GEOMETRICAL NON-LINEAR EFECTS IN STEEL FRAME STRUCTURES

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ABSTRACT

This dissertation aims to study the influence of geometrical non-linear effects on steel structures behaviour through a finite element model specifically developed and implemented by the author, performing comparative analysis with regulatory procedures of Eurocode 3.

A brief theoretical formulation of the numerical model is given presenting the background concepts and theories. The stiffness matrices were derived through the energetic method of Rayleigh-Ritz. To validate the numerical model a set of simple examples consisting of columns and frames with known analytical solutions were analysed. The model was then applied to the analysis of both beam-columns and frames.

Two beam-columns are analysed: a simple supported and a continuous beam with equal spans. A set of analysis are performed, allowing to observe how the structural behaviour is influenced by geometrical imperfections and applied loadings, comparing the results with the interaction formulas provided by Eurocode 3.

The influence of local and global imperfections on the global structural behaviour of a frame is analysed comparing the results with the Amplification of the Sway Effects Method. A bracing system of frames is also studied, being the influence of the local imperfections and the stiffness of the bracing's element on the global frame behaviour analysed.

Keywords: *Geometrical non-linear effects in beam-columns; Stability analysis of beam-columns; Safety verification of frames and beam-columns; Influence of geometrical imperfections*.

1. Introduction

Structures have the main function of transmitting to the foundations the effects of applied forces, simultaneously ensuring safety, economics and aesthetics conditions.

The linear-elastic analysis is one of the first options adopted for analysis of structures; however in some cases can lead to errors that are not negligible due to geometrical and physical non-linear effects.

The geometrical non-linear effects, particularly relevant in steel structures due to its slenderness, are related to deformations that are non-proportional to the applied forces, which can cause instability phenomena.

In order to consider the geometrical non-linear effects in the numerical modelling of structures it is necessary to use a stiffness matrix that incorporates these effects on its formulation; an alternative and a classic approach is to adopt semi-analytical methods based on a Fourier series, amplifying the displacement in order to consider second order effects.

The present dissertation aims to study, adopting a numerical model developed and implemented by the author in *Matlab*, the effects of the imperfections and loadings on the behaviour of steel structures, comparing the numerical results with the procedures of EN19931-1 (EC3) for verification of beam-columns and frames.

The study is focused only on the elastic behaviour of structures and the geometrical imperfections, being considered the effects of residual stresses in the form of equivalent geometrical imperfections.

2. Structural analysis and numerical formulation

A finite element model is derived and implemented in order to perform buckling analysis and geometrical non-linear analysis of columns and beam-columns.

2.1 Linear elastic analysis

The linear elastic analysis model is derived by considering principles of physical and geometrical linearity. The stiffness matrix for the linear elastic column and for the linear elastic beam is derived considering the Rayleigh-Ritz method.

The equilibrium equation, compatibility condition and constitutive relation are obtained for the element represented in Figure 2.1 as a function of the total potential energy (V), axial strain (ε), axial displacement (u), axial force (N), cross sectional area (A) and Young's modulus (E), being written as follow (Fish & Belytschko, 2007):

$$\text{Equilibrium equation: } \delta V = \delta U - \delta W = 0 \qquad (2.1)$$

$$\text{Kinematic condition: } \varepsilon(x) = \frac{du}{dx} \qquad (2.2)$$

$$\text{Constitutive relation: } N(x) = \varepsilon(x)\, EA \qquad (2.3)$$

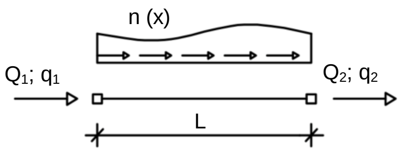


Figure 2.1 – Column element.

The corresponding energy of deformation (U) and the external work associated with loadings (W) are given by:

$$U = \frac{1}{2}\int_0^L EA\left(\frac{du}{dx}\right)^2 dx \qquad (2.4)$$

$$W = \int_0^L u(x).n(x)\,dx + N_1 u(0) + N_2 u(L) \qquad (2.5)$$

Considering an infinitesimal variation of the field displacements defined by the addition of an arbitrary infinitesimal function (ς(x)) to this field, it is obtained the following expressions for the portions of the variation of the potential energy:

$$\delta U = \int_0^L E(x)\,A(x)\,\frac{d\varsigma(x)}{dx}\frac{du(x)}{dx}dx \qquad (2.6)$$

$$\delta W = \int_0^L \varsigma(x).n(x)\,dx - N_1\,\varsigma(0) - N_2\,\varsigma(L) \qquad (2.7)$$

Defining the following linear polynomials as approximate functions of the field displacement:

$$\psi_1(x) = 1 - x/L \qquad (2.8)$$

$$\psi_2(x) = x/L \qquad (2.9)$$

it is possible to develop an approximate solution defined by the linear combination of the independent displacements and the approximate functions:

$$u(x) = q_1\psi_1(x) + q_2\psi_2(x) = \{\psi\}\{q\} \qquad (2.10)$$

Defining the infinitesimal function ($\varsigma(x)$) as a function of the partial derivatives of the displacement field in order to the independent displacements, it is observed that these are also given by a linear combination of the approximate functions:

$$\varsigma(x) = \frac{\partial u(x)}{\partial q_i} dq_i = \{\psi\}\{dq\} = \{\psi\}\{\varsigma\} \qquad (2.11)$$

Inserting the previous definitions of the displacement field and the infinitesimal function in equations (2.6) and (2.7) it is obtained the following expressions for the variation of the portions of the potential energy.

$$dU = \left[\int_0^L E(x)\, A(x)\, \{\psi'\}^T\{\psi'\}\, dx\right]\{q\} \qquad (2.12)$$

$$dW_n = \int_0^L \{\psi\}\, n(x)\, dx \qquad (2.13)$$

$$dW_N = \sum_i Q_i\{\psi(0)\} + \sum_j Q_j\{\psi(L)\} \qquad (2.14)$$

Considering the approximate functions defined in equations (2.8) e (2.9) the equilibrium equation can be written as follows:

$$[K]\{q\} = \{Q\} - \{Q_0\} \qquad (2.15)$$

where, K represents the stiffness matrix, q represents the independent displacements vector, Q_0 represents the forces vector from the distributed load and Q the forces vector from concentrated forces on the ends of the element, being the stiffness matrix given by:

$$[K] = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix} \qquad (2.16)$$

The equilibrium, the kinematic conditions and the constitutive relation for the beam element, represented in Figure 2.2, are derived considering the following set of equations (Fish & Belytschko, 2007):

$$\text{Equilibrium equation: } dV = 0 \qquad (2.17)$$

$$\text{Kinematic condition: } \frac{1}{R}(x) \cong \frac{d^2w}{dx^2} \qquad (2.18)$$

$$\text{Constitutive relation: } M(x) = EI\,\frac{1}{R} \qquad (2.19)$$

being, V the potential energy, R the radius of curvature, w the transverse displacement, M the bending moment and I the cross sectional inertia.

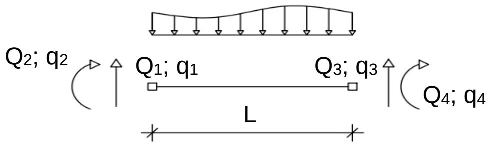


Figure 2.2– Beam element.

The energy of deformation and the external work caused by the applied loads are given by:

$$U = \frac{1}{2}\int_0^L E(x) I(x) \left(\frac{d^2w}{dx^2}\right)^2 dx \qquad (2.20)$$

$$W = \int_0^L w(x)\, p(x)\, dx + \sum Q_i\, w_i \qquad (2.21)$$

Applying the previous procedure to the potential total energy, considering its differentiation, the variation of the energy of deformation and the variation of the external work are given by:

$$dU = \left[\int_0^L E(x) I(x)\, \{\psi''\}^T\{\psi''\}\, dx\right]\{q\} \qquad (2.22)$$

$$dW_n = \int_0^L \{\psi\}.p(x)\, dx \qquad (2.23)$$

$$dW_N = \sum_{i=1,2} Q_i.\{\psi(0)\} - \sum_{j=3,4} Q_j.\{\psi(L)\} \qquad (2.24)$$

Considering the Hermite's polynomials as approximate functions, the stiffness matrix of prismatic bars is given by:

$$[K] = \frac{EI}{L}\begin{bmatrix} 12/L^2 & 6/L & -12/L^2 & 6/L \\ 6/L & 4 & -6/L & 2 \\ -12/L^2 & -6/L & 12/L^2 & -6/L \\ 6/L & 2 & -6/L & 4 \end{bmatrix} \qquad (2.25)$$

2.2 Geometrical non-linear analysis

The geometrical non-linear analysis considers a linear physical constitutive relation and assumes that the geometrical effects are not negligible, considering them into the equilibrium analysis.

The external work due to the external forces and the deformation must be determined and included in the definition of the potential energy, which for the element of a beam column represents the product between the axial displacement (not considering the axial deformation) and the axial force.

Considering a quadratic approximation of the axial displacement per unit length, the external work due to geometrical effects is given by the integration of this approximation along the element length multiplied by the axial force (Reis & Camotim, 2001):

$$W_P = P\int_0^L \frac{1}{2}\left(\frac{dw}{dx}\right)^2 dx \qquad (2.26)$$

Differentiating the total potential energy, the stiffness matrix is defined by two tensorial quantities: the linear elastic stiffness matrix (K_e), which is equal to the linear elastic stiffness matrix; and the geometrical stiffness matrix (K_{geo}), which simulates the effects in the stiffness of the element due to geometrical effects.

Considering an approximate solution of the displacement field given by a linear combination of the Hermite's polynomials and introducing the equation (2.26) into the equilibrium equation (2.17), the geometrical stiffness matrix is defined by the following components:

$$[K_{geo.}] = -P\begin{bmatrix} 6/5L & 1/10 & -6/5L & 1/10 \\ 1/10 & 2L/15 & -1/10 & -L/30 \\ -6/5L & -1/10 & 6/5L & -1/10 \\ 1/10 & -L/30 & -1/10 & 2L/15 \end{bmatrix} \qquad (2.27)$$

The exact stiffness matrix can be derived considering the differential equation of equilibrium of a beam written for the deformed shape as follows (Reis & Camotim, 2001):

$$\frac{d^4w}{dx^4}\,E(x)I(x) + P\frac{d^2w}{dx^2} - q = 0 \qquad (2.28)$$

The general solution of equation (2.28) allows to obtain the exact approximate functions needed to be adopted for the definition of the displacement field, which allows to derive the following matrix.

$$[K] = \frac{EI}{L}\begin{bmatrix} 12\phi_1/L^2 & 6\phi_2/L & -12\phi_1/L^2 & 6\phi_2/L \\ 6\phi_2/L & 4\phi_3 & -6\phi_2/L & 2\phi_4 \\ -12\phi_1/L^2 & -6\phi_2/L & 12\phi_1/L^2 & -6\phi_2/L \\ 6\phi_2/L & 2\phi_4 & -6\phi_2/L & 4\phi_3 \end{bmatrix} \qquad (2.29)$$

The functions ϕ_i represent the stability functions that consider the geometrical non-linear effects on the stiffness of elements.

$$\beta = \frac{L}{2}\sqrt{\frac{P}{EI}} \qquad (2.30)$$

$$\phi_1 = \beta.\phi_2.\text{cotg}(\beta) \qquad (2.31)$$

$$\phi_2 = \frac{\beta}{3\,(1-\beta.\text{cotg}(\beta))} \qquad (2.32)$$

$$\phi_3 = \frac{3}{4}\phi_2 + \frac{1}{4}\beta.\text{cotg}(\beta) \qquad (2.33)$$

$$\phi_4 = \frac{3}{2}\phi_2 - \frac{1}{2}\beta.\text{cotg}(\beta) \qquad (2.34)$$

Considering the dependency of the stiffness matrix on the forces of the elements, it was established the Newton-Raphson method in order to determine the values of the matrix with a small error. The method consists on an incremental procedure followed by an iterative procedure, which when applied together have the capacity to determine geometrical non-linear equilibrium paths (Chan & Chui, 2000).

3. Standard code (EN1993-1-1)

In this section, a brief introduction of some requirements imposed by EC3 regarding the safety verification considering the geometrical non-linear effects on structures is presented.

The geometrical effects are divided into global effects (P-Δ) and local effects (P-δ).

The global effects have to be considered depending on the relation between the applied load and the critical load (α_{cr}). In fact, the EC3 in § 5.2.1 considers that: for $\alpha_{cr}>10$ P-Δ effects can be neglected; for $3<\alpha_{cr}<10$ P-Δ effects have to be considered by the Amplification of the Sway Effects Method; and for $\alpha_{cr}<3$ the structure should be studied by a 2nd order analysis.

The P-δ effects are usually considered indirectly in interaction formulas.

The analyses, according to EC3, have to consider local and global imperfections considering the effects of geometrical imperfections and residual stresses, which are defined in § 5.3.1, as represented in Figure 3.1.

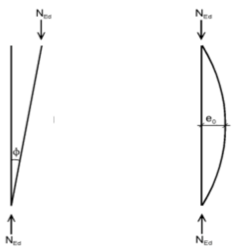


Figure 3.1 – EC3 imperfections.

The EC3 in § 5.3.2 defines both imperfections in function of structure geometry and type of section.

For the safety verification of compressed bars EC3 provides two interaction formulas that approximately describe the geometrically non-linear behaviour regarding P-δ effects.

$$\frac{N_{ed}}{\chi_y N_{rd}} + k_{yy}\frac{M_{y,ed}}{M_{y,rd}} + k_{yz}\frac{M_{z,ed}}{M_{z,rd}} \leq 1,0 \qquad (3.1)$$

$$\frac{N_{ed}}{\chi_z N_{rd}} + k_{zy}\frac{M_{y,ed}}{M_{y,rd}} + k_{zz}\frac{M_{z,ed}}{M_{z,rd}} \leq 1,0 \qquad (3.2)$$

In this dissertation, the interaction factors k_{ij} will be obtained through Method 2 (EC3 – Annex B). The implicit imperfections in the formulas will be taken into account in the following studies.

4. Validation of the numerical model

The numerical model was validated comparing the corresponding numerical results with classic analytical solutions.

Critical loads obtained for columns with different boundary conditions and a frame are compared with classic analytical results.

The columns and the frame analysed are represented in Figure 4.1 corresponding to: a column with both ends fixed; a column with a fixed end and a fixed end with sway displacements; a cantilever with linear variation of cross-section dimensions; and a frame with columns with both bases fixed.

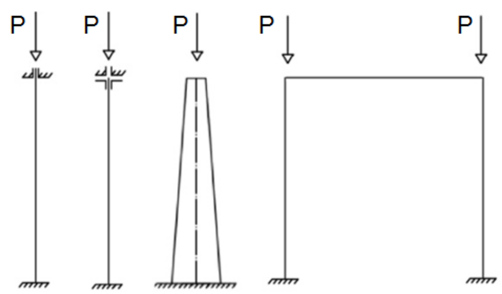


Figure 4.1 - Analysed structures.

The examples considered have a height of 4 m, and in the case of the frame it is considered a length of 4 m.

An analysis of the columns and frame was performed through the exact stiffness matrix by evaluating the corresponding determinant.

The results obtained are presented in Figure 4.2 for the prismatic columns, being the corresponding bifurcation loads defined by the "zeros" of the curves. The values of the obtained bifurcation loads are given in Table 4.1 being achieved a good agreement with analytical results (Reis & Camotim, 2001).

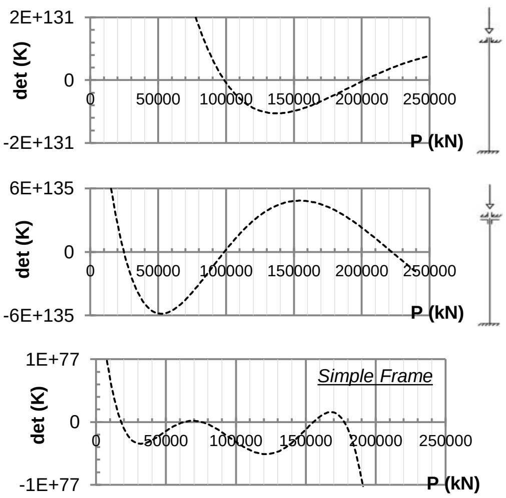


Figure 4.2 – Determinant of exact matrices.

Table 4.1 – Critical loadings with exact matrix.

Structure	$P_{cr,1}$ (kN)	$P_{cr,2}$ (kN)
Column: Both ends fixed	98696	201907
Column: End with sway displacements	24674	98696
Simple Frame	18237	63358

The bifurcation loads obtained by the numerical model that considers a linear approximation of the stiffness matrix are compared with the results obtained by considering the exact stiffness matrix. The relative errors between these approaches are presented in Figure 4.3 for different discretization of the columns and the frame. The numerical solutions (linear approach) have an excellent convergence to the exact solution, being concluded that with only 3 elements per bar the relative error is reduced to less than 1%.

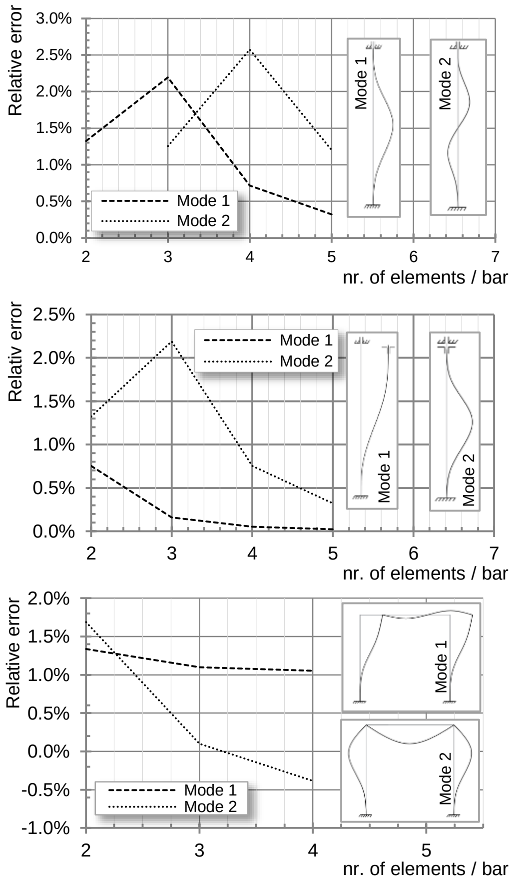


Figure 4.3 – Relative errors.

Regarding the column with a linearly variable cross section, the exact matrix derived in the previous chapter cannot be adopted. To this end, two different approaches to evaluate the accuracy of the model were considered.

In the first approach the column is modelled by considering a stiffness matrix derived for a finite element with linear variable cross section. The integration of the stiffness matrix is performed numerically by adopt Hermite polynomials as approximate functions.

In the second approach, the column was modelled with finite elements of constant cross section.

The errors between the results obtained from the numerical model and classic results (Theodore Von Kármán, 1940) are represented in Figure 4.4 for both approaches.

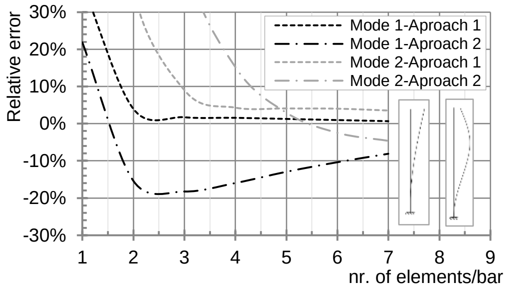


Figure 4.4 – Relat. error: variable cross section.

It can be concluded that the model allows a good agreement of results for bars with variable cross section when is applied the *Approach 1* with a discretization of three or more finite elements per bar.

The results from *Approach 2* (even with an adequate refinement of the discretization) correspond to significant errors.

5. Application of the model

The applications of the model consist in a set of second order analyses which aims to determine the influence of loads and geometrical imperfections in the geometrical non-linear behaviour of beam-columns and frames. The examples of application of the model are divided in two sets of analyses.

In the first, it is studied the geometrical non-linear behaviour of beam-columns, considering a beam-column simple supported and a beam-column with three equal spans with pinned supports, for which are analysed the effects of geometrical imperfections and applied loads in their non-linear behaviour; it is also made a comparison between the numerical results and the formulas (3.1) and (3.2) from EC3.

In the second set of analyses it is studied two frames: one without a bracing system and another with a bracing system; being analysed how the bifurcation loads and the geometrical non-linear behaviour of the frames are affected by (i) the stiffness of the bracing bar, (ii) the geometric and mechanical properties of the frame itself and finally (iii) the geometrical local imperfections. For these cases it is also compared the numerical results with the EC3 procedures.

5.1. Double-pinned column

A beam-column, simple supported, with a span of 4 m and an IPE200 cross-section in a steel grade S355 is analysed. The beam-column is represented in Figure 5.1.

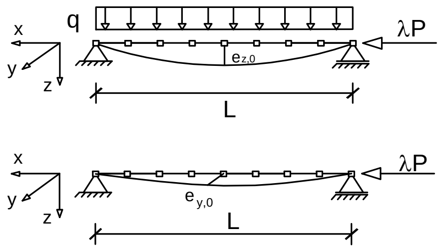


Figure 5.1 – Beam-column model in the y and z axis.

The geometrical imperfections considered for the y and z axis are defined on the Table 5.1, in accordance with the imperfections implicit on the definition of the formulas (3.1) e (3.2) from § 6.3.1.2 of EC3.

Table 5.1 – Geometrical imperfections (EC3: § 6.3.1.2).

Eixo y		**Eixo z**	
L / w_0	**w_0 (m)**	**L / w_0**	**w_0 (m)**
L/667	0,006	L/571	0,007

5.1.1. Case 1 – Influence of applied forces on equilibrium paths

The purpose of Case 1 is to analyse the effect due to applied loading on the equilibrium paths of simple-supported beam-columns.

The study is developed considering two sets of applied forces: the first set considers an initial and uniform distributed force applied along the span and an increasing compression; the second set considers an initial and uniform compressive load and an increasing distributed force along the span.

The equilibrium paths are represented in Figure 5.2 and Figure 5.3 for both sets of loads.

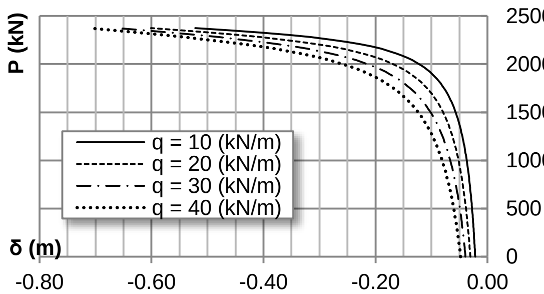


Figure 5.2 – Equilibrium paths for uniform distributed force.

It can be concluded that the effect of the application of distributed forces is similar to an increase of the initial geometrical imperfection.

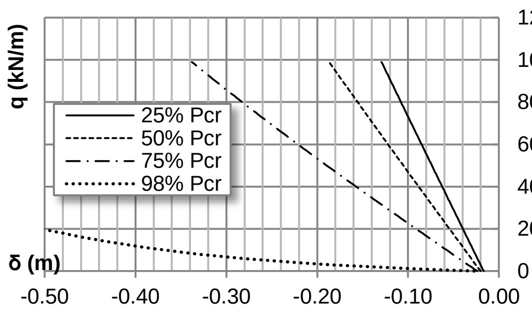


Figure 5.3 – Equilibrium paths for uniform compression force.

From the second set of analysis it is concluded that the axial compression of the beam-column element causes a decrease of the corresponding bending stiffness.

In these analyses, for the case in which it is applied a compressive loading of 98% of P_{cr}, it is possible to observe the post-buckling resistance of the beam.

5.1.2. Case 2 – Comparison with EC3

The interaction formulas of EC3 regarding the verification of the buckling of beam-columns are compared with the numerical results obtained from the model.

The simple-supported column was analysed by the derived numerical model, considering a geometrical non-linear analysis; the (geometrical) imperfections were considered separated for each plan of buckling.

The criterion adopted for evaluate the resistance of the numerical model is the yielding criterion of the cross-section.

The structure was analysed considering geometrical imperfections applied in only one direction a time.

The numerical results together with the results from the interaction formulas are represented in Figure 5.4.

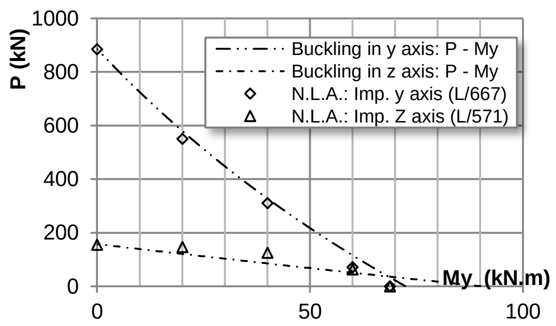


Figure 5.4 – Numerical results and interaction formulas: double-pinned beam column.

The results of the non-linear analyses are in a good agreement with the results obtained from the interaction formulas.

On the left side of Figure 5.4 it is observed the “column” behavior of the beam, which is quite different for the two bending planes. This behavior is significantly influenced by the geometrical imperfection of the column and by the direction considered.

On the right side it is possible to observe the beam behavior of the structure which is not much affected by the direction of the imperfection, and which converge to the yielding moment.

5.1.3. Case 3 – Comparison with EC3 for a three span beam-column

Case 3 has the purpose of completing the previous comparison between numerical results and the results from EC3’s interaction formulas. In this example of application of the model it is considered a beam-column with three equal spans with a length of 4 m. The displacements in the y direction are considered to be fixed.

Two comparisons are made, being represented in Figure 5.5 and Figure 5.6: the first comparison (Comparison 1), considers a single-span modelled as a beam-column, having the distributed load applied along the span and concentrated moments at the respective ends. The second comparison (Comparison 2) considers a global model of the beam-column, i.e., the 3 spans are considered with the corresponding boundary conditions.

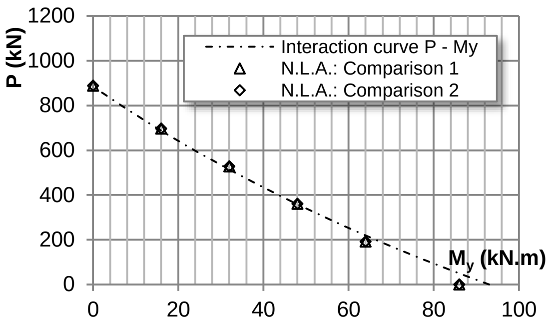


Figure 5.5 – External span: interaction formula P-M_y and the yielding points (L/667).

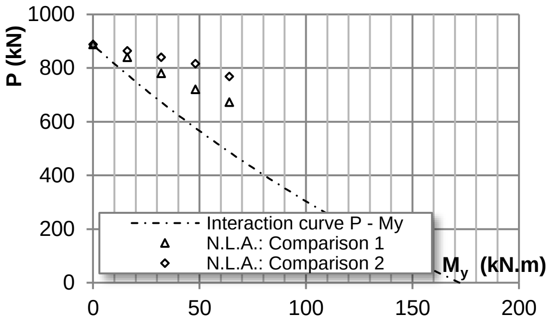


Figure 5.6 - Internal span: interaction formula P-M_y and the yielding points (L/667).

The results obtained, from both comparisons, for the external span are in a close correlation with those obtained by the interaction formulas from EC3.

For the internal span the numerical results deviate from the interaction curves, not following the progress of these, yet the formulas keep a conservative position relative to the obtained numerical results. The difference between the two comparisons may be explained by the effects in the internal span behaviour due to the external span deformation, which combined with the compressive load causes a lever effect.

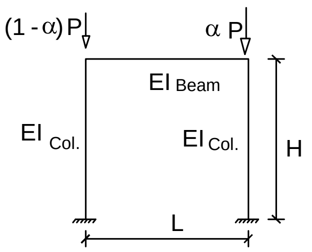
5.2. Frame structures

The examples of applications aiming the study of frames consider always a frame with a length of 4 m and a height of 4 m.

5.2.1. Case 4 – Influence of element stiffness on the critical loading

The objective of this example of application of the model is to analyse the influence of the relation between the stiffness of the beam and the stiffness of columns, as well as the influence of the loading asymmetry on the values of the critical load of the frame.

The frame represented in Figure 5.7, with the identified geometrical and mechanical properties, is used to define the parameter β, which defines the relation between the column stiffness and the beam stiffness.



$$\beta = \frac{EI_{beam}}{EI_{columns}} \frac{H}{L}$$

Figure 5.7 – Analysed frame.

A set of buckling analyses were performed being the corresponding results represented in Figure 5.8.

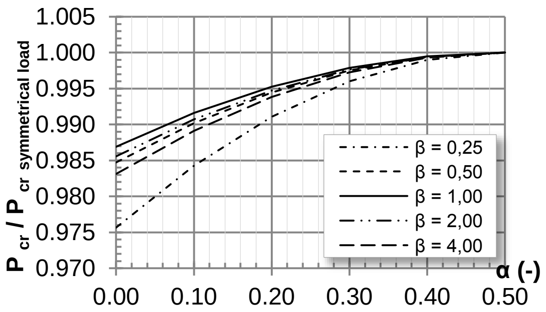


Figure 5.8 – Frame's critical loading.

The non-symmetrical load reduces the value of the frame's critical loading due to the less well distributed axial compression through the columns, being the vertical loads concentrated in one column while the other has less axial force.

Simultaneously the consideration of different stiffness's for the beam and the columns also implies a decrease of the frame's critical loading, due to a less aggregated behaviour between columns and beam.

5.2.2. Case 5 – Parametrical study of frame's behaviour considering a bracing system

A parametric analysis of the frame is performed considering its lateral stiffness as a function of: (i) the bracing system stiffness; (ii) the local imperfections of the bracing system; (iii) and the axial compression in the frame's columns.

The frame as well as its bracing system are represented in Figure 5.9, in which: $e_{0,column}$ represents the local imperfection of the columns; $e_{0,diag.}$ represents the local imperfection of the bracing bar; ϕ represents the global imperfection of the frame, which was previously presented in Figure 3.1.

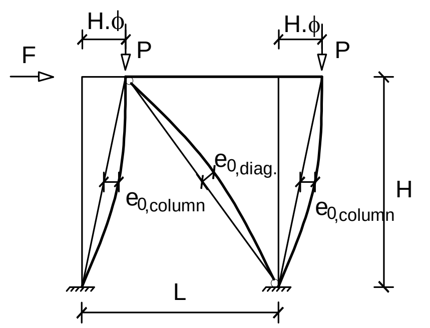


Figure 5.9 – Analysed frame.

The geometrical and mechanical properties of the structure are defined considering the two following parameters:

$$\kappa = \frac{EI_{frame}}{EA_{diag.} L^2_{diag.}} \quad \bar{\lambda} = \frac{L_{diag.}}{i_{diag.}\, 93{,}9\, \varepsilon} \quad (5.1)$$

Two sets of results considering different values of k and $\bar{\lambda}$, are represented in Figure 5.10 and Figure 5.11. For these cases there is no vertical loading and it is assumed a constant local imperfection of the frame and of the bracing system with a value of L/150.

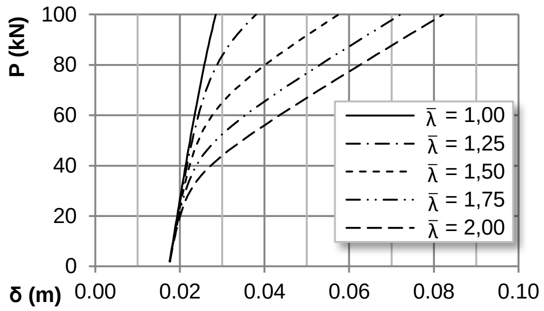


Figure 5.10 – Equilibrium path: k = 0,001

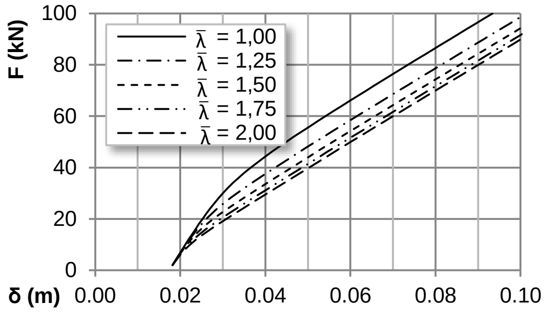


Figure 5.11 - Equilibrium path: k = 0,006.

As observed, the stiffness of the bracing bar represents an important parameter in the lateral stiffness of the frame. In cases which the stiffness of the bracing bar is bigger it is observed that the variation of the slenderness represents a big variation in the lateral behaviour of the frame. Otherwise, the slenderness variation doesn’t affect so much the lateral stiffness of the frame, but still being important especially when it has small values.

In Figure 5.12 and Figure 5.13 are shown the graphs for the cases with a variable local imperfection a vertical loading, respectively.

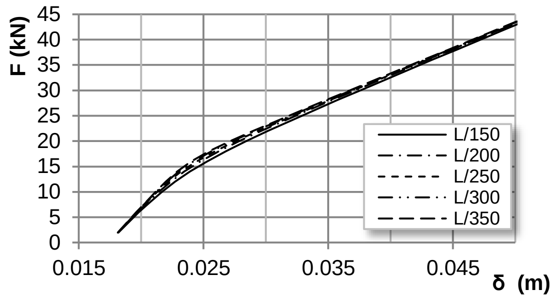


Figure 5.12 – Equilibrium path: k = 0,06; $\bar{\lambda}$ = 1,5.

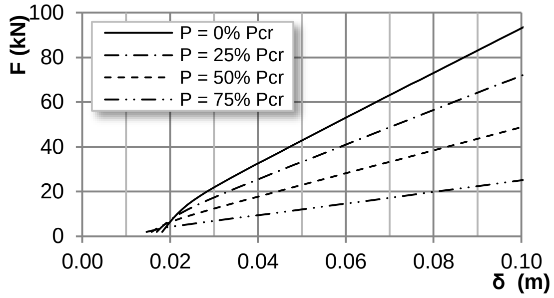


Figure 5.13 – Equilibrium path: k = 0,06; $\bar{\lambda}$ = 1,5.

As observed the local imperfection of the bracing bar does not affect much the global behaviour of the frame, representing only small differences between the equilibrium trajectories.

The column’s compression represents a big influence in the global behaviour, causing a reduction in the global lateral stiffness after the buckling of the bracing bar.

6. Conclusions

A numerical model that allow to perform geometrical non-linear analysis of steel structures was developed. The model was implemented in Matlab.

The model was derived through the Rayleigh-Ritz method, developing a set of four structural analyses consisting in a:

1. Linear-elastic analysis;
2. Approximated linear buckling analysis;
3. Exact linear buckling analysis;
4. Geometrical second order analysis.

Three types of stiffness matrix (linear-elastic; approximated geometrically non-linear; exact geometrically non-linear) were used to simulate the structural behaviour.

Regarding the buckling analysis, and aiming the validation of the model, four types of structures were analysed. It was studied the influence of the model discretization on the convergence of critical loads when using approximate stiffness matrices. It was concluded that considering a refined discretization, the values of critical loadings converge to their exact solutions.

The critical loads of the analysed prismatic structures corresponded to a relative error of 1%, when using discretization of more than 4 elements per bar. The prismatic structures analysed with constant cross sections the critical loadings have a relative error of 1% when using discretization equal or bigger than 4 for elements per bar.

It was also analysed the influence of the beam and columns stiffness on the critical loads of the first buckling mode of a simple frame, considering a set of asymmetric vertical loads.

A geometrical non-linear analysis of a simple-supported beam-column was performed, allowing to evaluate the influence of the applied loads on the corresponding behaviour. The results were compared with the results from the interaction formulas from EC3.

A non-linear analysis of a simple frame was also performed, being the numerical results compared with the EC3's interaction formulas.

Finally, it was analysed a frame with a bracing system, performing a parametric study of the global lateral stiffness of the frame. Three parameters were considered to the analyses: (i) the geometrical local imperfections of the bracing bar; (ii) the vertical applied loads on columns; (iii) and the relation between the bracing system's stiffness and the frame stiffness.

Considering this, it was determined a set of equilibrium paths which represented the influence of each parameter on the lateral global behaviour of the frame.

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