Suborbital flight and other long distance travel alternatives

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Abstract

The present thesis is focused on the energy and efficiency analysis of a suborbital flight. A general design framework has been developed in order to compare the different configurations of this method of transportation and other long distance alternatives. First of all, a re-usable multistage rocket has been designed, this will constitute the base for future configurations. Secondly a Waverider and Wingbody type vehicles will be considered for re-entry. Thanks to their capability of generating lift they will perform a Skip re-entry manoeuvre, which will improve overall range and efficiency. As an alternative to suborbital flights a Hypersonic aircraft will be considered. The preliminary design of this vehicle will follow historical trends and data, as well as developing technologies. This configuration will use Turbine-based combined engine, in order to achieve speeds over Mach 5. Finally, this alternatives will be compared with current transportation, such as Jetliners, trains and cars. This will provide a comparison framework for every alternative, giving the best solution for a given range, flight time and number of passengers.

Keywords: Suborbital, hypersonic, efficiency, energy, optimization
Resumo

Na presente tese, a energia e eficiência de um voo suborbital são analisadas. Um projeto geral foi desenvolvido para comparar as diferentes configurações deste método de transporte e outras alternativas de longa distância. Em primeiro lugar, um foguete multiestágios reutilizável foi definido, de modo a constituir uma base para futuras configurações. Em segundo lugar, veículos do tipo Waverider e Wingbody foram considerados na reentrada. Graças à sua capacidade de gerar sustentação, foi considerada como opção uma manobra de “Skip Re-entry”, que melhora o alcance e a eficiência geral. Como alternativa aos voos suborbitais, uma aeronave hipersônica foi também considerada. O projeto preliminar deste veículo foi definido tendo em conta dados históricos, bem como o desenvolvimento de tecnologias. Essa configuração utilizará o motor combinado baseado em turbina para atingir velocidades superiores a Mach 5. Por fim, essas alternativas foram comparadas com o transporte atual como aviões, comboios e carros. Isso fornece uma estrutura de comparação para todas as alternativas, oferecendo a melhor solução para um determinado intervalo, tempo de voo e número de passageiros.

Palavras-Chave: Suborbital, hipersônico, eficiência, energia, otimização
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Chapter 1

Introduction

In this thesis the energy and efficiency requirements for a suborbital flight are analysed. Results will be compared with other transportation alternatives. Also, different possibilities have been taken into consideration for the most optimal path and launch system.

1.1 Current state of suborbital flights

Currently, to the author’s knowledge, there are no plans for the production of an affordable Suborbital flight transportation network. Both the public and private sector have been working on this mean of transportation for the last decade. Although most of the conducted research and development has been made on the space tourism topic Point to point (P2P) flights have been also analysed [4, 5].

The first space-plane to ever be designed for suborbital flights was the Boeing X-20 Dyna-Soar. Although it did not achieve its manufacturing phase, it was the first design of a series of important spacecraft such as the Space Shuttle. Its main purposes were reconnaissance and bombing. Further development was conducted by Boeing with the X-37 series [6]. The X-37 was designed for both suborbital and orbital flights, depending on the mission requirements. The X-37 completed a total of 5 missions, but due to it’s military nature they have not yet been publicly disclosed.

The private sector has also made great investments in the advancements of such technologies. One of the most notable projects was the space-plane called, Spaceship One, winner of a $10,000,000 prize by the X Prize Foundation in 2004 [4, 7]. The prize was to be awarded to the first company to launch a reusable vehicle into space two times in the short period of two weeks. Furthermore, the company Space X has recently announced that they are planning on using their new Big Falcon Rocket (BFR) for commercial suborbital flight, but further development is still needed [1, 8, 9]. The British company Reaction Engines Limited is also developing a space-plane both capable of orbital and suborbital flights, the Skylon. The Skylon is a single stage to orbit (SSTO) spaceship. It uses a hybrid propulsion system, being capable of both rocket and air breathing propulsion. Once it becomes operational, the Skylon will be capable of transporting up to 30 passengers to any location on Earth. Its first flight is expected to be on the year 2025 [10, 11].

1.2 The problem of suborbital flights

With the prospect of creating a new market to the public, different studies have been conducted in order to estimate the magnitude of this future industry. Previous work showed that most people do not fully
understand the concept of a suborbital flight. This study also showed that there are two potential opportunities for technology development. The first one being tourism and the second P2P transportation, which will be the main topic for this thesis. Suborbital transportation does not come without any limitations. Some authors assure that range plays an important role in efficiency. Flights with a range of under 3500km are not efficient enough to be considered as a possibility, with flights starting to be most efficient at around ranges of 7000km. Based on this results it can be assumed that longer ranges provide higher efficiencies.

For the engineering problem, there are a variety ways of approaching the problem. Some authors have compared the different alternatives available, from space-planes to fully ballistic flights. Some results indicate that space-planes which use a boost-glide manoeuvre achieve maximum range. Said study also confirms that while a boost-glide manoeuvre gives the vehicle a greater range, a rocket assisted flight with the same take off weight (TOW) could be more practical, since the later would be cheaper and more reliable. Companies such as Space X seem to be focusing more on the concept of a fully reusable rocket.

Most authors have preferred to use complex numerical methods in order to obtain accurate results. While this is the most optimal method for obtaining very precise results, it lacks an easy approach for understanding the obtained results. This thesis, will focus on giving solution to this problem while maintaining simple assumptions and only using numerical methods when there is no other way of resolution.

1.3 Thesis objectives

This thesis main focus will be on efficiency and energy optimization. Different alternatives will be analysed, advantages and disadvantages will be taken into account and the most practical solution for each situation will be determined.

The first solution that will be analysed is going to be a fully ballistic rocket. The problem of ballistic flights has been extensively studied in the past for military purposes. While the objective on this case is really different from this thesis’s it will be considered to be a good starting point for the analysis. Once this case has been studied, different variations and optimizations that can be made in order to further extend the range and increase the total efficiency of the system will be analysed. The most notable of this alternatives is the "skip-reentry" manoeuvre.

Different vehicles will also be compared, such as space-planes, SSTO systems and multi-stage rockets. As with most engineering problems, a solution could be more efficient than another with an initial set of variables, such as number of passengers and maximum range, so it is necessary to compare them in different situations and arrive at a final conclusion. Once analysed, these alternatives will also be compared with other means of transportation, such as hypersonic flights.

As mentioned above, past studies have been conducted using a numerical methods. The results obtained with these kind of simulations are precise, but lack practical way of understanding the physics behind the solution. In this thesis numerical resolution methods will also be used, but with simpler hypothesis and models. With this, it will be possible to further understand the problem of suborbital flight and how the different variables affect efficiency and energy consumption. The most optimal trajectory in order to minimize both gravity and drag losses will be analysed. For the rocket, the number of stages needed, the difference between both reusable and non-reusable rockets, the type of engine and fuel, all of this variables will be taken into consideration in this study.

The final goal is the comparison between suborbital flights and other current long distance travel alternatives, in terms of efficiency and energy expenditure. Other parameters will also be taken into ac-
count, such as flight time, structural and propellant mass. This analysis is of vital importance to conclude if suborbital flights are a competitive alternative to other methods of transportation. It is also necessary to know in which margins one method is superior to another, depending on number of passengers, range and time of flight among other variables.

Furthermore, future technology developments will be analysed. With this it will be possible to see the projection of this method of transportation. It will also be studied how different variables affect the overall efficiency of the vehicle. The most important are: specific impulse ($I_{sp}$), structural efficiencies ($\epsilon$), maximum allowed acceleration and number of passengers.
Chapter 2

Alternative configurations and Trajectory descriptions

2.1 Multi-stage rocket with ballistic trajectory

With the latest advancements on rocket re-usability by companies such as SpaceX and Blue Origin it is clear that a multi-stage rocket could be a suitable solution \[18, 19\]. The vehicle, for the sake of simplicity, will follow a vertical trajectory followed by a gravity turn. After this manoeuvre it will be injected into an elliptical orbit. The heat absorption for the re-entry phase will not be analysed in this thesis. Based on the results of previous work, it seems that controlling this parameter as a constraint does not greatly vary the maximum range, so it will not be taken into account. Important variables that will be analysed are: number of stages, structural efficiencies and specific impulse \[13, 16\].

It will also be analysed how re-usability affects the total efficiency and mass of the launch vehicle. The landing phase for a reusable rocket is the most critical part of the mission. Due to its complexity, it will not be fully analysed, instead, it will assumed that the rocket will carry extra fuel for this process based on the percentage of fuel that Space X has used of past launches. This percentages are 10% for a landing back at launch pad and 6% for a landing on a maritime platform \[20\]. This is due to the extra energy required for changing the ballistic trajectory. SpaceX places its maritime platform on the end of the ballistic trajectory that the vehicle would follow, if the stage is required to land back at the launch pad this trajectory must be changed, and thus, an extra 4% is used (see figure 2.1 for reference).

2.2 Multi-stage Rocket with Skip-Reentry

One way to further optimize the range obtained with the last configuration is to provide the last stage with lifting surfaces. This elements will provide an increase in the total weight of the payload, which will mean a bigger rocket. It will be studied if the extra range provided by this method will suffice the range lost by the extra weight. In the re-entry the vehicle will make a manoeuvre called “Skip-Reentry”. The vehicle will assume the angle of attack which provides the best lift to drag coefficient. As it enters the atmosphere and density increases the vehicle will start to gain altitude again, until the atmosphere becomes too thin and its trajectory is determined by an ellipse again (see figure 2.2). This will occur again until the vehicle loses enough speed due to drag losses when it will glide to its final destiny. For the geometry and structure of the flying vehicle two different configurations will be studied, a lifting-body and Waverider, based on previous author’s work. The Waverider is design that improves its lift-to-drag ratio by using the shock waves generated by its own movement. This method will also provide another
important advantage. Since the vehicle is going to gradually lose speed on each re-entry it will not absorb as much energy in the form of heat. This means that the trajectory can be optimized without the consideration of overheating problems [13, 16].

2.3 SSTO with Skip Re-entry

In terms of re-usability, a single stage to orbit would seem like the most practical solution. The problem with this system relies on the capability of attaining a sufficient maximum range. Energy efficiency improves greatly with the addition of additional stages, specially with the first two [21]. This means that the payload which a SSTO system can carry is much smaller in terms of mass. As it will be shown in later chapters, the boost glide manoeuvre will be needed in order to reach a sufficient range. It will also be assumed that the vehicles produce zero lift when its angle of attack $\alpha$ is equal to zero. Rockets are designed to sustain the minimum amount of force on its perpendicular axis, with the last assumption it will be possible to minimize the structural coefficient of the vehicle, improving its efficiency.

An important demonstrator for this technology is the X-33 [22]. The X-33 would have used a revolutionary engine called the "Aerospike", which would have improved the overall efficiency of the vehicle. The possibility of this type of engine will be considered when analysing this configuration [23].
2.3.1 Hypersonic aircraft

As mentioned in chapter 1, a hypersonic aircraft is an important alternative for long distance transportation. While the analysis of this type of vehicles is not the main focus of this thesis, it can be an important point of comparison for suborbital flights. The technology for hypersonic transportation has not yet been fully developed. The main developments have been made by NASA with the X-43 and X-51 series [24, 25]. The X-43 was an experimental unmanned vehicle which purpose was the experimentation at hypersonic speeds. It used a scramjet engine to achieve a maximum speed of Mach 9.6 [13, 26]. The X-51 also used the scramjet engine, using a Waverider configuration which improved its aerodynamic capabilities. Both vehicles showed promising results, but its scale made them not viable to carry any passengers. One of the main problems that both vehicles encountered was the high temperatures reached on their surfaces. This problem will not be analysed in this thesis, and it will be assumed that the materials will be chosen correctly for this matter. A Waverider is a hypersonic aircraft configuration which maximizes lift-to-drag ratio using the shock waves created by the vehicle. This design will be used in the aerodynamic analysis of both the re-entry vehicles and the hypersonic aircraft [26].

Figure 2.3: Artist’s conception of the X-33 after engine shutdown [3].
Figure 2.4: Example of existing hypersonic aircraft [26].
Chapter 3

Simplifying Assumptions

The problem of rocket launch calculation and trajectory optimization requires, generally, powerful computers and long processing times. Since the objective of this thesis is not creating a simulation program, but calculating general energy cost and efficiencies, simplified models will be used in order to keep the solution simple as well.

3.1 Physical models

3.1.1 Earth model

This thesis objective is to arrive to a general solution for the problem of suborbital flight, for this reason it is not convenient to analyse specific trajectories, i.e., Lisbon to Los Angeles. It would be more appropriate to calculate the energy required for a trajectory without taking into account variables such as longitude and latitude. The approximation given by a perfect non-rotating Earth model will be considered satisfactory for obtaining adequate results with low error. The flight trajectory will be described by a two-dimensional curve, and since the earth rotational speed is not taken into account the trajectory for any two points that are the same distance apart will be the same [13, 27].

3.1.2 Gravitational model

With the last assumption that the earth is a perfect sphere, Newton’s gravitational law of gravity. The acceleration caused by this force will be given by the following equation [27, 28]:

\[
g(H) = g_0 \left( \frac{R_0}{R_0 + H} \right)^2
\]  

(3.1)

With \( g_0 \) being the acceleration of gravity on Earth at sea level and \( R_0 \) the radius of Earth:

\[
g_0 = 9.81; R_0 = 6371
\]  

(3.2)

3.1.3 Atmospheric model

An exponential atmospheric model will be used for the resolution of this problem [21]. The results obtained with this model

\[
\rho(H) = \rho_0 \times \exp\left(-\frac{H}{K}\right)
\]  

(3.3)
With $K$ being a constant with value: $K = 7500$.
Nonetheless, for the hypersonic aircraft case the standard atmosphere model will be used. The hypersonic aircraft will be assumed to maintain a constant cruise altitude of 20km, and for this reason only one operation will be required to calculate the air density and temperature [27].

### 3.2 Capsule and launcher design

The complete design of the rocket and passenger capsule escapes the intended reach of this thesis. Existing technologies will be used for the design of our vehicle. The only other commercially available rockets with a reusable design are the Falcon9 and Falcon Heavy. With this in mind, most of the technologies used in the design of the Falcon 9 rocket will be used for this thesis's models. For the 2-stage launcher the Falcon 1 values for structural efficiency, engine specific impulse and rocket diameter will be used. For the capsule weight estimation a linear regression taking into account the weight and number of passengers of existing capsules will be used. With the linear regression will provide a function that will give return an approximate value of the capsule mass depending on the number of passengers [1, 18].

![Figure 3.1: Linear regression for the capsule weight determination](image)

When considering the capsules with lifting surfaces the extra weight that this structures provide must be taken into account. For this calculations we will make an estimate of the extra mass based on models provided by Michael N. Beltramo and Donald L. Trapp [29]:

$$M_{Wings} = 0.112TOW - 780.1789$$ \hspace{1cm} (3.4)

$$M_{Landinggear} = 0.0439TOW - 929.8644$$ \hspace{1cm} (3.5)

For all intents and purposes, it will be assumed that our vehicle will fall into the category of a medium military aircraft. The data used for this calculation can be seen in figure [3.1] which gives the following formula for the capsule weight ($W$) depending on Number of passengers ($N$):

$$M_{capsule} = 682.21(N + 2) + 1406.9$$ \hspace{1cm} (3.6)
Solving for the re-entry vehicle with no lifting surfaces the total payload mass \((M_{pl})\) is:

\[
M_{pl} = (682.21(N + 2) + 1406.9) + ((N + 2)M_{passengers})
\]  

(3.7)

And for the vehicles with wings:

\[
M_{pl} = \frac{(682.21(N + 2) + 1406.9) + ((N + 2)M_{passengers}) + 1710.0433}{0.8441}
\]  

(3.8)

It is being assumed that the crew will only consist of a pilot and copilot, so there are two extra passengers aboard the vehicle.

Another important variable that must be taken into account is the structural coefficient of the rocket, which is defined as follows:

\[
\epsilon = \frac{m_{empty}}{m_{empty} + m_{propellant}}
\]  

(3.9)

The structural coefficient is a good measurement of the efficiency of a rocket, the lower it is the more propellant mass the vehicle carries so it is able to transport a bigger payload. It is an important variable when calculating the total mass of the rocket, and must be known prior to its calculation. Since its estimation can only be done following historical data, values of existing two and three stage rockets will be used. The Falcon 1 rocket was chosen for its similarity with the Falcon 9 launcher, using the same engine and number of stages. The Delta II rocket was chosen because its total mass being in the same order of magnitude as the Falcon 9 rocket [30].

<table>
<thead>
<tr>
<th>Structural Coefficients ((\epsilon))</th>
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<tr>
<td>Falcon 1 First stage: 0.056</td>
</tr>
<tr>
<td>Second stage: 0.112</td>
</tr>
<tr>
<td>Delta II First stage: 0.056</td>
</tr>
<tr>
<td>Second stage: 0.136</td>
</tr>
<tr>
<td>Third stage: 0.094</td>
</tr>
</tbody>
</table>

Table 3.1: Structural coefficients of the Falcon 1 and Delta II rockets

For the rocket diameter the Dragon II capsule of Space X will be used, since is the one used by the Falcon 9. Its diameter is equal to to 3.66 meters [18]. This value will be considered constant for any given number of passengers and initial TOW. The reason for this choice is to minimize the number of variables to take into account when comparing the different alternatives.

### 3.2.1 Passenger mass estimate

Once the capsule and rocket weight have been calculated it is necessary to calculate what each passenger and its luggage will weight. When estimating this values for aircraft a conservative approximation is 100 kilograms for each passenger and its luggage. An extra weight for vital systems will be considered for the passengers. Our vehicle will reach an altitude where there is no atmosphere, so this kind of system is necessary for the comfort and security of the passengers. The A7L suit used in the Apollo program will be used as a reference for the mass of vital systems required. Its weight is approximately 29.3 kgs [31] [32].

The total weight per passenger will be:

\[
M_{passenger} = 129.3
\]  

(3.10)
### 3.2.2 Engine choice

The models and calculations will be based on the Merlin rocket engine, used on the Falcon rocket family. The reason for this choice is that since the calculations and models are mainly based on the reusable rockets of the Falcon series it can be assumed that it meets all the demands for a re-usable rocket [1]. As it will be later shown in future chapters, the Thrust provided by this kind of engine will not be enough for the SSTO configuration. For this reason the Aerospike engine mentioned in the last chapter will be used as an alternative.

<table>
<thead>
<tr>
<th>Merlin Rocket Engine</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Sea Level Thrust</td>
<td>653.89 kN</td>
</tr>
<tr>
<td>Vacuum Thrust</td>
<td>716.16 kN</td>
</tr>
<tr>
<td>Sea Level $I_{sp}$</td>
<td>282s</td>
</tr>
<tr>
<td>Vacuum $I_{sp}$</td>
<td>311s</td>
</tr>
<tr>
<td>Mean $I_{sp}$</td>
<td>296.5s</td>
</tr>
</tbody>
</table>

Table 3.2: Merlin rocket engine characteristics

<table>
<thead>
<tr>
<th>X-33 Linear Aerospike Rocket Engine</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea Level Thrust</td>
<td>916.33 kN</td>
</tr>
<tr>
<td>Sea Level $I_{sp}$</td>
<td>339.9</td>
</tr>
<tr>
<td>Vacuum $I_{sp}$</td>
<td>429.8</td>
</tr>
<tr>
<td>Mean $I_{sp}$</td>
<td>384.8500</td>
</tr>
</tbody>
</table>

Table 3.3: Linear Aerospike rocket engine characteristics

$I_{sp}$ depends on the external pressure as it can be seen in the table. For simplicity it will be assumed to be constant for each stage. For the first stage of the rockets the first value will be used and for later stages the second one. For the SSTO vehicle the arithmetic mean will be used.

### 3.3 Aerodynamic model

Drag coefficient depends on Mach number, peaking at $M \simeq 1$ with a value of $C_D = 0.2$. The drag coefficient will be considered constant and equal to this value. With this choice it can be expected to get conservative results, as it is the worst case scenario [33].

For the re-entry vehicles with lift two different types or aircraft will be used. The first one will be a vehicle with a waverider configuration [13, 26]. The following equations describe the Lift and Drag coefficients of said configuration [13]:

\[
C_L = -0.03 + 0.75\alpha \tag{3.11}
\]

\[
C_D = 0.012 - 0.01\alpha + 0.6\alpha^2 \tag{3.12}
\]

\[
\left(\frac{C_L}{C_D}\right)_{max} = 3.5441; \alpha = 10.5825^\circ \tag{3.13}
\]

A wing-body will be considered as a second configuration [13]:

\[
C_L = -0.034 + 0.93\alpha \tag{3.14}
\]
\[ C_D = 0.037 - 0.01\alpha + 0.736\alpha^2 + 0.937\alpha^3 \]  
(3.15)

\[ \frac{C_L}{C_D}_{\text{max}} = 2.1225; \alpha = 12.4217^\circ \]  
(3.16)

As it can be seen, both functions depend on the angle of attack \(\alpha\) only. This is considered to be true at hypersonic speeds. All models were obtained from previous work presented by H. Linshu and X. Dajun [13].

The wing load will be calculated with the following equation based on the work of previous authors [13]:

\[ \frac{M}{A_w} = 400 \]  
(3.17)

With \(M\) being total initial mass of the aircraft and \(A_w\) the wing area.

### 3.4 Re-entry and landing

The landing method for the rocket stages will be based on the one used by Space X. After detachment, the engines will be reignited in order to land back at the launch pad or on a maritime platform. Instead of making a full simulation a simpler method will be used. It will be assumed that the rocket will be carrying an extra propellant weight based on the one Space X uses for the Falcon9 rocket, and that this fuel is enough in order to slow down and land the vehicle. This percentages will be considered 10% for a landing back at the launch pad and 6% for a maritime platform [20].

### 3.5 Hypersonic aircraft

Following the design followed by NASA with the X-51 series [25], the hypersonic aircraft, will have a Waverider configuration. This will maximize the aerodynamic efficiency. The weight of the vehicle for a given range and number of passengers will be estimated. This results will be compared with the other alternative suborbital configurations. In order to compare the energy expenditure between the two means of transportation the amount of propellant used by each vehicle will be compared. For propulsion at cruise a scramjet engine based on previous work will be used [31, 34].

The main parameter that must be known prior to any calculation for the vehicles T\text{OW} is the specific fuel consumption (SFC). The \(I_{sp}\) can be estimated [34], which is directly related to the SFC:

\[ I_{st} = \frac{1}{(g_0 \times SFC)} \]  
(3.18)

With figures 3.2 and 3.3 \(I_{sp}\) can be estimated based on cruise altitude and engine type. This values will be used in chapter 4 to calculate the MT\text{OW} and the fuel mass of the aircraft.
3.6 Propellants and fuel

In order to compare the energy expenditure between different alternatives it is necessary to know the specific energy of each of their fuel and propellants. The Merlin rocket engine uses RP1/LOX as propellant. The RP1 is a high quality kerosene, for this reason its specific energy will be considered the...
same than jet fuel. The Aerospike engine uses LH2/LOX for propellant. Liquid hydrogen posses higher energy density than kerosene, but must be stored in pressurized containers. Rockets must carry both the fuel and the oxidizer as opposed to other methods of transportation. For this reason it is necessary to calculate the percentage of fuel accounted in the total propellant mass of the rocket. The total mass of a rocket propellant is divided between its oxidizer and fuel mass:

\[
M_p = M_f + M_o
\]  

(3.19)

The oxidizer-to-fuel mass ratio for RP-1/LOX is 2.56 and 4.7 for LH2/LOX. Equation 3.19 can then be rewritten as follows:

\[
M_f = \frac{M_p}{3.56}
\]  

(3.20)

\[
M_f = \frac{M_p}{5.7}
\]  

(3.21)

Finally, when compared with land vehicles such as cars the specific energy of gasoline will be needed. All this values can be seen in table 3.4.

<table>
<thead>
<tr>
<th>Storage Material</th>
<th>Specific Energy ( \frac{MJ}{kg} )</th>
<th>Specific Energy ( \frac{MJ}{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen (compressed at 700 bar)</td>
<td>142</td>
<td>9.17</td>
</tr>
<tr>
<td>Diesel</td>
<td>48</td>
<td>35.8</td>
</tr>
<tr>
<td>Gasoline</td>
<td>46.4</td>
<td>34.2</td>
</tr>
<tr>
<td>Jet Fuel (Kerosene)</td>
<td>42.8</td>
<td>37.4</td>
</tr>
</tbody>
</table>

Table 3.4: Energy densities of common energy storage materials
Chapter 4

Trajectory Calculations

In order to simplify the numerical resolution of this problem the solution will be separated into two. First, the forces that act on the vehicle at launch will be studied. Afterwards, the free flight and atmospheric re-entry will be analysed.

4.1 Launch

Following the work of previous authors, a gravity-turn trajectory of a vehicle that does not produce lift is governed by the following set of equations:\[16\]:

\[
\begin{align*}
\frac{m}{dt} & = T - D - \left( mg - \frac{mX^2}{R_0 + H} \right) \sin \gamma \\
\frac{mV}{dt} & = -\left( mg - \frac{mX^2}{R_0 + H} \right) \cos \gamma \\
\frac{dX}{dt} & = V \cos \gamma \\
\frac{dH}{dt} & = V \sin \gamma
\end{align*}
\] (4.1)

Where \( X \) is the ground distance in meters, \( V \) the speed of the rocket, \( T \) the thrust generated by the engines, \( D \) the drag generated by the vehicle, \( m \) the total mass of the rocket at any given moment, \( \dot{X} \) the ground speed of the rocket, \( H \) the altitude of any given time and \( \gamma \) the relative angle to the horizon.

As stated in chapter 3, this equations will be solved numerically. A forward Euler discretization will be applied:

\[
\begin{align*}
\frac{m_i V_{i+1} - V_i}{t_{i+1} - t_i} & = T_i - D_i - \left( m_i g_i - \frac{m_i \dot{X}_i^2}{R_0 + H_i} \right) \sin \gamma_i \\
\frac{m_i V_i \gamma_{i+1} - \gamma_i}{t_{i+1} - t_i} & = -\left( m_i g_i - \frac{m_i \dot{X}_i^2}{R_0 + H_i} \right) \sin \gamma_i \\
\frac{X_{i+1} - X_i}{t_{i+1} - t_i} & = V_i \cos \gamma_i \\
\frac{H_{i+1} - H_i}{t_{i+1} - t_i} & = V_i \sin \gamma_i
\end{align*}
\] (4.5) (4.6) (4.7) (4.8)
Solving for $V_{i+1}$, $\gamma_{i+1}$, $H_{i+1}$ and $X_{i+1}$ and taking the time step constant and equal to $\Delta t = t_{i+1} - t_i$:

$$V_{i+1} = V_i + \left( \frac{\bar{T}_i}{m_i} - \frac{D_i}{m_i} - (g_i - \frac{\dot{X}_i^2}{R_0 + H_i}) \sin \gamma_i \right) \Delta t$$  \hspace{1cm} (4.9)

$$\gamma_{i+1} = \gamma_i - \left( g_i - \frac{\dot{X}_i^2}{R_0 + H_i} \right) \frac{\cos \gamma_i}{V_i} \Delta t$$  \hspace{1cm} (4.10)

$$X_{i+1} = X_i + V_i \cos \gamma_i \Delta t$$  \hspace{1cm} (4.11)

$$H_{i+1} = H_i + V_i \sin \gamma_i \Delta t$$  \hspace{1cm} (4.12)

Expanding the drag ($D$):

$$D_i = q_i C_D A$$  \hspace{1cm} (4.13)

And with $q$ being the dynamic pressure equal to:

$$q_i = \frac{1}{2} \rho_i V_i^2$$  \hspace{1cm} (4.14)

And $\rho$ being the air density at any given altitude as stated in the last chapter:

$$\rho_i = \rho_0 \exp\left(-\frac{H_i}{7500}\right)$$  \hspace{1cm} (4.15)

Finally, according to Tsiolkovsky rocket equation the thrust of a chemical rocket is given by the following equation\[16, 21\]:

$$T_i = I_{sp} \times g_0 \dot{m}_i$$  \hspace{1cm} (4.16)

Where $\dot{m}_i$ is the fuel mass rate of the rocket.

Being $g_0$ the acceleration of gravity at sea level and $I_{sp}$ the specific impulse of our engine. The value of $I_{sp}$ will vary depending on each stage. The initial value for each of the independent variables are:

$$V_0 = 0; H_0 = 0; \gamma_0 = 90^\circ;$$  \hspace{1cm} (4.17)

The initial value of $\dot{m}_i$ will be chosen depending on the initial Thrust to Weight ratio. Since this ratio will vary from simulation to simulation it will be defined by the following equation:

$$\dot{m}_0 = \left( \frac{T}{W} \right)_0 \frac{M_0}{I_{sp}}$$  \hspace{1cm} (4.18)

With equation (4.17) the resulting acceleration that our vehicle experiences can be seen. As stated in past chapters, it is of vital importance to define a maximum allowed acceleration for the sake of comfort and security for the passengers. According to equation (4.16) and (4.18) thrust will be constant through the launch, and since the vehicles mass decreases with time this means that the rocket will start to gain acceleration as it losses weight. This effect will be amplified by the fact that both drag and gravity decrease with altitude. This will be solved by creating a constraint of maximum acceleration, making the fuel flow $\dot{m}$ drop once it reaches the defined maximum acceleration.

The code will also take into account the stage detachment. Since the fuel flow is known $\dot{m}_i$, it can be calculated how the total and propellant mass vary. When the mass reaches a certain determined value the script will subtract the total mass of the stage, giving us a new set values for all variables.
Another important issue is how the gravity turn will be carried out. As it can be seen in equation 4.2 with the initial value of $\gamma = 90^\circ$, $\frac{d\gamma}{dt}$ will always be equal to zero. The gravity turn manoeuvre will begin at a given time $t = 3s$ when a change in the lifting surfaces or the engine nozzle will create a small perturbation giving $\gamma$ a new value equal to $\gamma = 90^\circ - \delta$ with $\delta \ll 1$. This will in return start said manoeuvre until the value of $\gamma$ reaches the needed value for the trajectory.

### 4.1.1 Ballistic flight optimization

The free flight trajectory of a ballistic rocket can be determined by two variables, the non dimensional parameter $Q$ at burnout and the trajectory angle at burnout $\phi$ \[38\]. $Q$ can be defined as follows, being $\mu$ the gravitational constant of the Earth:

$$Q = \frac{V^2(R_0 + H)}{\mu} \tag{4.19}$$

The values of these variables that maximize range in the ballistic flight must be calculated with the following set of equations \[38\]:

$$\phi_{bo} = \frac{1}{4}(180^\circ - \Psi) \tag{4.20}$$

$$Q_{bo} = \frac{2\sin\left(\frac{\Psi}{2}\right)}{1 + \sin\left(\frac{\Psi}{2}\right)} \tag{4.21}$$

With $\Psi$ being the angle in degrees of the free flight portion of the trajectory.

### 4.1.2 Multistage optimization

As stated before, a single stage rocket will generally not be capable of possessing the required efficiency for a long distance ballistic flight. A multi-stage rocket is then a more suitable solution. One of the problem that must be solved is the optimization of the propellant mass carried by each stage. This will be solved following a Lagrange multiplier optimization\[21\].

$$h = N_{\text{stages}} \sum_{i=1}^{N_{\text{stages}}} \left[ \ln(1 - \epsilon_i) + \ln \Lambda_i - \ln(1 - \epsilon_i n_i) \right] - \eta(V_{bo} - N_{\text{stages}} \sum_{i=1}^{N_{\text{stages}}} c_i \ln n_i) \tag{4.22}$$

$$\frac{\partial h}{\partial \Lambda_i} = 0 \tag{4.23}$$

$$\frac{\partial h}{\partial \eta} = 0 \tag{4.24}$$

Where $c$ is the effective exhaust velocity and $\Lambda$ is the mass ratio of each stage which are defined:

$$c = I_{\text{st}} g_0 \tag{4.25}$$

$$\Lambda = \frac{M_0}{M_f} \tag{4.26}$$

Which yields the following general solution, that must be solved iteratively for $\eta$:

$$\sum_{i=1}^{N_{\text{stages}}} c_i \ln \frac{c_1 \eta - 1}{c_i \epsilon_i \eta} = V_{bo} \tag{4.27}$$
Once this equation is solved the result can be used to calculate the optimum mass ratio for each stage:

\[ \Lambda_i = \frac{c_i \eta_i - 1}{c_i \epsilon_i \eta_i} \]  

(4.28)

4.2 Free-flight and Re-entry

The free flight and re-entry problem will be modelled with the following set of equations, where a lift component \((L)\) for the vehicles with lifting surfaces will be added. It will also be assumed that Thrust \((T)\) is equal to zero[16]:

\[ \frac{m \, dV}{dt} = -D - (mg - \frac{m \dot{X}}{R_0 + H}) \sin \gamma \]  

(4.29)

\[ mV \frac{d\gamma}{dt} = L - (mg - \frac{m \dot{X}}{R_0 + H}) \cos \gamma \]  

(4.30)

\[ \frac{dX}{dt} = V \cos \gamma \]  

(4.31)

\[ \frac{dH}{dt} = V \sin \gamma \]  

(4.32)

Following a forward Euler scheme:

\[ V_{i+1} = V_i - \left( \frac{D_i}{m_i} + \left( g_i - \frac{\dot{X}_i^2}{R_0 + H_i} \right) \sin \gamma_i \right) \Delta t \]  

(4.33)

\[ \gamma_{i+1} = \gamma_i + \left( \frac{L_i}{m_i V_i} - \left( g_i - \frac{\dot{X}_i^2}{R_0 + H_i} \right) \frac{\cos \gamma_i}{V_i} \right) \Delta t \]  

(4.34)

\[ X_{i+1} = X_i + V_i \cos \gamma_i \Delta t \]  

(4.35)

\[ H_{i+1} = H_i + V_i \sin \gamma_i \Delta t \]  

(4.36)

Expanding the lift force \((L)\):

\[ L_i = q_i C_L V_i^2 \]  

(4.37)

4.3 Computer simulation

For the resolution of this numerical problem a Matlab script was used. The equations showed in this chapter must be solved iteratively. First of all, our ballistic flight optimization depends only on one parameter, the free flight range \((\Psi)\). This value will be fixed as an initial parameter, which will in return give a value for the non-dimensional parameter \(Q\) and \(\varphi\) at burnout. As the value of \(Q\) raises the bigger the rocket will be, for this reason the initial guess for the delta-V required will be lower than the expected one. This value will be used for the multi-stage optimization, which will in return give us the total mass of the vehicle. With this value the simulation can be started, once it finishes, the script will return a final value for \(Q_{bo}\), if this value is lower than the required one in the ballistic optimization, the simulation will
be repeated using a higher value for delta-V. This operation will be repeated until the error between the required value and the obtained value is close to zero. This process is further explained in figure 4.1.

**Figure 4.1: Matlab script explanation**

### 4.4 Hypersonic aircraft

In order to estimate the total and fuel mass for this vehicle models and estimations provided by Thomas C. Corke will be followed [31]. The different mass fractions are calculated for each phase of the travel as following:

- **Take off and Landing:**
  \[ 0.97 < \frac{W_f}{W_i} < 0.975 \]  
  \( (4.38) \)

- **Climb and acceleration to Cruise:**
  \[ \frac{W_f}{W_i} = 0.96 - 0.03(M_{cruise} - 1) \]  
  \( (4.39) \)

- **Cruise to destination:**
  \[ \frac{W_f}{W_i} = \exp\left(-\frac{SFC_{min} \times g \times Range}{V_LD}\right) \]  
  \( (4.40) \)

- **Landing:**
  \[ 0.97 < \frac{W_f}{W_i} < 0.975 \]  
  \( (4.41) \)

Where \( W_f \) and \( W_i \) are the final and initial mass of the vehicle at each phase, respectively.
It will be assumed that the vehicle adopts its most efficient configuration, in order to maximize its lift to drag coefficient ($\frac{L}{D}$). For the waverider aerodynamic model this value is the following [26]:

$$\frac{L}{D} = 3.5441; \alpha = 10.5825^\circ$$

Then, the total fuel for the aircraft can be calculated with the following equation. It is necessary to take into account that the rocket must carry an extra 6\% of fuel, 1\% would be the unusable trapped fuel and the other 5\% reserve fuel.

$$W_{fueltotal} = (MTOW_1 - W_f)(1 + \frac{6}{100})$$

The available empty weight (AEW) will then be calculated, where Non Exp is the non expendable cargo. It is assumed that the aircraft will only carry passengers and their luggage.

$$AEW = MTOW_1 - W_{fueltotal} - NonExp$$

The required empty weight (REW) will now be calculated. The REW is the structure weight it can be expected from a particular type of aircraft. Since this formula relays on historical trends it will be assumed that our vehicle will be similar to a supersonic cruiser type of aircraft [31]:

$$REW = MTOW_1 \times SF$$

$$SF = A \times W_{C_{TO}}$$

$$A = 1.02; C = -0.06$$

With this data the expected MTOW can now be calculated as following:

$$MTOW_1 = W_{fueltotal} + REW + NonExp$$

The next step is calculating the difference between the initial guess value and the expected MTOW

$$\delta_1 = MTOW_1 - MTOW_1$$

If this value does not tend to zero the iteration will be repeated with the resulted MTOW until both values are the same.

$$MTOW_2 = MTOW_1 - \delta_1$$

With this procedure, the initial MTOW can be estimated. Afterwards an equilibrium of forces can be applied in order to estimate the wing area and the required Thrust ($T$). It will be assumed that the engines will provide the required thrust needed for the flight (see figure 4.2 for reference).

$$T \cos(\alpha) - D \cos(\alpha) - L \sin(\alpha) = 0;$$

$$T \sin(\alpha) - D \sin(\alpha) + L \cos(\alpha) - W = 0;$$

With drag ($D$) and lift ($L$) having the following values, where $A_w$ is the area of the wing:

$$D = qC_D A_w$$
Figure 4.2: Forces acting on an aircraft.

Source: http://www.phy6.org/stargaze/Sflight2.htm. Dr. David P. Stern

\[ L = qC_L A_w \]  
(4.52)
Chapter 5

Results and Discussion

In this chapter all results obtained with the procedures and models explained in past chapters will be presented. The mass ratio will be used as an efficiency measurement. This value is equal to the fraction between the initial total mass and the final mass of the vehicle, and must always be larger than one. An efficient rocket demands less propellant mass to achieve a required delta-V, meaning that its mass ratio is lower. The mass ratio is defined as follows:

\[ \Lambda = \frac{M_0}{M_f} \]  

(5.1)

It will be later shown in this chapter that the variation of this value is directly proportional to the variation of total mass at launch. For this reason both values are proper for efficiency measurement.

5.1 Reusable rocket

First, the reusable rocket configuration will be analysed. This configuration uses the same optimization that the Waverider and Wingbody re-entry vehicles use at launch. For this reason results obtained for this vehicle can be applied to all vehicles and configurations. In this analysis different variables and parameters will be modified to see how they affect the rocket efficiency, as well as other parameters.

![Reusable rocket ballistic trajectory](image)

Figure 5.1: Reusable rocket ballistic trajectory
5.1.1 Effect of maximum acceleration

An important variable that must be taken into account is the maximum allowed acceleration. Previous work showed that an average human can tolerate and properly function over an hour with an acceleration of $3g$, with higher accelerations starting to be problematic for long periods of time [14].

As it can be seen in figure 5.2, the total rocket mass and mass ratio ($\Lambda$) decrease with maximum $g$, but for ranges of 2-3 $g$ this variation is almost negligible. It can also be observed that for ranges of approximately 20,000km the total initial mass is the same. For this reason, for following calculations the maximum acceleration will be 2.5$g$, as it is a conservative value for both efficiency and passenger comfort.

![Figure 5.2: Total mass in tons and mass ratio ($\Lambda$) depending of maximum allowed acceleration](image)

It can also be noted in figure 5.3 that this decrease in efficiency is due to our launch vehicle being injected in a more eccentric elliptic orbit, which requires more energy. Our time of flight also increases with lower maximum accelerations. For figure 5.3 the total flight time in minutes are:

$$T_{1g} = 54.6; T_{2g} = 50.6; T_{3g} = 50.3 \quad (5.2)$$
Finally, as it can be seen in figure 5.4, the maximum allowed acceleration is only achieved by the third stage for values bigger than 2g. This result on the small differences on total mass and efficiency between the different values of maximum acceleration. This is an important result, as it can be seen that the vehicles Thrust can be further optimized in order to achieve better results.

5.1.2 Effect of number passengers and stages

The analysis of number of passengers is equivalent to the analysis of total payload mass. As it can be seen in figures 5.5 and 5.6, efficiency improves with the number of stages. This results were expected based on previous work [21]. Efficiency also improves with total number of passengers as well as the
total weight. It is also important to note the scale of the results obtained for total mass. Even though efficiency increases slightly with number of passengers the total mass reaches values never before seen in space launchers. For comparison, the Saturn V total mass at launch was around 2,900 tons. The Saturn V is still the heaviest rocket ever built. Space X BFR is expected to have a total mass of 4,400 tons and will carry around 100 passengers to Mars after being refuelled in orbit. For conservative results, following calculations will assume the total number of passengers to be 20 when this value is not mentioned.

![Figure 5.5: Total mass and mass ratio depending on number of passengers, range and stages (1)](image)

![Figure 5.6: Total mass and mass ratio depending on number of passengers, range and stages (2)](image)

The primary advantage that suborbital flights offer is the possibility of being anywhere in the world in under one hour. As it can be seen in figure 5.7 time of flight ranges from 30 to 60 minutes. It increases slightly with the number of stages and slightly decreases with number of passengers. The relationship
between flight range and distance is almost linear.

Figure 5.7: Time of flight depending on number of passengers, range and stages

Finally, it is important to analyze how the energy consumption per passenger varies with number of stages and passengers. Figure 5.8 shows that the energy consumption decreases with number of stages and passengers, confirming results observed in figure 5.6. This increase in efficiency is smaller every consequent passengers. This results will later be compared with other configurations.

Figure 5.8: Specific energy per passenger as a function of Range

5.2 Skip re-entry

The skip re-entry manoeuvre improves the overall efficiency and range of the vehicle. The increase in range provided by the lifting surfaces outweighs the losses due to a bigger cargo. In this simulations it was assumed that the launch vehicle had 3 stages and 20 passengers. The first stage would land back at the launch pad, the second at a maritime platform and the third would be incorporated in the re-entry vehicle and land at destination.
The Waverider configuration proves to be the most efficient overall. Its total mass for any given range is the lowest of any configuration, as well as its mass ratio. The Wingbody configuration also shows good results, being more efficient than the reusable rocket but falling behind the Waverider in terms of efficiency and total weight. These results can be seen in figures 5.10 and 5.11.
It can be noted that the difference in total weight and efficiency is smaller in the 3-stage configuration. This difference also grows with the total number of passengers. For this reason it can be confirmed that the gain in efficiency also grows with the number of passengers.

Figure 5.10: Comparison between re-usable rocket, waverider and wingbody configuration for 2 stages

Figure 5.11: Comparison between re-usable rocket, waverider and wingbody configuration for 3 stages
The flight time for these configurations is also considerably higher as it can be seen in figure 5.12. While the Waverider configuration shows to be the most energy efficient of the three, its flight time is also the highest. The Wingbody also shows a higher flight time, with results in-between the other configurations.

![Flight time for different vehicles with 20 passengers](image)

Figure 5.12: Flight time for different vehicles with 20 passengers

It can also be noted in figure 5.13 that the speed achieved by the Waverider is lower that the Wingbody configuration, which is also lower than the ballistic flight. It is important to also observe how speed is lost gradually with the Skip re-entry manoeuvre. This will translate as less energy absorbed as heat in re-entry as well as lower decelerations.

![Speed as a function of distance for three different configurations with three stages and 20 passengers](image)

Figure 5.13: Speed as a function of distance for three different configurations with three stages and 20 passengers

The specific energy consumption decreases with the number of passengers and stages, like in the reusable rocket configuration. The total value for specific energy is lower than the last configuration, as it can be seen in figure 5.14.

![Specific energy consumption](image)
Figure 5.14: Specific energy consumption for the Skip re-entry manoeuvre

Figure 5.15: Trajectories for a reusable rocket, a Waverider and a Wingbody re-entry vehicle for a range of 17.000km and a launcher with 3 stages and 20 passengers

5.3 SSTO vehicle

The analysis for this alternative will follow a different approach. As stated in chapters 1 and 2, there has never been a vehicle built with this configuration, and for this reason there is no historical data regarding structural ratio and engine characteristics. As seen in the last section, it is necessary for this vehicle to be capable of performing a Skip re-entry manoeuvre. This means that the extra weight provided by this structures must be also taken into account. Following the results of the previous section of this chapter the SSTO vehicle will have a Waverider configuration which has proven to attain longer ranges with a smaller initial mass.

Based on work by previous authors the total mass of a single stage rocket can be written as a function
of the payload mass, the structural ratio and mass ratio as follows[^21]:

$$M_0 = \frac{\Lambda(1 - \varepsilon)}{1 - \varepsilon \Lambda} M_{\text{payload}}$$

(5.3)

The values of these variables must follow these constraints:

$$0 < \varepsilon < 1$$

(5.4)

$$\Lambda > 1$$

(5.5)

And since mass must have a positive value, the denominator of equation 5.3 must follow the constraint:

$$\Lambda < \frac{1}{\varepsilon}$$

(5.6)

Finally, taking into account Tsiolkovsky rocket equation:

$$\Delta V = I_{\text{sp}} g_0 \ln(\Lambda)$$

(5.7)

With this constraints the maximum range that a rocket can achieve can be calculated, given a structural ratio and payload mass. For this study, the payload mass will be considered to be the sum of the passengers total weight and the capsule weight where they will be placed. The weight provided by the lifting surfaces, landing gear and other required systems will be included in the structural ratio, as opposed to being included in the payload mass like in the Waverider and Wingbody configurations. According to previous work, the structural ratio of a single stage rocket must have a value between 0.08 and 0.5. With the specific impulse provided by the Merlin engine the trajectory for a flight with 20 passengers and a maximum TOW of 4000 tons can be seen in figure 5.16[^40].

![Figure 5.16: Trajectory of an SSTO vehicle depending on its structural ratio](image)

With the ideal structural ratio a rocket 1000 tons bigger than the Saturn V would not be capable of performing a trip to the opposite side of the Earth. To further optimize this configuration the use of a more efficient engine is necessary. Using the Aerospike engine exposed in past chapters the initial total mass can be calculated as a function of the structural factor and the maximum range.
It can be seen in figure 5.17 that for structural factors greater than 0.11 a rocket may not be capable of performing a trip around the Earth. Important rockets whose structural ratio is approximately this value are numerous, such as SpaceX Falcon1 Stage 2, Saturn V AS-501 stage 2 and Ariane ESC-B\[30]. Overall, the SSTO configuration appears to be possible for only a small window of structural ratios. This ratio depends entirely on the complete design of the vehicle and it cannot be known prior to this process. For this reason it is not possible to confirm or reject this configuration as a real possibility with today’s technologies.

Finally, the specific energy consumption heavily depends on the structural ratio as well. Even though this values tends to infinity for high structural ratios, the obtained results for ranges of 10000km and lower are the best of all the past configurations (see figure 5.18 for reference). This results will be further discussed in the last section of this chapter.
5.4 Hypersonic aircraft

For this study, the aerodynamic models exposed in chapter 3 will be used. As stated in chapters 2 and 3, the Wingbody vehicle proves not to be efficient enough to be used for this purpose. As it can be seen in figure 5.19, the total weight for this configuration ranges between 3500 and 4500 tons. Comparing this value with the A380-800 MTOW of 560 [41] tons it can be assumed that the production of this theoretical vehicle would not be possible.

![Figure 5.19: MTOW as a function of the number of passengers for a vehicle with a Wingbody configuration at Mach 6](image)

For the Waverider configuration this value ranges from 80 and 700 tons, depending on number of passengers and cruise Mach, as it can be seen in figure 5.20. This alternative offers more realistic results in comparison with the Wingbody configuration.

![Figure 5.20: MTOW and fuel weight as a function of the number of passengers for a vehicle with a Waverider configuration at different cruise speeds](image)
Another important factor is the maximum range design point. As it can be seen in figure 5.21, the relation between maximum range and MTOW is exponential and rapidly increases for ranges of over 16000km.

Figure 5.21: MTOW and fuel weight as a function of the maximum range for a vehicle with a Waverider configuration at different cruise speeds

Finally it is important to note that the scramjet engine used for this simulation is most efficient at a cruise speed of Mach 6, as it can be seen in figure 3.3. Flight time ranges from 180 to 120 minutes depending on cruise Mach. This is a 50% difference, while the difference in total weight is over 70%. Losses in efficiency are greater that gains in flight time.

In figure 5.22 it can be observed how the specific energy consumption greatly varies with cruise Mach. The total for this alternative is an order of magnitude lower than past configurations.

Figure 5.22: Total cruise time depending on cruise Mach
5.5 Overall comparison

In this section the overall characteristics of every alternative will be compared between each other and with other current methods of transportation.

Figures 5.24a and 5.24b show the specific energy expenditure of this configurations for 20 and 30 passengers. Some interesting results can be seen. For the suborbital flight configurations increasing the number of passengers increases efficiency, as well as the Mach 6 Hypersonic flight. The SSTO configurations are unaffected and the Mach 8 Hypersonic flight specific energy consumption increases. This results indicates that increasing the number of passengers makes suborbital flight a more competitive alternative. Unfortunately, as it can be seen in figure 5.10, the total mass of the suborbital vehicle also raises exponentially, making bigger rockets impossible to be constructed. In figure 5.25 this difference
in weight becomes more apparent. The mass required for the suborbital configurations is up to 10 times higher than for other alternatives.

![Figure 5.25: Total initial mass comparison between all alternatives with 20 passengers](image)

When comparing the flight time suborbital configurations are the most efficient option among all other alternatives. The Reusable rocket configuration offers flight times between 40 and 60 minutes, in comparison to 70 and 190 minutes for the Hypersonic Mach 6 flight.

![Figure 5.26: Flight time comparison between all alternatives](image)

It is also important to compare this results with other already available alternatives. Hypersonic and suborbital flights will now be compared with general aviation, private cars and trains. For the jet airliner the B-787 has been chosen for its long range and its high efficiency. It proves to be the most energy efficient option, but its maximum range is only 15200km. For the car the Toyota Prius was chosen, as it is considered one of the most energy efficient commercially available cars. It has a consumption of approximately $4.7 \frac{\text{L}}{100 \text{km}}$. Multiplying this value by the specific energy of gasoline and assuming that it
can carry up to 5 passengers the specific energy per kilometre and passenger is:

\[ q_{\text{car}} = 32.148 \frac{MJ}{km \times \text{passenger}} \]  \hspace{1cm} (5.8)

This value on the same order of magnitude than the waverider configuration for a range of 7000km. For longer ranges the Waverider configuration surpasses the car alternative. Based on previous work the specific energy consumed by trains per passenger is\[42]:

\[ q_{\text{train}} = 0.092 \frac{MJ}{km \times \text{passenger}} \]  \hspace{1cm} (5.9)

This value is the best value of any of the alternatives shown in this thesis. The problem with trains is its low speed. At a average speed of 140 \( \frac{km}{h} \) it would take this method of transportation over 6 days to go from one side of the Earth to the opposite, without taking into account the problem of the need for rails and infrastructure. In long distance transportation jet airliners are still the most energy efficient option. The advantages offered by the hypersonic and suborbital flights are clear in terms of flight time, but further optimizations are needed for them to be considered competitive in terms of efficiency. Results will be further discussed in the next chapter.
Chapter 6

Conclusions

We have calculated the efficiency and total energy expenditure for the different alternatives considered for a suborbital flight, using a set of simplified assumptions.

A Reusable Multi-stage rocket proves to be capable of transporting a small number of passengers with efficiencies superior to other current methods of transportation. Although results confirmed that existing Jet-Airliners have significantly superior efficiencies the difference in flight times puts suborbital transportation in a different niche market. It also must be noted that the launch problem can be further optimized. Points that could be further improved using more complex algorithms are the acceleration profile at launch, the gravity turn and Skip re-entry manoeuvre. This could result in more realistic results.

The Skip re-entry manoeuvre used by the last stage of the Waverider and Wingbody configurations proves to improve overall efficiency at the expense of longer flight times. Between the Wingbody and the Waverider re-entry vehicles the Waverider offers better general characteristics. It maximizes efficiency and minimizes overheating problems on re-entry. Furthermore, since this configuration can glide to its destination it could theoretically be capable of landing in standard airports, as opposed to the Multistage Re-usable rocket, which would need specialized platforms for landing.

The results obtained in the analysis of the SSTO suborbital vehicle were given as a function of the structural ratio ($\varepsilon$). This parameter is calculated once the full vehicle is designed, and can only be estimated based on historical trends and data. Since there has never been a fully constructed vehicle of this kind it is hard to confirm if the required value is doable with today’s materials and technologies. Current rocket engines also prove not to be sufficient for this configuration. The Linear Aerospike engine used in this alternative shows to greatly improve overall performance. The development of this engine was stopped due to the cancellation of the X-33 program, so unfortunately it can only be thought as a future alternative.

The Hypersonic aircraft showed also promising results in terms of efficiency. Compared with current generation airliners its energy expenditure per passenger was 2 to 6 times higher, but its flight times were lower up to 8 times lower. Although this results look promising, hypersonic transportation is in need of further development. Surface overheating is one of the main problems that this alternative faces, as well as the higher complexity of the engines required for this purpose.

Finally, we can conclude that suborbital flights are not a practical alternative to current Jet airliners. Results showed that the energy expenditure for suborbital flights is on average 30 times higher than current alternatives, this makes suborbital flights not a competitive option for a regular passenger. If this technologies were to be fully developed we could expect the creation of a new niche market for wealthy individuals and businessmen. Hypersonic transportation, on the other hand, is not to far from current alternatives in terms of efficiency. Gains, in economic and energy terms, provided by shorter flight times could out-scale losses due to lower efficiencies.
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