Hydrological-economic risk analysis in small hydropower schemes based on synthetic series of daily flows

Daniel Filipe Henriques Violante

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Supervisor: Prof. Dr. Maria Manuela Portela Correia dos Santos Ramos da Silva

Examination committee

Chairperson: Prof. Dr. Rodrigo de Almada Cardoso Proença de Oliveira

Supervisor: Prof. Dr. Maria Manuela Portela Correia dos Santos Ramos da Silva

Member of the committee: Helena Margarida Machado da Silva Ramos

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Abstract

This study aims to perform a risk analysis concerning the energy production in run-of-river small hydropower schemes (SHS) by evaluating the effect of the natural variability of the flow regime in their expected incomes.

To accomplish this objective, a procedure to generate annual and daily synthetic streamflow series was applied to a data set of 14 Portuguese river daily streamflow samples.

This procedure encompasses a probabilistic model, namely the log-Pearson III distribution, for generating values at the annual level and a disaggregation model, namely the method of fragments, to disaggregate these values into the daily level. Based on the historical samples of mean daily flows, the previous procedure was used to generate $M = 5000$ synthetic daily flow series with lengths (N) equal to the ones of the historical samples.

For different design discharges and for the global period, a methodology to simulate the daily exploitation of run-of-river SHSs and obtain the daily and annual energy productions and its respective incomes was applied to both the synthetic (M) and historical (1) series. By applying economic criteria to the $(M + 1)$ annual revenues, its cumulative present value is determined. The resulting collection of revenues were statistically analyzed using the Pearson-III distribution in order to estimate the SHS revenues as non-exceedance probabilities.

As a result, it was concluded that the usual design of SHS based on average conditions, this is, based on a reference revenue, corresponds to a suitable design as the referred revenue represents a good estimate of the expected income in SHS.

Keywords: hydrological risk; flow regime temporal variability; small hydropower scheme; synthetic series; disaggregation model.

Resumo

O presente estudo visa a análise do risco associado à produção de energia em pequenas centrais hidroelétricas (PCHs) com exploração a fio de água, avaliando o efeito da variabilidade temporal natural do regime hidrológico nas suas receitas esperadas.

Para tal, recorreu-se à geração de séries sintéticas de escoamentos anuais e diários em 14 secções da rede hidrométrica nacional.

Este procedimento engloba um modelo probabilístico (lei log-Pearson III) para gerar valores a nível anual e um modelo de desagregação (método dos fragmentos) desses escoamentos em escoamentos diários. Em cada um dos locais geraram-se $M = 5000$ séries sintéticas, cada uma com dimensão N, igual à da correspondente série histórica.

No pressuposto de exploração a fio de água puro, de diferentes valores do caudal de dimensionamento e de período de licenciamento coincidente com o período global de registos, procedeu-se à simulação da exploração diária de cada hipotética PCH, tendo por base as séries sintéticas (M) e histórica (1) de caudais médios diários, com estimativa das correspondentes produções diárias de energia e receitas anuais. Através da aplicação de critérios económicos às $(M + 1)$ receitas anuais, obteve-se o seu valor acumulado atualizado. Tais receitas foram, seguidamente, adimensionalizadas por consideração de uma receita de referência e analisadas estatisticamente, utilizando a distribuição Pearson III, a fim de as associar a probabilidades de não-excedência.

Como resultado, conclui-se que o dimensionamento de PCHs com base no critério de projeto mais frequentemente utilizado, ou seja, com base na receita de referência, constitui um bom estimador das receitas esperadas.

Palavras chave: risco hidrológico; variabilidade temporal hidrológica; pequeno aproveitamento hidroelétrico; série sintética; modelo de desagregação.

Contents

List of Figures

List of Tables

List of abbreviations

English

AR	Autoregressive model
ARMA	Autoregressive-moving average model
CPV	Cumulative present value
EU	European Union
GHG	Greenhouse gas
HP	Hydropower
HS	Hydropower scheme
IDF	Inverse distribution function
PAR	Periodic autoregressive model
PARMA	Periodic autoregressive-moving average model
PRNG	Pseudorandom number generator
RSD	Relative standard deviation
SHS	Small hydropower scheme
UNIDO	United Nations Industrial Development Organization

Portuguese

APA	Agência Portuguesa do Ambiente
DL	Decreto-lei (decree-law)
PCH	Pequena central hidroelétrica (small hydropower scheme)
SNIRH	Sistema Nacional de Informação de Recursos Hídricos

List of symbols and notation

Uppercase latin

$\boldsymbol{C}$	Generic monetary annuity
$\boldsymbol{C}$	Number of fragments classes
$\boldsymbol{C_i}$	Generic monetary flux
$\boldsymbol{C_j}$	Class of fragments number j
$\boldsymbol{C_v}$	Coefficient of variability or relative standard deviation
$\boldsymbol{E_{med}}$	Annual average energy production
$\boldsymbol{E_{ref}}$	Reference energy production
$\boldsymbol{E_s^{(k)}}$	Annual energy production for a given daily flow series
$\boldsymbol{H}$	Water net head
$\boldsymbol{\overline{H}}$	Mean annual flow depth
$\boldsymbol{K_k = K_P}$	Probability factor of the Pearson III distribution
$\boldsymbol{M}$	Number of generated synthetic series
$\boldsymbol{N}$	Number of year of the historical series / global period of records
$\boldsymbol{N^*}$	Concession/licensing period
$\boldsymbol{P}$	Hydropower scheme installed capacity
$\boldsymbol{PV}$	Present value or worth factor
$\boldsymbol{Q}$	Flow discharge
$\boldsymbol{Q_{ave} = Q_{mod}}$	Average flow or modulus flow
$\boldsymbol{Q_{i,k}}$	Daily streamflow
$\boldsymbol{Q_{max}}$	Design or maximum discharge/Maximum flow that can be turbinated
$\boldsymbol{Q_{min}}$	Minimum flow that assures turbines operation
$\boldsymbol{Q_{ref}}$	Cumulative reference turbinated volume
$\boldsymbol{Q_{res}}$	Reserved or ecological flow
$\boldsymbol{Q_s^{(k)}}$	Cumulative annual turbinated volume pertaining a given daily flow series
$\boldsymbol{Q1_{ref}}$	Cumulative reference turbinated volume when the daily flows are smaller than Q_{max}
$\boldsymbol{Q1_s^{(k)}}$	Cumulative annual turbinated volume when the daily flows of a given series are smaller than Q_{max}
$\boldsymbol{Q2_{ref}}$	Cumulative reference turbinated volume when the daily flows are equal or bigger than Q_{max}
$\boldsymbol{Q2_s^{(k)}}$	Cumulative annual turbinated volume when the daily flows of a given series are equal or bigger than Q_{max}
$\boldsymbol{R}$	Annual revenue from the sale of energy

$\boldsymbol{R_i}$	Energy revenue in year i
$\boldsymbol{R_{med}}$	Annual average revenue
$\boldsymbol{R_{ref}}$	Reference revenue accumulated for N years
$\boldsymbol{R^*_{ref}}$	Reference revenue accumulated for N^* years
$\boldsymbol{R^{PV}}$	Cumulative present value of the energy revenue
$\boldsymbol{R^{PV}_s}$	Cumulative present value of the reference energy revenue of a given daily flow series
$\boldsymbol{R^{(k)}_s}$	Annual revenue of a given daily flow series
$\boldsymbol{V_{med}}$	Average annual turbinated volume
$\boldsymbol{X}$	Random variable
$\boldsymbol{\overline{X}}$	Mean of random variable X
$\boldsymbol{\widehat{X_j}}$	Annual flow pertaining the non-exceedance probability of fragment class j
$\boldsymbol{X_k}$	Annual streamflow
$\boldsymbol{X_{k,i}}$	Daily streamflow
$\boldsymbol{X^{ss}_k}$	Synthetic annual streamflow
$\boldsymbol{X^{ss}_{k,i}}$	Synthetic daily streamflow
$\boldsymbol{\overline{Y}}$	Mean of the natural logarithm of the annual streamflows
$\boldsymbol{Y_k}$	Natural logarithm of the annual streamflow
$\boldsymbol{Y^{ss}_k}$	Natural logarithm of the synthetic annual streamflow

Lowercase latin

$\boldsymbol{c}$	Energy selling price
$\boldsymbol{c}$	Constant to avoid null flows
$\boldsymbol{c.c.}$	Correlation coefficient
$\boldsymbol{f_k}$	Fragment
$\boldsymbol{g_x}$	Skewness coefficient of sample x
$\boldsymbol{g_y}$	Skewness coefficient of the natural logarithm of the annual streamflow
$\boldsymbol{s}$	Standard deviation
$\boldsymbol{s}$	Order of the daily flow series
$\boldsymbol{ss}$	Order of the synthetic flow series
$\boldsymbol{s_y}$	Standard deviation of the natural logarithm of the annual streamflow
$\boldsymbol{s_{\theta_{ss}}}$	Standard deviation of a generic statistic of the synthetic series
$\boldsymbol{t}$	Discount rate
$\boldsymbol{x_i}$	Values of a random variable
$\boldsymbol{z}$	Normally distributed random variable
$\boldsymbol{z_k}$	k^{th} order of the normally distributed random variable z

$\boldsymbol{z}_{\left(\mathbf{1}-\frac{\boldsymbol{\alpha}}{\mathbf{2}}\right)}$	$\left(1-\frac{\alpha}{2}\right)$ quantile of the Normal standard distribution

Lowercase greek

$\boldsymbol{\alpha}$	Significance level
$\boldsymbol{\gamma}$	Water unit weight
$\boldsymbol{\varepsilon_t}$	$N(0,1)$ random variable
$\boldsymbol{\eta}$	Powerhouse average global efficiency
$\boldsymbol{\theta}$	Statistic of the sample series
$\boldsymbol{\theta_{ss}}$	Generic statistic of the synthetic series
$\overline{\boldsymbol{\theta_{ss}}}$	Mean of a generic statistic of the synthetic series
$\boldsymbol{\theta_{ss}^i}$	Statistic of the i^{th} synthetic series
$\boldsymbol{\mu}$	Mean
$\boldsymbol{\mu_x}$	Mean of sample x
$\boldsymbol{\xi}$	Natural random number between 1 and the number of fragments inside a given class of fragments
$\boldsymbol{\xi_t}$	$N(0,1-\phi^2)$ random variable
$\boldsymbol{\sigma}$	Standard deviation
$\boldsymbol{\sigma_x}$	Standard deviation of sample x
$\boldsymbol{\sigma_x^2}$	Variance of sample x
$\boldsymbol{\phi}$	Parameter

Others

$(\mathbf{1}-\boldsymbol{\alpha})$	Confidence level
$[\boldsymbol{f}]$	Array of fragments

1. Introduction

1.1. General framework

In our days, the demand for energy, particularly electricity, is rapidly increasing. Consequently, water demand, as a green way to produce electricity, is growing significantly[1]. Due to their small construction time and exploration costs, but especially due to their easier integration in the local ecosystems, run-of-river small hydropower schemes (SHSs) represent a good alternative to schemes based on large dams and reservoirs aiming at regulating the river flows[2].

As a run-of-river small hydropower scheme does not have storage capacity, it only produces energy when the natural river discharges allow the turbine to operate. Its design is usually made assuming a constant annual turbinated volume, equal to the average of the annual turbinated volumes provided by the available historical series of river flow discharges. However, the temporal variability that characterizes such series is not compatible with the assumption of a constant annual turbinated volume but instead with yearly variable volumes, and consequently with yearly variable energy productions and incomes. When applying economic analysis criteria to variable incomes, the corresponding cumulative present value necessarily differs from the one based on the assumption of a constant annual turbinated volume. For this reason, one of the major issues when designing SHSs is the uncertainty that results for not considering the natural temporal variability of the river discharges and its impact on the expected revenues[3–5].

To understand the relevance and the effect of the natural temporal variability of the river regime in the profitability of small hydropower schemes there were performed some studies[3,4,6–8], based on different approaches that aimed at extracting as much information as possible from the available samples of river discharges.

Due to the enormous potential for the construction of new SHSs, it was considered relevant to continue the previous studies by applying a recently developed approach that utilizes information that goes beyond the available samples to address the effect of the temporal variability of the flow regime, namely based on synthetic daily flow series.

Under the assumption of hydrological stationarity, synthetic daily flow series can be seen as alternative flow series with the same probability of occurrence and, expectably, with statistical characteristics similar to the observed/historical ones[9].

The generation of synthetic series can be made concurrently at different time levels by using a disaggregation technique. This technique assumes a combination of two models: a first model for generating values at a given time level, for example year, and a second one for disaggregating these values into a lower time level, for example month or day, preserving the main statistics of the samples, such as the mean, standard deviation and skewness coefficient at both time levels[9,10].

1.2. Aims of study

Taking into account the previous studies that proved the good results yielded by the method of the fragments to generate synthetic flow series, the present study aimed at using such series to perform a risk analysis in small hydropower schemes by assessing the effect of the natural flow regime variability in the expected incomes. For this purpose, 14 hypothetical case studies located at river gauging stations with long daily discharge records were adopted.

In order to accomplish the previous objective, a methodology to simulate the daily exploitation of run-of-river small hydropower schemes was applied to the synthetic streamflow data. Therefore, the energy productions and consequently the revenues for different design discharges were obtained.

The procedure implemented to generate the synthetic flow data combined a probabilistic generation model at the annual level, namely the log-Pearson III distribution, and a disaggregation model directly from that level to the daily level, namely the method of fragments, as mentioned before.

By considering different design discharges in each case study, a large number of synthetic daily flow series were generated and utilized to compute the cumulative present values of the expected revenues based on the assumption of a licensing period equal to the length of the historical sample to which the case study refers.

By applying a statistical model, namely, the Pearson III distribution, non-exceedance probabilities were assigned to those revenues aiming at expressing their plausibility. Furthermore, the aforementioned simulation was also applied for two other periods smaller than the global period of records, specifically ten and twenty-five years, to evaluate the susceptibility of SHSs to the temporal variability of the flow regime for different periods.

1.3. Structure

The present document is organized in six chapters. In chapter 1 a brief introduction is made focusing on the scope and objectives of the study carried out. The structure of the document is also presented.

Chapter 2 introduces the main concepts associated with small hydropower schemes and presents the theoretical background related to the evaluation of their expected revenues. The concept of synthetic series and disaggregation models is also defined and explained.

In chapter 3 the data set used in this study is presented and characterized.

Chapter 4 describes the proposed methodology for the generation of annual and daily synthetic series and the algorithm to simulate the daily exploitation of a run-of-river SHS.

Chapter 5 presents the results obtained in each case study. It starts by showing the good results of the disaggregation model, based on its capacity to preserve the main statistics of the samples, followed by the presentation of the revenues yielded by each hypothetical SHS for different periods.

Finally, chapter 6 states a synthesis of the conclusions and emphasis the importance of considering synthetic series as alternative scenarios when analyzing the profitability of SHSs. Lastly, the future perspectives are presented.

2. Background

Greenhouse emissions are responsible for trapping the heat in the atmosphere and, therefore, contributing for increasing Earth's temperature. The most important greenhouse gas is carbon dioxide (CO_2), produced mainly by transports and electricity production[11].

Faced with an accelerated increase of these gases, European Union (EU) countries have made a commitment to reduce their emission to values 20% below 1990 levels by 2020 and 40% below by 2030. Additionally, they would have to guarantee that, at least, 20% of their energy comes from renewable sources by 2020[12], a value that increases to 27% ten years later[13]. To accomplish this reduction, EU countries will have to promote energy efficiency and increase the investment on renewable energy sources[2].

Hydropower (HP) is the name given to the energy produced by hydropower schemes (HS). This schemes are the most important renewable source used to generate electricity, having represented 12% of all EU electricity production in 2015 (Figure 1) [14].

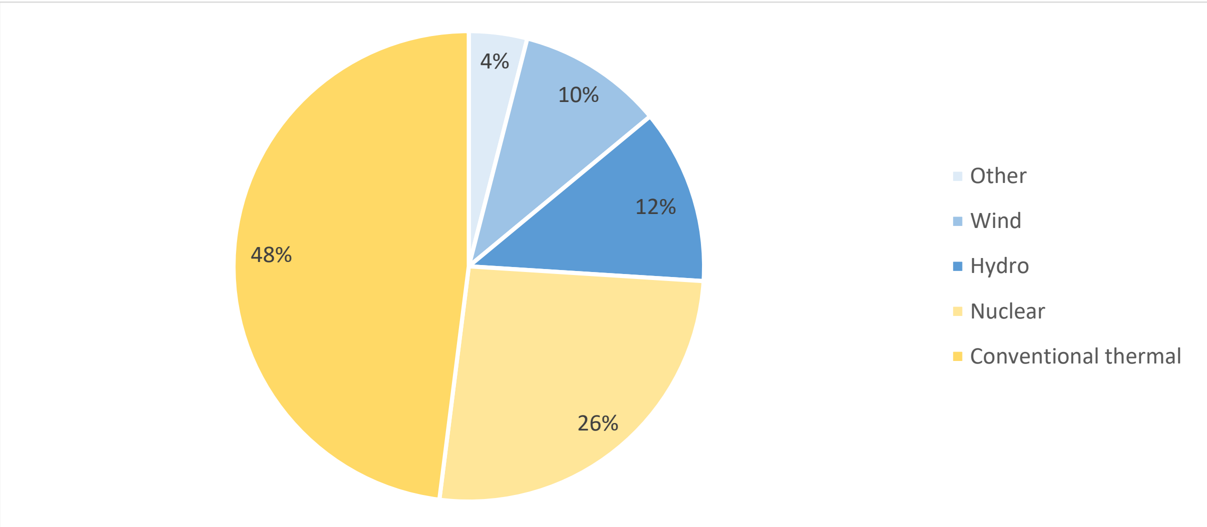

Figure 1: EU 28 electricity production by source in 2015 (adapted [14]).

2.1. Hydropower schemes

A hydropower scheme is used to convert potential energy from water - flowing in a determined stream and with a certain topographic fall (also defined as "head") - into electric energy by making a turbine or a group of turbines turn[2,15]. The electric energy generated is proportional to the water discharge multiplied by the head[15].

Hydropower schemes can be classified according to some parameters, such as their installed capacity, head and location of the powerhouse.

According to their installed capacity, these systems can be classified as large, medium, small, mini and micro. However, these definitions vary among the countries[16]. Usually there are no defined limits for a medium or large scheme and, in most cases, not even a distinction between these two terms. Given that, it is considered as a medium/large hydropower scheme all that exceed the installed capacity defined as the limit for a small hydropower by each country. Most countries establish this limit in 10 MW of installed capacity, with some exceptions like Sweden that considers a lower limit of just 1.5 MW and countries like Canada and China that go up to 50 MW. For the majority of countries, a small hydropower scheme (SHS) can also be divided in mini and micro if it has an installed capacity lower than 2 MW and 0.5 MW, respectively[15–19].

According to the head, hydropower schemes can be defined as high head (100m or above), medium head (30 – 100 m) and small head (2 – 30 m)[2].

In what concerns the location of the powerhouse and the storage capacity, they can be classified as schemes with the powerhouse located at the foot of the dam (creating a reservoir with or without storage capacity) or as run-of-river schemes (without storage capacity and having the powerhouse located or not at the foot of the dam). The first type is more expensive, so usually, only bigger hydropower plants are associated to dams. These are built to increase the head, thus producing more energy, but mainly, to create artificial reservoirs which regulate the natural stream flows allowing the energy production even during dry periods (Figure 2a). In some cases, where the dam has already been built for another purposes, like irrigation or flood control, it can be possible to install there a powerhouse to produce electricity[2,19].

In the most common layout of the second type, the water is diverted from the river to a pressure intake or forebay, by a low slope canal, and then to the powerhouse, by a pressurized penstock (Figure 2b). Despite its lower construction cost, they have the particularity of generating electricity only when the water is available and provided by the river. When the discharge falls below a certain limit, called the technical or minimum flow, the generation stops. Consequently, the energy generation is not ensured during all year, especially in rivers with a high temporal flow variability, making this the greatest disadvantage of the run-of-river schemes[2,16]. In some cases, this type of schemes are built downstream bigger dams with storage capacity thus benefiting from the flow control upstream[15].

Due to its particularities, small hydropower schemes will be better discussed in the next chapter.

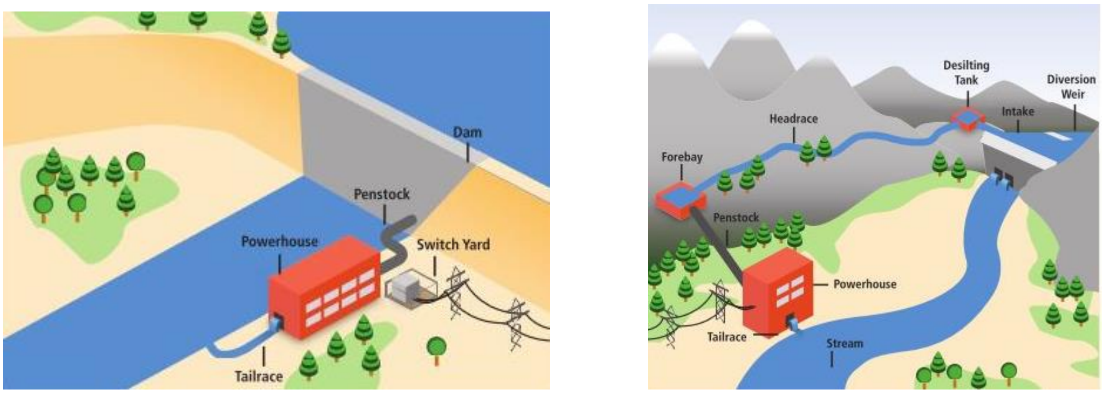

Figure 2: a) scheme with a powerhouse located at the foot of a dam; b) run-of-river scheme[19] .

2.2. Run-of-river small hydropower schemes

Run-of-river small hydropower schemes are a good alternative to the well-known large hydropower facilities with big dams, mainly because they have lower costs and can be easily integrated into the local ecosystems[16]. Like the majority of countries, in Portugal, the maximum installed capacity of a hydropower scheme to be considered as small is 10 MW. This definition is very important because, for example, small hydropower systems are generally covered by specific support policies that can dictate the success or failure of the investment[2,15].

In Portugal, small hydropower schemes have a major role in the generation of electricity. In 2013 there were 157 small hydropower plants totalizing an installed capacity of 450 MW. According to United Nations Industrial Development Organization (UNIDO), by 2020, the aim is to have around 250 plants and an installed capacity of 750 MW[18,20]. At a world level, the same organization estimates that there is an installed capacity of 75 GW with a potential of 173 GW and, at a European level, an estimated installed capacity close to 18 GW and a potential of near 27 GW[18]. As it can be seen in Figure 3, the potential of small hydropower has not yet been exhausted, leaving space for the constructions of new plants[18].

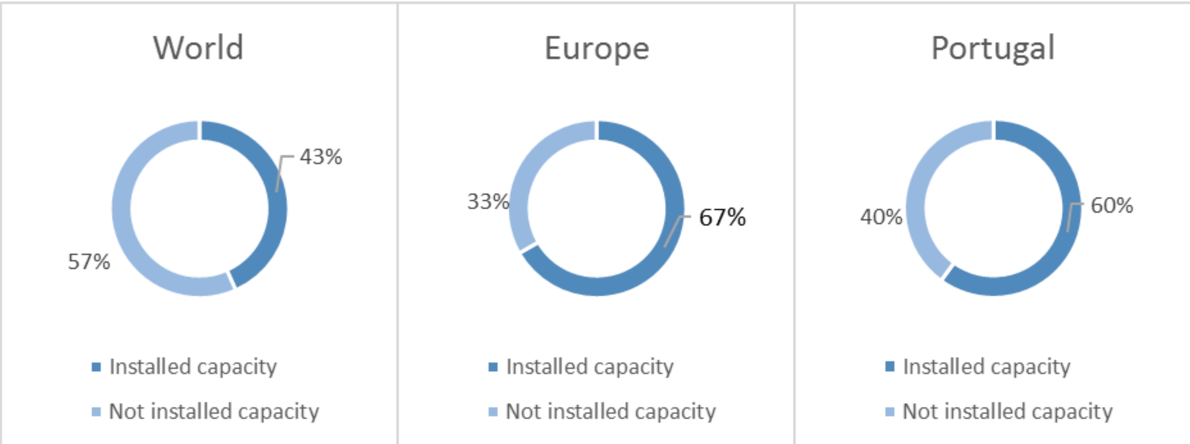

Figure 3: Percentage of installed small hydropower capacity in the world, Europe and Portugal, according to UNIDO (adapted[18]).

2.2.1. Advantages and disadvantages

It is easy to understand that one of the major advantages either of the big or of the small hydropower schemes is not producing greenhouse gases (GHG) like CO_2, nor other types of pollutants, although there may occur some GHG emissions during its construction. However, small hydropower schemes have some negative impacts on ecosystems and biodiversity such as the modification of the landscape and habitat, and river connectivity destruction, especially between the diversion weir, where the water is diverted, and the tailrace, where the water is discharged back to the river[5]. In any case, as small hydropower plants are mainly run-of-river schemes, their environmental impacts, namely those related to the river flow regime, are much smaller than large scale hydraulic infrastructures[15].

They also contribute to reduce the external energy dependency, especially in countries that do not have fossil fuels, like Portugal. In 2014, this country imported around 74% of its energy consumption[16,5].

Another advantage of small hydropower schemes compared to energy production based on fossil fuels and on larger hydropower plants is that they can be installed in off-grid rural areas, by public or private entities allowing them to deliver energy to small communities and rural residents with a much lower cost[16].

Despite being cheaper to explore, they are expensive to build. Nowadays with the investments made in the research of new solutions, like more efficient turbines, replacement of steel penstock pipes by plastic ones and lower cost of speed and load systems, that situation is changing. In any case, the investment is still higher than the one in fossil fuels systems with equivalent capacity, although their low operation/maintenance costs and long life associated with renewable energy incentives could make them a profitable investment[16].

One of the greatest challenges of small hydropower schemes is to establish a sustainable project, ensuring that all the impacts are assessed and mitigated in order to take fully advantage of their tremendous potential[15]. However, their planning and design have some complexities that should be discussed and analyzed.

2.2.2. Design of small hydropower schemes

Achieving the final design of a small hydropower plant is a complex process, as the different technical alternatives need to attend social, economic and environmental requirements[2].

It is easy to understand that the main component of these systems is the water, so its characteristics must be verified and a precise preliminary characterization must be done prior any further and more detailed studies. Consequently, topographic surveys and hydrological studies to identify the gross head and, not only the water availability, but also their temporal variability along the year and among years, and, consequently, the energy production should be performed. The definition of the available head can utilize topographic maps and field visits, while a general, albeit precise, appraisal of the water availability and variability can be assessed based on mean daily flow duration curves. These curves give the number of days during which a certain flow, Q, is achieved or exceeded, as represented in Figure 4. They are frequently made dimensionless by dividing the daily flows by the average annual or multiannual flow, Q_{ave} (also called modulus flow, Q_{mod}). In order to guarantee the representativeness of the hydrological regime, this curve should refer to a sufficiently long period[2,3,21].

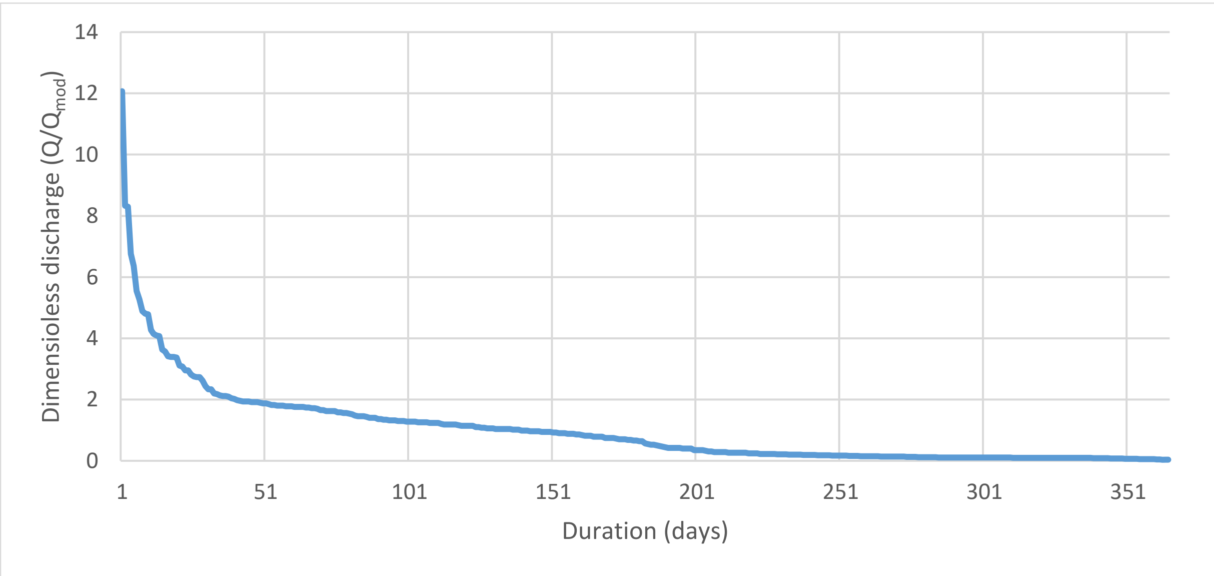

Figure 4: Example of a mean daily flow duration curve.

The installed capacity in the hydro plant, $P\ (kW)$, can be calculated by:

$$P = \gamma H Q \eta \qquad [1]$$

where $\gamma\ (kN/m^3)$ represents the water unit weight, $H\ (m)$ represents the net head, $Q\ (m^3/s)$ represents the discharge and η represents the power plant average global efficiency.

Knowing the previous parameters, the turbines can be selected. Due to their elevated price, that can reach 50% of a small hydropower plant total cost, they have to be carefully selected, in order to obtain the best efficiency. The turbine efficiency depends on the characteristics of the turbines, but mostly on the flow values which should be in the higher part of the turbine efficiency curve in order to maximize its performance (Figure 5)[3,22].

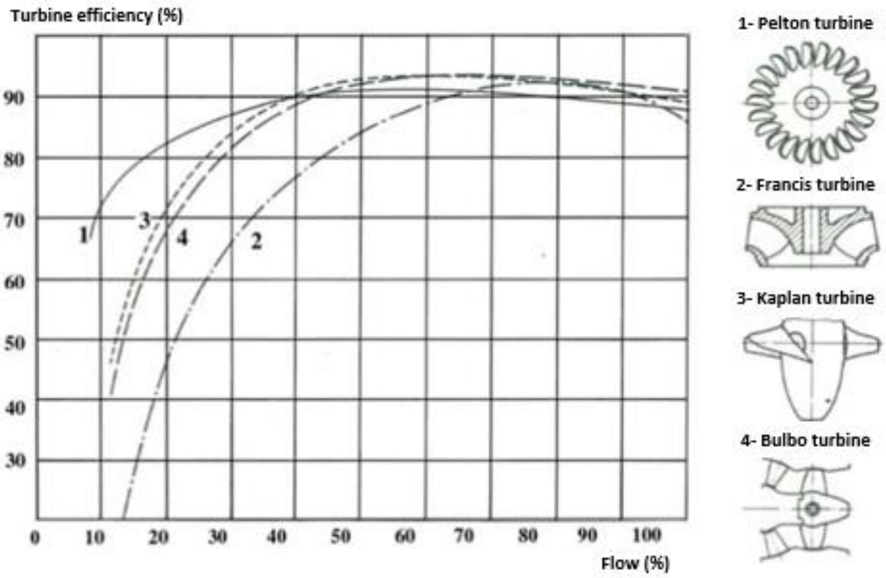

Figure 5: Efficiency of different turbine types (adapted[22]).

The design of a run-of-river small hydropower scheme can be seen graphically in Figure 6. The blue and green areas represent the average annual turbinated volume, $V_{med}\ (m)$, this is, the sum of the daily flows that are equal or smaller than the design or maximum discharge, Q_{max}, plus the reserved discharge, Q_{res}, ($Q_{max} + Q_{res}$), subtracted by the reserved volume (presented in yellow in the same figure). This volume represents the minimum amount of water that needs to be available for priority uses if a constant reserved discharge, Q_{res}, is assumed. This volume will be discussed in more detail further. Naturally, in run-of-river schemes, this flow cannot be assured when the natural river discharges are smaller than Q_{res}. The pondage volume (presented in green) represents the volume during the period of time where the turbinated flow is equal to Q_{min}, that is the minimum flow that assures the turbines operation, which according to Figure 5 is comprehended between 10% and 20 % of the maximum flow[3,22].

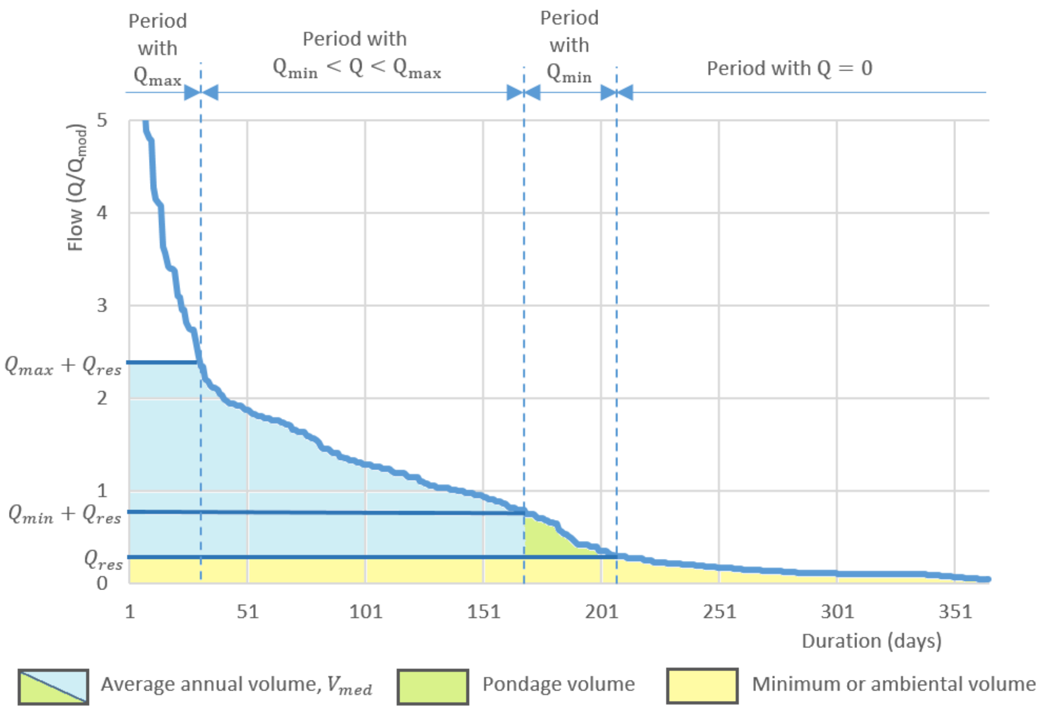

Figure 6: Energy production in a small hydropower scheme from a mean daily flow duration curve (adapted from [22]).

The annual energy production, E_{med} (GWh), for a given year, can be determined according to equation [2]:

$$E_{med} = \frac{\frac{V_{med} H}{3600}}{9.81\eta} \qquad [2]$$

being V_{med} (m) the average annual volume, H (m) the water net head and η the powerhouse global efficiency.

Therefore, the annual revenue, R $(m.u.^*)$, also for a given year, is obtained by multiplying equation [2] by an energy selling price, c $(GWh/m.u.)$, as it can be seen in equation [3][3,4,22]:

$$R = E \times c \qquad [3]$$

Once estimated the annual energy production, the total energy income during the lifetime of a small hydropower plant can be determined by performing an economic analysis. One of its main problems is the difficulty to estimate the evolution of the inflation rate during the project lifetime and its effect on the total revenue of the scheme[21].

To overcome this problem, a simple economic approach based on constant market prices referred to the first year of the plant exploitation phase, also called year 0, can be made. This approach has the particularity of not accounting the inflation, by assuming that it will have the same effect in the different monetary fluxes[4,21].

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* $m.u.$ represents a monetary unit and depends on the currency used.

This system is based on the fact that a given monetary unit, for example, one euro, worth more in the present than in the future, this is, the future value of a monetary flux that has a present value of one will be greater than one[21].

The decision to transfer money from the present to the future and vice-versa, that is, the decision of keeping the money to allow future expenditures or, on the contrary, of spending the money because it is not worth to keep it for future expenditures is measured by the discount rate, t (%). The value of these rates depends mostly on the risk that the investment involves, the state of the economy and the expected inflation rate[6,8,21].

Let $N\ (years)$ be the length of the period of analysis (generally the lifetime of the project) and R_i the energy revenue in year i. According to the concept of discount rate and as represented in Figure 7, one monetary flux unit of today will be exchanged by $(1+t)^N$ monetary units in year N, while one monetary flux unit of year N will be exchanged in the present by $\frac{1}{(1+t)^N}$ monetary units[4,6,8,21].

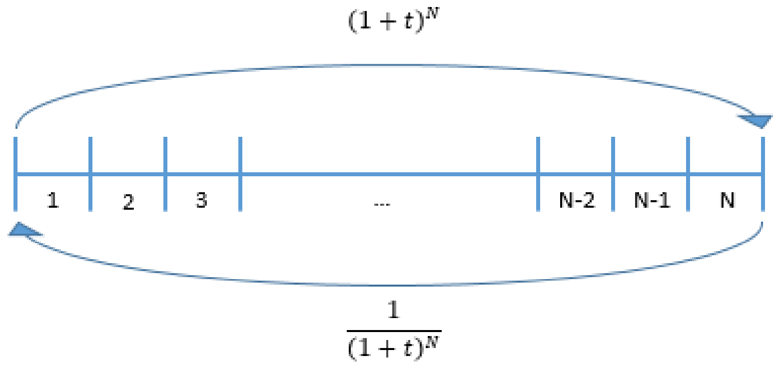

Figure 7: Transference of monetary fluxes by means of the discount rate, t.

The factor $\frac{1}{(1+t)^N}$ represents the depreciation suffered by future monetary fluxes when transferred to the present and is called present value or worth factor, $PV\ (m.u.)$[21].

The present value factor of a generic monetary flux, $C_i\ (m.u.)$, occurring in a future year i, PV, is given by equation[4]:

$$PV = \frac{1}{(1+t)^i} C_i \qquad [4]$$

The accumulated present value of a continuous sequence of futures annual monetary fluxes is given by equation [5]:

$$PV = \sum_{i=1}^{N} \frac{1}{(1+t)^i} C_i \qquad [5]$$

If the monetary fluxes are constant and equal to an annuity, C, the present value is given by equation [6]:

$$PV = \frac{(1+t)^N - 1}{(1+t)^N t} C \qquad [6]$$

Having this in mind, the accumulated present value of the energy revenue, R^{PV}, is given by equation [7]:

$$R^{PV} = \sum_{i=1}^{N} \frac{1}{(1+t)^i} R_i \qquad [7]$$

where R_i is the energy revenue in year i [4,8,21].

In our days, environmental impacts are also very important and should be rigorously evaluated, since they can dictate the failure of the project. The design of every small hydropower must assure a minimum flow, called ecological or reserved flow, Q_{res} in order to maintain water quality, especially during drier months, and to protect downstream habitats. Fish-passages are also essential to maintain aquatic life migration routes.

Hence, a compromise between maximum energy production and the maintenance of aquatic system's equilibrium must be settled[21] .

The social impacts also have a big importance since they determine how people feel towards the project. When negative social impacts increase, the population's concerns also increase, threatening the execution of the project. Generally, the impacts are higher during the construction phase, as a result of the disturbances made by civil works, reducing substantially in the exploitation phase. Besides that, there are some positive social impacts that should be referred, as the job creation, not only by the construction, but also by the exploitation of the plant and the potential economic development of the area[21].

Finally, the technical alternatives are generally limited, not only by the local conditions and constraints, but also by economic criteria. As stated in previous points, the global profit of the project will strongly depend on the available water and head, and how the first changes during the year, especially in run-of-river schemes, where there is no impounding nor dams to regularize the river flows[21].

The design of small hydropower plants is usually made assuming a constant annual turbinated volume, equal to the average of the annual turbinated volumes computed based on the available historical streamflow samples.

However, even if the time series of the river discharges can be considered homogeneous, which is unlikely in the context of climate change, the temporal variability that characterizes such series will lead to different annual productions of energy, with direct consequences on the plant's profit. For example, having very dry years in the beginning of exploitation, where the investment impact is bigger, could put the project feasibility at risk. Thus, it is very important to take into account the hydrologic temporal variability of the streamflows in the design of a small hydropower scheme[5].

2.3. The effect of hydrologic variability

As stated before, the design and the management of small run-of-river hydropower schemes require a deep knowledge about the temporal variability of the river discharges. However, the available historical data is insufficient to predict accurately the pattern of the future streamflow series[3,5].

Therefore, in the last two decades, there were some studies to assess the effect of that variability in the profitability of small hydropower schemes.

In what concerns those studies for Portugal, there were utilized some river gauging stations with available flow data, namely mean daily flows. Hypothetical hydropower plants were placed there. Based on the available mean daily flows series, economic analyses were performed.

The main objective of these analyses was to estimate the effects of having very dry years in the beginning of the exploitation phase, when the interest expenditures are higher. They concluded that the expected incomes in a small hydropower scheme can be significantly different depending on the variability of the mean daily flow series. For example, a very dry period at the beginning of the exploitation period will, most likely, lead to problems related to the plant profitability[4,7,23].

More recently, a study covering 23 river gauging stations used another approach to estimate the so called hydrologic risk due to the unpredictably of the temporal pattern of the future streamflows series: statistical and exploitation simulation analysis. This study also introduced the concept of non-cyclic and cyclic time series. A cyclic time series consists in a virtual extension of the records considering that the first mean daily flow of the original series would follow the last one. Adopting, for example, a licensing period of 10 years for the exploitation a SHS and having 15 years of continuous mean daily flows data, considering the non-cyclic series, the study would be confined to only 6 different 15-years' series. Considering the cyclic series, and as it can be seen in Figure 8, more 9 series would be added to the study, to a total of 15 series. This artifice for "generating" new mean daily flow series increased the amount of data that can support the economic analysis, thus leading to more accurate results when applying statistical tools. The study concluded that the design of a small hydropower assuming constant conditions, equal to the average annual conditions, generally slightly underestimates the expected energy production[8,23].

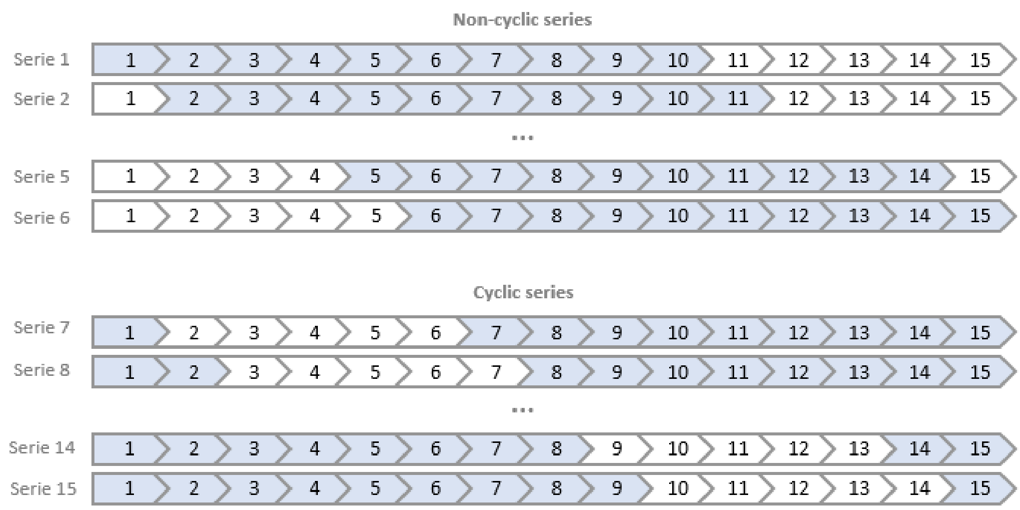

Figure 8: Cyclic and non-cyclic series (adapted [8]).

2.4. Synthetic flow series generation

2.4.1. An overview of time series modeling

The mathematical modeling in hydrology may follow two opposite approaches: deterministic and non-deterministic. In the first one there is no element of chance as there is a fixed link between a given input and the resulting output. On the contrary, the second approach admits randomness which introduces some indeterminacy in the model. This means that, for the same given input, the model may take different paths that lead to different outputs, being some paths more probable than others[3,10,24–26].

The deterministic models can be further divided in empirical models, based on experience, when there is a cause-effect relationship between the input and output variables and in physically based models with the intention of reproducing physical laws to which natural processes are subjected to [3,10,24–26].

Similarly, the non-deterministic models can also be divided in probabilistic models when the hydrological processes are not affected by the temporal and/or spatial sequence of the variables, having a purely random behavior, and stochastic models, when cumulatively with the random component, there is a serial correlation structure[3,10,24–26].

This categorization of mathematical hydrologic models is summarized in Figure 9.

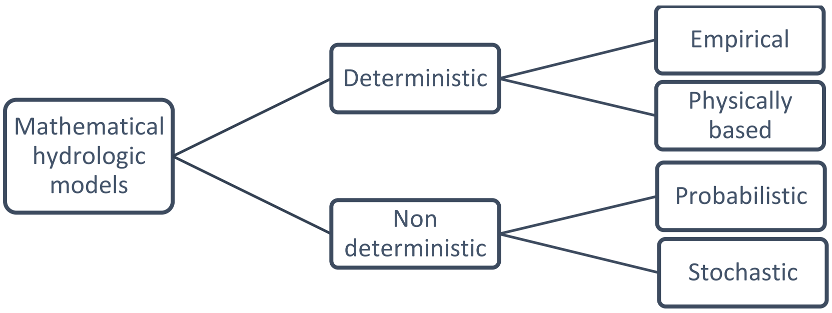

Figure 9: Categorization of mathematical hydrologic models[24].

Among the non-deterministic stochastic models, the generation of synthetic time series is a very reliable tool for planning and designing a water resource system. Synthetic time series are obtained from the historical time series and they represent alternative events with the same probability of occurrence as for the historical ones. By producing a huge number of synthetic time series based on a unique hydrological event, this method highly decreases the uncertainty associated with hydrological characteristics of the phenomena[9,10,25].

Considering a time series with N years of annual flows, X, following a normal distribution, with average μ_x and standard deviation σ_x, the mathematical model that provides an estimate of the variable for any year t, X_t, can be written as[27]:

$$X_t = \mu_x + \sigma_x \varepsilon_t \quad , \quad t = 1,2,\dots,N \qquad [8]$$

where ε_t is a random variable with mean zero and variance one. Equation [8] represents a non-deterministic probabilistic model where the variable X_t is a function only of the independent variable, ε_t, since μ_x and σ_x are constants, i.e., they do not vary in time. This model would become a stochastic model if ε_t would be represented by an equation such as the following one[27]:

$$\varepsilon_t = \phi\epsilon_{t-1} + \xi_t \qquad [9]$$

where ξ_t is an independent series with mean zero and variance $(1-\phi^2)$, being ϕ the parameter of the model and ϵ_t a function of both ξ_t and ϵ itself at a time $(t-1)$. In this case the parameters of the model X_t would be μ_x, σ_x^2 and ϕ. As these parameters are constant in time, the model is considered as a stationary sequential generation model. On the opposite, non-stationary models use time-varying parameters[27].

The quality of the generated series is measured by comparing the synthetic series to the observed or historical series and by analyzing if their main statistics, like the average, standard deviation and skewness coefficient, have been preserved. In other words, if the synthetic series preserves all the main statistics of the historical series from where they were derived, the synthetic series can be applied for the purpose they were generated for[3,10,27].

The comparison of the statistic, θ_{ss}^i $(i = 1,2,\dots,M)$, of a set of M synthetic series, each one with length equal to the length of the historical series with the statistic of the last series, θ, is made according with equation [10]:

$$\left]\overline{\theta_{ss}} - z_{\left(1-\frac{\alpha}{2}\right)} s_{\theta_{ss}}; \overline{\theta_{ss}} + z_{\left(1-\frac{\alpha}{2}\right)} s_{\theta_{ss}}\right[\qquad [10]$$

in the applications carried out $\overline{\theta_{ss}}$ and , $s_{\theta_{ss}}$ were the mean and the standard deviation (defined by equations [38] and [39] in Appendix A, respectively) of θ_{ss} and $z_{\left(1-\frac{\alpha}{2}\right)}$ represents the $\left(1-\frac{\alpha}{2}\right)$ quantile of the normal standard distribution, being α the significance level.

If the parameter θ belongs to the interval defined in equation [10], then it is preserved in the synthetic series.

Daily synthetic flow series can be used to perform a risk analysis in small hydropower schemes. This series can be generated by combining a generation model for the annual synthetic series with a disaggregation model to obtain the corresponding daily flow series[3,10].

2.4.2. Generation models

The selection of a synthetic series generation model depends on several factors such as, the type of problem and study to be developed, the availability of data and the characteristics of the historical samples. On the one hand, if the sample is purely random, a non-deterministic probabilistic model can be used, based on the identification of the probability distribution function that best fits the sample. On the other hand, if the sample is time dependent or has any kind of serial dependency, a non-deterministic stochastic model has to be adopted.

Some of the most utilized models that aim at preserving the serial temporal correlation with satisfactory results are:

- Autoregressive models, $AR(p)$, and periodic autoregressive models, $PAR(p)$, of order p, where the value of a given variable at the present depends on the values at previous times. These are short memory models capable of preserving low-order moments of the sample series and, due to its low correlation, they are suitable for annual synthetic series generation.
- Autoregressive-moving average models, $ARMA(p, q)$, and periodic autoregressive-moving average models, $PARMA(p, q)$, add a moving average component of order q to the previous models. These are longer memory models making them more suitable to samples with high correlation, as it is the case of monthly synthetic series[3,10,27,28].

Each one of the previous models is prepared to preserve the statistical characteristics of the samples (average, standard deviation and correlation coefficient) at its time level (year or month): a model applied to a sample of annual flows will preserve the above statistics of the annual flows, but it may not preserve the statistics at the monthly level. Similarly, the same happens to monthly flow series. In other words, they are not prepared to preserve statistical characteristics at more than one time level.

However, another kind of generation models, namely, disaggregation models are designed to preserve these characteristics at more than one level of aggregation. They assume a combination of a model for generating values at a given time level, for example year or month, and a model for disaggregating these values into a lower time level, like year to month or month to day, preserving the statistics of the samples at both time levels. This disaggregation can be made by applying one of the several existing models that are summarized below[3,10,28].

- The Valencia and Schaake model, proposed by Valencia & Schaake in 1972, was the first disaggregation model, becoming frequently used in stochastic hydrology. This model is based in the principle of simultaneous generation of seasonal or monthly flows of a given year, by disaggregation of the respective annual flow, preserving the sample characteristics. Its additive property allows that the aggregation of the generated seasonal flows yields the generated annual flow. However, this model has some disadvantages, like the fact that it cannot preserve the correlation between the last season and the first season of consecutive years and the large number of parameters involved in the process[3,10,28].
- The stepwise model, proposed by Emídio Santos in 1983, resides on a sequential disaggregation of say annual series into a first season and the sum of the remaining series. After, the sum of the remaining series is disaggregated into a second season and the sum of the remaining seasons until the last season is obtained. For instance, to disaggregate yearly flows into monthly flows, it is necessary 11 steps. This model also preserves the seasonal flows additive property and has the advantage of having less parameters than the previous model to preserve the characteristics of the historical flow series[3,10,28].
- The method of fragments, proposed by Svanidze in 1961, where the observed seasonal flows are standardized by dividing the seasonal flows in a given year by the annual flow volume of that year. The resulting series of standardized seasonal flows is called a fragment. In the opposite, by

multiplying generated annual flow volumes by the seasonal ordinates of the fragments, a new seasonal flow series is obtained. The assignment of the fragments to the mean annual generated volumes is very important because it takes in account the within-the-year variability of the streamflows. In this way, they cannot be assigned randomly to the annual generated volumes under penalty of generating seasonal flows that do not preserve the historic series statistic characteristics. However, a procedure generally used consists on the clustering of the fragments into classes defined by probability intervals. This procedure takes in account a similar distribution of the flows in a given superior time level when two flows at that same time scale are similar, making possible that, for instance, two seasonal flows of years with analogous flows being similar [3,10,28,29].

This last model although in an improved version was the one applied in the research carried out. Its development is presented in more detail in chapter 4.1.3.

2.5. Statistical analysis

To properly evaluate the risk of a small hydropower scheme having (or not) a given profit, it is necessary to perform a statistical analysis to the generated samples which, in this case, are the synthetic streamflow series. This can be done by using one of the well-known available statistical laws, such as the normal distribution or the Pearson type III distribution, described below.

2.5.1. Normal law

A normal distribution can be represented by a random variable X, composed by a sample of N values, $X_1, X_2, \dots X_N$. An estimate, $X_n$could be obtained by equation [11]:

$$X_n = \bar{X} + z\,\sigma \qquad [11]$$

where $\bar{X}$ and σ represents the mean and standard deviation of the sample, obtained by equations [38] and [39] in Appendix A, respectively, and z corresponds to the normally distributed random variable.

However, this statistical law can only be applied if a time series is normally distributed, this is, if the skewness coefficient of that time series is equal to zero. This hypothesis can be verified by a number of methods, like the Snedecor & Cochram test, described below[8].

2.5.2. Snedecor & Cochram normality test

The Snedecor & Cochram test is based on the fact that the skewness coefficient for a normal variable is zero[27,30]. The skewness coefficient, g_x, of a given time series, x_i, where $i = 1, \dots, N$, can be estimated by equation [12]:

$$g_x = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^3}{\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right]^{\frac{3}{2}}} \qquad [12]$$

where $\bar{x}$ is the sample mean. If the series comes from a normal distribution, g_x is asymptotically normally distributed with mean equal to zero and variance $6/N$. For that, the $(1-\alpha)$ probability limits, i.e., the confidence interval limits on g_x, may be defined by expression [13]:

$$\left[-z_{\left(1-\frac{\alpha}{2}\right)}\sigma \ \ , \ \ z_{\left(1-\frac{\alpha}{2}\right)}\sigma\right] \Leftrightarrow \left[-z_{\left(1-\frac{\alpha}{2}\right)}\sqrt{\frac{6}{N}} \ \ , \ \ z_{\left(1-\frac{\alpha}{2}\right)}\sqrt{\frac{6}{N}}\right] \qquad [13]$$

where $z_{\left(1-\frac{\alpha}{2}\right)}$ is the $\left(1-\frac{\alpha}{2}\right)$ quantile of the normal distribution[27,30].

Therefore, and for a given confidence level, if g_x falls within the limits presented in equation [13], the hypothesis of normality of the time series is accepted. On the contrary, if g_x falls off the limits, the hypothesis is rejected and the time series cannot be considered as normally distributed[27].

2.5.3. Pearson type III law

A Pearson III distribution can be represented by a random variable X, composed by a sample of N values, $X_1, X_2, \dots X_N$. An estimate, $X_p$could be obtained by equation [14]:

$$X_p = \bar{X} + K_p\sigma \qquad [14]$$

where $\bar{X}$ and σ have the meaning presented in chapter 2.5.1 and K_p is the probability factor of the Pearson III distribution obtained by applying the Wilson-Hilferty transformation[31] to z, as represented in equation [15]:

$$K_p = \{[k(z-k)+1]^3 - 1\}\frac{2}{g_x} \qquad [15]$$

where z is the normally distributed random variable and k is a factor given by equation [16]:

$$k = \frac{g_x}{6} \qquad [16]$$

being g_x the skewness coefficient with bias correction that is given by equation [17]:

$$g_x = \frac{N^2\sum_{i=1}^{N}(x_i-\bar{x})^3}{(N-1)(N-2)\,N\,\sigma'^3} \qquad [17]$$

If g_x is zero, then $K_p = z$, i.e., if the skewness coefficient is zero, there is no asymmetry and the sample is normally distributed[8].

3. Streamflow data

To analyze the risk related to the temporal variability of the river flow regime in small hydropower schemes based on synthetic series of daily flows by disaggregation models it is necessary to have sufficiently long historical samples of mean daily flows that characterize well enough the natural regime. For that, there were selected 14 gauging stations spread mainly over the north of Portugal, as shown in Figure 10. These stations were selected from a list of 54 stations used in previous studies[32,33], although some samples were updated taking into account the more recent data.

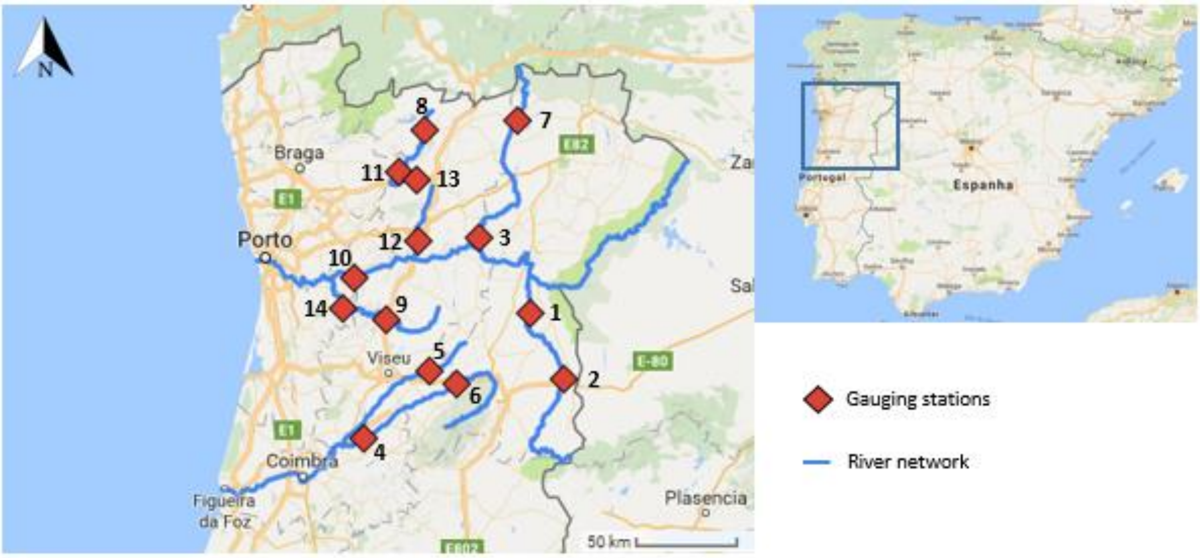

Figure 10: Schematic location of the stream gauging stations used in the study.

These river gauging stations were selected due to their long continuous period of records, but mainly because they represent natural flow regimes with different temporal patterns. As some studies suggest, in Portugal, the temporal variability of the flow regime is related to the mean annual flow depth, $\overline{H}$, this is, the greater the flow depth, the more regular the regime is. Those studies reached that conclusion by computing the coefficient of variability, C_v, of the annual flows for each one of their case studies and by making a regression analysis to determine the relationship between $\overline{H}$ and C_v, using the form presented in equation [18]. The coefficient of variability, C_v, or the relative standard deviation, RSD, of a series can be understood as a measure of its relative variability and is defined as the ratio of the standard deviation, s, to the mean, μ, of the series.

$$C_v = \alpha \overline{H}^{-\beta} \qquad [18]$$

The graphical representation of those analysis, as well as the equations that express each one of the relationships and the correlation coefficient, $c.c.$, can be seen in Figure 11. The figure shows a comparison between the results achieved by Portela & Quintela[34*,35*,36*,37*] based on a set of 24 samples , by Portela &

[*] References not consulted directly. Consulted through[10].

Quintela[32*,38*,39*] based on a set of 54 samples, and by Silva[10] based on a set of 54 samples, of which 53 samples were used in the latter study, though with slightly different recording periods.

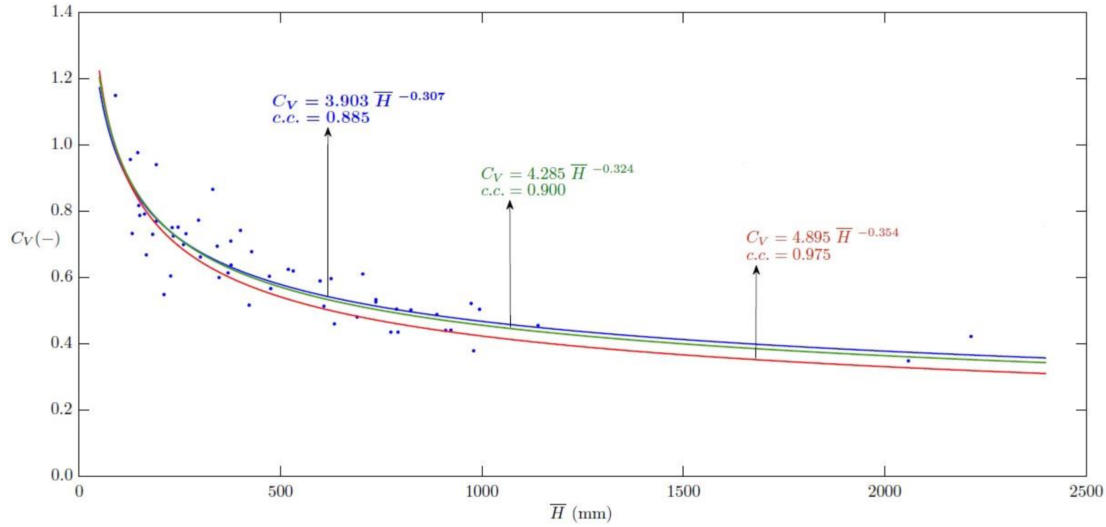

Figure 11: Relationship between the coefficients of variation of annual flows, C_v, and the mean annual flow depth, $\overline{H}$. Results from different studies: blue curve[10], green curve[32,38,39] and red curve[34–37] (adapted from[10]).

The curves presented in Figure 11 do not differ much from each other, and express a high correlation between C_v and $\overline{H}$. Furthermore, all of the samples used in the current research were contained in the data set of the mentioned studies, however, with also slightly different recording periods. Consequently, the hypothesis of the mean annual flow depth, $\overline{H}$, being closely related to the temporal variability of the flow regime is validated for the river gauging stations used.

Table 1 summarizes some of the streamflow samples main characteristics, namely, the name and code of each gauging station, the period with records and the main catchment/river where they are located, as well as the area and mean annual flow depth of each catchment.

The 14 stations were ranked according to the mean annual flow depth, which range from 300 mm to around 1000 mm, covering a wide variety of flow regimes, from more irregular to more regular, respectively. It is important to refer that the mean daily flow series were provided by the *Agência Portuguesa do Ambiente* (APA), via the public database *Sistema Nacional de Informação de Recursos Hídricos* (SNIRH)[40], which is the main source of Portuguese hydrological and hydro meteorological data and has very high quality standards.

The main statistical characteristics, namely, the average and standard deviation of the annual streamflow samples are presented in Table 2. It is important to notice that the statistics presented are referred to the period of records without the 29th of February of every leap year. As a matter of fact, all leap days were removed before any studies in order to standardize the samples and to make the procedure described in section 4 easier to apply. It is also important to notice that the data collected from SNIRH had some gaps that were fulfilled by regionalization methods, according to the methodology developed by Portela[41].

[*] References not consulted directly. Consulted through[10].

Table 1: Streamflow samples and some of their characteristics.

Gauging station				Catchment area (km²)	Period of records (number of years)	Mean annual flow depth (mm)
Sample nº	**Code**	**Name**	**Main catchment/ /river**			
1	08O/02H	Cidadelhe	Douro/Côa	1685	1955/56 - 2003/04 (49)	300
2	10P/01H	Castelo Bom	Douro/Côa	897	1957/58 - 2003/04 (47)	347
3	06M/01H	Castanheiro	Douro/Tua	3718	1958/59 - 2005/06 (48)	366
4	11I/06H	Ponte Tábua	Mondego/ Mondego	1552	1937/38 - 1978/79 (42)	421
5	10K/01H	Pte. Santa Clara Dão	Mondego/Dão	177	1921/22 - 1972/73 (52)	454
6	10L/01H	Pte. Juncais	Mondego/ Mondego	604	1918/19 - 1984/85 (67)	481
7	03N/01H	Rebordelo	Douro/Rabaçal	857	1955/56 - 2002/03 (48)	597
8	03K/01H	Vale Giestoso	Douro/Beça	77	1957/58 - 2005/06 (49)	703
9	08J/01H	Castro Daire	Douro/Pavia	291	1945/46 - 2003/04 (59)	736
10	07I/04H	Cabriz	Douro/Ribª S. Paio	17	1966/67 - 1997/98 (31)	773
11	04J/04H	Cunhas	Douro/Beça	338	1949/50 - 2005/06 (57)	823
12	06K/01H	Ermida-Corgo	Douro/Corgo	291	1956/57 - 2005/06 (50)	887
13	05K/01H	Sta. Marta do Alvão	Douro/Louredo	52	1955/56 - 2005/06 (51)	922
14	08H/02H	Fragas da Torre	Douro/Pavia	660	1956/57 - 2005/06 (50)	1016

Table 2: Average and standard deviation of the historical annual flows.

Gauging station		Annual flow volumes	
Sample nº	**Name**	**Average (hm³)**	**St deviation (hm³)**
1	Cidadelhe	506.26	335.37
2	Castelo Bom	311.37	187.07
3	Castanheiro	1361.60	895.10
4	Ponte Tábua	653.14	338.26
5	Pte Santa Clara Dão	80.37	55.09
6	Pte Juncais	290.29	176.75
7	Rebordelo	511.95	302.42
8	Vale Giestoso	54.16	33.13
9	Castro Daire	214.19	114.39
10	Cabriz	13.15	5.74
11	Cunhas	278.12	139.89
12	Ermida-Corgo	258.22	126.41
13	Sta Marta do Alvão	47.95	21.21
14	Fragas da Torre	670.49	344.92

4. Methodology

4.1. Generation of synthetic daily streamflow series

4.1.1. General considerations

The methodology used in this work consists on a two-level approach containing a generation model for the annual synthetic streamflow series and a disaggregation model to create the corresponding daily streamflow series.

The hydrological year (starting on October 1st, in Portugal) was adopted as the annual time-step to ensure the statistical independence of the annual flows. In fact, the annual streamflow, when referred to a hydrological year, is an independent time series, thus, the modelling of the annual flow series may use a non-deterministic probabilistic model[42].

Probabilistic models often generate negative flows, due to the hydrological temporal variability, representing a physically impossible situation. For this reason, the generation model is based on the probability density function of the log-Pearson III law, which applies the Pearson III law to the logarithms of the random variable. As this distribution only allows positive numbers, the generation of negative flows is avoided. Moreover, this law has a flexible distribution ensuring the treatment of skewed data, by assuming different shapes depending on its three parameters, namely, mean, standard deviation and skewness of the sample.

The disaggregation model used to obtain the corresponding daily flows was the method of fragments proposed by Svanidze in 1961 and briefly descripted in chapter 2.4.2.

The quality of the synthetic daily flow series depends upon their capability of preserving the main statistical characteristics of the historical series to which they refer. To solve this problem a large number, M, of synthetic time series, each one with length equal to the one of the historical sample is generated. A given historical statistical characteristic is preserved if it is contained in the confidence interval defined for that characteristic based on the M synthetic series. In the applications carried out, $M = 5000$ synthetic series were generated for each case study. As the average length of the historical sample is 50 years, there were generated around 250 thousand annual flows and 91 million daily flows for each case study.

4.1.2. Synthetic annual streamflows generation

As referred before, the annual flows were generated through a random sampling of the log-Pearson III distribution, which means that if $\ln(X)$ follows the Pearson III law, then X follows the log-Pearson III law. Therefore, the first step is to compute the series of natural logarithms of each annual streamflow series (equation [19]):

$$Y_k = \ln(X_k) \qquad [19]$$

where Y_k represents the series of the natural logarithm of the annual streamflows, X_k and $k = 1, \ldots, N$, where N represents the number of years with data of a given sample. Having in mind that X_k is a random value that can take values between 0 and $+\infty$ and $\ln(0) = undefined$, equation [19] needs to be corrected to avoid null flows. For that, as presented in equation [20], a constant c has to be added. This constant should be very small to minimize its effect on the characteristics of the sample. The application carried out consider $c = 0.0001$.

$$Y_k = \ln(X_k + c) \qquad [20]$$

Once the logarithms of the annual flows of a given sample, Y_k, were obtained, their statistical parameters, namely the mean, $\bar{Y}$, the standard deviation, s_y, and the skewness coefficient, g_y, were computed by applying the method of moments – equations [38], [39] and [40] presented in Appendix A.

Based on those parameters, the synthetic series of the N logarithms of annual flows, Y_k^{ss}, are next calculated according to equation [21]:

$$Y_k^{ss} = \bar{Y} + K_k s_y \qquad [21]$$

where K_k represents the probability factor of the Pearson III distribution that can be obtained by applying the Wilson-Hilferty transformation[31] to z_k. z_k represents the k^{th} order of the standard normally distributed random variable, z, with mean zero and variance one, where k takes values between 1 and N, and N represents the length of the sample (equation [22]):

$$K_k = \left\{\left[\frac{g_y}{6}\left(z_k - \frac{g_y}{6}\right) + 1\right]^3 - 1\right\}\frac{2}{g_y} \qquad [22]$$

The values of z_k are obtained by generating random values between 0 and 1, which are assigned to non-exceedance probabilities. The value of z that corresponds to each of these probabilities is obtained by inverting the distribution function of the standard normal distribution for that non-exceedance probability.

The random values are generated by using a built-in function in Matlab named RAND. This function uses a pseudorandom number generator (PRNG) that generates numbers from a uniform distribution, having a return period of approximately 2^{1492}.

The sequence generated by this type of functions is determined by the state of the generator that is the same unless the program is restarted. In the contrary, setting the generator to a fixed state, which can be made using a seed state, allows computations to be repeated, even if the program is restarted, thus allowing to achieve the same results every time the program is executed [43].

To ensure that for each streamflow sample, the random number generated would be the same, a unique seed for each sample was created, as it can be seen in Table 3.

Table 3: Seed number to create pseudorandom numbers for each sample in Matlab.

River gauging station		Seed number
Sample nº	**Name**	
1	Cidadelhe	83650
2	Castelo Bom	29846
3	Castanheiro	80914
4	Ponte Tábua	92548
5	Pte Santa Clara Dão	76544
6	Pte Juncais	56740
7	Rebordelo	46273
8	Vale Giestoso	86232
9	Castro Daire	56023
10	Cabriz	83702
11	Cunhas	56881
12	Ermida-Corgo	40769
13	Sta Marta do Alvão	18566
14	Fragas da Torre	51885

Figure 12 describes a sequence of one hundred pseudorandom generated numbers using the function RAND in Matlab with a seed no. 12345 and Figure 13 represents the inverse distribution function (IDF) of the standard normal distribution for each non-exceedance probability assigned to the previous sequence of random numbers.

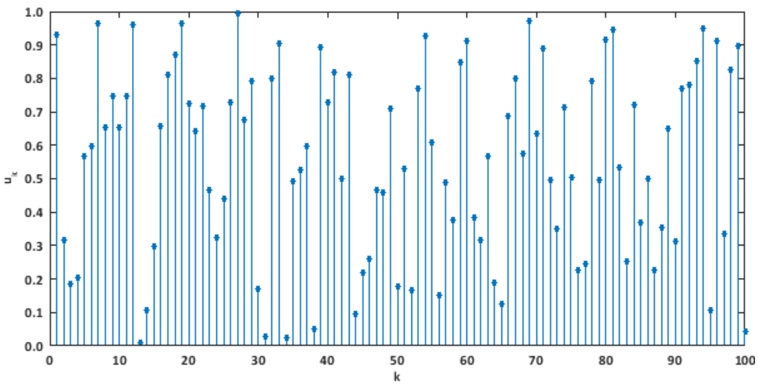

Figure 12: 100 generated numbers using RAND function in Matlab with seed no. 12345.

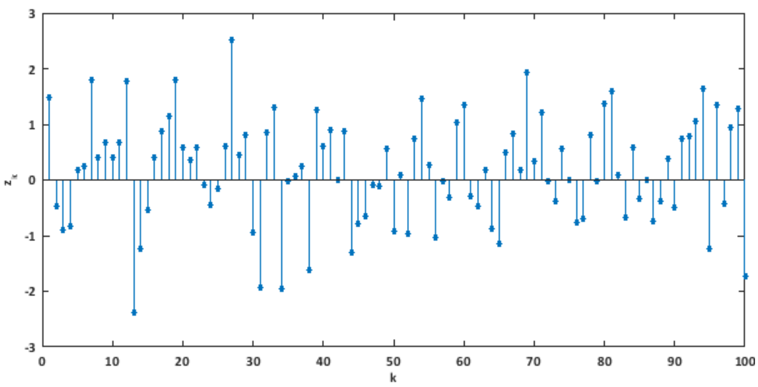

Figure 13: Inversion of the distribution function of the standard normal distribution function for each non-exceedance probability assigned to the sequence of random numbers presented in Figure 12.

Finally, now that it is possible to obtain Y_k^{ss}, the annual synthetic streamflows, X_k^{ss}, are obtained by inverting the logarithmic transformation, and subtracting the constant c, (added in equation [20]) [23]:

$$X_k^{ss} = e^{Y_k^{ss}} - c \qquad [23]$$

The described model is applied M times for each sample of length N, thus generating 5000 synthetic annual streamflow series, each with length N.

4.1.3. Daily disaggregation of annual streamflows

As referred before, the model adopted to disaggregate the synthetic annual flows is the method of fragments. The main assumption of this method is that the temporal distribution of a given hydrologic parameter over a defined time interval is similar for identical values of that hydrologic parameter. This means that, for instance, in years with similar annual streamflow volumes, the within-the-year distribution of the monthly or of the daily streamflows should be similar.

Based on a historical sample of annual and daily flows, the daily flows in each year, k, $X_{k,i}$, with $i = 1, \dots, 365$ are divided by the respective annual streamflow, X_k, resulting in a fragment, f_k, composed by 365 values. A fragment can be seen as a series of dimensionless flows at a time scale shorter than the year, in this case, at a daily scale. The sum of all the values of a fragment is obviously one. A generic fragment, f_k, for a given year, is described by equation [24]:

$$f_k = \frac{X_{k,i}}{X_k} = \left[\frac{X_{k,1}}{X_k} \; \frac{X_{k,2}}{X_k} \; \dots \; \frac{X_{k,364}}{X_k} \; \frac{X_{k,365}}{X_k}\right] \qquad [24]$$

where $X_{k,i}$ represents the daily flows for the year k $(k = 1, \dots, N)$ and the day i $(i = 1, \dots 365)$.

Naturally, for a sample with N years of records, an array of N fragments, f_k, $(k = 1, \dots, N)$, is obtained as expressed by equation [25]:

$$[f] = \begin{bmatrix} f_1 \\ \dots \\ f_N \end{bmatrix} = \begin{bmatrix} \frac{X_{1,1}}{X_1} & \dots & \frac{X_{1,365}}{X_1} \\ \vdots & \ddots & \vdots \\ \frac{X_{N,1}}{X_N} & \dots & \frac{X_{N,365}}{X_N} \end{bmatrix} \qquad [25]$$

The application of the method of the fragments implies that the fragments are constituted and assembled into classes beforehand. For that purpose, prior to computation of equation [25], the N annual flows are ranked from smallest to highest.

In Figure 14, three typical fragments of consecutive years for Cidadelhe gauging station can be seen, as an example of the variation of the daily flows within-the-year and among years.

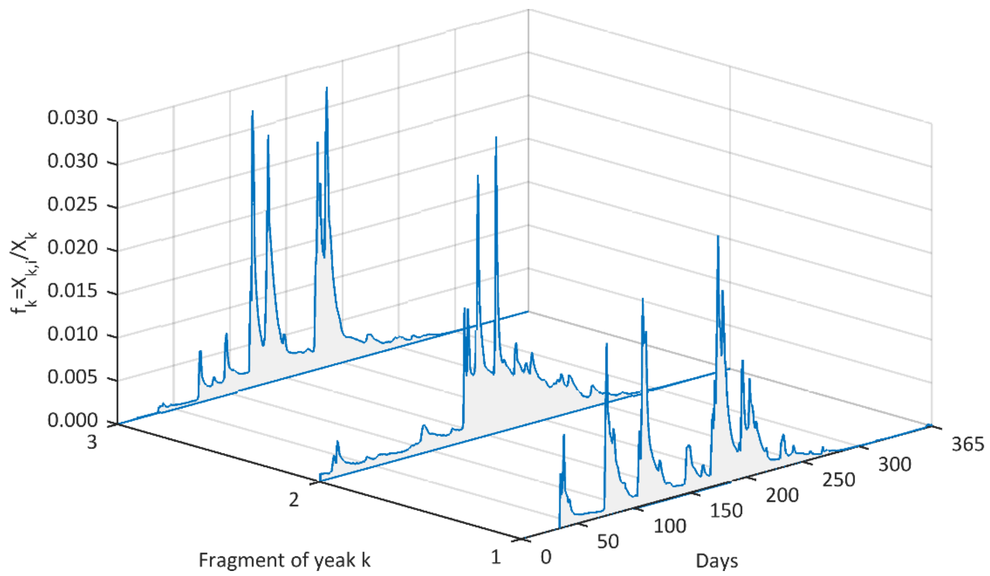

Figure 14: Daily fragments for three consecutive years (from 1955/56 to 1957/58) at Cidadelhe gauging station.

The classes of fragments are created by assembling the fragments of annual flows with similar values into classes. The number and size of the adopted classes are a big source of ambiguity, since there is no known rule for the optimal values of these two parameters.

After performing an analysis involving different criteria to create and define the fragment classes when using the method to disaggregate annual flows into monthly flows, Arsénio suggested that the definition of classes should be done based on a trial and error procedure, aiming at finding the assembly of fragments that best preserves the statistical characteristics of the samples[44].

To avoid this procedure, Silva[10] and Portela & Silva[25,45,46] suggested the definition of the fragments classes as probability intervals for the disaggregation of annual flows into monthly flows. In this way, the authors considered classes of fragments, each one delimited by two annual flows in a way that the difference between the non-exceedance probabilities of each one was constant and equal to $100\%/C$, being C the number of classes. For example, for 10 classes of fragments, the amplitude of the intervals would be 10%. If the precedent procedure results in one or more empty classes, i.e., one or more classes without fragments, half of the probability amplitude that defines the empty class is attributed to the two adjacent classes. This step is valid only for the internal classes. If the empty class is the first or the last one, then that class is included in the next or in the previous class, respectively, by changing its upper or lower limit. The authors also concluded that this method leads to good results in what concerns the preservation of the statistical parameters of the samples at the monthly level.

Motivated by the good results at this time level, Pinto[3] applied the precedent method to the disaggregation of annual flows directly into daily flows, attesting the preservation of the statistics at the daily level in most of the cases.

Although the preceding procedure starts with classes of fragments equally-spaced in terms of non-exceedance probability limits, the possible existence of empty classes and their consequent re-allocation will end up by changing the limits of some classes. Thus, when applying this method to a large number of samples, it is difficult to categorize the different class definitions and, consequently, to observe its consequences in the performance of the model. In this way, the study conducted by Portela & Silva[9,47] introduced a new way to define the number and amplitude of the fragments classes related to the previous one - classes always equally-spaced in terms of the non-exceedance probabilities of their limits.

This procedure was the one applied in this study for ten and twenty initial classes of fragments. To simplify the following description, only the procedure with ten initial classes of fragments will be explained, since both are very similar.

1. Establishment of $C = 10$ classes, delimited by constant non-exceedance probabilities of the annual streamflow and equal to $(100\%/C)$. As $C = 10$, the amplitude of the classes is 10%, as shown in Figure 15.

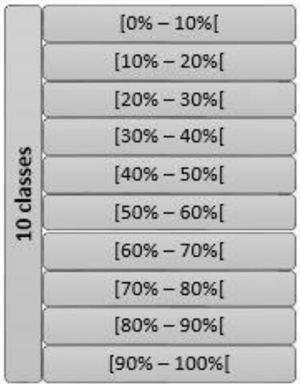

Figure 15: Ten classes with equally spaced non-exceedance probability intervals of the annual streamflow.

2. Estimation of the annual flows, $\widehat{X_j}$, for the extremes non-exceedance probabilities defined in Figure 15 by inverting the probability density function of the log-Pearson III law, according to equations [21] and [22], followed by the inversion of the logarithmic transformation. This step results into ten classes, with limits given by annual flows, instead of probabilities (Figure 16).

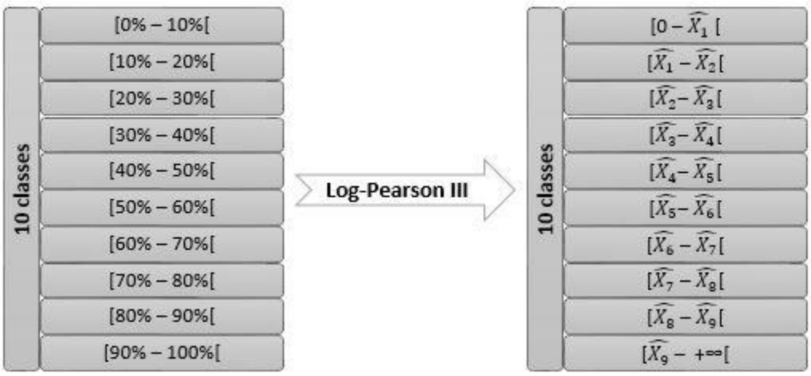

Figure 16: Definition of the classes in terms of intervals of annual flows by inverting the log-Pearson III law.

3. Allocation of the fragments among the successive classes within whose limits the annual flow of each fragment is comprehended (Figure 17).

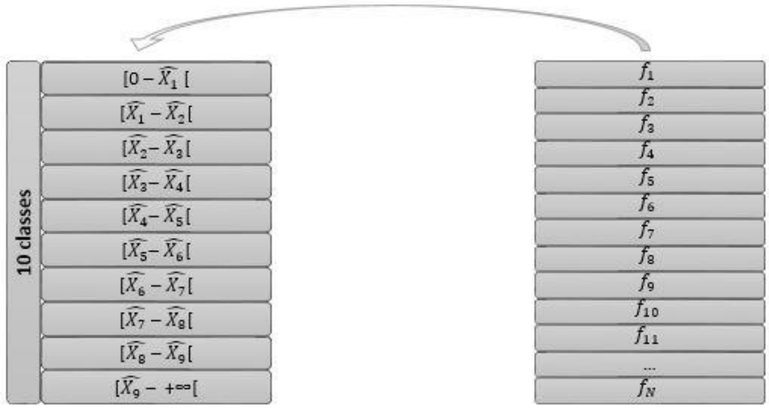

Figure 17: Allocation of the fragments to the classes.

If all the ten classes have at least one fragment, the procedure for the definition of the fragments classes is complete. If there is one or more empty classes, the number of classes is decreased by one (in this case, from ten to nine classes) and the previous steps are repeated, as suggested by Figure 18.

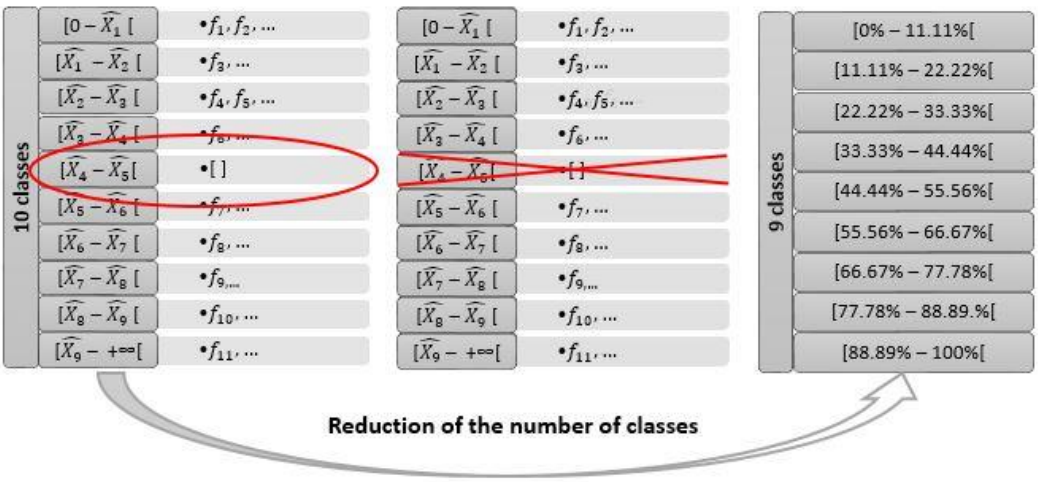

Figure 18: Redefinition of the classes if there is an empty class.

Once all the fragments are clustered into classes with at least one fragment, a synthetic annual flow series can be disaggregated into a synthetic daily flows series. This process is accomplished by identifying, for each year k, the class, C_j, to which the annual flow belongs.

After that, one of the fragments of that class is selected to disaggregate that flow, according to the following procedure:

- If the class to which the annual flow belongs has only one fragment, that fragment is used.
- If the class has more than one fragment, one of them is randomly selected. For that:
  - The fragments in each class are sorted from the smallest to largest value of the corresponding annual flows.
  - A natural random number, ξ, between 1 and the number of fragments belonging to that class is generated, using the Matlab built-in function RANDi. The initial state of this Pseudo random number

generation algorithm is reset at the beginning of the generation model for each streamflow sample and the seed numbers used were those of Table 3.

- The fragment in the position ξ is selected.

The random selection of fragments from a given class is done without replacement until the class runs out of fragments. When this happens, all the fragments that composed the class are restored with the same exact order as they were in the beginning.

Each synthetic annual flow is disaggregated into synthetic daily flows according to equation [26]:

$$X_{k,i}^{ss} = X_k^{ss} f_k \quad [26]$$

where the indices $k = 1, \ldots, N$ and $i = 1, \ldots, 365$ refer to the year and the day, respectively, X_k^{ss} represents the synthetic annual flow, $X_{k,i}^{ss}$ represents the correspondent synthetic daily flows, ss the order of the synthetic flow series and f_k the fragment randomly selected to disaggregate X_k^{ss}.

This procedure is repeated until M N-year synthetic daily streamflow series are obtained. As said before, for each sample $M = 5000$ synthetic daily streamflow series were generated.

4.1.4. Evaluation of the generated synthetic series

The quality of the generated synthetic series obtained by the above generation and disaggregation models were assessed by evaluating the preservation of the historical statistical parameters at the different time levels, as explained briefly in paragraph 2.4.1.

In this way, the statistical parameters of an historical series, θ, are preserved by a given synthetic series when they are included in the confidence interval given by equation [10].

The results from the generation model applied to the annual level are logarithms of the annual flows which means that, from a theoretical point of view, the performance of that model should be evaluated in the field of the logarithms, which has little meaning. Accordingly, prior the analysis of the preservation of the statistical characteristics, the flows were returned to their original field by inverting the logarithm transformation. By other words, the analysis of the performance of the model considered the annual flows instead of the logarithms of those flows. At a day level, the assessment of the performance of the disaggregation model was based in the mean, standard deviation and skewness coefficient of the series of each daily flow.

Taking advantage of the additive property of the method of fragments, the synthetic monthly flow series were obtained by the simple monthly accumulation of the respective daily flows. The performance of the model was evaluated at this time level as well, based once again, in the mean, standard deviation and skewness coefficient of the series of each monthly flow.

At all temporal levels, it was adopted a confidence level of $100\% \times (1 - \alpha) = 95\%$ for the confidence intervals. Therefore, expression [10] can be represented by equation [27]:

$$\overline{\theta_{ss}} - 1.96 s_{\theta_{ss}} < \theta < \overline{\theta_{ss}} + 1.96 s_{\theta_{ss}} \quad [27]$$

Once attested their quality, the energy production, and consequently, the revenue yielded by every series for each sample can be computed.

4.2. Analysis of the effect of the hydrological variability (hydrological risk) for the global period

4.2.1. General Considerations

To simulate the exploitation of a small hydropower scheme and to determine its revenue during its lifetime period is a very expeditious procedure. In this study, the simulation considers the most common scenario in a SHS, which is a run-of-river exploitation regime. For this reason, no regularization volumes are considered. In order to make the procedure even more generic, some SHS's characteristics are considered identical and equal to a reference value. More precisely, the net head, the global efficiency of the powerhouse and the unitary selling price of the energy are considered equal to one. These three parameters do not have any impact on the following procedure and the fact that they are equal for every sample allows a more straightforward comparison among the results from the different case studies.

As referred in section 3, the simulation considers that every hypothetical hydropower plant is located in a river gauging station, making flow data easier to access.

One of the main objectives of this study is to analyze the effect of the maximum flow that can be turbinated or design discharge, Q_{max}, in the production of energy and, consequently, in the SHS revenue. For that purpose, there were adopted values for $Q_{max} = Q/Q_{mod}$ between 1.0 and 3.0, with increments of 0.2, $(1.0 \leq Q/Q_{mod} \leq 3.0)$. This interval of flows includes the most utilized criteria when designing a SHS, which is commonly around $2.0\ Q_{mod}$. In any case, no minimum or ecological flows are considered.

Due to the lack of appropriate legislation regulating the licensing period of small hydropower schemes, in a first approach, the period of analysis considered in each case study was coincident with the length of the corresponding flow sample.

In economic terms, the present values of the annual revenues resulting from the sale of the energy produced in these hypothetical SHS are determined based in a constant market price system referred to the first year of the exploitation of the schemes (year 0). The discount rate, t, is set in 7%, being a common value in this type of projects.

The economic analysis in each case study utilized sequences of dimensionless annual revenues, obtained by dividing the revenue provided by the models for each year by a reference revenue obtained from the historical series, thus allowing comparing results among the different case studies. The following sections present the methodology applied to determine both the reference and the synthetic series revenues.

4.2.2. Determination of the reference revenue

The reference revenue is obtained from the mean annual energy production derived from the historical series, based on the mean annual turbinated volume that naturally depends on the design discharge, Q_{max}. For each value of Q_{max}, that volume can be represented on a mean annual daily flow duration curve as exemplified in Figure 19, for a $Q/Q_{mod} = 2.0$.

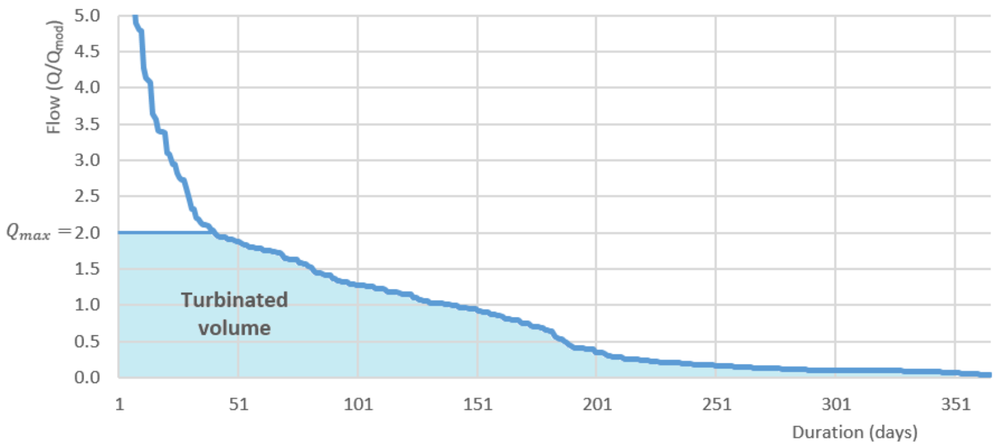

Figure 19: Mean annual flow duration curve and turbinated volume if $Q_{max} = 2.0$.

In mathematical terms, the reference turbinated volume can be given by the sum of equations [28] and [29], described below, which do not consider the release of ecological or minimum flows, as previously mentioned.

- Sum of the historical daily flows, $\sum Q_{i,k}$, whenever those flows are smaller than the design discharge, Q_{max} [28]:

$$Q_{1_{ref}} = \sum_{i=k=1}^{365,N} Q_{i,k} \quad if\ Q_{i,k} < Q_{max} \qquad [28]$$

- Sum of the design discharge, $\sum Q_{max}$ whenever the historical daily flows, $Q_{i,k}$ are equal or higher than that discharge, Q_{max} [29]:

$$Q_{2_{ref}} = \sum_{i=k=1}^{365,N} Q_{max} \quad if\ Q_{i,k} \geq Q_{max} \qquad [29]$$

In the previous equations, $Q_{1_{ref}} [m^3/s]$ and $Q_{2_{ref}}$ $[m^3/s]$ are cumulative reference turbinated volumes. The indices i and k represent the number of days and years of the historical series, respectively. Naturally, the total reference turbinated volume, Q_{ref} $[m^3/s]$ is given by the sum of $Q_{1_{ref}}$ and $Q_{2_{ref}}$.

The average annual turbinated volume, V_{med} is then given by equation [30]:

$$V_{med} = \frac{Q_{ref} \times 24 \times 3600}{N} \qquad [30]$$

where N represents the historical series length in years.

It should be stressed that the previous approach considers that all the daily discharges smaller than the design discharge can be turbinated which may not be the case as there is always a minimum discharge below which the turbine is unable to work. However, many SHS have a small storage capacity, either in the reservoir created by the dam or in the conveyance system, that allows to concentrate the inflows smaller than the minimum discharge so they can be utilized for energy production. The analysis carried out assumed that such storage capacity was available

Based on equation [30], the annual average energy production, E_{med} is given by equation [2].

As described in section 2.2.2, the annual average revenue, R_{med} represents the incomes with the sale of the produced energy, this is, the energy production multiplied by its unitary sale price. As this price considered equal to one, R_{med} is given by equation [31]:

$$\boldsymbol{R_{med}} = E_{med} \times c = E_{med} \times 1 = \boldsymbol{E_{med}} \qquad [31]$$

Having in mind that R_{med} represents an annuity, it is very easy to calculate its cumulative present value (CPV), also called reference revenue, R_{ref}, by applying equation [6], as summarized by equation [32]:

$$R_{ref} = R_{med} \frac{(1+t)^N - 1}{(1+t)^N t} \qquad [32]$$

As referred before, this model is applied for a range of values of Q/Q_{mod}, comprehended between 1.0 and 3.0 with increments of 0.2, resulting in 11 reference revenues for each case study, each one for a given value of Q/Q_{mod}.

4.2.3. Determination of the synthetic series revenues

The procedure implemented to obtain the revenues from the sale of energy in each case study took into account the temporal variability of the turbinated volumes given by equations [28] and [29], according to the following steps, that were applied to each one of the daily flow series.

This process can be applied not only to the $M = 5000$ daily flow synthetic series but also to the historical one, as both represent events equally likely to happen. Therefore, the below procedure is made for $M + 1$ daily flow series as described below:

- In each year, sum of daily flows, $\sum Q_{i,k}$, for flows smaller than the design discharge, Q_{max} [33]:

$$Q_{1_s}^{(k)} = \sum_{i=1}^{365} Q_{i,k} \quad if\ Q_{i,k} < Q_{max} \qquad [33]$$

- In each year, sum of the design discharge, $\sum Q_{max}$ for daily flows, $Q_{i,k}$ equal or higher than the design discharge, Q_{max} [34]:

$$Q_{2_s}^{(k)} = \sum_{i=1}^{365} Q_{max} \quad if\ Q_{i,k} \geq Q_{max} \qquad [34]$$

$Q_{1_s}^{(k)}[m^3/s]$ and $Q_{2_s}^{(k)}[m^3/s]$ represent the cumulative turbinated volume in each year. The indices i and k stand for the number of days and years of each series, respectively, and s for the order of the series (from 1 to $M+1 = 5001$). Again, the total annual turbinated volume resulting from the $M+1$ series, $Q_s^{(k)}[m^3/s]$ is given by the sum of $Q_{1_s}^{(k)}$ and $Q_{2_s}^{(k)}$.

The energy production, $E_s^{(k)}$, and, consequently, the energy income, $R_s^{(k)}$, vary from one year to another. Accordingly, the cumulative present value of the incomes resulting from each one of the 5001 daily flow series took into account equation [4], by replacing C_i by the annual revenues, resulting in equation [35]:

$$R_s^{PV} = \sum_{k=1}^{N} \frac{1}{(1+t)^k} R_s^{(k)} \qquad [35]$$

Each value of R_s^{PV} were next made dimensionless by dividing by the reference revenue.

Based on the $M+1$ daily flows series as many dimensionless cumulative present values of the revenues were obtained for each Q/Q_{mod}. The meaning of those $M+1$ values was analyzed by applying a statistical model, as described in the next item.

4.2.4. Statistical analysis

In order to decide what statistical model should be applied to each set of $M+1 = 5001$ values of R_s^{PV}, the Snedecor & Cochram skewness coefficient test was applied – equation [13]. Having in mind the number of values of R_s^{PV} ($M+1 = 5001$) and adopting a confidence level of 5%, equation [13] becomes:

$$1.96\sqrt{\frac{6}{5001}} < g_y < 1.96\sqrt{\frac{6}{5001}} \Leftrightarrow -0.0679 < g_y < 0.0679 \qquad [36]$$

This means that, according to the Snedecor & Cochram test, to consider a series of 5001 dimensionless cumulative present values of the revenues as normally distributed, its skewness coefficient has to be contained between]-0.0679 ,0.0679[. However, the majority of the series showed skewness coefficients outside this interval. Consequently, and also to standardize procedures, the Pearson type III distribution was applied to those series aiming at obtaining the revenues for different probabilities of non-exceedance. The model applied for that purpose was also based on the probability factor.

4.3. Analysis of the effect of the hydrological variability (hydrological risk) for subperiods

The analysis described in section 4.2 considered that the licensing period of each SHS was equal to the length of the sample of daily flows in the stream gauging station to which the case study refers. As it can be seen in Table 1, each case study has around 50 years of records. This period normally exceeds the usual expected licensing period of a SHS. In fact, and despite the current lack of legislation, until a few years ago, 35 years was a common licensing period. This period was subsequently reduced to 15 years eventually plus 10 years[48], and more recently, to 20 years eventually plus 5 years[49]. The decree-laws that supported the previous periods are no longer in force and the replacing legislation was not yet produced[50].

Therefore, for a more realistic perception of the economic risk on SHSs, the procedure described in the previous chapter was performed for periods of 25 years.

However, the smallest the period is, the more susceptible the SHS becomes to the temporal variability of the flow regime, meaning that the expected revenue can be considerably apart from the reference one. In this way, periods of 10 years were also considered in order to analyze the susceptibility of the investments to adverse hydrological conditions during the first years of exploitation.

For that purpose, the model previously presented was applied to determine, for each daily flow series of N years, the maximum and minimum revenues CPV yielded by licensing periods of $N^* = 25$ and $N^* = 10$ years.

In this way, for a given licensing period, N^*, and for each one of the $M + 1$ daily flow series of N years, all the possible sequences of N^* years of daily flows were determined, as represented in Figure 20. For instance, for a global period of $N = 50$ years and a licensing period of $N^* = 25$ years, each daily flow series of N years can be divided in 26 sequences of N^* years of daily flows. The first sequence includes the daily flows of year 1 to year 25, while the second one is from year 2 to year 26. The 26th sequence goes from year 26 to year 50.

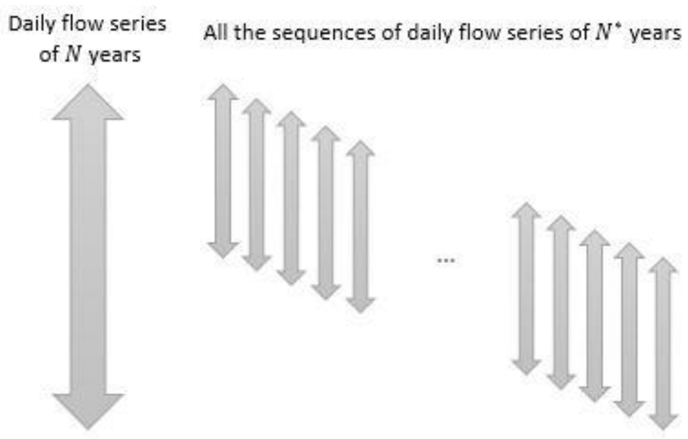

Figure 20: Schematic representation of all the possible sequences of daily flow series of N^* years ($N^* < N$), obtained from a given daily flow series of N years.

The daily simulation procedure, described in section 4.2. for the global period of records, was applied to each one of these sequences of N^* years in order to determine the cumulative present value of the revenue obtained for the sale of the energy produced in each one. Accordingly, these revenues were made dimensionless by dividing by the reference revenue in N^* years, R^*_{ref}, as presented in equation [37]. The reference revenue, R^*_{ref} , was determined based on the assumption of an annuity obtained from the mean annual energy production computed for the total recording period, N.

$$R^*_{ref} = R_{med} \frac{(1+t)^{N^*} - 1}{(1+t)^{N^*} t} \quad [37]$$

Finally, for each daily flow series of N years, the sequences of N^* years that lead to the maximum and minimum cumulative present value of the revenues were identified, as Figure 21 suggests.

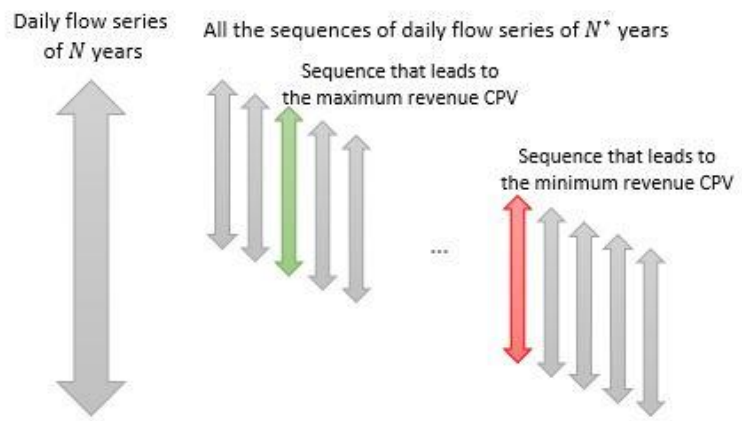

Figure 21: Schematic representation of the sequences of daily flows of N^* years ($N^* < N$) that lead to the maximum and minimum revenue CPV, obtained from a given daily flow series of N years.

Similarly to the analysis for the global period, this procedure was applied to values of Q/Q_{mod} comprehended between 1.0 and 3.0 with increments of 0.2.

5. Results and discussion

5.1. Assessment of the quality of the generated synthetic streamflow series

5.1.1. General considerations

The methodology for the generation of synthetic streamflow series described in chapter 4 was applied to the samples presented in Table 1 of chapter 3. In this table, the samples are ranked from the smallest to the highest mean annual flow depth, $\overline{H}$, that is, from the more irregular to the more regular flow regime.

As described in the previous section, a two-time-level generation model was used to generate 5000 synthetic annual and daily streamflow series, each with the length of its respective sample.

In what concerns the generation of synthetic daily flows, and as referred in chapter 4.1.3, the method of fragments is based on the clustering of fragments into classes defined by probability intervals.

There were considered and compared two possible initial number of classes of fragments: 10 and 20 classes, each with constant non-exceedance probability limits and equal amplitudes (10% for 10 classes and 5% for 20 classes).

From a theoretically point of view, the expected number of fragments in each class should be $\frac{N}{n^{\underline{o}}\ of\ classes}$, where N represents the length of each sample. Table 4 and Table 5 summarize, for each sample, the real number of fragments in each class that resulted from the application of the approach for ten and twenty initial classes of fragments, respectively. It is possible to see that, in practice, the number of fragments in each class varies. Furthermore, there are samples with less classes than the initially considered, consequence of the existence of one or more empty classes. This situation is explained by the temporal irregularity of the flow regime and the inexistence of annual flows within the limits of a certain class. In fact, it can be easily seen in Table 5, that the samples with a lower mean annual flow depth and, consequently, with a more irregular regime have their fragments spread for fewer classes than the ones with a higher mean annual flow depth.

To evaluate the results of the models, it is necessary to verify the capacity of the generated synthetic series to preserve the main statistics of the corresponding samples at the different time levels, namely year, month and day. It is important to note that the synthetic monthly series were obtained from the synthetic daily flows, ensuring the additivity, not only between day and month, but also between month and year and day and year.

The evaluation of the performance of the models is made by comparing the statistics (mean, standard deviation and skewness coefficient) of the synthetic series with those derived from the samples. As referred before, this comparison utilized confidence intervals, expressed by equations [10] and [24] for a confidence level of $100\% \times (1-\alpha) = 95\%$.

Table 4: Number of fragments in each class and schematic representation for their relative values for each sample (ranked from the smallest to the highest $\overline{H}$), for ten initial classes of fragments.

Sample number	Class number: 1	2	3	4	5	6	7	8	9	10	Schematic representation
	Number of fragments in each class										
1	6	2	8	1	7	4	7	5	7	2	
2	6	3	5	2	6	7	4	3	7	4	
3	6	4	3	5	10	1	4	2	9	4	
4	3	5	4	5	7	2	4	3	4	5	
5	6	6	2	8	8	5	9	1	7	-	
6	6	7	6	7	7	10	5	6	6	7	
7	5	5	3	5	9	2	3	4	8	4	
8	6	5	6	2	5	6	2	5	8	5	
9	6	8	2	7	7	6	4	4	10	5	
10	3	2	3	6	3	2	3	3	2	4	
11	6	3	7	6	8	6	5	3	8	5	
12	5	6	5	5	3	7	5	2	6	6	
13	6	5	3	6	6	5	3	5	6	6	
14	6	4	4	4	10	4	1	5	7	5	

Table 5: Number of fragments in each class and schematic representation of their relative values for each sample (ranked from the smallest to the highest $\overline{H}$), for twenty initial classes of fragments.

Sample number	Class number: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Schematic representation
	Number of fragments in each class																				
1	4	2	1	3	5	1	1	4	3	2	3	4	3	2	5	3	1	2	-	-	
2	5	2	2	6	1	5	4	4	4	2	5	5	2	-	-	-	-	-	-	-	
3	6	2	3	4	4	9	1	2	3	5	6	3	-	-	-	-	-	-	-	-	
4	2	1	4	1	3	1	1	4	2	5	1	1	2	2	1	2	3	1	4	1	
5	6	6	2	8	8	5	9	1	7	-	-	-	-	-	-	-	-	-	-	-	
6	3	3	4	3	1	5	3	4	4	3	6	4	3	2	2	4	3	3	4	3	
7	3	2	3	2	2	1	4	2	6	3	1	1	2	1	3	4	4	2	2	-	
8	2	6	1	2	1	5	1	1	4	2	4	2	1	2	2	6	2	3	2	-	
9	5	1	5	3	1	1	3	4	5	5	3	2	2	3	1	5	5	2	3	-	
10	2	1	2	1	3	4	3	1	1	2	2	2	1	2	1	3	-	-	-	-	
11	4	2	1	2	3	4	4	2	5	3	4	2	1	4	1	2	7	1	1	4	
12	3	2	3	4	3	1	4	2	2	5	2	3	2	2	3	1	5	3	-	-	
13	4	2	2	3	3	2	3	3	3	2	3	3	1	3	2	4	2	4	2	-	
14	3	4	1	3	3	1	2	6	5	2	2	1	2	2	3	4	3	3	-	-	

5.1.2. Results at the annual level

As referred in section 4.1.2, the generation of annual flows is made in the logarithm domain, thus the confidence interval and the statistics of the samples represented in Figure 22 refer to logarithms.

The analysis of the previous figure reveals that the historical values of the statistics of each sample are contained within the respective confidence interval, meaning that all statistical parameters are preserved at this time level, proving that the probabilistic model applied (log-Pearson type III distribution) for the generation of annual flows yields good results.

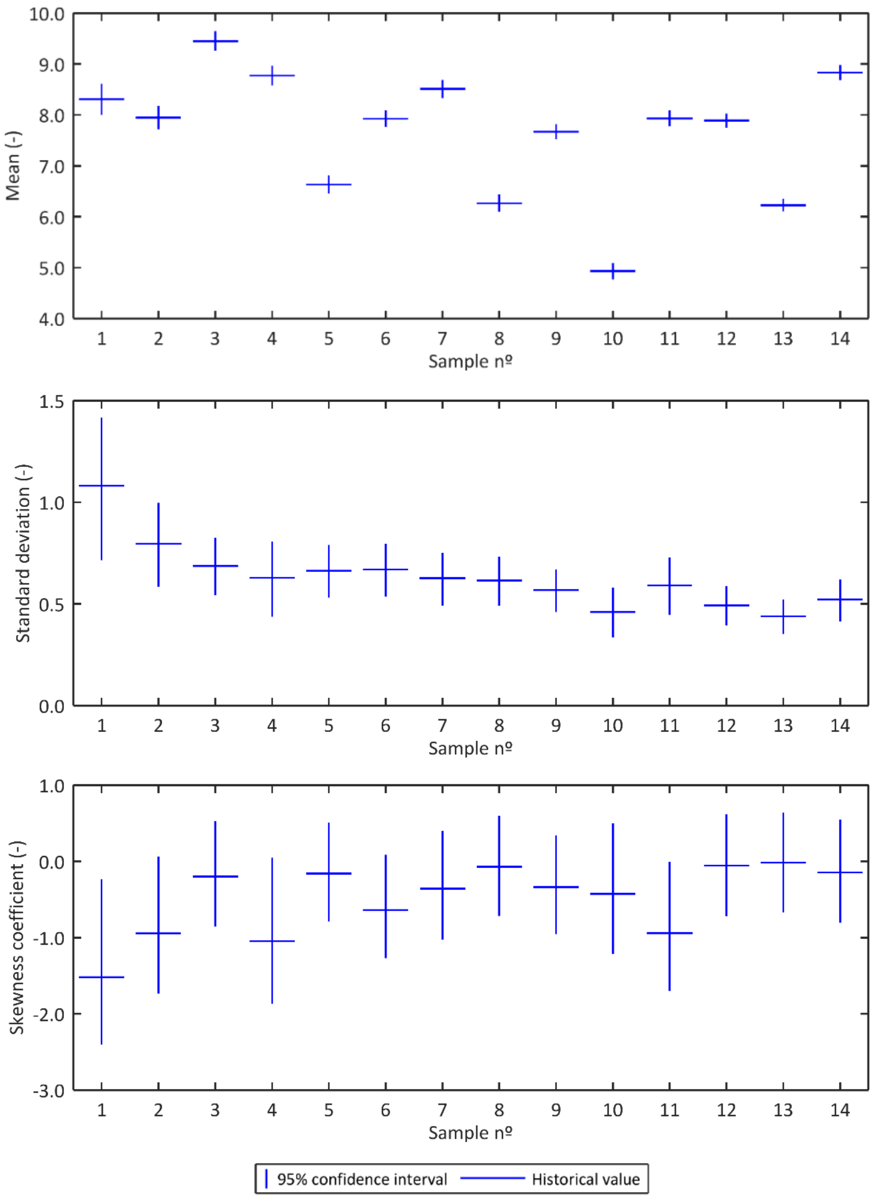

Figure 22: Historical value and 95% confidence intervals of the mean, standard deviation and skewness coefficient of the annual streamflows for each sample.

The same analysis was performed in the domain of the annual streamflows, after the flows were returned to their original field by inverting the logarithm transformation.

However, the representation of the respective results in a figure is difficult to perceive due to the amplitude of the values, especially in what concerns the means and the standard deviations. In any case, this analysis also proved the preservation of the aforementioned statistics of the annual flows for all case studies, showing that the transformation applied did not compromise the quality of the results at the annual level.

5.1.3. Results at the monthly level

To perform an analysis at a monthly level, both, the synthetic and historical monthly flows had to be obtained from the respective daily flows, taking advantage on the additivity property.

The statistical parameters analyzed were the mean, standard deviation and skewness coefficient of the monthly flows.

Due to the great number of variables under consideration for each case study, combined with two possible initial number of classes of fragments (10 and 20 classes), the results were not present in a figure but, instead, summarized in Table 6 which shows, for each sample, the number of months and the months where a given historical statistical characteristic, θ, was not preserved, as well as, the final number of equidistant fragment classes.

This table clearly shows the good performance of the model at the monthly level. For both initial number of classes of fragments, the model was able to preserve all the means and the majority of the standard deviations and skewness coefficients. It is noticeable that for twenty initial classes of fragments, the model leads to a better preservation of the standard deviations and skewness coefficients of the samples, maintaining the good results on what concerns the means. Ponte Santa Clara Dão river gauging station (sample nº 5) is the exception, with only 9 equally-spaced classes of fragments, regardless the initial number of classes.

It is also possible to infer that the majority of months where the standard deviations and skewness coefficients were not preserved occurred during the drier season.

In order to exemplify graphically the results at the monthly level, Figure 23 compares, month-by-month (from October to September), the referred statistics for the river gauging station of Cidadelhe (sample nº 1), Ponte Juncais (sample nº6) and Fragas da Torre (sample nº 14), respectively, and for twenty initial classes of fragments – situation that leads to the best results.

5.1.4. Results at the daily level

Like the results at the monthly level, the statistical parameters analyzed for the daily flows were again the means, standard deviations and skewness coefficients. Table 7 summarizes, for each sample, the number of days without preservation of the historical statistics, θ.

Once again, Table 7 demonstrates the good performance of the model. Contrarily to the monthly level, in some cases, the model does not always ensure the preservation of the means at the daily level, as is the case of Ponte Santa Clara Dão (sample nº 5) and Vale Giestoso (sample nº 8) river gauging stations, this last, only when twenty initial classes of fragments are considered. Nevertheless, the unpreserved averages represent around $12/365$ ≅3% of the number of averages in the first case and less than 1%, in the second one. The number of days without preservation of the standard deviation and skewness coefficient is slightly higher in some samples. However, even in the river gauging station with the highest number of days without preservation of the standard deviation – Ponte Santa Clara Dão (sample nº 5) – that number is less than 7%.

Table 6: Performance of the model at the monthly level for ten and twenty initial classes of fragments.

River gauging station		Number and months without preservation of the historical statistical characteristics							
		10 initial classes of fragments				20 initial classes of fragments			
Nº	**Name**	**Effective number of classes**	**Mean**	**Standard deviation**	**Skewness coefficient**	**Effective number of classes**	**Mean**	**Standard deviation**	**Skewness coefficient**
1	Cidadelhe	10	0	0	0	18	0	0	0
2	Castelo Bom	10	0	0	**1 (September)**	13	0	0	0
3	Castanheiro	10	0	**1 (May)**	**1 (August)**	12	0	0	**1 (August)**
4	Ponte Tábua	10	0	0	0	20	0	0	0
5	Ponte Santa Clara Dão	9	0	**2 (May, June)**	**2 (December, June)**	9	0	**2 (May, June)**	**2 (December, June)**
6	Ponte Juncais	10	0	0	**1 (January)**	20	0	0	0
7	Rebordelo	10	0	0	0	19	0	0	0
8	Vale Giestoso	10	0	0	0	19	0	0	0
9	Castro Daire	10	0	0	0	19	0	0	0
10	Cabriz	10	0	0	0	16	0	0	0
11	Cunhas	10	0	0	0	20	0	0	0
12	Ermida Corgo	10	0	0	0	18	0	0	0
13	Santa Marta do Alvão	10	0	0	0	19	0	0	0
14	Fragas da Torre	10	0	**1 (June)**	**1 (July)**	18	0	0	0

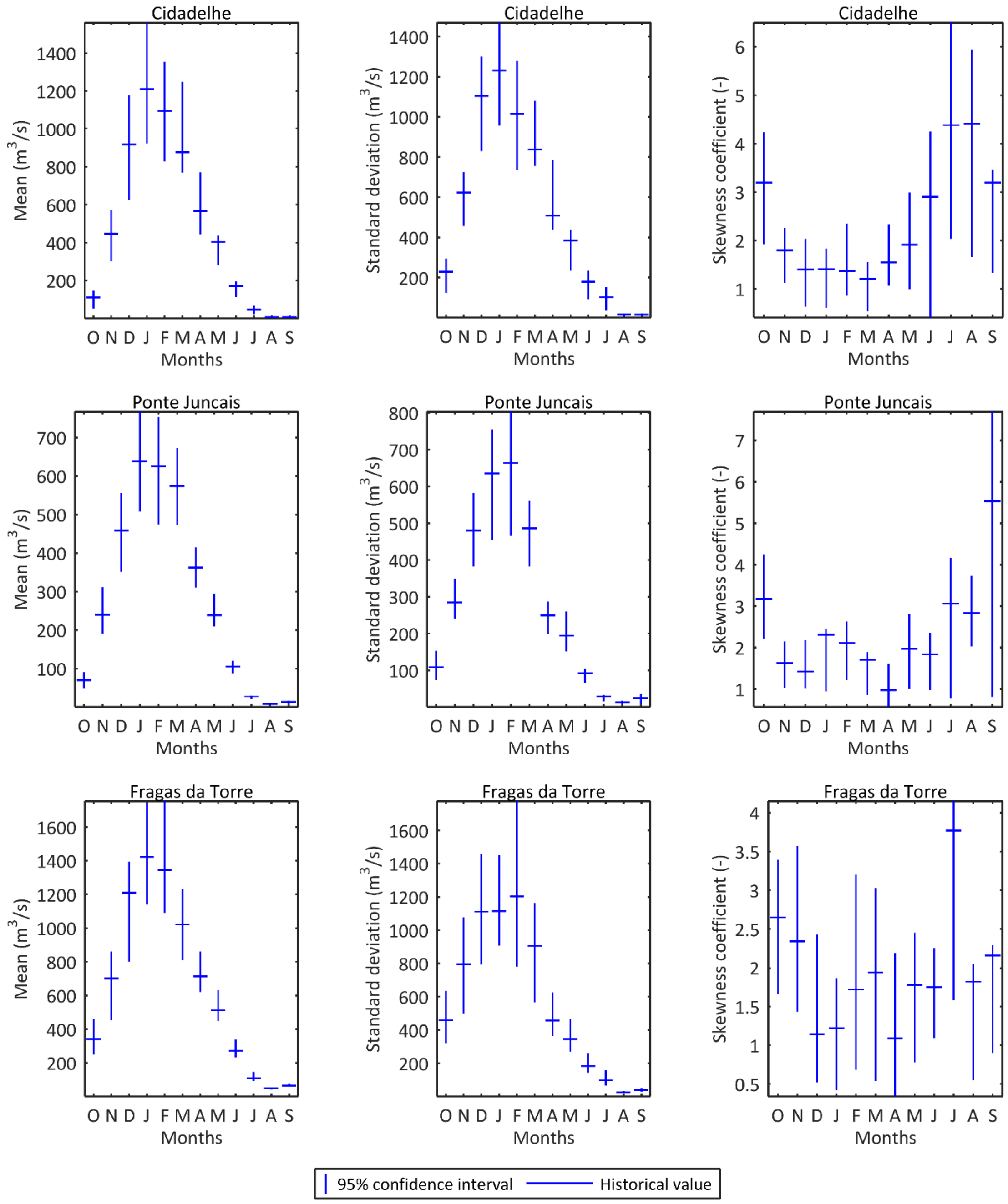

Figure 23: Confidence intervals at 95% of the means, standard deviations and skewness coefficients of the monthly flows in Cidadelhe, Ponte Juncais and Fragas da Torre river gauging stations.

Similarly to the analysis at the monthly level, twenty initial classes of fragments leads to better results in almost all of the samples, except the river gauging stations of Rebordelo (sample nº 7), Vale Giestoso (sample nº 8), Castro Daire (sample nº 9) and Cunhas (sample nº 11). In these cases, it is not clear why an increase in the number of initial classes results in an increase in the number of days without preservation of one or more statistics. One justification is that, increasing the initial number of classes, may result in some unused classes, thus reducing the variability of the synthetic streamflow series, affecting the standard deviation and the skewness coefficient of the synthetic streamflow series. This issue requires further investigation, especially regarding the correct identification of an optimal number of classes.

Table 7: Performance of the model at the daily level for ten and twenty initial classes of fragments.

River gauging station		Number of days without preservation of the historical statistical characteristics							
		10 initial classes of fragments				20 initial classes of fragments			
Nº	**Name**	**Effective number of classes**	**Mean**	**Standard deviation**	**Skewness coefficient**	**Effective number of classes**	**Mean**	**Standard deviation**	**Skewness coefficient**
1	Cidadelhe	10	0	**6**	**6**	18	0	0	**1**
2	Castelo Bom	10	0	0	**2**	13	0	0	0
3	Castanheiro	10	0	**21**	**26**	12	0	**6**	**24**
4	Ponte Tábua	10	0	0	0	20	0	0	**1**
5	Ponte Santa Clara Dão	9	**12**	**24**	**16**	9	**12**	**24**	**16**
6	Ponte Juncais	10	0	0	**4**	20	0	0	0
7	Rebordelo	10	0	0	0	19	0	0	**2**
8	Vale Giestoso	10	0	0	0	19	**2**	0	0
9	Castro Daire	10	0	0	**1**	19	0	0	**4**
10	Cabriz	10	0	0	0	16	0	0	0
11	Cunhas	10	0	0	**2**	20	0	**1**	**2**
12	Ermida Corgo	10	0	0	**2**	18	0	0	0
13	Santa Marta do Alvão	10	0	0	0	19	0	0	0
14	Fragas da Torre	10	0	**15**	**10**	18	0	0	0

5.2. Analysis of the effect of the hydrological variability (hydrological risk) for the global period

5.2.1. General considerations

Once confirmed the good results yielded by the applied generation models, it is possible to perform an analysis on the expected incomes of a small hydropower scheme based on the generated synthetic daily flows. Due to the better results previously achieved, only the case for twenty initial classes of fragments will be considered.

As presented in section 4.2.2, the reference revenue is based on the mean annual turbinated volume that depends on the river regime, which can be synthetized in the form of mean annual daily flow duration curves.

Figure 24 represents, for each sample, the dimensionless mean annual daily flow duration curves obtained from the historical and the 5000 generated synthetic daily flow series, as well as, their two enveloping curves. These last two curves represent the maximum and minimum values of the dimensionless daily flows for each day and provide an expectation regarding the extreme variability of the daily flow regime. In order to allow the comparison among case studies, the duration curves were made dimensionless by referring the mean daily flows to the respective modulus.

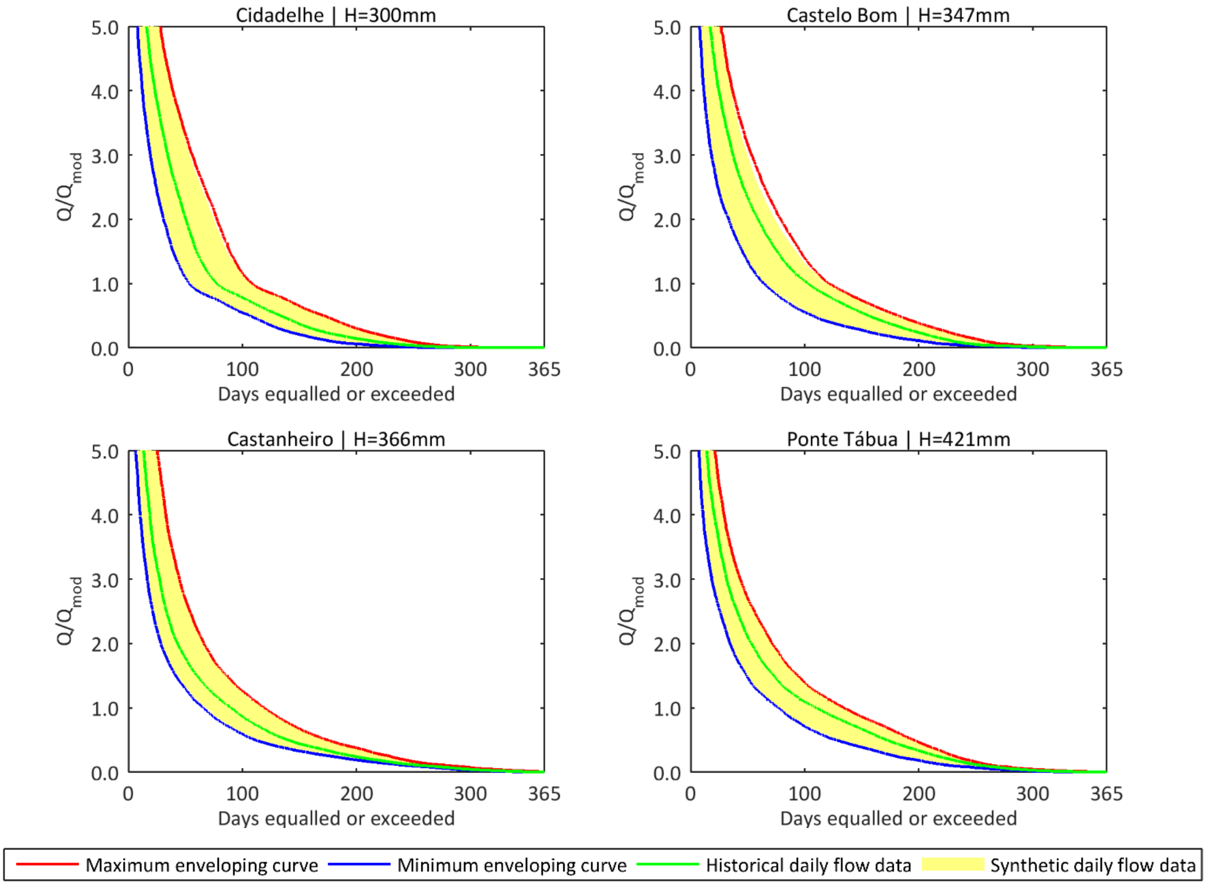

Figure 24: Dimensionless mean annual daily flow curves obtained from the historical and the synthetic series, as well as their enveloping curves for each sample (H stands for mean annual flow depth).

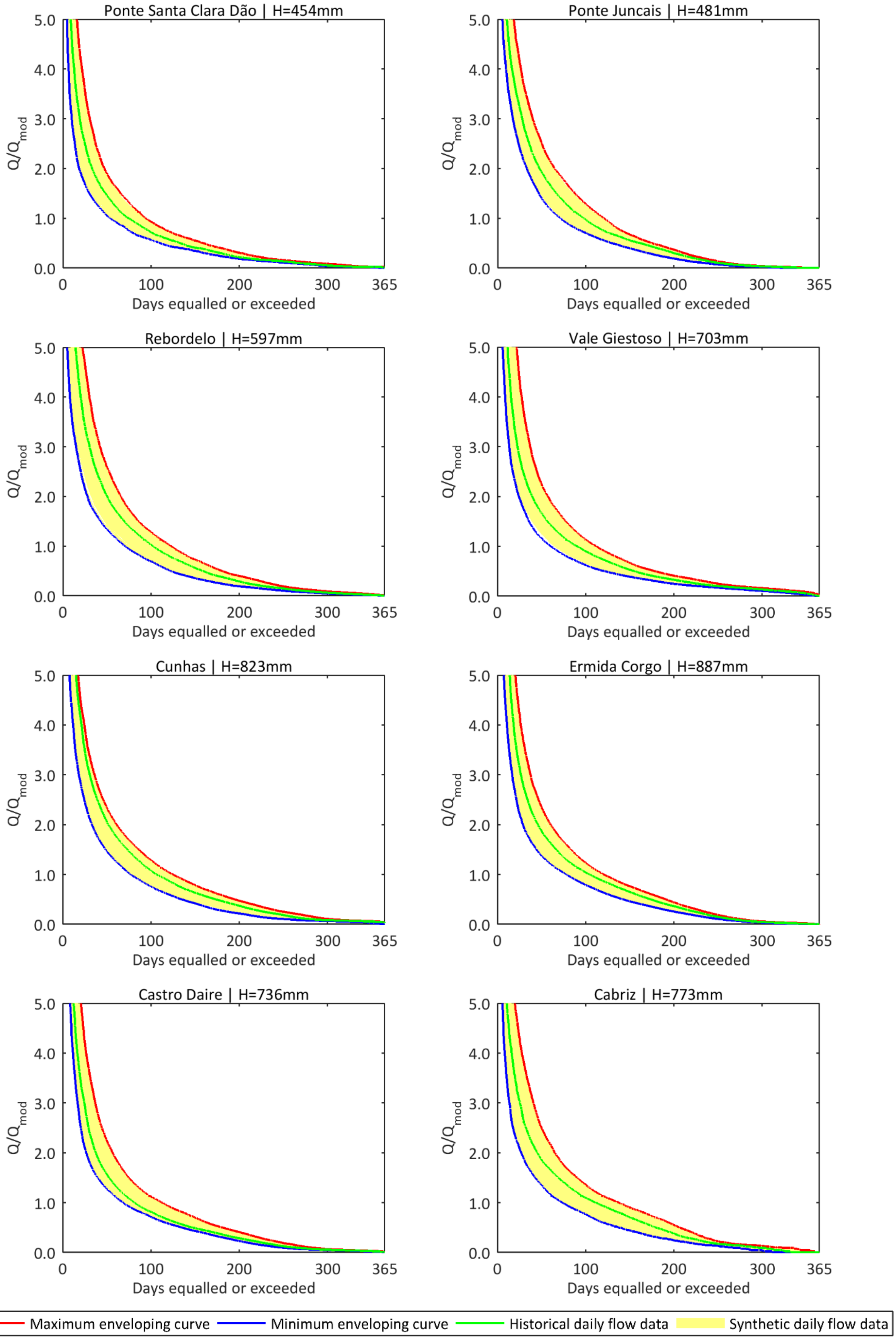

Figure 24 (cont.): Dimensionless mean annual daily flow curves obtained from the historical and the synthetic series, as well as their enveloping curves for each sample (H stands for mean annual flow depth).

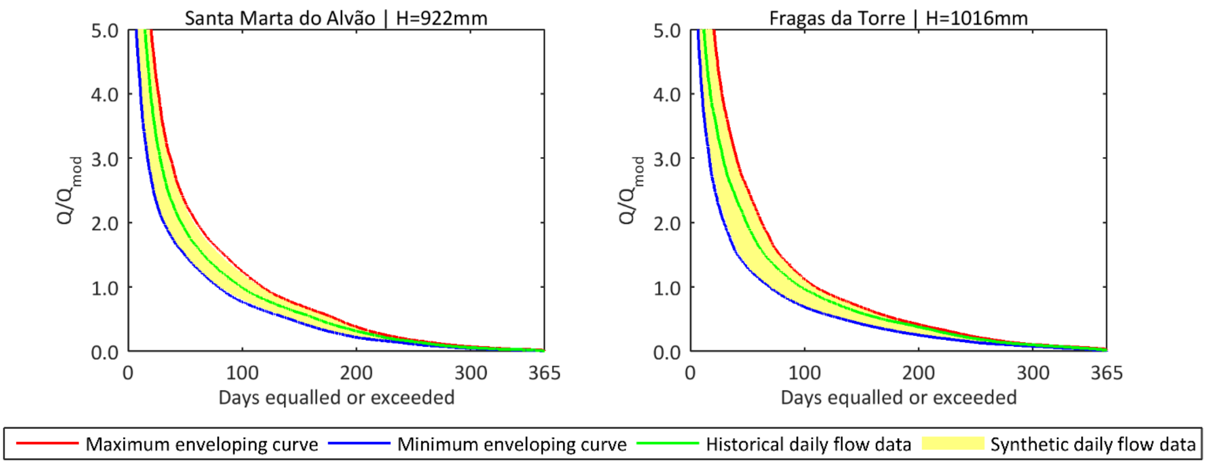

Figure 24 (cont.): Dimensionless mean annual daily flow curves obtained from the historical and the synthetic series, as well as their enveloping curves for each sample (H stands for mean annual flow depth).

As the synthetic streamflow series can be regarded as alternative hydrological events that mimic the historical samples, the distance between each two enveloping curves is nothing but a direct measure of the variability of the hydrological regime at that case study.

Having in mind that the samples presented in the figure above are ranked from the lowest to the highest value of mean annual depth, it is easy to conclude that, for the samples with lower mean annual flow depths and, consequently, higher temporal variabilities, the enveloping curves are further apart. For a better perception, Figure 25, presents the dimensionless mean daily flow duration curves of the river gauging stations of Cidadelhe (sample nº1), with $\overline{H} = 300\ mm$ and Fragas da Torre (sample nº 14), with $\overline{H} = 1016\ mm$. The discrepancy between the area contained within the maximum and minimum daily flow for each duration in these samples is remarkable, being much smaller in the second one, reinforcing the idea that the mean annual flow depth is closely related to the temporal variability of the flow regime.

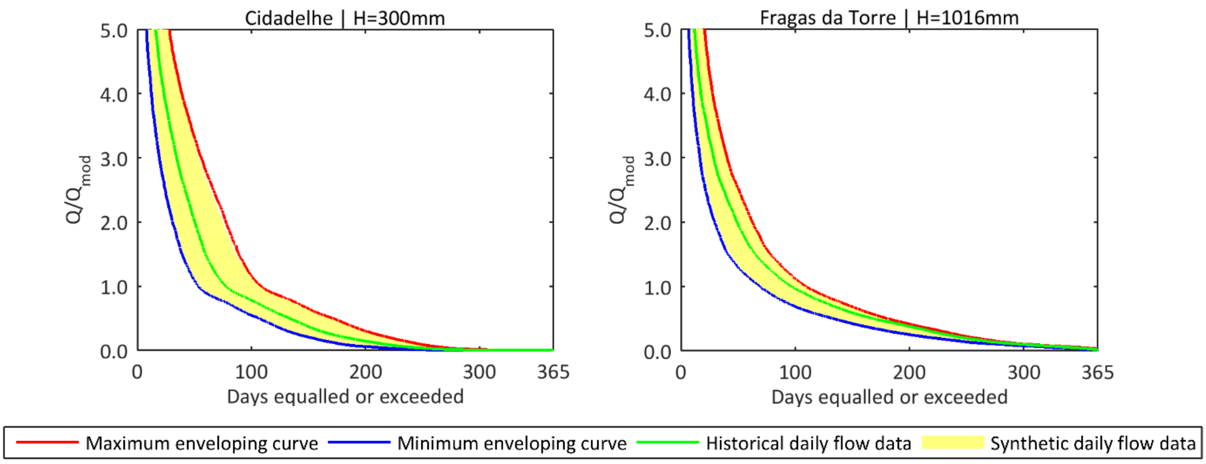

Figure 25: Dimensionless mean daily flow curves obtained from the historical and synthetic series and their enveloping curves for Cidadelhe and Fragas da Torre river gauging stations (H stands for mean annual flow depth).

5.2.2. Dimensionless revenue series provided by the historical series

Once the mean annual turbinated volume and consequently, the reference revenue, are obtained for each case study, the sequence of N annual revenues based on the respective historical series and its cumulative present value were calculated.

In order to allow a comparison between case studies, each one of these last revenues is divided by the correspondent reference revenue leading to dimensionless revenues. Figure 26 represents the ratio between the revenue CPV from the historical series and the reference revenue for the global period of N years in each case study. As expected, the quotient between them is close to one, as the only difference between the procedures applied to obtain the aforementioned revenues is the consideration or not of the temporal variability of the turbinated volumes.

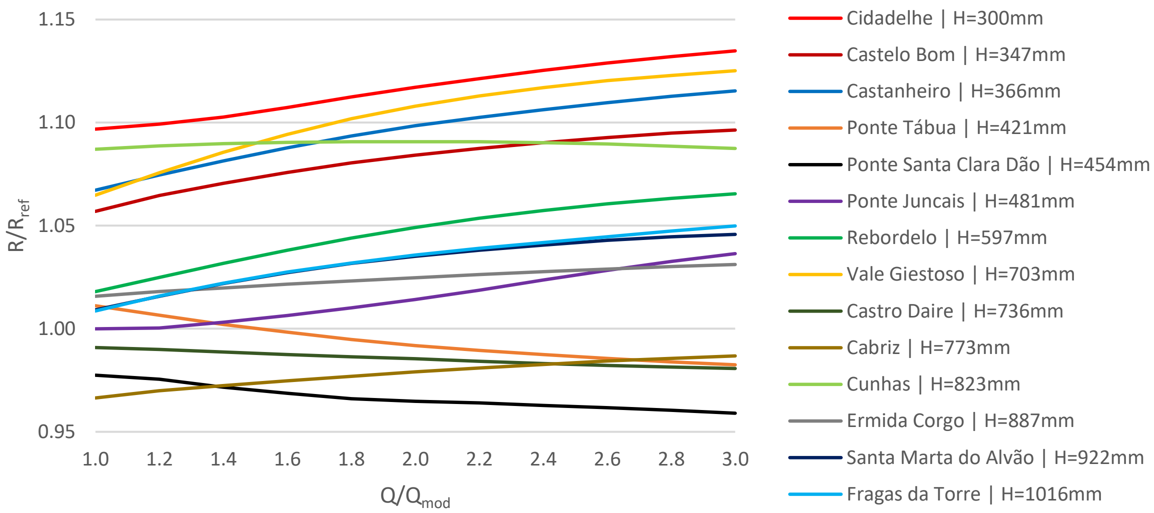

Figure 26: Ratio between the revenue CPV yielded by the historical series and the reference revenue for each case study for the global period (H stands for mean annual flow depth).

In fact, without the additional information derived from the generation of synthetic series or other methods, like the generation of cyclic series (briefly discussed in section 2.3), the results presented in the preceding figure characterized the most relevant constraints that can be extract from the historical sample, regarding the effect of the temporal variability of the flows on the revenues of a SHS.

According to the figure, for example, the construction of a small hydropower scheme located in Cidadelhe river gauging station (sample nº1), for a $Q/Q_{mod} = 2.0$, yields a revenue of about $1.12\ R_{ref}$. By its turn, the construction of a SHS in Fragas da Torre river gauging station (sample nº 14) for the same design discharge would yield to a revenue of about $1.03\ R_{ref}$. These results are related to the effect of the mean annual flow depth, $\overline{H}$ on the relative temporal variability of the river regime: the smaller $\overline{H}$ is (as in Cidadelhe comparatively to Fragas da Torre) the more pronounced the temporal variability is around the average, either upwards or downwards.

However, for each case study, Figure 26 can be regarded as representing one possible hydrological event – the sequence of N years of daily streamflows – and, consequently, only one of the possible patterns of the infinite number of series of N years of daily flows, each producing a given cumulative present value for the revenues. Because the synthetic series are alternative hydrological events as probable as the one really observed, the determination of the incomes produced by those series is a very helpful tool to provide a better characterization of the risk in SHSs profitability due to the temporal variability of the river regime.

5.2.3. Dimensionless revenue series provided by the historical and synthetic series

For each case study, the dimensionless cumulative present values of the revenues based on the historical series of N years of daily flows and on the generated synthetic series, each also with the length of N years, were obtained according to the methodology presented in section 4.2.

Thereby, Table 8 shows, for each sample and for each value of Q/Q_{mod}, the maximum, minimum, average and standard deviation of the dimensionless revenue CPV series. All the series and periods of analysis of the economic analysis that support the results presented in the table have lengths N equal to the lengths of the historical samples.

This table clearly shows that the standard deviation of dimensionless revenue CPV series increases as the temporal variability of the regime increases, consequence of the decrease in the mean annual flow depth, $\overline{H}$. This situation is even more noticeable as the design discharge, Q/Q_{mod}, increases. Likewise, the maximum and minimum values of the dimensionless revenue CPV series also deviate more from the reference value (represented by one in dimensionless terms) as the variability increases, being also more sensitive as Q/Q_{mod} increases.

For a more extensive analysis of the aforesaid, a statistical distribution, namely, the Pearson type III distribution was applied to the above-mentioned samples of dimensionless revenue CPV series for each design discharge. Each one of this series has 5001 values (5000 from the synthetic series plus 1 from the historical sample).

Thereby, Figure 27 represents, for each sample, the cumulative present value of the dimensionless revenue series obtained from the historical and the generated synthetic daily flow series, as well as, their maximum, minimum and mean values. It also shows the dimensionless revenue CPV for non-exceedance probabilities, F, of $99.9\%, 99.0\%, 95.0\%, 50.0\%, 5.0\%, 1.0\%$ and 0.1%. The data used to obtain the figure is presented in Appendix B.

Table 8: Characteristics of the dimensionless revenue cumulative present value series for each sample and Q/Q_{mod} for series and analysis periods with lengths N equal to the lengths of the historical samples.

		Case study													
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Q/Q_{mod}	**Characteristics of the dimensionless revenue cumulative present value series**														
1.0	Maximum	1.254	1.237	1.167	1.154	1.182	1.147	1.202	1.132	1.149	1.168	1.213	1.143	1.131	1.129
	Minimum	0.698	0.740	0.793	0.767	0.841	0.805	0.807	0.808	0.822	0.853	0.791	0.847	0.875	0.815
	Average	0.995	0.983	0.995	0.994	0.999	0.998	1.016	0.975	0.994	1.014	1.017	1.008	1.000	0.990
	Standard deviation	0.077	0.065	0.054	0.053	0.045	0.051	0.052	0.050	0.044	0.048	0.052	0.041	0.036	0.042
1.2	Maximum	1.264	1.250	1.182	1.173	1.206	1.163	1.214	1.153	1.167	1.177	1.229	1.156	1.146	1.149
	Minimum	0.691	0.729	0.776	0.755	0.827	0.791	0.798	0.799	0.811	0.837	0.783	0.834	0.866	0.800
	Average	0.995	0.984	0.995	0.995	0.997	0.998	1.016	0.975	0.993	1.011	1.017	1.007	1.000	0.990
	Standard deviation	0.081	0.069	0.058	0.057	0.049	0.056	0.056	0.055	0.048	0.051	0.055	0.044	0.039	0.046
1.4	Maximum	1.273	1.260	1.195	1.188	1.226	1.180	1.220	1.171	1.183	1.184	1.241	1.166	1.159	1.167
	Minimum	0.687	0.720	0.763	0.745	0.815	0.780	0.790	0.789	0.803	0.824	0.778	0.823	0.856	0.788
	Average	0.996	0.985	0.996	0.996	0.995	0.998	1.016	0.976	0.993	1.009	1.017	1.006	1.000	0.991
	Standard deviation	0.084	0.073	0.062	0.061	0.053	0.060	0.060	0.058	0.051	0.054	0.058	0.048	0.042	0.050
1.6	Maximum	1.287	1.271	1.206	1.200	1.249	1.198	1.224	1.186	1.199	1.191	1.251	1.177	1.169	1.183
	Minimum	0.683	0.713	0.753	0.738	0.805	0.770	0.781	0.782	0.797	0.814	0.773	0.815	0.847	0.779
	Average	0.996	0.986	0.996	0.996	0.994	0.999	1.016	0.976	0.993	1.007	1.017	1.005	0.999	0.992
	Standard deviation	0.087	0.076	0.066	0.064	0.056	0.064	0.063	0.062	0.054	0.057	0.061	0.050	0.045	0.053
1.8	Maximum	1.303	1.280	1.217	1.210	1.270	1.214	1.228	1.200	1.213	1.197	1.258	1.189	1.178	1.196
	Minimum	0.681	0.706	0.745	0.732	0.793	0.761	0.769	0.777	0.792	0.806	0.769	0.808	0.838	0.771
	Average	0.997	0.987	0.996	0.997	0.991	0.999	1.015	0.977	0.992	1.006	1.016	1.004	0.998	0.993
	Standard deviation	0.090	0.079	0.069	0.067	0.059	0.067	0.065	0.065	0.056	0.059	0.063	0.053	0.047	0.055
2.0	Maximum	1.318	1.287	1.227	1.219	1.290	1.229	1.241	1.213	1.226	1.210	1.262	1.199	1.186	1.208
	Minimum	0.675	0.701	0.739	0.727	0.784	0.753	0.758	0.773	0.783	0.799	0.766	0.802	0.831	0.764
	Average	0.998	0.988	0.996	0.997	0.990	1.000	1.015	0.978	0.992	1.005	1.015	1.003	0.998	0.993
	Standard deviation	0.093	0.082	0.072	0.070	0.062	0.070	0.068	0.067	0.059	0.061	0.065	0.055	0.049	0.058

Table 8 (cont.): Characteristics of the dimensionless revenue cumulative present value series for each sample and Q/Q_{mod} for series and analysis periods with lengths N equal to the lengths of the historical samples.

		Case study													
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Q/Q_{mod}	**Characteristics of the dimensionless revenue cumulative present value series**														
2.2	Maximum	1.332	1.293	1.238	1.226	1.309	1.242	1.253	1.225	1.238	1.221	1.266	1.209	1.192	1.219
	Minimum	0.669	0.696	0.734	0.722	0.776	0.747	0.749	0.770	0.775	0.793	0.763	0.797	0.825	0.758
	Average	0.999	0.989	0.997	0.998	0.989	1.000	1.015	0.979	0.992	1.004	1.014	1.002	0.997	0.994
	Standard deviation	0.095	0.084	0.074	0.072	0.064	0.073	0.070	0.070	0.061	0.063	0.066	0.057	0.051	0.060
2.4	Maximum	1.345	1.299	1.251	1.231	1.325	1.254	1.264	1.235	1.249	1.231	1.268	1.218	1.197	1.228
	Minimum	0.663	0.692	0.729	0.718	0.768	0.742	0.741	0.767	0.768	0.787	0.761	0.792	0.819	0.753
	Average	1.000	0.989	0.997	0.998	0.987	1.000	1.015	0.980	0.992	1.004	1.014	1.002	0.997	0.994
	Standard deviation	0.097	0.087	0.076	0.075	0.066	0.075	0.072	0.072	0.063	0.065	0.068	0.058	0.053	0.062
2.6	Maximum	1.357	1.304	1.262	1.236	1.340	1.264	1.274	1.244	1.259	1.240	1.276	1.226	1.202	1.237
	Minimum	0.658	0.688	0.724	0.715	0.762	0.738	0.734	0.766	0.762	0.782	0.760	0.788	0.815	0.749
	Average	1.001	0.990	0.998	0.998	0.986	1.000	1.014	0.981	0.992	1.004	1.013	1.001	0.997	0.995
	Standard deviation	0.099	0.089	0.079	0.076	0.068	0.078	0.074	0.074	0.064	0.066	0.069	0.060	0.054	0.063
2.8	Maximum	1.367	1.309	1.273	1.240	1.351	1.275	1.283	1.253	1.268	1.251	1.282	1.234	1.208	1.246
	Minimum	0.654	0.685	0.720	0.712	0.756	0.735	0.727	0.764	0.757	0.777	0.758	0.784	0.810	0.744
	Average	1.001	0.991	0.998	0.999	0.984	1.000	1.014	0.982	0.992	1.003	1.012	1.001	0.997	0.996
	Standard deviation	0.101	0.091	0.081	0.078	0.070	0.080	0.076	0.075	0.066	0.067	0.070	0.061	0.055	0.065
3.0	Maximum	1.377	1.313	1.282	1.244	1.360	1.284	1.290	1.260	1.277	1.260	1.288	1.241	1.215	1.254
	Minimum	0.649	0.683	0.717	0.709	0.750	0.733	0.721	0.762	0.752	0.772	0.757	0.781	0.806	0.741
	Average	1.002	0.992	0.999	0.999	0.982	1.000	1.014	0.983	0.992	1.003	1.012	1.000	0.997	0.996
	Standard deviation	0.103	0.092	0.083	0.080	0.072	0.081	0.077	0.077	0.067	0.069	0.071	0.062	0.057	0.066

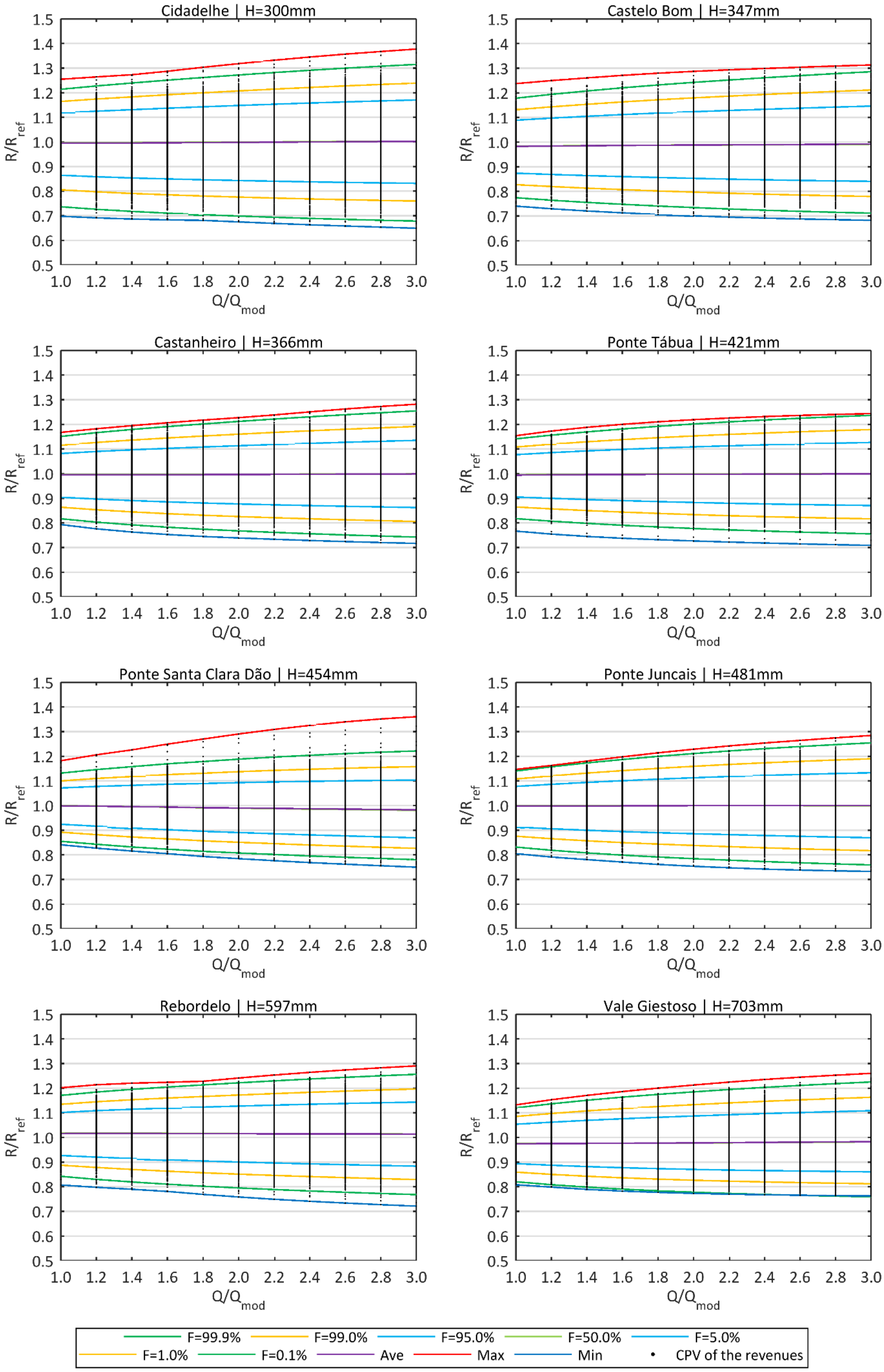

Figure 27: Present values of the dimensionless revenue CPV series obtained from the historical and the generated synthetic daily flow series: maximum and minimum values, average and estimates for different non-exceedance probabilities, F, for each Q/Q_{mod} and for each sample (H stands for mean annual flow depth).

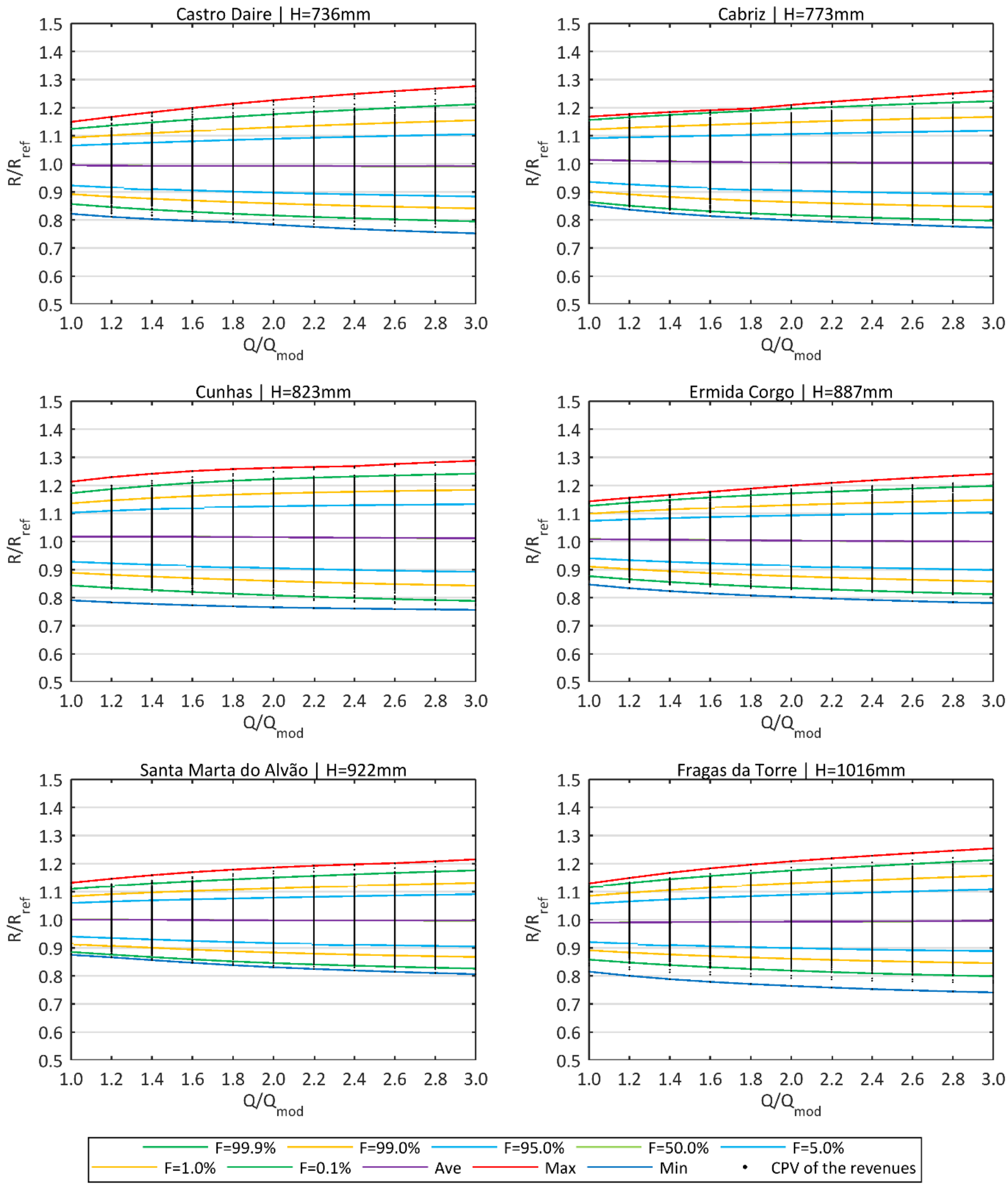

Figure 27 (cont.): Present values of the dimensionless revenue CPV series obtained from the historical and the generated synthetic daily flow series: maximum and minimum values, average and estimates for different non-exceedance probabilities, F, for each Q/Q_{mod} and for each sample (H stands for mean annual flow depth).

Figure 27 shows that, in every sample, the average of the revenues obtained from the aforementioned series is almost equal to the reference revenue. Likewise, the revenue associated to a 50% non-exceedance probability, i.e., the median of the revenues, is almost equal to the average of the revenues, which suggests the good adjustment of the Pearson III distribution to the samples. In some cases, and possibly due to the variability of the regime, the average and median of the incomes can be slightly different from the reference revenue, as in Rebordelo (sample nº 7), Vale Giestoso (sample nº 8) and Cunhas (sample nº 11) cases studies, highlighting the importance of the use of synthetic series to properly perform a risk analysis on SHS.

For all case studies, the incomes associated to 95% and 5% non-exceedance probabilities are around $1.1R_{ref}$ and $0.9R_{ref}$, respectively, indicating that the probability of yielding revenues 10% bigger or smaller than the reference revenue is only around 5%.

Based on the previous results it can be said that the design of a small hydropower plant based on the reference revenue – the most common way to project a SHS – provides an accurate enough approach, provided that the economic analysis period coincides with the recording period and that this last period is long enough to fully capture the temporal variability of the river regime.

Figure 27 also shows that the analysis based on synthetic series allows the determination of the maximum and minimum incomes, as well as their variation with Q/Q_{mod}.

In fact, the greater the design discharge, the greater the probability of obtaining revenues further away from the reference ones. It is also noteworthy that for a fixed value of the ratio Q/Q_{mod}, the previous amplitude decreases as the region is more humid, i.e., as the mean annual flow depth increases.

This situation is even more noticeable in Figure 28, where it is represented, for three different values of Q/Q_{mod}, the maximum, minimum, average and non-exceedance probabilities of the dimensionless revenue CPV series for all case studies.

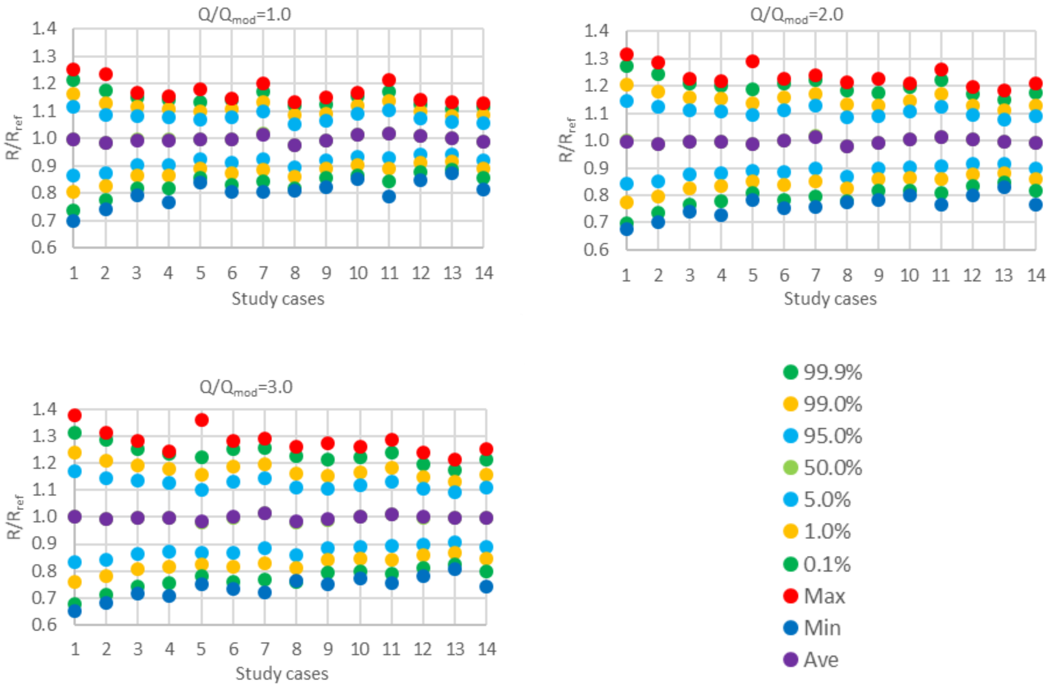

Figure 28: Maximum, minimum, mean and non-exceedance probabilities of the dimensionless revenue CPV series obtained from the historical and the generated synthetic daily flow series for each sample and for a) $Q/Q_{mod} = 1.0$, b) $Q/Q_{mod} = 2.0$ and c) $Q/Q_{mod} = 3.0$

Once again, Ponte Santa Clara Dão (sample nº 5) is clearly an exception, with maximum incomes definitely higher than the ones for river gauging stations with similar flow depths. However, this was the sample with the lowest performance of the generation model, that is, with the highest percentage of non-preserved days in what concerns the statistical characteristics (see section 5.1), which may indicate that the sample may have any problem that requires a more detail analysis of its behavior.

If one takes, for example, Cidadelhe river gauging station (sample n. º 1), Figure 26 indicates that the cumulative present value of the revenues that results from the historical sample for a design discharge of twice the modulus, $Q/Qmod = 2.0$, is $1.12R_{ref}$. However, Figure 27 and Figure 28 indicate that the non-exceedance probability of such income is approximately 95%. This means that, the probability of having incomes equal or higher than $1.12R_{ref}$ is only about 5%, which suggest that the design of the SHS based on that revenue may not be a good criteria, as it may overestimate the expected incomes, at least for a period of the economic analysis equal to the recording period.

5.3. Analysis of the effect of the hydrological variability (hydrological risk) for subperiods

As discussed in section 4.3, the analysis of the results from the exploitation of a SHS based on the assumption of a period larger than the expectable duration of the licensing one may lead to an unrealistic perception of the economic benefits of the scheme.

Furthermore, the smallest the period is, the more susceptible the incomes from the SHS become to the temporal variability of the flow regime, meaning that the first years are critical for the economic feasibility of the investment.

In this way, it was decided to compute the income series for two scenarios of the length of the licensing period, N^*, and to compare the results thus achieved taking into account the temporal variability of the flow regime with the reference revenue for the same period: $N^* = 25$ and $N^* = 10$ years.

As referred before, until a few years ago, 35 years was a common licensing period. This period was subsequently reduced to 15 years eventually plus 10 years[48], and more recently, to 20 years eventually plus 5 years[49]. The decree-laws that supported the previous periods are no longer in force and the replacing legislation was not yet produced[50].

Figure 29 represents, for each sample and value of Q/Q_{mod}, the average of the maximum and minimum dimensionless revenue CPV series for sequences of $N^* = 10$ and $N^* = 25$ years.

These dimensionless revenues are obtained by identifying, for each $M + 1$ daily flow series (the M synthetic series plus the historical one), the sequences of N^* years that lead to the maximum and minimum cumulative present value of the revenue. The data utilized in the figure is presented in Appendix C. The mean annual energy utilized to define the reference revenue in each case study refers to the global period of records. The corresponding cumulative present of the income was computed based on that energy and on the length $N*$.

Figure 29 confirms that, the smaller the exploitation period is, the more susceptible the SHS becomes to the temporal variability of the flow regime, as the expected incomes deviate more from the reference revenues. It is also possible to conclude that, similarly to Figure 27 and Figure 28, as the temporal variability of the regime decreases (higher $\overline{H}$), the variability of the dimensionless incomes decreases. Likewise, the greater the design discharge, Q/Q_{mod}, the greater the variability of the incomes.

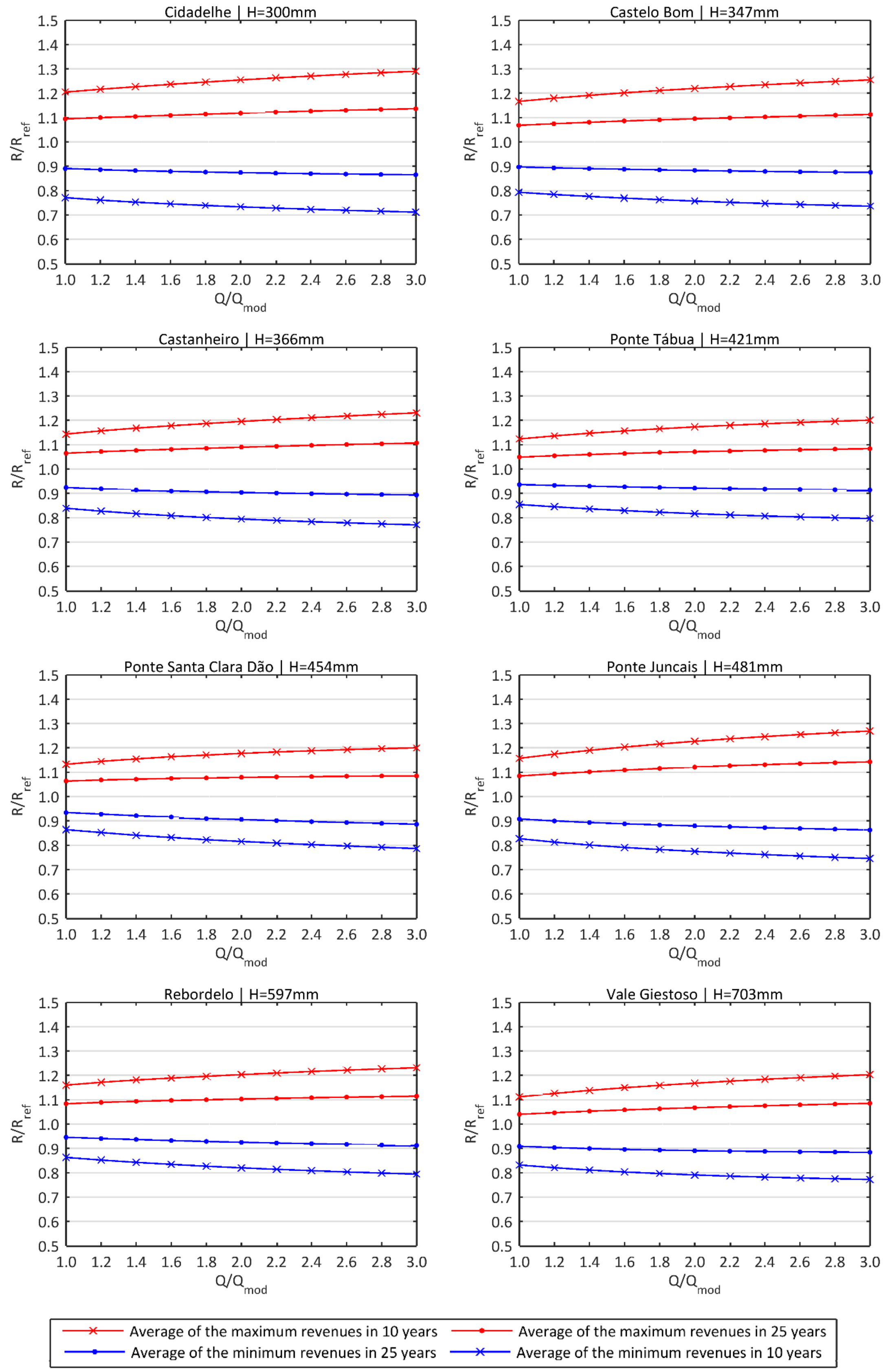

Figure 29: Average of the maximum and minimum dimensionless revenue CPV series for $N^* = 10$ and $N^* = 25$ years, for each sample and Q/Q_{mod}. (H stands for mean annual flow depth).

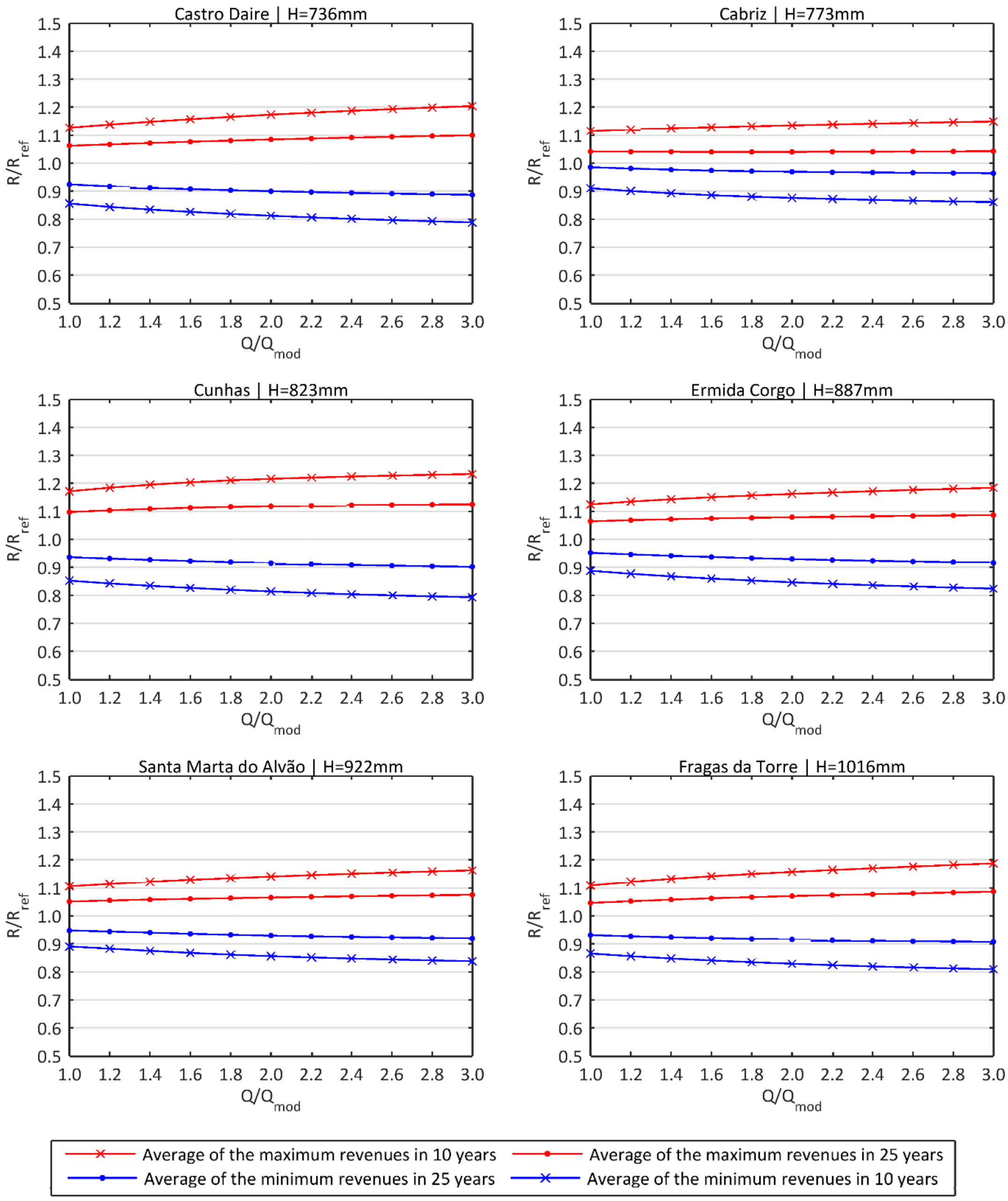

Figure 29 (cont.): Average of the maximum and minimum dimensionless revenue CPV series for $N^* = 10$ and $N^* = 25$ years, for each sample and Q/Q_{mod}. (H stands for mean annual flow depth).

Another important aspect is that the effect of the temporal variability in the expected incomes comparatively to the incomes based on the reference income is more pronounced as the length of the economic analysis period, N^*, decreases comparatively to the length of the recording period, N, utilized to compute the constant mean annual production that defines the reference income. This is the case, for instance, of Castro Daire river gauging station (sample nº 9, with 59 years of records), and Cabriz river gauging station (sample nº 10, with 31 years of records). In the first sample, comparatively to the second one, the differences between the reference income and the average of the best and worst income series for both N^* of 10 and 25 years is greater. The difference between the mean annual flow depths in those two stations is almost negligible ($\Delta\overline{H} = 37\text{mm}$), and, accordingly, it cannot be the reason for the difference between the incomes.

Based on the preceding figure, and due to the consideration of synthetic series, it is possible to analyze the impact on the revenue of a SHS of an exploitation period equal to the last legal licensing period (20+5 years).

Taking for example, Cidadelhe river gauging station (sample nº1), Figure 26 indicates that the revenue CPV resulting from the historical sample for a design discharge of $2.0Q_{mod}$ is $1.12R_{Ref}$. However, in Figure 29 it is possible to see that this value corresponds to approximately the average of the best incomes for a licensing period of $N^{*} = 25$ years and definitely not the average of the incomes for a 25 years' period. In other words, the cumulative present value of the revenue based only on the historical series is of the same magnitude as the average of the maximum cumulative present values of the revenues in periods of $N^{*} = 25$ years, which suggests that the design of the SHS based on that revenue may overestimate the expected incomes.

6. Conclusions and future developments

6.1. Conclusions

In this dissertation, it was assessed the effect of the natural streamflow variability in the expected incomes of run-of-river small hydropower schemes. Given the stochastic nature of the problem and due to previous studies that proved the good results of the method, the study utilized synthetic series.

It was developed and tested a procedure for generating annual and daily synthetic streamflow series, which encompasses a probabilistic model, namely the log-Pearson III distribution at the annual level (which prevents the generation of negative flows) and a disaggregation model, specifically, the method of fragments, at the daily level. The procedure was applied to the samples of daily streamflows at 14 Portuguese river gauging stations with different recording periods, resulting in the analysis of around 255 thousand daily flows, which were utilized to generate more than 1 billion of synthetic daily flows.

Similarly to previous studies, the probabilistic model used for generating the annual flows proved to be effective, since its application to the different samples always resulted in the preservation of all the main historical statistics.

The disaggregation model used to disaggregate the annual flows into daily flows – the method of fragments – also proved its efficiency by preserving most of the historical statistical characteristics at this time level, especially for the larger initial number of classes of fragments. The synthetic monthly flow series, obtained by the accumulation of the synthetic daily flows also ensured the preservation of most of the historical statistics, proving the additivity between daily and monthly flows without requiring any additional model.

In sum, the proposed two-level generation model succeeded to preserve the main statistical characteristics of the historical samples, namely the mean, standard deviation and skewness coefficient, in most of the cases, thus confirming its capacity for generating, not only annual, but also monthly and daily synthetic flow series.

Based on the good results provided by the model, an algorithm to simulate the daily exploitation of run-of-river small hydropower schemes was applied to both the historical and the generated synthetic daily series for different design discharges, obtaining the daily incomes that results from the selling of the energy produced in each one of those series. By accumulating, for the global period, the daily incomes, the annual incomes were obtained as well as their cumulative present values (CPV), by means of economic analysis criteria. In each case study, the series of the CPV of the incomes has as many values as the number of synthetic flow series, plus one – the CPV that results from the sample. These series were analyzed based on the Pearson III distribution, aiming at assigning non-exceedance probabilities to the cumulative present value of the revenues.

In order to evaluate the susceptibility of the SHSs to different exploitation periods and, especially, to adverse hydrological conditions in the first years of exploitation, the aforementioned simulation was applied to two periods shorter than the global one, namely 10 and 25 years.

Due to the enormous number of synthetic series generated, the study carried out became much more general than the previous ones and allowed some new conclusions.

In what concerns the natural variability of the flow regime, it was possible to conclude that as it increases (due to the decrease in the mean annual flow depth), the variability of the expected revenues also increases, which is particularly visible as the design discharge also increases.

Nevertheless, the average of the CPV of the revenues for the different design discharges considered are of the same order of magnitude of the reference revenues for almost all case studies. Furthermore, the probability of obtaining revenues 10% bigger/smaller than the reference revenue is only 5%, which indicates a small risk when designing a SHS based on the reference revenue.

In what concerns the effect of the exploitation period on the susceptibility of SHS, it is possible to conclude that, the smaller the period of exploitation, the higher the variability of the CPV of the incomes. Even for the case studies with higher mean annual flow depths and, consequently, with more regular river regime, the revenues when considering an exploitation period of 10 years could be 20% higher/lower than the reference ones.

In a general way, it is possible to conclude that, if the period of exploitation is long enough and close to the historical one, the synthetic series do not add much information to the historical series, yielding to revenues almost equal to the reference ones. This is a direct consequence of the preservation by the models of the mean of the daily streamflows and of all the derived entities, such as the reference revenues, which only account for that mean. Nevertheless, the analysis based on the synthetic series allows a much more detailed characterization of the temporal variability of the revenues.

So, as a general conclusion, it may be stated that the reference revenue is a good estimation for the design a small hydropower scheme with run-of-river exploitation provided that the recoding period to which the daily flows refer is long enough and close to the licensing period. However, a study based on synthetic flows series should be performed to better characterize the temporal variability of the expected incomes. This issue becomes crucial for small exploitation periods, because the consequences of the temporal variability of the river flows are no longer negligible. As the mean annual flow depth decreases, denoting a more irregular flow regime, the run-of-river scheme becomes even more susceptible.

6.2. Future developments

It should be stressed that the results briefly presented for exploitation periods smaller than the recording ones only took into account average conditions. However, a statistical analysis could also be applied to the cumulative present value of the revenue series under those conditions in order to better evaluate the susceptibility of a SHS to the length of the exploitation period. This can be part of future developments of the research initiated with this thesis.

Although the present study was focused on Portuguese river gauging stations, it is believed that in other European regions, especially in the south of Europe, where the hydrological regime is similar to the Portuguese, analogous procedures may be developed in order to obtain similar type of results. Furthermore, these procedures could also be tested in countries with hydrological characteristics different from those prevailing in Portugal.

It would be very interesting to apply this procedure to a real case study in order to properly compare the cumulative present value of the revenues obtained on that SHS with the ones that result from the design considering daily synthetic streamflow series.

Some of the assumptions made in the current research are not realistic, as is the case of the assumption related to the selling price of the energy produced. Therefore, it would be also convenient to accomplish an analysis considering a more realistic structure for that price. For the time being this is not possible due to the lack of applicable legislation.

As a last possible, yet challenging future development, it would be interesting to explore the capability of the applied methodology to take into account some of the expected consequences of climate change. Theoretically, that could be done at the annual level, namely by applying a model capable of expressing the increase/decrease of the annual flows and/or the change in their inter annual variability. For the disaggregation model, this is, for the daily flows and their temporal variability, the subject would require extensive additional analysis.

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Appendix A

Basic statistical parameters

The most relevant basic statistical parameters of a series of N random variables, x_i, used in this dissertation are presented as it follows:

1. The mean, $\bar{x}$, is a measure of central tendency of a series with length N:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{38}$$

2. The standard deviation, s_x, is a measure of the degree of variation of a data set around its mean value. The unbiased estimation of the standard deviation is obtained by:

$$s_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{39}$$

3. The skewness coefficient, g_x, is a measure of the asymmetry of the probability distribution function (PDF) of a data set, this is, the asymmetry of the distribution of the values of x_i. The unbiased estimation of this parameter is obtained by:

$$g_x = \frac{N}{(N-1)(N-2)s_x^3}\sum_{i=1}^{N}(x_i - \bar{x})^3 \tag{40}$$

Appendix B

Non-exceedance probability for the cumulative present value of the dimensionless revenue series for each sample and each Q/Q_{mod}.

	Q_{mod}	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
	$P(X < x)$	**Non-exceedance probability for the CPV of the dimensionless revenue series**										
Cidadelhe	99.9%	1.214	1.227	1.239	1.251	1.262	1.272	1.282	1.291	1.300	1.308	1.315
	99.0%	1.164	1.174	1.183	1.192	1.200	1.207	1.214	1.221	1.228	1.233	1.239
	95.0%	1.118	1.125	1.131	1.137	1.143	1.148	1.153	1.158	1.163	1.167	1.170
	50.0%	0.998	0.998	0.998	0.998	0.999	1.000	1.000	1.001	1.002	1.002	1.003
	5.0%	0.865	0.858	0.853	0.850	0.846	0.843	0.840	0.838	0.836	0.833	0.832
	1.0%	0.805	0.797	0.790	0.785	0.780	0.776	0.772	0.768	0.765	0.762	0.760
	0.1%	0.737	0.726	0.718	0.711	0.704	0.699	0.694	0.689	0.685	0.682	0.678
	Average	0.995	0.995	0.996	0.996	0.997	0.998	0.999	1.000	1.001	1.001	1.002
	Maximum	1.254	1.264	1.273	1.287	1.303	1.318	1.332	1.345	1.357	1.367	1.377
	Minimum	0.698	0.691	0.687	0.683	0.681	0.675	0.669	0.663	0.658	0.654	0.649
Castelo Bom	99.9%	1.178	1.194	1.207	1.220	1.232	1.242	1.252	1.261	1.270	1.278	1.286
	99.0%	1.131	1.143	1.153	1.163	1.172	1.179	1.187	1.193	1.200	1.206	1.211
	95.0%	1.089	1.097	1.105	1.112	1.118	1.123	1.128	1.133	1.138	1.142	1.146
	50.0%	0.983	0.985	0.986	0.987	0.987	0.988	0.988	0.989	0.990	0.991	0.991
	5.0%	0.874	0.869	0.864	0.860	0.856	0.853	0.850	0.847	0.845	0.842	0.841
	1.0%	0.827	0.820	0.813	0.807	0.802	0.797	0.792	0.789	0.785	0.782	0.779
	0.1%	0.774	0.764	0.755	0.748	0.741	0.734	0.728	0.723	0.719	0.715	0.712
	Average	0.983	0.984	0.985	0.986	0.987	0.988	0.989	0.989	0.990	0.991	0.992
	Maximum	1.237	1.250	1.260	1.271	1.280	1.287	1.293	1.299	1.304	1.309	1.313
	Minimum	0.740	0.729	0.720	0.713	0.706	0.701	0.696	0.692	0.688	0.685	0.683
Castanheiro	99.9%	1.151	1.166	1.179	1.191	1.201	1.212	1.221	1.230	1.239	1.247	1.255
	99.0%	1.115	1.126	1.136	1.145	1.152	1.160	1.167	1.174	1.180	1.186	1.191
	95.0%	1.082	1.090	1.097	1.102	1.108	1.113	1.118	1.123	1.127	1.131	1.135
	50.0%	0.996	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.999
	5.0%	0.904	0.897	0.891	0.885	0.881	0.877	0.873	0.870	0.867	0.865	0.862
	1.0%	0.864	0.853	0.845	0.837	0.831	0.826	0.821	0.817	0.813	0.809	0.806
	0.1%	0.817	0.803	0.792	0.782	0.774	0.767	0.761	0.756	0.751	0.747	0.742
	Average	0.995	0.995	0.996	0.996	0.996	0.996	0.997	0.997	0.998	0.998	0.999
	Maximum	1.167	1.182	1.195	1.206	1.217	1.227	1.238	1.251	1.262	1.273	1.282
	Minimum	0.793	0.776	0.763	0.753	0.745	0.739	0.734	0.729	0.724	0.720	0.717

	Q_{mod}	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
	$P(X < x)$	Non-exceedance probability for the CPV of the dimensionless revenue series										
Ponte Tábua	99.9%	1.141	1.156	1.169	1.181	1.192	1.201	1.210	1.218	1.225	1.231	1.236
	99.0%	1.108	1.119	1.129	1.138	1.146	1.153	1.159	1.165	1.170	1.174	1.179
	95.0%	1.077	1.085	1.092	1.098	1.104	1.109	1.113	1.117	1.120	1.124	1.126
	50.0%	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
	5.0%	0.905	0.900	0.895	0.890	0.886	0.883	0.880	0.877	0.875	0.872	0.870
	1.0%	0.865	0.857	0.850	0.844	0.838	0.834	0.829	0.825	0.822	0.819	0.816
	0.1%	0.817	0.807	0.798	0.790	0.783	0.777	0.772	0.767	0.763	0.759	0.755
	Average	0.994	0.995	0.996	0.996	0.997	0.997	0.998	0.998	0.998	0.999	0.999
	Maximum	1.154	1.173	1.188	1.200	1.210	1.219	1.226	1.231	1.236	1.240	1.244
	Minimum	0.767	0.755	0.745	0.738	0.732	0.727	0.722	0.718	0.715	0.712	0.709
Ponte Santa Clara Dão	99.9%	1.131	1.146	1.158	1.169	1.178	1.188	1.197	1.204	1.211	1.217	1.221
	99.0%	1.100	1.110	1.118	1.125	1.131	1.137	1.143	1.147	1.151	1.155	1.158
	95.0%	1.071	1.077	1.082	1.087	1.089	1.093	1.096	1.098	1.100	1.102	1.103
	50.0%	0.999	0.998	0.995	0.994	0.991	0.989	0.987	0.985	0.984	0.982	0.980
	5.0%	0.924	0.916	0.908	0.902	0.895	0.890	0.886	0.881	0.877	0.873	0.869
	1.0%	0.892	0.882	0.872	0.865	0.857	0.851	0.845	0.840	0.835	0.831	0.826
	0.1%	0.855	0.843	0.832	0.823	0.814	0.808	0.801	0.795	0.790	0.785	0.780
	Average	0.999	0.997	0.995	0.994	0.991	0.990	0.989	0.987	0.986	0.984	0.982
	Maximum	1.182	1.206	1.226	1.249	1.270	1.290	1.309	1.325	1.340	1.351	1.360
	Minimum	0.841	0.827	0.815	0.805	0.793	0.784	0.776	0.768	0.762	0.756	0.750
Ponte Juncais	99.9%	1.140	1.157	1.173	1.187	1.200	1.211	1.221	1.231	1.239	1.247	1.254
	99.0%	1.108	1.120	1.132	1.142	1.151	1.159	1.167	1.173	1.179	1.185	1.190
	95.0%	1.078	1.086	1.094	1.101	1.107	1.113	1.118	1.122	1.126	1.130	1.133
	50.0%	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	0.999	0.999
	5.0%	0.913	0.906	0.900	0.894	0.890	0.886	0.882	0.878	0.875	0.872	0.869
	1.0%	0.876	0.865	0.857	0.850	0.843	0.838	0.833	0.828	0.824	0.820	0.817
	0.1%	0.831	0.818	0.808	0.799	0.791	0.784	0.778	0.773	0.767	0.763	0.759
	Average	0.998	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	Maximum	1.147	1.163	1.180	1.198	1.214	1.229	1.242	1.254	1.264	1.275	1.284
	Minimum	0.805	0.791	0.780	0.770	0.761	0.753	0.747	0.742	0.738	0.735	0.733
Rebordelo	99.9%	1.171	1.184	1.195	1.205	1.213	1.222	1.230	1.237	1.244	1.250	1.256
	99.0%	1.135	1.145	1.153	1.160	1.166	1.172	1.178	1.183	1.188	1.193	1.197
	95.0%	1.101	1.109	1.114	1.119	1.123	1.127	1.131	1.135	1.138	1.141	1.143
	50.0%	1.017	1.017	1.017	1.017	1.016	1.016	1.016	1.015	1.015	1.014	1.014
	5.0%	0.927	0.920	0.915	0.909	0.904	0.900	0.896	0.893	0.890	0.886	0.883
	1.0%	0.887	0.878	0.870	0.863	0.857	0.851	0.846	0.841	0.837	0.833	0.829
	0.1%	0.842	0.830	0.819	0.810	0.802	0.795	0.789	0.783	0.777	0.773	0.768
	Average	1.016	1.016	1.016	1.016	1.015	1.015	1.015	1.015	1.014	1.014	1.014
	Maximum	1.202	1.214	1.220	1.224	1.228	1.241	1.253	1.264	1.274	1.283	1.290
	Minimum	0.807	0.798	0.790	0.781	0.769	0.758	0.749	0.741	0.734	0.727	0.721

	Q_{mod}	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
	$P(X < x)$	Non-exceedance probability for the CPV of the dimensionless revenue series										
Vale Giestoso	99.9%	1.120	1.136	1.151	1.164	1.175	1.185	1.195	1.203	1.211	1.218	1.225
	99.0%	1.085	1.097	1.108	1.117	1.126	1.133	1.140	1.147	1.152	1.158	1.163
	95.0%	1.054	1.062	1.069	1.076	1.082	1.087	1.092	1.097	1.101	1.105	1.109
	50.0%	0.975	0.975	0.976	0.976	0.977	0.978	0.978	0.979	0.980	0.981	0.982
	5.0%	0.894	0.887	0.882	0.877	0.873	0.870	0.868	0.866	0.864	0.862	0.861
	1.0%	0.859	0.850	0.843	0.836	0.831	0.826	0.823	0.819	0.817	0.814	0.812
	0.1%	0.820	0.808	0.798	0.790	0.783	0.778	0.773	0.769	0.765	0.762	0.759
	Average	0.975	0.975	0.976	0.976	0.977	0.978	0.979	0.980	0.981	0.982	0.983
	Maximum	1.132	1.153	1.171	1.186	1.200	1.213	1.225	1.235	1.244	1.253	1.260
	Minimum	0.808	0.799	0.789	0.782	0.777	0.773	0.770	0.767	0.766	0.764	0.762
Castro Daire	99.9%	1.124	1.136	1.147	1.158	1.167	1.176	1.185	1.192	1.199	1.206	1.212
	99.0%	1.093	1.101	1.109	1.117	1.124	1.130	1.136	1.141	1.146	1.151	1.155
	95.0%	1.065	1.070	1.075	1.080	1.085	1.089	1.093	1.096	1.099	1.103	1.105
	50.0%	0.995	0.994	0.993	0.993	0.992	0.992	0.992	0.991	0.991	0.991	0.991
	5.0%	0.923	0.915	0.910	0.905	0.901	0.897	0.894	0.891	0.888	0.886	0.883
	1.0%	0.892	0.883	0.875	0.869	0.864	0.859	0.854	0.850	0.847	0.844	0.841
	0.1%	0.857	0.846	0.837	0.829	0.822	0.816	0.811	0.806	0.802	0.798	0.795
	Average	0.994	0.993	0.993	0.993	0.992	0.992	0.992	0.992	0.992	0.992	0.992
	Maximum	1.149	1.167	1.183	1.199	1.213	1.226	1.238	1.249	1.259	1.268	1.277
	Minimum	0.822	0.811	0.803	0.797	0.792	0.783	0.775	0.768	0.762	0.757	0.752
Cabriz	99.9%	1.156	1.166	1.174	1.181	1.189	1.196	1.202	1.208	1.213	1.218	1.223
	99.0%	1.122	1.128	1.134	1.138	1.143	1.148	1.152	1.156	1.160	1.164	1.167
	95.0%	1.091	1.094	1.097	1.100	1.103	1.106	1.109	1.111	1.114	1.116	1.118
	50.0%	1.014	1.011	1.009	1.007	1.006	1.005	1.004	1.003	1.003	1.003	1.003
	5.0%	0.935	0.927	0.919	0.913	0.909	0.905	0.901	0.898	0.896	0.893	0.891
	1.0%	0.902	0.891	0.882	0.874	0.868	0.864	0.859	0.856	0.852	0.849	0.847
	0.1%	0.864	0.851	0.840	0.831	0.824	0.818	0.813	0.808	0.804	0.800	0.797
	Average	1.014	1.011	1.009	1.007	1.006	1.005	1.004	1.004	1.004	1.003	1.003
	Maximum	1.168	1.177	1.184	1.191	1.197	1.210	1.221	1.231	1.240	1.251	1.260
	Minimum	0.853	0.837	0.824	0.814	0.806	0.799	0.793	0.787	0.782	0.777	0.772
Cunhas	99.9%	1.172	1.187	1.199	1.209	1.217	1.222	1.227	1.232	1.235	1.239	1.242
	99.0%	1.136	1.146	1.155	1.162	1.167	1.171	1.175	1.177	1.180	1.182	1.184
	95.0%	1.103	1.110	1.115	1.120	1.123	1.126	1.128	1.129	1.131	1.132	1.133
	50.0%	1.018	1.018	1.018	1.017	1.016	1.015	1.014	1.014	1.013	1.012	1.011
	5.0%	0.928	0.922	0.918	0.913	0.909	0.905	0.902	0.899	0.896	0.894	0.892
	1.0%	0.889	0.882	0.875	0.870	0.864	0.859	0.855	0.852	0.848	0.845	0.843
	0.1%	0.844	0.835	0.827	0.820	0.814	0.808	0.803	0.799	0.795	0.792	0.789
	Average	1.017	1.017	1.017	1.017	1.016	1.015	1.014	1.014	1.013	1.012	1.012
	Maximum	1.213	1.229	1.241	1.251	1.258	1.262	1.266	1.268	1.276	1.282	1.288
	Minimum	0.791	0.783	0.778	0.773	0.769	0.766	0.763	0.761	0.760	0.758	0.757

	Q_{mod}	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.6**	**2.8**	**3.0**
	$P(X < x)$	**Non-exceedance probability for the CPV of the dimensionless revenue series**										
Ermida Corgo	99.9%	1.127	1.138	1.148	1.157	1.164	1.171	1.177	1.183	1.189	1.194	1.198
	99.0%	1.099	1.107	1.114	1.120	1.125	1.130	1.134	1.138	1.142	1.145	1.148
	95.0%	1.073	1.079	1.083	1.087	1.090	1.093	1.095	1.098	1.100	1.102	1.104
	50.0%	1.009	1.008	1.007	1.006	1.005	1.003	1.002	1.002	1.001	1.000	0.999
	5.0%	0.940	0.934	0.928	0.923	0.918	0.914	0.910	0.907	0.904	0.901	0.899
	1.0%	0.911	0.902	0.894	0.887	0.882	0.876	0.872	0.868	0.864	0.861	0.858
	0.1%	0.877	0.865	0.856	0.848	0.841	0.834	0.829	0.824	0.820	0.816	0.813
	Average	1.008	1.007	1.006	1.005	1.004	1.003	1.002	1.002	1.001	1.001	1.000
	Maximum	1.143	1.156	1.166	1.177	1.189	1.199	1.209	1.218	1.226	1.234	1.241
	Minimum	0.847	0.834	0.823	0.815	0.808	0.802	0.797	0.792	0.788	0.784	0.781
Santa Marta do Alvão	99.9%	1.110	1.120	1.129	1.137	1.144	1.150	1.156	1.162	1.167	1.171	1.176
	99.0%	1.084	1.091	1.097	1.103	1.108	1.112	1.117	1.121	1.124	1.127	1.131
	95.0%	1.060	1.065	1.069	1.073	1.076	1.079	1.081	1.084	1.087	1.089	1.091
	50.0%	1.001	1.001	1.000	0.999	0.998	0.998	0.997	0.997	0.997	0.997	0.997
	5.0%	0.940	0.935	0.930	0.925	0.920	0.917	0.914	0.911	0.909	0.907	0.905
	1.0%	0.914	0.907	0.900	0.894	0.888	0.883	0.879	0.876	0.873	0.870	0.867
	0.1%	0.885	0.876	0.867	0.859	0.852	0.846	0.841	0.836	0.833	0.829	0.826
	Average	1.000	1.000	1.000	0.999	0.998	0.998	0.997	0.997	0.997	0.997	0.997
	Maximum	1.131	1.146	1.159	1.169	1.178	1.186	1.192	1.197	1.202	1.208	1.215
	Minimum	0.875	0.866	0.856	0.847	0.838	0.831	0.825	0.819	0.815	0.810	0.806
Fragas da Torre	99.9%	1.116	1.131	1.144	1.156	1.166	1.175	1.184	1.191	1.199	1.206	1.213
	99.0%	1.085	1.096	1.106	1.115	1.122	1.129	1.136	1.141	1.147	1.152	1.157
	95.0%	1.058	1.065	1.073	1.079	1.084	1.089	1.093	1.097	1.101	1.105	1.108
	50.0%	0.990	0.991	0.991	0.992	0.992	0.993	0.993	0.993	0.994	0.995	0.995
	5.0%	0.920	0.915	0.910	0.906	0.902	0.899	0.896	0.894	0.892	0.890	0.888
	1.0%	0.891	0.883	0.876	0.871	0.866	0.861	0.857	0.853	0.850	0.848	0.845
	0.1%	0.858	0.847	0.839	0.831	0.825	0.819	0.814	0.809	0.805	0.802	0.799
	Average	0.990	0.990	0.991	0.992	0.993	0.993	0.994	0.994	0.995	0.996	0.996
	Maximum	1.129	1.149	1.167	1.183	1.196	1.208	1.219	1.228	1.237	1.246	1.254
	Minimum	0.815	0.800	0.788	0.779	0.771	0.764	0.758	0.753	0.749	0.744	0.741

Appendix C

Average of the maximum and minimum dimensionless CPV of the revenue series for $N^* = 10$ and $N^* = 25$ years, for each sample and Q/Q_{mod}.

Sample	Q/Q_{mod}	Average of the maximum dimensionless revenue CPV series for		Average of the minimum dimensionless revenue CPV series for	
		10 years	25 years	25 years	10 years
Cidadelhe	1.0	1.2045	1.0944	0.8908	0.7708
	1.2	1.2161	1.0996	0.8858	0.7607
	1.4	1.2264	1.1043	0.8817	0.7522
	1.6	1.2363	1.1092	0.8787	0.7451
	1.8	1.2456	1.1138	0.8759	0.7388
	2.0	1.2543	1.1181	0.8735	0.7330
	2.2	1.2626	1.1223	0.8714	0.7278
	2.4	1.2702	1.1261	0.8694	0.7230
	2.6	1.2775	1.1298	0.8678	0.7188
	2.8	1.2840	1.1332	0.8662	0.7149
	3.0	1.2901	1.1363	0.8648	0.7113
Castelo Bom	1.0	1.1667	1.0679	0.8974	0.7929
	1.2	1.1800	1.0747	0.8940	0.7839
	1.4	1.1915	1.0805	0.8910	0.7760
	1.6	1.2022	1.0860	0.8885	0.7691
	1.8	1.2116	1.0907	0.8860	0.7627
	2.0	1.2201	1.0950	0.8837	0.7570
	2.2	1.2279	1.0987	0.8815	0.7517
	2.4	1.2353	1.1024	0.8797	0.7470
	2.6	1.2423	1.1059	0.8782	0.7429
	2.8	1.2490	1.1094	0.8769	0.7392
	3.0	1.2551	1.1126	0.8758	0.7359
Castanheiro	1.0	1.1437	1.0647	0.9235	0.8397
	1.2	1.1571	1.0711	0.9181	0.8277
	1.4	1.1682	1.0763	0.9134	0.8177
	1.6	1.1779	1.0809	0.9094	0.8091
	1.8	1.1870	1.0851	0.9060	0.8016
	2.0	1.1956	1.0893	0.9031	0.7952
	2.2	1.2038	1.0933	0.9006	0.7895
	2.4	1.2113	1.0969	0.8983	0.7843
	2.6	1.2185	1.1004	0.8963	0.7796
	2.8	1.2252	1.1036	0.8943	0.7751
	3.0	1.2315	1.1066	0.8925	0.7710

Sample	Q/Q_{mod}	Average of the maximum dimensionless revenue CPV series for		Average of the minimum dimensionless revenue CPV series for	
		10 years	25 years	25 years	10 years
Ponte Tábua	1.0	1.1239	1.0491	0.9350	0.8523
	1.2	1.1365	1.0551	0.9316	0.8430
	1.4	1.1473	1.0600	0.9283	0.8348
	1.6	1.1568	1.0642	0.9253	0.8274
	1.8	1.1654	1.0680	0.9227	0.8210
	2.0	1.1730	1.0713	0.9202	0.8151
	2.2	1.1796	1.0741	0.9181	0.8100
	2.4	1.1858	1.0768	0.9163	0.8056
	2.6	1.1913	1.0793	0.9149	0.8018
	2.8	1.1962	1.0814	0.9135	0.7983
	3.0	1.2007	1.0835	0.9124	0.7952
Ponte Santa Clara Dão	1.0	1.1321	1.0631	0.9343	0.8636
	1.2	1.1443	1.0681	0.9276	0.8517
	1.4	1.1541	1.0712	0.9208	0.8405
	1.6	1.1634	1.0747	0.9155	0.8313
	1.8	1.1700	1.0762	0.9094	0.8221
	2.0	1.1773	1.0789	0.9051	0.8150
	2.2	1.1833	1.0808	0.9009	0.8083
	2.4	1.1882	1.0820	0.8966	0.8017
	2.6	1.1931	1.0836	0.8930	0.7961
	2.8	1.1972	1.0846	0.8895	0.7908
	3.0	1.2004	1.0851	0.8858	0.7855
Ponte Juncais	1.0	1.1572	1.0838	0.9074	0.8260
	1.2	1.1743	1.0927	0.8995	0.8119
	1.4	1.1897	1.1008	0.8930	0.8001
	1.6	1.2032	1.1079	0.8875	0.7901
	1.8	1.2156	1.1145	0.8829	0.7816
	2.0	1.2267	1.1204	0.8788	0.7740
	2.2	1.2369	1.1258	0.8750	0.7673
	2.4	1.2459	1.1304	0.8713	0.7609
	2.6	1.2542	1.1346	0.8680	0.7552
	2.8	1.2617	1.1384	0.8649	0.7500
	3.0	1.2685	1.1418	0.8621	0.7453

Sample	Q/Q_{mod}	Average of the maximum dimensionless revenue CPV series for		Average of the minimum dimensionless revenue CPV series for	
		10 years	25 years	25 years	10 years
Rebordelo	1.0	1.1600	1.0829	0.9454	0.8640
	1.2	1.1719	1.0885	0.9405	0.8530
	1.4	1.1815	1.0929	0.9360	0.8435
	1.6	1.1895	1.0962	0.9318	0.8353
	1.8	1.1968	1.0992	0.9280	0.8278
	2.0	1.2039	1.1022	0.9246	0.8212
	2.2	1.2106	1.1051	0.9215	0.8151
	2.4	1.2168	1.1077	0.9187	0.8096
	2.6	1.2224	1.1100	0.9161	0.8046
	2.8	1.2275	1.1120	0.9136	0.7999
	3.0	1.2322	1.1139	0.9112	0.7954
Vale Giestoso	1.0	1.1130	1.0407	0.9074	0.8306
	1.2	1.1271	1.0474	0.9022	0.8193
	1.4	1.1396	1.0535	0.8979	0.8100
	1.6	1.1505	1.0588	0.8943	0.8021
	1.8	1.1601	1.0637	0.8914	0.7957
	2.0	1.1687	1.0680	0.8890	0.7900
	2.2	1.1769	1.0722	0.8871	0.7853
	2.4	1.1843	1.0761	0.8856	0.7812
	2.6	1.1909	1.0796	0.8843	0.7777
	2.8	1.1970	1.0828	0.8832	0.7746
	3.0	1.2028	1.0859	0.8823	0.7718
Castro Daire	1.0	1.1263	1.0622	0.9233	0.8549
	1.2	1.1373	1.0671	0.9164	0.8428
	1.4	1.1474	1.0719	0.9111	0.8332
	1.6	1.1565	1.0764	0.9066	0.8250
	1.8	1.1651	1.0806	0.9026	0.8177
	2.0	1.1730	1.0843	0.8991	0.8112
	2.2	1.1802	1.0877	0.8959	0.8055
	2.4	1.1866	1.0908	0.8930	0.8004
	2.6	1.1927	1.0938	0.8905	0.7958
	2.8	1.1984	1.0966	0.8883	0.7917
	3.0	1.2036	1.0991	0.8861	0.7878
Cabriz	1.0	1.1146	1.0420	0.9850	0.9100
	1.2	1.1199	1.0415	0.9803	0.9000
	1.4	1.1238	1.0408	0.9762	0.8919
	1.6	1.1275	1.0404	0.9730	0.8852
	1.8	1.1311	1.0404	0.9706	0.8799
	2.0	1.1346	1.0407	0.9689	0.8756
	2.2	1.1378	1.0411	0.9674	0.8718
	2.4	1.1408	1.0415	0.9662	0.8684
	2.6	1.1437	1.0420	0.9651	0.8654
	2.8	1.1463	1.0425	0.9643	0.8627
	3.0	1.1488	1.0431	0.9636	0.8604

Sample	Q/Q_{mod}	Average of the maximum dimensionless revenue CPV series for		Average of the minimum dimensionless revenue CPV series for	
		10 years	25 years	25 years	10 years
Cunhas	1.0	1.1718	1.0971	0.9361	0.8519
	1.2	1.1845	1.1031	0.9309	0.8419
	1.4	1.1951	1.1082	0.9264	0.8334
	1.6	1.2039	1.1123	0.9223	0.8258
	1.8	1.2110	1.1154	0.9185	0.8190
	2.0	1.2164	1.1175	0.9149	0.8129
	2.2	1.2209	1.1192	0.9116	0.8075
	2.4	1.2248	1.1207	0.9088	0.8029
	2.6	1.2281	1.1219	0.9063	0.7987
	2.8	1.2311	1.1230	0.9041	0.7950
	3.0	1.2339	1.1241	0.9021	0.7918
Ermida-Corgo	1.0	1.1246	1.0641	0.9517	0.8877
	1.2	1.1343	1.0683	0.9460	0.8768
	1.4	1.1422	1.0715	0.9411	0.8676
	1.6	1.1493	1.0743	0.9368	0.8597
	1.8	1.1552	1.0764	0.9328	0.8526
	2.0	1.1606	1.0783	0.9292	0.8464
	2.2	1.1655	1.0800	0.9260	0.8408
	2.4	1.1701	1.0816	0.9232	0.8358
	2.6	1.1744	1.0832	0.9207	0.8314
	2.8	1.1783	1.0846	0.9184	0.8274
	3.0	1.1820	1.0860	0.9163	0.8237
Santa Marta do Alvão	1.0	1.1057	1.0509	0.9477	0.8897
	1.2	1.1144	1.0550	0.9438	0.8818
	1.4	1.1219	1.0581	0.9397	0.8741
	1.6	1.1287	1.0607	0.9356	0.8668
	1.8	1.1348	1.0630	0.9319	0.8604
	2.0	1.1405	1.0653	0.9289	0.8550
	2.2	1.1459	1.0676	0.9265	0.8505
	2.4	1.1508	1.0698	0.9245	0.8465
	2.6	1.1553	1.0718	0.9226	0.8430
	2.8	1.1594	1.0736	0.9209	0.8397
	3.0	1.1634	1.0754	0.9194	0.8367
Fragas da Torre	1.0	1.1106	1.0476	0.9303	0.8644
	1.2	1.1229	1.0537	0.9260	0.8547
	1.4	1.1339	1.0592	0.9225	0.8465
	1.6	1.1435	1.0641	0.9196	0.8395
	1.8	1.1517	1.0682	0.9169	0.8333
	2.0	1.1592	1.0721	0.9145	0.8279
	2.2	1.1660	1.0755	0.9124	0.8229
	2.4	1.1722	1.0786	0.9104	0.8184
	2.6	1.1783	1.0818	0.9089	0.8146
	2.8	1.1841	1.0849	0.9077	0.8113
	3.0	1.1897	1.0879	0.9066	0.8082