsteady one. It is defined as follows

$$\Phi(\tau) = \begin{cases} 0, & \tau \leq 0 \\ \frac{\tau + 2}{\tau + 4}, & \tau > 0 \end{cases},$$

where $\tau$ is the dimensionless time, defined as the time $t$ divided by the time it takes the flow to cover half
a chord of the aerodynamic surface, or

\[ \tau = \frac{2Vt}{c}. \]  

(2.68)

At this point there is the need to define the lift and drag coefficients that will be used. Similarly to Truong, Walker uses the values defined by Dickinson (see equations 2.39 and 2.40). The model then describes the forces it predicts will be acting on the wing, which fall into one of two categories: circulatory-and-attached-vortex force (or rotational, for short) and added mass force, both explained in detail next.

**Circulatory-and-attached-vortex**

Force that results from the circulation around the wing, while also accounting for the influence of the attached vortex through the Wagner function. Its calculation requires several steps, starting with the definition of the infinitesimal lift and drag forces, respectively

\[ dL_s = \frac{1}{2} V^2 c \Phi C_l \, dr , \]  

(2.69)

\[ dD_s = \frac{1}{2} V^2 c \Phi C_d \, dr. \]  

(2.70)

It is now necessary to transform these values into forces parallel and normal to the wing chord, namely

\[ dF_{\perp} = dL_s \cos(\alpha_s) + dD_s \sin(\alpha_s), \]  

(2.71)

\[ dF_{\parallel} = dL_s \sin(\alpha_s) - dD_s \cos(\alpha_s). \]  

(2.72)

The final step is to project these forces into their vertical and horizontal components, in order to obtain the total lift and drag

\[ dL_{\Gamma} = dF_{\perp} \cos(\theta) + dF_{\parallel} \sin(\theta), \]  

(2.73)

\[ dD_{\Gamma} = dF_{\perp} \sin(\theta) + dF_{\parallel} \cos(\theta). \]  

(2.74)

**Added Mass**

The explanation of this force has already been given in the previous model (see subsection 2.3.1) but its formula changes slightly. Here it is defined as

\[ dF_A = \frac{\pi}{4} \rho c^2 a_{\perp} \beta \, dr, \]  

(2.75)

so there is the introduction of the term \( \beta \), the added mass coefficient of the wing section. Having this force, the only necessary procedure is to transform it into a vertical (lift) and horizontal (drag) force

\[ dL_A = dF_A \cos(\theta), \]  

(2.76)

\[ dD_A = dF_A \sin(\theta). \]  

(2.77)
Total Forces

Finally, the total instantaneous drag and lift forces are obtained through an integration along the span of the wing

\[
L = \int_{0}^{R} (dL_A + dL_T), \quad (2.78)
\]

\[
D = \int_{0}^{R} (dD_A + dD_T). \quad (2.79)
\]
Chapter 3

Preliminary Power Consumption Analysis

This section aims to perform a preliminary, simple comparison between the three flying configurations under scrutiny, based solely on the formulas derived in subsection 2.1.1 and some assumptions about the vehicles’ performances. A parametric study is also introduced, to analyse how every parameter affects each configuration and its performance results. The objective is to later combine these results with those of a more refined analysis, to achieve a definitive comparison between configurations.

3.1 Fixed vs. Rotary

At first glance, this comparison might seem unnecessary. These are the most common approaches to powered flight and the advantages and drawbacks of each configuration are very well known. If one wants a vehicle capable of flying long distances and achieve higher speeds, fixed wings are the solution. If, however, the mission requires the vehicle to hover, then the answer is the rotary wing. It is also fairly obvious that a helicopter has a higher power consumption than a plane, but it is surprisingly hard to find studies that quantify this difference in a non-subjective way. This, as the author sees it, opens up the possibility of performing a head-to-head comparison that gives the user hard data about power consumption in terms of several parameters such as speed, weight and all other possible variables. Another aspect should also be taken into account; the technology miniaturization currently under way leads to the appearance of aerodynamic effects that change the so-called “normal” behaviour of a vehicle. Therefore, some certainties that seem obvious about the different configurations’ performances in medium and big-sized devices become more fragile when talking about MAVs, justifying the more in-depth look taken here at this unusual comparison.

This process starts by looking at the expressions that describe each configuration’s power expendi-
Recalling equations 2.3 and 2.14, respectively

\[ P_{\text{fix}} = \frac{\rho}{2\eta_{\text{prop}}} A_w V_\infty^3 \left( C_{D0} + \frac{C_T^2}{\pi A e} \right), \]

\[ P_h = \rho A_h V_{\text{tip}}^3 \left[ \frac{\kappa C_T^2}{2 \sqrt{\mu^2 + \lambda^2}} + \frac{\sigma C_{D0}}{8} \left(1 + K \mu^2\right) \right], \]

the similarities become apparent, like the dependence of the respective areas, the density of the fluid and the presence of some of the same parameters in both equations. This indicates that some simplifications could be assumed in order to allow for a meaningful comparison. The first one is that the wingspan of the fixed wing is equal to the diameter of the rotary wing’s rotor, or

\[ b = 2R, \quad (3.1) \]

Secondly, the weights of both configurations will be considered to be the same, meaning that the thrust of the rotary wing equals the lift of the airplane. With this in mind, it is possible to manipulate the expression in order to eliminate the variable \( C_L \) through the following procedure

\[ L = T \quad \iff \quad 1/2 \rho A_w V_\infty^2 C_L^2 = \rho A_h V_{\text{tip}}^2 C_T \quad \iff \quad C_L = \frac{2A_h}{A_w A^2} C_T. \quad (3.2) \]

in which the advance ratio \((\mu = V_\infty / V_{\text{tip}})\) definition was used to relate the speeds of both vehicles. Resorting to Equation 3.1, to the aspect ratio relation \((A_w = b^2 / R)\) and remembering that \(A_h = \pi R^2\), Equation 3.2 can be further simplified to read

\[ C_L = \frac{\pi A}{2\mu^2 \kappa} C_T. \quad (3.3) \]

It can also be assumed that the same airfoil is used in both configurations, leading to them both having the same value of profile drag coefficient \((C_{D0,\text{fix}} = C_{D0,h} = C_{D0})\). To further decrease the number of variables of this analysis, one might resort to the use of the \( C_T \) value that, according to Leishman [11, p.80], maximizes the power loading, a parameter that relates the thrust produced with the power required to generate it. Said value of \( C_T \) is given by the relation

\[ C_T = \frac{1}{2} \left( \frac{\sigma C_{D0}}{\kappa} \right)^{2/3}. \quad (3.4) \]

where \( \kappa \) is an empirical correction factor for profile power. Finally, introducing these considerations into
a power ratio equation results in

\[
\frac{P_h}{P_{fix}} = \frac{\rho A_b V_{tip}^3}{\frac{\rho}{2\eta_{prop}} A_w V_\infty^3} \left[ \frac{\kappa C_T^2}{2\mu^2 + \lambda^2} + \frac{\sigma C_{D0,b}}{8} (1 + K\mu^2) \right]
\]

\[
= \frac{\eta_{prop} \pi \rho A_w V_\infty^3}{16\mu^3} \frac{\kappa}{\sqrt{\mu^2 + \lambda^2}} \left[ \frac{2C_{D0}}{\pi \kappa} \right]^{4/3} + \sigma C_{D0} (1 + K\mu^2),
\]

(3.5)

an equation that depends on the following parameters:

- \(\eta_{prop}\) - efficiency of the fixed wing’s propeller;
- \(A\) - aspect ratio of the fixed wing;
- \(\kappa\) - helicopter’s empirical correction factor for induced power;
- \(\mu\) - advance ratio, here relating the speeds of both devices;
- \(\alpha\) - rotor angle of attack, influences the value \(\lambda\) (see section 2.1.2);
- \(N_b\) - number of blades, part of the rotor solidity \(\sigma\);
- \(A_{rb}\) - aspect ratio of the rotary wing’s blades, part of the rotor solidity \(\sigma\);
- \(C_{D0}\) - profile drag coefficient of both the fixed wing and the rotary wing blade;
- \(K\) - helicopter’s empirical correction factor for profile power;
- \(e\) - Oswald’s efficiency coefficient.

The expression also indirectly depends on the tip speed of the blades, \(V_{tip}\), through the advance ratio. So even though this value does not have a direct influence on the analysis, to know the forward speed correspondent to each \(\mu\), \(V_{tip}\) must be available, therefore demanding a method for its calculation to be explained. The approach chosen for this work was to associate the tip speed to the thrust coefficient for maximum power loading, even though this means assuming a mass for the device and therefore introducing yet another variable. Starting from Equation 2.6, it can be defined as

\[
V_{tip} = \sqrt{\frac{mg}{\rho A_w C_T}}.
\]

(3.6)

At this point Equation 3.5 can be manipulated, and some useful information can be extracted by leaving two of the parameters as variables while assuming typical values for the rest of them. The required manipulation is to equal said expression to 1, therefore indicating the points where the power consumption of both vehicles is the same. If then one variable is isolated and chosen as the dependent one, and another variable is left free to vary, it will be possible to plot a 2D graph of the their variation with one another. Please note that the choice of these two variables is completely up to the person performing the analysis, in this work the propeller efficiency \(\eta_{prop}\) was chosen because that was the approach adopted by Sachs [4], and \(\mu\) was selected as the varying parameter because speed is probably the most relevant variable in this analysis, while also being an intuitive concept to perceive as variant.
Performing this step results in

\[
\frac{P_h}{P_{fix}} = \frac{\eta_{prop} \pi R^2}{16 \mu^3} \frac{D}{\sqrt{\mu^2 + \lambda^2}} \left(\frac{\sigma C_{D0}}{\kappa}\right)^{4/3} + \sigma C_{D0} \left(1 + K \mu^2\right) = 1 \Leftrightarrow (3.7)
\]

\[
\Leftrightarrow \eta_{prop} = \frac{16 \mu^3}{\pi R^2} \frac{D}{\sqrt{\mu^2 + \lambda^2}} \left(\frac{\sigma C_{D0}}{\kappa}\right)^{4/3} + \sigma C_{D0} \left(1 + K \mu^2\right),
\]

meaning that there is the possibility of assuming values for every parameter aside from \(\eta_{prop}\) and \(\mu\), and the curve that appears as a result separates the zones where each configuration has a lower power consumption, making them preferable. The standard values are the following:

- \(A = 3\) - typical \(A\) value of a MAV, Davis et al. [16];
- \(\kappa = 1.75\) - measured value for very small helicopters, Leishman [11, p.336];
- \(\alpha = 2°\) - even though it changes with increasing speed, assuming a small value is acceptable because the variation is very subtle;
- \(N_b = 2\) - small helicopters tend to have the least number of blades possible, because of the very high values of profile drag, Leishman [11, p.335];
- \(A_b = 4\) - value derived from the typical value \(\sigma = 0.167\), Leishman [11, p.336];
- \(C_{D0} = 0.035\) - typical MAV value, Leishman [11, p.336];
- \(K = 4.6\) - helicopter’s typical value Leishman [11, p.219];
- \(e = 0.8\) - typical Oswald’s efficiency value.

With these values the curve shown in Figure 3.1 can be plotted

![Figure 3.1: Plot of the curve that divides the areas where the fixed wing is preferable (above the curve) from the area where the rotary wing is more efficient (below the line).](image-url)
Now for the extraction of the information that this curve conceals; firstly, the main thing to retain is that the curve separates the area where rotary wings are preferable (below the curve) from the area where the fixed wing configuration is more efficient (above the curve). This is due to the fact that, since the equation was built from equalling a power consumption ratio to $1$, the line indicates the points where the power expenditure is the same for both devices. So the idea of this graph is to represent how, for a given advance ratio (which corresponds to a forward speed through the relation $V_\infty = \mu V_{tip}$), the propeller efficiency of the fixed wing configuration has to equal at least a given value for the fixed wing to consume less power. So, for example, if both configurations are flying at a speed that corresponds to $\mu = 0.25$ (which means, assuming a mass for both vehicles of 50 g, a forward speed of 8.58 m/s), the propeller needs to have an efficiency of at least $\eta_{prop} = 0.47$ for the fixed wing to be more efficient than the rotary wing. One intuitive way to understand why the area above the curve is the zone where the fixed wing is more efficient is to start by isolating any constant advance ratio line. Since along this line only propeller efficiency changes, that means the part of the graph above the curve represents the zones with higher propeller efficiency, therefore favouring the fixed wing configuration. This kind of analysis, albeit not very usual, is arguably the best way to compare configurations that have very distinct ways to achieve flight, because it is as generic as possible and especially because it lets the user choose the variables and find the critical efficiency values for several parameters.

Having established the foundations for the analysis, the study can be taken one step further by seeing how every parameter influences the behaviour of this curve. To do this, several curves are calculated sequentially where only one parameter changes, and this procedure is repeated for every parameter. Such an analysis led to the results shown in Figure 3.2. Observing Figure 3.2(d) it becomes apparent that the rotor’s angle of attack barely has an impact on the rotary wing performance, as all the curves are very similar. This validates the idea that said parameter can be assumed to be small and constant throughout all different flight speeds, even though in reality this angle increases with the forward velocity. Increasing the aspect ratio of the fixed wing resulted, perhaps unsurprisingly, in a larger area where the fixed wing solution is better. As seen in Figure 3.2(a), for larger speeds this analysis shows that the difference becomes extremely relevant, with an $A = 10$ requiring just a propeller efficiency of $\eta_{prop} = 0.2$ to be more efficient than a rotary wing, while a contraption of $A = 2$ would need an efficiency of at least 0.6 to be superior to its rotary wing counterpart. Regarding the profile drag coefficient, analysed in Figure 3.2(b) the difference is notorious when speaking about lower flight speeds. The bigger the $C_{D0}$, the smaller the area where the fixed wing is better. Especially when approaching the fixed wing’s stall speed, it can be seen that increasing the $C_{D0}$ shifts the curve upwards, and for really low speeds the $\eta_{prop, crit}$ is roughly 50% bigger for a $C_{D0}$ of 0.05 than for a coefficient five times smaller. For higher speeds this difference becomes increasingly irrelevant, as the lift induced drag becomes the main source of drag for both vehicles. The number of blades is an interesting case: looking at Figure 3.2(e) it can be seen that, while having fewer blades proves to be penalizing at lower speeds, with the increase in velocity it actually expands the area where the rotary wing is better. Increasing the number of blades has the result of shifting the curve to the right, making the rotary wing more efficient at low speeds but leading it to perform worse when going faster.
Finally, Figure 3.2(f) shows that blades with a higher aspect ratio, even though performing worse at lower speeds, actually make the rotary wing so much more efficient at higher speeds that in several cases this configuration is more efficient than the fixed wing no matter its propeller efficiency.
3.2 Fixed vs. Flapping

The same kind of analysis can be applied to compare a fixed wing against a flapping wing solution. This part of the work is heavily influenced by Sachs [4], even though here the study is taken several steps further. Following the same logic as in section 3.1, recalling the equations that define the power consumption of both the fixed and flapping wing, respectively Equations 2.3 and 2.27

\[
P_{\text{fix}} = \frac{\rho}{2\eta_{\text{prop}}} A_w V_{\infty}^3 \left( C_{D0} + \frac{C_{L}^2}{\pi Re} \right),
\]

\[
P_{\text{flap}} = \frac{\rho}{2} A_w V_{\infty}^3 \gamma_{av}^{-3/2} \left( C_{D0} + \frac{C_{L}^2 \gamma_{av}^2}{\pi Re_{\text{flap}}} \right).
\]

The equations are noticeably similar, since the configurations share so many traits. The only differences are the presence of the propeller efficiency \(\eta_{\text{prop}}\) in the fixed wing equation, along with the modified Oswald coefficient \(e_{\text{flap}}\) and the average flapping angle \(\gamma_{av}\) in the flapping one. These equations are similar enough not to require any further manipulation when calculating the ratio, unlike the previous case. So, dividing one of the equations by the other, equalling the ratio to 1 and isolating the variable \(\eta_{\text{prop}}\), results in

\[
\frac{P_{\text{fix}}}{P_{\text{flap}}} = \frac{\frac{\rho}{2\eta_{\text{prop}}} A_w V_{\infty}^3 \left( C_{L}^2 + C_{D0} \right)}{\frac{\rho}{2} A_w V_{\infty}^3 \gamma_{av}^{-3/2} \left( C_{L}^2 + \frac{C_{D0}^2 \gamma_{av}^2}{\pi Re_{\text{flap}}} \right)} = 1 \Leftrightarrow
\]

\[
\eta_{\text{prop}} = \frac{\gamma_{av}^{3/2} \left( C_{L}^2 + C_{D0} \pi Re \right)}{C_{L}^2 \left( 1 + \Delta^2 \right) + C_{D0} \pi Re} \Leftrightarrow \eta_{\text{prop}} = \frac{\gamma_{av}^{3/2} \left( C_{L}^2 + C_{D0} \pi Re \right)}{C_{L}^2 \left( 1 + \Delta^2 \right) + C_{D0} \pi Re}.
\]

an equation that depends on the following factors:

- \(\gamma_{\text{max}}\) - maximum flapping angle, parameter that defines \(\gamma_{av}\) through Equation 2.21. The standard value used for the analyses is \(\gamma_{\text{max}} = 60^\circ\), since it the value used by Sachs [4];
- \(\rho\) - fluid density. Standard value is the Standard Sea Level (SSL) air density, \(\rho = 1.224 \text{ kg m}^{-3}\);
- \(m\) - wing’s mass. Standard value is \(m = 50 \text{ g}\);
- \(AR\) - wing aspect ratio. Standard value is \(AR = 3\), like in the the previous case [16];
- \(b\) - wingspan. Standard value is \(b = 20 \text{ cm}\);
- \(C_{D0}\) - profile drag coefficient of both the configuration’s wings. Standard value is \(C_{D0} = 0.035\);
- \(e\) - Oswald efficiency coefficient. Standard value is \(e = 0.8\);
- \(\Delta\) - nondimensional factor that quantifies the changes in the value of the lift vector during the course of the flapping cycle. The standard value is the one used by Sachs [4], \(\Delta = 1\).

Running the simulation with just the standard values and not varying any parameter, the curve shown in Figure 3.3 is obtained, where the area above the curve represents the zone where the fixed wing is
more efficient, while the area below illustrates the values for which the flapping wing has the edge. The curve has some interesting aspects to it, namely the values that it tends to for both very small and very high velocities, which can be related to the way $C_L$ is defined. Since the lift coefficient depends on the speed squared, it is very sensible to any change to this variable, and for the most extreme values the behaviour of the curve is determined by the parameters $\gamma_{\text{max}}$ and $\Delta$ because they do not directly relate to the speed but their effect can be felt all throughout the speed span.

![Figure 3.3: Plot of the curve that divides the area where the fixed wing is preferable (above the curve) from the region where the flapping wing consumes less power (below the curve).](image)

Figure 3.3: Plot of the curve that divides the area where the fixed wing is preferable (above the curve) from the region where the flapping wing consumes less power (below the curve).

If the parameters are now varied, some conclusions can be withdrawn about the behaviour of each configuration and how their performances compare. So, fixing every parameter aside from the one under scrutiny and repeating the analysis for all the variables gives rise to the results shown in Figure 3.4. Now for the analysis of the results, to learn about the aerodynamic behaviour of the configurations and to infer which are the aspects that most heavily affect their performances: beginning with the flapping angle, shown in Figure 3.4(a), it becomes clear that this is one of the factors that most heavily determines if the flapping configuration is more efficient than its fixed wing counterpart. For small velocities the difference is already appreciable, and the difference between curves only tends to grow with the increase in speed. It can also be seen that an increase in the flapping angle means a worse performance overall for the flapping wing configuration, as the curve is shifted downwards (diminishing the area for which the flapping wing is preferential). This is due to the fact that a greater flapping angle means that a higher percentage of the total lift generated will not be vertical. The aspect ratio shows not to play a great role in distinguishing the performance of both vehicles, and the same goes for the profile drag coefficient, as seen in Figures 3.4(b) and 3.4(c), respectively. When talking about the mid-range velocities, increasing the aspect ratio benefits the fixed wing and increasing $C_{D0}$ benefits the flapping wing, but for both small and high values of velocity these parameters become irrelevant as the curves all collapse into the same line. Interestingly, looking at Figure 3.4(d) it can be seen that increasing the
wingspan benefits the flapping wing for the most part, as the curve is shifted upwards sooner than for higher values of \( b \), while increasing the mass has the opposite effect, as shown in Figure 3.4(f).

The parameter \( \Delta \), shown in 3.4(e), requires a more complex analysis. It can be observed that the case of \( \Delta = 0 \) is the ideal scenario where the Oswald coefficient would not change for the flapping case, meaning that there would be no loss of efficiency due to the movement of the wings. Since this is impossible and given that an observation of the graph yields that a slight change to this parameter has
a severe impact on the flapping wing efficiency, this variable gains another relevance. For small speeds (around 5 m/s), for example, while a $\Delta = 0.5$ means that the fixed wing would only be more efficient with a propeller of $\eta_{prop} = 0.61$, a value of $\Delta = 2$ means that its enough for the fixed wing to have a propeller of $\eta_{prop} = 0.15$ for it to be more efficient, a very significant difference. For higher values of speed, however, these differences lose importance as all the curves tend to the $\Delta = 0$ efficiency value.

### 3.3 Rotary vs. Flapping

This is arguably the most interesting comparison of the chapter. The reality is that these are the two most promising technologies in terms of technology miniaturization, and they share most of their capabilities. Both can be made to hover and fly forward, for example, and both can be used for the same type of surveillance missions. Furthermore, on a small scale, it is not immediately obvious which configuration is going to perform better in different circumstances and what factors are going to influence such behaviours, all factors that contribute to the added value of this analysis.

Recalling the equations for rotary wing power consumption, Equation 2.14, and flapping wing power expenditure, Equation 2.27

$$P_h = \rho A_h V_{tip}^3 \left[ \frac{\kappa C_T^2}{2\sqrt{\mu^2 + \lambda^2}} + \frac{\sigma C_D0}{8} (1 + K\mu^2) \right],$$

$$P_{flap} = \frac{2}{3} A_w V_{\infty}^{-3/2} \gamma_{av} \left( C_{D0} + \frac{C_T^2}{\pi Re_{flap}} \right),$$

it becomes clear that this is going to be the most complicated comparison, due to the amount of different variables arising from the completely different paradigms of flight that each configuration represents. Again, calculating the ratio of power consumptions and resorting to the already defined relations shown in Equations 3.1, 3.2 and 3.4, the following manipulation can be performed

$$\frac{P_h}{P_{flap}} = \frac{\rho A_h V_{tip}^3}{\frac{2}{3} A_w V_{\infty}^{-3/2} \gamma_{av}} \left[ \frac{\kappa C_T^2}{2\sqrt{\mu^2 + \lambda^2}} + \frac{\sigma C_D0}{8} (1 + K\mu^2) \right] \left( C_{D0} + \frac{C_T^2}{\pi Re_{flap}} \right)$$

$$= \frac{2 A_h}{A_w} \gamma_{av}^{-3/2} \left( \frac{\kappa C_T^2}{2\sqrt{\mu^2 + \lambda^2}} + \frac{\sigma C_D0}{8} (1 + K\mu^2) \right) \left( C_{D0} + \frac{C_T^2}{\pi Re_{flap}} \right)$$

$$= \frac{\pi Re_{flap}}{4 A_h^3} \left( \frac{\kappa C_T^2}{2\sqrt{\mu^2 + \lambda^2}} + \frac{\sigma C_D0}{8} (1 + K\mu^2) \right) \left( C_{D0} + \frac{C_T^2}{\pi Re_{flap}(1 + \Delta^2)} \right).$$

(3.10)

Keep in mind that both $C_T(\sigma, \kappa, C_{D0})$ and $\sigma(N_b, Re)$ have further dependencies on the variables shown in parenthesis, relations that are not shown on the equation simply for readability issues. The parameters varying here are exactly the same as the ones from the previous cases, only now they are all together and the only variable specific of the fixed wing configuration, $\eta_{prop}$, is not present. This forces the analysis to choose any other variable to isolate as the dependent parameter. The one chosen
here was $\Delta$, the factor that quantifies the changes in the value of the lift vector during the course of the flapping cycle, because this is the most difficult quantity to calculate and the analysis will be able to output its value, so any later estimation will have a reference value to be compared to. With this in mind, the performed algebraic manipulation starts by equalling Equation 3.10 to 1, followed by isolating the $\Delta$ term

$$1 = \frac{\pi A R^{3/2}}{4\mu^3} \left( \frac{\kappa C_L^2}{\sqrt{\mu^2 + \lambda^2}} + \frac{C_{D0}}{4} \right)$$

$$\Leftrightarrow \frac{C_L^2}{\pi A R (1 + \Delta^2)} + C_{D0} = \frac{\pi A R^{3/2}}{4\mu^3} \left[ \frac{\kappa C_L^2}{\sqrt{\mu^2 + \lambda^2}} + \frac{C_{D0}}{4} (1 + K \mu^2) \right] \Leftrightarrow$$

$$\Leftrightarrow \Delta = \sqrt{\frac{4\epsilon \mu^4}{\pi A R C_T^2}} \left[ \frac{\pi A R^{3/2}}{4\mu^3} \left[ \frac{\kappa C_L^2}{\sqrt{\mu^2 + \lambda^2}} + \frac{C_{D0}}{4} (1 + K \mu^2) \right] - C_{D0} \right] - 1,$$

(3.11)

The parameters present in this section are all stated in section 3.1 and section 3.2, as well as the standard values used throughout the analysis, due to the fact that there are no new terms to this expression. This equation, when plotted as a function of the advance ratio $\mu$ and using the standard values previously stated, results in the curve shown in Figure 3.5.

Figure 3.5: Curve that divides the area where the rotary wing is preferable (above the curve) from the area where the flapping wing consumes less power (below the curve).

Several conclusions can be made by looking at the curve. Firstly, for very high advance ratios (since in this case $V_{tip} = 56.14 \text{ m/s}$, this means forward speeds of approximately $18 \text{ m/s}$) there is no case where the flapping wing is more efficient than the rotary wing. For moderate speeds, however, there is a large zone where the flapping wing is preferable, meaning that for speeds until $18 \text{ m/s}$ the flapping wing is less power demanding, if during the flapping cycle the $C_L$ curve does not deviate more than around $100\%$ on average. Finally, it becomes clear that if the lift deviation is significant, there is no speed for
which the flapping wing will be more efficient, immediately excluding almost every case with a very high flapping angle $\gamma_{\text{max}}$. Following the same procedure as in both previous analyses, every factor present in Equation 3.11 was varied to see the influence of every parameter on the overall performance of both configurations. Doing this produced the results visible in Figure 3.6.

Figure 3.6: Flapping vs. rotary wing parametric study.

Some of the conclusions here present are the same ones that other analyses have already yielded; the rotor's angle of attack, shown in Figure 3.6(f), practically does not make a difference in terms of
the overall performance. The aspect ratio of the flapping wing, analysed in Figure 3.6(b), plays a very significant role in its performance as small variations to this value mean completely different results, something that can be concluded by looking at some specific cases: for an $\mathcal{R} = 2$, after $\mu = 0.27$ ($V_\infty \approx 15$ m/s), there is no case whatsoever where the flapping wing is more efficient. For an $\mathcal{R} = 10$, however, there is practically no case where the rotary wing is a better option in terms of power consumption, which goes to show the influence of this parameter.

The flapping angle, seen in Figure 3.6(a) also shows some influence on the performance of the flapping wing, as it can be observed that its increase reflects itself on a decrease of the favourable area of the flapping wing. However, this growth is only more pronounced for bigger flapping angles, as small angles produce almost identical results. This is in accordance with what one might predict, since the same increment in the flapping angle does not have an equal impact on the tilting of the lift vector for small angles (see lines defined by $\gamma = 0^\circ$, the fixed wing case, and $\gamma = 15^\circ$, where the difference is barely noticeable) as it has when the angles are already close to the vertical (see lines defined by $\gamma = 75^\circ$ and $\gamma = 90^\circ$, where the same increase of $15^\circ$ resulted in a much smaller area where flapping is favourable). This result has, however, to be thought about sceptically, because a wing that has an aperture of $90^\circ$ will hit the other wing both at the beginning and end of the stroke, giving rise to aerodynamic effects like the clap-and fling mechanism that improve its performance, but are not accounted for in this simple model.

The rotary wing’s number of blades, when increased, shows to worsen the performance of the configuration, observation already made in the previous analysis involving rotary wings and observed here again in Figure 3.6(e). The profile drag coefficient, visible in Figure 3.6(c), shows a relatively small impact on the power consumption; a smaller value of $C_{D0}$ appears to be advantageous for the flapping wing option, but for higher velocities this value becomes irrelevant as all the curves collapse into the same one. This is also explainable, as the profile drag is constant and therefore more pronounced at smaller speeds, where the lift induced drag is not yet relevant. Increasing the speed results in a higher percentage of drag being attributable to the lift generated vorticity, to the point where the profile drag becomes irrelevant (at $\mu \approx 0.32$, or $V_\infty \approx 18$ m/s). The aspect ratio of the blade, $\mathcal{R}_b$, has the same effect as the aspect ratio in a normal wing. An increase in this value (meaning a more elongated blade) results in a better performance overall by the rotary wing. As seen in 3.6(d), changing this value drastically changes the areas where each configuration is favourable, as one can tell by looking at the curves of $\mathcal{R}_b = 3$ and $\mathcal{R}_b = 15$, with the latter showing a curve where the rotary wing is almost always preferable, contrary to the first case.
Chapter 4

Development of an UBET Model for Flapping Wing Forward Flight

In order to assess if the results of the previous chapter are valid, a method to study flapping forward flight has to be developed. Like previously stated, a BET based analysis is commonplace when it comes to rotary wings, but the case of flapping wings is not yet deeply understood nor developed. This chapter aims to develop exactly that, a tool that mimics the conditions of a flapping wing when flying forward, resorting to the Blade Element Theory and to unsteady aerodynamics concepts.

It starts by implementing the models for hovering flapping flight developed by Truong and Walker, discussed in subsection 2.3.1 and 2.3.2, and then modifying them to accommodate for the changes that forward flight demands. As for the validation, all models are put to the test through a comparison with a CFD analysis of a 2D airfoil performing simultaneously a plunging and a rotating movement, as studied by Choi et al. [9]. Finally, a combination of the two models is created, since this proves to achieve better results than each of the individual modified models.

4.1 Hovering Models

Due to the non-existence of forward flight UBET models for flapping flight, it is necessary to adapt the currently existing models to handle that specific task. The most similar case is hovering, a case that has been studied far more than the topic at hand and for which exist several options in terms of aerodynamic models. Two of these solutions were explained before, and in this section those same hovering models are implemented as described in their respective papers, in order to test and validate them. This is simply the first step, as later these solutions will be modified to represent forward flight accurately.

4.1.1 Truong’s Hovering Model

As explained in subsection 2.3.1 and resorting to the MATLAB tool, a model that calculates the forces acting on a hovering flapping wing is implemented. The wing itself was built by Dickinson et al. [5], and
the setup is shown in Figure 4.1. The wing is 25 cm from tip to root, is immersed in mineral oil to achieve the necessary $Re = 136$, flaps at 0.145 Hz in accordance with the movement shown in Figure 2.4 and its lift and drag coefficients are defined by Equations 2.39 and 2.40. From this assembly came the measurements that all of Truong’s work is compared to.

Figure 4.1: Scheme of the experimental setup used by Dickinson et al. [5].

Regarding the implementation of Truong’s model, it follows the BET logic, so it essentially divides the wing into $i$ elements and evaluates the forces in all elements for $j$ time intervals of the flapping cycle. Since this model assumes that there is no induced velocity and that only the translational speed of the wing influences the calculations, the angle of attack across all of the wing is the same for any given instant. This in turn means that the lift and drag coefficient values depend only on the time, not wing position, and that they can be calculated outside any loops, improving the performance of the tool. It is also important to mention that the routine uses the midpoint rule for the integration, since the objective of this algorithm is to be simple and fast to run, and the midpoint rule offers that option without compromising the accuracy of the results, provided that there are enough integration points. The optimum number of discretisation points was calculated to be around $i, j = 80$ by Ponte [17] in his thesis about BET models applied to helicopters, so this will be the chosen number of intervals. The MATLAB routine developed here works according to the following logic:

- Definition of the integration points: starting from the values of wingspan and flapping period, the program takes them and divides them by the $i$ and $j$ number of intervals, respectively. Then it calculates the midpoint between every interval, by averaging their values. This explains why, for the calculations, there are $i - 1$ wing elements and $j - 1$ time intervals;

- Calculation of the chord, according to the procedure by Ellington [18]. This is to achieve the wing shape of a *Drosophila*, to validate the results against the work by Dickinson et al. [5];

- Using the points extracted from Figure 2.4, the program builds the angular and translational velocity vectors, then calculates the acceleration vectors through a division of the speed difference between adjacent points by their respective time step, and finally calculates the instantaneous
angles through a simple integration;

- Up next comes the transformation of the angles, which is a necessary step because the upstroke and downstroke happen in different directions, thus requiring the angle of attack relative to the horizontal, $\theta$, to be transformed by the equation $\theta = \pi - \theta$ for all values of the downstroke;

- Calculation of the $C_l$ and $C_d$ values, using the new values of $\theta$;

- Calculation of the forces acting in every integration point. For every wing element and time interval, the velocities are determined and then used in the calculation of the translational (see Equations 2.44 and 2.45), added mass (see Equation 2.52) and rotational forces (see Equation 2.46);

- Integration of the results along the wingspan;

- Plotting of the results as a function of dimensionless time, $t/T_{cycle}$.

The routine outputs the several components of the generated forces, which are individually presented in Figure 4.2 and Figure 4.3 because of the value that there is in analysing the different mechanisms at play during the stroke.

![Figure 4.2: Lift components according to Truong’s model.](image)

Observing said images, it can be seen how the unsteady effects play a significant role in the generation of the total force. Particularly at the points of stroke reversal (meaning at the middle and end of the cycle) the rotating wing generates the extra force it needs through the predicted unconventional methods, in this case from pushing the air around it (added mass force) and taking advantage of the circulation established by the wing’s own previous movement (rotational force). The combined effect of these mechanisms explains the peaks in force that are seen around the stroke reversal points, contributing to the general increase in the net force. While this is advantageous when talking about lift, the increase in drag is penalizing for the overall performance of the beetle, yet this is the price to pay for the extra lift generation.
Comparing now the total lift and drag values to the ones measured by Dickinson [5], as seen in Figure 4.4, it becomes clear that the model does offer a good approximation of the forces that the wing generates. The peaks and valleys are similar, identically located, and the curves in general follow the measurements closely. The main visible issue with the model resides in the fact that the lift measurements during the downstroke ($t/T_{cycle} > 0.5$) present variations which are not predicted by the calculations. The fluctuations can be explained by the interaction of the wing with its previously shed wake, which is something not taken into account by this simple model, and could only be predicted resorting to a more sophisticated solution like a CFD simulation of the wing. Nevertheless, putting aside the relatively small shortcomings of the model, it is reasonable to say that this is an adequate tool for modelling the aerodynamic mechanisms involved in flapping hovering flight.
4.1.2 Walker’s Hovering Model

The theory of this model was explained previously in subsection 2.3.2, and the implementation of the model is very similar to the one done for Truong’s. Therefore the present subsection will be focused more on the differences that there are between the two approaches (that reflect themselves in slightly different implementations), the reason for these discrepancies, and finally how all of this impacts the final results.

Again resorting to MATLAB, the procedure starts in the exact same way as for Truong’s model. The wing is divided into several elements, the time steps are defined and, for coherence sake, the number of discretisations is kept the same. The chord is again calculated for every element, followed by the extraction of the translational and rotational velocity of the wing, and the first difference appears when defining the velocities and the effective angle of attack. Walker divides the speed into two categories, parallel and normal to the wing (see Equations 2.65 and 2.64), and with these values calculates the angle of incidence through Equation 2.66. So the definition of the angle of attack is the main difference in implementation between the two models, as Truong’s model doesn’t admit the scenario of velocity-induced changes to the angle of attack, while Walker’s does. This is something that will become extremely relevant down the line, when modifying the models, but for the time being it is important to emphasize that this assumption by Truong does not impact the hovering results in any significant way. This also leads to a slight change in the implementation; since here the angle of incidence depends on the velocities, and given that different parts of the wing move at different rates, now every point of the wing will have a specific angle and consequently its own lift and drag coefficients. This means that $C_l$ and $C_d$ are now individually calculated for every element, and are no longer constant throughout the wing for a given instant. Even though this is an added layer of complexity to the routine, it is such a simple calculation and the number of wing elements is so small that the difference in processing power this change represents is negligible. Having calculated the lift and drag coefficients, then comes the calculation of the forces acting on each element; this is done in the same way as for Truong’s model, the only change being that now the force components are different. Instead of the three components that the first model predicted (translational, rotational and added mass), here it is assumed that there are just two (rotational and added mass). To finish the process one must again resort to the integration along the wingspan, and the last step is the plotting of the results.

Regarding the output of the MATLAB routine, firstly there is the plot of the force components, both for lift (Figure 4.5) and drag (Figure 4.6). In this particular model, since the main component is considered to be the rotational one (which integrates the unsteady effects through the Wagner function), the unsteady aerodynamic effects are harder to directly observe. The added mass force is mainly the same as in Truong’s model, which was already discussed, but now the rotational force is the main responsible for the force generation. This difference in the models, at least in terms of force definitions, is essentially just about semantics: while Truong separates translational and rotational effects, Walker just considers them to be the same mechanism and names it rotational force. The real difference resides in the velocities and angles definition, as will become apparent later on.

The total values, when compared to Dickinson’s measurements (see Figure 4.7), present some of
the same issues that Truong’s model faced. The curve generally follows the measurements closely, but in this case the model overshoots its prediction of the maximum values for the drag peaks, while the problem of fluctuating lift remains unsolved by this method. This is predictable, as this approach does not differ much from the previous one and it is still not able to account for the effects of the shed wake. This is, nonetheless, a good model as it proves to be reliable enough to be used in the hovering case, similarly to the previous model.
4.2 Adaptation of the Hovering Models to the Forward Flight case

There are several types of flapping wing forward flight; perhaps the simplest one is the type where the animal or vehicle just tilts the hovering plane, which is the type of flight that is executed by most insects and even some small birds like the hummingbird. They opt for this solution because this movement is very similar to the normal hovering state, the mode they spend the most time in, the only difference being that the tilting of the plane generates a force that has not only the vertical component (still equalling the weight), but also has a component with a given direction, which corresponds to a thrust vector and makes it fly forward. This is the same logic that helicopters employ, where the tilting of the force vector is responsible for the forward movement of the aircraft. This type of flight is, however, very similar to the previously studied case of hovering, the only difference being the higher thrust required to account for both the vertical and the directional components. The forward velocities involved are negligible when compared to those of the wing, therefore the whole mode can be approximated by the hovering one without any significant loss of accuracy, and so this is not the most interesting case to study.

Another type of forward flight, however, presents some major challenges that render it much more interesting to study. This is the most common flapping motion that bigger birds execute, the normal flapping motion that one tends to think of where the wing is mainly aligned with the stream. Figure 4.8 depicts this mode more clearly by showing the wing and its trajectory, along with its wake. Taking the same image as the reference, the body of the vehicle would be aligned with the $x$ axis, the wing would flap up and down in the $z$ direction, and at the same time there would be a rotation movement around the $y$ axis. This case resembles more that of an airplane with its wings fixed than that of a rotary wing, but at the same time has the added complexities of unsteady aerodynamics, induced angles of attack and tilting vectors.

To accurately describe this flight mode the ideal would be, then, to combine the previously described hovering models, which tackle those exact same problems, with an adequate description of the overall environment of the wing and the speeds involved in this type of flight. This would allow for the use of
a computationally light method, namely BET, while taking into account all the complex unsteady forces that this flight mode involves. This is the justification to the approach adopted in this work to model flapping flight, which in short corresponds to a change in the reference axis and a different definition of the speeds relating to the wing.

Figure 4.8: 2D trajectory of a flapping wing in forward flight, along with its wake (adapted from [19]).

4.2.1 Modified Truong Model for Forward Flight

Recalling Figure 2.5, there is the possibility of simulating the effect of a free stream by adding a velocity that is oriented vertically. This is only possible because the model is blind to the orientation of the forces, and the only input that it has from the environment around itself is the orientation and magnitude of the speeds. If the frame is then rotated, purely for convenience reasons, a model that matches the physical case of a flapping wing is obtained. Now for the velocities definition, since Truong considered the induced velocity to be zero, the very same logic will be followed here. It is also important to note that the logic of ignoring wing rotation for the definition of the speeds is maintained, as the chosen reference point coincides with the rotation axis of the wing. With this in mind, and again resorting to the reference axis depicted Figure 2.5, the new speeds are

\[ V_\zeta = V_\infty, \]  
\[ V_\eta = \dot{\psi} r. \]

Even though Truong did not consider the effects of the induced angle of attack, not including them in this modified model is not an option, as the free stream and flapping speeds are of the same magnitude and that results in very high induced angles of attack (for some of the elements at the tip of the wing, where the flapping results in higher speeds, those angles can be in excess of 80°). Therefore the model accounts for this effect, and calculates the induced angle of attack for each element through the equation

\[ \phi = \arctan \left( \frac{V_\eta}{V_\zeta} \right). \]

Then, following the procedure explained in section 2.2, this angle is subtracted to the geometric angle of attack, resulting in the effective angle of attack. Having this angle, the lift and drag coefficients can be evaluated. This raises another question, however, as the expressions developed by Dickinson for the \( C_l \)
and $C_d$ are no longer valid, due to the increase in the Reynolds number (from $Re \approx 10^2$ to $Re \approx 10^4$) and because now there is the need to account for the impact of other unsteady effects such as delayed stall.

This is perhaps the greatest shortcoming of a BET model, relying on drag and lift coefficient data that must be obtained from other sources, and depending on the situation this can prove to be very challenging. In this very case the problem is accentuated by the fact that it requires measurements from a wing being oscillated and rotated at the precise frequency that the simulation is performed, which is next to impossible to calculate without resorting to a much more sophisticated approach like a CFD simulation. Proceeding in this fashion will defeat the whole purpose of this method, so this is not an option. This means a simplifying assumption will have to be made, in order to achieve reliable results without compromising the relative simplicity of the model, and in this case such objective was achieved by assuming that the profile of the wing is a NACA 0012 airfoil, for two reasons: firstly, there is an abundance of information about this specific profile, making it easier to find cases similar to the one being analysed. Secondly, the paper used for the validation assumed this very same profile for the wing of the vehicle, further encouraging its use.

Due to how essential their definition is to the whole success of the model, the lift and drag coefficients proved to be very challenging to calculate accurately enough. Even though some work has been developed on the subject of force coefficients in oscillated airfoils [20–22], the research mainly focuses on the range of relatively small AoAs (normally $\alpha < 20^\circ$, which is the region where there is the onset of the delayed stall). Since in this work there is the need to deal with really extreme angles of attack, both positive and negative, another solution needed to be found that could provide the values for a wide range of angles of attack. The solution to this problem came in the form of the work by Leishman [11, p. 409] on the static lift and drag coefficients of certain airfoils at extreme angles of attack and low Mach values, which is also the approach used by Pourtakdoust and Aliabadi [23] in their study of the aeroelastic properties of flapping wings. In this chapter Leishman analyses how the NACA 0012 and other profiles behave in the whole spectrum of angles of attack, since this is relevant for the reverse-flow region that is generated in forward flying helicopters. The values obtained by this analysis allow the model to overcome the biggest issue that it is faced with, induced angles of attack that go almost up to $90^\circ$. It is important to emphasize that this is not an ideal solution, as the $C_l$ and $C_d$ values are measured in static conditions and therefore don’t take into account the unsteady effects created by the plunging and rotating movements, but the rest of the model partially compensates that by predicting the unsteady effects and integrating them in the calculations. Following the measurements, Leishman developed the following equations for the lift, drag and moment coefficient of the NACA 0012 profile

\begin{align}
C_l &= 1.1 \sin(2\alpha) , \\
C_d &= 1.135 - 1.05 \cos(2\alpha) , \\
C_m &= -0.5 \sin(\alpha) + 0.11 \sin(2\alpha) ,
\end{align}

which are depicted graphically in Figure 4.9 for clarity. Having solved this issue, the calculation of forces
is done exactly in the same way as it was done in the hovering case.

![Figure 4.9: Lift, drag and moment coefficients of a NACA 0012 profile used for the implementation of the model.](image)

**Validation**

The validation of the model has to be done with care, since this will determine the accuracy of every value outputted by the routine from now on. The first criterion is that it is necessary to find a work in which a 2D profile is analysed, since the model here developed is no more than several two-dimensional elements stacked together. Any 3D model would most likely output values that have the influence of factors that this model does not try to include, like wing-wing interaction and crossflow around the wing. Then it has to involve wing kinematics that somewhat resemble the ones that will be defined later on, meaning that it must employ a pronounced rotation and oscillation of the wing. Luckily, such a work exists and was developed by Choi et al. [9], where, resorting to a 2D CFD model of a NACA 0012 airfoil, the study finds the cases where the unsteady aerodynamic mechanisms efficiently maximize lift and thrust generation. This is done for the cases of forward flight, hovering and high-thrust flight, and the case of forward flight is precisely what is needed to validate the present work. The study has the added value of also providing the values that maximize the performance of the flapping wing in a given setting, so, by using these values, the future analyses will be able to use the most efficient flapping mode possible.

Choi performs the optimization process of its model using the parameters $f_{flap} = 9.55$ Hz, reduced frequency $k = 1$ (a dimensionless number that defines how unsteady the movement of the wing is, calculated through $k = (\pi f_{flap})/V_\infty$), $V_\infty = 3$ m s$^{-1}$ and $Re = 10^4$. With the results of the process he is able to write the equations that describe the optimum wing motion for forward flight, both in terms of
rotation and translation. These equations are

\[ \theta(t) = 30 \cos(\omega t + 97.6624) , \quad (4.7) \]
\[ \gamma(t) = 53 \cos(\omega t) . \quad (4.8) \]

Resorting to a CFD simulation of the wing plunging and rotating according to the movement these equations describe, the thrust and lift coefficient curves that the simulation outputs, during a flapping cycle, are depicted in Figure 4.10.

![Figure 4.10: Lift and thrust coefficient outputted by a CFD model of a NACA 0012 profile plunging and rotating according to the optimized parameters of forward flight [9].](image1)

In order to validate the model there is the need to isolate just one wing element of the algorithm, apply to it all the optimized values that Choi used and finally compare the output of the model to that of the original study, shown in Figure 4.10. Doing so yields the results visible in figure Figure 4.11.

![Figure 4.11: Results yielded by Truong’s modified model, compared to the results obtained by Choi.](image2)

As it can be seen, even though the values do follow the overall trend of the the curve obtained by Choi, there are some problems with the model. Regarding lift, not only does the model underestimate
the peak value by more than 100% of its value, an unacceptable result by all measures, but there is also a noticeable offset between the curves, suggesting that the model might not be accounting for the effects of the forces immediately, or that there might be a problem with the estimation of the angles or the velocities that result in a delayed prediction of the aerodynamic loads. Drag also shows values that do not seem fitted to the reference curve, with a big underestimation of the peaks (almost by 50%) and a clear offset of the curve to the left, probably attributable again to an inaccurate definition of its aerodynamic angles. For now, since this model will inspire another one, the important thing to retain is that the calculations are able to predict the general shape of both the lift and thrust curve, and the refinements are left to another section of this work.

4.2.2 Modified Walker model for Forward Flight

The modification of this model follows the same logic as the one performed on Truong’s algorithm: alteration of the velocities, followed by a convenient rotation of the axis and finally the calculation of the new $C_l$, $C_d$ and $C_m$ values. They are, however, implemented slightly differently. Given that Walker looks at the angles of the wing in a different way, this difference is reflected in the definition of the velocities. Whereas for Truong the velocities were defined in terms of the local reference frame, Walker defines them in terms of being parallel and normal to the wing, while also accounting for the wing rotational speed by choosing a reference point that is not coincident with the rotation axis. So, recalling Equations 2.64 and 2.65

$$V_\perp = V_T \cos(\alpha) + (x_{\text{ref}} - x_f) c \dot{\alpha},$$

$$V_\parallel = V_T \sin(\alpha),$$

they can be modified to include the velocity of the free stream, resulting in the following set of equations

$$V_\perp = V_T \cos(\alpha) + (x_{\text{ref}} - x_f) c \dot{\alpha} - V_\infty \sin(\alpha),$$

$$V_\parallel = V_T \sin(\alpha) + V_\infty \cos(\alpha).$$ (4.9) (4.10)

This is a better approach than Truong’s, as it accounts for the rotation of the wing. It also means that the induced angle of attack, which in this model is calculated simply by $\alpha_s = \arctan \left( \frac{V_\perp}{V_\parallel} \right)$ will in principle be more accurate than the calculation of the effective angle of attack by Truong. Aside from the velocities, the only difference in implementation is again how the two explain the forces differently.

Validation

Following the exact same procedure as before, meaning that one element was also isolated and given all the properties that Choi described, results in the curves shown in Figure 4.12. Analysing the image it becomes clear that the problems, albeit slightly diminished, have not disappeared. The curves follow the general trend of the reference values and the problem of the underestimation of the thrust
peaks (as seen in Truong’s algorithm) is now a little less pronounced. It is undeniably a model with some problems, but at the same time it has room for improvement, as it will become apparent later on.

Figure 4.12: Results yielded by Walker’s modified model, compared to the results obtained by Choi.

4.2.3 Joint Model

Analysing the two previous models, the issues with both of them become apparent: topping the list there are factors like curve offsetting and peak force underestimation, making it clear that some adjustments have to be made. At the same time, since these models have been adapted in a very rudimentary way, it is fairly obvious that there is room for improvement. It only seems reasonable that, in order to build a more advanced UBET model for flapping wing forward flight, the best approach will consist in isolating the best aspects of each of the previous models, combine them into just one and try to adjust the equations to better describe the physical reality of a flapping wing MAV.

The procedure followed here was relatively simple. The joint model includes the best aspects of each of the previous models, namely the forces definition by Truong et al. [8] (added mass, circulatory and translational) that proved to describe the reality in a consistent way, and on the other hand it makes use of the velocities and angles as they were defined by Walker [7]. The implementation of this new model is in every way similar to the others before it, so a detailed explanation will not be presented here. Instead, a brief overview will be given, touching all the relevant subjects. Using Truong’s model as the base, since in its core the new model will resemble it more than Walker’s, the velocities were changed to follow the latter’s approach. This has the immediate effect of changing the angle calculation, which was arguably the weakest point in this model. The new speeds also impact the rotational force calculation, since the velocity is an integral part of its equations, and affect the translational force in several ways, given that this component (which is the biggest contributor to the total force) depends directly on the speeds, angles and finally on the drag and lift coefficients (which are calculated directly through the induced angle of attack, again emphasizing the importance of the angle calculation). So, in short, the logic is the following:
• For each element, the speeds are calculated through Walker’s method, meaning that Equations 4.9 and 4.10 are employed;

• The flow incidence angle $\phi$ is calculated through the same model, resorting to Equation 2.66;

• The Wagner function is calculated, similarly to the previous implementation of Walker’s model, through Equation 2.67;

• The lift and drag coefficients are calculated through the Equations 4.4 and 4.5, using the incidence angle $\phi$;

• As for the forces calculation, the new model now strays away from Walker’s approach. Adapting Truong’s equations to this model (that are no longer defined in terms of the inertial axis, instead are defined locally, therefore requiring an axis transformation), the incremental translational forces become

\[
dF_{T,\zeta} = \frac{1}{2} \rho \Phi V^2 c \left[ C_l \cos(\theta) - C_d \sin(\theta) \right] dy, \\
dF_{T,\eta} = \frac{1}{2} \rho \Phi V^2 c \left[ C_l \sin(\theta) + C_d \cos(\theta) \right] dy;
\]  

(4.11)  

(4.12)

• The added mass force is given by Equations 2.53 and 2.54, as Truong defined them;

• Rotational force is also calculated in the same fashion, meaning that Equations 2.50 and 2.51 are employed. Unlike the added mass force, which renders the exact same results for the normal Truong model and this one, the rotational force will output a different result in the two cases since the velocities are different;

• Results are then integrated along the wingspan, providing the total lift and drag results.

This algorithm produces the plot of the force coefficients shown in Figure 4.13. This figure confirms that joining the models and applying the described changes culminates in an undoubtedly better result than that of the previous separated models.

In order to start preparing for the next chapter and also to objectively quantify each model’s accuracy, the average difference between each one of the thrust results and Choi’s values were calculated. Knowing that the average thrust value given by Choi’s model is $T_{av} = 0.1404 \text{ N}$, the results obtained by each model, as well as their respective errors, are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Average Thrust (N)</th>
<th>Average Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truong</td>
<td>0.0763</td>
<td>49</td>
</tr>
<tr>
<td>Walker</td>
<td>0.1280</td>
<td>12</td>
</tr>
<tr>
<td>Joint</td>
<td>0.1321</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 4.1: Average thrust value of each model, as well as its respective error when compared to Choi’s average value.

The lack of precision by Truong’s model is evident. A model with an average error of 49% is without a doubt a very weak model, and this is due to the the way that the wing velocities were defined. Assuming
that the reference point is the center of rotation has the consequence of ignoring how the angle of incidence changes also due to the instantaneous rotation, and while this was an acceptable approximation for hovering, in this case it no longer is.

Walker’s model shows, on the other hand, a surprising precision in terms of the thrust estimation. Again due to the better method of velocity and angle calculation, and perhaps due to a better approach to unsteady effects (the Wagner function seems to impact Walker’s model in a more effective way than the unsteady model developed by Dickinson impacts Truong’s. This is only true for the forward flight case, suggesting that the Wagner function is more general and therefore more useful in this specific situation), Walker’s model is dramatically more precise than Truong’s. This is the justification for having chosen to use both Walker’s angle definition and unsteady modelling for the joint model.

The joint model’s results confirm, firstly, how Walker’s model was already really satisfactory. The marginal improvement of 3% in thrust calculation is a good result and confirms that there was room for improvement, but an error of 9% is still non-neglectable and shows the limitations that an UBET model possesses when compared to more sophisticated models. Secondly, it proves how both definitions of the forces by the different models were valid approaches, as the only thing that changes between Walker’s and the joint model is the way forces are defined, and the difference is small. In short, given that the MATLAB routine of the joint model runs instantaneously on old hardware (8-year old PC, running on a 1st generation i7 processor and 4GB RAM), these results are really satisfactory and prove that this might be a viable option to quickly estimate the thrust and power consumption of a flapping wing vehicle when in forward flight, without having to resort to vastly more time and resource-consuming methods.

4.3 Tool Development

One of the main objectives of this work is to facilitate the comparison between different configurations, namely fixed, flapping and rotary wings. With this in mind, and given that the biggest obstacle in this process is to evaluate the power expenditure of a flapping wing, it was only natural that the work de-
veloped during this thesis culminated in the development of a simple, user-friendly tool that summarized all the concepts here studied and allowed the user to estimate the lift and thrust generated by a flapping wing in a given flight condition, as well as its power consumption. The tool that was developed resorts to the joint model that was developed in subsection 4.2.3, as this model yielded the best results with only a small increase in complexity and no penalty in calculation time, and was built using MATLAB’s App Designer functionality. The logic followed is exactly the same as the one explained before, since this is an adapted UBET model running in the background in the form of a MATLAB script, only in this case there is a GUI (Graphical User Interface) on top of it in order to simplify the task of the person performing the analysis. The tool has three main functionalities: firstly, it can calculate the lift, thrust and power curves of a certain user-designated combination of parameters, during the whole flapping cycle, in what is designated an Absolute Analysis. Not only that, it also offers the possibility to fix every parameter except one, in order to study how said parameter influences the overall performance of the vehicle, in what is designated as a Variable Dependant Analysis. Finally, it can discover the equilibrium point, in terms of forces, for a given set of values, in what is called Steady State Analysis. In Figure 4.14, which shows the overall look of the tool right after being opened and before any interaction with the user, the numbers 1, 2 and 3 denote the tabs which the user must choose to perform each one of the types of analyses. A more in-depth look at the workings of each sub-menu is presented next.

Figure 4.14: View of the start menu, corresponding to the Absolute Analysis, before having had any interaction with the user.

4.3.1 Absolute Analysis

When the user selects the tab signalized with the number 1, the interface that is presented is the one shown in Figure 4.14. The user then has the possibility to change every parameter that is shown in the box marked with the number 4, meaning that there is a total of 9 variables that can be changed and
studied. These variables are all explained in detail in section 5.1, if further explanation is required. After having inserted the desired values in the respective field boxes, in the zone identified by the number 5, the user can choose from several options in the form of checkboxes. These options are:

- **Show Wing Movement** - Checking this box will result in, after pressing the Calculate button, the appearance of a pop-up figure that will show step-by-step the movement of the scaled wing, along with the instantaneous forces that the wing is generating. This is no way changes the analysis, it serves mainly as an intuitive guide to check if the inserted parameters are correct and adjusted to the simulation.

- **Coefficient Form** - This option will present all the results (lift, thrust and power) in their coefficient form, as well as the curves, instead of the normal absolute values.

- **Keep Previous Plots** - By default the tool will erase any plot that is previously drawn in the figure, and checking this option will prevent it from doing so. This allows the user to keep an unlimited number of plots from different simulations, as long as the box is checked, which is particularly useful in terms of comparison.

- **Use suggested speed & flapping frequency** - This option requires some background explanation; according to Pennycuick [24], birds follow several trends when it comes to flapping frequency and speed for optimum efficiency. These relations are

  \[ f_{\text{flap}} \propto m^{1/3} g^{1/2} b^{-1} A^{-1/4} \rho^{-1/3}, \]  
  \[ V_{\infty} \propto (mg)^{1/2} b^{-1} \rho^{-1/2} , \]  

and they come in very useful when it comes to projecting a vehicle with maximum efficiency in mind. These relations, when applied to the case at hand, did show to be adequate and outputted values that maximized thrust while not increasing the power consumption by large amounts. In order to be useful for this routine, an actual curve had to be found, so a fitting process was undertaken. This was done by adjusting for lift, meaning that the objective was to find the values of \( f_{\text{flap}} \) and \( V_{\infty} \) that yielded the correct values of lift for each combination, and from then the coefficients for each equation were derived. The resulting relations are

  \[ f_{\text{flap}} = 2.16 m^{1/3} g^{1/2} b^{-1} A^{-1/4} \rho^{-1/3} , \]  
  \[ V_{\infty} = 5.12 (mg)^{1/2} b^{-1} \rho^{-1/2} , \]  

and were included in the tool to offer the possibility to use the optimized values of \( f_{\text{flap}} \) and \( V_{\infty} \) for the other parameters, since these two are, by far, the two variables that most heavily influence the performance of the device. So, in short, checking this box will disable the edit fields of the flapping frequency and the forward speed, and underneath the optimized values will appear. The program will then use these values in the calculations. It is also important to emphasize that if the box is ticked and the user changes either the mass, wingspan or AR edit fields, the values will
automatically be updated.

The user can then press the Calculate button, which will trigger the analysis. The resultant lift, thrust and power values will appear in the 6, 7 and 8 plots, respectively, as a function of dimensionless time $t/T_{cycle}$. The program also calculates the average values of lift, thrust and power during one cycle, and presents them in the boxes identified by the number 9.

### 4.3.2 Variable Dependant Analysis

This capability was developed in response to the problems faced in the next chapter, particularly the necessity to evaluate the impact of changing each parameter on the overall performance of the vehicle. So the user can choose to study any of the nine parameters by leaving it as a variable, instead of fixing its value.

![Figure 4.15: View of the Variable Dependant Analysis menu, before having had any interaction with the user.](image)

The overall look of the menu is shown in Figure 4.15. The user begins by, in the section 10, selecting the variable from a dropdown box. Doing so will block the respective edit field in section 10, and the number "0" will appear to signal that the variable will not have a fixed value. Still in the box number 10, the user can also define both the lower and upper limits of the variable.

Then, in the box marked with number 11, the user can choose the values for every parameter, aside from that of the variable that was singled out, similarly to what was done in the Absolute Analysis menu. The options shown in section 12 are also the same as before, requiring no further explanation aside from one special case; when the user chooses either the Flapping Frequency or the Horizontal Speed from the dropdown box and then checks the Use suggested speed and flapping frequency option, the tool will still use the suggested values, but only for the variable that was not singled out.
Finally, after having defined every parameter and checking the desired options, the user presses the *Calculate* button and the values of lift, thrust and power appear respectively in the 13, 14 and 15 plots, respectively, as a function of the chosen variable.

### 4.3.3 Steady State Analysis

The algorithm presented in this tab is dedicated to finding the optimum point of a given configuration, which is also called steady state case. The explanation here is done from a functional perspective only as a more in-depth look at the workings of the algorithm is presented in subsection 5.1.10. To run the algorithm, the user must fill in the desired parameters in the box 16, and select the upper and lower values for velocity in the field 17. Having done this, the user can press the *Calculate* button, prompting the tool to start the calculations. Since the analysis takes around 10 seconds to run and there was no visual cue as to if the program was working, the lamp marked with the number 18 was added, which turns red while the algorithm is performing the calculations. Upon completion, it turns to green again.

When the calculations are complete, the results in terms of lift and thrust are presented in figure 19, power in figure 20 and the flapping frequency in figure 21. The results regarding the steady state point are all also conveniently presented in the fields signalled with the number 22.

![Figure 4.16: View of the Steady State Analysis menu, before having had any interaction with the user.](image)
Chapter 5

Refined Power Consumption Analysis

At this point of the work, all of the basic concepts necessary to evaluate the power consumption of all the three vehicle configurations have been established. A basic, rudimentary comparison of them all has also been performed to serve as a guide and, most importantly, there was the development of the tools to evaluate the power consumption of the most complicated case of the three, the flapping wing forward flight case. This next section is aimed at wrapping up all the gathered knowledge and developed work, in order to present the reader with the summarized information about how the configurations compare to one another.

It is not possible, unfortunately, to adopt the same approach to the comparisons as in chapter 3, since numerical expressions that can be manipulated with relative ease do not exist any more. Instead of that, there is an UBET algorithm that precisely calculates flapping wing power consumption, and it can be used to study how every parameter influences the overall performance of the flapping wing. Then the previously developed work can be used to compare its performance to that of a rotary and fixed wing of similar proportions and specifications, to analyse how advantageous such a configuration can be in terms of power consumption.

5.1 Flapping Wing Performance Analysis

As explained in section 4.3, a simple and easy tool to predict how a flapping wing behaves in different scenarios can now be used. The second part of the tool, Variable Dependant Analysis, was actually developed to suit the exact demands of this part of the work, where one parameter is varied between a range of values and the performance of the device is evaluated at each and every point. This means each parameter of the configuration can be varied while keeping all other values fixed, and therefore the impact of every specific change to the wing’s design will be known. This is a particularly useful feature in terms of optimization, because learning about the specific impact of each parameter is of the utmost importance when trying to perfect a design in terms of efficiency.

It is important to emphasize that this is just an approximation, as varying one parameter has a range of implications which are being simplified into thrust, lift and power variations. For example, when a
vehicle is flying its lift must always match its weight, and modifying these values will change this delicate balance. If, by any chance, a parameter were varied in a way that increased lift to above the weight value, what would happen in reality is that the vehicle would climb up until the density of the air dropped enough to allow for it to perform a levelled flight. There is also the fact that the term thrust is, in fact, an abuse of language; when speaking about thrust in this chapter, this is actually referring to the net horizontal force applied on the wing, so theoretically it should be zero for it to fly at a steady speed. However, since a vehicle cannot be composed of solely the wings, like the model is, it is assumed that some amount of thrust will be needed to account for the drag imposed by such an extra body, and in general it is considered that more thrust for the same power is beneficial. This, in itself, is also an approximation, as increasing the thrust too much means that the vehicle will tend to speed up and then all of the performance will change ever so slightly, rendering all of the study useless. Considering all of these scenarios is, for obvious reasons, borderline impossible, so these approximations are used to make sense of what is in reality a really complicated process.

The parameters that will be studied are presented next, along with the standard values (SV) that they take when not varying themselves. All of standard values were chosen taking into account the optimization work developed by Choi et al. [9], but also the article by Au et al. [25]. This being said, the parameters that are going to be studied are:

- Mass (m) - SV = 0.05 kg;
- Aspect Ratio (AR) - SV = 4;
- Flapping Range ($\gamma_{max}$) - SV = 60°;
- Maximum Rotation Angle ($\theta_{max}$) - SV = 40°;
- Average Rotation Angle ($\theta_{av}$) - SV = 10°;
- Nondimensional center of rotation of the wing ($x_r$) - SV = 0.2;
- Wingspan (b) - SV = 0.4 m;
- Forward speed ($V_\infty$) - SV = $2.16 m^{1/3} g^{1/2} b^{-1} A^{-1/4} \rho^{-1/3} m/s$;
- Flapping Frequency ($f_{flap}$) - SV = $5.12 (mg)^{1/2} b^{-1} \rho^{-1/2} Hz$.

The two final parameters, forward speed and flapping frequency, are not kept fixed because that would not be representative of the reality. The fact is that when parameters like the wingspan, the aspect ratio or the mass are varied, there is going to be an obvious difference in terms of the optimum flight speed and flapping frequency. Fortunately, there are two relationships developed by Pennycuick [24] (and explained previously in section 4.3) that describe how these parameters vary in bird flight, so the model will use a slight variation of these relations in order to approximate the results to what one could see in a real life scenario. Using this specific set of values, the flapping frequency is $f_{flap} = 13.02 Hz$ and the forward speed is $V_\infty = 8.10 m/s$. As for the movement that the wing will follow, it will be defined
by the equations

\[
\gamma = \gamma_{\text{max}} \cos \left( \frac{2\pi t}{T_{\text{cycle}}} \right),
\]
\[
\theta = \theta_{\text{max}} \sin \left( \frac{2\pi t}{T_{\text{cycle}}} \right) + \theta_{\text{av}},
\]

as was defined by Choi et al. [9], which, using all of the standard values, produces the movement seen in Figure 5.1.

Figure 5.1: Scaled movement of the tip of the wing, during a flapping cycle, using the reference values.

5.1.1 Mass

The mass of the vehicle is going to have an obviously large impact on all three parameters, thrust, lift and power consumption. As Figure 5.2 shows, there is a pronounced increase in lift and thrust generation accompanying the increment in mass, and the lift roughly equals the weight of the vehicle during all the instances of the study. This is important because if, by any chance, the generated lift were to be inferior to the vehicle’s mass, it would enter a state of stall and the results would be useless, as the aircraft would not be able to remain airborne under those circumstances. An extra lift creation, however, would not be a problem. If the vehicle generates more lift than that of its weight it can simply tilt itself forward, effectively turning a portion of the lift into thrust and therefore helping to propel the vehicle forward.

Thrust, on the other hand, does not vary so significantly in the same range of mass values. This may or may not be a problem, depending on the speed it is required to fly at, but in general what can be taken from this analysis is that the mass influences lift much more than thrust, and that the mass can be changed almost freely without worrying about the impact that it will have on the propulsion of the aircraft.

Power suffers a positive variation as well, as expected, due to the increasing forces necessary to keep the vehicle flying. Just as a comparison term, a vehicle with a mass of 100 g has an average power
consumption of around 31 W, while an aircraft weighing 1 kg with exactly the same parameters (except for $f_{flap}$ and $V_\infty$, which are optimized for each scenario) requires a power input of 364 W. This tenfold increase is simply explained by the variation in flapping frequency (see Figure 5.3) that Pennycuick’s model imposes, and with a large increase in flapping frequency comes necessarily a big increment in power consumption (see subsection 5.1.8 for further explanation on the matter). It is also worth mentioning that, since mass does not directly influence the calculations, if varying values of speed and frequency were not used, there would not be any variation in the plots of Figure 5.2, instead being just straight lines. This further justifies the use of Pennycuick’s relations, as not having them would render this analysis useless.

As a final remark on the subject, it is important to emphasize how, according to Pennycuick [24], the average wingspan of a bird with a weight of 1 kg is approximately 1.1 m. Therefore the fact that the weight of the vehicle is being increased without varying any other parameter, especially the wingspan, renders the results for the higher values of mass unrealistic, as it makes no sense for a vehicle with a
wingspan of 0.4 m to support a mass of 1 kg. They were kept, however, for academic purposes, as it is interesting to see what would happen if such a scenario were possible, and also as a reminder that the tool will still yield seemingly reasonable results even when the parameters that it is given do not have a plausible physical equivalent.

5.1.2 Aspect Ratio

The aspect ratio, $AR$, ought to be a compelling case to study. Since higher aspect ratios in fixed wings translate to a better efficiency and therefore better power consumption, it will be interesting to see if this is observed in the case of flapping wings as well. The reason for this not to be obvious relates to the fact that varying the $AR$ impacts almost all other factors. For example, it directly affects the wing shape, which influences the forces generation in ways difficult to predict, while also changing the optimum speed and frequency values. All of this leads to a certain level of uncertainty before running this analysis.

For the sake of comparison, the value that will be kept constant during the study is the wing area, defined through the standard values as being $A_f = \frac{b^2}{AR} = 0.04 \text{ m}^2$, meaning that the variation of the aspect ratio translates itself into different combinations of wingspan and chord values that always produce a wing with the same area.

The results are presented in Figure 5.4, and it can be observed that the aspect ratio does play a key role in terms of the flapping wing performance. While the smallest values of aspect ratio meant greater thrust and lift values, this came at the cost of an increased power consumption when compared to higher values of $AR$. Going from a wing with $AR = 2$ to $AR = 6$ means decreasing the power from 15.2 W to 12.4 W (which represents an estimated 18% decrease in power consumption), while losing only around 17% of the lift. Thrust, however, suffers more with this increase in the aspect ratio, as its value decreases approximately by 62%.

In conclusion, higher aspect ratio values translate into less generated thrust, which in turn means that the aircraft will fly slower. However, since this comes accompanied by a decrease in power consumption, it is preferable from a power saving perspective. An increase in the aspect ratio also implies a slight
decrease in lift generation, which in a real life scenario might mean that the vehicle will need to fly at a lower altitude or decrease its payload. This may or may not be relevant for the design of the MAV, but it is always important to keep it in mind. So, to wrap up this section, one can say that the best possible procedure to decrease the power consumption is to increase the aspect ratio, without compromising lift and thrust generation, which might come as no surprise as it is identical to the fixed wing’s case.

5.1.3 Maximum Flapping Angle

The flapping amplitude, represented by the letter $\gamma_{max}$, is likely to be a very important factor in the overall performance of the configuration. For clarity, it is important to note that this angle represents the maximum angle that the wing can ever reach, measured from the horizontal plane, so in fact a $\gamma_{max} = 60^\circ$ actually means a total wing aperture of $120^\circ$. Keeping all other parameters the same, a greater angle means a faster flapping movement and also tilting the lift vector during a longer portion of the flapping cycle. Both of these effects mean a greater power expenditure, so this is the main expected result of increasing the flapping angle.

![Graphs of Average Thrust and Lift, Average Power](image)

Figure 5.5: Results provided by the joint BET model for varying values of maximum flapping angle.

In order to test this possibility, the regular analysis was carried out where the varying parameter was the maximum flapping angle. This angle was tested from a relatively low angle, $20^\circ$, which is below what would usually be seen but was done to make sure that every possible scenario was covered, to $80^\circ$, which was chosen because going any higher than that would represent a scenario of wing-wing interaction (which in nature is actually used as a way to generate extra lift through the clap-and-fling mechanism). Since the means to simulate that effect are not available, $\gamma_{max} = 80^\circ$ was deemed to be the maximum possible flapping angle. The results are presented in Figure 5.5, and they show that increasing the flapping angle does indeed have a penalizing effect on the power consumption. Nevertheless, this is not a straightforward conclusion, as the lift being generated is not enough for the vehicle to withstand sustained flight until reaching $\gamma_{max,crit} = 35^\circ$, where the lift finally equals the weight of the device (since $m = 50\,g$, this means that the lift must be at least $L = mg = 0.4905\,N$). So it can be stated that an increase in the maximum flapping angle results in a higher power consumption, as predicted, but a
minimum flapping range is needed to create the forces that the vehicle requires to fly. As for thrust, even though it does not have a variation as relevant as the one that lift presents, it still increases in an appreciable way. Finally, looking at the power curve, it can be concluded that, as expected, this variation had an undeniable effect on the power consumption of the vehicle. It goes from a $P_{\text{crit}} = 3$ W to $P = 26$ W at $\gamma_{\text{max}} = 80^\circ$, which represents an approximately eightfold increase in power consumption.

All of these results can simply be explained by the hypothesis that was presented in the beginning of this analysis. A greater flapping angle, keeping every other parameter the same, means greater speeds and more tilting of the lift vector in the same time interval. Since the speed or the accelerations are the main factor in all of the force components, this means the generation of forces with a bigger magnitude. Since power is directly linked to the forces, it is only natural that it increases substantially when the flapping angle is raised.

In general, what can be extracted from this analysis is that for every configuration there is going to be a minimum value of maximum flapping angle $\gamma_{\text{max,crit}}$ that generates just enough lift for the aircraft not to stall, and this angle also corresponds to the point where the vehicle requires less power to fly. This turns the critical angle into a very relevant point in terms of designing the device for optimum performance, and it is highly recommended to perform a previous study on any aircraft project of this type to find said value.

5.1.4 Maximum Rotation Angle

The maximum rotation angle, $\theta_{\text{max}}$, is defined as the maximum pitch angle that the wing can reach, and has an impact which is difficult to predict. This value mainly controls the induced angle of attack, which is directly linked to the instantaneous lift, drag and moment coefficients. The outcome is difficult to predict because, for example, increasing the rotation angle results in a greater lift value, presumably increasing the power consumption, but at the same it might have conflicting impacts regarding thrust creation, which might result in a smaller speed and therefore a lower power expenditure. So a not so pronounced variation of the results is expected, perhaps with an increase in this angle resulting in a slight power demand increment.

The test was carried out and the results are presented in Figure 5.6. The range of values for this analysis was extended up until unusually large values, as $\theta_{\text{max}}$ does not normally exceed $40^\circ$, but some interesting trends appeared near the top values in the preliminary analyses (which went only up to this range), leading to the decision to extend the $\theta_{\text{max}}$ values until $60^\circ$ just to evaluate if some strange effect would occur.

So, looking at the results, it can be seen that there is a minimum $\theta_{\text{max}}$ that ensures that enough lift is generated. In this case the angle is $\theta_{\text{max,crit}} = 19^\circ$, meaning that any value below that would result in a stalling aircraft. Overall, the lift curve follows the trend that would be expected, showcasing a peak perhaps surprisingly late. This peak may be related to the extra rotational and added mass forces that a growing maximum rotation angle means (a greater amplitude of rotation in the same period represents a higher angular speed, which directly affects these components of the overall lift) up to a point where
the variation is so extreme that this effect is simply outweighed by the diminishing translational force.

Thrust, on the other hand, has a peak more in line with what could be predicted, somewhere around $\theta_{\text{max}} = 27^\circ$. It is important to mention that, provided the peak is always present, this means that for every possible combination of parameters there might be a value of $\theta_{\text{max}}$ that maximizes the speed of the vehicle while decreasing the overall power consumption, which is really interesting from an optimization perspective. This specific peak might be related to the apparent angle of attack of the airfoil during the whole cycle; small $\theta_{\text{max}}$ values mean that barely no rotation takes place, which can be prejudicial in terms of the translational forces by forcing the airfoil to attack the flow with an excessively positive angle during the downstroke and with a highly negative angle during the upstroke. Regarding the opposite side of the spectrum, very high values of $\theta_{\text{max}}$ have the exact opposite effect, leading to the deterioration in performance that can be seen. The most extreme $\theta_{\text{max}}$ values even show a very interesting behaviour by producing a thrust force that is directed backwards, meaning that in these specific cases the net force applied on the wings is actually drag, not thrust, so the vehicle will slow down.

In short, the study of the maximum rotation angle shows to be of great relevance in terms of the performance of the vehicle. Given that the overall thrust and power values vary by a great deal depending on the maximum flapping angle, the success or failure of such an aircraft may depend on the fine-tuning of this parameter. Therefore a study aimed at finding the optimum $\theta_{\text{max}}$ is recommended, in order to obtain the best possible performance out of a FWMAV.

### 5.1.5 Average Pitch Angle

This parameter defines the pitch angle around which the rotational movement is developed, and it is referred to as $\theta_{\text{av}}$. It is a relevant quantity because of the difference that there is between performing a cycle where the pitch angle revolves around zero, meaning that the wing spends precisely half of the period with a negative angle of attack and the other half with a positive one relative to the horizontal line (not to be confused with the induced angle of attack that could still be positive when $\theta$ is negative and vice versa), and to perform a cycle where, for example, the wing spends the majority of the period with
a positive pitch angle.

Intuitively, one could say that a rotational movement where the average pitch angle is positive would mean more drag and thus a higher power demand, but the truth is that this angle’s influence extends much further than that. Altering this angle results mainly in changes to the induced angle of attack (which may or may not be favourable to the overall performance) and to the direction of application of the unsteady forces (which are always aligned parallel and perpendicular to the wing and could therefore hurt the overall performance of the vehicle by generating forces contrary to that of the direction of flight).

![Graph](image1)

(a) Average Thrust and Lift

![Graph](image2)

(b) Average Power

Figure 5.7: Results provided by the joint BET model for varying values of average pitch angle.

The analysis was performed and the results (shown in Figure 5.7) were, to some extent, unexpected. Firstly there is the fact that negative values of $\theta_{av}$ actually produced negative lift, meaning that the flapping of the wing was not able to change the effective angle of attack enough for the resulting force to be positive, completely discarding this range of values as design options. As for the positive values of $\theta_{av}$, what can be observed is that there is a range of values that cannot be used due to insufficient lift generation, from around $\theta_{av} = 0^\circ$ up until $\theta_{av,\text{crit}} = 5^\circ$. Aside from that, thrust remains relatively undisturbed for the whole range of tested values, only showing a small decrease near the top end of the spectrum, suggesting that this is not a parameter that has much influence over the speed of the device, instead affecting more the lift.

Regarding power consumption, the first important observation is that it has two different regions. Discarding the negative values of the average pitch angle because of its unfavourable lift generation, the first region spans from $\theta_{av} = 0^\circ$ to $\theta_{av} = 6^\circ$, and the second covers the highest average pitch angle values. The first region is characterized by the invariance in power consumption, while the second one is defined by an almost linear 39% increase in power demand. This might have a relatively similar explanation to that of the maximum pitch angle, $\theta_{\text{max}}$, as small changes to this value mean that the wing is going to face the oncoming stream of air in different ways, particularly at the midpoint of both the upstroke and the downstroke, where its speed hits the peak value. So it is natural that small variations produce different power consumptions, and what can be observed here is that there is a critical point
where those changes do start to be noticeable.

In conclusion, it can be said that this is a very relevant parameter in the sense that will very much define the overall lift that the wings produce, given that small changes produce huge changes in lift. At the same time, any increase above a specified \( \theta_{av} \) will result in a higher power consumption overall, so the best strategy to increase the performance of a flapping wing vehicle is to find the critical value for lift creation, \( \theta_{av,crit} \), and use it as the design point, since this represents the lowest possible power consumption and comes with the added advantage of not sacrificing the speed of the plane, as the thrust is actually greater for small values of \( \theta_{av} \).

5.1.6 Wing’s Rotation Center

The rotation center of the wing should, in principle, influence the loads on the wing in a relevant way. Since both of the non-conventional mechanisms for aerodynamic forces generation heavily depend on the accelerations of the wing [6], a slight change to the wing’s rotation axis should result in a variation of the inertial loads of the wing, which in turn would result in differing loads and consequently varying power consumptions. The particular interest of this analysis is to observe how that influence reflects itself on the results, to then theorize as to why that might be happening.

![Average Thrust and Lift](image1.png)

![Average Power](image2.png)

Figure 5.8: Results provided by the joint BET model for varying values of the center of rotation of the wing.

The results of the analysis are presented in Figure 5.8. For the sake of good interpretation of the graph, it is important to clarify that \( x_f = 0 \) refers to a leading edge-placed center of rotation, while \( x_f = 0.5 \) means that the center of rotation is located precisely in the midpoint of the chord. Now for the graph analysis in itself, the first and main conclusion that can be drawn from the plot is that the rotation center of the wing has a relatively small impact on the results; lift is practically unaffected by this variation of \( x_f \) (the variation does not ever exceed 2% of the minimum value) and there is only a 13% difference between the minimum and maximum values of power consumption. Thrust, interestingly, is the only quantity that seems to be somewhat affected by the variation of \( x_f \). It seems that a wing which is rotating about its leading edge (LE) has a greater capability of generating a forward force, which might
be connected to the fact that it reaches higher absolute speeds when its rotating about this point. This, combined with the fact that the wing spends most of its trajectory with a non-horizontal attitude, might explain why the thrust goes from \( T = 0.76 \text{ N} \) to \( T = 0.34 \text{ N} \), which represents a 55\% decrease in thrust generation.

In short, this is a parameter that will not impact dramatically the power consumption, so the choice of its value should be based purely on the thrust and speed requirements of the vehicle.

5.1.7 Wingspan

The size of the vehicle’s wings, or the wingspan \( b \), is obviously a very important parameter to define its overall performance. Since there is a fixed mass and given that the relations developed by Pennycuick [24] are being used (Equations 4.15 and 4.16), varying \( b \) actually changes both the optimum flapping frequency and the horizontal speed of the device, whose fluctuation is shown in Figure 5.9.

![Figure 5.9: Variation of the optimum forward speed and flapping frequency values with the increasing wingspan, according to Pennycuick’s model.](image)

The results, shown in Figure 5.10, display some interesting trends. This is a similar case to that of the mass, in the sense that increasing it too much without adding mass is unrealistic (at a certain point the wing itself would have to weigh more than the specified 50 g). Aside from this, what is observable is that both thrust and lift, along with power consumption, increase almost linearly with the varying wingspan, which is a relatively predictable behaviour. Even though the flapping frequency decreases in the same interval, which could lead to the prediction that an increase in area might actually reduce the loads and power consumption, what is instead observed is that the bigger area and speeds that the increasing \( b \) imposes mean bigger forces, outweighing the effect of a smaller flapping frequency and forward speed.

Also interesting to note is the fact that there is a lowest possible value of wingspan to generate enough lift, in this case \( b_{\text{crit}} = 0.14 \text{ m} \), meaning that for this set of parameters there would not be the option to decrease the size of the aircraft any further without stalling the vehicle. This point is also important from a performance perspective as it represents the lowest possible power consumption, so, from a design point of view, this point should be used to minimize power expenditure.

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5.1.8 Flapping Frequency

The flapping frequency is, without a doubt, one of the parameters that is expected to have the greatest influence in the overall performance of the vehicle. This is intuitively understood by thinking that more beats per second mean greater wing speeds in the same period of time, which lead to greater forces. The combination of higher speeds and forces also means, due to the way power consumption is defined, that the power intake of the vehicle is going to dramatically increase with the increment in flapping frequency.

The results can be seen in Figure 5.11. As predicted, the power curve tends to become steeper for higher values of frequency, but the same trend is observable on the thrust and lift curves, meaning that this parameter is of the utmost importance to the performance. On one hand, it defines if the wings are generating enough lift to be above the stall threshold (which in this case happens around $f_{\text{flap, crit}} = 7$ Hz) and heavily influences the thrust, while also exponentially increasing the power consumption. For example, taking the power consumption at $f_{\text{crit}}$ ($P_{\text{crit}} = 4$ W) it can be studied how costly it is to increase...
the flapping frequency just by comparing it to the value of power for $f_{flap} = 15$ Hz, which, at $P = 20$ W, represents a 400% increase in power consumption by just doubling the flapping frequency. For higher values of flapping frequency the variation becomes increasingly more pronounced, with the power consumption reaching values two orders of magnitude bigger than $P_{crit}$. It is worth mentioning that the higher values of frequency are not especially representative of the reality, because they are values that would be really hard to achieve with the currently available technology and would present no particularly great benefit aside from the added thrust. They are presented here because it is interesting to see how the curve behaves, and because these values might be achievable in the future.

The flapping frequency, while being one of the most easily adjustable elements of the device, is also one of the parameters that most heavily dictates how it behaves. What can be inferred from this analysis is that, when designing such a device with efficiency in mind, a previous study has to be performed to find the optimum flapping frequency. The vehicle will then have to be tuned to fly in cruise conditions with said value of flapping frequency, reserving the increase in flapping frequency to manoeuvre or momentarily increase speed, because proceeding in any other way will result in a much higher power consumption.

### 5.1.9 Forward Speed

Since up until now the velocity has been varying according to Pennycuick’s relation (see Equation 4.16), which maximizes the efficiency for every case, this analysis is interesting because it will show the vehicle’s behaviour for all others speeds, when the performance is not optimized. An increase in power consumption with speed is expected, and as for thrust, it will likely decrease with the increase of speed as it approaches its balanced state of zero net horizontal force.

![Figure 5.12](image)

**Figure 5.12:** Results provided by the joint BET model for varying values of forward speed.

The results shown in Figure 5.4 represent exactly that. While the lift and power increase with the speed, thrust actually begins to decrease substantially. This happens because the increase in velocity is not accompanied by the required variation in flapping, and while lift does not suffer with this, the net horizontal force applied on the wings slowly starts to suffer from not enough tilting of the wing and
increasing drag from the wings themselves. Lift increases in an almost linear way with speed because of the translational forces that it depends on, which in turn depend directly on the speed of the wing. The power curve also reflects the same effect, with the increasing loads meaning higher and higher power consumptions, though not as substantially as the flapping frequency.

It is worth noting that, in physical terms, actually none of the results where the thrust is different from zero represent a steady-state case. Because of the way that the program is structured, every time that the results show that there is a positive net force this means that the configuration is not in a balanced state and will tend to accelerate. Similarly, if the net force is negative, it will tend to slow down. This opens up the door to another possibility in terms of analysis, which is to find the precise point for every configuration where the flapping frequency and the speed are precisely right, meaning that the actual point of equilibrium for a specific set of parameters was found, where the weight equals the lift and the net horizontal force is zero, implying that the vehicle will not tend to accelerate nor decelerate. Perhaps surprisingly, this point also corresponds to the lowest power consumption possible, since no excessive forces are being generated, so it is clearly in the best interest of this work to find it. This explains the final and most refined analysis here undertaken, presented in the next section. As a final remark on the subject, it is important to emphasize how normally, in a real life scenario and if faced with an instantaneous increase in any force, a bird or flapping wing MAV would just vary slightly one parameter (say, for example, the maximum flapping or pitch angle) to achieve a balanced state at that particular flapping rate and forward speed. Since this is a much more complex analysis than what can be can put to the test, due to the sheer amount of variables that can be modified, and given that there is still value to this analysis (one could be designing a MAV that is not able to change any parameter aside from flapping frequency mid-flight), the steady state analysis will be performed nonetheless.

5.1.10 Steady State Analysis

This kind of analysis was implemented in the tool under the name Steady State Analysis, and it has the objective of finding the point for which the standard set of values produces a null net force, meaning that this point is actually the only one that has physical representation and could fly normally at that speed. As for drag, it was decided that the goal should be for it to be zero, since this is the most general test possible and indicates the limit case where there is no fuselage attached to the wings.

The algorithm works in a rather simple way: it starts by, at a certain speed, testing every frequency until it fulfills two conditions, both of which must be verified at the same time for it to move on to the next velocity. The conditions are that the lift must be greater than the weight and the thrust ought to be greater than zero. When the program finally finds the frequency that meets the two requirements, it reduces the step and performs the analysis again, around the result previously found, and in this way it is able to relatively easily find the flapping frequency with three decimal places of accuracy, and it does so for 100 points in between the minimum and maximum values that were chosen.

What happens in all cases except for one is that those conditions are not verified at the same time, leading, for example, to the tool trying to achieve a lift greater than the weight when the thrust is already
positive, or vice-versa. These points do not have any interest because they have no physical equivalent, but the inflexion point, however, represents the steady state case that was being searched for. This is simply because it is the point where both conditions are verified at the same iteration, and so the lift equals the weight and the net horizontal force is practically null.

![Graph](image1.png)

Figure 5.13: Variation of the minimum required power for every speed.

![Graph](image2.png)

Figure 5.14: Results provided by the Steady State Analysis for the standard set of parameters.

Inserting the standard values into the tool, the resulting power curves are the ones shown in Figure 5.13, and the variation of the flapping frequency, as well as lift and thrust, is shown in Figure 5.14. The curves are really enlightening, in the sense that they allow the visualization of what had been theorized previously. It can be seen that for the velocity values until $V_{\infty,ss}$, the limiting factor was the lift, since the thrust was clearly positive through this period and the routine only had to keep working to find the frequency that corresponded to the desired lift. Then comes the steady-state point, corresponding to $V_{\infty,ss} = 15.6 \text{ m/s}$, which is truly the interesting point. Here the conditions were met at the same time, which can be verified looking at Figure 5.14, since at this point both the thrust and lift curves change their behaviour abruptly. Then comes the zone where the thrust is the limiting factor, but again these values have no physical correspondent, so they have no real interest to the analysis.
The most valuable part of this analysis is that the minimum power consumption point was just uncovered. In this case it was found that the optimum flying mode corresponds to a wing flying at 15.6 m/s with a frequency of 9.601 Hz, which results in a thrust of 0.4905 N, a horizontal net force of $1.92 \times 10^{-3}$N and a power consumption of 6.155 W.

5.2 Configuration Comparison

Having found out how every parameter influences the behaviour and performance of the flapping wing, and having identified what the optimum strategy in terms of power consumption might be, a rudimentary optimization of the initial values can be performed, having the previous sections as a basis. One can then look at how these results fare against those of vehicles with different configurations and similar proportions and missions. The point of this work will then be to evaluate under which circumstances each approach might be more adequate, if any at all.

5.2.1 Fixed vs. Flapping Wing

To start this process, first the comparison term must be found. For the fixed wing this is a relatively simple task, as there is an abundance of information on the subject. The chosen reference in terms of power consumption was a study by Ostler et al. [26], where the power curve of a flying wing is calculated. The parameters of said wing are the following:

- Mass: 0.95 kg
- Wingspan: 1.06 m
- Aspect Ratio: 3.5

And the assumed motor-propeller efficiency is $\eta = 0.37$, which is the average value between $\eta_{\text{min}} = 0.22$ and $\eta_{\text{max}} = 0.53$ that Ostler calculates through wind tunnel testing. The power curve, as a function of speed, is shown in Figure 5.15 as the dotted line.

Now for the flapping wing, a first case where the wing shares the exact same parameters as the fixed wing is going to be assumed, them being the mass, the wingspan and the aspect ratio. This is not ideal, but will be done so that the advantages of the optimization process can be quantified. The next case corresponds to fixing only the mass and wingspan, so that there is one more variable (AR) to work with. Finally, in the third case only the weight of the vehicle will be maintained, so that the wingspan can be optimized as well. The objective is to compare the results of every optimization to the ones of the fixed wing, to see if at any point there will be a superior performance by the flapping wing.

The simple optimization process was done resorting to the Variable Dependant Analysis capability of the tool. Every parameter was varied to find its value that produced the lowest power consumption possible, which is not necessarily the point of highest efficiency. This was purposely done to find if, in absolute terms, there can be a flapping wing configuration that requires less power than its fixed wing counterpart. Then, the defined values were fed into the Steady State Analysis, which outputs the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ [kg]</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$b$ [m]</td>
<td>1.06</td>
<td>1.06</td>
<td>0.92</td>
</tr>
<tr>
<td>$AR$</td>
<td>3.5</td>
<td>2.7</td>
<td>2.2</td>
</tr>
<tr>
<td>$f_{flap}$ [Hz]</td>
<td>5.22</td>
<td>4.28</td>
<td>4.78</td>
</tr>
<tr>
<td>$V_{\infty}$ [m/s]</td>
<td>14.2</td>
<td>13.9</td>
<td>14.4</td>
</tr>
<tr>
<td>$xf$</td>
<td>0.22</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>$\gamma_{max}$ [°]</td>
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<td>39</td>
<td>39</td>
</tr>
<tr>
<td>$\theta_{max}$ [°]</td>
<td>42</td>
<td>40</td>
<td>39</td>
</tr>
<tr>
<td>$\theta_{avg}$ [°]</td>
<td>15</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Thrust [N]</td>
<td>0.085</td>
<td>0.093</td>
<td>0.209</td>
</tr>
<tr>
<td>Power [N]</td>
<td>82.21</td>
<td>72.37</td>
<td>71.40</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters and configuration performance metrics resulting from the optimization process.

The optimum flight point for the configurations at hand, namely the set of values shown in Table 5.1. As for the comparison with the fixed wing, Figure 5.15 was plotted to simplify the visualisation of the results. In it are included the fixed wing power curve, according to Ostler’s specifications [26], and the power results of the three different cases.

![Figure 5.15: Comparison between three different optimized flapping wings and a fixed wing with similar specifications.](image)

What can be observed is that, through this rudimentary optimization, values of power consumption which are similar in magnitude to those of the fixed wing were obtained. Not only that, it can be seen how more freedom in terms of parameter variation, meaning more flexibility design-wise, allowed for the discovery of increasingly better solutions to save power. The results in terms of general optimization were not, however, as pronounced as desired, with the most efficient flapping wing still consuming 104% more power than the equivalent fixed wing, roughly twice as much power. This further reinforces that a more sophisticated optimization process is required to achieve better results. It is not unreasonable to assume that there might be a combination of parameters that allows the flapping wing to outperform the fixed wing, but, as far as can be extracted from this test, the performances are really quite distinct.
Nevertheless, what can be drawn from this study is that for approximately the same specifications, the fixed wing has the edge in terms of power consumption.

5.2.2 Rotary vs. Flapping Wing

The case of rotary wings is somewhat more complicated than the first one. The problem resides mainly in the lack of information about small devices, which is the reason why the already established rotary wing parameters used in section 3.1 are resorted to. In order to accurately calculate the power consumption of a rotary wing vehicle, a simple BET model was built following Leishman’s theory, explained in detail in section 2.2. The power curve of the helicopter is shown as the dotted line in Figure 5.16.

For the flapping wing, the procedure was very similar to that of the previous analysis. Two cases were defined: in the first one the flapping wing shares both the mass and the size (the wingspan of the flapping wing is equal to the rotor’s diameter) with the rotary wing vehicle, and in the second one they only have the same mass. Aside from these factors, every other flapping design variable was free to change at will. Optimizing for the smallest possible power consumption and resorting to the Steady State Analysis resulted in the values shown in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ [kg]</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$b$ [m]</td>
<td>0.15</td>
<td>0.26</td>
</tr>
<tr>
<td>$R$</td>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>$f_{flap}$ [Hz]</td>
<td>24.1</td>
<td>12.4</td>
</tr>
<tr>
<td>$V_{\infty}$ [m/s]</td>
<td>23.4</td>
<td>14.7</td>
</tr>
<tr>
<td>$x_f$</td>
<td>0</td>
<td>0.16</td>
</tr>
<tr>
<td>$\gamma_{max}$ [$^\circ$]</td>
<td>77</td>
<td>48</td>
</tr>
<tr>
<td>$\theta_{max}$ [$^\circ$]</td>
<td>40</td>
<td>43</td>
</tr>
<tr>
<td>$\theta_{av}$ [$^\circ$]</td>
<td>14</td>
<td>13</td>
</tr>
</tbody>
</table>

| Lift [N]    | 0.491 | 0.491 |
| Thrust [N]  | $2 \times 10^{-4}$ | $9 \times 10^{-3}$ |
| Power [N]   | 5.76  | 3.43  |

Table 5.2: Parameters and configuration performance metrics resulting from the optimization process.

The objective of decreasing the power consumption was achieved, but the results varied wildly as there was a really significant save in power (around 40%) from just being able to use the wingspan as a variable. The comparison of these configurations with the rotary wing is shown in Figure 5.16. The plot conceals some interesting results, showing that a simple alteration like increasing the wingspan made a very big difference in terms of flapping wing optimization. There is still not a point where the flapping wing outperforms the rotary wing, but the second case shows a power consumption only 22% higher than that of a rotary wing carrying the same payload and moving at the same speed. Taking into account that there is a lot of room for improvement regarding the wing optimization, the only logical conclusion that can be drawn is that the possibility of designing a flapping wing which is actually preferable in terms of power consumption, compared to a rotary wing, is a very real one.
Figure 5.16: Comparison between two different optimized flapping wings and a rotary wing with similar specifications.
Chapter 6

Conclusions

The objective of this work started by being to find out how the flapping wing solution fared against other vehicles, namely the traditional fixed and rotary wing. In this spirit, the first efforts of the author were directed at trying to find an analytical and de facto way to compare different configurations, something that is quite unusual. Somewhere along the line, however, the focused shifted from that, in itself a relatively limited topic, to a much more specific and challenging matter, flapping wing simulation and the implementation of models that could describe such behaviour without resorting to any sort of computationally-heavy method. This opened up several unexpected options like the possibility to modify and implement already existing aerodynamic models of hovering flapping wing flight, even creating a whole new model based on the observations made while implementing the simpler ones. From then the natural development was to transform the perfected model into an actual tool, so that it could be used in future studies of flapping wings, and putting it to use by studying how every aspect of the wing design affected its overall success. Then, having found out every parameter’s influence on the overall performance, a simple optimization was done, and given that the original objective was to compare said results to those of the typical configurations, this was the final step in this thesis. In the end this comparison showed that it was possible to obtain a flapping wing that consumed twice as much power as an equivalent fixed wing, and 22% more than a similar helicopter, values that can likely be reduced with an adequate wing optimization process.

6.1 Achievements

This work produced results in several different fields; firstly, a relatively simple and quick analytical method for comparing different configurations was developed, and the results showed that there were, in fact, situations where each and every configuration was preferable.

On the field of BET models implementation, several aspects should be emphasized. Firstly, two different hovering models were implemented, and its results were validated. They were then modified to suit the forward flight case, to test if they were a good option in terms of describing the overall performance of a flapping wing vehicle. When that was proven to be the case, the best parts of each model
were taken and fused into a new model, which showed to be very reliable and was therefore used for the latter part of this work.

The model development also led to the creation of a tool that can be easily used by anyone trying to study flapping wings. By being user-friendly and running quickly, the tool opens up several possibilities in terms of optimization studies and preliminary wing design. This is the legacy of this work in terms of an interactable, useful, physical product that anyone will be able to use in the future.

Finally, on the topic of the optimization of the flapping wing, the tool was put to the test and a simple optimization was performed. The values that resulted from the optimization could then be collected into two perfected flapping wing designs, and these optimized wings were then compared the other configurations, finally providing the answer to the question “is a flapping wing really worth it?”, which is, in short and in the right circumstances, probably.

6.2 Future Work

The topic of flapping wings is a fascinating one partly because of how unexplored it is. The present thesis was an attempt at understanding how the aerodynamic mechanisms used by birds and insects work in their favour, to study if those same benefits can be transported to FWMAVs, effectively analysing how viable the implementation of flapping flight to man-made creations is. While some conclusions were drawn in terms of the optimization of flapping wings, this is perhaps the biggest question mark that is left by this work. Can the flapping wing be further optimized and, if so, what is the best procedure to do so?

Furthermore, it would be interesting to put the joint model to the test by seeing how it fares against a CFD simulation, to actually put numbers on the difference that there is between the methods. This would be the definitive test in terms of reliability and accuracy, and, if proven that the difference is not relevant, the present work could actually be considered a viable option to predict the behaviour, the loads and the power consumption of a flapping wing vehicle.
Bibliography


