Strength analysis of corroded pipelines subjected to internal pressure and bending moment

Andrey Augusto Barbosa

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Naval Architecture and Marine Engineering

Supervisor: Professor Ângelo Manuel Palos Teixeira

Examination Committee

Chairperson: Professor Carlos António Pancada Guedes Soares

Supervisor: Professor Ângelo Manuel Palos Teixeira

Member of the Committee: Professor José Manuel Antunes Mendes Gordo

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“Para mim, o sentido da vida é sermos elos fortes em uma cadeia muito maior que é a história da humanidade.” (Neil deGrasse Tyson)
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Abstract

The failure of oil pipelines can lead to serious human and environmental damages. Therefore, the predictability of pipeline failure is an important research field. This study focuses on the evaluation of the burst pressure and maximum bending moment that a pipe is capable to sustain by Finite Element Method (FEM) simulations.

In real operating conditions, the presence of corrosion is an important factor that significantly limits the lifespan of a pipeline as well as its capability to carry loads. For this reason, the effect of corrosion defined as a local thinning area in the model, i.e., a region in which the thickness has a uniform reduction, is also studied. Firstly, a parametric study of the main dimensions of the corrosion defect is performed by applying solely pressure and, after, solely bending moment. Afterwards, for a fixed corrosion damage, simulations are performed considering, simultaneously, the presence of internal pressure and an incremental bending load, which is applied until the pipe reaches the failure condition.

The obtained results have shown that the presence of internal pressure significantly limits the ultimate bending moment of both external corroded and non-corroded pipelines. Such effect is proportional to the percentage of burst pressure applied and for the intact case, for instance, leads to a 30% reduction in the original bending moment obtained for single load numerical calculations.

**Key-words:** corroded pipelines; bending moment; combined load; internal pressure; finite element method; pipeline failure modes.
Resumo

A falha de dutos transportadores de petróleo pode acarretar sérios riscos às pessoas e ao meio ambiente. Portanto, a previsão de sua falha é um tópico relevante de investigação. Este estudo foca-se na determinação da pressão de ruptura e momento de flexão máximo que uma tubulação é capaz de sustentar através da análise numérica por elementos finitos. Estudos anteriores mostraram que o método dos elementos finitos é uma ferramenta válida para determinar os efeitos de diferentes carregamentos, tais como pressão interna ou momento de flexão.

Em condições reais, a presença de corrosão é um fator importante que limita de forma significativa a vida de dutos, assim como sua capacidade para resistir a carregamentos. Por essa razão, o efeito da corrosão, definida como uma área de redução uniforme de espessura no modelo, é também estudada. Primeiramente, é realizado um estudo paramétrico das dimensões principais do defeito num duto sujeito unicamente a pressão e, posteriormente, unicamente a momento de flexão. Em seguida, para um certo defeito, são feitas simulações considerando, simultaneamente, a presença de pressão interna e uma carga incremental de momento de flexão, que é aplicada até ser atingida a condição de falha.

Os resultados obtidos mostram que a presença de pressão interna limita, de forma significativa, a resistência de dutos à flexão com e sem corrosão externa. Este efeito é proporcional à percentagem da pressão de falha aplicada, e para o caso intacto, leva a uma redução de mais de 30% em relação ao momento máximo suportado pelo duto sujeito unicamente a momento fletor.

Palavras-chave: Dutos corroídos; momento fletor; carregamentos combinados; pressão interna, método dos elementos finitos, modos de falha de dutos.
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Nomenclature

**Latin Characters – UPPERCASE**

\[ A: \text{auxiliar factor for calculation of burst pressure} \]
\[ A_{\text{Elem}}: \text{area referring to one element in the cover section} \]
\[ A_{\text{section}}: \text{area of the "cover" cross section} \]
\[ D_o: \text{outside diameter} \]
\[ D_i: \text{internal diameter} \]
\[ F: \text{longitudinal force acting on the pipe} \]
\[ F_i: \text{limit longitudinal force} \]
\[ F_{\text{node}}: \text{force applied to one node located at the extreme section} \]
\[ I: \text{second moment of inertia of the pipe's section} \]
\[ L: \text{length of the pipe} \]
\[ L_A: \text{accidental loads} \]
\[ L_d: \text{factored design load} \]
\[ L_E: \text{environmental loads} \]
\[ L_F: \text{functional loads} \]
\[ L_i: \text{interference loads} \]
\[ M: \text{applied bending moment} \]
\[ M(t): \text{total bending moment being applied in a given time } t \]
\[ M_{\text{Allowed}(F,p)}: \text{allowable bending moment} \]
\[ MAOP: \text{maximum operating pressure} \]
\[ M_c: \text{bending moment capacity of corroded pipelines} \]
\[ M_{\text{effective}}: \text{resultant effective bending moment in a given time } t \]
\[ M_G: \text{general yielding moment} \]
\[ M_p: \text{plastic bending moment capacity} \]
\[ M_{\pi i}: \text{ultimate bending moment in the presence of internal pressure } p_i \]
\[ M_{p=0}: \text{ultimate bending moment in the presence of no internal pressure} \]
\[ M_{\text{SD}, \text{design bending moment}} \]
\[ M_T: \text{auxiliar factor} \]
$N_{\text{elem}}$: number of elements in the cover section

$P$: pressure which cannot be exceed

$P_b$: burst pressure according to Netto’s equation

$P_{co}$: collapse pressure of non corroded pipelines according to Netto’s equation

$P_{cor}$: collapse pressure of corroded pipelines according to Netto’s equation

$Q$: auxiliary factor in the equation

$R$: average radius

$R_d$: factored design resistance

$R_i$: internal radius

$R_o$: outside radius

$R_k$: unfactored loads

$S$: longitudinal spacing between adjacent defects (mm)

$S_p$: axial plastic stress

$SMTS$: Specified Minimum Tensile Stress

$SMYS$: Specified Minimum Yield Stress

$Size_{\text{center\_corrosion}}$: meshing at the center of the corroded region

$Size_{\text{outside}}$: meshing spacing size outside the corroded region

$S_{SD}$: design effective axial stress

**Latin Characters - LOWERCASE**

c: width of the defect region in radians or degrees

c’: width of the defect region in mm

d$_g$: degradation factor

$d$: corroded thickness

$d_{\text{max}}$: maximum depth of the defect

$f$: appropriate design factor

$f_{cb}$: minimum of $\frac{\sigma_j; \sigma_u}{1.15}$

$fy_{\text{temp}}$: temperature derating factor of the yield strength

$fu_{\text{temp}}$: temperature derating factor of the tensile strength
$f_0$: ovality

$g$: acceleration of gravity

$h_1$: local test height

$h_{\text{ref}}$: reference height

$k_1$: temperature derating factor

$l$: longitudinal length of corroded region

$l_{\text{max}}$: maximum extent of the corroded area

$p$: pressure acting on the pipe

$p_b$: burst pressure

$p_c$: external collapse pressure

$p'_{c}$: characteristic internal pressure

$p_d$: design pressure

$p_e$: external pressure

$p_{el}$: elastic collapse pressure of the pipe

$p_h$: hydrostatic test pressure

$p_i$: internal pressure

$p_l$: refining meshing factor

$p_i'$: limit pressure

$p_{li}$: local incident pressure

$p_{li}$: local teste pressure

$p_{\text{MIN}}$: minimum internal pressure that can be sustained (normally taken as zero)

$p_p$: plastic buckling pressure

$t$: thickness

$t_{\text{CORR}}$: effective corroded thickness of pipe in the Net Section Criteria

$t_{\text{MIN}}$: minimum thickness in the pipeline

$t_{\text{new}}$: new element thickness after insertion of corrosion

$t_2$: pipe wall thickness. Prior to operation equal to ($t$). In operation equal to ($t - d$)

$\text{width}_{\text{defect}}$: width of the defect referred in the parametric study

$\text{width}_{\text{original}}$: base case taken for the width of the defect

$y$: is the vertical position in relation to the $y$ global axis
Greek Characters

α: correction factor  
α_c: flow stress parameter  
α_p: Pressure factor used in combined loading criteria  
α_m: material strength factor available  
β: half angle at the neutral axis  
β': Factor used in combined loading criteria  
ε: factor for defining a fractile value for the corrosion depth  
γ: accidental safety factors  
γ_c: condition load effect factor  
γ_e: environmental safety factors  
γ_f: functional safety factors  
γ_m: material safety factor  
γ_inc: incidental to design pressure ratio  
γ_sc: class safety factor  
η: usage factor for equivalent stress (equal to 0.96)  
η_fab: fabrication factor  
η_h: usage factor  
η_RM: strength usage factor for bending moment  
η Rp: strength usage factor for internal pressure  
v: Poisson’s ratio  
ρ_con: density of the contained fluid  
ρ_t: density of the test fluid  
σ: is the stress subjected to the node  
σ_f: flow stress  
σ_u: ultimate tensile stress  
σ_h: hoop stress  
σ_t: longitudinal (axial) stress
$$\sigma_{net}: \text{net section stress at cracked section}$$

$$\sigma_R: \text{radial stress}$$

$$\sigma_y: \text{yield stress}$$

$$\tau: \text{torsional shear stress}$$

$$\theta'(t): \text{curvature in a given time t}$$

$$\theta'': \text{half angle of the local wall thinning}$$
1. Introduction

Due to the increasingly demand for hydrocarbons, the exploration and exploitation of offshore oil resources has moved into deepwater and ultra-deepwater scenarios. In these cases, pipelines are one of the safest and cost-effective means of transportation of oil and gas to offshore and onshore storage facilities. However, accidents with pipelines may pose a serious risk to human lives and to the environment, with everlasting consequences. According to the US Department of Transportation, corrosion in pipelines is a significant individual cause of accidents in the United States, as it can be seen in Figure 1. Indeed, corrosion can contribute to a system’s failure by thickness penetration, fatigue cracks, brittle fracture or unstable failure.

![Figure 1. Serious Incident Cause Breakdown 20 Years Totals (1996-2015)](https://hip.phmsa.dot.gov/analyticsSOAP/saw.dll?PortalGo)

Besides being a cause of accidents with human and environmental consequences, corrosion may also provoke economic losses due to reduced operating pressure, to loss of production during downtime as well as to repairs or to replacement. For this reason, even when corrosion has been detected, some pipelines are kept in operation by recalculating the operating pressure of the product been transported.

Regarding the effects of corrosion, it may induce uniform or non-uniform thinning at a pipe’s cross section (Teixeira, Zayed, & Guedes Soares, 2010). The presence of local thinning areas can limit the capacity of a pipe to sustain bending moment and pressure. In the light of this knowledge, the accurate evaluation of the remaining strength of a corroded pipeline can bring substantial benefits. Currently, research studies in this field have focused on the action of internal pressure, which have resulted in the

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1 OFFICE OF PIPELINES SAFETY, Pipelines statistics. US Department of Transportation; available in: https://hip.phmsa.dot.gov/analyticsSOAP/saw.dll?PortalGo
development of general guidelines and design codes, such as DNV (2015) and ASME B31G (2009). However, the analysis of the effect of combined loads (bending moment, axial forces, internal/external pressure) in the presence of corrosion is still an open-topic, which has been increasingly sought after.

Corrosion is a process of uncertain nature. Therefore, reliability analysis can be of great value to understand its effect on the residual strength and safety of pipelines. Nonetheless, these analyses depend on the evaluation of limit state functions that include several strength and load variables, which define the failure problem of a corroded pipe, in order to evaluate the reliability of corroded pipelines.

The topic of failure of corroded pipelines subjected to combined loads is still an open field of study. The main design codes and guidelines have focused particularly on burst pressure failures, and other failure modes that involve the presence of axial, bending loads and combined loads have not been fully addressed by them. Therefore, it is a major issue to comprehensively understand the effect of bending moment loads, which occur due to thermal variations in operation, in order to provide simple guidelines for the design of pipelines, maintenance or replacement of corroded pipelines.

This thesis aims to study the influence of internal pressure on the ultimate bending capacity of corroded pipelines. Firstly, a literature review of previous studies and of the main design codes regarding the failure modes of intact and corroded pipelines is performed. Secondly, a numerical finite element structural model of a pipeline is developed with adequate element type, model dimensions and material properties. Thirdly, the process of applying boundary conditions and loads to the model is presented in detail. Afterwards, numerical simulations of pipelines subjected to single and combined loads are performed. The results are then presented and analysed and a comparison with published results from other authors is performed. Finally, the limitations of the model and suggestion of future studies are presented.

The study is structured as follows: chapter 2 presents the state of the art related to the main failure modes of intact and corroded pipelines, with emphasis on finite element studies on which the model created was based; chapter 3 presents the methodology followed to the development of the model and applied loads, chapter 4 presents the main results obtained for single and combined load simulations and the comparison to other reference studies; chapter 5 presents the limitations and the conclusions of the study; and finally chapter 6 provides some suggestions for further studies.
2. Review of pipeline design approaches

2.1. Design methods

This work incorporates the definition of failure provided by DNV-OS-F101 as “An event affecting a component or system and causing one or both of the following effects: loss of component or system function; (...) to such an extent that the safety of the installation, personnel or environment is significantly reduced.” By this definition, a failure can be associated with either the loss of functional capacity by the system or the undermining of its safety.

Many of the most traditional design codes for pipelines and risers were traditionally based on the Allowable Strength Design (ASD), also known Permissible stress design, in which the unfactored stress, calculated from the most adverse combination of loads, must not exceed a permissible stress, which is equal to the ultimate stress divided by a safety factor. However, in the last years the Limit State Design (LSD), also known as load and resistance factor design (LRFD), has been successfully used as a design alternative and incorporated in important codes, such as DNV-OS-F101 and API (2009), which has ultimately led to more economical, but still reliable, design solutions. This approach considers a statistical distribution of the characteristic load and a statistical distribution of the characteristic structural strength. By the applicability of partial safety factors, load is factored up and the resistance is factored down until adequate probability of failure is obtained, which is defined as the region where the factored design load is equal or greater than the factored design resistance:

\[
L_d \leq R_d
\]

\[
L_d = L_F \gamma_F \gamma_C + L_E \gamma_E + L_I \gamma_F \gamma_C + L_A \gamma_A \gamma_C
\]

\[
R_d = \frac{R_k(f_k)}{\gamma_S \gamma_m}
\]

\(L_A\): accidental loads
\(L_d\): factored design load
\(L_E\): environmental loads
\(L_F\): functional loads
\(L_I\): interference loads
\(R_d\): factored design resistance
\(R_k\): unfactored loads

and \(\gamma_F, \gamma_C, \gamma_A, \gamma_E, \gamma_S, \gamma_m\) are defined in DNV-Os-F101 (2013) and stand for functional safety factor, environmental load safety factor, interference safety factor, accidental safety factor, class safety factor and material safety factor, respectively. Figure 2 provides a representation of this method, where the probability of failure is calculated as area of the region in which the load distribution \(L(x)\) is greater than the resistance distribution \(R(x)\).
In addition, LSD incorporates Stress Based Design and Strain based design. Depending on the surrounding mean and the capability of the pipe to bend due to loads applied, the system can be considered in a Displacement or Load Control condition, further detailed.

2.1.1. Stress and Strain based design

Stress based design was applied to the majority of pipelines installed around the world (Bai et al., 2014). It consists in limiting relevant stress to a fraction of the Specific Minimum Yield Stress (SMYS) of the material used. On the other hand, the strain based design sets strain limits rather than stress limits that can be reached by the material of the pipe and is applicable in displacement controlled pipelines, i.e. the ones which movement is restrained by the surrounding medium, such as buried or reeled pipelines.

2.1.2. Displacement control and Load Control

In displacement controlled pipelines, additional bending is prevented by the presence of external medium or barriers, in this case, it is advisable the use of strain based design in order to control deformations. On the other hand, pipes that can bend due to external loads are said to be load controlled pipelines (Bai et al., 2014). For instance, this is true in cases where the intensity of the load is independent from the displacement, such as in free-spanning pipelines in which stress based design criteria is commonly used.

2.2. Design of High Pressure and High Temperature pipes

As far as the design of High Pressure and High Temperature pipes (HPHT) is concerned, an approach using limit state design is more suitable than the traditional stress based design. The DNV-OS-F101 addresses this issue and provides proper design guidelines regarding this method.
According to DNV-OS-F101, the Limit State Design divides the pipe response to external factors in a safe region and in an unsafe region. In a pipeline design, four relevant Limit States should be checked, they are: Ultimate Limit State (ULS), Serviceability Limit State (SLS), Fatigue Limit State (FLS) and Accidental Limit State, however, the scope of this study will be the analysis of only the ULS for single and combined loads.

In brief, the Design will be focused on the evaluation of the Ultimate Limit State (ULS) for an arbitrarily defined case study of a pipeline in which considerations will be drawn regarding mainly the following failures modes:

- Burst of the pipe;
- Maximum Bending Capacity;
- Ultimate state reached by the combined action of internal pressure and bending moment.

2.3. Ultimate Limit State

For the present study, some assumptions are made in order to choose a case to focus on. It is assumed that the pipe is laying on the seabed in a load-controlled condition since according to DNV (2015): “A load controlled design criterion can always be applied in place of a displacement controlled design criterion”.

2.4. Burst Pressure of Intact pipelines

According to Bai et al., (2014), bursting is defined as “the point at which the uncontrolled tearing of the pipe wall occurs”. For this reason, this event is considered as an Ultimate limit state failure whereas the yielding of the section is a Serviceability limit state since it does not result in an immediate failure of the pipe.

Moreover, majors codes, such as ASME. B31.8 (2003), commonly advise the prevention against failure by limiting the internal hoop stress, represented in Figure 3, to a fraction of the Specified Minimum Yield Stress (SMYS) of the pipe. Firstly, the hoop stress for thin walls and thick walls can be defined according to the following formulations, respectively (Bai et al., 2014).

\[ \sigma_h = \frac{(p_l - p_e)D_o}{2t} \]  
\[ \sigma_h = (p_l - p_e)\frac{(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)} - p_e \]
Figure 3. Representation of loads acting in a shell cylinder subjected to internal pressure

$\sigma_h$: hoop stress
$D_o$: outside diameter
$D_i$: internal diameter
$p_e$: external pressure
$p_i$: internal pressure

Secondly, the previously mentioned hoop stress criteria is part of the stress based design approach and limits the load to a fraction of the SMYS, as described:

$$\sigma_h \leq \eta_h k_t SMYS$$ \hfill (6)

In the equation (6), $\sigma_h$ is the hoop stress, $k_t$ is the temperature derating factor, $\eta_h$ is the usage factor and SMYS stands for the specified material minimum yield stress. The usage factor is assumed as 0.72 for all major codes as established by ASME. B31.8 in 1958 and the derating factor is specified in table 841.116A of ASME. B31.8 (2003).

Stewart, Klever, & Ritchie (1994) studied the behavior of pipelines subjected to internal pressure and bending moment and stated that for pipes in displacement-control conditions the hoop stress criteria provides good control in order to prevent bursting failure. However, for pipes in load-control conditions an equivalent stress criteria, which limits the Von Mises stress to a fraction of the SMYS, should additionally be taken into account if axial loads are presented.

$$\sigma_e = \sqrt{\sigma_h^2 + \sigma_l^2 - \sigma_h \sigma_l + 3\tau^2} \leq \eta_e SMYS$$ \hfill (7)

$\sigma_e$: equivalent stress
$\sigma_h$: hoop stress

---

2 Image obtained from: http://nptel.ac.in/courses/112107146/lects%20&%20picts/image/lect15/3.jpg
\( \sigma_l: \) longitudinal (axial) stress

\( \tau: \) torsional shear stress

\( \eta_e: \) usage factor for equivalent stress (equal to 0.96) (Hauch & Bai, 2000)

**SMYS:** Specified Minimum Yield Stress

Another well-known code, DNV-OS-F101, (2013), focus its analysis on defining a relation to which a pipe in operation must fulfill. The local test pressure \( p_{lt} \) shall fulfill the following criteria:

\[
 p_{lt} - p_e \leq \min \left( \frac{p_b}{\gamma_m \gamma_{sc}}; p_h \right)
\]

where \( p_b, p_e, p_h \) are the containment pressure, the external pressure, the hydrostatic test pressure, respectively and \( \gamma_m \) and \( \gamma_{sc} \) are the material and class safety factor, respectively. Further information regarding the determination of these variables is provided in the Annex.

### 2.5. Burst Pressure of Corroded Pipelines

Corrosion effects in pipelines have a significant impact in the reduction of the burst pressure. The classical ASME B31G (2009), the most used design code, provides guidelines for the maximum extent of corrosion defect taking into account the depth of the defects and thickness of pipelines. Another possible approach, according to the same reference, is the evaluation of burst pressure of the pipe including a term to take into account the main dimensions of the defect.

\[
P_{31G} = 1.1 P \left[ \frac{1 - \left( \frac{2}{3} \right) \left( \frac{d_{\text{max}}}{t} \right)}{1 - \left( \frac{2}{3} \right) \left( \frac{d_{\text{max}}}{t} \right) M_T^{-1}} \right]
\]

where,

\[
M_T = \sqrt{1 + A^2}
\]

\[
A = 0.8 \frac{d_{\text{max}}^2}{D_o d_{\text{max}}}
\]

\[
P = \max \left( MAOP; \frac{2 \sigma_y f k_t}{D_o} \right)
\]

\( d_{\text{max}}: \) maximum depth of the defect

\( l_{\text{max}}: \) maximum extent of the corroded area

\( MAOP: \) maximum operating pressure

\( t: \) thickness of the pipe

\( d_{\text{max}}: \) maximum corroded thickness allowed
In addition, equation (9) is only applicable when the corroded thickness is bigger than 10% and smaller than 80% of the original thickness and factor A is smaller or equal than 4. Further explanation about the equation (9) may be found in Annex.

The DNV design code (DNV, 2015) recommends an assessment of the burst pressure of corroded pipelines considering the presence of compressive loads as well as the main dimensions of the corroded region, in a similar way as ASME 31G. The formula is presented below:

\[
P_{DNV} = \gamma_m^2 t \frac{SMTS \left(1 - \gamma_d \left(\frac{d}{t}\right)^*\right)}{(D_o - t) \left(1 - \left(\frac{\gamma_d \left(\frac{d}{t}\right)^*}{Q}\right)\right)}
\]

where,

\[
Q = \sqrt{1 + 0.31 \left(\frac{l}{\sqrt{D_o t}}\right)^2}
\]

and

\[
\left(\frac{d}{t}\right)^* = \left(\frac{d}{t}\right)_{med} + \epsilon_d S D_o \left(\frac{d}{t}\right)
\]

With:

d: corroded thickness

\[
\gamma_m: Partial safety factor for longitudinal corrosion model prediction,
\]

\[
\gamma_d: Partial safety factor for corrosion depth,
\]

Q: auxiliar factor in the equation

\[
\epsilon_d: Factor for defining a fractile value for the corrosion depth,
\]

S: Longitudinal spacing between adjacent defects (mm),

l: Longitudinal length of corroded region (mm)
SMTS: Specified Minimum Tensile Stress

Despite the efforts to provide concise and effective guidelines by the main codes, recent experiments showed that some safety factors recommended can lead to over conservative assessment of burst pressure of pipes what may impose unnecessary maintenance or replacement (Cronin & Pick, 2002). For this reason, Cronin & Pick (2002) recommended a new method considering weighted depth differences (WDD) for the evaluation of the failure pressure in the presence of complex-shape defect, which is applicable to any defect shape in ductile materials. The obtained results have shown to be more accurate than the ones from B31G, using the burst test of 40 pipe sections with actual defects.

Netto et al. (2005) also developed a simple procedure for estimating the burst pressure of corroded pipelines. Firstly, they performed a series of small-scale tests experiments used to calibrate a nonlinear numerical model. Figure 4 depicts an example of a test performed to evaluate the burst pressure of a specimen. Afterwards, a series of numerical simulations were run for different corrosions dimensions. On the basis of both experimental and numerical results, an analytical formula was derived.

\[
P_b = 1 - 0.9435 \left( \frac{d}{t} \right)^{1.6} \left( \frac{l}{D_o} \right)^{0.4}
\]

for \( \frac{c'}{D_o} \geq 0.0785 \), \( \frac{l}{D_o} \leq 1.5 \) and \( 0.1 \leq \frac{d}{t} \leq 0.8 \).

![Figure 4. Test specimen after burst (Netto et al., 2005)](image)

\( P_b \): burst pressure according to Netto equation

\( c' \): projection width of the defect region

Taking into account that corrosion is an uncertain process, many authors have developed models to evaluate the failure probability of pipelines considering the burst pressure as the failure function and using a statistical approach for the variables involved. A complete reliability analysis of the Netto’s equation was carried out by Teixeira et al. (2008) using the first-order reliability method (FORM). They performed a sensitivity analysis of the influence of internal pressure and the main dimensions of the
corroded defect taking into account the failure function provided by Netto et al. (2005). It was shown that the B31G code is over conservative compared to the Netto’s model, in spite of the latter lead to smaller reliability indices concerning large defects.

Still in the reliability field, Teixeira et al. (2010) presented a reliability analysis of non-uniform corroded pipelines under internal pressure, using the FORM, as an alternative method to the uniform reduction of thickness. Some findings were that this approach yields to lower burst pressure than the uniform approach considering equivalent reduction of thickness, furthermore, the burst pressure plays a role so important as the operational pressure in the reliability assessment.

2.6. Collapse Pressure of intact pipelines - Local Buckling

The local failure of the cross section, also known as local buckling is considered as an Ultimate State Limit. Commonly, it is induced by geometrical imperfections in the cross section since, in theory, the increase of the uniform external pressure would not affect the roundness of the cross section. However, in practice, some factors such as material properties, geometrical imperfections such as out-of-roundness (ovality), pipe-diameter-to-thickness ratio and thickness variation along the pipe contribute for a flattening of the pipe’s cross section and consequently increase the risk of a catastrophic failure, according to (Bai et al., 2014), Yeh & Kyriakides (1986), apud Netto et al. (2007,p.2).

Several analytical formulations have been proposed in order to determine a minimum collapse pressure in intact pressure vessels. Among them, the equation proposed by Timoshenko & Gere (1961), as well as the one proposed by Haagsma & Schaap (1981), are the most widespread. The first one provides a lower bound for the pressure of failure, while the second one provides an upper bound and further describes the limits for low D/t ratios (Bai et al., 2014). The Haagma and Schaap’s equation is described in equation (17).

\[ p_c^3 - p_{el}p_c^2 - \left( p_p^2 + p_{el}p_p f_0 D_o t \right) p_c + p_{el}p_p^2 = 0 \]  \hspace{1cm} (17)

where,

\( p_c: \text{collapse pressure (external pressure)} \)

\( p_{el}: \text{elastic collapse pressure of the pipe} = \frac{2E}{1-\nu^2} \left( \frac{t}{D_o} \right)^3 \)

\( p_p: \text{plastic buckling pressure} = \eta_f a_b S M Y S \frac{2t}{D_o} \)

\( f_0: \text{ovality} \)

\( \nu: \text{Poisson’s ratio} \)

DNV-Os-F101 (2013) refers to two main causes of local buckling: local buckling caused by only external pressure and local buckling originated by combined loads. In DNV, the use of Haagsma and Schaap’s equation is advised for the first scenario in order to evaluate the collapse pressure (\( p_c \)), when
only external pressure is verified. In a similar way as it has been proposed for the burst pressure, the pressure load must fulfill the following criteria:

\[ p_e - p_{\text{min}} \leq \frac{p_c}{\gamma m \gamma_{SC}} \]  

(18)

where,

\( p_e \): external pressure

\( p_{\text{min}} \): minimum internal pressure (normally taken as zero)

Nevertheless, in order to evaluate the contribution of different loads to the local buckling, an allowed bending moment must be checked. Further description of this methodology is given in the annex.

2.6.1. Collapse Pressure of corroded pipelines - Local Buckling

Collapse pressure for corroded pipelines is a topic of study not so readily available in the open literature. Fatt (1999) proposed an exact solution for the elastic buckling pressure and the plastic yield pressure of a shell cylinder with non-uniform circumferential cross sectional thickness. The formula derived is an extension of the Timoshenko & Gere, (1961) approach for shell cylinder with uniform thickness. Comparison with Finite Element (FE) results were not submitted in that paper.

Netto et al. (2007) through the combination of small-scale experiments with nonlinear numerical analysis, using FE models, showed that not only the geometry of the defect is relevant, but also the type of defect - interior or exterior – and its position relative to ovalized cross section, in order to determine the collapse pressure of corroded pipelines, as shown in Table 1. Results suggested that the collapse's shape could assume different modes depending on the geometry of the defect. Moreover, the worst case scenario was obtained when the defect coincides with the most compressed fibers of the collapsed cross-section.
Table 1. Influence of the position of the defect with respect to the ovalized cross-section in the collapse pressure Netto et al. (2007)

<table>
<thead>
<tr>
<th>Ovality / Defect position</th>
<th>0.2%</th>
<th>0.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{d}{t} )</td>
<td>( \frac{d}{t} )</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>( \hat{P}<em>{\text{COR}} / \hat{P}</em>{\text{CO}} ) (Collapse configuration)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shallow defects:</td>
<td>0.95</td>
<td>0.88</td>
</tr>
<tr>
<td>Moderately deep defects:</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Deep and narrow defects:</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Later, Netto (2009) studied the detrimental effects of corrosion in pipelines regarding the collapse pressure. From the analysis of experimental and numerical results, the following formula for the prediction of the collapse pressure with long and narrow defects was proposed.

\[
P_{\text{cor}} = \left[ \frac{1 - \frac{d}{t}}{1 - \frac{d}{t} \left( 1 - \left( \frac{c'}{\pi D_o} \right)^{0.4} \left( \frac{l}{10D_o} \right)^{0.4} \right)} \right]^{2.675} \]  \tag{19}

However, this formula is only valid for defects whose geometry is comprised inside the ranges:

**Shallow defects:**

\[
0.1 \leq \frac{d}{t} \leq 0.2 \quad \text{and} \quad \frac{c'}{\pi D_o} \leq 0.1 \]  \tag{20}

**Moderately deep defects:**

\[
0.2 \leq \frac{d}{t} \leq 0.4 \quad \text{and} \quad \frac{c'}{\pi D_o} \leq 0.1 \]  \tag{21}

If \( \frac{c'}{\pi D_o} \geq 0.05 \), set \( \frac{c'}{\pi D_o} = 0.05 \)

**Deep and narrow defects:**

\[
0.4 \leq \frac{d}{t} \leq 0.6 \quad \text{and} \quad \frac{c'}{\pi D_o} \leq 0.05 \]  \tag{22}

If \( \frac{c'}{\pi D_o} \geq 0.025 \), set \( \frac{c'}{\pi D_o} = 0.025 \)

\( P_{\text{CO}} \): collapse pressure of non corroded pipelines according to Netto’s equation

\( P_{\text{cor}} \): collapse pressure of corroded pipelines according to Netto’s equation

\( c' \): projection width of the defect region
Netto, (2010) has compared the predictions of equation (19) with a new set of experimental results published by Sakakibara et al. (2008). The results obtained through the application of the formula showed good correlation with the studied set of data.

2.7. Maximum bending moment of pipelines

One of the first studies of the influence of pure bending moment loads in pipelines was presented by Brazier (1927) in which it was suggested a relation between the allowable bending moment and the curvature of the cross section. He also showed that the collapse occurs when the load reaches a certain value proportional to the variation in the cross section radius. After, Belke (1984) expressed the failure bending moment in a relative simple way as function of the stress for a nonlinear stress-strain curve for austenitic and ferritic materials subjected to maximum deformation. The results were particularly accurate for specific ranges of diameter and thickness ratios.

2.7.1. Net Section criteria (NSC)

Analytical formulations were also proposed aiming to predict the maximum moment a corroded pipe could sustain. Kanninen et al. (1976), as quoted by Han et al. (1999, p.1), proposed a formulation to evaluate the occurrence of pipe leaks and break loads, indicating the critical net-stresses at crack initiation and maximum loads; what would be after known as the net-section criteria (NSC). This model is based on some assumptions such as:

- stress distribution is considered constant at the cross section;
- center of the defect is considered coincident with the diametric plane at the local of maximum bending load.

The formulation is presented below for the more common situation, where $\beta \leq \pi - \theta$. Figure 5 illustrates the meaning of each variable in the NSC model:

$$M_c = 2\sigma_{net}R^2t(2\sin\beta - \frac{d\sin c}{2t})$$

$$\beta = \frac{1}{2}\left(\pi - \frac{d}{2t}\right)$$
Figure 5. Model consider for the net section criteria (NSC)

\( M_c: \) bending moment capacity of corroded pipelines

\( M: \) applied bending moment

\( R: \) average radius

\( R_o: \) outside radius

\( R_i: \) internal radius

\( c: \) width of the defect region in radians or degrees

\( \beta: \) half angle at the neutral axis

\( \sigma_{net}: \) net section stress at cracked section

For intact pipelines, assuming \( \frac{d}{t} = \beta = c = 0 \), equation (23) yields to the general yielding moment of the pipe:

\[
M_G = 4\sigma_yR^2t
\]

\( M_G: \) general yielding moment

\( \sigma_y: \) yielding stress

Han et al. (1999) studied the subject using the NSC and compared the results obtained for different geometries with numerical simulations. They obtained interesting results such as that the limit moment is influenced not only by the width but also by the length of the defect; and the NSC is a valid approach to complement available codes when the defect is relatively long, i.e. \( \frac{d}{\sqrt{R}} > 1.5 \).

Miyazaki et al. (2002) proposed that better correlation with experimental data could be achieved, at a cost of slightly non-conservative estimation for some cases, if the flow stress \( \sigma_f \) - calculated as the average value of the yield stress \( \sigma_y \) and the ultimate tensile stress \( \sigma_u \) - were replaced by only the ultimate tensile stress \( \sigma_u \). According to him, pipes with deeper defects tend to fail when the ultimate stress is reached at the defect region and pipes with shallower defects tend to fail when the whole section reaches the yield stress.

Taking into account the limitations of the NSC, a modified expression for the NSC criteria was proposed by Zheng et al. (2004), which suggested the replacement of the original thickness for the
effective corroded thickness, which accounts for the length of the corrosion. This variable was calculated in the same way as the effective thickness suggested by ASME B31G (2009) for burst pressure analysis. However, the modified criteria only provided better results for relatively long defects in pipes.

\[
t_{\text{CORR}} = \frac{t - d}{1 - \frac{d}{t} \left(1 + 0.8 \left(\frac{l}{D_o t}\right)^2\right)}
\]

\(t_{\text{CORR}}\): effective corroded thickness of pipe

2.7.2. Alternative analytical formulations

Two alternative analytical formulation to estimate the maximum bending moment in the presence of other loads are mentioned in this work: one is provided by DNV-Os-F101 (2013) and the other one is described by Bai et al. (2014). According to Bai et al., the maximum allowed bending moment which can lead to local buckling failure can be analytically estimated considering the combination of applied loads on the Limit Stress Surface when the following assumptions are valid:

- Geometrically perfect pipe except for initial out-of-roundness;
- Elastic-perfect plastic material;
- Entire cross section has reached the limit stress;
- No change in cross-section geometry before the limit stress is reached.

Additionally, to account for modeling and input uncertainties, usage factors were inserted in the equation. The equation (27) was validated through FE analysis and provided a target reliability complying with Dnv (2015) and API (2000).

\[
M_{\text{Allowed}(F,p)} = \frac{\eta_{RM}}{\gamma_c} M_p \sqrt{1 - (1 - \alpha^2) \left(\frac{p}{\eta_{Rp} p_l}\right)^2} \cos \left[\frac{\pi \gamma_c F}{\eta_{(RP)}^2 p_l} - \alpha \frac{p}{\eta_{RP} p_l} \right] \sqrt{1 - (1 - \alpha^2) \left(\frac{p}{\eta_{RP} p_l}\right)^2}
\]

\(M_{\text{Allowed}}\): allowable bending moment

\(M_p\): plastic moment

\(p_l\): limit pressure

\(p\): pressure acting on the pipe

\(F_l\): limit longitudinal force

\(F\): longitudinal force acting on the pipe

\(\alpha\): correction factor

\(\gamma_c\): condition load factor

\(\eta_{RM}\): strength usage factor for bending moment

\(\eta_{Rp}\): strength usage factor for internal pressure
On the other hand, DNV-Os-F101 (2013) is the only found code to provide guidelines for combined loads. It recommends the following equation for pipes subjected to bending moment, effective axial force and internal pressure:

\[
\left\{ \gamma_m \gamma_{Sc} \frac{|M_{sd}|}{\alpha_p M_p(t_2)} + \left\{ \gamma_m \gamma_{Sc} \frac{S_{sd}(p_i)}{\alpha_c S_p(t_2)} \right\}^2 + \left( \frac{\alpha_p}{\alpha_c} \frac{p_i - p_e}{p_e(t_2)} \right)^2 \right\}^2 \leq 1
\]

(28)

This applies for

\[ 15 \leq \frac{D}{t_2} \leq 45, p_i > p_e, \left| \frac{S_{sd}}{S_p} \right| < 0.4 \]

where:

- \( M_{sd} \): design moment
- \( S_{sd} \): design effective axial force
- \( p_i \): internal pressure
- \( p_e \): external pressure
- \( p_b \): burst pressure
- \( S_p \): axial plastic capacity for a pipe
- \( M_p \): bending moment plastic capacity for a pipe
- \( \gamma_m \): material safety factor
- \( \gamma_{Sc} \): class safety factor
- \( \alpha_p \): pressure factor used in combined load criteria
- \( \alpha_c \): flow stress parameter
- \( t_2 \): pipe wall thickness. Prior to operation equal to \( t \). In operation equal to \( (t - d) \)

For pipes subjected to bending moment, effective axial force and external pressure:

\[
\left\{ \gamma_m \gamma_{Sc} \frac{|M_{sd}|}{\alpha_p M_p(t_2)} + \left\{ \gamma_m \gamma_{Sc} \frac{S_{sd}(p_i)}{\alpha_c S_p(t_2)} \right\}^2 + \left( \frac{\alpha_p}{\alpha_c} \frac{p_e - p_{MIN}}{p_e(t_2)} \right)^2 \right\}^2 \leq 1
\]

(29)

This applies for:

\[ 15 \leq \frac{D}{t_2} \leq 45, p_e > p_{MIN}, \left| \frac{S_{sd}}{S_p} \right| < 0.4 \]

where \( p_{MIN} \): minimum internal pressure that can be sustained (normally taken as zero)

\( p_e \): characteristic internal pressure
However, formulations proposed by this main code may not have, at least in this form, the simplicity to be applied in real operation conditions to easily recalculate operating pressure for corroded pipelines subjected to combined loads.

2.7.3. Finite-element modeling and analysis of pipelines

According to Roy et al. (1997) who performed full-scale tests in corroded oil transmission pipes subjected to combined loads, as quoted by Yu et al. (2011, p.1), the application of ASME B31G (2009) guidelines for internal pressure combined with bending moment could lead to non-conservative results. Therefore, further studies were developed by other authors in order to better understand the influence of bending moment in the failure of pipelines.

Some studies were performed based on the comparison of numerical simulations with experimental results. Yu et al. (2011) and Mohd et al. (2015) analyzed the influence of thinning areas in the pure bending loading capacity of pipelines, as figure 6 depicts. Yu et al. (2011) considered the presence of external corrosion in regions subjected to compression, in which the buckle moment was used as the carrying capacity of the corroded section. He compared the results obtained by FE analysis with full-scale experimental tests and concluded that the former represents an accurate approach to predict maximum bending moment capacity and the buckled shape on the defect region. Moreover, other findings were that quadratic, brick, or tetrahedral elements are recommended for simulations; accurate modeling of the edge smoothness of the LTA is not important and significant influence of secondary effects was detected during physical experiments. The model of corrosion as an area with uniform reduction in thickness is shown in Figure 6. The key parameters used for a corrosion defect are the corroded thickness $d$, the span of the damaged region $c$ and the length $l$ of the defect.

Yu et al., (2011) also showed that modelling edges as smooth or rigid edges, in the corroded area, only yields to a small difference of 4.5% between the reaction moment obtained in function of the pipes extremes (Figure 7).
In the literature review, tests were performed considering the corrosion located either on the tensile or on the compressive side, as Figure 8 depicts.

Other common practice is the use of symmetry boundaries to reduce the computational time required in numerical simulations, represented by the highlighted region A in Figure 9. In addition, the insertion of a “reference point” in which the bending moment is applied is an alternative for applying bending moment to the pipe, as represented in figure by the point B.
Yu et al. (2011) performed tests for specimens with different corroded dimensions. The width of the defect ranged from 15 degrees to 90 degrees and thickness reduction were of three types: 50%, 25% or Pit corrosion, as shown in Table 2.

The obtained experimental results confirmed the expected tendency that corrosion limits maximum bending moment capacity, as can be seen in Figure 10. Specimen number 1 to 4 indicate the maximum bending moment curve decreases with the increase of the corroded width; moreover, specimen 5 and 6 show this reduction is also observed when the corroded thickness increases. In addition, those tests revealed the fact that not always a clear bending moment peak is observed and a failure criteria, in this case the local buckling, should be adopted.

Table 2. Specimen and corrosion main dimensions (Yu et al., 2011)

<table>
<thead>
<tr>
<th>Testing group</th>
<th>Specimen no.</th>
<th>LTA area (ε)</th>
<th>LTA depth (d')</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>15 deg</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>30 deg</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>60 deg</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>90 deg</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>30 deg</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Virgin</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>Virgin</td>
<td>1/2</td>
<td>Bending moment</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>30 deg</td>
<td>1/2</td>
<td>Bending moment and internal pressure</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>45 deg</td>
<td>Pit corrosion</td>
<td>Bending moment</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Pit corrosion</td>
<td>Pit corrosion</td>
<td>Bending moment</td>
</tr>
</tbody>
</table>

Figure 9. Main boundary conditions for numerical bending moment simulations (Mohd et al., 2015)
Furthermore, when the defect is located in the compressive side, the ultimate sustained moment was found to be smaller than in the tension side, due to the local buckling that appears in the defect (Yu et al., 2011), as shown in Figure 11.

In the light of the non-conservative nature of available codes for combined loads, Mohd et al. (2015) aimed to correlate the ultimate strength interaction between internal pressure and bending moment for pipes with different local corroded thinning areas (LTA) sizes. Firstly, single-load simulations revealed that the detrimental effect of corrosion depth was greater to the burst pressure capacity than to bending moment capacity. In the first case, the obtained reduction of the burst pressure for pipes with 80% of...
locally corroded thickness were found to be 68% smaller than in a similar one with no corrosion. For those tests, represented in Figure 12, five pipes with different levels of corrosion were analyzed. Details of main dimensions of the defect can be found in Table 3 where Do and Di are the external and internal diameter of the pipe, t is the thickness, c is the width represented as an arc in degrees, d/t is the percentage of corroded thickness, l is the length of the defect and L is the total length of the pipe.

<table>
<thead>
<tr>
<th>Pipe no.</th>
<th>Do [mm]</th>
<th>Di [mm]</th>
<th>t [mm]</th>
<th>c [dgs]</th>
<th>d/t [-]</th>
<th>l [mm]</th>
<th>L [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>291</td>
<td>265.6</td>
<td>12.7</td>
<td>40</td>
<td>0</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.2</td>
<td>292</td>
<td>265.7</td>
<td>12.7</td>
<td>40</td>
<td>0.2</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.3</td>
<td>293</td>
<td>265.8</td>
<td>12.7</td>
<td>40</td>
<td>0.4</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.4</td>
<td>294</td>
<td>265.9</td>
<td>12.7</td>
<td>40</td>
<td>0.6</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.5</td>
<td>295</td>
<td>265.10</td>
<td>12.7</td>
<td>40</td>
<td>0.8</td>
<td>200</td>
<td>2,000</td>
</tr>
</tbody>
</table>

On the other hand, a reduction of only 14% in the ultimate bending moment was obtained for the same specimens studied, even when considering the same corrosion levels. The main results for all thicknesses analyzed are presented in Figure 12.

Afterwards, combined loads tests were performed for the same specimen detailed in Table 3 showing that internal pressure has a more significant effect on the ultimate bending moment capacity of pipes with deeper corrosion defects than shallower defects. Firstly, an internal pressure equal to a percentage of the burst pressure previously obtained was applied; secondly, the pipe was bend until the
failure is detected and the ultimate bending moment registered. In an intact pipe whose dimensions are in Table 3, the presence of 60% of the burst pressure reduced the ultimate bending moment in only 28%; nonetheless, when the corroded thickness was equal to 80% of initial thickness, the applicability of the same pressure reduced the ultimate moment in 69%, as shown in Figure 13. Pipes with intermediate corrosions levels presented reductions following a similar tendency.

![Figure 13. Detrimental effect of internal pressure and corrosion in the ultimate bending moment of an intact pipe and a pipe with 20% of corroded thickness](image)

Altogether, the final objective of these previously mentioned studies is the development of simplified closed formulas, similar to the ones of ASME B31G (2009) for the burst pressure pressure, which would comprise the combined effect of different load types, such as internal and external pressure, axial loads and bending moments, which is currently the state-of-art of this subjected.
3. Description of the methodology

The main goal of this study is the evaluation of the influence of combined loads on the residual strength of corroded pipelines through non-linear finite element simulations. In brief, this project could be considered a recreation/extension of the work done by Yu et al., (2011) and, particularly by Mohd et al., (2015).

The maximum bending moment a corroded pipe is able to sustain and its burst pressure depends on the material properties, the main dimensions of the pipe and the dimensions of the corrosion defect. Finite elements models have been widely used in literature and provide a valid tool for assessing the response of pipelines due to applied loads (Yu et al., 2011). However, certain cares must be taken in the pursuit of creating a successful numerical model, such as: meshing refinement study, proper boundary conditions, load application, proper element choice, use of true stress-strain material curve and others which are described in Section 3.

Moreover, regarding the simulation of the model, it is worth to describe the methodology adopted to determine the failure point and the sequence of application of the loads in the combined load simulations as well as the key concept of quasi-static simulations. This topic will be addressed deeper in sub-section 3.7 where the techniques employed for the generation of the model are detailed.

Finally, the main dimensions of the pipe such as: external diameter, thickness, length and material properties are the same used by Mohd et al. (2015) since the recent analysis performed by them can be used for purposes of verification, as summarized in Table 4.

<table>
<thead>
<tr>
<th>Table 4. Main geometrical and material dimensions of the pipe model</th>
</tr>
</thead>
<tbody>
<tr>
<td>External diameter ($D_o$)</td>
</tr>
<tr>
<td>Thickness ($t$)</td>
</tr>
<tr>
<td>Length ($L$)</td>
</tr>
<tr>
<td>Yield Stress ($\sigma_y$)</td>
</tr>
<tr>
<td>Poisson Coefficient ($\nu$)</td>
</tr>
<tr>
<td>Elastic Modulus ($E$)</td>
</tr>
</tbody>
</table>

3.1. Boundary Conditions

The application of adequate boundary conditions plays an important role in the numerical analysis. The boundary conditions are applied to the rigid section located at the extremes of the pipe and are identical in each of them. They are:
• The node located at the center of the section has its displacement constrained in the vertical (y) and in the lateral direction (x), therefore, only displacement in the pipe’s axial direction (z) is allowed as well as rotation in the x direction;

• The nodes located at 0º and 180º at the rigid section have their vertical displacement (y) constrained. This is done to prevent the undesired rotation of the section around the z axis – torsional motion, as the Figure 14 depicts.

![Figure 14. Boundary conditions at z=2,000 mm](image)

• The same is performed to the section located at z=0. Nonetheless, the additional constrain of axial motion (z displacement) is also applied to the three nodes located at y=0, what is done to avoid rigid-body motion of the model.

In Figure 15 it is shown the incremental rotation of the pipeline as the result of bending moment loads applied (translated into loads in red). One can see that the constraint applied to the node A limit its movement in the X and Y direction, although permits translation along the axial Z direction as the section rotates.
It is common practice to simulate only half length of the pipe by using symmetrical boundary conditions. In this study, however, the author chose to model the full length of the pipeline as this would lead to a simpler model to be implemented. However, the implementation of these conditions could substantially reduce the computation time and it is a valid alternative to improve the model.

3.2. Loads applied to the model

The model is subjected, in a first set of numerical calculations, to only bending moment or only to internal pressure. Later, the combined effect of those loads is assessed. In the modelling phase, the internal pressure is applied to all internal elements’ surfaces except to those parts of the rigid section, at the extremes. On the other hand, to the nodes belonging to this section axial forces are applied according to their vertical position in order to impose a pure bending moment to the section.

Notwithstanding, the application of a bending moment may induce large forces on few nodes which may bring out the issue of concentrated stresses. For this reason, the loads should be applied to a reasonable large quantity of nodes. In the present study, all nodes located at the cover in a distance equal to its radius were employed, as illustrated in Figure 16.
The force $F_{node}$ value ultimately depends on the node vertical position, on the bending moment applied and on the second moment of inertia of the whole section. Firstly, the nodes stresses are calculated.

$$\sigma = \frac{M}{I}$$

(30)

where,

$$I = \frac{\pi}{64} (D_0^4 - D_i^4)$$

(31)

$\sigma$: is the stress subjected to the node.

$y$: is the vertical position in relation to the $y$ global axis

$M$: is applied bending moment

$I$: is the second moment of inertia of the pipe's section

$D_0$: outside diameter

$D_i$: internal diameter

Secondly, the section area is obtained according to equation (32). The element area is the ratio of this value by the number of elements in the section.

$$A_{section} = \frac{\pi}{4} (D_0^2 - D_i^2)$$

(32)

$$A_{Elem} = \frac{A_{section}}{N_{elem}}$$

(33)

$A_{Elem}$: area referring to one element in the cover section

$A_{section}$: area of the "cover" cross section

$N_{elem}$: number of elements in the cover section

Finally, the force is obtained by multiplying the node stress of equation (30) by the element area of equation (33).

$$F_{node} = \sigma A_{Elem}$$

(34)

$F_{node}$: force applied to one node located at the extreme section

Moreover, one can observe that the section rotates due to the presence of longitudinal loads. This rotation, which is half the curvature angle of the pipe, reduces the effective bending moment applied.
perpendicularly to the section. Therefore, the effective moment in a certain step \( t \) of simulation can be calculated using the rotation \( \theta'(t) \) around the Z-local-axis of the node A, as following:

\[
M_{\text{effective}}(t) = M \cos[\theta'(t)]
\]

\( M_{\text{effective}} \): resultant effective bending moment in a given time \( t \)

\( \theta'(t) \): curvature in a given time \( t \)

### 3.3. Element type and main dimensions

In this study, a full-length pipe is modeled in ANSYS 16.2 software where the quasi-static response of the model is simulated. According to Yu et al., (2011), either quadratic elements, brick or tetrahedral elements are recommended for bending moment analysis. In this study, the quadratic element SHELL181 with six degrees of freedom in each node was chosen, which is well suited for linear, large rotation, and/or large strain nonlinear applications. Figure 17 depicts the SHELL181 geometry.

Last but not least, SHELL181 in ANSYS 16.2 has also keys, i.e. additional features that can be altered. In the model, the key option 3 was set to full integration with incompatible modes.

#### Chosen Pipe diameter and thickness

The main dimensions of the model are the external diameter, thickness and length of the pipe as shown in Table 5. The chosen external pipe diameter and thickness are the same used by Mohd et al., (2015) which will be, afterwards, used for verification of the obtained results by the model created.

---

3 Image obtained from ANSYS 16.2 Help View.
Table 5 - Main geometrical dimensions of the model

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>External diameter ($D_o$)</td>
<td>291.0 [mm]</td>
</tr>
<tr>
<td>Thickness ($t$)</td>
<td>12.7 [mm]</td>
</tr>
<tr>
<td>Length ($L$)</td>
<td>2,000 [mm]</td>
</tr>
</tbody>
</table>

3.4. Material properties

As far as the material properties are concerned, the steel considered in the analysis was STEEL API 5L X42, which properties such as yield stress $\sigma_y$, Poisson coefficient $\nu$ and elastic modulus $E$ are summarized in Table 6. This material is uniform distributed along the pipe, except in the extreme covers of the model.

Table 6. Material properties

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Stress</td>
<td>290</td>
</tr>
<tr>
<td>Poisson Coefficient</td>
<td>0.3</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>210</td>
</tr>
<tr>
<td>Tensile Stress</td>
<td>[MPa]</td>
</tr>
</tbody>
</table>

The true stress-strain curve was obtained by the linear interpolation of points taken from real tests, available in Mohd et al., (2015).

Figure 18. True Stress-Strain curve for STEEL API 5L X42
In previous studies that performed numerical analyses in ANSYS software, the application of bending moments has been done by using a rigid ring at the extreme of the pipe where forces will be in fact set, as done by Giordano & Guarracino (2002). This approach may affect displacement solution for nodes located near the added material, however, as the corroded defect is placed far from the affect region such effect will be neglected in the analysis.

Figure 19. ANSYS end section constraint model (Giordano & Guarracino, 2002)

In the present study, a rigid material disk, also called “cover”, is employed which has the important function of allowing the transmission of bending moment efforts uniformly from the nodes as well as to avoid out-of-plane movement by them. Stiffened section and the base-material (STEEL X42), are shown in Figure 16.

If loads had been applied directly to Steel X42, excessive strains due to concentrated stresses would have interfered in the rotation of the extreme. For this reason, such rigid material has an Elastic Modulus equal to a hundred times the base-material, as shown in Table 7.

<table>
<thead>
<tr>
<th>Table 7. Rigid material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Stress ($\sigma_y$)</td>
</tr>
<tr>
<td>Poisson coeficiente ($\nu$)</td>
</tr>
<tr>
<td>Elastic Modulus ($E$)</td>
</tr>
</tbody>
</table>
3.5. Corrosion modelling

The corroded region is modeled as a local thinning area (LTA) located at the pipe mid-length. This way of modelling the defect translates into a conservative approach in which any defect shape is represented by the affected region. Therefore, the presence of corrosion can be defined by three main parameters: the length $l$, the width $c$ represented by an arc in degrees and the corroded thickness $d$.

![Figure 20. Main dimensions of the corroded region in the model implemented](image)

Other important aspect is whether the LTA is located at the compressive or at tensile side when the pipe is bend by the applied load. The former location is capable of taking into account buckling effects, whereas the latter results in the failure of the section due to yielding. In this study, the defect will be located at the tensile side, at the mid length of the pipe, as Figure 21 depicts and as was done by Mohd et al., (2015), since the first goal would be the validation of the model by the verification of the results obtained by them.

![Figure 21. Corrosion located at the tensile side (Mohd et al., 2015)](image)
Corrosion thickness can be set in the model through two steps: by, firstly, shifting of the nodes located in the corroded area and, secondly, by the reduction of the element thickness. As this reduction occurs in both sides, in order to achieve a corroded thickness equal to $d$ the new thickness of the element should be $t_{\text{new}}$ and the element must be shifted $d' = \frac{d}{2}$, as done by Teixeira et al., (2008). This procedure can be seen in Figure 22, where $c$ corresponds to $d$.

$$t_{\text{new}} = t - d$$  \hspace{1cm} (36)

![Figure 22 - Relation between the eccentricity and the corrosion (Teixeira et al., 2010)](image)

In order to evaluate the effect of corrosion thickness, five degrees of corrosion were selected: intact, 20%, 40%, 60%, 80%. Regarding the width and length of the defect, the same values adopted by Mohd et al., (2015) will be considered for this study, as shown in Table 8. The arc length of 100 mm, adopted by Mohd, corresponds to an arc of about 40 degrees considering a diameter of 291 mm, as shown in Table 5.

$$c = \left( \frac{100}{291} \right) \frac{180}{\pi} \approx 40 \text{ degrees}$$  \hspace{1cm} (37)

Table 8. Width ($c$) and length ($l$) of the defect

<table>
<thead>
<tr>
<th>$c$</th>
<th>40.0 [degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>200.0 [mm]</td>
</tr>
</tbody>
</table>

Moreover, the refinement of the mesh at the corroded region, where the highest stresses are expected, is particularly important for the accuracy of the results. Therefore, the choice of an adequate meshing can be done by performing a meshing analysis to take into account the impact of a finer mesh on the obtained results. This topic is further detailed in sub-section 3.6. An example of refinement for a burst pressure simulation, where half pipe was modeled by symmetric conditions, is shown in Figure 23.
3.6. Meshing refinement and convergence

An analysis of meshing convergence is crucial to determine a proper refinement for the general model and, especially, the LTA. This refinement influences the slope between corroded and non-corroded region. Although may seem important to accurately model the LTA, one should evaluate the influence of the refinement in the results of single load simulations for bending moment and internal pressure to determine an efficient meshing for the model.

Firstly, three arbitrary selected meshing refinements were considered which correspond to the number of divisions on the circumferential and longitudinal direction along the whole model. These meshes were simulated for two cases, in the first one no corrosion was considered. On the other hand, in the second case, a significant corrosion of 60% of the original thickness was considered. The main objective is to assess if a thinner refinement at the meshing region leads to more accurate results having as reference (and assumed corrected) the results from Mohd et al., (2015). As shown in Table 9, no significant difference in the results was found when the meshing was thinner. The meshing of 36/20 divisions in the circumferential and longitudinal directions yields to a reasonable modelling of the corroded defect without leading to a too much longer time for simulation, henceforth, it will be selected for performing other simulations.
Table 9. Deviation between reference result from Mohd et al., (2015) and obtained results for different refinements

<table>
<thead>
<tr>
<th>Type</th>
<th>INTACT</th>
<th>Max Moment [kN.m]</th>
<th>Difference</th>
<th>0.6 DEFECT</th>
<th>Max Moment [kN.m]</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>R18/10</td>
<td>368.6</td>
<td>10.8%</td>
<td></td>
<td>360.1</td>
<td>4.7%</td>
<td></td>
</tr>
<tr>
<td>R36/20</td>
<td>370.8</td>
<td>10.2%</td>
<td></td>
<td>355.2</td>
<td>6.0%</td>
<td></td>
</tr>
<tr>
<td>R72/40</td>
<td>371.2</td>
<td>10.1%</td>
<td></td>
<td>359.3</td>
<td>4.9%</td>
<td></td>
</tr>
<tr>
<td>MOHD</td>
<td>413.0</td>
<td>0%</td>
<td></td>
<td>378.0</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Figure 23, it is common to use a thinner meshing at mid length of the pipe to more precisely model the corroded region and reduce the slope between the corroded and non-corroded areas. For this reason, a factor responsible for the increase of the refinement is defined as $p_l$, which corresponds to how many times the average size of the element at the center of the corrosion is smaller compared to an outside element.

$$ p_l = \frac{\text{Size}_{\text{outside}}}{\text{Size}_{\text{center corrosion}}} $$ (38)

$pl$: refining meshing factor

$\text{Size}_{\text{center corrosion}}$: meshing at the center of the corroded region

$\text{Size}_{\text{outside}}$: meshing spacing size outside the corroded region

It is important to enhance the fact that refinement is not uniform inside the affected region because a continuity in the discretization should exist between the most refined and least refined areas. Therefore, meshing is thinner at the center of the geometry delimited by the defect and becomes coarser as it moves towards the outside region until it becomes uniform and equal to 36/20 divisions. Figure 24 shows the different regions and the refinement along the pipe.

![Figure 24. Meshing longitudinal divisions along pipe](image-url)
Different values for $pl$ were considered, such as: 2, 4 and 6. For each case the results obtained for the burst pressure simulations ($P_b$) were compared to Mohd et al., (2015) and the deviations are shown in Table 10. One can see that $pl = 4$ yields to slightly smaller deviations than $pl = 2$, however, this trend is not observed for $pl = 6$. For this reason, the refinement adopted throughout the simulations will be equal to $pl = 4$. This simulation was done for the burst pressure since analysis for the ultimate bending moment showed that results were not sensible to further refinement of meshing. A visual comparison between the three levels of refinement is shown in Figure 25.

**Table 10. Deviation of $P_b$ in percentage from Mohd et al., (2015), per degree of refinement and level of corrosion**

<table>
<thead>
<tr>
<th>d/t</th>
<th>$pl = 2$</th>
<th>$pl = 4$</th>
<th>$pl = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.1%</td>
<td>1.0%</td>
<td>-31.5%</td>
</tr>
<tr>
<td>0.2</td>
<td>5.7%</td>
<td>5.7%</td>
<td>5.5%</td>
</tr>
<tr>
<td>0.4</td>
<td>8.8%</td>
<td>6.6%</td>
<td>5.4%</td>
</tr>
<tr>
<td>0.6</td>
<td>11.2%</td>
<td>9.3%</td>
<td>9.4%</td>
</tr>
<tr>
<td>0.8</td>
<td>16.7%</td>
<td>13.6%</td>
<td>13.2%</td>
</tr>
</tbody>
</table>

Finally, the meshing on the circumferential direction is also $pl$ thinner in the corroded region. Nonetheless, this refinement extends along the whole pipe, as shown in Figure 26.
3.7. Simulation methodology

The simulation procedure is performed through a quasi-static method, i.e an incremental load is applied to each step of simulation until the maximum load set by the user is reached, for each instant of simulation the static solution is calculated. Steps are determined by the variable \( TIME \) that does not represent a period of time but a fraction of the maximum load, as Table 11 shows. The numerical calculations have considered more steps and smaller time increments than that showed in Table 11, however, the minimum load step can be adjusted by the user.

Table 11. Step, \( TIME \) and corresponding applied moment for an ultimate bending moment simulation

<table>
<thead>
<tr>
<th>Step</th>
<th>TIME</th>
<th>Moment Applied [kN.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>250</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>300</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>350</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
<td>400</td>
</tr>
<tr>
<td>9</td>
<td>0.9</td>
<td>450</td>
</tr>
<tr>
<td>10</td>
<td>0.10</td>
<td>500</td>
</tr>
</tbody>
</table>
Additionally, some methods can be used to determine the point at which simulation finishes. In the present study, it ends when the structure becomes unstable. In order to accomplish this, the `ARCTRM,L` command is used. According to ANSYS “`ARCTRM L terminates the analysis if the first limit point has been reached. The first limit point is that point in the response history when the tangent stiffness matrix becomes singular (i.e., the point at which the structure becomes unstable)`. On the other hand, other possible approach is the usage of `ARCTRM,U` command which finishes the simulation when a specified degree of displacement is reached. In the present case, the value of 0.3 rad was chosen for the rotation of the section located at the pipe’s extreme, this value surpasses the rotation correspondent to the failure point. Hence, the maximum reaction load obtained during simulation is considered as the ultimate bending moment of the pipe. This second approach requires the previous knowledge of a good estimative for the failure point in order to avoid unnecessary simulations. Last but not least, Newton-Raphson Method can apply incrementing loads and is employed to determine the burst pressure in single-load simulations. It also assumes the failure point as the one in which the stiffness matrix becomes singular.

Finally, the approach employed in combined load numerical calculations is to first apply a constant fraction of the burst pressure of the specimen (previously determined by single load simulation) followed by the application of an incremental bending moment load until the failure of the specimen. The maximum load registered at this step is assumed to be the ultimate bending moment that the pipe is able to sustain in the presence of internal pressure.
4. Results and discussion

This section aims to present the main results obtained by the FE structural model. Firstly, the validation is done by comparing the results obtained for single load numerical calculations for bending moment and internal pressure for corroded thickness ranging from 20% to 80%, with results obtained by Mohd et al., (2015). Moreover, analytical formulations from main codes are used for comparison between obtained results and reference from Mohd et al., (2015). This validation is also part of a parametric study performed for the main dimensions of the defect such as thickness, length and width. Finally, the results for combined load numerical calculations in the presence of internal pressure and bending moment are described.

4.1. Parametric study

A parametric study regarding the main dimensions of the defect was done. By this analysis, one aims to understand the effect parameters, such as thickness, length or width of the defect on the ultimate bending moment capacity and burst pressure of a given corroded pipeline.

4.1.1. Analysis of thickness variation

First of all, main dimensions used in the analysis of thickness variation are summarized in Table 12. Those dimensions are the same chosen by Mohd et al., (2015) whose results, also compared with analytical formulations mentioned in the state of art section, will make possible the validation of the model.

<table>
<thead>
<tr>
<th>Pipe no.</th>
<th>Do [mm]</th>
<th>Di [mm]</th>
<th>t [mm]</th>
<th>c [dgs]</th>
<th>d/t [-]</th>
<th>l [mm]</th>
<th>L [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>291</td>
<td>265.6</td>
<td>12.7</td>
<td>40</td>
<td>0</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.2</td>
<td>292</td>
<td>265.7</td>
<td>12.7</td>
<td>40</td>
<td>0.2</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.3</td>
<td>293</td>
<td>265.8</td>
<td>12.7</td>
<td>40</td>
<td>0.4</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.4</td>
<td>294</td>
<td>265.9</td>
<td>12.7</td>
<td>40</td>
<td>0.6</td>
<td>200</td>
<td>2,000</td>
</tr>
<tr>
<td>No.5</td>
<td>295</td>
<td>265.10</td>
<td>12.7</td>
<td>40</td>
<td>0.8</td>
<td>200</td>
<td>2,000</td>
</tr>
</tbody>
</table>

In addition, Table 13 summarizes the results for the ultimate bending moment simulations and respective deviation in comparison with Mohd et al. (2015) One can see that deviation starts with its maximum value of about 7.0% and decreases to -1.1% as the corroded thickness increases. Differences can be mainly due to the methodology employed for the application of loads. In the reference, loads are applied as a remote point centered at extreme section, as point B in Figure 9. In the present study, however, they were applied through a rigid cover located at the extreme of the pipe, as shown in Figure 16.
Some reasons that explain the reduction on the deviations may be related to the fact that when thickness is reduced, failure is reached faster and the impact of using different models becomes less significant as shown in Table 13.

In addition, comparison of results from Net Section Criteria (NSC) shows a more conservative prediction for the maximum load than the ones obtained from the model and from the reference. However, results for the NSC did not presented a constant tendency of being closer either to the created model or to the adopted reference.

Table 13. Obtained results and respective deviation from Mohd et al., (2015) for the ultimate bending capacity for different corroded thicknesses (p=4). *NSC considering (Kim, Oh, Park, & Hasegawa, 2006)

<table>
<thead>
<tr>
<th>Pipe no.</th>
<th>d/ t</th>
<th>M_obtained [kN.m]</th>
<th>M_Mohd [kN.m]</th>
<th>Deviation</th>
<th>M_NSC* [kN.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>0</td>
<td>384</td>
<td>413</td>
<td>7.0%</td>
<td>407</td>
</tr>
<tr>
<td>No.2</td>
<td>0.2</td>
<td>380</td>
<td>405</td>
<td>6.2%</td>
<td>381</td>
</tr>
<tr>
<td>No.3</td>
<td>0.4</td>
<td>375</td>
<td>395</td>
<td>5.1%</td>
<td>356</td>
</tr>
<tr>
<td>No.4</td>
<td>0.6</td>
<td>367</td>
<td>379</td>
<td>3.1%</td>
<td>330</td>
</tr>
<tr>
<td>No.5</td>
<td>0.8</td>
<td>359</td>
<td>355</td>
<td>-1.1%</td>
<td>304</td>
</tr>
</tbody>
</table>

Figure 27 shows the results obtained, reference data and corresponding deviation, as a function of the corroded thickness of the pipe. The developed model reveals a smaller detrimental influence of the presence of the defect than the one of Mohd et al., (2015). This can be seen by a smaller slope of the curve in Figure 27.

Figure 27. Comparison between simulation results, Mohd et al., (2015) results and NSC results for the ultimate bending moment for different corroded thicknesses
Nonetheless, the numerical results obtained for the burst pressure show a different and opposite trend. As the depth of the defect increases, the deviation from Mohd et al., (2015) increases, as shown in Table 14. Moreover, analysis of prediction according to ASME B31G (2009) for only internal pressure may lead to the conclusion that this code can be perceived as overconservative for the corroded dimensions studied. Especially for intact pipelines, for instance, one can observe a difference of around 30% between the average of model numerical results and Mohd, and burst pressure predicted according to the main code.

Table 14. Obtained results and respective deviation for the burst pressure for different corroded thicknesses \((pl=4)\). \(P_{b_{31G}}\) and \(P_{b_{Netto}}\) are reference pressure according to B31G.

<table>
<thead>
<tr>
<th>Pipe no.</th>
<th>d/ t</th>
<th>(P_{b_{obtained}}) [MPa]</th>
<th>(P_{b_{Mohd}}) [MPa]</th>
<th>Deviation</th>
<th>(P_{b_{B31G}}) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>0</td>
<td>39.4</td>
<td>39.0</td>
<td>-1.0%</td>
<td>27.8</td>
</tr>
<tr>
<td>No.2</td>
<td>0.2</td>
<td>36.9</td>
<td>34.9</td>
<td>-5.7%</td>
<td>25.2</td>
</tr>
<tr>
<td>No.3</td>
<td>0.4</td>
<td>29.6</td>
<td>27.8</td>
<td>-6.6%</td>
<td>22.3</td>
</tr>
<tr>
<td>No.4</td>
<td>0.6</td>
<td>22.5</td>
<td>20.6</td>
<td>-9.3%</td>
<td>19.2</td>
</tr>
<tr>
<td>No.5</td>
<td>0.8</td>
<td>14.0</td>
<td>12.4</td>
<td>-13.6%</td>
<td>15.7</td>
</tr>
</tbody>
</table>

Figure 28. Obtained results compared to Mohd et al., (2015) results and ASME B31G (2009) formulations for the burst pressure at different corroded thicknesses.
4.1.2. Analysis of length variation

The considered base case is shown in Table 15. A specific level of corrosion, equal to 60%, was chosen since it corresponds to a high level of corrosion.

**Table 15. Main defect dimensions for the parametric study**

<table>
<thead>
<tr>
<th>$D_o$</th>
<th>External Diameter</th>
<th>291.0 [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Thickness</td>
<td>12.7 [mm]</td>
</tr>
<tr>
<td>$L$</td>
<td>Length pipe</td>
<td>2,000 [mm]</td>
</tr>
<tr>
<td>$c$</td>
<td>Width</td>
<td>40.0 [degrees]</td>
</tr>
<tr>
<td>$l$</td>
<td>Length defect</td>
<td>200.0 [mm]</td>
</tr>
<tr>
<td>$\delta / \ell$</td>
<td>Corroded %</td>
<td>60% [-]</td>
</tr>
</tbody>
</table>

The results when the length of the defect is increasing are represented in Table 16. As one can see, length variations ranging from +60% to -60% have a little effect, of only around -1% and +0.8%, respectively, in the final ultimate bending moment.
Table 16. Variations of the ultimate bending moment by increasing the length of the defect

<table>
<thead>
<tr>
<th>Defect Length/Original</th>
<th>Variation from reference</th>
<th>Max Bending Moment [kN.m]</th>
<th>Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-60%</td>
<td>372.0</td>
<td>0.8%</td>
</tr>
<tr>
<td>0.6</td>
<td>-40%</td>
<td>370.6</td>
<td>0.5%</td>
</tr>
<tr>
<td>0.8</td>
<td>-20%</td>
<td>370.2</td>
<td>0.4%</td>
</tr>
<tr>
<td>1.0</td>
<td>0%</td>
<td>368.8</td>
<td>0.0%</td>
</tr>
<tr>
<td>1.2</td>
<td>20%</td>
<td>368.4</td>
<td>-0.1%</td>
</tr>
<tr>
<td>1.4</td>
<td>40%</td>
<td>365.9</td>
<td>-0.8%</td>
</tr>
<tr>
<td>1.6</td>
<td>60%</td>
<td>365.1</td>
<td>-1.0%</td>
</tr>
</tbody>
</table>

Figure 29 depicts the curves obtained for these numerical calculations. One can see that the curves are extremely close and almost overlapped. This makes possible to conclude that such variable of the defect plays a secondary role in decreasing the ultimate bending capacity of the pipe.

Figure 29. Effect of the length of the defect on the bending capacity.
Sample input ranged from +60% to -60% of the original length

As far as the length of the defect is considered in the effect of burst pressure, some interesting results were obtained, as shown in Table 17. One can see that increasing the length leads to a smaller burst pressure and decreasing it leads to a greater pressure capacity. Moreover, the gain in capacity is larger in comparison to the reduction in strength for the same percentage of variation. Variations ranged from 21.2% and -6.7% for -60% and +60% of the original defect length of 200 mm, respectively.
Table 17. Variation of the burst pressure by increasing the length of the defect

<table>
<thead>
<tr>
<th>Defect Length/Original length</th>
<th>Variation from reference</th>
<th>Burst Pressure [MPa]</th>
<th>Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-60%</td>
<td>27.23</td>
<td>21.2%</td>
</tr>
<tr>
<td>0.6</td>
<td>-40%</td>
<td>25.74</td>
<td>14.6%</td>
</tr>
<tr>
<td>0.8</td>
<td>-20%</td>
<td>24.71</td>
<td>10.0%</td>
</tr>
<tr>
<td>1.0</td>
<td>0%</td>
<td>22.46</td>
<td>0.0%</td>
</tr>
<tr>
<td>1.2</td>
<td>20%</td>
<td>22.07</td>
<td>-1.7%</td>
</tr>
<tr>
<td>1.4</td>
<td>40%</td>
<td>21.11</td>
<td>-6.0%</td>
</tr>
<tr>
<td>1.6</td>
<td>60%</td>
<td>20.96</td>
<td>-6.7%</td>
</tr>
</tbody>
</table>

Curves for the internal pressure and radial displacement of a node located at the mid length of the pipe in relation to its original position can be analyzed in Figure 30. One can observe that different defect lengths only influence the reaction pressure behavior after 12.5 MPa.

4.1.3. Analysis of width variation

As one can see at Table 18, width variations ranging from +60% to -60% have also little effect in the final ultimate bending moment, although they are greater than the ones obtained by the length
scenario. The increase and decrease in the final bending moment shows a tendency in which wider defects are associated with slightly smaller bending capacity. However, results did not make possible to establish a proportional correlation between the two factors. In some cases, for instance, when $\frac{\text{width}_{\text{defect}}}{\text{width}_{\text{original}}} = 0.6$ led to a bigger increase in the bending capacity than when width was equal to $\frac{\text{width}_{\text{defect}}}{\text{width}_{\text{original}}} = 0.4$.

Table 18. Variations of the ultimate bending moment by increasing or decreasing the width of the defect

<table>
<thead>
<tr>
<th>Defect Width/Original Width</th>
<th>Variation from reference</th>
<th>Max Bending Moment [kN.m]</th>
<th>Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-60%</td>
<td>376.2</td>
<td>2.0%</td>
</tr>
<tr>
<td>0.6</td>
<td>-40%</td>
<td>380.0</td>
<td>3.0%</td>
</tr>
<tr>
<td>0.8</td>
<td>-20%</td>
<td>376.5</td>
<td>2.1%</td>
</tr>
<tr>
<td>1.0</td>
<td>0%</td>
<td>368.8</td>
<td>0.0%</td>
</tr>
<tr>
<td>1.2</td>
<td>20%</td>
<td>366.3</td>
<td>-0.7%</td>
</tr>
<tr>
<td>1.4</td>
<td>40%</td>
<td>358.5</td>
<td>-2.8%</td>
</tr>
<tr>
<td>1.6</td>
<td>60%</td>
<td>358.0</td>
<td>-2.9%</td>
</tr>
</tbody>
</table>

The obtained curves of Figure 31 show a similar behavior of the bending reaction moment as far as the rotation of the section located at the extreme of the pipe increases. Such tendency indicates small influence of variations of this parameter.

![Figure 31. Effect of the width of the defect on the ultimate bending moment. Sample input ranged from +60% to -60% of the original length](image)
A similar analysis has been carried out for the burst pressure considering a range of variation of the width of the defect summarized in Table 19. A pattern of increase of strength capacity can be observed when the defect area is reduced. However, no proportional correlation could be established between the reduction of width and increase of the burst pressure, since for 1.4 and 1.6 ratios yield to similar values.

Table 19. Variation of the burst pressure by increasing the width of the defect

<table>
<thead>
<tr>
<th>Defect Width/Original width</th>
<th>Variation from reference</th>
<th>Burst pressure [MPa]</th>
<th>Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-60%</td>
<td>23.32</td>
<td>3.9%</td>
</tr>
<tr>
<td>0.6</td>
<td>-40%</td>
<td>23.23</td>
<td>3.4%</td>
</tr>
<tr>
<td>0.8</td>
<td>-20%</td>
<td>22.94</td>
<td>2.1%</td>
</tr>
<tr>
<td>1.0</td>
<td>0%</td>
<td>22.46</td>
<td>0.0%</td>
</tr>
<tr>
<td>1.2</td>
<td>20%</td>
<td>22.19</td>
<td>-1.2%</td>
</tr>
<tr>
<td>1.4</td>
<td>40%</td>
<td>22.00</td>
<td>-2.1%</td>
</tr>
<tr>
<td>1.6</td>
<td>60%</td>
<td>22.00</td>
<td>-2.0%</td>
</tr>
</tbody>
</table>

Moreover, obtained curves for the internal pressure and radial displacement of a node located at the mid length of the pipe in relation to its original position can be analyzed in the horizontal axis, as Figure 32 depicts. The maximum pressure of the curve corresponds to the burst pressure taken.

Figure 32. Effect of the width of the defect for burst pressure. Sample input ranged from +60% to -60% of the original length
From this analysis, becomes clear that variations in the length of the defect have a greater influence in the final burst pressure of the system than variations in the width. Moreover, the width and length of the defect have a small influence on the ultimate bending capacity of the corroded pipe.

4.2. Combined loads

Combined load numerical calculations are an important way to measure the impact of two or more types of loads in the failure of pipelines. This study is focused on the evaluation of the effect of internal pressure in the ultimate bending moment of corroded pipelines. In order to accomplish that, numerical calculations are carried out for different corroded thicknesses and different internal pressures. The approach adopted to choose the intensity of internal pressure to be applied was the same used by Mohd et al., (2015) in which 20%, 40%, 60% and 80% of the burst pressure from single loads simulations from Table 14 were applied to a given pipe.

In turn, the corroded thickness simulated ranged from intact condition to 40% of the original thickness. Main dimensions of the pipe, main dimensions of the defect and burst pressure previously obtained from Table 14 are summarized in Table 20 and will be used to perform combined load simulations.

<table>
<thead>
<tr>
<th>Pipe no.</th>
<th>Do [mm]</th>
<th>Di [mm]</th>
<th>t [mm]</th>
<th>c [dg]s</th>
<th>d/t [-]</th>
<th>l [mm]</th>
<th>L [mm]</th>
<th>Pb_obtained [MPa]</th>
<th>Pb_Mohd [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>291</td>
<td>265.6</td>
<td>12.7</td>
<td>40</td>
<td>0</td>
<td>200</td>
<td>2,000</td>
<td>39.4</td>
<td>39.0</td>
</tr>
<tr>
<td>No.2</td>
<td>292</td>
<td>265.7</td>
<td>12.7</td>
<td>40</td>
<td>0.2</td>
<td>200</td>
<td>2,000</td>
<td>36.9</td>
<td>34.9</td>
</tr>
<tr>
<td>No.3</td>
<td>293</td>
<td>265.8</td>
<td>12.7</td>
<td>40</td>
<td>0.4</td>
<td>200</td>
<td>2,000</td>
<td>29.6</td>
<td>27.8</td>
</tr>
</tbody>
</table>

Firstly, internal pressure condition is simulated by a Newton-Raphson method in which the static equilibrium is calculated. Secondly, an increasingly bending moment is applied until the failure of pipeline is detected. The maximum bending moment detected during the simulation is used as the ultimate bending moment for such corroded thickness and for a particular ratio of internal burst pressure.

Table 21 for the intact condition shows that the application of internal pressure corresponds to a decrease in the strength capacity of the model expressed by the degradation factor for combined loads. The degradation factor is defined in equation (39) as the ratio between the ultimate bending moment found when a certain pressure is applied and the one when no internal pressure is present in the pipe for the same corroded thickness.

\[
\text{degradation factor} = \frac{M_{pi}}{M_{p=0}}
\] (39)
In Table 21, columns correspond, respectively, to the percentage of the burst pressure applied to the system, the obtained maximum ultimate bending moment for the numerical simulation, the ultimate bending moment predicted by Mohd et al., (2015), the degradation factor and the ratio of burst pressure used by the reference in terms of previously defined burst pressure, as shown in Table 14.

**Table 21. Ultimate bending moment obtained for combined load tests in the presence internal pressure and bending moment for an intact pipeline**

<table>
<thead>
<tr>
<th>% of Pb</th>
<th>( M_{\text{obtained}} ) [kN.m]</th>
<th>( M_{\text{MOHD}} ) [kN.m]</th>
<th>( \frac{M_{\text{p}<em>\text{i}}}{M</em>{\text{p}_{\text{in}}}} ) (%)</th>
<th>( \text{Pb}_{\text{MOHD}} ) (% of Pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>384.00</td>
<td>413.00</td>
<td>0.0%</td>
<td>0.00</td>
</tr>
<tr>
<td>0.20</td>
<td>358.45</td>
<td>367.36</td>
<td>6.7%</td>
<td>0.20</td>
</tr>
<tr>
<td>0.40</td>
<td>322.95</td>
<td>332.84</td>
<td>15.9%</td>
<td>0.40</td>
</tr>
<tr>
<td>0.60</td>
<td>291.74</td>
<td>296.49</td>
<td>24.0%</td>
<td>0.59</td>
</tr>
<tr>
<td>0.80</td>
<td>265.51</td>
<td>259.16</td>
<td>30.9%</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Figure 33 depicts the behavior of the maximum ultimate bending moment as the ratio of burst pressure increases. One can see that internal pressure significantly limits the strength of intact pipelines from more than 30% when it corresponds to 80% of the failure pressure. Moreover, results are similar to the ones obtained by Mohd et al., (2015).

**Figure 33. Ultimate bending moment for combined load simulations for different ratios of the original burst pressure applied to an intact pipeline. Line in orange represents the results of Mohd et al., (2015) at the same conditions**
The results of the numerical calculations for a 20%-corroded-pipe reveal the similar tendency (Table 22). The internal pressure plays an important role in the failure of both intact or corroded pipelines, although, the obtained degradation factor is slightly smaller. These results shown in Figure 34 allow to conclude that the ultimate bending moment decreases as the ratio of internal pressure increases. In addition, the results obtained seem to deviate from the ones of Mohd et al., (2015) as the level of corrosion of the pipeline increases.

Table 22. Ultimate bending moment obtained for combined load tests in the presence internal pressure and bending moment for a 20% corroded thickness

<table>
<thead>
<tr>
<th>% of Pb</th>
<th>M_ obtained [kN.m]</th>
<th>MOHD [kN.m]</th>
<th>( \frac{M_{pi}}{M_{p=0}} ) (%)</th>
<th>Pb_MOHD (% of Pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>380.00</td>
<td>405.00</td>
<td>0.0%</td>
<td>0.00</td>
</tr>
<tr>
<td>0.20</td>
<td>356.80</td>
<td>343.97</td>
<td>6.1%</td>
<td>0.19</td>
</tr>
<tr>
<td>0.40</td>
<td>324.46</td>
<td>310.31</td>
<td>14.6%</td>
<td>0.38</td>
</tr>
<tr>
<td>0.60</td>
<td>295.18</td>
<td>268.52</td>
<td>22.3%</td>
<td>0.57</td>
</tr>
<tr>
<td>0.80</td>
<td>270.51</td>
<td>225.38</td>
<td>28.8%</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Figure 34. Ultimate bending moment for combined load simulations for different ratios of the original burst pressure applied to a pipeline with 20% corroded thickness.

*Line in orange represents the results of Mohd et al., (2015) at the same conditions*
The results obtained for pipes with 40% corroded thickness shown in Table 23 confirm the tendency of reduced bending moment capacity with increasing internal pressure and a larger deviation from the results obtained by Mohd et al., (2015). One might argue that a possible reason for the obtained deviations is the modelling of the defect region, which seems to be more sensitive to simulations with internal pressure, as shown by Table 14, in which a deviation of 13.6% was obtained for 80% corroded thickness. However, further investigation must be carried out in order to fully understand the causes of such a deviation or the support of the results obtained in the present study.

**Table 23. Ultimate bending moment obtained for combined load tests in the presence internal pressure and bending moment for a 40% corroded thickness**

<table>
<thead>
<tr>
<th>% of Pb</th>
<th>M_obtained [kN.m]</th>
<th>MOHD [kN.m]</th>
<th>( \frac{M_{pl}}{M_{p=0}} ) (%)</th>
<th>Pb_MOHD (% of Pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>375.00</td>
<td>395.00</td>
<td>0.0%</td>
<td>0.00</td>
</tr>
<tr>
<td>0.20</td>
<td>356.91</td>
<td>333.22</td>
<td>4.8%</td>
<td>0.19</td>
</tr>
<tr>
<td>0.40</td>
<td>331.29</td>
<td>297.71</td>
<td>11.7%</td>
<td>0.38</td>
</tr>
<tr>
<td>0.60</td>
<td>308.12</td>
<td>246.88</td>
<td>17.8%</td>
<td>0.56</td>
</tr>
<tr>
<td>0.80</td>
<td>286.28</td>
<td>177.28</td>
<td>23.7%</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Figure 35. Ultimate bending moment for combined load simulations for different ratios of the original burst pressure applied to a pipeline with 40% corroded thickness.**

*Line in orange represents the results of Mohd et al., (2015) at the same conditions*
5. Conclusions and limitations of the study

The main conclusions of single load numerical calculations are that corrosion has a detrimental effect in the burst pressure and ultimate bending moment of pipelines. In this study the obtained burst pressure decreased from 39.4 MPa to 14.0 MPa, from the intact case to the 80% corroded-thickness scenario. However, the ultimate bending moment capacity decreased from 384 kN.m to 359 kN.m, comparing a non-corroded and a 80% corroded thickness pipe, which corresponds to only 6.5% capacity reduction. In addition, comparison of obtained results with main codes guidelines showed that they are conservative in the prediction of either burst pressure or ultimate bending moment.

A parametric study showed that increasing the length of the defect in 60% and decreasing it at the same amount only led to -1.0% and +0.8% less and more capacity, respectively. This means that the length of the defect plays a secondary role for this particular mode of failure. Moreover, variations in the width from -60% to +60% of the original dimension led to similar results of +2.0% and -2.9% in terms of capacity. Nonetheless, the simulations performed for single internal pressure loads indicate that the length of the defect is a more significant variable for burst pressure. The same range of variation led to an increase of 21.2% in capacity when the defect was -60% narrower and a decrease of 6.7% when the defect was +60% wider, respectively. Results for width variations with the same range indicated that such effects are similar to the ones observed for ultimate bending moment, with 3.9% and -2.0%, respectively greater and smaller burst pressure values. Altogether, the set of results reveals that corroded thickness is the most significant variable regarding the strength of corroded pipelines in terms of both burst pressure and ultimate bending moment, however, it is more significant for the former than for the latter.

As far as the combined load simulations are concerned, the presence of internal pressure has a detrimental effect on the bending moment capacity. This effect is more significant as the level of applied initial pressure increases. The results of the numerical calculations showed that such effect affects distinctly pipes with different levels of corrosion, being more intense when the corroded thickness is smaller.

The results obtained for the maximum ultimate bending capacity of the pipes with internal pressure are similar to ones of Mohd et al., (2015) for intact and slightly corroded pipes, but tend to deviate as the level of corrosion increases.

Modelling of the defect region seems to play an important role in the ultimate bending moment in the presence of internal pressure. Therefore, a thinner meshing could be used in order to better model the slope of the defect as well as symmetric boundary conditions, which would reduce the computational time required by the simulations. Although FE modelling is perceived as a valid tool for assessing strength of corroded pipelines, small and full scale tests in laboratory could be performed not only to verify differences between the results obtained and the ones reported in the literature but also to allow full control of all variables relevant to the problem.
The model developed does not cover other possible combinations of loads that might be significant to fully represent real problems. Therefore, other load combinations could be investigated in future studies. One of them is the fact that axial loads are not considered as well their effects in the presence of other loads such as bending moment and internal pressure. Furthermore, failures in pipelines due to external pressure as well the effects of internal corrosion were not addressed in this study and may be a major issue for specific industries such as offshore oil & gas industry, particular for production and transport pipelines in deep and ultra-deep scenarios.

Moreover, other failure modes classified as serviceability limit states such as global buckling and upheaval buckling are major concerns in real operational conditions and the model could be adapted to comprise these types of failure.
6. Suggestion for further studies

In the light of what has been said in the preview section, slight adaptations in the model would make possible to study the behavior of pipelines subjected to external pressure and axial forces. Future parametric studies could address their effects by single or combined load numerical calculations.
References


ASME B31G. (2009). By Authority Of, 552(c).


Kanninen, M. F., Broek, D., Marschall, C. W., Rybicki, E. F., Sampath, S. G., Simonen, F. A., &


Teixeira, A. P., Zayed, A., & Guedes Soares, C. (2010). Reliability of Pipelines with Non-uniform...


Annex

Partial safety factors DNV-Os-F101 (2013)

Table 24. Load effect factor combinations (DNV-Os-F101 2013)

<table>
<thead>
<tr>
<th>Limit State / Load combination</th>
<th>Load effect combination</th>
<th>Functional loads(^1)</th>
<th>Environmental load</th>
<th>Interference loads</th>
<th>Accidental loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULS</td>
<td>a System check(^2)</td>
<td>1.2</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b Local check</td>
<td>1.1</td>
<td>1.3</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>FLS</td>
<td>c</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>ALS</td>
<td>d</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

1) If the functional load effect reduces the combined load effects, \(\gamma\) shall be taken as 1/1.1.
2) This load effect factor combination shall only be checked when system effects are present, i.e. when the major part of the pipeline is exposed to the same functional load. This will typically only apply to pipeline installation.

Table 25. Condition load effect factors

<table>
<thead>
<tr>
<th>Condition</th>
<th>(\gamma_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipeline resting on uneven seabed</td>
<td>1.07</td>
</tr>
<tr>
<td>Reeling on and J-tube pull-in</td>
<td>0.82</td>
</tr>
<tr>
<td>System pressure test</td>
<td>0.93</td>
</tr>
<tr>
<td>Otherwise</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 26. Material resistance factor

<table>
<thead>
<tr>
<th>Limit state category(^1)</th>
<th>SLS/ULS/ALS</th>
<th>FLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_m)</td>
<td>1.15</td>
<td>1.00</td>
</tr>
</tbody>
</table>

1) The limit states (SLS, ULS, ALS and FLS) are defined in D.
Table 27. Safety class resistance factors

<table>
<thead>
<tr>
<th>Safety class</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure containment</td>
<td>1.046</td>
<td>1.138</td>
<td>1.308</td>
</tr>
<tr>
<td>Other</td>
<td>1.04</td>
<td>1.14</td>
<td>1.26</td>
</tr>
</tbody>
</table>

1) The number of significant digits is given in order to comply with the ISO usage factors.
2) Safety class low will be governed by the system pressure test which is required to be 3% above the incidental pressure. Hence, for operation in safety class low, the resistance factor will effectively be minimum 3% higher.
3) For system pressure test, \( \alpha_3 \) shall be equal to 1.00, which gives an allowable hoop stress of 96% of SMYS both for materials fulfilling supplementary requirement U and those not.
4) For parts of pipelines in location class 1, resistance safety class medium may be applied (1.138).

Detailed calculation of Burst Pressure

The containment pressure:

\[
p_b = \frac{2t}{D_o - t} \cdot f_{cb} \cdot \frac{2}{\sqrt{3}}
\]  

(40)

Where,

\[
f_{cb} = \text{Min} \left[ f_y, \frac{f_u}{1.15} \right]
\]  

(41)

- \( f_y \): yield strength to be used in design
- \( f_u \): tensile strength to be used in design

Where they are defined according to the following formulations.

\[
f_y = (\text{SMYS} - f_y, \text{temp}) \cdot a_u
\]  

(42)

\[
f_u = (\text{SMTS} - f_u, \text{temp}) \cdot a_u
\]  

(43)

- \( f_y, \text{temp} \): temperature derating factor of the yield strength
- \( f_u, \text{temp} \): temperature derating factor of the tensile strength

\( \text{SMYS} \): Specified Minimum Yield Stress

\( \text{SMTS} \): Specified Minimum Tensile Stress

Tensile strength and \( a_u \) is the material strength factor available in table 5-4. DNV-OS-F101 provide values for common materials in figure 2 of Section 5.

Equivalent Stress

If \( \frac{D_o}{t} > 20 \):

If \( \frac{D_o}{t} < 20 \):
\[ \sigma_e = \sqrt{\frac{1}{2} \left[ (\sigma_h - \sigma_l)^2 + (\sigma_l - \sigma_R)^2 + (\sigma_h - \sigma_R)^2 \right]} \] (44)

\( \sigma_e \): equivalent stress
\( \sigma_h \): hoop stress
\( \sigma_l \): longitudinal (axial) stress
\( \sigma_R \): is the radial stress

**Calculation of the local test pressure and local incident pressure**

The called local pressure \( p_{lt} \) is used as the local test pressure. It differs from the test pressure and the incident pressure, \( p_t \) and \( p_{li} \), respectively, because they take into account the difference of elevation at the local point analyzed, so:

\[
p_{lt} = p_t + \rho_t \cdot g \cdot (h_{ref} - h_l)
\] (45)

\[ p_{lt} \geq \alpha_{spt} \cdot p_{li} \] (46)

\( p_{lt} \): local test pressure
\( p_{li} \): local incident pressure
\( p_t \): test pressure
\( g \): acceleration of gravity
\( h_l \): local test height
\( h_{ref} \): reference height
\( \alpha_{spt} \): medium safety level
\( \rho_t \): density of the test fluid

Where \( h_{ref} \) and \( h_l \) are the reference and respective local height. According to table 5-9 from (DNV-OsF101, 2013), \( \alpha_{spt} \) is equal to 1.05 for a medium safety level.

\[
p_{li} = p_{inc} + \rho_{cont} \cdot g \cdot (h_{ref} - h_l)
\] (47)

\[ p_{inc} = p_d \cdot \gamma_{inc} \] (48)

\( p_d \): design pressure
\( \rho_{cont} \): density of the contained fluid
\( p_{inc} \): incidental pressure
Where $p_d$ is the design pressure. From table 3-1 of the same reference, $\gamma_{inc}$ is equal to 1.10 for normal pipe systems.

From Swamee, (1993) the density of the oil can be taken as 830 kg/m$^3$.

$$\rho_{cont} = 830 \text{ kg/m}^3$$

The hydrostatic (mill) pressure test $p_h$ shall be calculated according to the following formula:

$$p_h = \frac{2t_{MIN}}{D_o - t_{MIN}} \cdot \min[SMYS \cdot 0.96; SMTS \cdot 0.84]$$  \hspace{1cm} (49)

Where $t_{MIN}$ is the minimum thickness in the pipeline.

**ASME B31G equation for burst pressure**

In equation (9), factor 1.1 arises from the interpolation for the curves of Figure 36 and may not be related to a safety factor.

![Figure 36. P' in function of P which is equal to the maximum allowed operating pressure (MAOP)](image)

**P**: pressure which must not be exceeded

**Local Buckling criteria according DNV**

$Ssd$ and $Msd$ are the design effective axial stress and the design bending moment, respectively. The following factors can be calculated by the formulas from Bai et al., (2014).
\[ S_p = f_y (D_o - t) \cdot t \]  
(50)

\[ M_p = f_y (D_o - t)^2 \cdot t \]  
(51)

\[ \alpha_c = (1 - \beta') + \beta' \frac{f_u}{f_y} \]  
(52)

\[ \alpha_p = \begin{cases} 
1 - \beta', & \frac{p_i - p_e}{p_b} < \frac{2}{3} \\
1 - 3\beta' \left(1 - \frac{p_i - p_e}{p_b}\right), & \frac{p_i - p_e}{p_b} \geq \frac{2}{3}
\end{cases} \]  
(53)

\[ \beta' = \frac{60 - \frac{D_o}{t}}{90} \]  
(54)

\( M_p \): plastic bending moment capacity

\( S_p \): axial plastic stress

\( \sigma_u \): tensile strength to be used in design

\( \sigma_y \): yield strength to be used in design

\( t \): pipe thickness

\( D_o \): outside diameter

\( \beta' \): Factor used in combined loading criteria

\( \alpha_c \): flow stress parameter

\( p_b \): burst pressure

\( p_e \): external pressure

\( p_i \): internal pressure

Where \( \alpha_c \) is a flow stress parameter and \( \alpha_p \) accounts for effect of \( \frac{D_o}{t} \) ratio.