GARCH processes and the phenomenon of misleading and unambiguous signals

Ana Beatriz Torres de Sousa
Instituto Superior Técnico — Universidade de Lisboa

December, 2015

Abstract

In Finance it is quite usual to assume that a process behaves according to a previously specified target GARCH process. The impact of rumors or other events on this process can be frequently described by a change in the variance or an outlier responsible for a shift in the process mean, thus calling for the use of joint schemes for the process mean and variance, such as the ones proposed by Schipper (2001) and Schipper and Schmid (2001).

Since changes in the mean and in the variance require different actions from the traders/brokers, this paper provides an account on the probabilities of misleading and unambiguous signals (PMS and PUNS) of those joint schemes, thus adding insights on their out-of-control performance.

Programs for the R statistical software were written to produce all the results and to provide striking illustrations throughout this paper.

Keywords: financial time series; statistical process control; simultaneous EWMA schemes; misleading and unambiguous signals.

1 Monitoring structural deviations in Finance

Time series models were initially introduced for descriptive purposes like prediction and seasonal correction or dynamic control (Gouriéroux, 1997, p. 1). C. Gouriéroux aptly adds that in the 1970s, the research in time series models essentially focused on ARMA processes, which were easy to implement back then.

Financial time series is among the field of applications for which standard ARMA fit is poor (Gouriéroux, 1997, p. 1). The chief cause of the inadequacy of ARMA models is closely related to several features of economic and financial time series.

- Many of them contain a clear *trend* (Franses, 1998, p. 9; Enders, 2010, p. 121).
- When economic and financial time series are observed each month or quarter, such series often display a *seasonal* pattern (Franses, 1998, pp. 13–14).
- Some economic and financial time series seem to meander in the sense that they show no particular tendency to increase or decrease (Enders, 2010, pp. 123–124).
- Distorting and aberrant observations can occur and have a major influence on financial time series modeling and forecasting (Franses, 1998, p. 20); moreover, these observations tend to appear in clusters and in some cases they can be interpreted as resulting, for instance, from the impact of certain news on a stock market (Franses, 1998, p. 24). This phenomenon is called volatility clustering or, otherwise, *conditional heteroscedasticity* (Franses, 1998, p. 24).

Indeed, many economic and financial time series feature this outstanding form of nonlinear dynamics, the strong dependence of the instantaneous variability of the series on its own past (Gouriéroux, 1997, p. 1).

1.1 GARCH processes

Since ARMA time series models prove to be inadequate to describe economic and financial time series that tend not to operate under the assumption of constant conditional variance, Engle (1982) introduced the autoregressive conditionally heteroscedastic (ARCH) processes. Moreover, the ARCH(q) model relies on the conditional variance h_t written in terms of a moving average of q previous shocks $(Y_{t-1}^2, \ldots, Y_{t-q}^2)$, leaving the unconditional variance constant (Tsay, 2010, p. 119; Bollerslev, 1986).

The extension of the ARCH model tacked in this subsection bears, as Bollerslev (1986) pointed out, much resemblance to the extension of the standard AR process to the general ARMA process. It seems natural and of *practical interest* to consider that the conditional variance h_t depends not only on those MA terms, but also on AR terms, the lagged conditional variances $(h_{t-1}, \ldots, h_{t-p})$, this leads to the generalized autoregressive conditionally heteroscedastic (GARCH) processes introduced by Bollerslev (1986).

A stochastic process $\{Y_t : t \in \mathbb{Z}\}$ is said to be a GARCH(p, q) process if it satisfies the following set of equations:

$$Y_t = \varepsilon_t \, h_t^{1/2} \tag{1}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, Y_{t-i}^2 + \sum_{i=1}^{p} \beta_i \, h_{t-i}, \tag{2}$$

where: $\{\varepsilon_t : t \in \mathbb{Z}\} \sim \text{GWN}(0,1)$; ε_t is independent of h_t, for all t; $q \geq 0$; $\alpha_0 > 0$, $\alpha_i \geq 0$, for $i = 1, \ldots, q$; $p \geq 0$; $\beta_i \geq 0$, for $i = 1, \ldots, p$ (Bollerslev, 1986). Moreover, $Y_t \mid \psi_{t-1} \sim N(0, h_t)$, where ψ_{t-1} is the past information set available at time $t-1$.

Bollerslev (1986) also proved that the GARCH(p, q) is stationary iff $(0 \leq) \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i < 1$, and in this case: $\text{E}(Y_t) = 0$; $\text{Cov}(Y_t, Y_{t-n}) = 0$, for $n \in \mathbb{N}$; and

$$\text{V}(Y_t) = \frac{\alpha_0}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{i=1}^{p} \beta_i}. \tag{3}$$

The simplest but frequently useful GARCH process is the GARCH(1, 1) (Bollerslev, 1986).

1.2 Target and observed processes

In several domains, suchlike Finance, it is crucial to detect deviations from a target process as soon as possible after their occurrence.

Control charts have been thoroughly used to detect the presence of assignable causes responsible for changes in parameters such as the process mean or variance that may cause a deterioration in the quality (Morais, 2012, p. 69) of a product or service. Expectedly, such a graphical tool can also be used to detect relevant structural deviations in the observed process $\{X_t : t \in \mathbb{N}\}$ underlying an economic or financial time series, from a target process $\{Y_t : t \in \mathbb{N}\}$ with mean μ_0 and variance σ_0^2.

As aptly noted by Schipper and Schmid (2001), the impact of rumors or other events on the target (G)ARCH process can be frequently described by a change in the variance of the returns or an additive outlier responsible for a shift in the process mean. Therefore it is reasonable to relate the observed and target processes as follows:

$$X_t = \begin{cases} Y_t & \text{for } t < \tau \\ \mu_0 + \theta\,(Y_t - \mu_0) + \delta\,\sigma_0 & \text{for } t = \tau \\ \mu_0 + \theta\,(Y_t - \mu_0) & \text{for } t > \tau, \end{cases} \tag{4}$$

where τ ($\tau \in \mathbb{N}$), δ ($\delta \in \mathbb{R}$) and θ ($\theta \in \mathbb{R}^+$) denote the time epoch of the structural change, the relative size of the *short-lived shift* (Montgomery, 2009, p. 183) in location due to the additive outlier and the magnitude of the sustained shift in scale, respectively.

Since the mean and variance of X_t are given by

$$\begin{aligned} \mathrm{E}(X_t) &= \mu_0 + \delta\,\sigma_0 \times \mathbb{1}_{\{\tau\}}(t) && (5)\\ \mathrm{V}(X_t) &= [1-(1-\theta^2)\times \mathbb{1}_{\{k\in\mathbb{N}:k\geq\tau\}}(t)]\,\sigma_0^2 && (6) \end{aligned}$$

(Schipper and Schmid, 2001), one can write $\delta \equiv \delta(t) = \frac{\mathrm{E}(X_t)-\mu_0}{\sigma_0} \times \mathbb{1}_{\{\tau\}}(t)$ and $\theta \equiv \theta(t) = \frac{\sqrt{\mathrm{V}(X_t)}}{\sigma_0} \times \mathbb{1}_{\{k\in\mathbb{N}:k\geq\tau\}}(t) + \mathbb{1}_{\{k\in\mathbb{N}:k<\tau\}}(t)$. It is also important to note that the process $\{X_t : t \in \mathbb{N}\}$ is said to be out-of-control if $\delta \neq 0$ or $\theta \neq 1$, and deemed in-control otherwise.

As referred by Ramos (2013, p. 15), one of the standard assumptions while designing a control chart is that the underlying output is independent. However, this assumption is not fulfilled when the target process is generated according to a GARCH model and it has dramatic consequences in the ability of the chart to detect changes (Knoth *et al.*, 2009; Morais, 2012, p. 70), as illustrated in the next section.

2 On the impact of falsely assuming independence

For illustration purposes,

- one assumes that the target process is governed by a GARCH(1, 1) model, with zero mean and variance equal to $\sigma_0^2 = \alpha_0/(1-\alpha_1-\beta_1)$, and
- yet a Shewhart control chart for individual measurements (Montgomery, 2009, pp. 260–261) is used admitting that $Y_t \overset{iid}{\sim} \mathrm{N}(0, \sigma_0^2)$.

The control statistic of this chart is nothing but X_t. The observed value x_t is plotted against time t and compared with a pair of control limits; a point lying outside the control limits indicates the potential presence assignable causes (Ramos, 2013, p. 3), thus, a signal should be triggered.

Unsurprisingly, the performance of this Shewhart control chart for individual measurements (or any other chart) is commonly assessed by making use of the run length (RL), which is the number of samples collected before a signal is triggered by the chart.

The control limits are usually set in such way that the in-control average run length (ARL) attains some desired (and reasonably large) value, say ARL^*. Since one assumed that $Y_t \overset{iid}{\sim} N(0, \sigma_0^2)$, the lower and upper control limits of the Shewhart chart for individual measurements are

$$\begin{aligned} LCL_S &= -\gamma_S \times \sigma_0 && (7)\\ UCL_S &= +\gamma_S \times \sigma_0, && (8) \end{aligned}$$

where the critical value γ_S is given by $\gamma_S = \Phi^{-1}[1-1/(2ARL^*)]$.

Furthermore, one assumes from now on that $\tau = 1$, thus the observed process is given by

$$X_t = \begin{cases} \theta \times Y_t + \delta \times \sigma_0 & \text{for } t = 1 \\ \theta \times Y_t & \text{for } t = 2, 3, \ldots \end{cases} \quad (9)$$

In the next proposition one can find not only the probabilities of a signal being triggered and the probability function, p.f. (resp. survival function, s.f.), of the RL of this Shewhart chart if the target process is indeed i.i.d, but also the associated ARL, standard deviation (SDRL), coefficient of variation (CVRL) and median (MdRL).

Proposition 2.1. — **Probabilities of triggering a signal and RL related performance measures when $Y_t \overset{iid}{\sim} N(0, \sigma_0^2)$**

Let $p \equiv p(\delta, \theta)$ (resp. $p' \equiv p'(\theta)$) be the probability that the first (resp. t^{th}, $t \in \mathbb{N}\backslash\{1\}$) observation is responsible for a signal, while using the Shewhart control chart for individual measurements with control limits (7) and (8) and $\tau = 1$. Then

$$p = 1 - \left[\Phi\left(\frac{\gamma_S - \delta}{\theta}\right) - \Phi\left(\frac{-\gamma_S - \delta}{\theta}\right)\right] \tag{10}$$

$$p' = 2\left[1 - \Phi\left(\frac{\gamma_S}{\theta}\right)\right]. \tag{11}$$

The p.f. and s.f. of the RL are given by

$$P(RL = t) = \begin{cases} p & \text{for } t = 1 \\ (1-p)(1-p')^{t-2}p' & \text{for } t \in \mathbb{N}\backslash\{1\} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

$$P(RL > t) = \begin{cases} 1 & \text{for } t < 1 \\ (1-p)(1-p')^{\lfloor t-1 \rfloor} & \text{for } t \geq 1. \end{cases} \tag{13}$$

The associated ARL, SDRL, CVRL and MdRL are equal to

$$\text{ARL}_{iid} = \frac{p' + 1 - p}{p'} \tag{14}$$

$$\text{SDRL}_{iid} = \sqrt{\frac{1-p^2}{(p')^2} - \frac{1-p}{p'}} \tag{15}$$

$$\text{CVRL}_{iid} = \frac{\sqrt{(1-p)(1+p-p')}}{p'+1-p} \tag{16}$$

$$\text{MdRL}_{iid} = \max\left\{\left\lceil\frac{\ln(1-p') - \ln(1-p) + \ln(0.5)}{\ln(1-p')}\right\rceil, 1\right\}. \tag{17}$$

•

Deriving an analogue of the previous proposition is rather difficult when the target process is governed by a GARCH model. Therefore one has to rely on Monte Carlo simulations to estimate ARL, SDRL, CVRL and MdRL and, thus, assess the performance of the Shewhart chart for individual measurements under the false assumption of an i.i.d. $\text{N}(0, \sigma_0^2)$ target process, when it is in fact a GARCH process with zero mean and variance equal to σ_0^2.

One assumed that $\{Y_t : t \in \mathbb{N}\} \sim \text{GARCH}(1,1)$ with: $\alpha_0 = 1.0$; $\alpha_1 = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $\beta_1 = 0.95 - \alpha_1$. For instance, one analyzes the scenario where there is a short-lived shift in the process mean accompanied by a sustained shift in the process standard deviation with magnitude $(\delta, \theta) = (0.5, 3)$ and one:

- simulated a time series $\{y_{i,1}, \ldots, y_{i,N}\}$ for each i ($i = 1, \ldots, rep$), drawn from the target process, where the number of replications is equal to $rep = 10^5$ and the number of observations simulated is equal to $N = 1000$;
- obtained the associated values of the control statistics $\{x_{i,1}, \ldots, x_{i,N}\}$ for each i ($i = 1, \ldots, rep$), compared them with the control limits defined in (7) and (8) and counted the number of observations (i.e., the RL) until a signal was triggered by the chart for each i and estimated the ARL, SDRL, CVRL and MdRL.[1]

[1]For estimation purposes, one dismissed the realizations of the observed process for which no observation was beyond the control limits.

One considered Shewhart charts for individual measurements calibrated in such a way that the in-control ARL (ARL^*) is equal to 60 and roughly reflects three months at the stock exchange (Schipper and Schmid, 2001).

Now, one can confront the ARL, SDRL, CVRL and MdRL estimates with the corresponding values obtained using Proposition 2.1. The plots of the estimated ARL, SDRL, CVRL and MdRL can be found in Figure 1, along with horizontal lines corresponding to $\text{ARL}_{iid}(0.5, 3)$, $\text{SDRL}_{iid}(0.5, 3)$, $\text{CVRL}_{iid}(0.5, 3)$ and $\text{MdRL}_{iid}(0.5, 3)$.

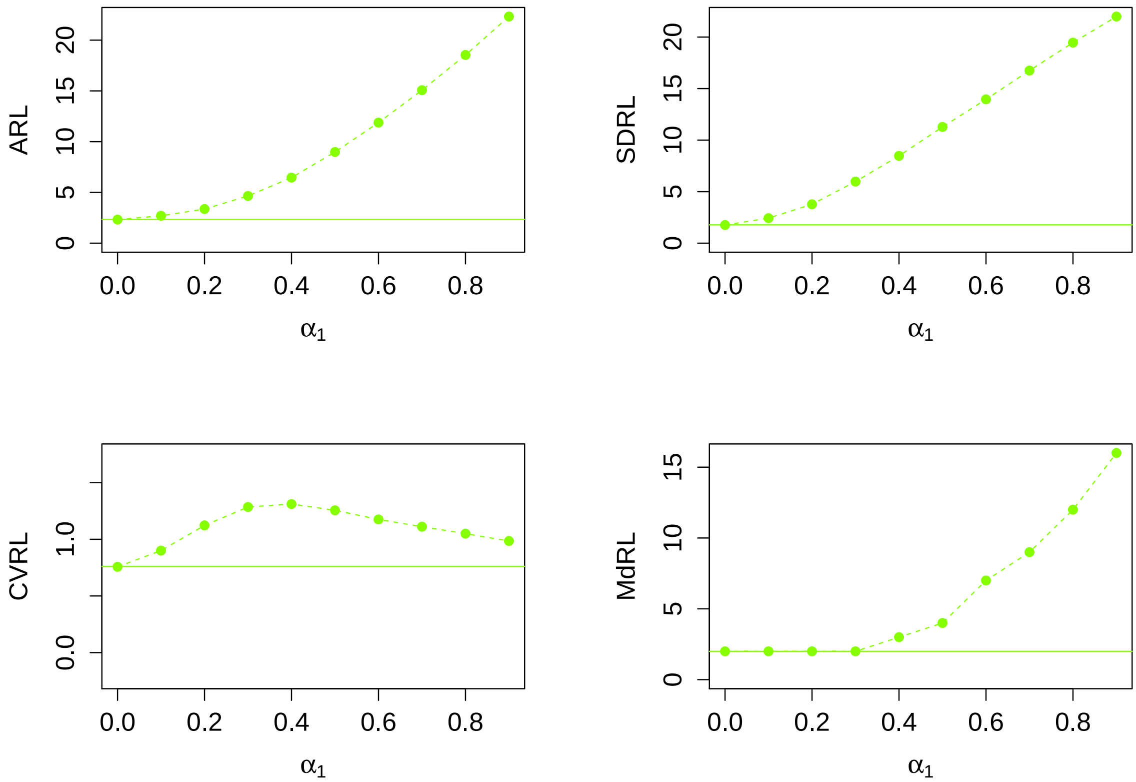

Figure 1: Estimates of ARL, SDRL, CVRL and MdRL, in the presence a short-lived shift in the process mean accompanied by a sustained shift in the process standard deviation with magnitude $(\delta, \theta) = (0.5, 3)$ — GARCH(1,1) model.

The true ARL, SDRL, CVRL and MdRL are usually underestimated by $\text{ARL}_{iid}(0.5, 3)$, SDRL_{iid} $(0.5, 3)$, $\text{CVRL}_{iid}(0.5, 3)$ and $\text{MdRL}_{iid}(0.5, 3)$.

Moreover, the estimates of ARL, SDRL and MdRL seem to increase with α_1, suggesting that the larger the ARCH parameter the more one underestimates the true ARL, SDRL and MdRL when one falsely assumes independent output and adopt a chart accordingly. For instance, $\text{ARL}_{iid}(0.5, 3) = 2.3386$, $\text{SDRL}_{iid}(0.5, 3) = 1.7806$ and $\text{MdRL}_{iid}(0.5, 3) = 2$, whereas the corresponding estimates for $\alpha_1 = 0.8$ are equal to 18.5388, 19.4648 and 12, respectively.

In light of the previous results, one concludes that ignoring the GARCH character of the target process while designing a chart can have dramatic consequences in its performance. This calls for alternative charts to control the process mean and variance, suchlike the simultaneous modified EWMA schemes introduced by Schipper (2001) and described in the next section.

3 Simultaneous modified EWMA schemes

The first control charts with the purpose of detecting changes in the mean (resp. variance) of a GARCH process can be traced back to 1999 (resp. 2001). However, in order to efficiently control a process, one needs to jointly monitor its location and scale and to do that joint (simultaneous or combined) control schemes are used (Ramos, 2013, p. 5). The standard practise is to use simultaneously two charts, one for the mean value (μ) and other for the variance (σ^2). That is the case of the simultaneous modified EWMA schemes proposed by Schipper (2001).

In Table 1, one finds the statistics and the control limits of one individual modified EWMA chart for μ and of four individual modified EWMA charts for σ based on the squared observations (I), the conditional variance (II), the exponentially weighted variance (III) and the logarithm of the squared observations (IV) (Schipper, 2001, pp. 34, 35), with smoothing parameters $\lambda_1, \lambda_2 \in (0, 1]$.

Table 1: Statistics and control limits — individual modified EWMA charts for μ and for σ.

	EWMA control chart for $\boldsymbol{\mu}$
Statistic	$Z_{1,t}^{(k)} = \begin{cases} \mu_0 & \text{for } t = 0 \\ (1-\lambda_1)Z_{1,t-1}^{(k)} + \lambda_1 X_t & \text{for } t \in \mathbb{N} \end{cases}$
LCL	$LCL_{E-\mu}^{(k)} = \mu_0 - c_1^{(k)}$
UCL	$UCL_{E-\mu}^{(k)} = \mu_0 + c_1^{(k)}$
	EWMA control chart for $\boldsymbol{\sigma}$ based on $\boldsymbol{X_t^2}$
Statistic	$Z_{2,t}^{(\text{I})} = \begin{cases} \sigma_0^2 & \text{for } t = 0 \\ (1-\lambda_2)Z_{2,t-1}^{(\text{I})} + \lambda_2 (X_t - \mu_0)^2 & \text{for } t \in \mathbb{N} \end{cases}$
LCL	$LCL_{E-\sigma}^{(\text{I})} = c_2^{(\text{I})}\alpha_0$
UCL	$UCL_{E-\sigma}^{(\text{I})} = c_3^{(\text{I})}\alpha_0$
	EWMA control chart for $\boldsymbol{\sigma}$ based on $\boldsymbol{\widehat{\sigma}_t^2}$
Statistic	$Z_{2,t}^{(\text{II})} = \begin{cases} \sigma_0^2 & \text{for } t = 0 \\ (1-\lambda_2)Z_{2,t-1}^{(\text{II})} + \lambda_2 \widehat{\sigma}_t^2 & \text{for } t \in \mathbb{N} \end{cases}$
LCL	$LCL_{E-\sigma}^{(\text{II})} = c_2^{(\text{II})}\alpha_0$
UCL	$UCL_{E-\sigma}^{(\text{II})} = c_3^{(\text{II})}\alpha_0$
	EWMA control chart for $\boldsymbol{\sigma}$ based on $\boldsymbol{(\sigma_t^*)^2}$
Statistic	$Z_{2,t}^{(\text{III})} = \begin{cases} \sigma_0^2 & \text{for } t = 0 \\ (1-\lambda_2)Z_{2,t-1}^{(\text{III})} + \lambda_2 (\sigma_t^*)^2 & \text{for } t \in \mathbb{N} \end{cases}$
LCL	$LCL_{E-\sigma}^{(\text{III})} = c_2^{(\text{III})}\alpha_0$
UCL	$UCL_{E-\sigma}^{(\text{III})} = c_3^{(\text{III})}\alpha_0$
	EWMA control chart for $\boldsymbol{\sigma}$ based on ln($\boldsymbol{X_t^2}$)
Statistic	$Z_{2,t}^{(\text{IV})} = \begin{cases} \text{E}(\ln[(Y_t - \mu_0)^2]) & \text{for } t = 0 \\ (1-\lambda_2)Z_{2,t-1}^{(\text{IV})} + \lambda_2 (\ln[(X_t - \mu_0)^2]) & \text{for } t \in \mathbb{N} \end{cases}$
LCL	$LCL_{E-\sigma}^{(\text{IV})} = c_2^{(\text{IV})}$
UCL	$UCL_{E-\sigma}^{(\text{IV})} = c_3^{(\text{IV})}$

Please note that according to Schipper (2001, pp. 12–14, 35):

$$\widehat{\sigma}_t^2 = \begin{cases} \sigma_0^2 & \text{for } t = 1 \\ \sigma_0^2 + (\alpha_1 + \beta_1)[(X_{t-1} - \mu_0)^2 - \sigma_0^2] + \dfrac{\beta_1}{r_{t-1}}[(X_{t-1} - \mu_0)^2 - \widehat{\sigma}_{t-1}^2] & \text{for } t \in \mathbb{N}\backslash\{1\}, \end{cases} \quad (18)$$

where $r_t = \frac{1-2\beta_1\alpha_1-\beta_1^2}{1-(\alpha_1+\beta_1)^2}$, for $t = 1$, and $r_t = 1 + \beta_1^2 - \frac{\beta_1^2}{r_{t-1}}$, for $t \geq 2$; and

$$(\sigma_t^*)^2 = \begin{cases} \sigma_0^2 & \text{for } t = 0 \\ 0.94\,(\sigma_{t-1}^*)^2 + 0.06\,(X_t - \mu_0)^2 & \text{for } t \in \mathbb{N}. \end{cases} \tag{19}$$

The simultaneous scheme gives a signal when at least one of its individual charts does, that is, its run length is defined as

$$RL = \inf\{t \in \mathbb{N} : |Z_{1,t}^{(k)} - \mu_0| > UCL_{E-\mu}^{(k)} \vee Z_{2,t}^{(k)} < LCL_{E-\sigma}^{(k)} \vee Z_{2,t}^{(k)} > UCL_{E-\sigma}^{(k)}\}, \tag{20}$$

where $k \in \{I, II, III, IV\}$.

In order to compute $c_1^{(k)}$, $c_2^{(k)}$ and $c_3^{(k)}$ for $k \in \{I, II, III, IV\}$, Schipper (2001, p. 34) defined a system of 3 nonlinear equations to obtain an unique solution solution satisfying the following conditions: the ARL^* is equal to a pre-specified value and $E(RL_1^{upper}) = E(RL_1^{lower}) = E(RL_2^{upper}) = E(RL_2^{lower})$, where $E(RL_1^{upper})$ (resp. $E(RL_1^{lower})$) represents the in-control ARL of an individual upper (resp. lower) one-sided chart for μ, and $E(RL_2^{upper})$ (resp. $E(RL_2^{lower})$) denotes the in-control ARL of an individual upper (resp. lower) one-sided chart for σ. This system can be solved using a 3-dimensional secant rule, having as starting values for the iteration the critical values of the individual one-sided charts (Schipper and Schmid, 2001).

4 Estimating PMS and PUNS via Monte Carlo simulation

The concept of misleading signal (MS) was firstly introduced by St. John and Bragg (1991) that identified the three following types of MS:

I. the process mean increases but the signal is triggered by the chart for the variance or on the negative side of the chart for the mean;

II. the process mean decreases but the signal is triggered by the chart for the variance or on the positive side of the chart for the mean;

III. the process variance has shift up but the signal is triggered by the chart for the mean.[2]

Feeling that what is more relevant is the misidentification of the parameter that has changed, Morais and Pacheco (2000) only studied the MS of Type III and defined a fourth type of MS that is related to MS of types I and II (Ramos, 2013, p. 7):

IV. the process mean shifts but the signal is triggered by the chart for the variance.

Additionally, Ramos (2013, p. 112) identified another class of valid signals[3] — the unambiguous signals (UNS) that were divided into two types by Ralha (2014, p. 25):

III. the process variance is out-of-control and the first chart that triggers a signal is the chart for variance;

IV. the process mean is out-of-control and the corresponding chart is the first to signal.

In this paper, the attention is restricted to the misleading and unambiguous signals of types III and IV.

Since the assignable causes on charts for μ can be different of those on charts for σ, the appropriate trading strategy that follows a signal can differ depending on whether the signal is triggered by the chart for μ or by the one for σ. Therefore a MS can lead to a reduction in the profit made by the traders/brokers

[2]Note that in this paper one uses an individual chart to detect upward and downward shifts in the process variance.

[3]An alarm is valid if the signal is triggered in the presence of assignable causes, i.e., when the process is indeed out-of-control (Ramos, 2013, p. 3).

if it led them to use an inappropriate trading strategy. Consequently, PMS and the PUNS should be used to assess the performance of simultaneous schemes that are used to monitor changes in the mean and variance of financial time series, such as the four simultaneous modified EWMA schemes introduced by Schipper (2001).

In this section, one uses the PMS and PUNS of types III and IV to compare the performance of the four EWMA simultaneous control schemes described in Section 3.

The PMS and PUNS of types III and IV are defined as follows:

$$\text{PMS}_{III}(\theta) = P[RL_\sigma(0,\theta) > RL_\mu(0,\theta)],\ \theta \neq 1 \tag{21}$$

$$\text{PMS}_{IV}(\delta) = P[RL_\mu(\delta,1) > RL_\sigma(\delta,1)],\ \delta \neq 0 \tag{22}$$

$$\text{PUNS}_{III}(\theta) = P[RL_\mu(0,\theta) > RL_\sigma(0,\theta)],\ \theta \neq 1 \tag{23}$$

$$\text{PUNS}_{IV}(\delta) = P[RL_\sigma(\delta,1) > RL_\mu(\delta,1)],\ \delta \neq 0, \tag{24}$$

where RL_μ and RL_σ are the RL of the individual charts for μ and for σ, respectively (Ralha *et al.*, 2015). When one assumes a GARCH target process, the expressions of these probabilities can not be simplified and one has to rely on Monte Carlo simulations to estimate them.

One assumed that $\{Y_t : t \in \mathbb{N}\} \sim \text{GARCH}(1,1)$ with $\alpha_0 = 0.1$, $\alpha_1 = 0.05$ and $\beta_1 = 0.9$, as in Schipper and Schmid (2001). The four simultaneous schemes were calibrated in such a way that ARL^* is equal to 60. The sets of values of smoothing parameters λ_1 for the chart for μ and λ_2 for the chart for σ considered are in Table 2 and coincide with the ones considered by Schipper (2001, p. 48) and Schipper and Schmid (2001). Moreover, the three critical values for each simultaneous scheme and for (λ_1, λ_2) can be found in Schipper (2001, tables A.3 to A.6, pp. 109–110).

Table 2: Pairs of smoothing parameters (λ_1,λ_2) — simultaneous modified EWMA schemes.

λ values	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
λ_1	0.10	0.25	0.50	0.75	1.00	0.10	1.00	1.00	1.00
λ_2	0.10	0.25	0.50	0.75	1.00	1.00	0.10	0.25	0.50

As far as the shifts in the parameters are concerned, two different out-of-control scenarios have been considered in order to obtain the estimates of the PMS and PUNS of types III and IV:

- in the first case, the process standard deviation suffers a sustained shift of magnitude $\theta = 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.5, 2.0, 3.0$ and the process mean stays on-target ($\delta = 0$);
- in the second situation, the process mean suffers a short-lived shift of magnitude $\delta = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$ and the process variance stays on-target ($\theta = 1$).

For each constellation of parameter values and for all the pairs of smoothing parameters (λ_1, λ_2) in Table 2, one proceeded with the simulations as before. However, in this case one:

- calculated for each i ($i = 1, \ldots, rep$) the associated values of the control statistics and compared them with the corresponding control limits;
- counted the number of misleading signals, the number of unambiguous signals and the number of signals triggered by the simultaneous schemes, and estimated the corresponding PMS and PUNS.

One desires that a control scheme triggers MS sporadically and triggers UNS as often as possible, therefore when several simultaneous control schemes are compared, one ought to choose the one that is associated with the minimum PMS or the maximum PUNS.

The minimum PMS_{III} (resp. PMS_{IV}) and the maximum $PUNS_{III}$ (resp. $PUNS_{IV}$) estimated in the presence of several sustained shifts in scale (resp. short-lived shifts in location) can be found in Table 3 (resp. 4). Moreover the estimates PMS (resp. PUNS) are followed by the label of the pair (λ_1, λ_2) at which the minimum (resp. maximum) is attained. The values corresponding to the minimum PMS_{III} (resp. PMS_{IV}) and the maximum $PUNS_{III}$ (resp. $PUNS_{IV}$), for a given θ (resp. δ) when the values obtained for the four charts are compared, are in **bold**.

Tables 3 to 4 surely deserve some comments.

When there is a sustained shift in the process variance, the minimum PMS_{III} and the maximum $PUNS_{III}$ were attained, in most cases, when one uses the simultaneous EWMA control scheme with a chart for σ based on the conditional variance. It is worth mentioning that, for some small shifts, the maximum values of $PUNS_{III}$ in **bold** are regretfully smaller that 0.5.

Table 3: Minimum $PMS_{III}(\theta)$ and maximum $PUNS_{III}(\theta)$ — simultaneous modified EWMA scheme with chart for σ listed in order corresponding to: squared observations (I), conditional variance (II), exponentially weighted variance (III) and logarithm of squared observations (IV).

Performance measures	θ								
	0.7	**0.8**	**0.9**	**1.1**	**1.2**	**1.3**	**1.5**	**2.0**	**3.0**
min(PMS_{III})	0.0163 (7)	0.0754 (7)	**0.2032** (9)	0.3778 (9)	0.3802 (9)	0.3600 (9)	0.2692 (1)	0.1175 (1)	0.0338 (1)
	0.0140 (5)	**0.0701** (5)	0.2078 (5)	0.4665 (5)	0.4345 (6)	0.3559 (6)	**0.2243** (6)	**0.0754** (6)	**0.0146** (6)
	0.0163 (5)	0.0744 (5)	0.2206 (5)	0.5145 (6)	0.4729 (6)	0.4002 (6)	0.2726 (6)	0.1285 (6)	0.0416 (6)
	0.0229 (7)	0.1065 (7)	0.2396 (5)	**0.3434** (5)	**0.3493** (5)	**0.3355** (5)	0.2811 (6)	0.1339 (6)	0.0409 (6)
max($PUNS_{III}$)	0.9810 (7)	0.9092 (7)	0.7170 (7)	0.4738 (6)	**0.4912** (6)	0.5219 (6)	0.5818 (6)	0.6696 (6)	0.6317 (6)
	0.9840 (8)	**0.9177** (7)	**0.7365** (7)	0.4465 (6)	0.4724 (6)	**0.5278** (6)	**0.6322** (6)	**0.7303** (6)	**0.6812** (6)
	0.9827 (8)	0.9150 (8)	0.7357 (7)	0.4190 (6)	0.4327 (6)	0.4856 (6)	0.5706 (6)	0.6348 (6)	0.5783 (6)
	0.9764 (7)	0.8889 (7)	0.7089 (7)	**0.4748** (6)	0.4899 (6)	0.5209 (6)	0.5854 (6)	0.6706 (6)	0.6323 (6)

In the presence of a short-lived shift in the process mean, the smallest PMS_{IV} (resp. largest $PUNS_{IV}$) were usually obtained when one uses a simultaneous Shewhart control scheme with a chart for σ based on the squared observations (resp. EWMA scheme with a chart for σ based on the exponentially weighted variance with $(\lambda_1, \lambda_2) = (1, 0.1)$ (pair (7))).

Table 4: Minimum $PMS_{IV}(\delta)$ and maximum $PUNS_{IV}(\delta)$ — simultaneous modified EWMA scheme with chart for σ listed in order corresponding to: squared observations (I), conditional variance (II), exponentially weighted variance (III) and logarithm of squared observations (IV).

Performance measures	δ					
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**
min(PMS_{IV})	**0.1554** (5)	**0.1475** (5)	**0.1320** (5)	**0.1097** (5)	**0.0786** (5)	**0.0448** (5)
	0.4598 (5)	0.4398 (5)	0.4009 (5)	0.3314 (5)	0.2430 (5)	0.1511 (5)
	0.4640 (5)	0.4421 (5)	0.4059 (5)	0.3327 (5)	0.2455 (5)	0.1500 (5)
	0.3663 (5)	0.3484 (5)	0.3114 (5)	0.2553 (5)	0.1840 (5)	0.1087 (5)
max($PUNS_{IV}$)	0.4719 (6)	0.4624 (1)	0.4543 (1)	0.4645 (7)	0.4621 (7)	0.4047 (7)
	0.5161 (7)	0.5355 (7)	0.5736 (7)	0.6440 (7)	0.7408 (7)	0.8345 (7)
	0.5159 (7)	**0.5375** (7)	**0.5765** (7)	**0.6488** (7)	**0.7416** (7)	**0.8407** (7)
	0.4972 (1)	0.5007 (1)	0.5303 (7)	0.6071 (7)	0.7112 (7)	0.8208 (8)

The main purpose of this paper was to assess the phenomenon of misleading and unambiguous signals (MS and UNS), in simultaneous schemes for the process mean and variance of GARCH processes. The reader should be reminded that these types of valid signals can result in money and time spent in attempting to identify inexistent causes of unnatural variation or misapplying a trading strategy.

References

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* **31**, 307–327.

Enders, W. (2010). *Applied Econometric Time Series.* 3rd edition. New York: John Wiley & Sons, Inc.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of variance of United Kingdom inflation. *Econometrica* **50**, 987–1007.

Franses, P.H. (1998). *Times Series Models for Business and Economic Forecasting.* Cambridge: Cambridge University Press.

Gouriéroux, C. (1997). *ARCH Models and Financial Applications.* New York: Springer.

Knoth, S., Morais, M.C., Pacheco, A., and Schmid, W. (2009). Misleading signals in simultaneous residual schemes for the mean and variance of a stationary process. *Communications in Statistics – Theory & Methods* **38**, 2923–2943.

Montgomery, D.C. (2009). *Introduction to Statistical Process Control.* 9th edition. New York: John Wiley & Sons.

Morais, M.C. (2012). Real- and integer-valued time series and quality control charts. Unpublished report.

Morais, M.C. and Pacheco, A. (2000). On the performance of combined EWMA schemes for μ and σ: a Markovian approach. *Communications in Statistics – Simulation and Computation* **29**, 153–174.

Ralha, T., Morais, M.C., and Oliveira, M.R (2015). On valid signals in joint schemes for the process mean and variance. *Economic Quality Control.* Accessed from `http://www.degruyter.com/view/j/eqc.ahead-of-print/eqc-2015-0001/eqc-2015-0001.xml` on 2015-11-11.

Ralha, T.I.C. (2014). *Sinais válidos em esquemas conjuntos para a localização e a dispersão com exemplos em R (Valid signals in joints schemes for location and spread with examples in R).* MSc thesis. Instituto Superior Técnico, Universidade Técnica de Lisboa (in Portuguese).

Ramos, P.A.A.C.F.P. (2013). *Performance Analysis of Simultaneous Control Schemes for the Process Mean (Vector) and (Co)Variance (Matrix).* PhD thesis. Instituto Superior Técnico, Universidade Técnica de Lisboa.

Schipper, S. (2001). *Sequential Methods for Detecting Changes in the Volatility of Economic Time Series.* PhD thesis. European University, Department of Statistics, Frankfurt (Oder), Germany.

Schipper, S. and Schmid, W. (2001). Control charts for GARCH processes. *Nonlinear Analysis: Theory, Methods & Applications* **47**, 2049–2060.

St. John, R. and Bragg, D. (1991). "Joint X-bar and R charts under shift in mu or sigma". *ASQC Quality Congress Transactions, American Society for Quality Control, Milwakee*, 547–550.

Tsay, R.S. (2010). *Analysis of Financial Time Series.* 3rd edition. Hoboken, New Jersey: John Wiley & Sons, Inc.