RISK BASED SHIP HULL STRUCTURAL DESIGN AND MAINTENANCE PLANNING

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Thesis to obtain the Master of Science Degree in

Naval Architecture and Marine Engineering

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July 2017
Abstract

Objective of this work is to develop a tool for a multi-purpose vessel that estimates optimal ship specifications, defines the optimal scantlings of the midship section and finds out the optimal maintenance strategy. The measure parameters for the optimization tasks are the cost, aiming to a consistent model that, based on structural safety, is the most convenient for the ship owner. The first task, through an optimization algorithm, finds the minimum of the Required Freight Rate by changing the design variables \( (L_{pp}, B, D, T, C_b, V_s) \), taking into a consideration of constraints related to the ship’s operational profile and stability. Employing MARS2000 software, a typical midship section of a multi-purpose vessel has been designed and the ultimate bending moment is evaluated.

The second task aims the optimization of the scantlings through the First Order Reliability Method, in which the probabilistic variables of the wave-induced, still water and ultimate bending moments and the respective model uncertainty factors are accounted for in the limit state function, that defines the bounds for the failure of the ship. The estimated reliability indexes are used to calculate the risk-associated costs being possible to find out its minimum.

The third task consists of analyzing six different maintenance policies, which, considering different levels of corrosion tolerances and repair consequences, define the optimal strategy based on the costs, downtime and availability.
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1 Introduction

Since engineering and architecture are applied to build any kind of vessel, the design process has been kept for years always more or less the same, with a lot of estimations, complex analyses, iterations and data extrapolations, leading to a big number of compromises, where the objective was a satisfactory good model, and not the optimal model. For this reason, it has been decided to apply a computer-aided optimization on the same methodologies, with the result of an increase in speed and quantity of outputs for the design process, i.e. the time invested for the conceptual design has been reduced from few months to few days [1], giving as output an optimum design for the ship model.

Besides the reduced time and higher reliability, the software-based process has become essential to engineers and naval architects as it allows to analyze a big number of options and solutions, achieving an optimized model that takes into account technical and economic issues. For this purpose, standard engineering principles have been applied with the objective to formulate a tool that can achieve an optimized design based on the costs, structural reliability and maintenance strategy, by considering constraints regarding the ship’s operation and ship’s integrity. As to design the model there are many challenges because of lack of data, model uncertainties and interdependability of different factors, in line with the traditional engineering design, assumptions and approximate estimations had to be carried out, in order to achieve a good balance between tool speed and reliability.

The optimization has been performed developing a tool through the use of MATLAB, a multi-paradigm numerical computing environment that with an iterative method finds the optimum by changing the objective variables. Two optimization algorithms have been used for the different tasks, which use will be explained further in the chapter “Objectives and methodology”.

The tool consists of three consecutive tasks, which are respectively the conceptual design, structural reliability assessment based on scantling modification and maintenance strategy policies (Figure 1-1).

Conceptual design is the first stage of the ship’s design and its function is to define the initial data of the model based on relations between ship’s characteristics, weights, transportable cargo and costs, subdivided into capital costs (CAPEX) and operative costs (OPEX). In the tool, it is implemented the possibility to personalize the optimization problem, by considering owner’s specification requirements, such as shipyard capacities, total distance and restrictions of the voyage, being taken as measure of merit the Required Freight Rate (RFR), a cost measure for the cargo freight [2]. In this way, it can be shaped the midship section using the software MARS2000 developed by Bureau Veritas in order to obtain midship section modulus and the ultimate bending moment, which are calculated based on stipulated methods based on the Classification Society.

More detailed ship design is achieved in a second step by employing the risk-based and maintenance-based models, as with these will be calculated the optimal scantlings and the optimal maintenance strategy. In the second task, the bending moments described by probabilistic distributions are calculated in the limit state function and the reliability index is computed through the First Order Reliability Method (FORM). The reliability index is calculated for different scantlings and having as measure of merit the total cost, the most convenient
scantlings is chosen [3]. At last six different maintenance policies have been adopted, which have as an objective the costs, downtime and availability, in order to estimate the maintenance strategy that allows to optimize ship’s operation. For this task, in order to take into consideration different probabilities related to the failure state of corroded plates and to the consequences of a repair, have been calculated the objective values computing the combination matrix between the different corrosion tolerance and repair consequence levels [4].

![Flowchart of the tasks in the code](image)

Figure 1-1: Flowchart of the tasks in the code

In order to be able to make more detailed assumptions and to analyze more in the specific the model problem, it has been chosen to focus the code on one ship category, which is the multipurpose ship. This type of vessel allows the carriage of many different cargoes, from wood to bulk, although in this tool the cargo tank is designed calculated based on TEU blocks.

As this chosen ship type would present a wide range of different solutions, the model had to be constrained even more to a range of dimensions, which will be discussed more in detail in the chapter “Conceptual design”.
2 Objectives and methodology

The tool for the three tasks is coded in MATLAB, which is a multi-paradigm numerical computing, allowing easy implementation of the optimization process.

Two optimization algorithms are used for this code, the non-linear optimization algorithm, that performs a sensitivity analysis for the variables and analyzes the gradient of the objective function to find the optimum, and the optimization with the use of a genetic algorithm, which categorizes the first solutions regarding the objective function and after keeps changing the variables until the objective value does not vary anymore converging to the optimum [5].

For the conceptual design, a genetic algorithm is adopted, which, although slower, is more reliable in the quality of the solution. The objective of this task is minimizing the Required Freight Rate (RFR), by modifying length between perpendiculars ($L_{pp}$), breadth ($B$), depth ($D$), draft ($T$), block coefficient ($C_b$) and service speed ($V_s$), defined as design variables. Moreover, constraints regarding ship’s operation and technical specifications are applied, in order to reduce the number of outputs that have to be analyzed. To relate the objective value with the design variables, in line with the traditional engineering, empirical formulas based on the regression from existing ships, formulas from classification society and estimations to reduce complexity of the problem are applied.

The midship section of the resulting ship from the first optimization process is designed by MARS2000. This program calculates midship section modulus and ultimate bending moment by shaping the typical midship section of a multipurpose vessel. Different scantlings are calculated to obtain the respective ultimate bending moment, used in the reliability model, and the midship section modulus.

The First Order Reliability Method (FORM) is used to calculate the reliability index for the respective ultimate bending moment with the non-linear optimization algorithm, as it delivers the same values as the genetic algorithm, being at the same time faster. As the different reliability indexes relative to the scantlings imply a modification in the costs regarding the risk of failure and investment in building the structure, the costs for the different scantlings are calculated and the solution with the lowest total costs is chosen.

A similar approach is also adopted for the maintenance model, where a function between the different strategies and the total final costs is calculated and the minimum chosen as an optimal model. In order to take into account material behavior and the differences in the acceptance, four different levels of corrosion tolerance and repair consequence are applied. For this task, different maintenance policies are employed, based on minimizing costs, downtime or on maximizing availability of the vessel.
3 Theoretical background

The present tool, coded in MATLAB, is based on three consecutive tasks, which focus respectively on the conceptual design, risk-based and maintenance-based analyses, and for this reason have a different theoretical background, which will be individually presented in the following chapters.

3.1 Conceptual design

The ship design consists in a sequence of tasks that aim to define all dimensions and specifications needed to build a ship. It is generally divided into five main stages: conceptual design, preliminary design, contract design, detail design and production design (Figure 3-1).

![Figure 3-1: Stages of the ship design](image)

The conceptual design is the initial step of the vessel design and its objective is to define the main dimensions, cargo capacity, speed and to perform an economic feasibility analysis, taking into account requirements set by the ship owner over operability and functionality of the vessel [6].

During the last century, the conceptual design has changed his role in the ship building, as from an estimative tool it has been transformed into a more consistent process up to the point where the estimations are more realistic and it is possible to create a series of models between which can be chosen the most convenient solution.
Harry Benford [7] applied to the decision-making process in building a ship the costs assessment, with which it is possible to estimate options for the vessel as cost dependent, as the ship needs to be a profitable investment. For this purpose, it has been introduced as measure of merit the Required Freight Rate (RFR), which is the minimum price of a unit of cargo (TEU in this tool) to be delivered from one port to another.

As initially engineers were manually calculating the relation between the design variables and the RFR, it was possible to define just few models for the same problem, as a higher number of iterations would be time consuming, leading to acceptable solutions. Lyon [8] adapted this process to a computer aided optimization, so that the quality of models is higher, by increasing the number of output models and choosing the most convenient between these, lowering drastically the time needed. In this way, it is possible to consider a wide range of interconnected models based on technical efficiency, costs, operation and priorities, with the objective to minimize the RFR.

Michalski [9] proposed a model where the design objective is the RFR based on the optimization of ship’s deadweight, which is faster and analyzes the problem under the perspective of the ship owner. To this has been preferred to set the ship main dimensions as design variables, as these are dynamically used to describe also the risk and the maintenance based-models.

As in reality the ship design problem can’t be described just by a single-objective optimization, but more objectives have to be taken into account, Ray and Sha [10] proposed a multi-objective model optimization. Indeed, as a ship can be also optimized based on e.g. maneuverability, sea keeping or specific ship type characteristics, the different optimizing objectives can be associated to a weightage factor defined with the Analytic Hierarchy Process, used to formulate mathematically the ship owner’s priorities. The proposed multi-objective model optimization is estimated to be more detailed than requested by this project, as the problem restricted to a single-objective optimization gives a satisfactory and more simple solution.

Chen [2] proposed a single-objective optimization model applied on container ship, where, besides the described standard procedure, adopted a smooth function to correlate the number of TEU available on the vessel and the ship main dimensions using the least square fitting method, to solve the problem of having a non-smooth function due to the integer number of TEU.

### 3.2 Risk-based model

As the loads, that a ship experiences during its lifetime and the strength of the structural material, present many uncertainties, it is not possible to predict if the vessel will fail. For this reason, Mansour [11] proposed a model based on statistically distributed random values, as also the structure response follows a such pattern. In the model have been considered the extreme values of the wave-load bending moment, as its variance is not negligible and, thus, also statistically distributed maxima have to be considered, while the still water bending moment is assumed to have a deterministic value.

Faulkner and Sadden [12] apply on such problem the limit state function and propose a ship design policy based on partial safety factors approach and on the safety index (beta-index), analyzed with the First Order Reliability Model (FORM). The limit state function is design criteria that, applied on the ship’s bending moments, defines
the limit between failure and safe region, where by an increase of wave-induced and still water bending moments increases also the probability of failure, thus the probability to overcome the limit state represented by the ultimate bending moment in a defined period.

More investigation has been done on the models that describe the structural behavior of a vessel subjected to loads, leading to an efficient solution by allowing to consider in the limit state function different types of distribution for the variables [13]. In this way, it is possible to consider the extreme values for the wave-load bending moment and to consider still water bending moment normal distributed.

Zayed et al [14] consider the probability of failure being time-dependent, as the reliability of the structure decreases with the time. Also in this model, using random variables and stochastic random processes, the failure is described by the up crossing of the limit state represented by the ultimate bending moment.

Garbatov and Soares [3] analyzed a model applied on a container ship, analyzing its typical midship section and taking into account loads in full, partial and harbor conditions. Moreover, the reliability index has been examined as dependent from the heeling angle.

Besides the FORM, have been analyzed also Second Order Reliability Models (SORM), but their application to this problem is rather inconvenient, as it defines a more complex limit state function as it is actually needed by the problem [15].

The FORM is a probabilistic reliability method based on the first-order expansion about mean value (Taylor expansion) and calculates the reliability index through analytical equations. If it is not possible to approximate the limit state function to a linear curve but rather to a quadratic function, it is needed to adopt the SORM, which is more complex but allows a better estimation of limit state functions of higher order than 1.

The Monte Carlo method is a simulation method that consists in generating sample data, considered as observations, calculates the solutions based on such samples and produces statistical distributions for the results, with higher precision if the number of data sampled is higher. This method is usually adopted if the probability distributions are not known and have to be assumed or the problem presents random variables.

The current state of investigation has been considered in parallel to the Formal Safety Assessment (FSA) [16], a systematic model used to analyze the risks and the associated costs and evaluate the benefits of changes in the structure.

The FSA methodology (Figure 3-2) identifies five steps where the hazard, the risk and the cost benefits are calculated and evaluated based on decision making recommendations.

The hazard is defined as “a potential to threaten human life, health, property or the environment” and the risk is the “combination of the frequency and the severity of the consequence” [16], and based on these concepts the aim of the problem is to minimize the costs reducing the risks.
3.3 Maintenance based model

The aging structures exposed to marine environments present a decrease in the reliability, being in this way more likely that the system fails after a period. For this reason, it has to be planned an interval after which some components of the structure have to be replaced (maintenance interval), in order to substitute it before lose strength and to guarantee the minimum design reliability of the system.

The main reason to perform maintenance is the corrosion, described as gradual loss of material caused by chemical or electrochemical reaction with the environment. It is of a major issue for the structural reliability as, besides the loss of weight and midship section modulus, it causes also loss of material strength properties.

Given that the corrosion presents a high number of uncertainties, Melchers [17] proposes a phenomenological model where the rate, extent, localization and variability of expected corrosion are analyzed, considering the loss of material due to corrosion as time-dependent.

Yamamoto and Ikagaki [18] define the corrosion process based on three stages: degradation of the coating, generation of corrosion pits and propagation of the corrosion pits until the structural failure. The model that describes the corrosion progression is based on probabilistic data obtained by analyzing the thickness decreases in the measured plates. In this way, the corrosion model designed results to be a non-linear time dependent process, where the progress of the depth of the corrosion is non-linear as well.

A more detailed model with consideration of non-linear behavior and random protection coating duration is presented by Guedes Soares and Garbatov [19], which define the failure when the depth of corrosion wastage reaches the corrosion tolerance of the component. Also, this model defines three stages (Figure 3-3), where the first, defined as \( t_{cr} \), is the time where the coating has effect, and for this reason there is no material wastage due...
to corrosion, the second defines the period $\tau_c$ when corrosion pits are generated with a non-linear increase of the thickness wastage and the third stage presents an approximate linear wastage of material. Thus, in this model the reliability is assessed to be time-dependent and for this reason it is easy to implement a replacement policy for the components that surpass the acceptable corrosion depth.

Considering the maintenance strategy, Singh [20] proposed three policies based on the minimization of costs and maximization of availability: replacement age, minimal repair with replacement age and minimal repair/failure replacement. In this model the failure rate, defined as the frequency at which one component or a system fails in a determined unit of time, is supposed to follow the Weibull distribution.

Garbatov [4] proposes six different maintenance policies: optimal replacement interval, optimal replacement age and optimal replacement age accounting for time required to perform replacement based on minimization of the total costs, optimal replacement interval and optimal replacement age based on minimization of downtime, and optimal replacement interval based on the availability of the system. Moreover, in this model are calculated also four different levels of corrosion tolerance and four of repair consequence, in order to consider the variations in the material strength and repair effectiveness.

The downtime is the period when the component or system is expected to be unavailable, while the availability is the probability that a component or system at a defined unit time is able to operate.
4 Conceptual design

The conceptual design is the first phase of a ship design where the initial dimensions and data of the model are estimated. In this work, the conceptual design has been aimed coding the single-objective constrained optimization with MATLAB, which consists in changing a number of variables within constrains in order to minimize or maximize an objective value through a genetic algorithm and a non-linear solver.

Initially, physical and empirical formulas have to be defined, in order to allow to estimate the first data of the ship starting from some assumptions explained along the chapter and few main dimensions, which for this case are:

- $L_{pp}$, length between perpendiculars in m
- $B$, breadth in m
- $D$, depth in m
- $T$, draft in m
- $C_h$, block coefficient
- $V_s$, vessel’s speed in kn

After that the structure of formulas that relate main dimensions to more detailed data has been created, the main dimensions listed above are set as variables so that they can change within constraints, which are chosen based on requirements on the ship, on physical limitations and on data from ships of similar type (more detailed description in Chapter “Single-objective constraint optimization”).

Finally, the variables have been set in order to minimize the Required Freight Rate $RFR$, which has been chosen because it resumes all the main data of the ship (dimensions, power, cargo capacity, weights and costs) and at the same time it is an easy way to evaluate the results, as it is the minimum value for a container (TEU) to be shipped.

![Figure 4-1: Conceptual design tool](image)
Under the point of view of the code execution (Figure 4-1), the program, starting from the default initial data, computes if the model presents the minimum objective value, if this shall not be the case, the code generates new random design variables. With these values estimations about power, weights and costs are done and the results are checked for the constrains imposed to the models. Once the program has found the minimum for the objective value, the code ends and displays the final optimized values.

4.1 Power calculation

In order to estimate the power associated with the main dimensions of the ship, a wide range of methods has been developed in the years adopting approaches more or less accurate, where to increasing level of accuracy of the results follow higher level of detailing and estimation of some data. An example of a quick and rather imprecise method to estimate the power is the so-called Admiralty formula, which depends from the speed, the displacement and the type of the ship.

To this method, it was preferred a more accurate and detailed approach, which consists first in estimating the total hull resistance in order to obtain the necessary effective power required for the given speed. Also, to estimate the total resistance acting on the hull have been developed several methods, among which has been chosen the Holtrop and Mennen’s method \[21\]. This method was chosen because has been proved to be more reliable concerning merchant ships, and having been updated few times, it is still nowadays a wide used method. The Holtrop and Mennen method subdivides the total resistance that acts on the hull in:

\[
R_{TOT} = R_F (1 + k_1) + R_{APP} + R_W + R_B + R_{TR} + R_A \tag{1}
\]

Where:

- \( R_{TOT} \) – Total resistance of the ship in N
- \( R_F \) – Frictional resistance in N
- \( 1 + k_1 \) – Form factor for viscous resistance
- \( R_{APP} \) – Appendages resistance in N
- \( R_W \) – Wave resistance in N
- \( R_B \) – Pressure resistance on bulbous bow
- \( R_{TR} \) – Transom resistance in N
- \( R_A \) – Correlation resistance from model to ship in N

The frictional resistance results from:
\[ R_F = 0.5 \rho V_s^2 C_f S \] (2)

where \( C_f \) is the frictional resistance coefficient calculated as:

\[ C_f = \frac{0.0075}{(\log(Rn) - 2)^2} \] (3)

and \( S \) is the projected wetted surface of the bare hull, calculated as follows:

\[ S = L_{pp} (2T + B) \sqrt{C_m} \left(0.453 + 0.4425 C_b - 0.2862 C_m - 0.003467 \frac{B}{T} \right) \] (4)

\[ + 0.3696 C_w \right) + 2.38 \frac{A_{wT}}{C_b} \]

\[ C_m = 1 - 0.062 \ Fn^{0.792} \] (5)

The term \((1 + k_1)\) represents the form factor of the hull and is defined by:

\[ 1 + k_1 = 0.93 + 0.487118 c_{14} \left( \frac{B}{L_{pp}} \right)^{1.06806} \left( \frac{T}{L_{pp}} \right)^{0.46106} \left( \frac{L_{pp}}{L_r} \right)^{0.0121563} \left( \frac{L_{pp}^3}{A} \right)^{0.36486} \] (6)

\[ - C_p \right)^{-0.60247} \]

\[ L_r = L_{pp} (1 - C_p + 0.06 C_p \ Lcb / (4 \ C_p - 1)) \] (7)

\[ Lcb = 8.80 - 38.9 \ Fn \] (8)

\[ C_p = \frac{C_b}{C_m} \] (9)

The resistance derived by the appendages \( R_{App} \) has been ignored and considered equal to 0 N, as assumed in [2].
In order to calculate the wave resistance, have been developed three formulas, which correspond to three ranges of Froude number [2]. If \( F_n > 0.55 \):

\[
R_{W-B} = \nabla \rho \ g \ c_{17} \ c_2 \ c_5 \ \exp \left[ m_3 \ F_n^d + m_4 \ \cos \left( \lambda_n \ F_n^{-2} \right) \right]
\]

With:

\[
c_{17} = 691.3 \ C_m^{-1.3346} \ \left( \frac{V}{L^2} \right)^{2.00977} \ \left( \frac{L}{B} - 2 \right)^{1.40692}
\]

\[
c_2 = \exp \left( -1.89 \ \sqrt{c_3} \right)
\]

\[
c_3 = 0.56 \ \frac{A_{BT}^{1.5}}{B \ T \ \left( 0.31 \ \sqrt{A_{BT}} + T_F - h_B \right)}
\]

\[
c_5 = 1 - 0.8 \ \frac{A_T}{B \ T \ C_m}
\]

\[
m_3 = -7.2035 \ \left( \frac{B}{L_{pp}} \right)^{0.326869} \left( \frac{T}{B} \right)^{0.605375}
\]

\[
d = -0.9
\]

\[
m_4 = c_{15} \ 0.4 \ \exp \left( -0.034 \ F_n^{-3.29} \right)
\]

\[
c_{15} = \begin{cases} 
-1.69385, & \frac{L_{pp}^3}{V} < 512 \\
\frac{L_{pp}^3}{V^2} - 8, & 512 < \frac{L_{pp}^3}{V} < 1726.91 \\
-1.69385 + \frac{2.36}{\sqrt{3}}, & \frac{L_{pp}^3}{V} > 1726.91 \\
0, & \frac{L_{pp}^3}{V} > 1726.91
\end{cases}
\]
\[ \lambda_R = \begin{cases} 
1.446 \, C_p - 0.03 \frac{L_{pp}}{B}, & \frac{L_{pp}}{B} < 12 \\
1.446 \, C_p - 0.036, & \frac{L_{pp}}{B} > 12 
\end{cases} \] (19)

If \( F_n < 0.4 \):

\[ R_{W-A} = \nabla \rho \, g \, c_1 \, c_2 \, c_5 \, \exp \left[ m_1 \, F_n^d + m_4 \, \cos (\lambda_R \, F_n^{-2}) \right] \] (20)

With:

\[ c_1 = 2223105 \, c_7^{3.70613} \left( \frac{T}{B} \right)^{1.07961} \left( 90 - t_e \right)^{-1.37565} \] (21)

\[ c_7 = \begin{cases} 
0.229577 \left( \frac{B}{L_{pp}} \right)^{0.33333}, & \frac{B}{L_{pp}} < 0.11 \\
\frac{B}{L_{pp}}, & 0.11 < \frac{B}{L_{pp}} < 0.25 \\
0.5 - 0.0625 \left( \frac{L_{pp}}{B} \right), & \frac{L_{pp}}{B} > 0.25 
\end{cases} \] (22)

\[ i_E = 1 - 89 \exp \left[ - \left( \frac{L_{pp}}{B} \right)^{0.80856} \left( 1 - C_w \right)^{0.30484} \left( 1 - C_p \right) \\
- 0.0225 \, L_{cb}^{0.6367} \left( \frac{L_R}{B} \right)^{0.34574} \left( 100 \, \frac{\nabla}{L_{pp}^3} \right)^{0.16302} \right] \] (23)

\[ m_1 = 0.0140407 \frac{L_{pp}}{T} - 1.75254 \frac{V_3^2}{L_{pp}} - 4.79323 \frac{B}{L_{pp}} - c_{16} \] (24)

\[ c_{16} = \begin{cases} 
8.07981 \, C_p - 13.8673 \, C_p^2 + 6.984388 \, C_p^3, & C_p < 0.8 \\
1.73014 - 0.7067 \, C_p, & C_p > 0.8 
\end{cases} \] (25)

If \( 0.4 < F_n < 0.55 \):
\[ R_W = R_{W-A0.4} + (10 F_n - 4) (R_{W-B0.55} - R_{W-A0.4})/1.5 \] (26)

Where:

- \( T_F \) – draft at the fore perpendicular, assumed to be equal to \( T \), as in a first estimation there is no trim
- \( Lcb \) – position of longitudinal center of buoyancy, estimated with the Schneekluth formula [22]. This value is positive forward the center of the ship and is expressed as percentage of \( L_{pp} \).
- \( C_m \) – midship coefficient, calculated with the formula from Alvarino, Azpiroz and Meizoso [23]
- \( C_p \) – prismatic coefficient
- \( A_t \) – transom area, assumed to be 4m²
- \( Lr \) – parameter for the length of the run
- \( \rho \) – salt water density equal to 1.027 kg/m³
- \( Rn \) – Reynolds number

The pressure resistance on the bow \( R_B \) is calculated with the following formula:

\[ R_B = 0.11 \exp \left( -3 P_B^{-2} \right) F_{ni}^3 A_{BT}^{1.5} \rho \ g/(1 + F_{ni}^2) \] (27)

\[ A_{BT} = 0.08 A_m \] (28)

\[ A_m = C_m B T \] (29)

Where:

- \( A_m \) – transverse midship section’s area
- \( A_{BT} \) – transverse sectional area of the bulb, approximated to be 8% of the midship section area

The coefficient for the emergence of the bow \( P_B \), the vertical position of the center of the bow \( h_B \) and the Froude number based on the immersion \( F_{ni} \) are determined by:

\[ P_B = 0.56 A_{BT}^{0.5} / (T_F - 1.5 h_B) \] (30)

\[ F_{ni} = V / [g (T_F - h_B - 0.25 A_{BT}^{0.5}) + 0.15 V^2]^{0.5} \] (31)

\[ h_B = 0.9 \frac{T}{2} \] (32)
If present, the immersed transom has influence on the resistance on the hull and is derived by the formula:

$$ R_{TR} = 0.5 V^2 A_T c_6 $$

(33)

With:

$$ c_6 = \begin{cases} 
0.2 \left( 1 - 0.2 F_{nT} \right), & F_{nT} < 5 \\
0, & F_{nT} \geq 5 
\end{cases} $$

(34)

$$ F_{nT} = \frac{V}{2 g \left( \frac{A_T}{B + B_C W} \right)^{0.5}} $$

(35)

$$ C_w = 0.95 C_p + 0.17 \left( 1 - C_p \right)^{\frac{1}{3}} $$

(36)

Where:

- $F_{nT}$ – Froude number related with the transom
- $C_w$ – waterline coefficient

As the formulas presented above are calculated based on a model, an additive resistance, the model-ship correlation resistance $R_A$, has to be added to obtain the total resistance. This is calculated as follows:

$$ R_A = 0.5 \rho V^2 S C_A $$

(37)

$$ C_A = 0.006 \left( L_{pp} + 100 \right)^{-0.16} - 0.00205 + 0.003 \left( \frac{L_{pp}}{7.5} \right)^{0.5} c_6^4 c_2 \left( 0.04 - c_4 \right) $$

(38)

$$ c_4 = \begin{cases} 
\frac{T_F}{L_{pp}}, & \frac{T_F}{L_{pp}} \leq 0.04 \\
0.04, & \frac{T_F}{L_{pp}} > 0.04 
\end{cases} $$

(39)

Having obtained the total resistance that acts on the hull and knowing the velocity, it is possible to calculate the required Effective Horsepower (EHP) and Shaft Horsepower (SHP):
\[ EHP = \frac{R_{TOT} V}{746.0} \]  
\[ SHP = \frac{EHP}{0.65} \]  

Where:

- EHP, SHP – in hp
- \( R_{TOT} \) – in N
- \( V \) – velocity in m/sec

### 4.2 Weight estimate

The total weight of a ship can be split into the proper ship’s weight, the lightweight \( W_{light} \), and the consumables and cargo weight, the deadweight \( W_{dead} \), for this reason while the lightweight will be calculated in order to get an estimation of the building costs, the deadweight is the determinant for operative costs of the ship. The lightweight is the sum of weights regarding the hull \( W_{hull} \), which include the main hull structure, superstructure and bulkheads of the ship, the outfit and hull engineering \( W_{oh} \), that takes into account hull insulation, joiner bulkheads, pipes, deck fittings, cargo booms, anchors, rudder, galley equipment and hatch covers, and the machinery \( W_{m} \), which is the sum of the weights regarding the whole propulsion system, from propeller to funnel [7].

The deadweight is composed by the weight of the cargo \( W_{con} \) (containers in this case), the fuel weight \( W_{fuel} \) and the weight of fresh water, lubricating oil, stores and crew and other weight related to the machinery being idle \( W_{misc} \).

The same subdivision has been done to estimate the weights [2]:

\[ W_{TOT} = W_{light} + W_{dead} \]  
\[ W_{light} = W_h + W_{oh} + W_m \]  
\[ W_{dead} = W_{con} + W_{fuel} + W_{misc} \]

The calculations to estimate the lightship weight are as follows:

\[ W_h = C_s \cdot C L_1 \cdot C L_2 \cdot C L_3 \cdot \left[ \frac{CN}{1000} \right]^{0.9} \]
\[ C_s = 8550 \]  

\[ CL_1 = 0.675 + 0.5 C_b \]  

\[ CL_2 = 1 + 0.36 \frac{L_s}{L_{pp}} \]  

\[ CL_3 = 0.00585 \left( \frac{L_{oa}}{D} - 8.3 \right)^{1.8} + 0.939 \]  

\[ CN = \frac{L_{oa} B D}{100} \]  

\[ KG_h = [48 + 0.15 (0.85 - C_b) \left( \frac{L_{oa}}{D} \right)^2 \frac{D_s}{D}] \]  

\[ W_{oh} = W_o + W_{he} \]  

\[ W_o = 2412 \left( \frac{CN}{1000} \right)^{0.825} \]  

\[ W_{he} = 1196 \left( \frac{CN}{1000} \right)^{0.825} \]  

\[ KG_{oh} = (1.005 - 0.0000689 L_{oa}) D \]  

\[ W_m = 215 C_f \left( \frac{SHP}{1000} \right)^{0.72} \]  

\[ KG_m = 0.47 D \]  

\[ KG_{light} = \frac{W_h K_G_h + W_{oh} K_G_{oh} + W_m K_G_m}{W_h + W_{oh} + W_m} + 0.3 \]
Where:

- $W_h$ – weight of hull structure in ton
- $W_{oh}$ – weight of outfit and hull engineering in ton
- $W_o$ – weight of outfit in ton
- $W_{ke}$ – weight of hull engineering in ton
- $W_m$ – machinery weight in ton
- $L_s$ – length of superstructure in m
- $KG_h$ – vertical center of gravity of the hull weight
- $\frac{d_z}{D}$ – increased depth because of shear and hatchway approximated as 1.008 [2]

Before calculating the deadweight of the ship, it is necessary to calculate the maximal number of containers (TEU) that the ship can carry with the following formulas [2]:

\[
TEU = TEU_b + TEU_d \tag{59}
\]

\[
TEU_b = S_b L_b B_b D_b \tag{60}
\]

\[
TEU_d = S_d L_d B_d TN_d \tag{61}
\]

\[
L_b = L_d = \text{int}\left(\frac{L_{con}}{L_0}\right) \tag{62}
\]

\[
B_b = \text{int}\left(\frac{B - 2 WB}{B_0}\right) \tag{63}
\]

\[
D_b = \text{int}\left(\frac{D - DBH}{D_0}\right) \tag{64}
\]

\[
B_d = \text{int}\left(\frac{B}{B_0}\right) \tag{65}
\]

\[
TN_d = \begin{cases} 
4, & B < 32.2m \\
4 + \frac{B - 32.2}{7.8}, & 32.2m \leq B \leq 40m \\
5 + \frac{B - 40}{3}, & 40m \leq B \leq 43m \\
6, & B > 43m
\end{cases} \tag{66}
\]
\[ S_b = 0.8479 \ C_b - 0.0918 \]  

\[ S_d = 0.7534 \]  

Where:

- \( TEU_{b}, TEU_{d} \) – numbers of containers respectively below and on the deck
- \( L_b, B_b, D_b, L_d, B_d \) – container number along length, breadth and height direction respectively below and on the deck
- \( TN_d \) – number of rows of containers above deck
- \( S_b, S_d \) – stowage factors for containers respectively below and on the deck
- \( L_{con} \) – effective length to carry containers
- \( WB \) – wing tank breadth, assumed to be 1.83m [2]
- \( DBH \) – double bottom height, assumed to be 1.83m [2]
- \( L_o, B_o, D_o \) – length, width and height of a container, respectively equal to 6.10m, 2.44m and 2.59m

The usage of the integer function in the formulas for the calculation of the number of maximum containers possible in the three dimensions delivers a stepwise function, which leads the optimization code to deliver wrong results, as the gradient of such function is always 0. For this reason, it has been used an estimation for the calculations above, based on regressions of existing ships [2]:

\[ TEU_{b, float} = S_b \ (0.0196 \ L_{oa} B \ D - 148.6129) \]  

\[ TEU_{d, float} = 0.050117 \ L_{oa} B \ TN_d - 82.6702 \]  

\[ TEU = TEU_{b, float} + TEU_{d, float} \]  

The length overall \( L_{oa} \) has been estimated to be 2% bigger than the length between perpendiculars \( L_{pp} \). The length at the waterline \( L_{wl} \) has been assumed to be equal to \( L_{pp} \).

The round-trip number \( NT \) is calculated with the formulas [2]:

\[ NT = \frac{Ot}{Drt} \]
\[Ot = 365 - oh\]  \hfill (73)

\[Drt = Lut + Pwt + St\]  \hfill (74)

\[Lut = 4 \frac{TEU}{TSLU N_{crane\_float}}\]  \hfill (75)

\[N_{crane\_float} = 0.0187 L_{oa} + 0.3572\]  \hfill (76)

\[St = \frac{Dst}{V}\]  \hfill (77)

Where:

- \(Ot\) – ship’s annual operation time
- \(oh\) – off-hire time of the ship, estimated equal to 15 days [2]
- \(Pwt\) – Port waiting time, estimated to be 2 days [2]
- \(TSLU\) – number of containers handled in one day by one crane, equal to 1440

\[KG_b = DBH + C_{kgb} D_b D_0\]  \hfill (78)

\[C_{kgb} = \begin{cases} 
\frac{1}{2}, & C_b = 1 \\
\frac{2}{3}, & C_b = 0.5 \\
\text{else, linear interpolation} & 
\end{cases}\]  \hfill (79)

\[KG_d = D + HCH + TN_d \frac{D_0}{2}\]  \hfill (80)

\[W_{fuel} = 1.1 \frac{SHP\ Dst\ SFC}{V}\]  \hfill (81)

\[KG_{fuel} = \frac{6.1}{3385} W_{fuel}\]  \hfill (82)
\[ W_{misc} = W_{cp} + W_{fw} + W_{lo} + W_{hc} \]  
\[ (83) \]

\[ W_{cp} = (6 \text{ Drt} + 50) C_f \]  
\[ (84) \]

\[ W_{fw} = 280 \ C_f \]  
\[ (85) \]

\[ W_{lo} = 50 \ C_f \]  
\[ (86) \]

\[ W_{hc} = 5 \text{ Pwt} \ C_f \]  
\[ (87) \]

\[ KG_{misc} = 0.5 \ D \]  
\[ (88) \]

\[ KG_{dead} \]  
\[ = \frac{WPC \ TEU_{b\_float} \ KG_b + WPC \ TEU_{d\_float} \ KG_d + W_{fuel} \ KG_{fuel} + W_{misc} \ KG_{misc}}{WPC^2 \ TEU_{b\_float} \ TEU_{d\_float} \ W_{fuel} \ W_{misc}} \]  
\[ (89) \]

Where:

- \( KG_b \) – vertical center of gravity of the containers below deck
- \( KG_d \) – vertical center of gravity of the containers above deck
- \( HCH \) – hatch cover height
- \( W_{fuel} \) – fuel weight in ton
- \( Dst \) – service range of ship in nm
- \( KG_{fuel} \) – vertical center of gravity of the fuel
- \( W_{cp} \) – crew and provisions weight in ton
- \( W_{fw} \) – fresh water weight in ton
- \( W_{lo} \) – lubricate oil weight in ton
- \( W_{hc} \) – machinery being idle related weight in ton
- \( C_f \) – conversion factor long tons to metric tons equal to 1.016 [2]
- \( WPC \) – weight per container, estimated to be equal to 12 ton [2]
4.3 Cost estimate

Once the weights of the ship are defined, it is possible to estimate the costs to build and operate the vessel with the addition of more values, such as the working time to build the vessel, labor cost and others. As suggested by Chen [2] the labor cost \( L_c \) has been assumed to be 20$ per man-hour. In accordance to this initial value, all the formulas used to estimate the cost of a ship are in USD and the weights in ton.

The total annual cost of the ship is subdivided into capital cost \( Acc \) (CAPEX) and operative cost \( Aoc \) (OPEX). The first derives from the building of the ship taking into account of material labor costs and profits, while the second takes into account the expenses relative to the operation of the ship, which means both the yearly expenses (insurance, crew salary, administration and maintenance) and the expenses related to the actual voyage (fuel and port fees). Both CAPEX and OPEX have been finally added in order to obtain the minimum selling cost of each container \( RFR \) in order to recover the expenses of the ship.

First has been calculated the capital costs with the following formulas [2]:

\[
Lhs = Mhs \, Lc \\
Mhs = Cmhs \left( \frac{W_h}{1000} \right)^{0.85} \\
Mats = Csh \, W_h \\
Lo = Mho \, Lc \\
Mho = Co \left( \frac{W_o}{100} \right)^{0.9} \\
Mato = Cof \, W_o \\
Lhe = Mhhe \, Lc \\
Mhhe = Chhe \left( \frac{W_{he}}{100} \right)^{0.75}
\]
\[
Mathe = Chull \, W_{he}
\]

\[
Lm = Mhm \, Lc
\]

\[
Mhm = Chm \left( \frac{SHP}{1000} \right)^{0.6}
\]

\[
Matm = Cmm \left( \frac{SHP}{1000} \right)^{0.6}
\]

\[
Miscc = 0.10 \, (Mats + Mato + Mathe + Matm)
\]

\[
Accoc = 180000 \, Ncrew^{0.56}
\]

\[
Ovhc = 0.70 \, (Lhs + Lo + Lhe + Lm)
\]

\[
Ytc = Mats + Mato + Mathe + Matm + Miscc + Accoc + Ovhc
\]

\[
Pr = 0.05 \, Ytc
\]

\[
Ybc = Ytc + Pr
\]

\[
Owe = 0.05 \, Ybc
\]

\[
Owc = Ybc + Owe
\]

\[
Acc = Owc \, Cr
\]
Where:

- $Lhs$ – labor cost hull
- $Mhs$ – man-hours for hull
- $Cmhs$ – coefficient for the effectiveness of the yard, assumed to be 3160 [2]
- $Mats$ – material cost for hull
- $Csh$ – cost of hull material per ton, assumed to be 400 USD/ton [2]
- $Lo$ – labor cost for outfit
- $Mho$ – man-hours for outfit
- $Co$ – outfit coefficient, assumed to be 8000 [2]
- $Mato$ – material cost for outfit
- $Cof$ – cost of outfit per ton, assumed to be 1500 USD/ton [2]
- $Lhe$ – labor cost for hull engineering
- $Mhm$ – man-hours for hull engineering
- $Chhe$ – hull engineering coefficient, assumed to be 20400 [2]
- $Mathe$ – material cost for hull engineering
- $Chull$ – cost of hull engineering per ton, assumed to be 3500 USD/ton [2]
- $Lm$ – labor cost for machinery
- $Mkm$ – man-hours for machinery
- $Chm$ – machinery coefficient, assumed to be 6773 [2]
- $Matm$ – material cost for machinery
- $Cmm$ – coefficient for machinery cost, assumed to be 38867 [2]
- $Miscc$ – miscellaneous cost
- $Accoc$ – accommodation cost
- $Ovhe$ – overhead cost
- $Ytc$ – yard’s total cost
- $Pr$ – yard’s profit
- $Ybc$ – yard’s building price
- $Owe$ – owner expense
- $Owc$ – owner cost
- $Acc$ – annual capital cost (CapEx)
- $Cr$ – capital recovery factor
- $Ir$ – interest rate
- $Sl$ – ship’s life

\[
Cr = \frac{(1 + Ir)^{st} Ir}{(1 + Ir)^{st} - 1}
\]

(111)
The operating costs are calculated based mostly on the number of crew members in the ship $N_{crew}$, on the cubic number $CN$ and on the Shaft Horsepower SHP, as follows [2]:

$$N_{crew} = C_{st} \left[ C_{dk} \left( \frac{CN}{1000} \right)^{\frac{1}{6}} + C_{eng} \left( \frac{SHP}{1000} \right)^{1/5} \right]$$

$$Wage = 37800 N_{crew}^{4}$$

$$S_{S} = \begin{cases} 
112 \left( \frac{N_{crew}}{10} \right)^{4}, & \text{if } N_{crew} < 50 \\
70000 + 5600 \left( N_{crew} - 50 \right), & \text{if } N_{crew} \geq 50 
\end{cases}$$

$$Ins_{1} = 1351 N_{crew}$$

$$Ins_{2} = 14000 + 0.0098 \left( Mathe + Mats + Mato + Matm + Lm + Lhs + Lo + Lhe \right)$$

$$Mrh = 151200 \left( \frac{CN}{1000} \right)^{\frac{2}{3}}$$

$$Mrm = 14000 \left( \frac{SHP}{1000} \right)^{\frac{2}{3}}$$

$$Port = \left[ 20 + 290 \left( \frac{CN}{1000} \right) \right] \text{Pwt NT}$$

$$Af c = W_{f} \text{ Fcost NT}$$

$$Aoc = Wage + Ss + Ins_{1} + Ins_{2} + Port + Af c$$

$$Aac = Acc + Aoc$$
\[ RFR = \frac{Aac}{NT \times TEU \times 0.8} \]  

(123)

Where:

\( C_{st} \) – steward department coefficient assumed to be 1.25 [2]
\( C_{dk} \) – deck department coefficient assumed to be 15.4 [2]
\( C_{eng} \) – engine department coefficient assumed to be 10 [2]
\( Ss \) – costs related to stores and supplies
\( In\:S_1 \) – protection and indemnity insurance
\( In\:S_2 \) – hull and machinery insurance
\( Mrh \) – maintenance and repair costs for the hull
\( Mrm \) – maintenance and repair costs for the machinery
\( Port \) – port expenses
\( Af\:c \) – annual fuel cost
\( Fcost \) – average fuel cost, assumed to be 80 USD/ton [2]
\( Aoc \) – annual operating cost (OpEx)
\( Aac \) – annual average cost
\( RFR \) – required freight rate, objective to minimized in the optimization

4.4 Stability

In order to make some assessments regarding the initial stability of the ship, has been used as parameter the minimum heeling arm \( GM_{min} \) calculated following the formula provided by [24]:

\[ GM_{min} = \frac{PAH}{\Delta \tan \theta} \]  

(124)

\[ P = 0.055 + \left( \frac{L_{pp}}{1309} \right)^2 \]  

(125)

\[ A = L_{oa} (D - T) + TN_d D_0 L_{oa} \]  

(126)

\[ H = \frac{0.5 L_{oa} (D - T)^2 + 0.5 L_{oa} (TN_d D_0)^2}{A} + 0.5 T \]  

(127)

\[ \Delta = \rho L_{pp} B T C_b \]  

(128)

Where:
$P$ – Pressure on transverse projected area subject to heeling angle $\theta$ in ton/m²

$A$ – projected lateral area in m²

$H$ – vertical distance between center of $A$ and center of underwater lateral area in m

$\Delta$ – displacement of the ship in ton

$\theta$ – angle of heel, not smaller than 14 degrees [2]

This value is compared to the actual heeling arm, calculated in the standard way [2]:

$$GM = KM - KG$$

(129)

$$KM = KB + BM$$

(130)

$$BM = (0.085 C_b - 0.002) \frac{B^2}{T C_b}$$

(131)

$$KB = T (0.9 - 0.3 C_M - 0.1 C_b)$$

(132)

$$KG = \frac{KG_{light} W_{light} + KG_{dead} W_{dead}}{W_{light} + W_{dead}}$$

(133)

Where:

$GM$ – transversal metacentric height

$KM$ – transversal metacentric radius, estimated with the Xuebin formula [25]

$KG$ – height of center of gravity

$KB$ – height of center of buoyancy, estimated with the Schneekluth formula [22]

4.5 Freeboard

Also, the minimum value for the freeboard has to be calculated with the following approximation [2]:

$$FB_{min} = c \ a + b$$

(134)

$$c = 0.025633 L_{oa}^{0.9146}$$

(135)
The actual freeboard $FB$ of the ship is calculated as:

$$FB = D - T$$  \hspace{1cm} (138)$$

Where:

$D$ – depth of the ship in m

$T$ – draft of the ship in m

### 4.6 Rolling period

Being the rolling period a measure for the accelerations on the ship, it has to be checked against a minimum value $(RollingPeriod_{\text{min}})$, in this case assumed to be equal to 15 seconds, in order to avoid discomfort of the crew caused by high accelerations. The rolling period of the ship is approximated with the following formula [2]:

$$RollingPeriod = 0.58 \sqrt{\frac{B^2 + 4KG^2}{GM}}$$ \hspace{1cm} (139)$$

### 4.7 Single-objective constraint optimization

The formulas presented above are mostly estimations that relate ship’s main dimensions to other parameters useful for the initial ship design, so that either the initial ship’s main dimensions are decided and fixed before the calculations are performed, or these are kept as variable in order to dynamically optimize the problem and find an optimal solution. The second procedure has been chosen, as it offers a more flexible design, being possible also to set constraints and limits to the data analyzed.

For this reason, have been chosen six variables as design variables, identified as the ship’s main dimensions, from which all the previous calculations depend. These are:

$L_{pp}$ – length between perpendiculars
As the design variables during the optimization could assume whatever value, it was necessary to constrain these in certain intervals that have been defined based on data of existing ships. The chosen constraints are shown in the following table:

<table>
<thead>
<tr>
<th>Table 1: Bounds for design variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lower bound</strong></td>
</tr>
<tr>
<td>$L_{pp}$ [m]</td>
</tr>
<tr>
<td>$B$ [m]</td>
</tr>
<tr>
<td>$D$ [m]</td>
</tr>
<tr>
<td>$T$ [m]</td>
</tr>
<tr>
<td>$C_p$</td>
</tr>
<tr>
<td>$V_s$ [kn]</td>
</tr>
</tbody>
</table>

Moreover, typical proportions between main dimensions of multi-purpose vessels have been analyzed and set as constraints in order to build a model which is closest as possible to reality. The constraints are shown in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Bounds for proportions between main dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lower bound</strong></td>
</tr>
<tr>
<td>$L_{pp}/B$</td>
</tr>
<tr>
<td>$B/D$</td>
</tr>
<tr>
<td>$B/T$</td>
</tr>
</tbody>
</table>

As the ship during one voyage trip could have to pass through some canals or shallow waters, also physical constraints had to be taken into consideration. For this purpose, has been estimated the Panama Canal to present the highest restrictions, so that its maximal draft ($T < 12.6\, m$) and maximal breadth ($B \leq 32.3\, m$) have been taken as well as constraints for the model. Besides these, have been used other constraints that need to be fulfilled for the ship to comply with Rules and Regulations, regarding metacentric height (GM) and freeboard (FB), and to avoid discomfort of the crew, regarding the rolling period. The constraints are:

\[ RollingPeriod > RollingPeriod_{\text{min}} \quad (140) \]
\[ GM > GM_{\text{min}} \]  \hspace{1cm} (141)

\[ FB > FB_{\text{min}} \]  \hspace{1cm} (142)

As the ship’s displacement \( \Delta \) and the total ship’s weight \( W_{\text{TOT}} \) have been calculated with two different procedures (respectively through the block coefficient \( C_b \) and with the formulas presented above) and as they have to be set as equal because of the Archimedes’ principle, it has been introduced another constraint, which is fulfilled if the equality presented below is verified:

\[ \Delta = W_{\text{TOT}} \]  \hspace{1cm} (143)

The tool as designed allows to implement the insertion of different variables that are specific to the shipyard (e.g. available scantlings or labor costs), to the voyage (e.g. voyage distance, off-time period or minimum service speed) or to the ship owner’s requirements (e.g. amount of shipped TEUs), so that a more specific model can be achieved. In the presented code has been considered just one input variable, shown in Table 3 below.

**Table 3: Input variable**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>totalDist</td>
<td>distance of one voyage</td>
<td>5000 nm</td>
</tr>
</tbody>
</table>

As the model is orientated to the ship owner, are presented four examples for an optimized model based on the requirements for the voyage (}
These models have been obtained setting a maximum number of TEU that have to be shipped, respectively 650, 700, 750 and 800 TEU.

The four models are presented as example in order to show which are the most convenient models that it is possible to design with the use of the proposed optimization tool by constraining the number of TEU. Although these are optimal solutions regarding the RFR, to perform the scantlings, risk and maintenance analysis, it has been decided to adopt a real ship model. The used model has been taken from a design of a multi-purpose ship by the Bulyard Shipbuilding Industry, being available the midship section and main data. A more detailed analysis has been performed for such ship, applying the formulas for the conceptual design over the provided initial data. The results are listed in Table 5.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>L&lt;sub&gt;pp&lt;/sub&gt;</td>
<td>137.20</td>
<td>129.02</td>
<td>128.73</td>
<td>131.94</td>
</tr>
<tr>
<td>Length overall</td>
<td>L&lt;sub&gt;oa&lt;/sub&gt;</td>
<td>139.94</td>
<td>131.60</td>
<td>131.3</td>
<td>134.57</td>
</tr>
<tr>
<td>Breadth</td>
<td>B</td>
<td>21.11</td>
<td>22.80</td>
<td>23.86</td>
<td>23.685</td>
</tr>
<tr>
<td>Depth</td>
<td>D</td>
<td>11.13</td>
<td>11.69</td>
<td>13.09</td>
<td>13.53</td>
</tr>
<tr>
<td>Draft</td>
<td>T</td>
<td>8.13</td>
<td>8.594</td>
<td>9.10</td>
<td>8.915</td>
</tr>
<tr>
<td>Service speed</td>
<td>V&lt;sub&gt;s&lt;/sub&gt;</td>
<td>14.39</td>
<td>14.794</td>
<td>15.271</td>
<td>14.41</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>C&lt;sub&gt;b&lt;/sub&gt;</td>
<td>0.795</td>
<td>0.749</td>
<td>0.713</td>
<td>0.729</td>
</tr>
<tr>
<td>Midship section coefficient</td>
<td>C&lt;sub&gt;m&lt;/sub&gt;</td>
<td>0.983</td>
<td>0.982</td>
<td>0.981</td>
<td>0.982</td>
</tr>
<tr>
<td>Displacement</td>
<td>Displ</td>
<td>19216</td>
<td>19452</td>
<td>20448</td>
<td>20862</td>
</tr>
<tr>
<td>Shaft horse power</td>
<td>SHP</td>
<td>2358</td>
<td>2496</td>
<td>2678</td>
<td>1975</td>
</tr>
<tr>
<td>Power</td>
<td>P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>1143</td>
<td>1210</td>
<td>1298</td>
<td>957</td>
</tr>
<tr>
<td>Number of crew</td>
<td>N&lt;sub&gt;crew&lt;/sub&gt;</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>31</td>
</tr>
<tr>
<td>Number of cranes</td>
<td>N&lt;sub&gt;crane&lt;/sub&gt;</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Metacentric height</td>
<td>GM&lt;sub&gt;t&lt;/sub&gt;</td>
<td>1.90</td>
<td>2.03</td>
<td>1.65</td>
<td>1.40</td>
</tr>
<tr>
<td>Freeboard</td>
<td>FB</td>
<td>3.00</td>
<td>3.09</td>
<td>3.99</td>
<td>4.61</td>
</tr>
<tr>
<td>Lightship weight</td>
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<td>10810</td>
<td>11048</td>
<td>11033</td>
</tr>
<tr>
<td>Deadweight</td>
<td>W&lt;sub&gt;dead&lt;/sub&gt;</td>
<td>8505</td>
<td>8643</td>
<td>9400</td>
<td>9829</td>
</tr>
<tr>
<td>CAPEX</td>
<td>A&lt;sub&gt;a&lt;/sub&gt;c</td>
<td>2.29*10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>2.35*10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>2.48*10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>2.39*10&lt;sup&gt;6&lt;/sup&gt;</td>
</tr>
<tr>
<td>OPEX</td>
<td>A&lt;sub&gt;a&lt;/sub&gt;c</td>
<td>1.18*10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>1.21*10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>1.26*10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>1.16*10&lt;sup&gt;6&lt;/sup&gt;</td>
</tr>
<tr>
<td>Number of containers</td>
<td>TEU</td>
<td>650</td>
<td>662</td>
<td>724</td>
<td>763</td>
</tr>
<tr>
<td>Required Freight Rate</td>
<td>RFR</td>
<td>214.73</td>
<td>212.21</td>
<td>199.739</td>
<td>191.95</td>
</tr>
</tbody>
</table>
### Table 4: Four models presented as example

<table>
<thead>
<tr>
<th>Definition</th>
<th>Variable</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>L_{pp}</td>
<td>137.20</td>
<td>129.02</td>
<td>128.73</td>
<td>131.94</td>
<td>m</td>
</tr>
<tr>
<td>Length overall</td>
<td>L_{oa}</td>
<td>139.94</td>
<td>131.60</td>
<td>131.3</td>
<td>134.57</td>
<td>m</td>
</tr>
<tr>
<td>Breadth</td>
<td>B</td>
<td>21.11</td>
<td>22.80</td>
<td>23.86</td>
<td>23.685</td>
<td>m</td>
</tr>
<tr>
<td>Depth</td>
<td>D</td>
<td>11.13</td>
<td>11.69</td>
<td>13.09</td>
<td>13.53</td>
<td>m</td>
</tr>
<tr>
<td>Draft</td>
<td>T</td>
<td>8.13</td>
<td>8.594</td>
<td>9.10</td>
<td>8.915</td>
<td>m</td>
</tr>
<tr>
<td>Service speed</td>
<td>V_s</td>
<td>14.39</td>
<td>14.794</td>
<td>15.271</td>
<td>14.41</td>
<td>kn</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>C_b</td>
<td>0.795</td>
<td>0.749</td>
<td>0.713</td>
<td>0.729</td>
<td>-</td>
</tr>
<tr>
<td>Midship section coefficient</td>
<td>C_m</td>
<td>0.983</td>
<td>0.982</td>
<td>0.981</td>
<td>0.982</td>
<td>-</td>
</tr>
<tr>
<td>Displacement</td>
<td>Displ</td>
<td>19216</td>
<td>19452</td>
<td>20448</td>
<td>20862</td>
<td>ton</td>
</tr>
<tr>
<td>Shaft horse power</td>
<td>SHP</td>
<td>2358</td>
<td>2496</td>
<td>2678</td>
<td>1975</td>
<td>hp</td>
</tr>
<tr>
<td>Power</td>
<td>P_d</td>
<td>1143</td>
<td>1210</td>
<td>1298</td>
<td>957</td>
<td>kW</td>
</tr>
<tr>
<td>Number of crew</td>
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<td>30</td>
<td>31</td>
<td>32</td>
<td>31</td>
<td>-</td>
</tr>
<tr>
<td>Number of cranes</td>
<td>N_{crane}</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Metacentric height</td>
<td>G_M_t</td>
<td>1.90</td>
<td>2.03</td>
<td>1.65</td>
<td>1.40</td>
<td>m</td>
</tr>
<tr>
<td>Freeboard</td>
<td>FB</td>
<td>3.00</td>
<td>3.09</td>
<td>3.99</td>
<td>4.61</td>
<td>m</td>
</tr>
<tr>
<td>Lightship weight</td>
<td>W_{light}</td>
<td>10711</td>
<td>10810</td>
<td>11048</td>
<td>11033</td>
<td>ton</td>
</tr>
<tr>
<td>Deadweight</td>
<td>W_{dead}</td>
<td>8505</td>
<td>8643</td>
<td>9400</td>
<td>9829</td>
<td>ton</td>
</tr>
<tr>
<td>CAPEX</td>
<td>A_ac</td>
<td>2.29\times10^6</td>
<td>2.35\times10^6</td>
<td>2.48\times10^6</td>
<td>2.39\times10^6</td>
<td>USD</td>
</tr>
<tr>
<td>OPEX</td>
<td>A_oc</td>
<td>1.18\times10^6</td>
<td>1.21\times10^6</td>
<td>1.26\times10^6</td>
<td>1.16\times10^6</td>
<td>USD</td>
</tr>
<tr>
<td>Number of containers</td>
<td>TEU</td>
<td>650</td>
<td>662</td>
<td>724</td>
<td>763</td>
<td>-</td>
</tr>
<tr>
<td>Required Freight Rate</td>
<td>RFR</td>
<td>214.73</td>
<td>212.21</td>
<td>199.739</td>
<td>191.95</td>
<td>USD/TEU</td>
</tr>
</tbody>
</table>
Table 5: Dimensions used for the scantlings, risk-based and maintenance-based models

<table>
<thead>
<tr>
<th>Definition</th>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>$L_{pp}$</td>
<td>115.07</td>
<td>m</td>
</tr>
<tr>
<td>Length overall</td>
<td>$L_{oa}$</td>
<td>117.37</td>
<td>m</td>
</tr>
<tr>
<td>Breadth</td>
<td>$B$</td>
<td>20.00</td>
<td>m</td>
</tr>
<tr>
<td>Depth</td>
<td>$D$</td>
<td>10.40</td>
<td>m</td>
</tr>
<tr>
<td>Draft</td>
<td>$T$</td>
<td>8.30</td>
<td>m</td>
</tr>
<tr>
<td>Service speed</td>
<td>$V_s$</td>
<td>15.98</td>
<td>kn</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>$C_b$</td>
<td>0.719</td>
<td>-</td>
</tr>
<tr>
<td>Midship section coefficient</td>
<td>$C_m$</td>
<td>0.980</td>
<td>-</td>
</tr>
<tr>
<td>Displacement</td>
<td>$\text{Displ}$</td>
<td>16591</td>
<td>ton</td>
</tr>
<tr>
<td>Shaft horse power</td>
<td>$\text{SHP}$</td>
<td>3557</td>
<td>hp</td>
</tr>
<tr>
<td>Power</td>
<td>$P_d$</td>
<td>1724</td>
<td>kW</td>
</tr>
<tr>
<td>Number of crew</td>
<td>$N_{\text{crew}}$</td>
<td>31</td>
<td>-</td>
</tr>
<tr>
<td>Number of cranes</td>
<td>$N_{\text{crane}}$</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Metacentric height</td>
<td>$G_{M_t}$</td>
<td>1.78</td>
<td>m</td>
</tr>
<tr>
<td>Freeboard</td>
<td>$\text{FB}$</td>
<td>2.10</td>
<td>m</td>
</tr>
<tr>
<td>Lightship weight</td>
<td>$W_{\text{light}}$</td>
<td>10531</td>
<td>ton</td>
</tr>
<tr>
<td>Deadweight</td>
<td>$W_{\text{dead}}$</td>
<td>6060</td>
<td>ton</td>
</tr>
<tr>
<td>CAPEX</td>
<td>$A_{ac}$</td>
<td>$2.34 \times 10^6$</td>
<td>USD</td>
</tr>
<tr>
<td>OPEX</td>
<td>$A_{oc}$</td>
<td>$1.34 \times 10^6$</td>
<td>USD</td>
</tr>
<tr>
<td>Number of containers</td>
<td>$\text{TEU}$</td>
<td>441</td>
<td>-</td>
</tr>
<tr>
<td>Required Freight Rate</td>
<td>$\text{RFR}$</td>
<td>294.35</td>
<td>USD/TEU</td>
</tr>
</tbody>
</table>
5 Scantlings through MARS2000

As the risk-based model estimates the failure state of the structure through the limit state function, the ultimate bending moment is needed and is calculated using the program MARS2000 developed by Bureau Veritas. To calculate the ultimate bending moment initially the midship section had to be geometrically drawn (Figure 5-1), with relative panels, stiffeners, compartments and materials, which data have been taken from the midship section of the multipurpose ship, initially designed by Bulyard Shipbuilding Industry. The estimation of the ship specifications is carried in chapter “Conceptual design” and the final values used are shown in Table 5.

![Figure 5-1: Materials in the plates of the midship section (green for HS, red for marine steel)](image)

As the midship section shall be checked with different scantlings in order to observe the influence of increased or decreased material, it has been decided to modify just the thickness of the high tensile steel plates (shown in green in Figure 5-1) over 2mm in a range ± 10mm from the original scantling. It has been decided to just modify the upper plates’ thickness as this is the part of the midship section that experiences the highest stresses, so that it is a good assumption that if the upper structure will not fail, nor will the rest of the midship section.

In order to have a dimensional factor that takes into account the change of thickness it has been introduced a Design Modification Factor (DMF)[26], that is the relation between the modified sectional area and the original one, as shown in Table 6.
Table 6: Relation between thickness modification and DMF

<table>
<thead>
<tr>
<th>Scantlings</th>
<th>Sectional area</th>
<th>DMF</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>1.528</td>
<td>0.94</td>
</tr>
<tr>
<td>-4</td>
<td>1.560</td>
<td>0.96</td>
</tr>
<tr>
<td>-2</td>
<td>1.593</td>
<td>0.98</td>
</tr>
<tr>
<td>0</td>
<td>1.626</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1.658</td>
<td>1.02</td>
</tr>
<tr>
<td>4</td>
<td>1.691</td>
<td>1.04</td>
</tr>
<tr>
<td>6</td>
<td>1.724</td>
<td>1.06</td>
</tr>
<tr>
<td>8</td>
<td>1.756</td>
<td>1.08</td>
</tr>
<tr>
<td>10</td>
<td>1.789</td>
<td>1.10</td>
</tr>
</tbody>
</table>

It has to be noted that although the thickness modification shall have a ±10mm range from the original value, in this section has been calculated the scantling modification just from −6mm to +10mm, as with a lower thickness the used program MARS2000 does not deliver any value.

With these inputs, it was finally possible to obtain the ultimate bending moments in hogging condition (as standard for container vessels) for both intact $M_{u,\text{intact}}$ and corroded $M_{u,\text{corroded}}$ sections for each of the DMF, as shown in Table 7 and displayed in Figure 5-2. In Table 8 are displayed the gross and net section moduli at deck for each of the DMF.

Table 7: Ultimate bending moment obtained from MARS2000

<table>
<thead>
<tr>
<th>DMF</th>
<th>$M_{u,\text{intact}}$ (MPa)</th>
<th>$M_{u,\text{corroded}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.94</td>
<td>772.79</td>
<td>592.98</td>
</tr>
<tr>
<td>0.96</td>
<td>877.37</td>
<td>716.42</td>
</tr>
<tr>
<td>0.98</td>
<td>989.39</td>
<td>797.29</td>
</tr>
<tr>
<td>1.00</td>
<td>1102.18</td>
<td>906.70</td>
</tr>
<tr>
<td>1.02</td>
<td>1214.78</td>
<td>1018.15</td>
</tr>
<tr>
<td>1.04</td>
<td>1324.68</td>
<td>1129.20</td>
</tr>
<tr>
<td>1.06</td>
<td>1431.80</td>
<td>1237.40</td>
</tr>
<tr>
<td>1.08</td>
<td>1515.86</td>
<td>1276.69</td>
</tr>
<tr>
<td>1.10</td>
<td>1540.82</td>
<td>1289.12</td>
</tr>
</tbody>
</table>
Table 8: Gross and net modulus at deck of the midship section obtained from MARS2000

<table>
<thead>
<tr>
<th>DMF</th>
<th>Gross section modulus (m³)</th>
<th>Net section modulus (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.94</td>
<td>3.098</td>
<td>2.56</td>
</tr>
<tr>
<td>0.96</td>
<td>3.396</td>
<td>2.859</td>
</tr>
<tr>
<td>0.98</td>
<td>3.695</td>
<td>3.158</td>
</tr>
<tr>
<td>1.00</td>
<td>3.995</td>
<td>3.458</td>
</tr>
<tr>
<td>1.02</td>
<td>4.294</td>
<td>3.758</td>
</tr>
<tr>
<td>1.04</td>
<td>4.595</td>
<td>4.084</td>
</tr>
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<td>4.895</td>
<td>4.359</td>
</tr>
<tr>
<td>1.08</td>
<td>5.196</td>
<td>4.661</td>
</tr>
<tr>
<td>1.10</td>
<td>5.497</td>
<td>4.963</td>
</tr>
</tbody>
</table>

Figure 5-2: DMF vs ultimate bending moment
6 Risk based model

Having estimated in the previous chapter the main dimensions of the model, it is possible to make further calculations about the reliability of the structure and find the optimal thickness based on the probability of failure. The reliability of the structure has been estimated taking into account the asymmetrical bending moments that act on the hull (still water and wave-load induced bending moment) and comparing it with the ultimate bending moment of the structure itself, obtained by the program MARS2000. This has been aimed by calculating the limit state function of the problem with the First Order Reliability Method (FORM), which provided the probability of failure. In this way, it was possible to calculate the cost associated with the risk of failure and, summing it with the cost resulting by changing the thickness, it results in the total expected cost as function of the change in thickness, so that the lowest value of this function presents the optimal thickness based on costs related to the probability of failure. The tool for the risk-based model is shown in Figure 6-1.

![Figure 6-1: Risk-based model flow chart](image)

6.1 Limit state function

In order to describe the reliability of the structure under the asymmetrical bending moments to which the ship is subject during its lifetime and to predict the probabilities of this to collapse, it has been used the First Order Reliability Method (FORM) [3]. This method has been adopted to evaluate the variables that have influence on
the failure of the structure and in this way to define a limit state function for the reliability \( G(X) \), that describes whether the ship is in the safe region \( (G(X) \geq 0) \).

It has been preferred the FORM to evaluate the reliability of the ship, because it takes into account different typologies of distribution of the different variables, as it is needed in the ultimate strength analysis, and it has been preferred this to the Second Order Reliability Method (SORM) as the linear method of FORM provides a simpler and satisfyingly accurate approach [15].

In this way, the limit state function performed with FORM over the ultimate strength analysis has as variables the ultimate, still water and wave-induced bending moments, and the related uncertainty factors, summed up in the equation

\[
G(X) = \chi_u M_u - (\chi_{sw} M_{sw} + \chi_{wv} M_{wv})
\]

Where:
- \( M_u \) – ultimate bending moment of the structure in MN.m
- \( M_{sw} \) – still water bending moment in MN.m
- \( M_{wv} \) – wave-load induced bending moment in MN.m
- \( \chi_u, \chi_{sw}, \chi_{wv} \) – model uncertainty factors for ultimate, still water and wave-load bending moment, see Table 9
- \( \chi_{ml} \) – model uncertainty factor for non-linear effects, taken as 1

The limit state function presented above takes into account different types of distributions for the different variables, so that it is necessary to describe the variables independently and show for each of them which is the procedure adopted relative to the typical distribution to obtain the mean value \( Mean \) and standard deviation \( StdDev \).

In order to get the limit state function, it was necessary to proceed through two steps: the definition of the mean value and standard deviation for the different variables (defined by the index 0), and the optimization of them in order to get the limit state function \( G(X) = 0 \), so that the combination of the variables is the optimal one and obtain in this way the minimum probability of failure possible (defined by the index 1).

The still water bending moment is assumed to be described by a Normal distribution [3, 42], which statistical descriptors depend from the ratio between deadweight and displacement of the ship \( W \) and the length of the ship \( L_{pp} \) [27, 42], while typical values for still water and wave induced bending moments have been calculated following the rules of Classification Societies (Det Norske Veritas rules [28]). Mean and standard deviation of the

<table>
<thead>
<tr>
<th>Mean</th>
<th>Deviation</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>1.00</td>
<td>0.1</td>
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</tr>
<tr>
<td>1.00</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 9: Model uncertainty factors
still water bending moment are calculated based on the formulas (145) and (146) developed by Guedes Soares and Moan [42].

\[
StdDev_{sw,0} = StdDev_{sw,max} M_{sw,cs} 10^{-2} \quad (145)
\]

\[
Mean_{sw,0} = Mean_{sw,max} M_{sw,cs} 10^{-2} \quad (146)
\]

\[
StdDev_{sw,max} = a_{0,dev} + a_{1,dev} W + a_{2,dev} L_{pp} \quad (147)
\]

\[
Mean_{sw,max} = a_{0,mean} + a_{1,mean} W + a_{2,mean} L_{pp} \quad (148)
\]

\[
W = \frac{W_{dead}}{W_{TOT}} \quad (149)
\]

\[
M_{sw,cs} = 175 n C L_{pp}^2 B (C_b + 0.7)10^{-6} - M_{wv,cs} \quad (150)
\]

\[
C = 10.75 - \left(\frac{300 - L_{pp}}{100}\right)^{1.5} \quad (151)
\]

Where:

- \(M_{sw,cs}\) – still water bending moment as given by the Classification Society [28]
- \(M_{wv,cs}\) – vertical wave-induced bending moment as given by the Classification Society [28]
- \(a_{0}, a_{1}, a_{0}\) – coefficients taken as show in Table 10
- \(W\) – relation between deadweight and total ship’s weight
- \(n\) – coefficient for the probability level, taken as 1.0 for the strength assessment corresponding to the probability of \(10^{-8}\)
\( C \) – wave parameter for \( 90 \leq L_{pp} < 300 \) [28]

**Table 10: values for the coefficients \( a_0, a_1, a_0 \)**

| \( a_{0,\text{dev}} \) | 17.4 |
| \( a_{1,\text{dev}} \) | -7.0 |
| \( a_{2,\text{dev}} \) | 0.035 |
| \( a_{0,\text{mean}} \) | 114.7 |
| \( a_{1,\text{mean}} \) | -105.6 |
| \( a_{2,\text{mean}} \) | -0.154 |

In a similar way have been obtained mean value and standard deviation for the vertical wave-induced bending moment \( M_{wv,cs} \), for which have been taken into consideration the stochastic extreme values at a random time point over a reference time period \( T_r \) for a mean wave period \( T_w \). For this reason, it has been fitted to a Gumbel distribution [3] and is calculated as follows:

\[
\text{Mean}_{wv,0} = \alpha_{m,wv} + \beta_{m,wv} \gamma_{wv}
\]

\[
\alpha_{m,wv} = w_{wv} \log(n_c)^k
\]

\[
\beta_{m,wv} = \frac{w_{wv}}{k} \log(n_c)^{\frac{1-k}{k}}
\]

\[
w_{wv} = \frac{M_{wv,cs}}{\log(10)^{1/k}}
\]

\[
M_{wv,cs} = 190 \, F m \, n \, C \, L_{pp}^2 \, B \, C_{b} \, 10^{-6}
\]

\[
k = 2.26 - 0.54 \log(L_{pp})
\]
\[ n_c = 0.35 \frac{T_r}{T_w} \]  \hspace{1cm} (158)

Where:
- \( \alpha_{m,ww} \) – parameter of the Gumbel distribution
- \( \beta_{m,ww} \) – parameter of the Gumbel distribution
- \( \gamma_{ww} \) – taken equal to 0.577 [26]
- \( w_{ww} \) – scale factor of the Weibull distribution [28]
- \( M_{ww,cs} \) – vertical wave-induced bending moment as given by the Classification Society [28]
- \( Fm \) – distribution factor, equal to 1 for bending moment at \( \frac{L_pp}{2} \)
- \( k \) – shape factor of the Weibull distribution [28]
- \( n_c \) – mean number of load cycles in a reference time period \( T_r \) for a mean wave period \( T_w \)
- \( T_r \) – reference time period, taken equal to 1 year [26]
- \( T_w \) – mean wave period, taken equal to 8 sec [26]

The ultimate bending moment distribution has been calculated, as shown in chapter “Reliability of the structure”, computing for each of the scantlings described in chapter “Scantlings through MARS2000” the ultimate bending moment for the hogging condition resulting from geometry of the midship section and still water bending moments. With these values is possible to calculate mean value and standard deviation of ultimate bending moment (intact and corroded) for the different Design Modification Factors (DMF).

Standard deviation \( StdDev \) and mean value \( Mean \) for ultimate bending moment are fitted to a Log-normal distribution [3] and are calculated in two steps marked with the indexes 0 and 1 (where the second is the final value of the equivalent normal mean and standard deviation used in the FORM optimization) and is valid for both intact and corroded condition with the relative ultimate bending moment shown in Table 7 as follows:

\[ StdDev_{u,0} = \sqrt{\exp(\sigma_{Mu}^2 - 1)) (\exp(2 \mu_{Mu} + \sigma_{Mu}^2))} \]  \hspace{1cm} (159)

\[ Mean_{u,0} = \exp \left( \mu_{Mu} + \frac{\sigma_{Mu}^2}{2} \right) \]  \hspace{1cm} (160)

\[ \mu_{Mu} = 2 \sigma_{Mu}^2 \log(0.05 M_u \sigma_{Mu} \sqrt{2\pi}) + \log (M_u) \]  \hspace{1cm} (161)
\[ \sigma_{Mu} = \sqrt{\log \left( \text{COV}_{u}^{2} + 1 \right)} \]  (162)

Where COV is the coefficient of variation, taken equal to 0.08 [3].

Second step to calculate the limit state function consists in the optimization of the objective variable with the FORM through the change of modification factors \( z^* \) to obtain the design points \( x^* \) in the limit state function, which represent the most likely combination of values of the parameters that lead the structure to fail. The design points are calculated for mean values of ultimate, still water and wave-induced bending moments and model uncertainty factors as follows:

\[ x^* = \text{Mean}_0 \cdot z^* \]  (163)

Initially these modification factors have all been set equal to 1, so that the calculated parameter and the design point were the same, and just in a second moment, during the optimization, they were changed to get the \( G(X) = 0 \) and the lowest beta-index possible, as this gives the combination of variables that represent the threshold between failure (negative \( G(X) \)) and safe region (positive \( G(X) \)). Input values for this optimization are mean values and standard deviation of the ultimate, wave-induced and still water bending moments and the respective model uncertainty factors, as shown and calculated in the first step. As these input values follow different distributions, different formulas were used to get the equivalent normal mean \( \text{Mean}_1 \) and the equivalent normal standard deviation \( \text{StdDev}_1 \).

As the still water bending moment and all the uncertainty factors are fitted to a Normal distribution, their equivalent normal mean and standard deviation correspond to the previously calculated mean and standard deviation:

\[ \text{Mean}_{sw,1} = \text{Mean}_{sw,0} \]  (164)

\[ \text{StdDev}_{sw,1} = \text{StdDev}_{sw,0} \]  (165)

The wave-induced load is fitted to a Gumbel distribution, so that its equivalent normal mean and standard deviation are as follows:
\[ Mean_{wv,1} = Mean_{wv,0} - StdDev_{wv,1} F^{-1}(F_{wv}) \] (166)

\[ StdDev_{wv,1} = f(F^{-1}(F_{wv}, 0, 1)/f_{wv}) \] (167)

\[ F_{wv} = \exp \left( -\exp \left( -\frac{Mean_{wv,0} - \alpha_{m,wv}}{\beta_{m,wv}} \right) \right) \] (168)

\[ f_{wv} = \frac{1}{\beta_{m,wv}} \exp \left( -\left( -\frac{Mean_{wv,0} - \alpha_{m,wv}}{\beta_{m,wv}} \right) \right) + \exp \left( -\frac{Mean_{wv,0} - \alpha_{m,wv}}{\beta_{m,wv}} \right) \] (169)

Where \( f(x) \) represents the normal probability density function and the \( F(x) \) is the normal cumulative density function.

The mean and standard deviation of the ultimate bending moment are calculated following the Log-normal distribution as shown below:

\[ Mean_{u,1} = x_u^* \left( 1 - \log (x_u^*) + \lambda_u \right) \] (170)

\[ StdDev_{u,1} = x_u^* \sqrt{\log \left( 1 + \left( \frac{StdDev_{u,0}}{Mean_{u,0}} \right)^2 \right)} \] (171)

\[ \lambda_u = \log(Mean_{u,0}) - 0.5 \log \left( 1 + \left( \frac{StdDev_{u,0}}{Mean_{u,0}} \right)^2 \right) \] (172)

In order to be able to calculate the reliability index (\( \beta \)-index) have been obtained the design points in the reduced coordinate system for all the parameters of the limit state function as follows:

\[ nx = \frac{x^* - Mean_1}{StdDev_1} \] (173)
Being the reliability index the minimum distance from the origin point of the reduced coordinate system to the limit state surface, it is calculated as follows:

\[
\beta = \sqrt{(nx)^2 (nx)}
\]  \hspace{1cm} (174)

To optimize the FORM problem presented above has been used a non-linear optimization algorithm coded on MATLAB, where the objective was the limit state function equal to \( G(X) = 0 \), obtained by changing the design points of the variables in order to get the lowest reliability index possible and to take into consideration the constraints and boundaries set. In order to keep the beta-index as low as possible, this has been constrained in the code and achieved with an iterative process.

The reliability index has been constrained in the code and, with an iterative process lowering the constraint of the maximal beta-index allowed but still keeping the \( G(X) = 0 \), it was aimed to get the lowest reliability index possible. It was decided to minimize this parameter as to a lower reliability index corresponds a higher probability of failure, so that being also this case in the “safe region” of the limit state function, it is assured that also all the other cases will have no higher probability of failure.

Moreover, in order not to allow the modification factors \( z^* \) to have every possible value, they were constrained to be between -2 and 2.

An assessment about the influence that the seven variables have over the limit state function can be done with a sensitivity analysis, where the sensitivities are calculated for each variable as follows:

\[
\alpha_i = - \frac{1}{\sqrt{\sum_{i=1}^{p} \left( \frac{\partial g(x)}{\partial x_i} \right)^2}} \frac{\partial g(x)}{\partial x} \hspace{1cm} (175)
\]

In Figure 6-2 is shown the sensitivity analysis, where it can be seen that the variables regarding the ultimate bending moment have high positive values, which means that by an increase of them, the limit state function is positively affected, while it is the opposite for wave-induced and still water bending moments.
6.2 Reliability of the structure

The reliability index has been calculated for the scantlings presented above for both intact and corroded conditions (Table 11 and Figure 6-3).
The \( \beta \)-indexes for intact and corroded condition can be unified under one common \( \beta \)-index if also the time is taken into account assuming the corrosion is a linear process for the ship’s lifetime (assumed to be 25 years [26]):

\[
\beta = -\beta_{\text{gross}} + (t - 1) \frac{\beta_{\text{gross}} - \beta_{\text{net}}}{24}
\]

(176)

Where:

\( t \) – life of the ship in years, with a maximum of 25 years

In this way, it was possible to obtain the probability of failure \( P_f \) by calculating the normal cumulative density function of the beta index as function of the time (Table 11 and Figure 6-4):

\[
P_f(t) = F(-\beta(t))
\]

(177)

<table>
<thead>
<tr>
<th>DMF</th>
<th>( \beta_{\text{gross}} )</th>
<th>( P_{f,\text{gross}} )</th>
<th>DMF</th>
<th>( \beta_{\text{net}} )</th>
<th>( P_{f,\text{net}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.94</td>
<td>1.435</td>
<td>7.56E-02</td>
<td>0.94</td>
<td>0.201</td>
<td>4.20E-01</td>
</tr>
<tr>
<td>0.96</td>
<td>2.269</td>
<td>1.16E-02</td>
<td>0.96</td>
<td>1.005</td>
<td>1.58E-01</td>
</tr>
<tr>
<td>0.98</td>
<td>3.176</td>
<td>7.46E-04</td>
<td>0.98</td>
<td>1.768</td>
<td>3.85E-02</td>
</tr>
<tr>
<td>1.00</td>
<td>3.484</td>
<td>2.47E-04</td>
<td>1.00</td>
<td>2.437</td>
<td>7.40E-03</td>
</tr>
<tr>
<td>1.02</td>
<td>3.928</td>
<td>4.28E-05</td>
<td>1.02</td>
<td>2.938</td>
<td>1.65E-03</td>
</tr>
<tr>
<td>1.04</td>
<td>4.345</td>
<td>6.96E-06</td>
<td>1.04</td>
<td>3.481</td>
<td>2.50E-04</td>
</tr>
<tr>
<td>1.06</td>
<td>4.826</td>
<td>6.98E-07</td>
<td>1.06</td>
<td>3.984</td>
<td>3.39E-05</td>
</tr>
<tr>
<td>1.08</td>
<td>5.285</td>
<td>6.30E-08</td>
<td>1.08</td>
<td>4.244</td>
<td>1.10E-05</td>
</tr>
<tr>
<td>1.10</td>
<td>5.558</td>
<td>1.37E-08</td>
<td>1.10</td>
<td>4.454</td>
<td>4.21E-06</td>
</tr>
</tbody>
</table>

Table 11: Beta-index and probability of failure for the different DMF for intact and corroded condition
6.3 Reliability cost model

To evaluate the cost associated with the modification of the structure has been performed a cost benefit analysis [26]:

\[
C_t^\beta = C_{T_f}^\beta + C_{me}^\beta
\]  

(178)

Where:

- \(C_t^\beta\) – total expected cost, USD
- \(C_{T_f}^\beta\) – cost associated with the failure of the ship, USD
- \(C_{me}^\beta\) – cost of implementing a structural safety measure, USD

The cost associated with the failure of the ship \(C_{T_f}^\beta\) is the sum of the cost of lost of the ship calculated in the year of life \(t\) \(C_n(t)\), the loss of cargo \(C_c\), the accidental spill of fuel oil \(C_d\) and the loss of human life \(C_v\), assuming that the operational life of the ship \(T\) is 25 years [26]:

\[
C_{T_f}^\beta (t) = \sum_{t=1}^{T} P_f (\beta) \left[ C_n (t) (C_c + C_d + C_v) \right] \exp (-\gamma t)
\]  

(179)
\[ C_n(t) = (Owc - (Owc - C_{scrap})) \left( \frac{t}{25} \right) \]  
\[ C_{scrap} = C_{material/ton} W_{light} \]  
\[ C_c = C_{container} \times TEU \]  
\[ C_d = W_{fuel} \times P_{spill} \times P_{sl} \times CATS \]  
\[ C_v = N_{crew} \times P_{crew} \times ICAF \]

Where:

- \( C_n(t) \) – cost associated with the lost of the ship in the year of life \( t \), in USD
- \( C_c \) – cost associated with the loss of cargo, in USD
- \( C_d \) – cost associated with the damages caused by the accidental spill of fuel oil, in USD
- \( C_v \) – cost associated with the loss of human life, in USD
- \( \gamma \) – discount rate, assumed to be 5%
- \( Owce \) – owner initial cost of the ship, calculated in Chapter 4.3, in USD
- \( C_{scrap} \) – scrapping value of the ship, in USD
- \( C_{material/ton} \) – cost of structural material per ton, assumed to be 277 USD
- \( C_{container} \) – value of one single container, estimated to be 2500 USD
- \( W_{fuel} \) – weight of fuel estimated in Chapter 4.2, in ton
- \( P_{spill} \) – percentage of fuel oil spilled after the structural failure, taken as 25%
- \( P_{sl} \) – probability that fuel oil spilled reaches the shore, taken as 10%
- \( CATS \) – Cost of Averting a Ton of Split, taken equal to 60000 USD
- \( P_{crew} \) – probability of loss of human life after structural failure, assumed as 25%
- \( ICAF \) – Implied Cost of Averting a Fatality, value given to human life assumed to be \( 3.85 \times 10^6 \) USD
The cost of implementing a structural safety measure $C_{me}$ takes into account the decreased or increased cost of the deck structure, if the DMF is taken respectively negative or positive and it is calculated with the following formula:

$$C_{me}^\beta = (DMF^\beta - 1) W_{structure} C_{work/ton}$$  \hspace{1cm} (185)

Where:
- $W_{structure}$ – weight of the cylindrical body of the ship, estimated to be 70% of the total lightship weight
- $C_{work/ton}$ – cost to work on ton of structural material, taken equal to 2500 USD

It has been observed that while the cost of implementing a structural safety measure is roughly proportional to the added material to the top part of the section, the cost associated with the structural failure of the ship highly depends on the probability of failure, which is almost negligible when $DMF \geq 1.02$, lowering this type of cost to a point where it has no influence more, as shown in Figure 6-5, where the points mark the values for every DMF from 0.94 to 1.10.

![Figure 6-5: Total costs as function of the beta-index](image)

Figure 6-5 and Table 12 show that while for a negative DMF, the total expected cost is influenced both by the cost associated with the failure of the ship and the cost of implementing a safety measure, for positive DMF the total cost is mostly determined by the cost of implementing a structural safety measure, being in this way inconvenient to have a bigger Design Modification Factor (DMF) than 1.00.
The scatter diagram above shows that the total cost as the function of the beta-index has the minimum between the DMF of 0.98 and 1.00, being the second just thousand USD cheaper than the first. For this reason, it has been estimated that the initial scantling of the proposed ship \((DMF = 1.00)\) is already in the optimum condition for the reliability, although with a continuous function of the cost as function of the beta-index the minimum would be between the DMF 0.98 and 1.00. This range of DMF of minimum total expected cost is verified between beta-index of 1.70 and 2.5, as a lower index would increase drastically the cost of a failure, being the probability of collapse too high, and a higher increase would lead to high cost of implementing a safety measure for no advantages in the reliability, as for \(DMF \geq 1.00\) the cost related to the failure of the ship are almost negligible.

Table 12: Values for the costs for each beta-index

<table>
<thead>
<tr>
<th>(\beta)-index</th>
<th>(C_{me})</th>
<th>(C_{tf})</th>
<th>(C_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.201</td>
<td>-897960</td>
<td>4053475</td>
<td>3155515</td>
</tr>
<tr>
<td>1.005</td>
<td>-598640</td>
<td>1519504</td>
<td>920864</td>
</tr>
<tr>
<td>1.768</td>
<td>-299320</td>
<td>371461</td>
<td>72141</td>
</tr>
<tr>
<td>2.437</td>
<td>0</td>
<td>71419</td>
<td>71419</td>
</tr>
<tr>
<td>2.938</td>
<td>299320</td>
<td>15941</td>
<td>315261</td>
</tr>
<tr>
<td>3.481</td>
<td>598640</td>
<td>2411</td>
<td>601051</td>
</tr>
<tr>
<td>3.984</td>
<td>897960</td>
<td>327</td>
<td>898287</td>
</tr>
<tr>
<td>4.244</td>
<td>1197280</td>
<td>106</td>
<td>1197386</td>
</tr>
<tr>
<td>4.454</td>
<td>1496600</td>
<td>41</td>
<td>1496641</td>
</tr>
</tbody>
</table>
7 Maintenance based model

During the life of a ship, which is generally designed to be 25 years [4], many structural components have to be replaced to guarantee full functionality and safety for the whole structure, as with the usage, these components may not be any more functional for their purpose and lead the system in which they are integrated to failure. As in this chapter the ship has been analyzed under the structural aspect, the thickness of the structural components has been checked with respect to planned maintenance.

The biggest issue for the metal components during the ship’s lifetime is corrosion, which is the cause for the material degradation and loss of strength [31]. Corrosion is defined as a time and environment dependent loss of material caused by a chemical or electrochemical reaction with the environment [32], for which reason its behavior has to be analyzed in order to prevent failures of the structure.

The process of corrosion leads to a failure of the component when the plate thickness is less than a minimum safety level defined taking into consideration structural safety levels. Indeed, the loss of material does not just cause loss of the thickness in the component and surface destruction, but it is also responsible for the loss of tensile strength and for non-linear reduction in yield strength. For these reasons, the most important consequences of corrosion on structural components are reduction of structural strength, unfavorable stress distribution (caused by corrosion pits) and reduced net sectional area [31, 33, 34, 4].

Consequently, the maintenance has been set as a central optimization objective, by minimizing the costs that derive from preventive and corrective maintenance, as these can be estimated with the help of stochastic models and listed between the OPEX. In this model have been considered both preventive and corrective maintenance, being the first the substitution of a component before it fails and the second the substitution of it after its structural failure. As it is not possible to inspect steadily the structure to predict the structural failure, statistical calculations based on Weibull distribution have been used and implemented in the cost calculations. With estimations about the consequences on the costs deriving from a failure and relating these with a risk level, it was possible to find the most convenient maintenance strategy regarding costs, downtime and availability, analyzed using different maintenance policies.

The Weibull family probability models have been adopted to describe the likelihood of a component to fail [35, 36] in order to define optimal replacement intervals and optimal replacement age (also taking into consideration the time required to perform the replacement itself), by minimizing the total cost, the total downtime of the system per unit time or by maximizing the availability of it.

The corrosion progression has been defined by the non-linear model proposed by Yamamoto and Ikagaki [18], although this is not sufficient to describe the behavior. Guedes Soares and Garbatov [19] and Paik et al [37] proposed a corrosion model for marine structures based on three steps: after a period when the protective coating has its effect, it starts to take place on the surface of the metallic component a non-linear corrosion process that leads to increasingly higher corrosion rates up to the point where the process gets stable and the rate is stabilized to a steady-state. Exceeding the limit on structural strength of one component causes the failure of it, which is also fitted to a Weibull distribution.
This model is based on previous studies done by Garbatov [4], analyzes and implements in the main model the probability and severity of structural failures caused by corrosion, calculates costs deriving from preventive and corrective maintenance, with the objective to minimize costs, downtime or maximize the availability, optimizing the replacement interval, replacement age and replacement age with consideration of the time needed to replace the component, factor that can lead to a better model design.

1.1 Probability and severity of structural failure due to corrosion

In order to estimate the probability of a component to fail as a consequence of corrosion, the failure rate (or hazard rate) \( h(t) \) has been analysed. This function gives the number of components that out of the totality fails in a determined unit of time, or, said in other words, it is the instantaneous probability of a component not to survive at age \( t \) [4, 20]. Following this definition, the failure rate is calculated as:

\[
h(t) = \frac{\int_{t}^{t+\delta t} f(t)dt}{\int_{t}^{\infty} f(t)dt} = \frac{F(t + \delta t) - F(t)}{1 - F(t)} = \frac{f(t)}{1 - F(t)}
\]

(186)

The hazard rate function can be subdivided into three regions with different behavior regarding the number of failures per unit time (Figure 7-1).

![Figure 7-1: Hazard rate as function of time, the so-called bathtub function.](image)

Indeed, while in the first region (Region A) the number of failures starting from a high rate decrease quickly, as the new built components need time to get stable, mostly regarding failures in the production and mounting (burn-in failures), the Region B is characterized by a constant hazard rate and by random failures, being the period when the components are in normal operation conditions. The third region, characterized by wear-out failures, represents the last period of life of a component, as with an increasingly higher hazard rate, it will fail or will be substituted. Consequently, the aim of this model is to avoid that components enter into the wear-out
region, done by identifying the moment in the life-cycle of a structural component when it statistically goes from Region B to Region C, as just in the wear-out region it is convenient and favorable to replace a component.

As stated above, the distribution of stochastic failures has been fitted to a Weibull distribution [4], with the probability density function, \( f(t) \) expressed as:

\[
f(t) = \begin{cases} 
\frac{\beta_w}{\eta_w} \left( \frac{t - \gamma_w}{\eta_w} \right)^{\beta_w-1} \exp \left( - \left( \frac{t - \gamma_w}{\eta_w} \right)^{\beta_w} \right), & \text{for } t > \gamma_w \\
0, & \text{for } t \leq \gamma_w 
\end{cases}
\]  

(187)

where the shape, \( \beta_w \), scale, \( \eta_w \) and location, \( \gamma_w \) parameters of the Weibull distribution are bigger than 0.

In order to estimate numerically the structural failure of a component, have been considered four grades of corrosion tolerance (low, moderate, high and extreme), which correspond to four permissible depths of corroded material, after which the component is considered to be completely failed. These correspond to four values for the three different Weibull distribution parameters, listed in Table 13:

Table 13: Corrosion tolerance limits

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_w )</td>
<td>4.43</td>
<td>4.75</td>
<td>4.13</td>
<td>4.18</td>
</tr>
<tr>
<td>( \eta_w ) [years]</td>
<td>11.13</td>
<td>12.19</td>
<td>17.22</td>
<td>22.06</td>
</tr>
<tr>
<td>( \gamma_w ) [years]</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Consequently, the hazard rate modelled by a Weibull distribution is equal to:

\[
h(t) = \begin{cases} 
\frac{\beta_w}{\eta_w} \left( \frac{t - \gamma_w}{\eta_w} \right)^{\beta_w-1} \exp \left( - \left( \frac{t - \gamma_w}{\eta_w} \right)^{\beta_w} \right), & \text{for } t > \gamma_w \\
0, & \text{for } t \leq \gamma_w 
\end{cases}
\]  

(188)

Based on the shape parameter \( \beta_w \) can be defined the bathtub function on the hazard rate, so that Region A is described by \( \beta_w < 1 \), Region B has a failure rate, not time-dependent with a \( \beta_w = 1 \) and the wear-out region characterized by a \( \beta_w > 1 \).

The integral of the probability density function, \( f(t) \) results to be the cumulative density function, \( F(t) \) fitted also to a Weibull distribution:

\[
F(t) = \int_0^t f(t) \, dt = 1 - \exp \left( - \left( \frac{t - \gamma_w}{\eta_w} \right)^{\beta_w-1} \right)
\]  

(189)

As also for the probability density function (pdf), if \( t \leq \gamma_w \), the value for the cumulative density function (cdf) is 0.
The reliability function (or survival function) \( R(t) \) is strictly related to the cdf, being the probability that a component will fulfill its function in the unit of time (for this model it has been chosen one year). The reliability function is defined as:

\[
R(t) = \int_{t}^{\infty} f(t) \, dt = 1 - F(t) \tag{190}
\]

For the different corrosion tolerances has been plotted the reliability of corroded plates as shown in Figure 7-2. It has been estimated that the preventive and corrective maintenance has a big influence over the total costs of a ship, responsible for 25% up to 35% of the operating costs (OPEX) [38], depending on factors such as the shipyard, geographical location of the ship, operational profile and more [4].

![Figure 7-2: Reliability of corroded plates](image)

In the calculation of the total costs of the model it has been included the time of stoppage due to preventive and corrective maintenance, being possible in this way to make assessments about the availability of the component or of the whole system, in this case the vessel.

In accordance to Dod [39], Kececioglu [40] and Langford [41], have been applied procedures to estimate the probability and severity of a failure, being possible to make assessments about different configurations. Methods that have been applied are based on the risk priority numbers, occurrence/severity matrix, risk ranking tables and criticality analysis [4].

These methods have been applied over the following parameters to be able to estimate and classify them on four levels (values in Table 14):

- Corrosion tolerance, CT
- Consequence of preventive replacement, CPR
- Consequence of failure/corrective replacement, CFR
- Consequence of replacements accounting for the inspection interval, CII
- Time required to make a repair or replacement, CR

**Table 14: Tolerance/consequence**

<table>
<thead>
<tr>
<th></th>
<th>Low i=1</th>
<th>Moderate i=2</th>
<th>High i=3</th>
<th>Extreme i=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CT_i$</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>0.25</td>
</tr>
<tr>
<td>$CPR_i$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$CFR_i$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$CII_i$</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$CR_i$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Based on the presented values, costs and time needed have been calculated for the model with the following formulas:

\[
C_{pi} = \text{Cost } CT_i \cdot CPR_i \cdot \text{StructuralWeight} \quad (191)
\]

\[
C_{fi} = C_{pi} \cdot CFR_i \quad (192)
\]

\[
TP_i = \frac{To \cdot CPR_i}{1 - CT_i} \quad (193)
\]

\[
TF_i = TP_i \cdot CFR_i \quad (194)
\]

\[
TI_i = \frac{ToI \cdot CII}{1 - CT_i} \quad (195)
\]

\[
TR_i = \frac{ToR \cdot CR_i}{1 - CT_i} \quad (196)
\]

where $C_{pi}$ is the total cost of the replacement before failure, Cost is the total repair cost per ton of structural material, assumed to be 1000 USD/ton [4], StructuralWeight is the weight of the material of the structure, taken as calculated in chapter “Conceptual design”, $C_{fi}$ is the total cost of a replacement after failure, $TP_i$ is the preventive replacement interval, $TF_i$ is the corrective replacement interval, $TI_i$ the inspection interval, $TR_i$ the interval of repair/replacement, and $i$ is the tolerance or consequence level, with $1 \leq i \leq 4$. 
1.2 Optimal replacement interval with minimization of total costs

Maintenance policy and planning for commercial vessels is critical for the operational costs, because it aims to find the optimum for maintenance, avoiding unnecessary costs, and reduces drastically the probability of failure of a vessel. For this reason, the objective is to find the best maintenance policy to maximize the effectiveness of the system (analyzed under the point of view of costs, downtime or availability) over operation and maintenance cycle of the vessel [20].

It has to be taken into account that an optimal maintenance is based on preventive maintenance performed when the life-cycle of a component goes from Region B to Region C, so that intervals shall not be designed too short, as this would reduce the time when the component is in the normal operation conditions, and at the same time they shall not be planned too long, as the probability of failure and thus the costs of corrective maintenance would be higher [4].

A first maintenance policy that has been analyzed is based on finding the optimal interval between preventive maintenance operations, which only target is the optimum moment when the component shall be replaced, without taking into consideration the corrective maintenance, performed whenever needed. This optimization is calculated by minimizing the cost to replace one component summing the cost of preventive maintenance \( C_p \) and the cost of corrective maintenance \( C_f \) times the probability of failure before time \( t_p \), divided by the length of the chosen interval \( t_p \):

\[
C(t_p) = \frac{C_p + C_f \cdot H(t_p)}{t_p} \tag{197}
\]

\[
H(T) = \sum_{i=0}^{T-1} \left[ 1 + H(T - i - 1) \right] \int_{i}^{i+1} f(t) \, dt, \quad T \geq 1 \tag{198}
\]

Renewal theory approach has been adopted to calculate \( H(T) \) that is the expected number of failures in the time interval \((0,T)\), with \( H(0) = 0 \), which means that when a component has been implemented in a system, it is new and thus there are no pre-existing failures at time 0.

It has been calculated the total cost as a function of the years of the component’s life for the four different levels of corrosion tolerances and repair consequences (Figure 7-3) and it has been found the minimum for each combination between the corrosion tolerance and repair consequence levels, as shown in Table 15.
Table 15: Optimal preventive replacement interval in years

<table>
<thead>
<tr>
<th>Corrosion tolerance</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Moderate</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>High</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Extreme</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

Also for this maintenance policy it has been aimed the minimization of the total costs resulting from both preventive and corrective replacement, but instead of the interval, it has been taken into consideration the age of the analyzed component [4]. In this way, the interval between two maintenance tasks does not take into consideration just the scheduled preventive maintenance, but it takes account as well of the corrective maintenance, to consider the actual age of a component and to avoid to replace a component that is still in Region A or B.

Hence, the objective of this policy is to optimize the age of a component after which it shall be replaced, keeping the total cost per unit time deriving from maintenance as low as possible and taking into account at the same time the benefits of such maintenance. This is obtained by summing the cost of preventive maintenance times

1.3 Optimal replacement age with minimization of total costs

Figure 7-3: Costs as function of the preventive replacement intervals
the reliability, summed with the cost of corrective maintenance times the probability of having a failure, everything divided by the unit time, which is the maintenance interval \( t_p \) plus the time after which a corrective replacement would have to be performed, as follows:

\[
C(t_p) = \frac{t_p R(t_p) + C_f [1 - R(t_p)]}{t_p R(t_p) + \int_{-\infty}^{t_p} t f(t) \, dt}
\]

(199)

The total costs deriving from maintenance are shown as function of replacement ages in Figure 7-4, and the optima for every combination between corrosion tolerances and repair consequences are displayed in Table 16.

<table>
<thead>
<tr>
<th>Table 16: Optima for replacement ages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>Moderate</td>
</tr>
<tr>
<td>High</td>
</tr>
<tr>
<td>Extreme</td>
</tr>
</tbody>
</table>

Figure 7-4: Total cost per unit time as function of the replacement ages

65
1.4 Optimal replacement age accounting for time required to perform replacement

As a preventive and a corrective maintenance require time for the replacement, this could have effects on the costs and lead to a different distribution of these. For this reason, it has been considered a policy that calculates the optimal replacement age taking into account the time required to perform the replacement [4]. This has been achieved by minimizing the total costs per unit time deriving from maintenance, as, if it would have been minimized the downtime, it would have optimized not only the replacement age, but also the time required for the maintenance, leading to incorrect results. The total costs result by implementing in the formula the time to perform the preventive and corrective maintenance, as shown in the following formula:

\[
C(t_p) = \frac{C_p R(t_p) + C_f [1 - R(t_p)]}{(t_p + T_p) R(t_p) + [M(t_p) + T_f] [1 - R(t_p)]}
\]  

(200)

where \(T_p\) is time required to perform preventive maintenance, \(T_f\) the time required to perform corrective maintenance and \(M(t_p)\) the mean time to failure after a replacement done at moment \(t_p\).

To be able to compute such optimization, the mean time to failure (MTTF), which is the expected age of a component before it fails, has been defined as:

\[
M(t) = \int_{-\infty}^{t_p} \frac{t f(t) dt}{1 - R(t)}
\]  

(201)

In this way, it was possible to display the function of total cost per unit time related to the replacement age in Figure 7-5, and to show the optima for every function in Table 17.

Table 17: Optimal preventive replacement time age accounting for the time required to perform replacement

<table>
<thead>
<tr>
<th>Corrosion tolerance</th>
<th>Repair consequence</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td></td>
<td>11</td>
<td>11</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Moderate</td>
<td></td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>High</td>
<td></td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Extreme</td>
<td></td>
<td>18</td>
<td>17</td>
<td>16</td>
<td>15</td>
</tr>
</tbody>
</table>
1.5 Optimal replacement interval with minimization of downtime

Another way to consider the problem is analyzing and minimizing, instead of the total costs, the downtime per unit time, as this can give a measure of the time needed to replace a component [4]. It has been minimized the downtime $D(t_p)$ by summing the time required to perform a corrective replacement times the hazard rate and the time required for the preventive maintenance, divided by the unit time $t_p + T_p$, as follows:

$$D(t_p) = \frac{H(t_p) T_r + T_p}{t_p + T_p} \quad (202)$$

It has been analyzed the relation between replacement intervals and downtime for the different levels of corrosion tolerance and repair consequence as done in the previous chapters, in order to understand this behavior (Figure 7-6) and find the minima for each of the combinations between corrosion tolerance and repair consequence Table 18.

Table 18: Minimization of the downtime by optimizing the replacement interval

<table>
<thead>
<tr>
<th>Corrosion tolerance</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Moderate</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>High</td>
<td>11</td>
<td>10</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Extreme</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>
1.6 Optimal replacement age with minimization of downtime

A similar approach has been applied to find out the optimal replacement age $t_p$ by minimizing the downtime per unit time [4]. This results by dividing the downtime in a cycle by the cycle length, where the downtime in a cycle is the multiplication of the time needed for a preventive replacement time the reliability of the component, plus the multiplication of the time needed for a corrective replacement times the probability of failure of the same component, as shown in the following formula:

$$D(t_p) = \frac{T_p R(t_p) + T_f [1 - R(t_p)]}{(t_p + T_p) R(t_p) + [M(t_p) + T_f] [1 - R(t_p)]} \quad (203)$$

The downtime as a function of the replacement age for all the combinations between the four levels of corrosion tolerance and repair consequence is shown in Figure 7-7 and its optima are displayed in Table 19.

Table 19: Minimization of the downtime by optimizing the replacement age

<table>
<thead>
<tr>
<th>Repair consequence</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>11</td>
<td>11</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
1.7 Optimal inspection interval with maximization of availability

Another optimization policy is to maximize the availability of the structure, by considering the intervals between inspections. As the corrosion development with a component delivers just estimations based on stochastic data and does not provide the actual state of it, it is necessary to plan inspections to measure the state of the structure. On this concept is based the optimal policy described in this chapter, considering the component after the inspection in as-new condition, as either the inspection reveals a failure, and the component has to be replaced, or shows that there are no major failures, hence no replacement is needed [4].

In this way, the inspection increases the reliability of the structure, so that the main goal is to identify the optimal so-called failure-finding interval $t_i$, which is the optimal interval between two inspections by maximizing the availability.

Availability $A(t)$ is defined as the most important measure for a system effectiveness and gives the probability that the component is ready to be used at any time in the interval $(0, t_i)$. It depends on the probability of failure,
the analyzed component has while it is used by the system and by the time and resources needed to bring the component back to an operational state [20], and the long-run availability is defined as:

\[ A = \frac{E(U)}{E(U) + E(D)} \]  \hfill (204)

This means that the total availability of a system (or component) is equal to the expected uptime \( E(U) \) divided by the total time considered \( E(U) + E(D) \), where the second term is the expected downtime of the system. Following this definition, it has been applied over the analyzed system, where \( t_i \) is the interval between inspections (i.e. the moment when the inspection is performed), \( T_i \) is the time needed for the inspection and \( T_r \) is the time to repair or replace the component, as shown in the following formula:

\[ A(t_i) = \frac{t_i R(t_i) + \int_{t_i}^{\infty} t f(t) \, dt}{(t_i + T_i) R(t_i) + (t_i + T_i + T_r) [1 - R(t_i)]} \]  \hfill (205)

The objective was to obtain the optimal interval between inspections so that the availability could have been maximized, by taking into account the costs deriving from the inspection and relating these to the benefits that the inspection gives.

Availability per unit time as a function of the inspection intervals for every combination of corrosion tolerance and repair consequence is displayed in Figure 7-8 and the optima (maxima) for each of the functions are listed in Table 20.

**Table 20: Optimal inspection intervals with maximized availability**

<table>
<thead>
<tr>
<th>Corrosion tolerance</th>
<th>Repair consequence</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Moderate</td>
<td>11</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Extreme</td>
<td>16</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
Figure 7-8: Availability per unit time as function of the inspection interval
8 Conclusions

The developed tool has fulfilled the objectives of the work, delivering a functionality that in an easy and fast way can estimate optimal main dimensions, scantlings and maintenance strategy for a multi-purpose vessel. The first task consists in calculating ship main dimensions based on rules by Classification Societies, estimations, regression analysis from existing vessels and approximations, obtained by previously developed models for conceptual design and implemented to deliver a consistent and reliable first estimation. The main dimensions calculated are obtained by optimizing through an iterative algorithm the objective function, which has been chosen to be the Required Freight Rate (RFR), as this variable sum most of the ship’s specifications and takes into account CAPEX, OPEX and number of TEU transported, allowing the ship owner to decide the amount of TEU to be shipped and to analyze if the proposed model is profitable. On the tool are imposed constraints regarding ship stability, physical restrictions (e.g. the Panama Canal) and voyage requirements.

Further, these constraints and results from formulas linked to the design values shall be compared to existing multi-purpose vessels, to validate the model and in case to correct some estimations done. Also, the code can be implemented with the insertion of more data by the user, in order to consider more requirements imposed by the ship owner regarding operational life of the ship. Another improvement to the suggested tool is to consider different computational formulas to estimate and analyze the proposed main dimensions, as e.g. to estimate the structural weight there have been developed a wide number of models, based on regression analysis, midship section modulus and more. By the results it has been noted the number of TEU that can be loaded on the ship model is bigger than the amount that a multi-purpose ship normally carries, as this type of vessel is loaded also with other type of cargo.

The second task is the optimization of the scantlings based on a risk analysis, after that the ultimate bending moment has been calculated through the software MARS2000 implementing the main dimensions of the model in a typical midship section of a multipurpose ship. The reliability of the model has been calculated applying the First Order Reliability Method (FORM) on the limit state function, being the boundary between safe and fail condition estimated to be linear [15, 3]. The variables that were considered in the limit state function are the wave-induced, still water and ultimate bending moments and the respective model uncertainty factors, and each has been described by the most suitable statistical distribution. This process has been calculated over scantlings in a range from $-6\text{mm}$ to $+10\text{mm}$ from the original with an interval of $2\text{mm}$, obtaining the respective reliability index ($\beta$-index) and thus the probability of failure. The different probabilities of failure have been used to calculate the risk-associated costs, which, summed with the cost of modifying the structure by the Design Modification Factor (DMF), resulted in the total costs associated to each DMF, being in this way possible to find out the solution that presents the lowest economical risk.

With this tool can be checked if another reliability-based method in substitution to FORM can be adopted, in order to link, make dynamically interactive and faster the processes to calculate the $\beta$-index and the risk-related costs. In addition to this, it is interesting to check if modifying the scantlings of the whole midship section would lead to more accurate values.
The third task is based on defining the optimal maintenance strategy based on minimizing costs, downtime or maximizing availability, calculating the optimal replacement interval, the optimal replacement age and the optimal replacement age accounting for time required to perform replacement. In this model have been considered four levels of corrosion tolerance combined with four levels of repair consequence, in order to take into account material behavior and variable factors.
Bibliography


