Optimisation and Hydrodynamic Analysis of a Bottom-Hinged Surge Wave Energy Converter

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Thesis to obtain the Master of Science Degree in Naval Architecture and Marine Engineering

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December 2017
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“Efficiency itself is of no concern when the gods pay for the waves”

Stephen Salter
Acknowledgments

Having reached this stage, there are many to whom I must thank both due to their support and to their help during these years.

First, I must thank my parents and my sister for their support which cannot be compared with anyone else during these seven long years.

Secondly, to all my friends and colleagues who transformed these long seven years in a virtually smaller period which will always be memorable.

Lastly, I must thank the ones who helped me during this last stage, which were Dr. José Miguel Rodrigues and Prof. Dr. Carlos Guedes Soares.

I must also thank Dr. Conceição Fortes for the possibility of using the facilities of the Portuguese National Laboratory of Civil Engineering and to Dr. Diogo Neves for his kind help during my stay there.
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Resumo

O objectivo desta dissertação é o de estudar o conversor de energia que, com base nos seus movimentos de oscilação, consegue extrair a energia do avanço das ondas. Este conversor, que é conhecido por oscillating surge wave energy converter, já se encontra em funcionamento nalgumas costas europeias.

De forma a conseguir a analisar os efeitos que a variação das dimensões deste conversor terá na potência gerada, recorreu-se a uma análise semi-analítica do escoamento potencial em torno deste dispositivo considerando o potencial de difração e de radiação das ondas através ao teorema do integral de Green.

Para validar os resultados obtidos com o método semi-analítico, foram realizadas análises experimentais que permitiram verificar quão próximo os valores teóricos se encontram dos práticos.

Foi realizada uma análise quanto às características que definem as ondas da costa portuguesa, podendo analisar como é que o dispositivo respondeia caso fosse colocado numa zona com estas características.

Palavras-chave: Conversor de energia das ondas, Energia renovável, Energia das ondas, OSWEC, Teorema do integral de Green
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Abstract

The objective of this dissertation is to study the energy converter that, based on its oscillatory movements, is able to extract the surge energy from the waves. This converter, which is known as oscillating wave surge energy converter, is already being used in some European coastlines.

In order to be able to examine the effects of varying the converter main dimensions in the amount of generated power, we turn to a semi-analytical analysis of the potential flow around the device, considering the diffraction and radiation potentials, by using the Green's integral Theorem.

To validate the results obtained with the semi-analytical method, experimental analyses were realised with the aim of verifying if the theoretical results follow the same tendencies as the experimental trials.

An analysis of the Portuguese coast waves’ characteristics was done so that the device’s responses could be analysed accordingly with these characteristics.

**Keywords:** Wave energy converter, Renewable Energy, Wave energy, OSWEC, Green’s integral Theorem
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Nomenclature

Greek symbols

\( \gamma \)   Euler constant.
\( \kappa \)   Wave number of the evanescent.
\( \lambda \)   Wave length of the incident waves.
\( \mathcal{F} \)  Hydrodynamic torque applied on the flap.
\( \mathcal{T} \)  Torque applied on the flap by the generator.
\( \mu \)   Added moment of inertia due to the radiated waves.
\( \mu_{pto} \)  Added moment of inertia due to the generator.
\( \nu \)   Damping coefficient due to the radiated waves.
\( \nu_{pto} \)  Damping coefficient due to the generator.
\( \omega \)   Incident wave frequency.
\( \phi^D \)  Velocity potential function of the diffracted waves.
\( \phi^I \)  Velocity potential function of the incident waves.
\( \phi^R \)  Velocity potential function of the radiated waves.
\( \rho \)   Density of the fluid.
\( \rho_f \)  Density of the flap.
\( \Theta \)   Amplitude of rotation.
\( \theta \)   Function of the angular motions of the flap.
\( \xi \)   Amplitude of motion of the horizontal water particle.

Roman symbols

\( A_D \)  Downstream complex amplitude of the wave.
\( A_U \)  Upstream complex amplitude of the wave.
$A_w$ Incident wave amplitude.

$C$ Hydrostatic restoring moment.

$c$ Dimension of the height of the hinge.

$c_g$ Group velocity of the propagating waves.

$C_{pto}$ Hydrostatic restoring moment.

$E_{max}$ Maximum theoretical efficiency.

$h$ Local water depth.

$h_f$ Dimension of the flap height.

$I_f$ Second moment of inertia.

$k$ Wave number of the incident waves.

$P_e$ Excitation force created by the incident waves.

$P_w$ Average generated power by the generator.

$R_w$ Reflected complex wave coefficient.

$T$ Period of the incident system of waves.

$t_f$ Dimension of the flap thickness.

$T_w$ Transmitted complex wave coefficient.

$w_f$ Dimension of the flap width.

**Subscripts**

$\infty$ Free-stream condition.

$i, j, k$ Computational indexes.

$x, y, z$ Cartesian components.

' Denotes physical quantities.
Glossary

2-D  Two dimensional.
AWS  Archimedes Wave Swing.
EPS  Expanded Polystyrene.
FFT  Fast Fourier Transform.
NLCE National Laboratory of Civil Engineering.
NOAA National Oceanic and Atmospheric Administration.
OSWEC Oscillating surge wave energy converter.
OWC  Oscillating water column.
PTO  Power take-off.
SWAN Simulating waves nearshore.
WAM  Wave Modelling.
WEC  Wave energy converter.
WW3  WaveWatch III.
XPS  Extruded Polystyrene.
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Chapter 1

Introduction

1.1 Motivation

Since the disclosure by Faraday, in the 1830s, of the electric generator which converted mechanical energy into electric energy, that mainly non-renewable fuels have been used to power the rotor of a generator. In 1879, Thomas Edison made a demonstration of the incandescent light bulb which triggered an increase on the demand for a supply of electrical power. Soon after, in 1882, he inaugurated the first commercial scale power plant and used an improved direct current generator that was powered by steam which was generated in the boilers by burning coal, as was described in the website referenced [Pea]. Since then, there has been some additions to the fuels that power the generators, or more recently the alternators, but the main fuel hasn’t changed and still is, the cheap and highly available, coal. There are many other resources, besides the main fossil fuels, that could power the alternators, like the renewable resources such as the solar energy which dynamises the wind energy and the wave energy.

The interest in studying wave energy converters aroused from the increased investigation that has been realised on devices that are able of extracting power from renewable energies. A recent analysis, published by Whittaker and Folley [60], concluded that the oscillating surge wave energy converter, from herein OSWEC, is an effective contender in the challenge of extracting energy from water waves.

The Portuguese coast has already been selected as the site for the experimental trials of wave energy converters. Until today, two types WECs have been tested in Portuguese waters however, they fit into two very different categories. One took advantage of the fluid particle motion, by being placed on the sea floor and occupying partially the water column, and the other had to float on the water surface taking advantage of the waves’ amplitudes. The first device has to be placed underwater in shallow water regions with depths that range between 8 m and 20 m, taking advantage of the almost horizontal motions of the water particles. For these depths, usually, the device is placed at a distance of 0.3 km up to 2 km from shore and, due to their proximity to shore, it is expected that their implementation costs will be smaller when directly compared with deep water devices.
The western side of the Portuguese coast, which is exposed to the Atlantic Ocean, presents a large amount of wave energy resources due to its exposure to the westerly winds that blow over the North Atlantic, creating and driving the waves to the western European countries. Mota and Pinto [36] published a report where they evaluated the wave energy potential along the western Portuguese coast and concluded that the main wave direction is from the north-west quadrant, ranging between $303^\circ$ and $309^\circ$, and the variability of these values ranged between $\pm 20^\circ$ and $\pm 25^\circ$. For such low variability, and since that it is beneficial to an OSWEC to be perpendicularly oriented to the waves' direction of propagation, the flap could be initially oriented in such a way that the incident waves would be, most of the time, perpendicular to it. There is an OSWEC, the Oyster 801, that is being designed so that this variability can be considered and it will enable the device to dynamically orient itself to the incoming waves.

An International Symposium was held in Lisbon, in 1985, and from the gathering of all the presentations a book was published in 1986 by Evans and Falcão [17]. The conference focused on the topic of hydrodynamics of ocean wave-energy utilisation and one of the publications was about the progress on flap-type wave absorbing devices, presented by Scher [50]. The author considered that one of the main reasons which explained why this type of WEC was not being more seriously developed would be due to its simple operation which was not technically eye-catching. Further reasons could be given to explain the low interest in these type of devices, and the main one was due to the theoretical 2-D efficiency, which imposes an upper limit to the ratio between the absorbed power and the incident wave power of 50% to all WECs with a single degree of freedom. Due to this, the single-flap device was widely judged as "inferior" by a factor of two, when compared with other WECs that have, at least, two degrees of freedom. Some investigators presented solutions, such as Salter [48], Scher [50] and, later, Whittaker and Folley [59] that could solve the low efficiency of devices with solely one degree of freedom.

Wave energy converters, besides the most obvious differences such as the type of motions that characterise them, can be defined by other factors, such as their dimensions. The only response of an OSWEC, to the incoming waves, is the pitch rotation along a horizontal hinge that is placed, ideally, perpendicularly to the direction of wave propagation. This hinge is located in the bottom part of the device, near the sea floor, inside the support that holds the flap, as can be seen in the Fig. 1.1(a).

Some scientific publications, such as the one published by Whittaker and Folley [60], have already concluded that to maximize the amount of generated power that an OSWEC is able to extract from the incident waves it should be a surface piercing device, like the one that can be seen in Fig. 1.1(b). Although, this conclusion doesn’t eliminate the other possible configurations and, like previously stated, a fully underwater OSWEC, such as the WaveRoller, was already tested in Portuguese waters. To be certain about which parameters affect mostly the generated power, an analysis will be made allowing to conclude about which one is more prone to increase the generated power when their dimensions are changed.
1.2 Objectives

The aim of this dissertation is to analyse the hydrodynamics of an OSWEC and to experimentally validate the equations that describe these devices. These were developed for the analysis of devices whose upper surface extends above the water’s free surface, more commonly identified as surface piercing devices. However, these equations, by changing the limits of their integrals, can be used to evaluate submerged devices, which are the ones whose top surface is always beneath the waves’ free surface. Yet, until today, no validation has been realised to these equations when evaluating submerged devices. So, after the analysis of the resultant values of these equations, experimental tests were realised enabling the chance of comparing them with theoretical results for both the surface piercing and the submerged devices.

Similar comparisons have already been realised yet solely for surface piercing devices. For example, experiments with this type of devices were developed by van’t Hoff [56] with the goal of validating his numerical calculations. However, these experimental results were used by Renzi et al. [43] to validate the results of the equations that describe this type of WECs. So, since that these equations have already been validated for surface piercing devices, the experiments realised for this thesis have the possibility to compare the obtained results with the ones of Renzi et al. [43].

For the experimental analysis it will be necessary to design the devices that will be tested as well as to choose the equipment that will be used for the measurement of the device’s oscillations and of the incident waves’ forces. For registering these measurements, it will be necessary to create programs that will enable the chance of registering the experimentally obtained values.
1.3 Thesis Outline

This thesis has been developed with the aim of explaining all the terms that are related with the OSWEC, allowing an easy understanding of the used terminology. The semi-analytical equations might be one of the more complicate parts to explain due to their complexity. However, an effort has been done so that the deductions of the equations can be easily understood.

Initially, in chapter 2 - State of the Art, it is possible to find what has been developed and investigated about the OSWECs. It also explains the differences between the different types of WECs and the main dimensions of an OSWEC that will be used in this dissertation.

Having settled what were the main characteristics of an OSWEC, it was possible, in chapter 3 - Analytical Analysis of the Oscillating Surge Wave Energy Converter, to use them to define the semi-analytical equations that describe the OSWEC and that were going to be used in this dissertation. These equations, to be fully understood, were explained step by step however, some less important steps were solely explained in appendix B - Deduction of the Equations. Along the text, references were made to this appendix when larger steps between the equations were being realised.

The semi-analytical equations were implemented in MatLab, which enabled the possibility of evaluating several devices with different dimensions. The results obtained from these tests were presented in chapter 4 - Semi-analytical Analysis of the WEC Parameters, mainly analysing the results of, for example, the added inertia, of the radiation and generator damping, of the response angle and of the excitation torque. Conclusions can be withdrawn about which dimension is more able to increase the device’s generated power.

These theoretical results, for both the submerged and for the surface piercing devices, needed to be validated, as previously stated. This can be accomplished by comparing the theoretical results with values obtained experimentally. So, in chapter 5 - Experimental Analysis it is possible to find the experimental results and the comparison between them and the theoretically obtained values.

Finally, in chapter 6 - Conclusions, it is possible to find the conclusions that can be withdrawn from this thesis and the planning for what it can be done in future works.
Chapter 2

State of the Art

In this chapter, a historical review on the topic of this thesis will be presented. The main goal is to analyse the hydrodynamics of an OSWEC and, consequently, to analyse the wave energy and the different categories of WECs. Since the beginning of the 1970s, there has been a larger focus on the development of theories that enabled the analysis of WECs. Those theories facilitated the possibility of calculating the amount of generated power by a WEC and concluded that it is directly related with the incident waves’ power. So, to be able to calculate the amount of generated power, it is convenient to be able to calculate the amount of incident power, allowing the possibility of calculating the conversion efficiency. Lastly, and after reviewing the advancements on the analysis of wave energy and on the development of WECs, a review on what was developed on the topic of the oscillating surge wave energy converter devices will be presented.

2.1 Wave Energy

The main contributor for the existence of energy in waves is the energy irradiated by the Sun. The dynamisation process begins with the warming of the air mass, that is closer to the Earth’s surface originating, consequently, a recirculation between the warmer and colder air, more commonly known as wind. The motion of the air particles can be directly captured by wind turbines which are devices capable of converting the winds’ kinetic energy into electrical power. The advantage of using the waves’ energy, as stated by [9, 46], is due to its concentrated form of wind energy. Yet, it is necessary to investigate the availability of this resource.

Several authors have studied this topic and one of the most recent publications was written by Gunn and Stock-Williams [24], where they unveiled their obtained results for the calculation of the global wave power resource which totalled $2.11 \ TW \pm 0.05 \ TW$. This result is comparable to other estimates, like the ones by Kinsman [28] and Inman and Brush [27] where they stated that the global wave power was $1.87 \ TW$ and $2.5 \ TW$, respectively. Inman and Brush [27] obtained their results by assuming that the average
wave height was of 1 m and that they would only transmit 10 kW per meter of wave crest. The resultant value for the global estimative of wave power was too high, on the order of 4 400 GW. So, and since that there are shores which are not exposed to the open ocean, they assumed that only half, or two-thirds, of this value would in fact reach the shores, resulting in 2 500 GW.

These calculations, at the time, were only estimates to understand the upper and lower limits of the wave power that would reach the shorelines. However, nowadays, to obtain more reliable results, it is necessary to resort to computational programs that can model waves. There are several programs that facilitate this investigation but it is necessary to understand their strong points and towards which depth they are oriented. Some of them are more fit to make analysis in deep waters and others are more recommendable for shallow water depths. The state-of-the-art programs for modelling waves in the offshore region are the WaveWatch III, WW3, or the Wave Modelling, WAM, and in the nearshore it is possible to use the Simulating Waves Nearshore, SWAN. Usually, when the goal is to analyse the waves’ power in the nearshore it is possible to make a combination between the offshore and the nearshore oriented programs, so that they complement each other. Gunn and Stock-Williams [24], when they were analysing the global wave power resource, used a combination between the WW3 model, for wave generation and swell propagation, and used their results as boundary conditions for the SWAN model, like Silva et al. [52] did for the analysis of the waves’ energy in the Iberian Peninsula. However, like stated previously, there are other possible combinations such as the WAM model for the boundary conditions of the SWAN model, as was done by Rusu and Guedes Soares [47] when estimating the spatial distribution of wave energy in the Portuguese nearshore.

The Portuguese western coast line has its most southern shore at a latitude of 37° and the most northern point is at 42°, as can be assessed by analysing Fig. 2.1. From the results obtained by Gunn and Stock-Williams [24], for the calculation of the global wave power resource, it is possible to conclude that the
western shores are predominantly exposed to by more energetic waves due to their primary direction of waves from west to east. Within the northern and southern poles there is a ribbon defined between the latitudes of $40^\circ$ and $60^\circ$ where the waves are known to have a higher energetic density, as can be concluded by the results shown in Gunn and Stock-Williams [24]. Silva et al. [52] published a study that had as an objective the calculation of local wave energy in the Iberian Peninsula, which is within this most energetic area. As stated earlier, they used the WW3 model, for the wave generation and for the swell propagation in the deep waters, and these results were used for the boundary conditions of the SWAN model. In the Portuguese coast, they analysed three different locations and validated the SWAN results with the values from the measurements of three buoys placed in Leixões, Lisbon and Sines. The comparison between these results presented a good agreement for both the wave height and wave period but for the mean wave direction the results showed a lower correlation. The resultant values, obtained by using SWAN, for the mean wave power in Figueira da Foz reached a maximum in February of 47 kW/m and a minimum in August of 5.5 kW/m.

In the 1970s, during the period when in the United Kingdom there was a set of large developments in the wave energy conversion devices, Glendenning which usually advocated for the employment of the wave energy conversion devices, stated, and was transcribed by Ross [46], that the waves are more ferocious and most powerful during the winter which, coincidently, is when we most need energy. This energy needs to be captured and that can be accomplished by using devices which are able of extracting the energy from the waves and transforming it into electrical energy.

### 2.2 Wave Energy Converters

Wave energy conversion technology started to be developed more than 200 years ago and the first known patent for a wave energy converter is dated to 1799. It was registered to a father and his son, who designed a wave energy converter with a gigantic lever, with its fulcrum on the shore and a “body” floating on the sea, Burman [7]. At the time, due to the fact that the generator hadn’t yet been revealed, the goal, for the use of this device, was to power tilt hammers, saws, mills, or other tools that required mechanical power.

Since then, and during the period between 1856 and 1973, it is estimated that the United Kingdom granted 340 patents on devices that claimed to be able to utilise wave energy as their source of power, as was stated by Leishman and Scobie [30]. In 1973 and due to the support of the United States to Israel during the 4th Arab-Israeli war, the Saudi-Arabians declared an oil embargo reducing their monthly oil production to 5%, which lead to an unreasonable increase in oil prices, shifting the attention to the renewable energies. At the time, the design of devices that could extract energy from waves restarted, in Europe, with the development of the Salter's duck, which had the name of its creator Salter [48]. He stated that a wave energy converter should be designed in such way that it should be able to convert the disperse, random and alternating forces of the waves into a concentrated, direct force by using a
mechanism that needs to be efficient at both the low wave heights and robust enough to withstand the worst conditions. For a device with the previous characteristics, he concluded that it should be placed below the water surface, as much as possible. Late in the year of 1973, before the development of the *Salter’s duck*, he began his studies on the topic of wave energy and managed to get a suitable site for his experiments, accompanied by Laurence Draper of the Institute of Oceanographic Sciences, enabling him to understand which design was better for a *WEC*.

Several authors, for example [15] and [46], spoke about the achievement made by Salter in the development of the shape of the *Salter’s duck*. He had been able to conclude, by analysing the governing equations of the maximum efficiency, that he needed to design the shape of the device in such a way that it had to oscillate without transmitting waves past the device, as can be deduced from Eq. 2.1:

\[
E_{\text{max}} = \frac{|A_U|^2}{|A_U|^2 + |A_D|^2},
\]

(2.1)

where the value of \(A_U\) is the amplitude of the generated waves upstream and the value of \(A_D\) is the amplitude of the generated waves downstream. If the amplitude of the waves downstream from the device, \(A_D\), are small then the value of \(E_{\text{max}}\) will tend to a unitary values.

David Ross, when he was writing his book [46], managed to speak with Stephen Salter who explained to Ross the development process around the *Salter’s duck*. He explained that he had begun with an obvious mechanism that bobbed up and down with an efficiency of 15%. By adding a hinge below the surface, he managed to increase the efficiency up to 60%. After this, he thought that by adding an underwater vertical flap it would increase the efficiency. However, it decreased to 40% because it was reflecting waves due to its own movement. He began by changing the shape of the device to decrease the amplitude of the reflected waves, both to the rear and to the front, and reached an efficiency of 90%.

The initial phase of the design process of a *WEC* starts by selecting the depth in which the device will be placed. While some of them are designed for the offshore region, there are others that need to be placed in shallow waters. Their motions can either be completely free or can oscillate around an axis. Due to their specific characteristics there are several methods that enable the categorisation of the wave energy converters. They can be categorised either by the location where they will be placed, or by their size and directional characteristics, or, even, by their working principle.

### 2.2.1 Placement of a Wave Energy Converter

This method of categorising *WECs* is based in the division of the ocean in three distinct areas, by using the water depth as a variable. The resultant division, that ranges between the deep waters to the onshore depth, is: offshore, nearshore and onshore devices.

The placement of devices in the offshore region, in depths larger than 40 m, is with the intention of
extracting more power from the more energetic seas. Due to their location, the reliability and survivability of these devices is a big problem because they are more prone of interacting with extreme seas which would impose higher loads on the structure. Since that they are more distant from the shore it is expected that their implementation and maintenance costs are higher when comparing with the devices that are closer to shore.

The advantage of placing devices in the nearshore region, where the depths range between 10 up to 25 m, is due to their smaller implementation and maintenance costs when comparing with the devices that are placed in the offshore region. This reduction of costs is a consequence of placing the devices in smaller depths which, usually, only occur in places nearer the shore. However, due to this proximity to the seas’ floor, the energy of the waves suffers more predominantly from the shoaling effects, as stated by Whittaker and Folley [59]. This can be compensated by the fact that, in this region, the devices are less prone to extreme wave events and the waves’ dimensions suffer less variability, due to the shoaling effects and to the depth limited wave height. Due to the smaller depths, the device can rest on the seabed avoiding the need for moorings which would be necessary for the devices that are placed in deep waters.

The last possibility, for the placement of a wave energy converter, is onshore. These devices can be placed above the sea, integrated in a breakwater, in a dam, or fixed to a cliff. The main advantage of these converters is their easy maintenance and installation because, in most cases, their location is easily accessible. Due to the proximity to the shore the waves are even more affected by the bottom friction which reduces the incident wave energy. The sites where these devices can be installed need to be thoroughly checked because the devices will probably reshape the shore and the bottom surface.

In this thesis, the device that is going to be analysed is usually placed in the nearshore region with water depths that range between 10 up to 25 m. As stated in 2.1 - Wave Energy, the loss of energy, due to the passage from deep waters to shallower waters, varies and depends of a couple parameters which are: the wave period, of the wave height and, also, of the total variation of depth. Whittaker and Folley [59] analysed the variation in wave energy due to the passage from depths of 50 m to depths of 10 m for four different bathymetry profiles: 1:50, 1:100, 1:200 and 1:500. From the results, they verified that the energy loss was higher for the cases that had a smaller bottom slope. Which means that the waves that covered sea slopes with an inclination of 1:500 would reach the 10 m depth with less energy than when they covered the slope with an inclination of 1:50, due to the shorter distance that they had to cover in the latter case. For waves advancing towards the shore, coming from deep waters with a bed slope of 1:100, the loss of energy can be analysed in the Fig. 2.2.

The almost linear lines that cover the Fig. 2.2 represent the losses due to the transition from depths of 50 m to depths of 10 m. For waves with power levels between 50 kW/m and 100 kW/m it can be said that the energy losses will vary between the 10 % and 20 % range which seems to settle some of the doubts about the feeling of great losses due to this transition.
Figure 2.2: Wave energy loss with a bottom slope of 1:100 from the deep to shallow waters (Whittaker and Folley [59])

2.2.2 Orientation of a Wave Energy Converter

The device’s dimensions and their orientation, with respect to the incoming waves, is another parameter that can be used to categorise the several types of WECs. With this categorisation method it is possible to aggregate devices that can be placed in different regions even if they use the same methods of extracting the energy from the waves. The devices can either be: an attenuator, a terminator or a point absorber.

The attenuator type of device has a long length, of the order of the wave length, and a width which is, comparatively, much shorter. The format of this device is identical to a snake formed by 4 or 5 elements which are interconnected between each other. Between each element there is a hydraulic system that will damp the device’s motion and extract the energy from their relative oscillations. When placing this device, and to extract the most energy from the incident waves, it must be oriented along the direction of waves’ propagation. The goal of this device, which is usually formed by a set of connected bodies, is to attenuate the motion of the waves. The most known device of this category is the Pelamis, a prototype converter with 750 kW that was placed in the Portuguese shore and it powered partially a Portuguese city called Póvoa de Varzim.

The dimensions of the terminator type of device are similar to the ones of the attenuator, which was previously described, however, instead of being oriented along the direction of waves’ propagation it should have its biggest side oriented parallel to the crest of the waves. Its main dimension, which is its width, can reach values of 26 m, like the renewed Oyster Aquamarine version 2. The height of this device, depending if it is a surface piercing or a completely underwater device, can reach values up to 11 m. This type of devices were already deployed, in the beginning of 2010, for testing purposes and, with the obtained results, improvements were made to the Oyster device originating a renewed
second version, like stated by Cameron et al. [8]. There are several examples of devices that fit into this category, such as the Salter’s duck, the WaveRoller [AWE] or the Oyster Aquamarine.

Finally, there is the point absorber type which, when compared with the previous types, has a smaller length and width but, probably, will have a larger height, when it is placed in deep waters. This type of device, instead of being oriented with respect to the direction of waves’ propagation, it can be freely placed because it is able to extract energy from all directions. The energy generation is done either by the bobbing of the device, in the upward and downward direction, or by using the variation of the height of the waves’ free-surface. There are a lot of devices that fit in this category because, usually, these devices are placed in deep waters where the available energy is higher.

In the ambit of this thesis, the device that is going to be analysed is classified as a terminator due to its perpendicular orientation with the direction of propagation of the waves. In the Fig. 2.3 it is possible to find the three different types of categories which were previously defined.

Figure 2.3: Categories of the WECs by their orientation: a) Point Absorber, b) Attenuator, c) Terminator (López et al. [32])

This thesis will focus in the device that is represented in the lower part of the Fig. 2.3, marked with the letter c. By knowing how the device will be oriented in relation with the incident wave direction it is necessary to define the different working principles by which the devices can be categorised.

2.2.3 Working Principle of a Wave Energy Converter

There are several types of devices and many ideas on how to classify their working principle. One of the first publishers who created a method to categorise them was Leishman and Scobie [30]. They concluded that, from the several possible approaches of classifying the devices, one of which is to consider how the wave energy manifests itself and then to divide the main types of wave power converters according to the fashion in which they appear to extract energy from the waves. The energy in ocean waves can be obtained by the wave power converters either by:

(A) Variations in surface profile, that can either be slope or height, of travelling deep-water waves;
(B) Sub-surface pressure variations;

(C) Sub-surface fluid particle motion;

(D) Unidirectional motion of fluid particles in a breaking wave which may be naturally or artificially induced;

(E) Other effects,

where the last item was inserted by Leishman and Scobie [30] to stimulate others to think about other possible ways of obtaining energy from sea waves and, until the time when [30] was published, all the systems were included in the categories A to D. Leishman and Scobie [30] evaluated 38 different devices and categorise them by using the categories that they had defined. They developed a very ostensive tree diagram which can be found in Appendix A.1 - National Engineering Laboratory.

A device can be categorised as one that uses the variations of the surface profile when it has a body which is put in motion by the wave profile, or when the motion is transmitted to a water level chamber or when converted into a standing waveform. An example of a device that uses this type of working principle is the one developed by Yoshio Masuda which is more commonly known as Oscillating Water Column, OWC, as stated by [32].

When a device uses the sub-surface pressure variations, it can be purely due to the vertical oscillations of the water level in submerged chambers, like the Archimedes Wave Swing, AWS, as described by [32], or because the pressure interacts directly with a diaphragm.

There is also the possibility of using the sub-surface fluid particle motion as the agent that powers the device. This type of working principle encompasses all the devices that move underwater, like a vertical flap, or a cylindrical rotor which can be set perpendicularly or parallel to the wave direction. The fluid particle motion, in the nearshore, can be approximated to an oval shape at the free-surface and is almost horizontal at depths close to the bottom surface. Both the WaveRoller [AWE], the Salter's duck and the Oyster Aquamarine fit into this category because they take advantage of the surging motion of the waves.

The last category, that has devices attributed to it, is the unidirectional particle motion in breaking wave. They operate by taking advantage of the kinetic energy of waves. Imagining the slope of a beach, it is known that when the waves break the fluid particles continue with their forward motion and, before the next wave breaks, all these particles swash backwards. All this kinetic energy can be extracted either by placing devices in deep waters or by placing them in an onshore structure. In the onshore region the breaking of the waves can be easily achieved. However, in deep waters, the devices need to be able to cause the waves to break, either to extract the forward motion of the particles or to direct the waves towards a reservoir, placed at a point higher than the free-surface. The result of the latter case causes the device to be over-topped by waves giving origin to the name overtopping devices. In the 60s, a similar device was designed for onshore operations. It had a ramp that would converge the waves to an
elevated reservoir. The water had to pass through a turbine, which was connected to a generator, before going back to the ocean.

The device that is going to be studied takes advantage of the third item specified previously, that is, it takes advantage of the sub-surface fluid particle motion. The device oscillates around an axis that is in the lower part of a vertical flap. In shallower waters the motion of the fluid particles, due to the shoaling effects, begins to describe an ellipse shaped path near the water surface and for smaller depths the fluid particle motion is almost, only, horizontal. The devices that are going to be studied take advantage of this motion and by using a damping system they are able of converting this mechanical motion into electrical power.

2.3 Oscillating Flap Device

The British Department of Energy, in 1975, noticed the increased interest in the development of wave power conversion devices and ordered a study to be made by the National Engineering Laboratory. The resultant report would become the first major feasibility study into the subject, as stated by Ross [46] in his book. He used NEL report while he was writing his book and considered it to be the basic source of information and to be, certainly, the most comprehensive study that had been produced, until that period. However, this opinion wasn’t consensual and Gordon Goodwin, who worked for the Department of Energy and was a big supporter of the Salter’s duck, wasn’t very enthusiastic with the outcome of the report because the National Engineering Laboratory which was, initially, supporting the Salter’s duck, started favouring the OWC device, a Japanese invention. Besides the device that the National Engineering Laboratory was supporting, they reinforced the idea that the wave power is an alternative power source that can be considered for large-scale production of energy which could be achieved by the development of the existing technology.

Even before the presentation of the Salter’s duck, that occurred in 1974 [48], there were others that started to publish their theories enabling the possibility of obtaining the values of the excitation forces applied on bodies by the waves. Haskind was interested in calculating the excitation forces without the laborious work of calculating the diffraction effects, or the forced-oscillation potential, which can be arduous to calculate, as stated, later, in 1962 by Newman [37]. The goal was to determine an expression that allowed the calculation of the excitation forces for a given incident-wave system by simply using the asymptotic characteristics of the velocity potential at a large distance from the body. Newman [37] decided to transcribe the expressions obtained by Haskind because, initially, his paper was published in Russian, impeding some of discovering such good findings. Newman considered these expressions to be extremely valuable because without them the determination of the excitation forces would be highly intractable [37].

The number of published documents on the topic of wave energy conversion and on wave power in-
creased during the period when the oil prices sky-rocketed, in the 1970s, due to the increased investment that was being done on new sources of energy. The influence that the oil had, and still has, in the global economy lead some countries to invest in the research of the renewable energies, like the British government did in 1976. They started by investing £1 million on a two-year feasibility study into wave energy. On the following year, they decided that the initial amount was going to be doubled and the two-year study would have an investment of £2.5 million, as stated by a British minister at the time, Tony Benn in an interview given to Weinstein [58]. In 1978, the investment, for this year solely, increased to £2.9 million however, still short from the £22 million that Japan invested on a seven-year plan, as plainly stated by Ross [46]. At the end of the 70s decade there was a change in the British government, foreseeing some changes in the investment plans that were being applied. In the following year, the government took a secret decision that would stop further investment in the two most promising British inventions, causing them to be limited to model testing, and to continue the investment on the Japanese device, the OWC, and on the three newest devices, the Lancanter Airbag, the Lanchester Clam and the Bristol Cylinder.

Later, and as a continuation on the topic of the 2-D efficiency, Budal [6] proved that for properly spaced bodies, with a single degree of freedom, the absorbed power per body increased by a substantial factor. He did so by using the general expression of the wave power absorbed by the system, in terms of the far-field velocity potentials, that were initially derived by Haskind. If the devices were placed in a linear row with a space between each other, of the order of the wave length, $\lambda$ or less, the power absorption per body would be raised by a factor that could be as large as $\pi$, when directly compared to a single isolated device. Budal [6] recognised that, due to the restrictions of his calculations, which only considered the linear theory, he could only analyse the cases that caused small amplitude responses. For higher wave amplitudes, or higher response amplitudes, non-linear effects would start to have more importance. Evans [16] states in his publication, that when calculating the performance by only considering the linear terms the resultant values represent an upper bound of the efficiency because it is generally accepted that the non-linear effects will tend to reduce it.

During this period, while the British government was investing on WECs, there were others that were focused in evaluating the amount of extractable power from the waves. There was a British investigator, from the Department of Mathematics of the University of Bristol, named Dr. David Evans, that was interested in studying the absorption of wave-power by oscillating bodies [15]. He was interested in developing a theory that could predict the amount of absorbed power from an incident sinusoidal wave train by means of a damped, oscillating, partly or completely submerged body. Evans published a couple of scientific documents where he exposed theories for the calculation of the wave-power absorption for a single oscillating body and for two oscillating bodies, [15] and [53], respectively. He stated that the efficiency of this kind of system could be calculated by a proportion between the extracted power and the incident wave power, which would clearly depend on the details of the coupling between the device and the fluid. He also stated that the calculation of the maximum efficiency could be obtained without
knowing the details of the coupling, either by using the Eq. 2.1 or by using a similar formula published by Newman [38], where he related the upstream complex coefficient that represents the wave amplitude, $A_U$, the downstream complex coefficient that represents the wave amplitude, $A_D$, with the coefficients of reflection and transmission of waves, $R_w$ and $T_w$, respectively. The obtained relation is given by Eq. 2.2.

$$A_U + \bar{A}_U \cdot R_w + \bar{A}_D \cdot T_w = 0,$$

(2.2)

the bar over a quantity represents the complex conjugate. This relation is obtained by applying the boundary-value problem which states that at upstream infinity the standing wave must vanish. With this relation, presented in Eq. 2.2, Evans [15] could relate the complex coefficients that represent the wave amplitude with the reflection and transmission coefficients. The resultant relations are presented in Eq. 2.3:

$$|R_w| = \delta, \quad |T_w| = \sqrt{\delta} \sqrt{(1 - \delta)},$$

$$E_{\text{max}} = 1 - R_w R_w - T_w T_w \Leftrightarrow E_{\text{max}} = 1 - \delta,$$

(2.3)

where the value of the $\delta$ is given in Eq. 2.4:

$$\delta = \frac{|A_D|^2}{|A_U|^2 + |A_D|^2}.$$

(2.4)

For a single oscillating flap with horizontal symmetry, and by using the Eq. 2.3 and Eq. 2.4, it is possible to calculate the maximum efficiency by knowing that the values of the upstream and downstream coefficients are symmetric, $A_U = -A_D$, for devices with horizontal symmetry. The resultant value, since the complex coefficients of the wave amplitude are symmetric, is $\delta = 0.5$ with a maximum efficiency of $E_{\text{max}} = 0.5$, as was concluded by [15, 34, 53].

Evans [15] showed that a single oscillating device would be able to absorb a maximum of 50% of the incident wave energy where 25% would be reflected and the other 25% would be transmitted past the device. This transmitted component, in case a second device is placed at a certain distance from the first one, will be absorbed with a 50% efficiency. The remnant energy will be divided in two parts where 25% will be reflected towards the first device and 25% will be transmitted past this device, towards infinity. This leads to a maximum of $\frac{2}{3}$ of the incident wave energy being absorbed, $\frac{1}{15}$ being reflected and the remainder $\frac{1}{15}$ part is transmitted past the device. However, such consideration is unsound because it doesn’t consider phase cancellation which can increase the total efficiency.

A method was proposed by Srokosz and Evans [53] that allowed a set of two WECs to extract almost all the incident wave energy. The plan was to place two independent devices and, each of them, would be able to oscillate in a single mode and be capable of absorbing energy solely in that mode. One of the early group of investigators that started to focus on the experiments of the twin-flap wave-energy absorbing device was Scher et al. [51]. The goal, that they had settled, was to make experiments with this type of device and then to compare them with the theoretical results published by Srokosz
and Evans [53]. They obtained a fair agreement between their results and the theoretical values, and were able to reach the maximum theoretical efficiencies, close to 100%. The results obtained with the experiments allowed Scher et al. [51] to conclude that the primary importance, when designing this type of devices, was the depth and the separation of the flaps. The flap draft must be chosen so that the longest and shortest waves of economic interest fall within the range of relatively high efficiency. The separation between the flaps should be less than approximately $\frac{1}{4}$ of the wavelength for the shortest wave of interest, in order to obtain the desired broad efficiency curve.

In 1993, Charlier and Justus [9] published a book that contained a summary of all the ocean energies and covered several topics such as the environmental, economic and technological aspects of alternative power sources. They, like many others, such as Ross [46], consider that the extraction of energy from the waves is more efficient than the direct collection of energy from the wind because the waves are a concentrated form of wind energy. The ocean waves can be of regular or irregular type and can either be short or long crested. The irregular waves are resultant from the overlapping of several composite waves.

In 2009 an OSWEC was deployed at the European Marine Energy Centre, in Orkney, Scotland, for testing purposes and it enabled the possibility of analysing the data with the goal understanding which parameters influenced the power generation the most. From this data, it was possible to withdraw conclusions about what can be done to maximise the generated power. The device that was being studied was the Oyster Aquamarine, a surface piercing device, that was rated for generating, at the time, 315 kW from the energy of the waves. The results from this study were published by Cameron et al. [8] and some of his conclusions are comparable to the ones presented by Scher et al. [51] in 1983. Concretely, the influence that the flap draft has on the amount of generated power. Besides this parameter there were others that were evaluated to understand their influence in the amount generated power, such as the flap thickness, the flap width and the flap freeboard.

To evaluate these parameters, and since that some results are difficult to interpret from model tests, it is necessary to use theoretical models that enable this analysis. Yet, it isn’t economically feasible to iteratively make changes on the full-scale flap to evaluate how they will affect the amount of generated power. So, with the results obtained from the full scale tests, or from the model scale tests, it is possible to validate the theoretical calculations. If the results from these calculations are comparable to the full scale or to the model scale tests then it is possible to use these formulas to evaluate the effects of changing the parameters that define the main dimensions of a flap.
Chapter 3

Analytical Analysis of the Oscillating Surge Wave Energy Converter

The method that is going to be used for the analysis of the oscillating surge wave energy converter, OSWEC, was discovered by Haskind, in the beginning of the 60s, and the goal was to determine the forces that are applied on a ship by an incident wave system. However, besides the necessity of knowing the hydrodynamic pressure of the incident waves, it is also necessary to know the effects made by the presence of the body on this pressure field. Haskind derived equations of the exciting forces and the moments on a fixed body which do not require the knowledge of the diffraction effects caused by the body's presence. These equations are based on a demonstration made by Haskind which confirmed that the velocity potential at a large distance from the body is sufficient to determine the exciting forces for a given incident wave system, as Newman stated in [37]. For many problems, this solution can expedite the calculation process of the exciting forces and the moments applied on a body because the near-field forced-oscillation potential or the diffraction potential can be very complex to obtain. So, this method is based on a linear relation obtained by Haskind between the exciting forces exerted by the incident waves on a fixed body and the amplitude of the far-field radiated waves generated by forced motions of the body in otherwise calm water.

3.1 Governing Equations

The method that is going to be explained enables the analysis of an oscillating surge wave energy converter, OSWEC, and is based on a semi-analytical model that assumes the fluid to be inviscid and incompressible while the flow must be irrotational and the perturbation time-harmonic. For a flow with such characteristics it is possible to define a velocity potential given by $\Phi'(x', y', z', t')$ that satisfies the Laplace equation:

$$\nabla^2 \Phi'(x', y', z', t') = 0,$$

(3.1)
the usage of primes is to represent the physical dimensions of the variables. The main dimensions that
will be dealt with, when using this method, are:

Table 3.1: Definition of the main parameters for the semi-analytical model

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Variable</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap width</td>
<td>$w'_f$</td>
<td>[m]</td>
</tr>
<tr>
<td>Flap height</td>
<td>$h'_f$</td>
<td>[m]</td>
</tr>
<tr>
<td>Flap thickness</td>
<td>$t'_f$</td>
<td>[m]</td>
</tr>
<tr>
<td>Height of the hinge</td>
<td>$c'$</td>
<td>[m]</td>
</tr>
<tr>
<td>Local water depth</td>
<td>$h'$</td>
<td>[m]</td>
</tr>
<tr>
<td>Wave amplitude</td>
<td>$A'_w$</td>
<td>[m]</td>
</tr>
<tr>
<td>Wave period</td>
<td>$T''$</td>
<td>[s]</td>
</tr>
</tbody>
</table>

The semi-analytical model, that is going to be described in this section, is based on the first-order
analysis of a higher-order phenomena. By considering the linearised theory, the resultant values will
represent a basis for further development. This theory imposes that the amplitude of the response angle
and the amplitude of the incident waves must be small, when directly compared with the main dimension
of what is being studied. In this case the ratio between the wave amplitude and the flap width must be
small, which can be represented as $A'_w / w'_f \ll 1$.

The velocity potential of the flow must satisfy some initial boundary conditions such as the impermeability
of the bottom surface and the velocity limitations at the free surface. For the latter location, there are
two boundary conditions that need to be enforced to the velocity potential at the free surface, which are
the kinematic and the dynamic conditions. By linearising these boundary conditions, it is possible to
obtain a combined boundary condition, at the free surface, that must be met by the velocity potential.
The resultant equation is:

$$\frac{\partial^2 \Phi'}{\partial t'^2} + g \frac{\partial \Phi'}{\partial z'} = 0,$$

(3.2)

where $g$ is the acceleration due to the gravity and this equation must be validated at the free surface,
specifically at $z = 0$, with the $z$-coordinate pointed upwards with its origin at the free surface.

With the boundary conditions defined at the free surface, it is also necessary to enforce that the velocity
potential must comply with the impermeability of the bottom surface. This means that the fluid does not
pass through this surface. So, the vertical component of the fluid velocity through the bottom surface
must be zero which gives the relation:

$$v_z \bigg|_{z=-h} = 0 \iff \left. \frac{\partial \Phi'}{\partial z'} \right|_{z=-h} = 0,$$

(3.3)

These boundary conditions must be imposed when a fluid is flowing in a channel with an impermeable
bottom and a free surface with vertical elevations due to the waves. However, when the goal is to analyse
the flow with a body immersed in it, it is necessary to impose more boundary conditions. These need
to be dependent of the motion of the device and of its shape. For the case that is going to be analysed, the device is idealised as a vertical flap with thickness $t_f'$, with a width $w_f'$ and a height $h_f'$, which can be larger or smaller than the local water depth $h'$. The flap can oscillate due to a hinge that is located at a height $c'$ above the bottom surface. In Fig. 3.1 it is possible to determine the location of these variables in the OSWEC:

![Diagram of OSWEC](image)

Figure 3.1: Model of the OSWEC that is going to be analysed (Renzi and Dias [44])

Similarly, to what was imposed at the bottom surface, it is necessary to impose the impermeability of the flap by stating that the fluid velocity must be equal to the oscillatory velocity of the flap. This boundary condition can be expressed by:

$$\frac{\partial \Phi'}{\partial x'} = - \frac{\partial \theta'}{\partial t'} \cdot (z' + h' - c') \cdot H(z' + h' - c'), \quad x' = 0, \quad -\frac{w_f'}{2} < y' < \frac{w_f'}{2},$$  \hspace{1cm} (3.4)

where $\theta'$ is the rotational motion of the flap and, since that the rotation is positive when it moves in anticlockwise direction, it is necessary to place a minus sign to impose a velocity to the fluid in the negative direction of the $x$-axis. When $\theta'$ is derived in relation to $t'$, the result will be the rotational velocity which, when multiplied by the distance between a certain point on the flap and the hinge, will result in the angular velocity at that point. The Heaviside's function is represented by $H$ which allows the possibility of imposing the no flux condition through the support, complying with the limits that were defined. This boundary condition must be verified inside the flap's volume which is defined by the flap's height, its thickness and its width. However, Linton and Mclver [31] concluded that the thickness of the flap can be neglected if compared with the flap's width and by considering the hypothesis of thin obstacle. In the calculations that were made by Renzi and Dias [44, 45] they also considered the thin-body approximation, neglecting the thickness of the flap when applying this boundary condition.

This semi-analytical solution was used by several authors such as Evans and Porter [18], Renzi et al. [43], Renzi and Dias [44, 45] and Sarkar et al. [49], among several others. Since some of these publica-
tions compared these theoretical calculations with the results obtained experimentally, they had to make all their calculations with dimensionless variables. These dimensionless variables will be represented by the same letters but without the prime symbol, \((x', y', z')\), being adimensionalised by the flap’s width. Through a dimensional analysis it is possible to perceive how each variable can be adimensionalised and in Eq. 3.5 it is possible to find the results.

\[
(x, y, z) = (x', y', z')/w_f; \quad t = t' \cdot \sqrt{g/w_f}; \quad \Phi = \Phi' \cdot (A'/\sqrt{g}w_f)^{-1}; \quad \theta = \theta' \cdot (w_f/A'),
\]

(3.5)

where \(A' = 1\) m is the amplitude scale of the incident wave. So, this value can also be adimensionalised by dividing it by a scale amplitude factor.

\[
(h, c) = (h', c')/w_f; \quad \omega = \omega' \cdot \sqrt{w_f/g}; \quad A_I = A'/A',
\]

(3.6)

where \(\omega\) is the incident wave’s frequency. It can be assumed that this frequency will only give rise to a single frequency of oscillation, as was stated by Evans [16]. The motion of the flap, due to what was previously stated, is defined by:

\[
\theta(t) = \text{Re}\{\Theta e^{-i\omega t}\},
\]

(3.7)

where \(\Theta\) is the unknown complex amplitude of flap’s rotation. The oscillation of the flap, \(\theta\), is dependent of several parameters such as the excitation torque of the waves, the radiation damping, the added torque due to the inertia and the flap’s buoyancy.

The velocity potential of the fluid flow is a function that is dependent of both the location and of the time. It can be defined by the sum of the velocity potentials that influence its propagation. This resultant velocity potential can be represented by:

\[
\Phi(x, y, z, t) = \text{Re}\{(\phi^I + \phi^D + \phi^R) \cdot e^{-i\omega t}\} = \text{Re}\{(\phi^S + \phi^R) \cdot e^{-i\omega t}\},
\]

(3.8)

where \(\phi^I\) is the incident wave potential, \(\phi^D\) is the diffracted wave potential and \(\phi^R\) is the radiated wave potential. The incident wave potential which is not dependent of the presence of the flap can be determined by:

\[
\phi^I = -\frac{iA_I}{\omega} \cdot \frac{\cosh(kh)}{\cosh(k(z + h))} \cdot e^{-ikx},
\]

(3.9)

and \(A_I\) is the dimensionless amplitude of the incident regular waves, \(h\) is the dimensionless local water depth, it has been assumed that the direction of the waves is from \(x = +\infty\) to \(x = -\infty\) and the dimensionless wave number, \(k\), is obtained by solving the dispersion relation.

Experimentally the scattered potential, \(\phi^S\), can be determined by maintaining the flap fixed while the incident waves hit the flap. This occurrence causes the waves to be diffracted which can be obtained by considering the resultant scattered potential and the incident wave potential, \(\phi^D = \phi^S - \phi^I\). The radiated potential, \(\phi^R\), can be determined by analysing the waves that are radiated by the flap when it is set on motion by forced oscillations without the presence of incident waves.

An incident regular wave is defined by two parameters which are the wave period and the wave amplitude which is the distance from the free surface up to the wave crest or trough. From the kinematic-dynamic
boundary condition at the free surface it is possible to obtain a relation between the waves’ length and its frequency, or period, which is called the dispersion relation. The dispersion relation is a property that depends of the incident waves and it can be obtained by substituting the dimensionless incident wave potential, $\phi^I \cdot e^{-i\omega t}$, in the obtained kinematic-dynamic boundary condition, Eq. 3.2. The result from the substitution is the dispersion relation, given by:

$$\omega^2 = k \cdot \tanh(kh). \quad (3.10)$$

In deep waters, the resultant value of the multiplication of the wave length by the local water depth, $kh$, assumes large values causing the hyperbolic tangent to tend to a unitary value, $\tanh(kh) \to 1$. So, for waves in this region, with larger depths, the hyperbolic tangent can be discarded. However, for waves that travel towards the shallow water region, when the values of $kh$ are smaller than 2.65 then the value of $\tanh(kh)$ assumes values that are smaller than 0.99. This occurrence imposes the need of considering this hyperbolic term, $\tanh(kh)$, in the calculation of the wave number, by using the Eq. 3.10.

Besides the calculation of the wave number of the incident waves for a certain depth, $h$, and a given wave frequency, $\omega$, it is necessary to consider the waves that are going to be radiated by the motion of a WEC. These waves can be radiated in a direction towards the origin of the incident waves or towards their original direction of propagation. However, there is a big difference in the propagation of these waves because they are characterised as being evanescent which, numerically, can be settled by using imaginary wave numbers, $k = i\kappa$. It is necessary to resort to these imaginary wave numbers because these evanescent waves need to decay exponentially instead of propagating towards infinity, with no variation. By replacing these imaginary terms in Eq. 3.10 and by using the relations between the trigonometric equations, it is possible to achieve the following relation:

$$\omega^2 = -\kappa \cdot \tan(\kappa h), \quad (3.11)$$

and this relation is needed to obtain the infinite amount of wave numbers of the evanescent waves which can be obtained by solving Eq. 3.11. There are an infinite number of branches resultant from the tangent term. These need to be intersected by $-\omega^2/\kappa$, as can be seen in Fig. 3.2.

![Figure 3.2: Graphical representation of the dispersion relation's roots with imaginary roots (Mei et al. [35])](image)
The dimensionless kinematic-dynamic boundary condition needs to be applied at the free surface, for \( z = 0 \). All the three potentials must comply with the boundary conditions and the Formula - 3.9 is the resultant velocity potential for the incident waves. The kinematic-dynamic boundary condition is enforced to the velocity potential of the radiated and diffracted waves, resulting in:

\[
\frac{\partial^2 \phi^{(R,D)}}{\partial t^2} + \frac{\partial \phi^{(R,D)}}{\partial z} = 0 \Leftrightarrow -\omega^2 \cdot \phi^{(R,D)} + \frac{\partial \phi^{(R,D)}}{\partial z} = 0, \quad z = 0,
\]

(3.12)

where the notation \( \phi^{(R,D)} \) means that this boundary condition is being imposed to both the radiated and the diffracted velocity potentials. The resultant equation was obtained by replacing the Eq. 3.9 in this boundary condition. Also, the impermeability boundary condition needs to be imposed to both these velocity potentials at the bottom surface, which is presented in:

\[
\frac{\partial \phi^{(R,D)}}{\partial z} = 0, \quad z = -h.
\]

(3.13)

The presence of the flap needs to be considered when imposing the dimensionless boundary conditions to the radiated velocity potential. The flap’s rotational velocity is given by:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial (\Theta e^{-i\omega t})}{\partial t} = -i\omega \Theta e^{-i\omega t}.
\]

(3.14)

So, the boundary condition that must be applied due to the motion of the flap, which needs to be solely imposed to the radiated velocity potential, takes the following format:

\[
\frac{\partial \phi^R}{\partial x} = -(-i\omega \Theta) \cdot (z + h - c) \cdot H(z + h - c),
\]

(3.15)

by defining \( V = i\omega \Theta \) it is possible to settle that the radiated velocity potential at the flap’s oscillating location is given by:

\[
\frac{\partial \phi^R}{\partial x} = V \cdot (z + h - c) \cdot H(z + h - c), \quad x = 0, \quad -\frac{1}{2} < y < \frac{1}{2}.
\]

(3.16)

With the boundary condition settled for the radiated velocity potential it is possible to start defining the diffracted velocity potential by knowing that, as for the real physical scale, it must be equal to the inverse of the incident wave potential:

\[
\phi^D = -\phi^I, \quad x = 0, \quad -\frac{1}{2} < y < \frac{1}{2}.
\]

(3.17)

While evaluating these boundary conditions, it is needed to consider that the equations obtained by Haskind impose that at the downstream infinity the velocity potential must have a zero-wave propagation and at the upstream infinity the velocity potential must be a combination between the incident and the reflected waves. The velocity potential, \( \Phi(x, y, z) \), varies with three variables which are: \( x \) that is directed along the waves’ direction of propagation, \( y \) that is directed perpendicularly to the waves’ direction of propagation and \( z \) that is directed perpendicularly in the upward direction in relation to the free surface.
This velocity potential can suffer a separation of variables and the vertical coordinate can be defined by a single function, \(Z(z)\). This function can be determined by applying the boundary conditions, that can be seen in Eq. 3.12 and in Eq. 3.13, and it is necessary to impose the Laplace equation, Eq. 3.1.

A problem where these boundary conditions need to be imposed is defined as of Sturm-Liouville type. The solution of such problem needs to be proportional to \(\cosh(k \cdot (z + h))\) and for this case is represented by:

\[
Z_n(z) = \frac{\sqrt{T} \cdot \cosh(k_n \cdot (z + h))}{\sqrt{h + \omega^{-2} \cdot \sinh^2(k_n \cdot h)}}, \quad n = 0, 1, 2, \ldots, N
\]  

(3.18)

This equation is usually called an eigenfunction and the positive real solutions for the incident’s wave number, \(k_0\), are the eigenvalues. However, when considering the radiated waves and to model them as being evanescent, it is necessary to use imaginary wave numbers, \(k_n = i\kappa_n\). These, when being replaced in Eq. 3.18, become:

\[
Z'_n(z) = \frac{\sqrt{T} \cdot \cos(k_n \cdot (z + h))}{\sqrt{h - \omega^{-2} \cdot \sin^2(k_n \cdot h)}}, \quad n = 1, 2, \ldots, N
\]  

(3.19)

Kreisel [29] has proven that any function \(G(z)\) defined in the interval \(z \in [-h, 0]\) can be represented by a Fourier series based on the set \(\{Z_n(z)\}, n = 0, 1, 2, \ldots, N\). The velocity potential, \(\phi\), can be decomposed into two different functions as it is expressed by:

\[
\phi^{(R,D)}(x, y, z) = \sum_{n=0}^{N} \varphi_n^{(R,D)}(x, y) \cdot Z_n(z),
\]  

(3.20)

By using Eq. 3.18 and Eq. 3.19 in Eq. 3.20, it is possible to represent the velocity potential by:

\[
\phi^{(R,D)}(x, y, z) = \varphi_0^{(R,D)}(x, y) \cdot Z_0(z) + \sum_{n=1}^{N} \varphi_n^{(R,D)}(x, y) \cdot Z'_n(z),
\]  

(3.21)

where the first term, \(n = 0\), corresponds to the propagating wave and the subsequent terms, for \(n > 0\), are the evanescent modes generated by the radiated waves. Renzi and Dias [44] concluded that the calculation of the velocity potential converges quickly for values of \(N \geq 4\) with virtually no error. So, by considering solely 4 evanescent wave numbers it is possible to obtain very small relative errors when calculating the velocity potential of both the radiated and the diffracted waves.

To obtain the radiated velocity potential caused by the motion of the flap, which spans from \(z_{bot} = -h + cf\) up to the upper part of the flap at \(z_{up} = -h + hf + cf\), it is necessary to execute some algebra, like it is presented in Appendix - B. For the radiated part, this algebra is based on the replacement of the Eq. 3.20 in the Eq. 3.15 and by taking advantage of the orthogonality of \(Z_n(z)\) along the flap’s surface. The obtained radiated velocity potential is given by:

\[
\frac{\partial}{\partial x} \sum_{n=0}^{N} \varphi_n^{(R)}(x, y) = \frac{\sqrt{2V} \cdot \left[ k_n h_f \cdot sh(k_n(h_f + cf)) - ch(k_n(h_f + cf)) + ch(k_n cf) \right]}{k_n^2 \cdot \sqrt{h + \omega^{-2} \cdot sh^2(k_n h)}} = V \cdot f_n,
\]  

(3.22)

By having the variable \(f_n\) defined this way, it is possible to analyse WECs that can be either completely
submerged or above the free surface, usually called surface piercing.

\[ f_n = \frac{\sqrt{2} \cdot \left[ k_n h_f \cdot sh(k_n(h_f + c_f)) - ch(k_n(h_f + c_f)) + ch(k_n c_f) \right]}{k_n^2 \cdot \sqrt{h + \omega^{-2} \cdot sh^2(k_n h)}} \]  

(3.23)

For the diffracted velocity potential, the calculations are more direct than for the radiated potential. It needs to be calculated at the free-surface, at \( z = 0 \), and because the incident waves are of the regular type, it is only necessary to consider the first mode, \( n = 0 \), which is equivalent to the propagating wave defined in Eq. 3.21.

\[ \frac{\partial}{\partial x} \sum_{n=0}^{N} \varphi_n^{(D)}(x, y) = A_I \cdot \frac{k_n \cdot \sqrt{h + w^{-2} \cdot sinh^2(k_n h)}}{\sqrt{2} \cdot \omega \cdot cosh(k_n h)} \]  

(3.24)

In the literature it is usual to define the following variable:

\[ d_n = \frac{k_n \cdot \sqrt{h + w^{-2} \cdot sinh^2(k_n h)}}{\sqrt{2} \cdot \omega \cdot cosh(k_n h)} \delta_{0n}, \quad n = 0, 1, 2, ..., N \]  

(3.25)

and, by using the Kronecker’s delta, it is possible to settle that this variable is zero for every value of \( n \) unless when \( n = 0 \).

The equations that enable the calculation of the flap’s motion require the hydrodynamic parameters to be determined. These are dependent of the previously defined functions, \( f_n \) defined in 3.23 and \( d_n \) defined in 3.25.

### 3.2 Hydrodynamic Parameters

In this analysis it has been considered that the analysed body is submerged in a fluid with density \( \rho \) and that it will be influenced by the unsteady flow that will be created around the body. This latter occurrence evidences the need of having to consider the additional effects that will result from the fluid acting on the body. These additional effects, which are dependent of the radiation of waves, were determined by Mei et al. [35] as being in phase with the body’s acceleration, \( \ddot{\theta} \), and with the body’s velocity, \( \dot{\theta} \). This enables the possibility of deducting that they are related with the added moment of inertia, \( \mu \), and with the radiation damping, \( \nu \), respectively. Beside these parameters, that are caused by the unsteady flow around the WEC while it is in motion, it is necessary to calculate the hydrodynamic parameters of the OSWEC such as the second moment of inertia of the flap, \( I_f \), the hydrostatic restoring moment, \( C \), and all the parameters that will be added to the system due to the presence of the generator. The second moment of inertia, for an idealised OSWEC with a rectangular parallelepiped shape, can be calculated by the following formula:

\[ I_f = \frac{1}{3} \cdot \rho_f \cdot t_f \cdot w_f \cdot h_f^3 \]  

(3.26)

where \( \rho_f \) is the volumetric density of the flap, \( t_f \) is the thickness of the flap, \( w_f \) is the width of the flap and \( h_f \) is the height of the flap, as was presented by Dhanak and Xiros [13].
The equation that enables the calculation of the hydrostatic restoring moment is present in Dhanak and Xiros [13] and it can be obtained by knowing that this force is caused by the floatation of the flap. So, it is necessary to take in consideration the flap’s and the fluid’s volumetric density.

\[ C = g h_f t_f w_f \cdot \left( \rho r_B - \rho_f r_G \right), \]  

(3.27)

where \( r_B \) and \( r_G \) are the centre of buoyancy and the centre of gravity of the immersed body, respectively.

For the case that is being analysed, it is known, due to the rectangular parallelepiped shape, that the centre of gravity is coincident with the centre of floatation, \( r_B = r_G = h_f/2 \). So, the hydrostatic restoring moment can be presented in the following way:

\[ C = g h_f t_f w_f \cdot \left( \rho \frac{h_f}{2} - \rho_f \frac{h_f}{2} \right) = \frac{1}{2} g t_f w_f h_f^2 \rho \cdot \left( 1 - \frac{\rho_f}{\rho} \right). \]  

(3.28)

The equation of motion of the flap is identical to the equation of a damped harmonic oscillator. By using the parameters that were previously defined, the equation of motion of the flap can be expressed by:

\[ I_f \ddot{\theta}(t) + C \dot{\theta}(t) = F + T_e(t), \]  

(3.29)

where the first function placed on the right side of the \( \text{Eq. 3.29} \) is called the hydrodynamic torque, \( F \). This function evaluates the pressure differential between the sides of the flap, at \( x = -t_f/2 \) and \( x = +t_f/2 \), along its exposed area which results in a force, \( F = p \cdot A \). By multiplying this force by an arm it will result in a torque, \( T = F \cdot h_f \). By using the linearised form of the Bernoulli equation, as presented in Renzi and Dias [44, 45] where \( \Delta p = -\frac{\partial (\Delta \phi)}{\partial t} \), the hydrodynamic torque can be defined by:

\[ F = \int_{-h+c}^{-h+hw_f} \int_{-1/2}^{1/2} \frac{\partial (\Delta \phi^{(R,D)})}{\partial t} \cdot (z + h - c) \, dy \, dz, \]  

(3.30)

The diffracted and radiated waves are considered in the equation of motion of the flap by using the velocity potential, defined in \( \text{Eq. 3.22} \) and in \( \text{Eq. 3.24} \). When evaluating the velocity potential, it is necessary to take in consideration the variation that will occur between the sides of the flap, at \( x = -t_f/2 \) and \( x = +t_f/2 \), which can be calculated by the following expression:

\[ \Delta \phi^{(R,D)} = \phi^{(R,D)}(-0, y, z, t) - \phi^{(R,D)}(+0, y, z, t) \Leftrightarrow \]

\[ \Leftrightarrow \Delta \phi^{(R,D)} = \left[ \phi^{(R)}(-0, y, z, t) + \phi^{(D)}(-0, y, z, t) \right] - \left[ \phi^{(R)}(+0, y, z, t) + \phi^{(D)}(+0, y, z, t) \right], \]  

(3.31)

where it has been used the thin-body approximation with \( t_f \approx 0 \) since that \( t_f \ll w_f \). With the Expression - 3.31 it is possible to integrate the velocity potential along the flap’s surface. By following the method used by Renzi and Dias [44, 45], the \( \text{Eq. 3.30} \) takes the following format:

\[ F = -\mu \ddot{\theta}(t) - \nu \dot{\theta}(t) + F, \]  

(3.32)

where the coefficient that is in phase with the body’s acceleration is called added moment of inertia, usually represented by \( \mu \), and is given by:
\[
\mu = \frac{\pi}{4} \cdot \text{Re}\left\{ \sum_{n=0}^{\infty} f_n \alpha_{0n} \right\}.
\]  
(3.33)

The coefficient that is in phase with the angular velocity of the flap, \( \dot{\theta}(t) \), acts as a damping component which results from the diffracted waves and it is represented by:

\[
\nu = \frac{\pi \omega}{4} f_0 \cdot \text{Im}\{\alpha_{00}\}.
\]  
(3.34)

Finally, the excitation torque, \( F \), is given by the following expression:

\[
F = -\frac{i \omega \pi}{4} A f_0 \cdot \beta_{00},
\]  
(3.35)

The resultant equation of motion of the flap which can be obtained by replacing the hydrodynamic torque, \( F \), and its parameters, in Eq. 3.29, will take the following format:

\[
(I_f + \mu) \cdot \ddot{\theta}(t) + \nu \cdot \dot{\theta}(t) + C \cdot \theta(t) = F + T_e(t),
\]  
(3.36)

The last term in the right side of the Eq. 3.36, \( T_e \), represents the function that defines the torque applied by the generator to the flap and it will be assumed as being a function of time. This function besides being a function of time can be assumed as being composed by three parts. These can be partly inertial, partly damping and, also, partly elastic. Each of these components is in phase with a single parameter which can be with the acceleration, the velocity or the angular displacement. The resultant function can be expressed by:

\[
T_e(t) = -\mu_{pto} \ddot{\theta}(t) - \nu_{pto} \dot{\theta}(t) - C_{pto} \theta(t),
\]  
(3.37)

where \( \mu_{pto} \), \( \nu_{pto} \) and \( C_{pto} \) are the inertial, the damping and the elastic properties of the generator, respectively. These parameters are a function of the chosen generator however, they can be optimised to guarantee that the amount of generated power is maximised. As presented by Pizer [42] the power extracted from the motion of a device can be calculated linearly estimated by multiplying the damping force, \( F_c = \nu_{pto} \cdot \dot{\theta}(t) \), by the device’s angular velocity, \( \dot{\theta}(t) \), which results in \( P_G = F_c \cdot \dot{\theta}(t) \). This formula can be used when calculating the average generated power over a period \( T = 2\pi/\omega \):

\[
P_G = \frac{1}{T} \int_0^T F_c \cdot \dot{\theta}(t) \, dt = \frac{1}{T} \int_0^T (\nu_{pto} \cdot \dot{\theta}(t)) \cdot \dot{\theta}(t) \, dt,
\]  
(3.38)

To calculate the average generated power, it is necessary to determine the equation of motion of the flap which can be achieved by replacing the generator parameters in the Eq. 3.36. This resultant equation, relates the flap’s dimensions with the generator’s parameters taking in consideration the diffracted and radiated waves caused by the flap’s motion. The resultant equation relates all the forces that are applied on the flap and it is represented by the following equation:

\[
(I_f + \mu + \mu_{pto}) \cdot \ddot{\theta}(t) + (\nu + \nu_{pto}) \cdot \dot{\theta}(t) + (C + C_{pto}) \cdot \theta(t) = F,
\]  
(3.39)

To obtain the equation that allows the calculation of the motion of the flap it is necessary to calculate the first and second time derivative of \( \theta \). By effectuating the derivatives, the equation will result in the following expression:

\[
\left[ -\omega^2 \cdot (I_f + \mu + \mu_{pto}) - i \omega \cdot (\nu + \nu_{pto}) + (C + C_{pto}) \right] \cdot \theta(t) = F,
\]  
(3.40)
So, the equation that defines the motion of the flap can be defined by the following expression:

$$\theta(t) = \frac{F}{-\omega^2 \cdot (I_f + \mu + \mu_{pto}) - i \omega \cdot (\nu + \nu_{pto}) + (C + C_{pto})},$$  \hspace{1cm} (3.41)

With this equation it is possible to determine the amplitude of oscillation of the flap when it is forced to oscillate by incident waves with a constant frequency, $\omega$. The presence of the generator influences the amplitude of response of the flap and, as can be concluded by analysing Eq. 3.38, the average generated power is linearly proportional to the damping coefficient of the generator and to the flap’s rotational velocity. It is possible to replace this latter equation in the formula that defines the average generated power, Eq. 3.38. This substitution was done in Appendix B.2 - Average Generated Power, were it is possible to follow all necessary steps to achieve the following equation:

$$P_G = \frac{1}{2} \frac{\omega^2 \cdot \nu_{pto} \cdot |F|^2}{[-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}]^2 + \omega^2 \cdot (\nu + \nu_{pto})^2},$$  \hspace{1cm} (3.42)

The Eq. 3.42 is essential for the calculation of the optimum damping coefficient of the generator. The amount of generated power, as can be concluded from the Eq. 3.38, is dependent of the angular velocity of the flap and of the generator’s damping coefficient. In a book authored by Falnes [20], the relation between the generated power and the angular velocity of the flap was analysed and it can be found a figure where the relation between these parameters were plotted.

Figure 3.3: Relation between the average generated power and the angular velocity of the flap (adapted from Falnes [20])

where $P_e$ is the excitation force created on the flap by the incident waves. The formula that defines the excitation force is shown in Fig. 3.3 and it takes in consideration the variation that will occur to the phase angle between the flap’s angular velocity and the excitation force, given by $\gamma = \gamma_\theta - \gamma_e$.

By varying the damping coefficient of the generator, the flap’s angular velocity will change which will cause the average generated power to also vary, as shown in Fig. 3.3. The damping coefficient can be chosen so that the flap oscillates at an optimum angular velocity, enabling the average generated power, over a cycle, to reach its maximum. The optimum damping coefficient can be calculated by calculating the following derivative:

$$\frac{\partial P_G}{\partial \nu_{pto}} = 0,$$  \hspace{1cm} (3.43)
The Eq. 3.43 needs to be applied to the Eq. 3.42 and some algebra needs to be done for the determination of the generator’s optimum damping coefficient. All the steps can be seen in Appendix B.3 - Optimum Damping Coefficient. The resultant equation that defines the optimum damping coefficient is:

\[ \nu_{pto} = \nu_{opt} = \sqrt{\frac{-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}}{\omega^2} + \nu^2}, \]  

(3.44)

By having these optimum variables defined, it is assured that the generator will extract the maximum possible energy from the incident waves. However, beside these variables, the natural frequency of the device should also be considered. The natural frequency of a body is usually calculated by the following expression:

\[ \omega_n = \sqrt{\frac{k_{eq}}{M_{eq}}} = \sqrt{\frac{C + C_{pto}}{I + \mu + \mu_{pto}}}, \]  

(3.45)

The device’s natural period is given by the following expression:

\[ T_n = 2\pi \cdot \sqrt{\frac{I + \mu + \mu_{pto}}{C + C_{pto}}}, \]  

(3.46)

The oscillations made by a body, when it is being excited by an external force, are defined as having the same frequency as the excitation force. The devices’ amplitude of oscillations is a function of the waves’ frequency and as close as they get to the natural frequency of the device, the amplitude of the oscillations will increase, as can be concluded by replacing Eq. 3.45 in Eq. 3.41.

\[ \theta(t) = \frac{F}{(I_f + \mu + \mu_{pto}) \cdot (-\omega^2 + \frac{C + C_{pto}}{I_f + \mu + \mu_{pto}}) - i \omega \cdot (\nu + \nu_{pto})} \Leftrightarrow \]

\[ \Leftrightarrow \theta(t) = \frac{F}{(I_f + \mu + \mu_{pto}) \cdot (-\omega^2 + \omega_n^2) - i \omega \cdot (\nu + \nu_{pto})}, \]  

(3.47)

It is possible to achieve a more common format for this equation, where the ratio between frequency of excitation and the natural frequency is employed:

\[ \Leftrightarrow \theta(t) = \frac{F}{\omega_n^2 \cdot (I_f + \mu + \mu_{pto}) \cdot \left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) - i \omega \cdot (\nu + \nu_{pto})}, \]  

(3.48)

When the parameters that define the natural frequency of the device, given by Eq. 3.45, are modified, the amplitude of device’s oscillations will be changed. For sea-states with smaller wave periods it might be necessary to add ballast to the flap which will cause a change in the position of the device’s centre of gravity. If this change causes a decrease to the device’s centre of gravity, then the hydrostatic restoring moment will decrease resulting in an increase to the device’s natural period.

However, beside these considerations and to evaluate the efficiency of the OSWEC it is necessary to assess the capture factor. This is a dimensionless variable and is defined as being the ratio between
the optimum power generated per unit flap’s width and the power of the incident wave per unit of crest
length.

\[ C_F = \frac{P_{opt}}{P_{inc}} = \frac{P_{opt}}{\frac{1}{2}A_t c_g}, \]  
(3.49)

where \( P_{inc} \) is the average work per unit of incident wave period, \( c_g \) is the group velocity of the incident
waves and \( A_t \) is the amplitude of the waves. The formula for the calculation of the waves’ group velocity is:

\[ c_g = \frac{\omega}{2k_0} \left( 1 + \frac{2k_0}{\sinh(2k_0h)} \right), \]  
(3.50)

The parameters defined previously in Eq. 3.33, Eq. 3.34 and Eq. 3.35 are hydrodynamic parameters
that can be determined by using the Green’s integral function. This function allows the possibility of
achieving a relation between the variables \( f_n \) and \( d_n \) with the variables \( \alpha_{pn} \) and \( \beta_{pn} \). The expression
that relates these variables is:

\[ \sum_{p=0}^{P} \left\{ \alpha_{pn} \beta_{pn} \right\} \cdot C_{pn}(v_0) = - \left\{ \begin{array}{c} f_n \\ d_n \end{array} \right\}, \]  
(3.51)

where \( f_n \) and \( d_n \) were defined previously and can be found in Eq. 3.23 and in Eq. 3.25, respectively. To
determine the values of \( \alpha_{pn} \) and \( \beta_{pn} \) it is necessary to calculate the matrix \( C_{pn}(v_0) \).

It has been found by Parsons and Martin [40] that equations like Eq. 3.51 have a fast numerically
convergence when the values of \( v_0 \) are the zeros of the first kind of Chebyshev polynomials, given by:

\[ v_{0j} = \cos \left( \frac{(2j + 1) \pi}{2P + 2} \right), \]  
(3.52)

where \( j = 0, 1, 2, 3, \ldots, P \) and \( P = 2N \). The value of \( N \) was defined in Chapter 3.1 - Governing Equations
when the equation of the velocity potential, given by Eq. 3.21, was being defined. For values of \( N \geq 4 \)
it has been determined by Renzi and Dias [44] that the calculation of the velocity potential equation
converges with virtually no error. So, the value of the variable \( C_{pn}(v_0) \) is defined by the following formula:

\[ C_{pn}(v_0) = -(p + 1) \cdot U_p(v_0) \cdot \frac{ik_n}{4} \int_{-1}^{1} \frac{\sqrt{(1-u^2)}}{1-u} \cdot U_p(u) \cdot R_n \left( \frac{v_0 - u}{\sqrt{v_0-u}} \right) du, \]  
(3.53)

where the function given by \( U_p \) represents the Chebyshov polynomials of the second kind and the in-
dex \( p \) indicates the polynomial that is being analysed. The coefficients of these polynomials can be
found widely on the internet and the ones used and that are shown in Appendix C - Coefficients of the
Second Kind Chebyshev Polynomial were found in [A05]. The function placed on the latter quotient of
Eq. 3.53 given by \( R_n \) is the remainder resultant from the series expansion of the Hankel function of first
kind presented by Gradshteyn and Ryzhik [23]. This function can be expanded by using the following
expression:

\[ H_1^{(1)} \left( \frac{1}{2} \kappa_n \cdot \omega \cdot |v_0-u| \right) = \frac{4}{i\pi} \cdot \frac{1}{\kappa_n \cdot \omega \cdot |v_0-u|} + R_n \left( \frac{1}{2} \kappa_n \cdot \omega \cdot |v_0-u| \right) \]  
(3.54)

By leaving the term \( R_n \) alone in one of the sides of the Eq. 3.54 it is possible to obtain its value since
that the Hankel function has a known expression. The expression that was used to express \( R_n \) is given by:

\[
R_n \left( \frac{1}{2} \kappa_n \cdot \omega \cdot |v_0 - u| \right) = H_{1}^{(1)} \left( \frac{1}{2} \kappa_n \cdot \omega \cdot |v_0 - u| \right) - \frac{4}{i \pi} \cdot \frac{1}{\kappa_n \cdot \omega \cdot |v_0 - u|},
\]

(3.55)

where \( H_{1}^{(1)} \) is the Hankel function of first kind, or it can also be called Bessel function of third kind. So, with this Bessel function of third kind the function \( C_{pn} \) can be determined allowing the possibility of calculating the terms \( \alpha_{pn} \) and \( \beta_{pn} \).

The equations that were shown in this chapter enable the possibility of analysing several configurations for an oscillating surge wave energy converter. For example, it can be a completely immersed or a surface piercing device. The flap's dimensions can be changed allowing the possibility of evaluating the effects of these changes in the evaluated outputs. There are several possible configurations that can be analysed by changing each of these parameters:

- Flap width;
- Flap height;
- Flap thickness;
- Support height;
- Local water depth;
- Wave period/frequency;

Each of these parameters will vary in a range that can be defined independently of each other. Their values need to be chosen accordingly with what has been published, or deployed, allowing the possibility of comparing the results with the ones that have been published by Whittaker and Folley [60], Renzi and Dias [44, 45], for example. By varying these parameters, it is expected that the angular motion, the average generated power and the capture factor will also vary. These variations will enable the possibility of comprehending the effects of varying the parameters.
Chapter 4

Semi-analytical Analysis of the WEC Parameters

The first steps for the formulation of an analysis is the definition of the parameters that will be evaluated which can be characterised as being input parameters and output values. The former, the output values, are the ones by which a device is usually analysed and that enable the possibility of concluding about the viability for further investigation or even for the device to be deployed in open waters. Since that the equations that were shown in 3.2 - Hydrodynamic Parameters are defined as being functions, they have an input parameter and an output value. The inputs will depend of the device’s dimensions which will affect the hydrodynamic parameters of an OSWEC, causing variations to the device’s response. Beside these hydrodynamic parameters, there are the output values, which are:

- Added Inertia;
- Radiation and Optimum Generator Damping;
- Response Angle;
- Excitation Torque;
- Average Generated Power;
- Capture Factor;

Each of these output values can be obtained by solving the semi-analytical equations however, it is necessary to interpret and evaluate the feasibility of their results. For example, this type of devices have their amplitude of oscillations restrained due to the physical constraints inherent to these devices so, it is necessary to establish an upper limit to the amplitude of response. When the incident waves’ frequency is comparable to the device’s natural frequency, the device will enter in resonance which, when theoretically calculated, results in amplitudes of oscillation that are physically impossible. Considering this type of device without the machinery that is needed for power generation, then the maximum amplitude of oscillation would be limited to $90^\circ$ or else the device’s flap would collide with the seabed. However, such
case isn’t reasonable because the only way of extracting the waves’ energy is by placing a power take-off. The PTOs that are used in this type of devices are composed by hydraulic cylinders that are coupled to the device’s flap, as described by López et al. [32]. The amplitude of motion between the piston and the cylinder is defined as stroke length and it will limit the device’s amplitude of motion. Therefore, the device’s maximum amplitude of motion is bounded by the hydraulic cylinder stroke length which will be assumed as being of 40°. The condition that is going to be imposed is:

1. The device’s amplitude of pitch oscillations cannot surpass \( \theta_{\text{max}} = 40^\circ \);

This imposition is commonly used in the articles that have been published on OSWECs. However, the value for the maximum amplitude of oscillation isn’t consensual among the published articles. There are some authors who have decided to use maximum values for the pitch oscillations of 30°, such as Tom et al. [55] and Whittaker and Folley [60], while Henry [25] used the values of 40° and Chehaze et al. [10] of 50°. While, in extreme seas the article of Wei et al. [57] established that the oscillations could go as high as 75°. These values are too widely spread to be able to conclude about which one should be chosen. So, by taking a conservative approach, only the values of the articles that have realised model experiments were considered, which are the ones of Henry [25] and Chehaze et al. [10].

For the determination of the output values it is possible to use an analytical method or a computational program or by making model experiments with these type of WECs. However, to comprehend how each of the inputs will influence the output values, it is necessary to vary the input variables independently between each other. The resultant number of combinations between each input variable is too large for realising experimental tests with these variables. The experimental tests become unfeasible both due to the costs of manufacturing several WEC models and to the time necessary to evaluate each parameter. So, to evaluate which parameters influence the most these output values, it is necessary to use a computational program or an analytical method, or both options. In this thesis, the initial evaluation of the effects caused by the input parameters to the output values will be obtained by solving the semi-analytical equations that were shown in 3.2 - Hydrodynamic Parameters. The input parameters that are going to be evaluated are the following:

- Wave period/frequency;
- Local water depth;
- Flap width;
- Flap height;
- Flap thickness;
- Support height;

Each of these input variables will vary within a specified range which has been selected taking in consideration the values that were published by several authors and the values that were used for the Oyster Aquamarine and for the WaveRoller [AWE] projects. However, to be able to analyse a certain input
variable and conclude about its influence in the output parameters, it is necessary to keep the remaining variables fixed while the parameter that is being analysed may vary. The values for these input variables need to be chosen taking in consideration the feasibility of each combination. So, while evaluating which combinations are feasible, it is necessary to impose some conditions:

1. The sum of the flap height, \( h_f \), with the support height, \( c \), cannot be higher than the local water depth: \( h \geq h_f + c \);

2. The difference between the local water depth, \( h \), and the sum of the flap height, \( h_f \), with the support height, \( c \), cannot be larger than 6 m: \( h - (h_f + c) \leq 6 \);

There are other parameters that will influence the device’s amplitude of oscillations and, subsequently, the average generated power. These are the flap’s density, \( \rho_f \), and the amplitude of the incident waves, \( A_I \). The value of the incident waves’ amplitude will be kept at 1 m and the flap’s density will be kept constant with it the value that is shown in Table 4.1.

Table 4.1: Values for the flap’s density, the water density and the acceleration due to the gravity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap’s density</td>
<td>300</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>Water density</td>
<td>1025</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>Gravity acceleration</td>
<td>9.806</td>
<td>[m/s²]</td>
</tr>
</tbody>
</table>

The flap’s density affects the hydrostatic restoring moment which, by its turn, will change how the device responds to the incident waves. This parameter can be used to change dynamically the natural period of the device by adding ballast to the device’s flap. However, this dynamic change of ballast will not be considered in this thesis.

The interval that was chosen for each of the input variables can be found in Table 4.2. These initial values do not take in consideration the conditions that were presented previously because the constraints only need to be considered when creating the combinations of the inputs.

Table 4.2: Variable intervals for the OSWEC analysis

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap width</td>
<td>8</td>
<td>24</td>
<td>2</td>
<td>[w_f]</td>
</tr>
<tr>
<td>Flap height</td>
<td>8</td>
<td>16</td>
<td>1</td>
<td>[h_f]</td>
</tr>
<tr>
<td>Flap thickness</td>
<td>1</td>
<td>4</td>
<td>0.5</td>
<td>[t_f]</td>
</tr>
<tr>
<td>Support height</td>
<td>0.5</td>
<td>2</td>
<td>0.5</td>
<td>[c]</td>
</tr>
<tr>
<td>Local water depth</td>
<td>10</td>
<td>16</td>
<td>1</td>
<td>[h]</td>
</tr>
<tr>
<td>Wave period</td>
<td>4</td>
<td>14</td>
<td>1</td>
<td>[T]</td>
</tr>
</tbody>
</table>

As stated previously, while evaluating the effects that a certain input parameter has on the output values, it is necessary to fix all the input variables that are not being analysed. So, and to allow the possibility of comparison between the changed variable and the initial values, it is possible to find, in Fig. 4.1, the results obtained for an OSWEC device with the minimum dimensions that were shown in Table 4.2.
replacing these values in the program that was developed with the semi-analytical equations, the results can be found in Fig. 4.1.

By analysing the values of the curves shown in Fig. 4.1(e), Fig. 4.1(f) and Fig. 4.1(c), it is possible to conclude that the generated power, the capture factor and the response angle suffered corrections. These can be attributed to the device’s amplitude of oscillations which has surpassed the established maximum value. By limiting the amplitude of oscillations, the generated power will diminish accordingly with the equation defined by Evans [16]:

\[
P = P_{\text{opt}} \cdot |2r - r^2|, \quad r = \frac{\theta_{\text{max}}}{\theta_{\text{opt}}},
\]

The peaks in amplitude of response are due to the proximity between device’s natural period and incident waves’ period. A device with these dimensions will have a natural period of approximately \( T_n \approx 10.02 \, \text{s} \) and, as can be concluded by evaluating Eq. 3.44, when the device is excited by waves with these periods, the optimum generator’s damping tends to the body’s radiation damping, \( \nu_{\text{opt}} = \nu \). The usage of Eq. 4.1 is mandatory for wave periods located between the 9 s and 13 s, due to the resultant amplitude of motions.

The optimum value for the generator’s damping can be visualised in Fig. 4.1(b) as well as the device radiation damping. In this initial stage, the value of the generator’s damping will be assumed as being equal to the calculated optimum value.
This type of submerged devices is highly dependent of the site’s conditions where it will be placed. So, it is necessary to establish the parameters that characterise the incident waves, such as the local wave amplitude, the local water depth and the local incident wave period.

4.1 Characterisation of the Deployment Site

The optimization of the output parameters, by varying the flap’s dimensions of an OSWEC, need to take in consideration the site where the device will be placed. The initial stage for the optimization of the device’s generated power is the study of the main parameters that characterise the waves in the area where it will be deployed. The area that is being studied is in the nearshore region along the Portuguese shore within the ribbon defined by Gunn and Stock-Williams [24] as being the area where the waves have a higher energetic density. These waves can be characterised by their amplitude, by their direction of propagation, by their period between crests, or troughs, and by the depth where they propagate. The waves’ amplitude will be considered as being unitary for simplicity. Besides the previous simplification, the variability of the waves’ direction of propagation will not be considered and it will be assumed that there exists perpendicularity between the waves’ direction of propagation and the devices’ frontal surface. So, the angle of encounter, $\Psi$, is null. The two remaining parameters will vary accordingly with the site and the characteristics of this region will be described in the following sections.

4.1.1 Incident Wave Period

The incident wave period is one of the inputs that varies depending on the devices’ deployment site. The waves’ period varies depending on several factors such as: the season of the year, the site’s latitude and longitude, the confluence with swell, among other parameters. To characterise the temporary condition of the waves, there is a term known as sea-state. When the sea-states are averaged into a wider period, it is called wave climate. The wave climate is a long term description of a site which contains the combined information of its many sea-states.

The values for the wave periods in the Portuguese near-shore were chosen based on the values published by Rusu and Guedes Soares [47], where it is possible to find the wave climate data for the years between 1994-2003. The values that are going to be used were obtained with in situ measurements which were made by four buoys that were placed along the Portuguese shore in the intermediate water region. One of which was placed in the southern shore and the remaining three were placed in the Portuguese western shore. These three were almost equally spaced along the shore, with one in Leixões, one in Figueira da Foz and one in Sines. The first two buoys, placed near Leixões and near Figueira da Foz, are within the $40^\circ - 60^\circ$ latitude ribbon that was defined by Gunn and Stock-Williams [24] as having more energetic seas. The values for the mean wave periods between 1994 and 2003 can be found in an article published by Rusu and Guedes Soares [47] which can be checked in the Fig. 4.2:
Figure 4.2: Histogram with the obtained wave periods during the period of 1994-2003 (adapted from Rusu and Guedes Soares [47])

Similar values were published by Costa et al. [12] and the waves’ direction of propagation can also be found in this article. As it can be deduced by comparing the values between the histograms (a) and (b) from Fig. 4.2, the waves’ period increases during the winter time and the most common wave period is placed between the range of 7-9 seconds while the mean value during the whole year is in the range of 5-7 seconds.

The semi-analytical method that is being used takes singular values of wave period assuming that the waves are of the regular type. Even though this analysis is based on regular waves, the values for the waves’ period need to be selected accordingly with the site where the device is going to be placed. So, and to evaluate the OSWEC response to the average wave periods in the Portuguese nearshore, the interval that is going to be analysed is settled between the 4 and 14 seconds, as defined in Table 4.3.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave period</td>
<td>4</td>
<td>14</td>
<td>1</td>
<td>[T]</td>
</tr>
</tbody>
</table>

By selecting this range of values, it is possible to evaluate the response of the device when being excited by waves with different periods. This parameter is intrinsic to the area that is being evaluated however, there are other parameters that characterise the chosen area, such as the local water depth and the slope of the seabed.
4.1.2 Seabed Slope and Depth Evaluation

Having defined the area that will be studied, it is important to figure out if the seabed has the proper conditions to be able to accommodate the device’s dimensions. Apart the need of analysing the waves’ period, it is necessary to analyse the depth of the site and the bottom slope in the proximities of the deployment site. These parameters have an impact in the incident wave power and their effects were evaluated by Whittaker and Folley [59]. The slope gradients analysed in [59] were of 1:50, 1:100, 1:200 and 1:500, for the waves that flow from offshore depths with 50 m up to the nearshore region, with a depth of 5 m. With the resultant values, they concluded that the waves that flow toward the sites with more gradual gradients will lose more power than in the cases with more abrupt gradients. They attributed this occurrence to the larger horizontal distance that is needed to ascend from depths of 50 m up to 5 m. So, the conclusion is that with more abrupt gradients the waves will lose less power. This conclusion can be extracted from the Fig. 4.3:

![Figure 4.3: Wave power loss for different bottom slopes (Whittaker and Folley [59])]  

In Fig. 4.3 the values that are shown in the horizontal axis are of the depth’s absolute value. This graph represents the losses for waves with a 10 s period that flow towards shallow water depths. It can be concluded that for depths larger than 8 m the main source of power loss is due to the bottom friction whilst for smaller depths, $0 \leq h < 8m$, it is due to depth induced wave breaking. This occurrence is more evident in depths smaller than 2.5 times the significant wave height.

The OSWEC type of devices, that have been deployed, were placed in sites with depths larger than 8 m. Thus, the analysis that is going to be developed is primarily focused in these depths and in regions with seabed gradients that range between 1:50 up to 1:200, where the amount of power loss is of, at most, 10%. In an article published by Whittaker and Folley [60] it has been stated that almost all the European coast lines have a slope gradient that range between 1:100 up to 1:200. This can be confirmed by analysing the Portuguese seabed’s bathymetry which can be consulted freely in the website referenced [GGSgc]. This website was created with the support of the European Commission and it enables the possibility of viewing and downloading the bathymetry of the European coast lines.
The test site for the *WaveRoller* in the Portuguese waters is placed near Peniche in the Almagreira beach at 39°23′04.1″ N, 9°18′12.0″ W. This device was developed with the idea of being placed in the nearshore region with the goal of not having the devices' flap disrupting the free surface by being always immersed. Besides the need of having to select a site with an easy interconnection with the national electrical grid, the gradient of the bottom was another parameter that was considered. By using the data that is available at [GGSgc], it is possible to confirm that the gradient's value in the surroundings of the Almagreira beach is of 1:100. With this gradient, and as was presented in *Fig. 4.3*, for depths larger than 8 m the losses are smaller than 5%.

With the bottom's gradient settled, it is possible to define the depth values for which this analysis will be performed. To minimise the amount of losses, both due to the bottom friction and due to the depth induced wave breaking, it has been established that, for a gradient of 1:100, the depth should not be smaller than 8 m. Hence, the local water depth values that are going to be evaluated will begin at the 10 m value with a step of 1 m and the highest value will be of 16 m. These values can be checked in the *Table 4.4*:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Water Depth</td>
<td>10</td>
<td>16</td>
<td>1</td>
<td>[h]</td>
</tr>
</tbody>
</table>

The upper limit was defined by analysing the values that were used by the deployed devices, such as the *WaveRoller* and the *Oyster Aquamarine*. The *WaveRoller* developed their devices with the goal of being completely submerged with the ambition of not disturbing the area and the normal marine navigation in the area. On the contrary, the *Oyster Aquamarine* was developed with the goal of maximising the amount of generated power [8] which can be accomplished by increasing the ratio between the flap height and the water depth. So, by choosing the interval that was presented in *Table 4.4*, the possibility of evaluating these two types of *OSWECs* is still possible.

The effects caused by the local water depth need to be analysed to conclude about which depth is going to be chosen. By keeping the device's dimensions fixed and by analysing the reasoning behind the dimensioning of the *Oyster Aquamarine*, it is expected that the increase in depth will diminish the amount of incident wave force causing a decrease in the excitation torque. However, the device's hydrodynamic parameters, such as the added inertia and radiation damping, may change due to the higher depths were the device will be placed.

The values that were obtained for the excitation torque, which can be verified in *Fig. 4.4(d)*, follow the predictions that were previously stated. The increase in local water depth resulted in a decrease of excitation torque which expectedly would lead to a decrease in the amplitude of oscillations, if the values of the added inertia and radiation damping kept unaltered. However, the increase of the local water depth, besides the decrease in excitation torque, caused a reduction in added inertia and radiation damping which leaded to the amplitude of oscillations to be almost unaltered. The effects caused by the
change of the added inertia and the radiation damping on the amplitude of oscillation can be understood
by evaluating the Eq. 3.41. While analysing the Fig. 4.4(a), it is possible to conclude that the added
inertia will reach a limit even though the depth is increased, as exhibited by the curves shown for the
depth of 12 m and for 14 m.

The decrease of added inertia and radiation damping, which are inversely proportional to the average
generated power, culminated in higher values of generated power. For this to be possible, it is necessary
that the parameters that are directly proportional to the generated power, do not decrease more than
the values which are inversely proportional. So, if this is considered, it is possible to say that it can be
beneficial to place the device in higher depths, as shown in Fig. 4.3. The main benefit is that the incident
wave power will register smaller losses but, for a constant seabed slope, higher depths occur at larger
distances from the shore.

In this stage of analysis, only regular waters are being considered so, it is not needed to consider the
variability that will occur to the waves’ period. Still, while dimensioning these devices, the device’s
natural period should be analysed with the goal of evaluating the proximity towards the most common
wave period. However, for regular waves, since there isn’t a most common wave period, the value of
interest is the mean value of generated power for the different wave periods. For this analysis, the
average generated power reaches a peak for wave periods of 12 s when the depth reaches its smallest
value, of 10 m, resulting in a surface piercing device. The mean generated power for each of these
depths, from smallest to largest, is: 115.8 kW, 100.6 kW and of 79.6 kW, respectively.
With these intervals for the waves’ period and for local water depth, it is possible to analyse the effects caused by the variation of the flap’s dimensions. These can be based on other devices while taking in consideration the possibility of evaluating other type of configurations, such as different ratios between the flap height and the local water depth, or by using different flaps’ width or by changing support height.

4.2 Individual Analysis of the Device’s Dimensions

With the location defined, and with the parameters that characterise the area settled, it is possible to evaluate the effects caused by the device’s dimensions in the capture factor and in the generated power. The intervals that will be evaluated are going to be based on the values that were used by similar devices, such as the WaveRoller and the Oyster Aquamarine. This can be accomplished because the equations that are used allow the possibility of evaluating both type of OSWECs, the surface piercing and the submerged devices. This analysis can be accomplished by increasing the ratio between the flap’s height and the local water depth. The parameters that will be evaluated are the three main dimensions of the device, which are width, height and thickness, and, finally, the height of the support, where the flap’s hinge is placed.

4.2.1 Flap Width

One of the first parameters that is thought about when the goal is to maximise the generated power is the flap’s width. This immediate reasoning can be explained by reviewing how the incident waves’ power is usually represented which is in power per wave crest length, \(kW/m\). Therefore, to increase the incident power, it can be inferred that the flap’s width should be increased. However, this is true while the relation between the flaps’ width and the wave length is within a certain limit or else there is another parameter that affects the increase of the flaps’ width which is the possibility of having incident waves with an angle of encounter that isn’t zero. In Fig. 4.5 it is possible to determine how the angle of encounter is determined.

If this angle is considered, and to minimise the amount of stresses applied to the flap’s structure, the width should be analysed taking in consideration the possible variability of the angle of encounter \([61]\), given by \(\Psi\). The angle of encounter is defined as being the angle made by the direction of wave propagation and the normal to the device’s flap. The possibility of having an angle of encounter which isn’t nearly zero will originate a torsional moment around the device’s vertical axis causing the flap to deflect, in the elastic regime expectedly. The device’s flap, due to having its surge motions restrained by the hinge, will deflect due to the different phase of the excitation forces along the flap’s width. This deflection may cause the malfunction of the power take-off system or, at least, reduce its life cycle. However, an angle of encounter different from zero, \(\Psi \neq 0\), will not be considered in this analysis so, an orthogonal incidence between the direction of waves’ propagation and the flap’s frontal surface will be assumed.
The largest value of an OSWEC flap's width in 1:20 model experiments was of 1.2 m which is equivalent to 24 m. These experiments were realised by Henry et al. [26] enabling the possibility of concluding about the effects caused by the flap's width in the natural period and in the average power output. There are several authors who have focused on the analysis of the effects caused by the flap's width on the device's response and one of them, which was authored by Henry et al. [26], presented the experimental results that they have obtained. Besides this article, there are other documents where this topic was evaluated, such as in the PhD thesis of Henry [25]. He analysed the hydrodynamic effects of varying the flap's dimensions using computational programs and through model experiments. His experimental models were designed as being at a scale of 1:20 and, others, of 1:40. His experimental results allowed the possibility to conclude that the widest flap, of 24 m, will have an average capture factor that is higher than the smaller devices of 6 m and 18 m. However, by analysing the phase angle between the wave force and the wave crest, which was obtained by Henry [25] with the computational program WAMIT, it is possible to conclude that the wave force is almost always in phase with the horizontal water particle acceleration for flaps with smaller widths. For flaps with a larger width, it can be concluded that for seas with higher periods there is a good agreement with the long wave approximation whereas, for smaller wave periods, the values of the phase angle leads the deduction that the long wave approximation isn’t as valid as for the larger wave periods [25], as can be understood by analysing Fig. 4.6.

The first order term of the long wave approximation of the wave force is based on it being in phase with the horizontal water particle acceleration and the second term as being in phase with the horizontal water particle velocity and radiation damping. So, by analysing the Fig. 4.6 it can be said that for higher wave periods the first term, which is in phase with the particle acceleration, dominates the excitation force while for smaller wave periods the first term becomes less dominant.

The values for the flaps' width that were used by Henry [25] through the computational program WAMIT were taken into consideration, to comprehend which values should be analysed with the semi-analytical
equations. Beside these values, there were other authors who were considered such as Tom et al. [55] and Whittaker and Folley [60], for example.

The values were chosen with the idea of allowing the possibility of evaluating the effects caused by the flap's width in both extremes. By choosing a smaller value and a larger value it is possible to evaluate the differences caused by the width in the output values. The smaller value enables the possibility of analysing the device’s response when excited by frequencies that are near to its natural frequency and the larger value will allow the possibility of analysing the changes in the amount of generated power and in the capture factor. In the article published by Renzi and Dias [45], the authors concluded that there are benefits in choosing a wider flap, rather than a smaller one, mainly due to the broadening of the capture factor. However, it is necessary to be judicious with the chosen value because in further stages it may be necessary to consider an angle of encounter different from zero [61]. The values that were selected can be checked in Table 4.5:

Table 4.5: Flap’s width interval for the semi-analytical analysis of an OSWEC

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap’s width</td>
<td>8</td>
<td>24</td>
<td>2</td>
<td>( w_f )</td>
</tr>
</tbody>
</table>

Conversely to the constraints that were imposed between the local water depth and the vertical dimensions of the device, the flap’s width doesn’t have any constraint. So, the combinations of the input variables, in relation to this dimension, won’t be refined. The values that were obtained by changing this variable within the defined limits can be verified in Fig. 4.7.

The obtained values for this input parameter, which can be checked in Fig. 4.7, were expected or, at least, are similar to the ones that were obtained by Henry [25], Renzi and Dias [45] and Whittaker and Folley [60]. These results already take in consideration the constraints that were imposed to the amplitude of oscillation which cannot be larger than 40°, as can be seen in Fig. 4.7(c).
The three flaps, due to the large difference between their widths, will have different natural periods of 12.2 s, 15.8 s and 18.0 s, from the shortest to the widest flap, respectively. When the flaps are excited by waves whose period is in the proximities of the device’s natural period, the theoretically calculated amplitude of oscillations will tend to a value that is not feasible for these devices. So, and as was previously stated, the amplitude of oscillations is constrained primarily by the generator’s stroke length and secondarily by the proximity to the seabed. Due to this constraint, the average generated power, which is proportional to the amplitude of response, as can be checked in Eq. 3.42, needs to be corrected. The effects caused by the constrained amplitudes of motion in the generated power were evaluated by Evans [16], where he concluded that the resultant value for this parameter could be approximated by Eq. 4.1. This equation empowers the conclusion that while the difference between the optimum amplitude of oscillations and the maximum amplitude increases, the actual generated power will decrease instead of reaching the optimum value, which can be calculated with Eq. 3.42.

The benefits of tuning an OSWEC towards the most common wave period isn’t very favourable when dealing with this type of WECs. They have their amplitude of oscillations highly constrained, either due to the stroke length of the hydraulic generators or due to the proximity to the seabed. So, it can be detrimental to design the device with solely the natural period in mind, while the increase of flap’s width can broaden the capture factor and increase the total amount of generated power, comparatively. The limitation to the amplitude of oscillation will not allow the device to generate its optimum values of power, as can be seen in Fig. 4.1 and as was concluded by Whittaker and Folley [60].
In sea-states with a smaller wave period, it can be concluded that the benefits of having a wider flap are not as evident as when analysing the flap in higher wave periods. This can be confirmed by analysing the Fig. 4.7(e) and Fig. 4.7(f). However, as previously stated, and by comparing the capture factors between each flap, it is possible to confirm the broadening effect and the increase of the capture factor caused by the widening of the device’s flap [45]. The mean value of the capture factor for the three flaps, from the narrowest to the widest, is of 0.29, 0.43 and 0.53, respectively. So, the increase in the flap’s width should be considered when dimensioning a device of this type because it will broaden and increase the device’s capture factor.

4.2.2 Flap Height

The height of an OSWEC is the characteristic by which these devices are commonly categorised. The two main devices that are associated with these type of WECs are the WaveRoller and the Oyster Aquamarine due to their different approaches in terms of selecting the device’s height. The former device, the WaveRoller, was developed with the idea of having a low visual impact on the water surface which could be achieved by modelling the flap’s height as being smaller than the local water depth. Yet, the selection of the flap’s height for the Oyster Aquamarine had the goal of maximising the amount of generated power without setting constraints to the device’s height. The main characteristics of an OSWEC, such as the flap’s width, height, thickness and support height, were evaluated by Henry [25] to understand which dimension should be changed with the goal of maximising the generated power. He concluded that to maximise the wave force, the water column should be completely blocked, that is, the sum of the flap’s height with the device’s support should be equal, or larger, than the local water depth. Any leakage under, over or through the flap would result in a reduction of the wave force and thus a loss of performance. Experiments undertaken at Queen’s University Belfast [25] have shown that the leakage under or through a flap would result in a power loss of up to 30%. This conclusion was achieved when comparing two flaps with different flap heights but with equal width and thickness, of 10 m and 3.2 m, respectively. The resultant freeboards, due to the different flap heights, were of 1 m and -0.5 m. Taking these results in consideration, and to compare them with the semi-analytical equations, the chosen values for the device’s height will allow the possibility of evaluating both cases, the submerged and the surface piercing device, as was expressed by the imposed conditions.

The semi-analytical method that is being used to calculate the device’s response to the incident waves, enables the possibility of evaluating both surface piercing and completely submerged devices. However, while evaluating the devices whose top surface surpasses the free-surface, the values of the radiation damping and of the added inertia will not vary. This is due to a limitation of the semi-analytical equations because they solely consider the influence of the submerged part of the device.

The semi-analytical equations that were shown in chapter 3 - Analytical Analysis of the Oscillating Surge Wave Energy Converter, were adapted from Renzi and Dias [44] and Renzi and Dias [45]. These authors
had their equations delineated for devices that blocked the whole water column, which could be achieved by defining the device’s height as being equal to the difference between the local water depth and the support height. The values that are going to be considered for the device’s height can be verified in Table 4.6:

Table 4.6: Flap’s height interval for the semi-analytical analysis of an OSWEC

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap’s height</td>
<td>8</td>
<td>16</td>
<td>1</td>
<td>( h_f )</td>
</tr>
</tbody>
</table>

To compare the different obtained values, it is necessary to define the initial dimensions of the device. While each parameter that is being evaluated can vary between the defined interval, all the remaining flap’s dimensions have their values fixed to enable an easier comparison between the results. The results obtained by varying the flap’s height can be seen in Fig. 4.8.

![Graphs showing added inertia, radiation damping, response angle, excitation torque, average generated power, and capture factor for different flap heights.](image)

Figure 4.8: Comparison of the results obtained with a flap of 8 m of width, height between 8 m and 16 m, a thickness of 1 m, with a support height of 1 m and in a water depth of 13 m.

As Henry [25] concluded with his experimental results, the increase in flap’s height did result in a higher excitation torque, as presented in Fig. 4.8(d). However, the solely variation of the flap’s height didn’t result in a higher power generation while making evaluations with a fixed water depth. Evaluating the results obtained with the variation of the flap’s width and of the flap’s height, it is possible to state that the flap’s width is the main contributor in the process of flap tuning [25].

The variation of the flap’s height caused variations to the output values, as can be confirmed by analysing the Fig. 4.8. These differences are due to the significant influence that the flap’s height has in the hydrodynamic parameters, such as in the hydrostatic restoring moment and in the flap’s inertia. An
increase in the flap’s height will cause a higher increase in the device’s natural period than when the flap’s width is increased. The mean value of the device’s natural period for the heights of 8 m, 10 m, and 12 m, while keeping the other dimensions fixed, is of 12.3 s, 14.4 s and 17.3 s, respectively. Due to this increase, the ratio between the incident waves’ period and the device’s natural period becomes smaller, causing the device’s inertia and added inertia to become more dominant in the calculation of the amplitude of oscillation and in the generated power, as can be understood by analysing the Eq. 3.48 and Eq. 3.42, respectively.

The change of the flap’s height, between 8 m and 16 m, caused a distinct variation to the average generated power, as can be seen in Fig. 4.8(e). To understand the effects caused by the variation of flap’s height, the mean value of the average generated power is going to be calculated along the range of excitation frequencies for the different flaps. These mean values allow the possibility of determining the height that generates the largest amount of power, within the 4-14 s of wave period, while the other flap’s dimensions are fixed. The resultant values for different heights can be seen in Fig. 4.9.

![Figure 4.9: Variation of the generated power for different flap's height with a constant depth of 13 m](image)

The aim of increasing the device’s height to ensure a greater blockage of the water column, can be defeated if the remaining dimensions are not increased accordingly, as can be concluded by analysing Fig. 4.9. While evaluating the surface piercing devices, it can be concluded, through the analysis of the Eq. 3.27 and Eq. 3.46, that the natural period of the device will remain more or less constant for the devices that have a positive freeboard, 17.3 s, 15.9 s and 15.6 s for the 12 m, 14 m and 16 m flap’s height, respectively. This consistent natural period values are solely for the surface piercing devices and it can be attributed to the constant added inertia’s values which are the dominant inertia term in the calculation of the natural period.

The effects caused to the device’s natural period by the flap’s width and height were evaluated by Whitaker and Folley [60] and he concluded that the flap’s height is much more forceful in the increase of the natural period. Similar conclusions can be derived by analysing the values obtained of the device’s natural period while varying the flap’s width and height with the semi-analytical equations. In the following figure, it is possible to analyse the variation caused by the change of the flap’s height and of the flap’s width, while keeping all the other parameters fixed, in a local water depth of 13 m.
The natural period increases much more rapidly with the increase of the flap's height than with the flap's width. So, with the solely increase of the flap's height, the device’s natural period will achieve values that will be much higher than the periods of the incident waves. The generated power reaches its maximum value for the wave periods that are close to the device’s natural period. However, for the remaining excitation frequencies, the inertia terms become too dominant in the calculation of the amplitude of oscillation and of the generated power, causing the decrease of the generated power and of the capture factor.

Generally, the articles that have been published on this topic do not evaluate the effects caused by the change of the flap's height for the devices that are completely submerged. They evaluate the effects that the other dimensions, such as the flap’s width, the flap’s thickness or the flap’s support height, have on the output values. In these cases, they assume that the sum of the flap height with the support height is larger than the local water depth, which results in a positive flap freeboard. So, to validate the theoretically obtained values, experiments will be realised with the goal of understanding if these semi-analytical equations can also be utilised when evaluating submerged devices.

### 4.2.3 Flap Thickness

The remaining dimension that defines the device’s flap is its thickness. While describing the semi-analytical equations, it was settled that this parameter was one of the limitations of this method due to the approximation realised while calculating the device’s hydrodynamic torque, presented in Eq. 3.30. To expedite the calculation of the device’s hydrodynamic torque, the thin-body approximation was used, similarly to the method used by Renzi and Dias [44, 45]. By using this approximation, the possibility of evaluating the effects caused by the change in flap's thickness, on the device’s radiation damping and on added moment of inertia are hindered. Anyhow, while calculating the device’s inertia and the hydrostatic restoring moment the flap’s thickness is considered. However, the resultant values need to be analysed with care, mainly because the added inertia dominates the inertia terms and, by not considering the...
flap’s thickness while calculating this property, it may give improper results. These may include the device’s natural period, which could be overestimated, the amplitude of response of the device, the capture factor and the average generated power.

The effects caused by the flap’s thickness were analysed by Henry [25] in both of his tests, in the computational, using WAMIT, and in the model experiments. With the results that he obtained computationally, he concluded that the thin-body approximation can be implemented with a small error for wave periods smaller than 7 s. However, for values higher than 7 s, the thickness will start to affect the added moment of inertia which isn’t accounted for in these semi-analytical equations. The values that Henry [25] obtained can be seen in Fig. 4.11:

![Figure 4.11: Variation of the added moment of inertia for different flaps’ thickness (Henry [25])]  

The values shown in Fig. 4.11 are for a surface piercing flap in a depth of 11 m, with the hinge positioned 1.5 m above the seabed and a width of 18 m. As declared previously, the added moment of inertia dominates the inertia terms by being of a magnitude, on average, 10 times larger than the flap’s inertia. The values shown in Fig. 4.11 can be compared with the results that were presented in Fig. 4.7(a), for the intermediate flap’s width, and with the Fig. 4.8(a), for the highest flap.

Considering these limitations and to understand the effects caused by the variation of the flap’s thickness to the remaining output values, this parameter will be analysed with the semi-analytical equations. The values that are going to be evaluated were selected taking in consideration the values used by other authors, such as Henry [25], Henry et al. [26] and Whittaker and Folley [60]. The selected values are presented in Table 4.7:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flaps’ thickness</td>
<td>1</td>
<td>4</td>
<td>0.5</td>
<td>[t_f]</td>
</tr>
</tbody>
</table>

These values allow the possibility of broadly evaluating the effects caused by the variation of the flap’s thickness. The smallest and largest values were used by Henry [25], in his computational analysis. There were others who evaluated this type of device and used values that fit between these limits. The results that were obtained by using the semi-analytical equations, can be verified in Fig. 4.12.

As expected, the variation of the thickness didn’t cause any variations to the device’s added inertia or
to the radiation damping, due to the thin-body approximation. This uniformity will hinder the possibility of calculating, with a small error, the device’s natural period, leading to an incorrect value of amplitude of oscillations. The amplitude of oscillations, are not the solely terms that are affected by the thin-body approximation, as can be concluded by analysing the Fig. 4.12(e) and Fig. 4.12(f). Besides being forced by the same excitation torque, the invariable values of the added inertia and of the radiation damping causes the average generated power to acquire values that are higher than expected, inducing the capture factor to assume improbable values.

Through the results that were obtained by using the semi-analytical equations, it is possible to make some broad deductions about the effects caused to the device’s natural period by the variation of the flap’s thickness. It has been concluded, by analysing the Fig. 4.11, that the variation caused by the flap’s thickness to the added inertia will not be as large as the variation caused by the flap’s width. So, a broad analysis to the device’s natural period caused by the thickness variation, can be seen in Fig. 4.13.

Even though the added moment of inertia dominates the inertia terms, and it is being calculated assuming the thin-body approximation, it is possible to state that the natural period of the device will decrease with the increase of the flap’s thickness. The conclusions that can be extracted from the analysis of the results shown in Fig. 4.13 are identical to the conclusions obtained by Henry [25], through his computational results. These results, presented in [25], enabled the possibility of evaluating the effects caused by the flap’s thickness in the device’s natural period, and for the analysed thicknesses, the results drive the prospect of saying that the increase of the device’s thickness will cause a decrease in the device’s
4.2.4 Flap Support Height

The singular degree of freedom of an OSWEC allows the device to pivot around an axis that is, ideally, perpendicular to the incident waves’ direction. This angular motion is guaranteed by a mechanical hinge, composed by a pin, a support and the device’s flap. Even though, the structure that holds the hinge’s support is resting on the seabed, the height of the axis of rotation will be at a distance $c$ above the seabed, as shown in Fig. 4.14. This distance does not affect directly the dimensions of the flap, while evaluating individually each parameter. However, a change in the height of the support causes a variation to the pivot height which could make the flap’s upper surface closer, or further away, to the free surface, as can be seen in Fig. 4.14.

Before the evaluation that is going to be developed on the effects caused by the change of the support height, it is of interest the deliberation about the space that spans between the seabed and the lower surface of the flap. Part of the incident wave energy will pass through this void space, as can be seen in Fig. 4.14. It seems that as large as this space becomes, greater will be the leakage of wave energy which can cause a change to the total amount of generated power. This potential leakage cannot be evaluated with the used theoretical model however, this was assessed in the PhD thesis of Henry [25].
During his experimental analysis, two different models were evaluated with the goal of understanding the effects caused by the leakage through the flap and through the area that fits between the bottom structure and the flap's lower surface. Via the analysis of the experimental results, he concluded that any leakage through the flap, or through the bottom region, would adversely affect the amount of generated power up to 15% [25].

During the design phase, the structure that will sustain the device’s flap can be dimensioned with different objectives, and one of which could be the possibility of designing a device that would diminish the incident wave force leakage. A solution to resolve such problem could be to increase the device’s support in such way that the gap between the flap's bottom surface and the support upper surface is minimised, as shown in Fig. 4.15.

The value of the support height and of the flap’s height is the same for both Fig. 4.14 and Fig. 4.15. However, the latter has been designed with the goal of decreasing the leakage through the void spaces which will possibly increase the total amount of generated power, as concluded by Henry [25] with his experimental results.

The values that were chosen for the support height took in consideration the values used by other authors and, also, the conclusions that they have taken from their tests. The height values are dependent
of the support structure which will enforce a minimum value, larger than zero. The values that are going to be studied, for the height of the support, which will directly affect the height of the axis of rotation, can be seen in the Table 4.8:

Table 4.8: Flap’s support height interval for the semi-analytical analysis of the OSWEC

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Lower Value</th>
<th>Upper Value</th>
<th>Step</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap’s support height</td>
<td>0.5</td>
<td>2</td>
<td>0.5</td>
<td>[c]</td>
</tr>
</tbody>
</table>

This interval enables the possibility of evaluating the effects caused by the variation of rotation’s axis height while keeping the flap’s height fixed. This analysis, as it is going to be realised, may direct to inaccurate conclusions because a lower support height will place the flap in deeper waters where the horizontal water particle amplitude of motion is smaller. This amplitude affects the surge wave force which can be calculated by using long-wave approximations [21], resulting in the following expression:

\[ F_s = (I + I_a) \cdot \omega^2 \xi, \]  \hspace{1cm} (4.2)

where \( \xi \) is the motion’s amplitude of the horizontal water particle and \( \omega \) is the angular wave frequency, with \( \omega^2 \cdot \xi \) being the water particle acceleration. So, due to the decrease of amplitude of motion in deeper depths, it can be concluded that the surge wave force will decrease proportionally, while using the long-wave approximation and assuming no variation in added inertia. By executing the program replacing the values of the support height by the values shown in Table 4.8, the following results were obtained.

Figure 4.16: Comparison of the results obtained with a flap of 8 m of width, a height of 8 m, a thickness of 1 m, with a support height of 0.5 m, 1 m, 1.5 m and 2 m in a water depth of 13 m
The results shown in Fig. 4.16 enable the possibility of concluding about the effects caused by placing a flap, without changing its dimensions, at different depths, while keeping fixed the local water depth. By solely increasing the support height, the flap will have its lower and upper surfaces shifted upwards to a closer distance to the free surface, where the fluid has a higher horizontal amplitude of motion, shown in Fig. 4.17, resulting in a higher excitation torque as presented in Fig. 4.16(d).

Figure 4.17: Amplitude of horizontal motion of the water particles in a wave with 1 m of amplitude and a period of 8 sec

However, the effects caused by the change in the amplitude of motions are not as significant as the change in added inertia, which can be checked in Fig. 4.16(a). The added inertia is associated with the acceleration of the surrounding fluid, due to the oscillations of the flap [25], which seems to increase with the upwards shift of the flap and of its rotation axis. Even with constant flap dimensions, the raise of the support height will cause the device to increase the blockage of the water column, affecting the added inertia.

As was concluded while evaluating the remaining input parameters, the excitation torque’s peaks that occur for smaller wave periods are compensated by peaks of radiation damping and added inertia, resulting in a limitation on the generated power. The change in height of the rotation axis does not seem to be one of the most effective parameters in increasing the generated power. However, for devices, such as the WaveRoller, that aim to have a ratio between the device’s height and the local water depth smaller than 1, $\frac{h_c + c}{h} < 1$, the decrease of the support height can be compensated by increasing the flap’s height, while keeping this ratio constant. This was proposed by Henry [25] while evaluating experimentally the effects caused by the pivot height, where he concluded that it should be as close to the seabed as possible, allowing for the increase of the flap’s height which would create a larger blockage to the incoming waves.

The evaluation of each individual parameter allowed the possibility of understanding the effects that they will have in the generated power. However, it is necessary to figure out if a combined change of the input parameters will have the same effects in the output values.
4.3 Effects of the Variation of the Device’s Dimensions

The values obtained by varying each input variable allowed the possibility of understanding the effects caused to the generated power, to the capture factor, to the added inertia and to the excitation torque. These results are similar to the ones published by Henry [25], Renzi and Dias [44], Renzi and Dias [45] and Whittaker and Folley [60].

The increase of the flap’s width will induce an increase in the average generated power and will broaden the capture factor's bandwidth. This occurrence, in irregular waves, can be further improved by tuning the device so that its natural period is analogous to the most common wave period. Which, in the results, acts as a horizontal shift of the capture factor’s peak towards the intended wave period. The broadening effect is reflected by having a more uniform capture factor for the different excitation periods. However, the benefits of increasing the width will start to fade when the angle of encounter is considered as being different from zero [61]. In this case, and imagining that a wave crest reaches firstly the frontal extremity that has a positive y-coordinate, then the flap will be forced to oscillate in a direction even though the other extremity is in another phase. This occurrence will originate a torsional moment around the vertical axis of symmetry, as shown in Fig. 4.5.

The analysis that was executed, on the effects caused by the variation of the device’s height while keeping the value of the local water depth fixed, allowed to conclude that the increase in flap’s height will decrease the amplitude of oscillations. This reduction can be attributed to the increase of the following parameters: the radiation damping, the added inertia and the hydrostatic restoring moment. If the aim to tune the flap’s natural period towards the most common wave period is kept, than the increase of the flap’s height is hindered because the natural period increases proportionally with the device’s height.

Having concluded about the effects caused by two of the three main dimensions of the device’s flap, it is necessary to settle the effects caused by the remaining dimension, the flap’s thickness. With this semi-analytical method, the thin-body approximation was employed allowing the possibility of considering that the variation between the upstream and downstream front surfaces’ velocity potentials is negligible. Therefore, by resorting to this approximation, the thickness does not influence these potentials, resulting in a constant value of added inertia, of radiation damping and of excitation torque. Yet, and as was concluded by Henry [25], the increase of the flap’s thickness brings the possibility of tuning the device’s natural period towards the most common wave period, when dealing with irregular waves. This possibility emerges because the increase in thickness results in a decrease of the device’s natural period. Due to this decrease, the device’s natural period assumed values that were within the range of the incident wave periods, causing the device to enter in resonance, as can be understood by the amplitudes of oscillation.

There is one last parameter and it defines the height of the pivot’s hinge which can be assumed as being
the height of the structure that will support the device’s hinge. The increase of the support height, while keeping the remaining dimensions fixed, will place the flap in slightly more energetic waters. This can be attributed to the modest changes that occur in the amplitude of horizontal water particles along the water column, as shown in Fig. 4.17. So, despite the slight variation in horizontal water particle’s amplitude of motion, the change in the output values is too small to ascertain that the increase of the support height is beneficial. However, the reduction of the support height allows the possibility of increasing the flap’s height while keeping the ratio between the device’s height and the local water depth fixed, as suggested by Henry [25].

The following table summarises the effects caused by the increase of each parameter to the main output values. The conclusions that can be extracted from the results, which were obtained with the described semi-analytical method, are the following:

<table>
<thead>
<tr>
<th>Device’s dimension</th>
<th>Generated Power</th>
<th>Capture Factor</th>
<th>Natural Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap’s Width</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
</tr>
<tr>
<td>Flap’s Height (submerged)</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
</tr>
<tr>
<td>Flap’s Height (surface piercing)</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
</tr>
<tr>
<td>Flap’s Thickness</td>
<td>↗</td>
<td>↗</td>
<td>↘</td>
</tr>
<tr>
<td>Support Height</td>
<td>≈ 0</td>
<td>≈ 0</td>
<td>≈ 0</td>
</tr>
</tbody>
</table>

As shown in Table 4.9, there exists the possibility of increasing the parameters that do maximise the generated power while tuning the device’s response. However, to accomplish such goal and to be certain that the theoretical values follow the practical results, it is necessary to validate them. This initial validation can be realised by simply comparing, for a couple of flaps, if the results obtained with the semi-analytical equations are similar to what is measured experimentally.
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Chapter 5

Experimental Analysis

The results that were presented and analysed, in the previous chapters, need to be evaluated experimentally to understand how closely the semi-analytical equations fit the experimental results. Such comparison, between the described semi-analytical equations and the experimental models, for surface piercing devices, was realised by van’t Hoff [56] during his PhD thesis, with the goal of validating the theoretical model results. Part of these comparisons can be found in an article authored by Renzi and Dias [44] where they managed to compare the semi-analytical equations with experimental and numerical results. This comparison is essential to comprehend how fit is the agreement between the semi-analytical equations and the experimental results. If these results are closely related then it is possible to fully optimise the device’s dimensions solely by using the semi-analytical equations, which will decrease the costs associated with model experiments.

One of the settled goals for this thesis was the analysis of devices with a negative freeboard using the semi-analytical equations that were previously described. However, these equations are mainly used, and have been solely validated, for devices that are surface piercing i.e., devices that block the whole water column. So, to evaluate the possibility of using these equations for submerged devices, experiments were realised in the Portuguese National Laboratory of Civil Engineering, NLCE, where they have a department that is mainly focused on the investigation of maritime hydraulics. Their facilities are composed by three irregular wave flumes and four wave tanks with the necessary equipment for wave generation and, partially, equipped for wave measurements.

5.1 Experimental Equipment

The process of assessing the required equipment for the experimental tests was initiated even before knowing which flume was available for the experiments. The goal was to measure the waves’ amplitudes and their excitation forces, and the device’s amplitude of oscillations. Besides the measurement of these parameters, it was fundamental to determine the scale at which the experiments were going to be
realised, the number of wave probes for the measure of both incident and reflected waves, the model had to be built taking in consideration the scale and the flume's width, among several other parameters.

The process of choosing the scale for the model experiments was entirely dependent of the flume's dimensions, mainly its width. So, the device's building process had to halted until the attribution of the wave flume. Apart from the assignment of the wave flume by the NLCE, the wave probes were going to be lent by them, for the duration of the experiments. All the remaining equipment could be prepared beforehand, such as, the apparatus that was going to be used to measure the device's oscillations, the gear that would measure the force that was being applied to the device' flap, and the device’s support structure.

5.1.1 Wave Flume

Due to the filled agenda of the NLCE's bigger wave flumes, only the smaller wave flume could be lent for a longer time duration. It was an irregular wave flume with a width that varied along its length but with a constant value of 0.60 m in the testing zone. The depth also varied along the flume's length however, in the testing area it had a constant slope of 1:136 which duplicates the slope that is usually found in intermediate or shallow waters, as was concluded by Whittaker and Folley [59].

This wave flume has already been used for other experiments, such as, the analysis of the variation caused to the reflection coefficients by different wave breaking beaches, by Conde et al. [11]. This flume was described as having a peculiar geometry which can be attributed to the different slopes along the flume's length and, also, due to how the width varies along the channel's length. Usually, a wave flume with width variation is attained by converging one of the side walls however, this wave flume, as can be analysed in Fig. 5.1, has both of its side walls converging towards the middle of the flume.

![Diagram of the wave flume with the location of the OSWEC](image-url)

Figure 5.1: Model of the wave flume with the location of the OSWEC
The wave flume has a total length of $38.0 \text{ m}$ and the wave maker occupies the initial $5.43 \text{ m}$ so, the experiences may be realised in the remaining length of $32.57 \text{ m}$. The flume has been designed with a specific zone where the experiences have better viewing possibilities due to the installation, in the side walls, of glass panes spanning from the longitudinal coordinates $x = 7.2 \text{ m}$ up to $x = 13.00 \text{ m}$. As it can be concluded by analysing Fig. 5.1, the device was placed between limits that define the testing area at $x = 10.7 \text{ m}$, granting the opportunity of inspecting visually the incident and transmitted waves.

The wave flume was connected, through two valves, to the city water supply network, allowing the possibility of filling the flume in a matter of minutes. However, due to this large flow-rate, and if the main valve was open, the flume's intake valve would leak a small amount of water into the channel. So, to remove the water from the flume it had a sump which allowed a better control of the water column level and, also, for it to be completely emptied.

### 5.1.2 Wave Maker

Having described the main dimensions of the wave flume, it is now possible to explain how the waves were generated and how they were measured. This wave flume is equipped with a wave maker that can generate regular and irregular waves. The wave maker is composed by an electrical motor that is coupled to a paddle whose motions can be approximated to horizontal oscillations due to the presence of hinges in the support structure, as shown in Fig. 5.2.

![Coupling between Electric Motor and Paddle](image)

**Figure 5.2:** Side view representing the system of the wave maker

The motor is connected to a monitoring computer through a *National Instruments* connection board allowing the chance of sending sinusoidal voltages to the electric motor, according to the pretended wave amplitude and period. The pretended wave amplitude is directly related with the motor's excitation voltages which are defined in a text file that is then inputted to the *LabVIEW SignalExpress* software. Due to the iterative process of calibrating a wave maker towards the pretended wave amplitude and its period, a *MatLab* code was created for faster generation of the signal that would be sent to the electric motor. In Fig. 5.3, it is possible to find a small snippet of the values that were sent to the electric motor when analysing the wave period with $1.48 \text{ sec}$. 

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While analysing the Fig. 5.3, it is possible to notice that the voltage that is sent to the electric motor increases steadily until reaching the intended value at $t = 30$ sec. This steady increase was a recommendation made by the technician that was in charge of the facilities where the experiments were realised. To increase the life span of this type of wave makers, it is recommended to increase steadily the voltage of the motor before reaching the pretended value. This steady increase is usually defined as the ramp-up period which, for the electric motor that was installed in this flume, was of 30 sec. This ramping period varies from device to device and for safety reasons it was the value used for these experiments.

The calibration of the wave maker had to be done whenever there was a big change to the level of the water column because it was necessary to determine the coefficient that related the motor’s voltage amplitude with the pretended wave amplitude and its period. These tests were iterative and the goal was to minimise the variation between the pretended and the measured wave amplitudes, for the different wave periods. Apart from using the monitoring computer to control the wave maker, it also registered the waves’ amplitude values measured with the wave probes. So, to illustrate how the coefficient that relates the motor’s voltage amplitude and the waves’ period and amplitude varies irregularly, the values of the excitation voltage can be seen in the following Table 5.1.

Table 5.1: Comparison between the pretended and the obtained values for the periods and the amplitudes of the waves in a depth of 13.3 cm

<table>
<thead>
<tr>
<th>Voltage [V]</th>
<th>$P_{PRE}$ [s]</th>
<th>$P_{OBT}$ [s]</th>
<th>$A_{PRE}$ [m]</th>
<th>$A_{OBT}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23</td>
<td>0.630</td>
<td>0.630</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.207</td>
<td>0.740</td>
<td>0.740</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.21</td>
<td>0.840</td>
<td>0.840</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.21</td>
<td>0.950</td>
<td>0.950</td>
<td>0.010</td>
<td>0.011</td>
</tr>
<tr>
<td>0.205</td>
<td>1.050</td>
<td>1.050</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.215</td>
<td>1.160</td>
<td>1.160</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.225</td>
<td>1.260</td>
<td>1.260</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.26</td>
<td>1.370</td>
<td>1.370</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>0.205</td>
<td>1.480</td>
<td>1.480</td>
<td>0.010</td>
<td>0.008</td>
</tr>
</tbody>
</table>

It may seem that the wave period is dependent of the excitation voltage but for smaller wave periods it
is the other way around and for larger wave periods they seem to be independent of each other. So, when creating the input file for the wave maker, the wave period is defined by simply defining the voltage crests, or troughs, with the pretended time separation, i.e., the wave period.

On the opposite side of the wave maker and to decrease the amplitude of the reflected waves, it is possible to use active and/or passive absorption techniques. On this particular wave flume, solely the passive absorption technique was used and the effects caused by the different absorption techniques in the amplitude of the reflected waves were analysed by Conde et al. [11]. He studied, using the method of Mansard and Funke [33], three different passive absorption techniques which were: a reflective brick wall, a porous brick wall with its holes aligned with the direction of wave propagation and, the last was, porous rugs that would increase bottom friction. He concluded that the usage of the porous rugs, in this flume, decreased the amplitude of the reflected waves effectively so, this was the technique that was installed in this flume.

Actually, the wave absorbing beach that is installed in this wave flume, is composed by two sections where the initial one has porous rugs which dissipates the wave energy through the increase of bottom friction and by smoothing the wave breaking. Closer to the end of the flume, there is the second section composed by rocks which will decrease the reflection that would occur if the waves broke in the vertical wall that’s limiting the flume. However, the rocks section is only used when experimenting with larger levels of water column, as can be seen in the Fig. 5.4.

![Figure 5.4: Representing the two sections of the passive wave absorption beach](image)

While studying experimentally these devices, two different depths were evaluated. The depths were measured in the longitudinal coordinate where the device was going to be placed, at \( x = 10.7 \text{ m} \). The lowest depth was of 13.3 cm and the waves could only reach the initial part of the porous rugs. Yet, while evaluating the highest water level, the depth was of 26.7 cm and the waves covered the whole length of the porous rugs and were breaking in the second section made of rocks.
5.1.3 Wave Measurement

Having explained how the waves were generated and then how the waves were absorbed, it is convenient to explain how the waves were measured. The goal was to measure and to determine the amplitude and the period of both the incident and the reflected waves. Due to the presence of the device, it is expected that some waves will be radiated due to the motion of the flap while some of the initial incident waves will keep propagating. So, to evaluate the variation caused by the presence of the device, it is necessary to place wave probes upstream and downstream of the device’s location.

There are several methods that are focused on the measuring of wave propagation. However, to be able to measure both the incident and the reflected waves, while minimising the error caused by the measuring of these waves, it is necessary to use the method described by Mansard and Funke [33]. It accomplishes the minimisation of the error through the use of the least square method. To accomplish this, it is necessary to place three probes that cannot be randomly distanced. The distance between each probe has to be calculated using the equations that are shown in the articles [5, 33]. So, to evaluate the effects caused by the device, as stated earlier, it is necessary to analyse the waves at 6 different locations, as shown in Fig. 5.5.

![Figure 5.5: Side view of the experiments using the method of Mansard and Funke [33]](image)

By placing the probes as shown in Fig. 5.5, it is possible to measure the amplitude of the incident waves, measured by the probes 1-2-3, and the transmitted waves, measured by the probes 4-5-6. Besides the amplitude of the incident and transmitted waves, this probe arrangement allows the possibility of measuring the amplitude of the waves that are reflected by the device, being measured by the probes 1-2-3, and the ones caused by the beach, which are more easily measured by the probes 4-5-6.

The equipment for the measurement of the waves was supplied by the NLCE. The probe that was used for the measurement of the waves was of the resistive type with a total length of 80 cm and it registered the values at a frequency of 25 Hz. A probe with this length enables the chance of easily measuring the waves with enough clearance between the waves’ free-surface and the probe’s electronic components. The probe’s electronic component can be seen in the upper section of Fig. 5.6. The electronics were
distanced more than 65 cm from the free-surface while the bottom part was partially submerged in the water.

5.1.4 Device’s Components

The experimental analysis of WECs passes through several phases before reaching the stage where it is feasible to analyse the device in open waters, as described by the European Marine Energy Centre [14]. So, the experiments that were realised are still in Phase 1 - Concept Evaluation, where the model’s scale fits between 1:25 and 1:100.

Prior to the construction of the device’s components, it was necessary to design them taking in consideration the flume’s conditions. The design process can be idealised as proposed by Evans [19], where it is necessary to repeatedly optimise the device’s parameters until meeting the requisites, which were:

1. The flap’s density had to be smaller than the water density, optimally it had to be a value lower than half the density of the fluid where it was immersed;
2. The material that formed the device’s flap had to enable the possibility of coupling it to an axle;
3. The material that formed the device’s flap had to have low water absorption coefficients;
4. The apparatus that was going to sustain the device’s flap had to allow the possibility of holding ballast;
5. The apparatus that was holding the device’s flap could not have a high friction between its supports and the flap’s axle;
6. The support of the device’s flap had to enable the possibility of easily securing the support for the measuring of the excitation forces;

Thus, the design process began by evaluating which material should be selected for the device’s flap. It had to have a low density and the possibility for it to be coupled to the rotating axle had to be assured so, it could either be balsa, foam or PVC. Yet, it needed to have a low water absorption’s coefficient which disqualifies the possibility of using balsa. Between the remaining two materials, the main differences can be resumed to their densities, to their possibilities of being machined and to their compressive strengths.
Due to the increased ease in machining foams, and since that the remaining parameters are more or less comparable, the device's flaps were going to be out of foam.

The solution was to consider both the \textit{EPS, Expanded Polystyrene}, foam and the \textit{XPS, Extruded Polystyrene}, foam which are indicated for underwater jobs due to their low water absorption coefficients. For short periods of time, these foams are equivalent in what matters to water absorption so, it was necessary to consider which of those could be more easily obtained, considering the dimensions of the device's flaps. The \textit{EPS} foam cannot be as easily obtained with smaller thicknesses so, it was necessary to use the \textit{XPS} foam for the device's flaps. For the coupling between the flaps and the rotating axle, it was necessary to take advantage of their high compressibility strength because it seemed implausible to drill a concentric hole through the whole width of the device for the placement of the rotating axle.

Having settled the material that was going to be used for the device's flaps, it was possible to dimension the support that was going to fix the flaps to bottom of the flume while ensuring that the flap's motions were restrained to a single degree of motion. The requisites for the support where mainly focused on its height and on the need of having a low friction between the supports and the rotating axle. Differently to what was imposed while choosing the material for the device's flaps, the support could have a high water absorption's coefficient which was going to aid the ballasting of the device. For easier machining, the material that was selected for the device's support was balsa and pine wood.

Two extra beams were fixed to each side of the device's support without compromising the flap's oscillations. For the measurements of the flap's oscillations, these beams allowed the potentiometers to be positioned far enough from the water while enabling them to be connected to the flap's rotating axle, via an almost weightless synthetic rope.

For the measurements of the excitation forces, it was necessary to consider the forces that were going to be applied to the load cells by the device's flaps. To assume that all the applied forces would be measured by the load cells, the support structure needed to be evaluated as being rigid. This could be attained by over-dimensioning the cross-sectional area of the beam that was going to support the load cells. So, it had 2 cm of thickness and a width of 6 cm which, for the excitation forces that were expected to be exerted, would allow the possibility of classifying the beam as being rigid.

### 5.1.5 Flap Oscillations Measurement

As explained earlier, an \textit{OSWEC} has solely one degree of freedom enabling it to oscillate around the \textit{y-axis}. These motions can be registered by different techniques however, some of them are more fit to these experiments than others. These oscillations can be measured by using one of the following possibilities: using an accelerometer coupled to a gyroscope, using rotary encoders or by using linear potentiometers. However, it is convenient to analyse which are more fit for the experiments that will be realised. In Table 5.2 it is possible to find the pros and cons of each system.
Table 5.2: Analysis of the pros and cons for the measurement of the flap oscillations

<table>
<thead>
<tr>
<th>Measuring Device</th>
<th>Cons</th>
<th>Pros</th>
</tr>
</thead>
</table>
| **Accelerometer + gyroscope** | - Had to be coupled to the flap;  
- Cannot be submerged;  
- Increases the flap’s inertia;  
- The amplitude of oscillations had to be integrated from the accelerations; | - Very high sensitivity; |
| **Rotary Encoder** | - High cost of acquisition;  
- Dampens the flap’s oscillations; | - Good sensitivity to small oscillations;  
- Direct calculation of the oscillations’ rate of motions;  
- Can be easily coupled to the flap’s axle of oscillations; |
| **Linear potentiometer** | - Dampens the flap’s oscillations; | - Relatively cheap;  
- Good sensitivity to small oscillations;  
- Direct calculation of the oscillations’ rate of motions;  
- Can be easily coupled to the flap’s axle of oscillations |

Even though the linear potentiometer isn’t completely free of damping, it will have, comparatively, a smaller damping coefficient than the rotary encoder. So, for these experiments, which will be realised at small scale, all the external damping coefficients must be minimised because these will reduce the device’s amplitude of oscillations. Thus, after evaluating all the pros and cons of each measuring device, only the latter seemed to gather the least amount of cons and the largest number of pros.

A linear potentiometer can be described as being a variable electric resistance. The electric resistance varies due to a rotating knob that is coupled to a wiper which will vary the output resistance of the potentiometer. This value can be measured by connecting the middle connection pin to one of the arduino’s analog entries while the outer pins have to be connected to ground and to $5\,\text{V}$. The output of these potentiometers can either be linear or logarithmic however, for these experiments, the potentiometers were bought as having a linear relation between the wiper rotation and the output voltage. The linearity of this relation isn’t dependent of which pin is connected to ground and to $5\,\text{V}$ thus, it can be assembled either way.

For reference, the linear potentiometer that was used had a maximum resistance of $10\,k\Omega$ between its terminals. The damping brought to the system by the linear potentiometers was relatively small so, both extremities of the rotation axle had a potentiometer coupled to it. This enabled the possibility of averaging the values allowing a more credible result.
For better visualisation of the device’s motions, an interface was created in Java that enabled the chance of viewing in real-time the amplitude of oscillations and their rate of change. Both potentiometers were connected to the arduino which was sending serial data to another computer and not to the main computer that was monitoring the wave maker and the wave probes. The program was reading the data that was being sent via the serial port and was converting the values of resistances into angles. This could be done, credibly, after calibrating the linear curve’s slope that described each potentiometer. The calibration process would be effectuated when the potentiometers were already coupled to the flap and then the flap would be angled towards known angles, for example, \(-45^\circ\) and \(45^\circ\).

![Wave Direction](image)

Figure 5.7: Graphic interface created in Java for measuring the flap’s oscillations

The calibration was repeated on a daily basis and the values obtained for the linear slope, that related the applied force to the load cells’ output voltage, were used solely during that day. While testing the device’s amplitudes of oscillations, each potentiometer was going to reach 2000 cycles of rotation in a relatively short time period. So, to guarantee that the potentiometers’ life period wasn’t surpassed, they were replaced by new potentiometers when different flaps were analysed.

5.1.6 Excitation Wave Force Measurement

Besides the determination of the resultant flap’s amplitude of oscillations, it was necessary to determine the excitation forces that were causing the flap to oscillate out of its vertical resting position. These forces are originated by the incident waves and they can be measured by restraining the motions of the flap at its vertical position using load cells. This can be done using different methods, for example, by using the method of Henry [25], through a clutched system, or by placing load cells in contact with both upstream and downstream surfaces of the flap, which was the chosen method for these experiments. To explain how the load cells were placed, it is possible to find in Fig. 5.8 a picture that was taken before
the beginning of the experiments that shows how the load cells were fixed to a rigid wood beam.

![Image of load cells fixed to a wood beam]

**Figure 5.8: Representation of how excitation forces were measured in the experimental tests**

The load cells, when are bought, they already come with 4 holes, where 2 are placed in the load cells’ upper part and the other 2 are placed in the lower part, as can be seen in Fig. 5.8. The load cells had a bolt screwed into one of their bottom holes enabling the opportunity of centering the device’s flap in the middle of the load cells.

Prior to the experimental tests, the semi-analytical equations were used to discern about the magnitude of the excitation forces that were going to be applied to the device’s flap. These results lead to conclude that load cells rated to 5 kg would suffice. Besides the required load cells, it was necessary to use the amplifier module HX711AD which would convert the analog signal and amplify it to a digital signal which could then be sent to the arduino.

Prior to the determination of the excitation forces, it was necessary to calibrate the load cells using calibrated weights. The load cells had to be placed in the same conditions at which they were going to be placed during the experiments because the load cells are slightly influenced by temperature and the water’s temperature affected the readings. During the experiments, the load cells were partially immersed in the flume’s water and its temperature was varying either due to warm temperatures that were felt during the experiments or due to the heterogeneity of the water temperature in the city mains and in the flume. For the calibration and to ensure that the registered value was accurately measured, calibrated weights were used, with a total mass of 150 g.

Using the Java code that was created for the measurement of the device’s oscillations and adapting it for the measurement of the applied masses, it was possible to get a program that enable real-time analysis of the applied masses. The interface is very similar yet, it shows the mass that is applied to the each load cell instead of averaging their values. It also calculated the rate of change of the applied mass however, it isn’t as significant as the rate of change calculated for the oscillations.

In the bottom part of the graphic interface, shown in Fig. 5.9, it is possible to find buttons and sliders...
Figure 5.9: Graphic interface created in Java for the measuring of the excitation forces disposed in a matrix shape, with different functionalities. The first line allowed the possibility of calibrating the load cells and it could also activate the tare functionality. For preventing any wrongful pressing of these buttons, they could only be activated if the slider was swiped to the right. The second line had a different functionality, it allowed the possibility of saving the registered data into a text file with a name that could be selected by the user. The same safety measures were taken, for preventing any wrongful pressing of these buttons.

5.2 Experimental Procedure

Prior to the realisation of the experimental tests, several articles were taken into account but there are two which were the most helpful, Henry [25] and Payne [41]. The former evaluated experimentally OSWEC devices and the latter has several recommendations that need to be taken into account while realising experimental tests.

The experiments were planned to take place solely during 2 days for the analysis of 3 devices with 11 different periods and 2 different heights, assuming that 6 wave probes could be attributed to these experiments. Yet, the experiments were realised during 28th of July and the 16th of August because, due to the lack of available wave probes, a single probe could be lent by the NLCE. So, the duration of the experiments had to be extended, at least, six-fold if everything was to be kept as initially planned. The following sequence of experiments had to be repeated for the three flaps

1. Calibrating the wave probes three times per day;
2. Calibrating the wave amplitude and period by placing the wave probe where the device will be later
placed \((x = 10.7 \ m)\);

3. Measuring the waves, without the device in the flume, at 6 different locations for each period;

4. With the device in the flume, measure the flap’s amplitude of oscillations while measuring the waves at 6 different locations for each period;

5. With the device in the flume, analyse the excitation forces while measuring the waves at 6 different locations for each period;

As stated previously, the goal of these experiments was to validate the results of the semi-analytical equations for both the submerged devices and for the surface piercing devices. While the results for surface piercing devices have already been compared by Renzi and Dias [44], experiments with the goal of validating the submerged devices’ theoretical results have not been realised. So, at the scale of \(1:90\), two flaps with different dimensions were studied, one that was surface-piercing, similar to the flaps evaluated experimentally by Henry [25], and the other was completely submerged, similar to the WaveRoller design. For the scale of \(1:45\), solely one surface-piercing flap was analysed.

By performing these experiments, it will be possible to effectuate comparisons, similar to the ones realised by Renzi and Dias [44], between the analytical results and the experimental results. The comparison that was realised by Renzi and Dias [44] compels to conclude that the semi-analytical model can be used for the initial analysis of a surface-piercing OSWEC. However, they did not evaluate if the same conclusions can be withdrawn for a submerged device. Therefore, the experiments that were realised will enable the possibility of evaluating if it is viable to use these semi-analytical equations for the analysis of a submerged OSWEC.

Having defined what was the procedure during the experiments, it is convenient to state that they were not realised as initially planned and intended. Mainly because the requisites defined by method of Mansard and Funke [33] could not be met unless assuming that the experiments had a perfect repeatability factor [Taylor and Kuyatt]. The method defines the number of wave probes that need to be used, which is three, while measuring, simultaneously, the propagating waves. However, solely one wave probe could be lent by the NLCE imposing the need of repeating the tests enabling the measure of the waves in different locations. By doing so, unless it is considered that the generated waves by the wave maker and their propagation in the flume is ideal, which implies a perfect repeatability factor [Taylor and Kuyatt], the analysis of the reflected wave amplitude is compromised.

### 5.3 Experimental Tests

The results that will be presented in the following sections were obtained with the experiments that were realised in the wave flume of the NLCE. The experiments were divided in two sections, one with devices at a smaller scale of \(1:90\) and the other was with larger devices at a scale of \(1:45\). This type of device has already been experimentally studied by Henry [25] and van’t Hoff [56] however, they only evaluated devices that crossed the water free-surface, more commonly known as surface piercing devices. So,
even though experiments have been realised with surface piercing devices, the main goal was to validate the results obtained with the semi-analytical equations for the completely submerged devices while the results obtained for the surface piercing devices would play as a monitor to verify if the conclusions of Henry [25] and van't Hoff [56] can be equally withdrawn.

The experiments were realised between the 28th of July and the 16th of August and totalised, at least, 486 individual experiments, without taking into account the number of necessary runs to calibrate the wave maker. This number takes into account the following experiments:

- Measuring the amplitude of the incident waves without the presence of the device in the flume;
- Measuring the amplitude of the device's oscillations while registering the incident and transmitted waves' amplitudes;
- Measuring the incident forces applied to the device's flap and the reflected and transmitted waves

Each of these experiments had to be repeated six times, due to the number of locations that needed to be evaluated, for each of the eleven periods. However, the wave maker presented some limitations in the lower range of the tested periods so, solely the highest nine periods will be presented.

The dimensions of the device had to be evaluated taking in consideration the width of the flume, as recommended by Renzi and Dias [45]. The flume has a width of 0.60 m in the area where the device was going to be placed so, and to keep the analysis within the 3D domain, it was necessary to choose a scale that caused the flap's width to be smaller than 2/5 of the flume's width. By studying devices with a higher width ratio, the excitation forces applied to the device's flap would start to become higher than the values obtained in open waters [45]. This can be attributed to both the 2D effects caused by the blockage of the flume's width and due to the variation of the incident waves' amplitudes.

The reasoning behind the selection of the devices' dimensions will be explained as well as the variations caused by the variation of the model's scale. Since that the semi-analytical equations do not take in consideration the effects caused by the flume's side walls, it may be possible to detect variations between the experimental results and the values obtained with these equations when flaps with a larger width are analysed.

The values that will be presented for the response angle and for the excitation force, are the resultant average of the six measurements.

### 5.3.1 The 90th model scale

Taking in consideration the limitations imposed to the flap's dimensions, the smaller scale models were dimensioned in such way that the flap's width would be of 0.20 m, which is equivalent to 1/3 of the flume's width. By doing so, the effects caused by side walls have been, ideally, minimised and the excitation forces that were applied to the device's flap would be more similar to the forces in open waters.
The selection of the remaining dimensions were based on what has been used for the devices that have already been deployed in open waters, such as the WaveRoller and the Oyster Aquamarine. The dimensions of the devices that were analysed at a scale of 1:90 can be seen in Table 5.3.

<table>
<thead>
<tr>
<th>Scale</th>
<th>1:1</th>
<th>1:90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Submerged</td>
<td>Surface Piercing</td>
</tr>
<tr>
<td>Flap's Width [m]</td>
<td>18.00</td>
<td>18.00</td>
</tr>
<tr>
<td>Flap's Height [m]</td>
<td>7.20</td>
<td>9.90</td>
</tr>
<tr>
<td>Flap's Thickness [m]</td>
<td>1.80</td>
<td>1.80</td>
</tr>
<tr>
<td>Support Height [m]</td>
<td>3.60</td>
<td>3.60</td>
</tr>
<tr>
<td>Wave amplitude [m]</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Depth [m]</td>
<td>12.00</td>
<td>12.00</td>
</tr>
</tbody>
</table>

As explained earlier, the space that spans between the upper surface of the device's flap and the water's free-surface is called freeboard and it can be either positive, negative or null. For the submerged devices the freeboard is negative while for the surface piercing devices the freeboard is positive, which can be confirmed by adding the flap's height with the support height and then subtracting the depth. The result for these two cases, when dealing with the scaled models, is of -0.013 m and of 0.017 m for the submerged and for the surface piercing devices, respectively.

Both models were placed at the longitudinal coordinate $x = 10.70 \text{ m}$, as can be seen in Fig. 5.10. At the device's location, the local water depth was of 0.133 m, resulting in an equivalent depth of 12.00 m, as shown in Table 5.3. With this depth, the waves solely reached the bottom part of the passive beach which is composed by porous rugs, able to decreased the amount of reflected waves resultant of the wave breaking.

![Figure 5.10: Representation where the submerged device was placed longitudinally $x = 10.7 \text{ m}$](image)

For this scale, the goal was to study eleven periods than ranged between 0.422 s and 1.472 s with a step of 0.105 s. However, the first two periods could not be generated by the wave maker because it seemed to be reaching its own natural period which was affecting the regularity of the generated waves. So, solely the remaining nine periods will be shown from now on, corresponding to the periods between 6 s and 14 s with a step of 1 s.
5.3.1.1 Submerged Device

The semi-analytical equations that are being used enable the possibility of evaluating the dimensions that were shown in Table 5.3, for the submerged device however, they need to be experimentally validated. The values shown on the column that is related with the scaled submerged device represent the exact dimensions of the device that was studied in the wave flume of the NLCE.

The submerged device, i.e., the one with a negative freeboard of -0.013 m, was the one with which it would be possible to conclude about the validity of using these semi-analytical equations for the analysis of submerged devices. The parameters that are going to be used for the validation of the results, were measured experimentally and were the response angle and the excitation force. The latter is usually used for the validation of the theoretical results by comparing them with the experimental results. Yet, the amplitude of the oscillations is also of interest because it allows the possibility to conclude about the dynamics of the device. The comparisons obtained for the response angle and for the excitation forces can be seen in Fig. 5.11(a) and in Fig. 5.11(b), respectively.

![Figure 5.11: Comparison between the experimental and theoretical results for the submerged device at a scale of 1:90](image)

For the smaller wave periods, while analysing Fig. 5.11(a), there’s a good fit between the theoretical and the experimental results however, for larger wave periods the results start to diverge. For the response angle, while the theoretical results continue increasing until reaching the maximum at the device’s natural period, the experimental results start to decrease sooner. This could have been attributed to the overtopping effects, which would cause a decrease of the forces applied to the flap, if the data of the excitation forces wasn’t available. Yet, the values shown Fig. 5.11(b), allow to conclude that the decrease of the excitation forces were expected but theoretically it was predicted that the device would enter in the vicinity of its natural period, leading to an increase of the response angle. Since that did not occur, it can either be due to a wrongful calculation of the device’s natural period or due to the increase of the added inertia caused by the flow around the device. Either way, the semi-analytical equations need to
take into consideration other parameters, for the larger wave periods, to be able to say that these can be used for the analysis of submerged devices.

As explained earlier, for the measure of the forces applied to the device’s flap, two load cells were placed on the upstream and downstream surfaces of the flap. The load cell that was measuring the forces that were being applied by the incident wave, identified by $LC_1$, registered higher values than the load cell that was measuring the forces on the opposite direction, as can be confirmed in Fig. 5.11(b).

5.3.1.2 Surface Piercing Device

Having concluded that these semi-analytical equations cannot be used for the analysis of the submerged devices as they were presented, it is of interest to analyse how fit will be the results for the surface piercing devices. Similar analyses have been presented by Henry [25], van’t Hoff [56] and Renzi and Dias [45] where a good agreement between the experimental and the theoretical results have been obtained so, it is expected to reach similar conclusions.

The experimental results shown in Fig. 5.12, are the resultant average of the six tests realised for each period. However, while the curve shown in Fig. 5.12(a) is the average of the two potentiometers, the curves shown in Fig. 5.12(b) represent the values obtained for each of the load cells, the one upstream and the one downstream of the device, as shown in Fig. 5.8.

Figure 5.12: Comparison between the experimental and theoretical results for the surface piercing device at a scale of 1:90

The values obtained with the semi-analytical equations for the response angle, shown in Fig. 5.12(a), seem to be a good fit with the experimental results. The theoretical values follow the same tendency as the experimental measurements, besides the slight variation that exists between them. This variation can be due to several motives and two of the main agents are the damping brought to the system by the potentiometers and the friction between the support and the flap’s axle. These are parasitic terms brought involuntarily to the system and will be assumed as being damping coefficients which could be
calculated through free-motion tests. However, the device’s floatation and the added inertia was so high that the flap oscillations would stop after a single cycle, hindering the possibility of measuring the damping coefficient. So, it was necessary to approximate the value of the external damping while using the semi-analytical equations and then fixing it for the three devices.

Similarly to what was concluded for the submerged device, the values obtained with the semi-analytical equations follow the tendency of the experimental measurements for the excitation forces, as represented in Fig. 5.12(b). Even though, the variation between the measurements of the two load cells is almost negligible, the same cannot be said about the variation between the theoretical and the experimental results. The bigger disparities occurred for the smaller wave periods, which could have been attributed to the development of stationary waves, if these disparities had not happened to the submerged device, as seen Fig. 5.11(b). While measuring the excitation forces applied to the surface piercing flaps, the phenomenon of stationary waves was registered for the smaller wave periods. The waves’ amplitudes would begin with the defined dimension and then would start to increase slightly, until reaching the state of standing waves.

5.3.2 The 45th model scale

Experiments with devices that have a small ratio between their width and the flume’s width, enabled the chance of evaluating the 90th model scale while minimising the effects created by the presence of the side walls. However, it can be beneficial to evaluate a device whose width ratio is larger. By doing that, it will be possible to evaluate the effects caused by the side walls while comparing with the semi-analytical results.

An article has been published where a comparison between the excitation forces obtained in a wave flume and in open waters was realised. The results lead to conclude that when the ratio between the flap’s width and the flume’s width was higher than 0.2, the excitation forces in the flume would start to become larger than the ones registered in open waters [45]. This would occur both due to the flume’s sloshing modes and due to the partial blockage of the channel width, leading to a 2D analysis. So, the flap’s width was limited 0.4 m obtaining a ratio of 0.6. This could cause a variation on the registered excitation forces but that would be analysed in the experimental measurements.

By keeping the real scale dimensions of the device fixed and just changing the model's scale to 1:45, the dimensions shown in Table 5.4 represent the obtained values. Basically these could be calculated by multiplying the values shown in the most right column of Table 5.3 by two.

For this model, the freeboard was equally two times larger than the one of the smaller device, resulting in a freeboard of 0.034 m. Yet, the scaling of time isn’t as direct as for lengths because, through Froude scaling, times scale with the square roots of the length ratio. For this larger model, the wave periods would range between 0.596 s up to 2.087 s with a time step of 0.1491 s. However, as occurred while
experimenting with the smaller devices, the wave maker wasn’t able to generate regular waves with the smaller periods. So, the results that will be presented will disregard the two smallest wave periods.

Table 5.4: Comparison between real scale and experimental values for the 45th model scale

<table>
<thead>
<tr>
<th>Scale Type</th>
<th>1:1</th>
<th>1:45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flap’s Width [m]</td>
<td>18.00</td>
<td>0.40</td>
</tr>
<tr>
<td>Flap’s Height [m]</td>
<td>9.90</td>
<td>0.22</td>
</tr>
<tr>
<td>Flap’s Thickness [m]</td>
<td>1.80</td>
<td>0.04</td>
</tr>
<tr>
<td>Support Height [m]</td>
<td>3.60</td>
<td>0.08</td>
</tr>
<tr>
<td>Wave amplitude [m]</td>
<td>0.90</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth [m]</td>
<td>12.00</td>
<td>0.266</td>
</tr>
</tbody>
</table>

The coordinates that define the location of the device in the wave flume did not change between experiments. Solely the dimensions of the flaps and the depth changed between experiments, as defined in Table 5.4. The water depth was always measured in the same location, at the location of the device’s flap, as shown in Fig. 5.13.

In Fig. 5.13, the presence of the rocks has not been contemplated however, for this water depth, the waves were breaking on the bottom part of the upper beach section, which is composed by rocks. For these experiments, the reflected waves reached significant values, mainly when measuring the excitation forces, causing standing waves.

5.3.2.1 Surface Piercing Device

The results obtained for the smaller scale surface piercing device, lead to conclude that the semi-analytical equations may by fit for the analysis of devices with a positive freeboard. So, the methods that were used previously for the measurement of the flap’s oscillations and of the excitation forces were kept unaltered, expecting that the same conclusions could be withdrawn for this model scale.

While analysing the results obtained experimentally and theoretically for the smaller models, it was possible to conclude that there was a disparity between these results. The problem of such variation was that the experimental measurements where larger than the results obtained theoretically, and since
that the load cells were designed taking in consideration the theoretical results, it could mean that the load cells were going to be overloaded, as proposed by Renzi and Dias [45].

The results obtained experimentally for the response angle, can be checked in Fig. 5.14(a), and, contrarily to what was obtained for the smaller surface piercing device, the experimental measurements are smaller than the theoretical results.

![Figure 5.14: Comparison between the experimental and theoretical results for the surface piercing device at a scale of 1:45](image)

Combining the information obtained with the smaller models and the information presented in Fig. 5.14, there’s a possible explanation to why, for the larger wave periods, the oscillations’ amplitudes are smaller than the theoretically calculated results. The semi-analytical equations have a few limitations, that have been discussed earlier, and one of which is the inability of considering the overtopping effects. For angles larger than 40°, the waves hurdle the upper surface of the device’s flap, causing a slight reduction in the excitation forces which isn’t contemplated in the semi-analytical equations.

As explained in chapter 3 - Analytical Analysis of the Oscillating Surge Wave Energy Converter, a limitation has been imposed to the maximum oscillation’s amplitude. However, as shown in Fig. 5.14(a), there exists the chance that the device will oscillate with higher values, if no limitation is imposed to the device’s amplitudes of motion.

The flap’s width, as shown in Table 5.4, blocks partially the flume’s width creating a barrier to the passage of the propagating waves, while the measurements of the excitation forces were being realised. This blockage results in a higher generation of reflected waves which could be visually confirmed during the experimental tests. The propagation of these waves, if they don’t have the same phase as the incident waves, could result in a decrease of the propagating waves’ amplitude. However, that wasn’t the case because the amplitude of the incident waves increased for certain periods causing the peaks that can be seen in Fig. 5.14(b).
5.4 Scaling of Excitation Forces

Having analysed devices whose dimensions are proportional, it is possible to scale, through Froude relations, the forces obtained for the smallest model and compare with the ones obtained for the largest model. In part, this analysis will enable the chance of checking if the blockage of the flume’s width had any effect on the excitation forces [45]. If it had been possible to analyse one more device, it would be of interest to test a smaller surface piercing device, with the goal of minimising, even more, the effects caused by the proximity to the side walls.

Using the Froude scaling relations, it is possible to scale the excitation forces of the smaller model by multiplying these values by the third power of the lengths’ relation, \(L_{p2}/L_{p1}\), given by Eq. 5.1.

\[
F_{p1} = F_{p2} \left( \frac{L_{p2}}{L_{p1}} \right)^3,
\]

Using the values obtained with the smallest model, and scaling them with the Eq. 5.1, it is possible to obtain an initial prediction of the excitation forces that would be obtained with the side wall effects were kept unaltered. However, as predicted by Renzi and Dias [45], the excitation forces exceeded the predictions for certain wave periods.

![Figure 5.15: Comparison between the experimental and theoretical results for the surface piercing device at a scale of 1:45](image)

Similarly to what was described earlier, the load cell identified by \(LC0\) was placed on the upstream side of the device’s flap while the \(LC1\) was placed in the downstream side. Due to the direction of waves’ propagation, the latter load cell registered larger values, as can be concluded in Fig. 5.12(b) and in Fig. 5.14(b). The difference between the scaled forces and the measured forces, can be attributed to two factors, the effects caused by the proximity of the side walls and the increase of the incident waves’ amplitude due to reflected waves. Either way, since the data obtained in open water experiments isn’t available, it cannot be concluded which of the results is closer to the open water results. However, it is possible to state that the smaller model has minimised the effects caused by the side walls.
5.5 Analysis of Wave Reflection

The analysis of the waves’ text files was realised using MatLab as was the analysis of the reflected waves. This code enabled the possibility of determining several parameters wave related however, the following three were the main ones for the analysis of the reflected waves which were the reflection coefficient, given by $C_R$, the amplitude of the incident and of the reflected waves, given by $Z_I$ and $Z_R$, respectively. The code was based in the equations defined by Mansard and Funke [33], which were rewritten in an article authored by Bak and Lykke [5].

The method defined by Mansard and Funke [33] has certain requirements that need to be met. One of which is that the waves should be measured simultaneously for error minimisation. However, due to the lack of available wave probes, that requisite could not be satisfied so, the waves had to be measured with a single wave probe while the remaining experiments were being realised. By measuring the waves at the different locations at different times, it can be argued that the propagating waves will be different both due to the interaction between the propagating waves and the long waves resultant from the previous tests.

The method of Mansard and Funke [33] requires the usage of the fast fourier transform, FFT, to convert a signal from the time domain into the frequency domain. This also enables the possibility of evaluating the amplitudes of both the incident and of the reflected waves, as can be seen in Fig. 5.16.

![FFT of the waves](image)

**Figure 5.16**: FFT of the waves with a period of 1.79 s while measuring the forces applied to the 45th model

When the waves were calibrated their amplitude was of 2 cm. However, both due to the presence of the
device and to the reflected waves, the measured amplitudes varied as expected. The probes’ location, specified on the legends of Fig. 5.16, followed the distribution defined in Fig. 5.5.

With the calculation of the FFT for all waves, it became possible to use the equations defined in the method of Mansard and Funke [33]. One of the results that can be obtained through this method is the coefficient that relates the reflected waves’ and the incident waves’ amplitudes, given by Eq. 5.2.

$$C_R = \frac{Z_R}{Z_I}, \quad (5.2)$$

Where $Z_R$ is the amplitude of the reflected waves and $Z_I$ is the amplitude of the incident waves. When this coefficient is unitary the amplitude of both waves is comparable as can be confirmed in Fig. 5.17.

Figure 5.17: Analysis of the reflection coefficient for the 90th model scale

It is possible to extract some conclusions from the Fig. 5.17 that are related with the amplitude of the reflected waves and in which configuration the device seemed to have a larger impact on the propagating waves. During the experimental tests, it seemed that the submerged device was generating smaller reflections however, due to the small values of wave amplitude, it is possible that smaller reflected waves were being disguised by the propagating waves. Yet, even when the devices weren’t in the channel, the reflection coefficient, for some wave periods, were registering large values biasing the analysis of this coefficient.

Besides the analysis for the smaller scale model, the reflection coefficient was also analysed for the
larger model. This one, expectedly, was going to generate more reflected waves both due to the greater blockage of the channel and to the region where the waves were breaking, as previously defined.

Through the analysis of the results shown in Fig. 5.18 it is possible to conclude that for most periods the amplitude of the reflected waves was larger than of the propagating ones. There were some cases where the amplitude of the reflected waves were smaller however, these occur with much less frequency.

Even though the results for the larger model’s reflection coefficient followed what was expected, it is necessary to reiterate that the used procedure for the waves’ measurement did not comply with the conditions stated by Mansard and Funke [33] neither with what initially planned. The waves should have been measured simultaneously or, at least, with three wave probes, registering the waves upstream and then downstream of the device.
Chapter 6

Conclusions

This dissertation began with the aim of analysing the equations that described an oscillating surge wave energy converter and it ended with the validation of their results. The analysis enabled the chance of evaluating the effects caused by the variation of the device’s main dimensions such as, its width, its height and, partially, its thickness. Besides these main dimensions that characterise the device’s flap, it was also possible to vary the height of the support, causing a variation to the distance between the flap’s upper surface and the water’s free-surface, known as freeboard.

The obtained theoretical results lead to conclude that the increase of the flap's width would result in a higher excitation torque leading to the chance of increase the generator's damping which would result in a higher generated power. Yet, the increase of the flap’s width will obligate the need of considering the device’s directionality in relation to the waves’ direction of propagation. Even though, it has been established that the increase of the flap’s width will enable the chance of generating more power, it was necessary to study if the increase of other dimensions would enable the same increase in generated power.

The increase of the flap’s height can also be used to increase the generated power [25]. This is accomplished by having a device whose flap covers the whole water column leading to a lower amount of wave energy surpassing the device’s flap. The flap’s height which is the vertical distance that spans between the lower and the upper surfaces of the device, excluding the support height, resulted in an increase of the generated power however, it was a smaller increase, when comparing with the results for the variation of flap’s width. Either way, these two dimensions could be both increased, as proposed by Henry [25].

The remaining dimension that defines the device’s flap is its thickness. Yet, due to the assumptions that have been settled for the semi-analytical equations, they are hindered to correctly calculate the effects caused by the variation of the flap’s thickness. While developing the equations, the thin-body approximation has been assumed easing the calculation of the velocity potential. By doing so, the velocity
potential is equal between both upstream and downstream surfaces of the flap thus not considering the flap’s thickness. Even though there’s this slight problem, and as was proposed by Henry [25], the increase of the flap’s thickness could be used to change the device’s natural period mainly due to the increase of its thickness which would result in an increase of the flap’s added inertia leading to a decrease of the natural period, assuming that the variation would be smaller for the remaining parameters.

There was one last dimension that was analysed which was the support height. This was one of the parameters that registered very small changes when varying its value. However, the variation of this parameter could be realised in accordance with the increase of the flap’s height, enabling the possibility of keeping a constant value for the freeboard. Actually, this idea of decreasing the support height while increasing the flap’s height was proposed by Henry [25] and it enabled the possibility of increasing the area that is going to absorb the incident waves’ energy.

These conclusions should have been confirmed experimentally however, solely a few experiments could be realised. So, and since that the flap’s height is one of the main characteristics that distinguishes this type of devices, the variation of the flap’s height was the parameter that was evaluated experimentally. The experimental results led to conclude that the theoretical equations, for the surface piercing cases, followed the correct tendency with the variation of the waves’ period. Yet, the experimental results obtained with the submerged device led to conclude that these equations should be revised in order to take into account the energy that passes over the device’s submerged flap.

6.1 Achievements

This dissertation gathered several different topics such as the analysis of the hydrodynamics of OS-WECs, the coding of the semi-analytical equations that describe an OSWEC, the design of experimental equipment with low-cost materials and the coding of the programs that allowed the possibility of measuring the experimental results. While some of these were clearly explained in this dissertation, there were others which could not be explained either due to their complexity or because they run out of the main topic that involves this thesis. Either way, this thesis enabled the chance of understanding the complexities related with the analysis of WECs either when analysing them theoretically or when experimentally.

These experimental tests allowed the chance of gathering the most important subjects that need to be known while realising experimental trials. For a good understanding of what is being measured, the best way is plan a priori what needs to be measured and how that can be accomplished. The idea of using potentiometers for the measurement of the flaps’ oscillations is one example how the simplest equipment can be used for other purposes.

The possibility of being able to work in the field of electronics with the need of coding programs to
measure experimental values while validating theoretical equations, was one of the big achievements of
this dissertation. Yet, there exists the chance of improving certain aspects.

6.2 Future Work

During the analysis of the semi-analytical equations and of the experimental tests, the aim was to be
as complete as possible however, there is some space for improvements. Even though some of the
possible corrections might have been referred to in the previous chapters, they need to be gathered for
easier accessibility.

One of the main topics that needs corrections, if the goal is to also analyse submerged devices, are
the semi-analytical equations. These, as was confirmed through the results of the experimental tests,
do not take in consideration the losses that occur when the waves surpass the device's flap. They
solely remove the small portion of wave energy between the upper device's surface and the waves' free-
surface. However, due to the motion of the device, this portion becomes larger which is not taken into
account.

With the correction of the semi-analytical equations, it is also recommended to improve the equipment
that was used for the experimental tests. For these initial tests the used equipment was sufficient yet, for
larger scale testing it is recommended to test the device in a tank or in a larger channel and to improve
the whole design of the device.
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Appendix A

Wave Energy Converter categories

There are several authors who developed systems which enable the categorisation of the different devices that are able to extract energy from the ocean waves. The most important systems will be shown here.

A.1 National Engineering Laboratory

The National Engineering Laboratory developed a system to categorise all the existent devices. The system was composed by 5 different categories which included:

(A) Variations in surface profile, that can either be slope or height, of travelling deep water waves;

(B) Sub-surface pressure variations;

(C) Sub-surface fluid particle motion;

(D) Unidirectional motion of fluid particles in a breaking wave which may be naturally or artificially induced;

(E) Other effects;
FIG 17 SUB-CLASSIFICATION OF WAVE POWERED GENERATORS

Types dependent on level or pressure fluctuations within enclosures

- Fluctuating water level part of larger structure
- Immobilised chamber
- Floating structure

Energy in water waves can be obtained from

- Transmitted to a sea level energy converted
- By energy converted to hydraulic pressure
- From a diaphragm on a buoy acting on a piston to produce hydraulic pressure supply by being connected to a hydraulic motor, Pelton wheel etc.
- Drawing off water level fluctuation by being part of a structure

- Fluctuating water level
- Immobilised chamber
- Structure part of shore

Displacing air in a chamber to drive an air turbine

Hydraulic pressure from direct compression on a diaphragm of a membrane device installed on sea bed

- Act directly on diaphragm by energy converted
- Displacing water level fluctuation partially or totally by being submerged pipes or chambers

Surface profile significant

- By energy converted from deep water level fluctuations
- By being connected to a sea bed

- By energy converted from standing wave form
- By being connected to a sea bed

- By fluctuating part of shore structure

- By fluctuating part of shore structure

- By standing by being connected to a sea bed

- By fluctuating part of shore structure

- By fluctuating part of shore structure

- By partially or total energy variation in chamber without

- By energy converted by sea bed

- By energy converted by a sea bed
Energy in water waves can be obtained from subsurface fluid particle motion. Energy converted by movement of an oscillating vane fixed to sea bed part of larger floating structure.

Rotation of rotor with horizontal axis normal to wave direction fixed to sea shore.

Rotation of rotor with horizontal axis parallel to wave direction floating raft.

Energy converted direct impact by of water mass compressing air in a chamber fixed to shore structure.

Energy concentrated by means of converging channels feeding water to inlet of water turbine.

Uni-directional particle motion in breaking wave either naturally or artificially induced.

Converted to static head in hydraulic ram fixed to shore structure.

Induced water to supply water turbine.
Appendix B

Deduction of the Equations

B.1 Radiated and Diffracted Parameters

The velocity potential of the radiated and diffracted waves $\phi^{(R,D)}$ can be decomposed into two functions $\varphi$ and $Z$ which will depend of $(x, y)$ and of $z$, respectively, and can be represented by:

$$\phi(x, y, z) = \varphi(x, y) \cdot Z(z), \quad (B.1)$$

The latter function, $Z(z)$, is solely dependent of the vertical variable, so it will have to comply with the vertical boundary conditions which can be found in Eq. 3.12 and Eq. 3.13. By complying with these conditions, it is possible to consider $Z$ as an eigenfunction that needs to be proportional to $\cosh(k(z + h))$ with $k$ as its eigenvalue. This eigenvalue can be obtained by solving the dispersion relation, $\omega^2 = k \cdot \tanh(kh)$, considering the wave frequency, $\omega$, and the local water depth, $h$, are fixed constants. By solving this problem, we will achieve two possible solutions for the wave number, $\pm k$, and, for this problem, only the positive solution will be considered. The $Z(z)$ function, for real wave numbers, is represented by Expression - B.2:

$$Z_0(z) = \sqrt{2} \cdot \cosh(k_0 \cdot (z + h)) \frac{1}{\sqrt{h + \omega^{-2} \cdot \sinh^2(k_0 h)}}, \quad \text{so that} \quad \int_{-h}^{0} Z_0^2(z) \, dz = 1. \quad (B.2)$$

Besides the necessity of taking into consideration the incident waves, with a constant real wave number, $k$, it is necessary to evaluate the waves that are radiated by the body due to its own motion. These waves can either be radiated towards their origin or towards their initial direction of propagation. These radiated waves are more commonly defined as evanescent waves which are characterised as waves that decay exponentially as they propagate away from their initial location. Aside from this propagation divergence, and in order to comply with this exponential decay, it is necessary for the wave numbers to be imaginary, $k = \imath \kappa$. By replacing this imaginary wave number in the dispersion relation and by using
the trigonometric relations, one achieves the Eq. B.3:

\[ \omega^2 = -\kappa \cdot \tan(\kappa h) , \]  

(B.3)

the roots of this equation, graphically can be seen as the intersections between \(-\omega^2/\kappa\) and the infinitely many branches of \(\tan(\kappa h)\), as can be seen in Fig. B.1.

![Figure B.1: Graphical representation of the roots of the dispersion relation with imaginary roots (Mei et al. [35])](image)

With imaginary eigenvalues, the hyperbolic functions that are used in Eq. B.2 take an equivalent form, given by Eq. B.4:

\[ Z_n(z) = \sqrt{2} \cdot \cos(\kappa_n \cdot (z + h)) \sqrt{h - \omega^{-2} \cdot \sin^2(\kappa_n h)} , \]  

(B.4)

It can be shown, by using the resultant eigenvalues of Eq. B.3, that distinct eigenfunctions, \(Z_n(z)\), are orthogonal between each other which results in:

\[ \int_{-h}^{0} Z_n(z) \cdot Z_m(z) \, dz = \delta_{nm} , \quad m, n = 0, 1, 2, 3, ... \]  

(B.5)

By analysing the eigenfunctions presented in Eq. B.2 and in Eq. B.4, Kreisel [29] concluded that a set of \(Z_n\) for \(n = 0, 1, 2, \ldots, N\) is complete so that a certain function \(G(z)\) in the interval \([-h, 0]\) can be represented by a fourier series based on \(Z_n\). With this conclusion, one can represent the radiated or the diffracted velocity potentials by using the Expression - B.6:

\[ \phi^{(R.D)}(x, y, z) = \varphi_0^{(R.D)}(x, y) \cdot Z_0(z) + \sum_{n=1}^{N} \varphi_n^{(R.D)}(x, y) \cdot Z_n(z) \]  

(B.6)

\[ \Leftrightarrow \phi^{(R.D)}(x, y, z) = \sum_{n=0}^{N} \varphi_n^{(R.D)}(x, y) \cdot Z_n(z) \Leftrightarrow , \]

The first term, for \(n = 0\), of the Eq. B.6 corresponds to the propagating wave while the other terms, for \(n > 0\) which are the evanescent waves, with imaginary wave numbers, only matter locally. By knowing
the formulas that define \( Z_0(z) \) and \( Z_n(z) \) and taking in consideration their orthogonal relations, it is possible to obtain the expressions of the \( \varphi^{(R,D)} \) which need to comply with the boundary conditions:

\[
\begin{align*}
\left\{ \frac{\partial \varphi^R}{\partial x}, \frac{\partial \varphi^D}{\partial x} \right\} &= \left\{ V \cdot (z + h - c_f) \cdot H(z + h - c_f), -\frac{\partial \phi^I}{\partial x} \right\} \\
\Leftrightarrow \quad \left\{ \frac{\partial \sum_{n=0}^N \varphi_n^{(R)}(x,y)}{\partial x}, Z_n(z) \right\} &= \left\{ V \cdot (z + h - c_f) \cdot H(z + h - c_f), \frac{\partial e^{-ik_nz}}{\partial x} \cdot \frac{iA_I}{\omega \cdot \cosh(k_n h)} \cosh(k_n(z + h)) \right\}
\end{align*}
\]

(B.7)

By multiplying both sides of the top branch equation, which is \( \varphi^R \), by \( Z_m \) and integrating across the whole flap, which does not take into account the height of the support, we will get the following relation:

\[
\left\{ \frac{\partial \sum_{n=0}^N \varphi_n^{(R)}(x,y)}{\partial x}, \int_{z_{bot}}^{z_{up}} Z_m(z) \cdot Z_n(z) \cdot dz \right\} = \left\{ V \cdot \int_{z_{bot}}^{z_{up}} (z + h - c_f) \cdot H(z + h - c_f) \cdot Z_m(z) \cdot dz \right\} \quad \Leftrightarrow \quad \left\{ \frac{\partial \sum_{n=0}^N \varphi_n^{(D)}(x,y)}{\partial x}, Z_n(z) \right\} = \left\{ \frac{V}{\omega \cdot \cosh(k_n h)} \cdot \cosh(k_n(z + h)) \right\}
\]

(B.8)

By making use of the orthogonal relation of the function \( Z_n(z) \), presented in Eq. B.5, it is possible to obtain the following relation for the calculation of the radiated velocity potential. For the diffracted wave potential, since the calculation is simply based on the propagating wave and not in the evanescent terms, it is only necessary to consider the real solutions of the wave number:

\[
\begin{align*}
\left\{ \frac{\partial \sum_{n=0}^N \varphi_n^{(R)}(x,y)}{\partial x}, \delta_{nm} \right\} &= \left\{ \sqrt{\frac{2V}{h + \omega^2 \cdot sh^2(k_n h)}} \cdot \int_{z_{bot}}^{z_{up}} (z + h - c_f) \cdot ch(k_n(z + h)) \cdot dz \right\} \\
&= \left\{ \frac{1}{k_n^2} \cdot \left[ k_n \cdot (z + h - c_f) \cdot sh(k_n(z + h)) - ch(k_n(z + h)) \right]_{z_{bot}}^{z_{up}}, \frac{A_I \cdot k_n \cdot e^{-ik_nz}}{\omega \cdot \cosh(k_n h)} \cdot \left[ \cosh(k_n(z + h)) \cdot \frac{1}{Z_n(z)} \right]_{z=0} \right\}
\end{align*}
\]

(B.9)

where the representation of \( \cosh \) and \( \sinh \) has been simplified and was replaced by \( ch \) and \( sh \), respectively. The integral needs to be calculated along the flap, where the bottom limit of the integral is given by \( z_{bot} = -h + c_f \) and the upper limit is \( z_{up} = -h + (h_f + c_f) \). It needs to be settled that if the value of \( z_{up} \) isn't negative, that is below the free surface, then it needs to be corrected because the value of the upper limit cannot be higher than zero. By evaluating each branch of the Eq. B.9 individually, inside the

B.7
defined interval, it is possible to achieve the following expression:

\[
\frac{\partial}{\partial x} \sum_{n=0}^{N} \varphi_{n}^{(R)}(x, y) \cdot \delta_{nn} = C_{1} \cdot \left[ \left( k_{n} \cdot (z_{up} + h - c_{f}) \cdot sh(k_{n}(z_{up} + h)) - ch(k_{n}(z_{up} + h)) \right) \right] 
\]

\[
\left( k_{n} \cdot (z_{bot} + h - c_{f}) \cdot sh(k_{n}(z_{bot} + h)) - ch(k_{n}(z_{bot} + h)) \right),
\]

where \( C_{1} \) does not depend of the vertical coordinate and is given by the following expression:

\[
C_{1} = \frac{\sqrt{2} V}{k_{n} \cdot \sqrt{h + \omega^{-2} \cdot sh^{2}(k_{n} h)}},
\]

If instead of replacing the value of \( z_{up} \) by the value which was defined previously of \( z_{up} = -h + \left( h_{f} + c_{f} \right) \) and replaced by \( z_{up} = 0 \), one would reach the formula defined by Renzi and Dias [45] in Eq. (23), for exclusively surface piercing WECs. The \( \delta_{nn} \) is the kronecker delta which is equal to 1 when \( n = m \). The radiated velocity potential is given by the Expression - B.12:

\[
\frac{\partial}{\partial x} \sum_{n=0}^{N} \varphi_{n}^{(R)}(x, y) = \frac{\sqrt{2} V \cdot \left[ k_{n} h_{f} \cdot sh(k_{n}(h_{f} + c_{f})) - ch(k_{n}(h_{f} + c_{f})) + ch(k_{n} c_{f}) \right]}{k_{n}^{2} \cdot \sqrt{h + \omega^{-2} \cdot sh^{2}(k_{n} h)}} = V \cdot f_{n},
\]

From Eq. B.9 it is possible to obtain the expression that enables the calculation of the diffraction velocity potential. However, since this equation was obtained from the kinematic boundary conditions at the flap, it is necessary to impose that the velocity potentials need to be verified at the flap. The diffracted velocity potential is given by Expression - B.13:

\[
\frac{\partial}{\partial x} \sum_{n=0}^{N} \varphi_{n}^{(D)}(x, y) = A_{I} \cdot \frac{k_{n} \cdot \sqrt{h + w^{-2} \cdot sinh^{2}(k_{n} h)}}{\sqrt{2} \cdot \omega \cdot cosh(k_{n} h)},
\]

and is more commonly presented as

\[
\frac{\partial}{\partial x} \sum_{n=0}^{N} \varphi_{n}^{(D)}(x, y) = A_{I} \cdot d_{n},
\]

As was presented earlier, the diffracted potential is based on the propagating wave which is obtained by solving the Expression - 3.10, resulting in positive real roots of the wave number, for \( n = 0 \). The diffracted potential can be presented by the Expression - B.15:

\[
\frac{\partial \varphi_{0}^{(D)}(x, y)}{\partial x} = A_{I} \cdot \frac{k_{0} \cdot \sqrt{h + w^{-2} \cdot sinh^{2}(k_{0} h)}}{\sqrt{2} \cdot \omega \cdot cosh(k_{0} h)},
\]

\[\text{B.2 Average Generated Power}\]

The total amount of energy of the propagating waves can be obtained by adding the waves’ kinetic and potential energy. The average energy density during one wave period is equally composed by a kinetic and a potential part. However, the amount of energy from the propagating waves is not directly related
with the amount of generated energy. This average generated energy can be calculated by multiplying the flap’s velocity by the damping force applied by the generator.

\[ P_G = F_c \cdot \dot{\theta}(t) = \frac{1}{T} \cdot \int_0^T (\nu_{pto} \cdot \dot{\theta}(t)) \cdot \dot{\theta}(t) \, dt \]  

(B.16)

By replacing the Eq. 3.7, which defines \( \theta \), in the previously defined equation, it is possible to obtain the following equation:

\[ P_G = \frac{1}{T} \cdot \int_0^T \nu_{pto} \cdot \frac{\partial \{ \text{Re}(e^{-i\omega t}) \}}{\partial t} \cdot \frac{\partial \{ \text{Re}(e^{i\omega t}) \}}{\partial t} \, dt \]  

(B.17)

In order to determine the real part of a function it is necessary to take use of its conjugate. The formula for the calculation of the real component is:

\[ \text{Re}(z) = \frac{1}{2} \cdot (z + z^*) \]  

(B.18)

where \( z^* \) represents the conjugate of \( z \). Using this notion in Eq. B.17, it will result in the following expression:

\[ P_G = \nu_{pto} \cdot \frac{\theta^2}{4} \cdot \int_0^T \frac{\partial (e^{-i\omega t} + e^{i\omega t})}{\partial t} \cdot \frac{\partial (e^{-i\omega t} + e^{i\omega t})}{\partial t} \, dt = \]

\[ = \nu_{pto} \cdot \frac{\theta^2}{4} \cdot \left( \frac{\partial (e^{-i\omega t} + e^{i\omega t})}{\partial t} \cdot \frac{\partial (e^{-i\omega t} + e^{i\omega t})}{\partial t} \right) \, dt \]  

(B.19)

By integrating the previous equation within the defined limits, the result can be written as shown in the following equation:

\[ P_G = \nu_{pto} \cdot \frac{\theta^2}{4} \cdot (0 + T \cdot 2\omega^2 - 0) = \nu_{pto} \cdot \frac{\theta^2}{2} \cdot \omega^2 \]  

(B.20)

With this result and by replacing the relation obtained in Eq. 3.41 in Eq. B.17 it is possible to obtain an expression where the average generated power is obtained through the amplitude of motion of the device.

\[ P_G = \frac{\omega^2 \cdot \nu_{pto}}{2} \cdot \left[ \frac{F}{-\omega^2 \cdot (I + \mu + \mu_{pto}) - i\omega \cdot (\nu + \nu_{pto}) + C + C_{pto}} \right]^2 \]  

(B.21)

After some algebra, the denominator can be written as

\[ P_G = \frac{1}{2} \cdot \frac{\omega^2 \cdot \nu_{pto} \cdot |F|^2}{-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}}^2 \]  

(B.22)

The terms used in Eq. B.22 were defined in 3.2 - Hydrodynamic Parameters and in order to calculate the average generated power they need to be determined anteriorly.
The average generated power, as expressed by Eq. B.22, is a function of the generator’s damping coefficient which in turn influences directly the angular velocity of the flap, \( \dot{\theta} \). The relation between the generated power and the angular velocity of the flap varies parabolically, as shown in Fig. 3.3. This figure shows that the average generated power has a peak which for a certain flap velocity will allow a maximum power generation. The value of the damping coefficient of the generator can be calculated by the following formula:

\[
\frac{\partial P_g}{\partial \nu_{pto}} = 0 \quad \Leftrightarrow \quad \frac{\partial}{\partial \nu_{pto}} \left( \frac{1}{2} \frac{\omega^2 \cdot \nu_{pto} \cdot |F|^2}{-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu + \nu_{pto})^2} \right) = 0 ,
\]

(B.23)

The derivative that will be applied to \( P_g \) needs to take use of the quotient rule.

\[
\frac{\omega^2 \cdot |F|^2}{2} \cdot \frac{\partial}{\partial \nu_{pto}} \left( \frac{\nu_{pto}}{-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu + \nu_{pto})^2} \right) = 0 \quad \Leftrightarrow \quad \frac{\omega^2 \cdot |F|^2}{2} \cdot \left( \frac{(-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu + \nu_{pto})^2) - \nu_{pto} \cdot 2\omega^2 \cdot (\nu + \nu_{pto})}{(-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu + \nu_{pto})^2)^2} \right) = 0 \quad \Leftrightarrow \quad (-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu + \nu_{pto}) \cdot ((\nu + \nu_{pto}) - 2\nu_{pto}) = 0 ,
\]

(B.24)

The sequence shown in Eq. B.24 is based in simple algebra and the goal is to obtain a formula where \( \nu_{pto} \) is defined by other parameters.

\[
(-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu + \nu_{pto}) \cdot (\nu - \nu_{pto}) = 0 \quad \Leftrightarrow \quad (-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot (\nu^2 - \nu_{pto}^2) = 0 \quad \Leftrightarrow \quad \omega^2 \cdot \nu_{pto}^2 = (-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2 + \omega^2 \cdot \nu^2 ,
\]

(B.25)

By dividing both sides by \( \omega^2 \) and afterwards by applying the square root to both sides, it is possible to achieve the following relation:

\[
\nu_{pto} = \nu_{opt} = \sqrt{\frac{(-\omega^2 \cdot (I + \mu + \mu_{pto}) + C + C_{pto}^2)}{\omega^2}} + \nu^2 ,
\]

(B.26)

By calculating the generator’s damping coefficient with Eq. B.26 and by using the resulting value in Eq. B.22 it is assured that the generator will extract the maximum amount of energy possible.
Appendix C

Coefficients of the Second Kind
Chebyshev Polynomial

The Chebyshev polynomials are identified by their different coefficients of the polynomials and their highest degree.

Table C.1: Coefficients of the Chebyshev polynomial of the second kind

<table>
<thead>
<tr>
<th>Polynomial Degree</th>
<th>Chebyshev Equation Order ($p =$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$x^0$</td>
<td>1</td>
</tr>
<tr>
<td>$x^1$</td>
<td>-</td>
</tr>
<tr>
<td>$x^2$</td>
<td>-</td>
</tr>
<tr>
<td>$x^3$</td>
<td>-</td>
</tr>
<tr>
<td>$x^4$</td>
<td>-</td>
</tr>
<tr>
<td>$x^5$</td>
<td>-</td>
</tr>
<tr>
<td>$x^6$</td>
<td>-</td>
</tr>
<tr>
<td>$x^7$</td>
<td>-</td>
</tr>
<tr>
<td>$x^8$</td>
<td>-</td>
</tr>
<tr>
<td>$x^9$</td>
<td>-</td>
</tr>
</tbody>
</table>

In order to exemplify how this table should be read, for the case when $p = 6$ the resultant Chebyshev polynomial is:

$$U_7 = -1 + 24 \cdot x^2 - 80 \cdot x^4 + 64 \cdot x^6$$ \hspace{1cm} (C.1)
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