Structural Design of Reinforced Concrete Deep Shafts
Inserted Into Fractured Rock Masses

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Extended Abstract

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Abstract: In the design of deep shafts inserted into fractured rock masses it is important to know the behavior and distribution of the vertical loads between the concrete structure and the rock mass. With the purpose to know this load distribution, this work develops an analytic model for the load distribution between the concrete and the rock mass, and the respective study of this analytic model, comparing it to a finite element method (FEM) model. The results reported by the analytic model provide a tool for estimating the forces in concrete for the structural design of a deep shaft.

Key-words: Vertical load distribution, structural design of deep shafts, fractured rock masses, reinforced concrete.

1. Introduction

The main goal of this work is to develop a theoretical/analytical model to obtain the load distribution between the rock mass and the reinforced concrete structure, in the design of deep shafts of circular cross-section, with a reinforced concrete lining, inserted in rocky environment (fractured rock mass), such as shafts created in some hydropower plants projects. It is important to identify the load distribution between the two materials given that the depth of these shafts can reach 300 meters and higher, and taking into account also the irregular contact surfaces between the reinforced concrete structure and the surface of the rock mass.

Shafts are structures that can be used for multiple purposes, such as access to high depth levels, collect underground resources or samples, conduct surveys, serve to support other large underground works and also have ventilation functions.

There are various types of lining for shaft structure, such as, reinforced concrete lining and steel lining. The use of reinforced concrete lining can bring several advantages such as the possibility of reinforcing “week” rock regions, water leakage control along the shaft, prevention of rock erosion, prevention of instability of the rock mass and the economic advantage of being less expensive than a steel lining (ASCE, 1989).

2. Deep Shaft Excavation

The excavation of deep shafts with large diameters, embedded in rock masses is usually performed through one of following methods: drilling and blasting method, Alimak method, and rotary drilling with Raise Boring Machine or Shaft Boring Machine.

The drilling and blasting method consists of two main phases: first, the excavation by use of percussion or rotation drilling methods, followed by blasting with explosives arranged in the excavation already performed (Grieves,
1996). This second phase cause an irregular surface on the rock wall (presenting wedges throughout the depth of the shaft) and fracturing of the rock in the shaft surroundings by the violent vibrations and gas releases provoked by the blasting (Fig. 1) (Hartmann, 1992).

Raise Boring Machine (Fig. 2) and Shaft Boring Machine are automated machines for performing excavation up to 1260 meters depth and a range of diameters from 0.7 to 8.5 meters. The first one bores a pilot hole, breaking through the rock from the surface to the level below, the pilot bit is then replaced by a reamer head, with the diameter planned for the shaft, then the machine pulls the reamer head upward, while rotating, to break a circular hole in rock (Ozdemir ,1986). The second one is for shafts excavation exclusive, and unlike the previous, performs the excavation from the top down, but also with the use of rotary drilling.

3. Analytical Model to Obtain the Load Distribution

The load sharing is possible by means of wedges existing in the rock wall (Fig. 3), that result from use of blasting excavation method, which have the capacity to bear part of the loads acting on the concrete. Together with the rock mass, the wedges show a spring type of behavior and have a certain stiffness, and it is through this stiffness that is to be determined which load absorbed either by the rock mass either by concrete. In order to calculate this stiffness is first necessary to calculate all the associated displacements.

As simplifying assumptions it is assumed that the wedges are similar to each other along the shaft, with a $t$ meters width and a $\alpha$ angle with the vertical axis, and that the rock mass characteristics do not vary with depth.
3.1 – Equilibrium at Slide Situation

To determine the displacements is first necessary to know the stresses and forces actuating in a single isolated wedge, when it is submitted to a force \( P \) (Fig. 4).

Supposing that the inner radium of the shaft is \( R_c \) and the concrete lining width is \( e \), the vertical force is given by (4).

\[
R_v = \frac{P}{2 \times \pi \times (R_c + e + \frac{t}{a_v})} \tag{4}
\]

From equation (2) it is possible to obtain the perpendicular force to the surface of the wedge (5).

\[
R_n = \frac{R_v}{\sin \alpha + \mu \times \cos \alpha} \tag{5}
\]

Replacing (5) in equation (1) it is possible to achieve the horizontal force on the wedge (6).

\[
R_b = R_v \times \frac{(\cos \alpha - \mu \times \sin \alpha)}{(\sin \alpha + \mu \times \cos \alpha)} \tag{6}
\]

At last from (3) it is possible to obtain the distance between the application point of \( R_n \) and point A (7).

\[
x = t \times \left( \frac{\sin \alpha + \mu \times \cos \alpha}{a_v} - \frac{\cos^2 \alpha}{2 \times \sin \alpha} - \frac{\mu \times \cos \alpha}{2} \right) \tag{7}
\]

It is also important to define the horizontal tension on the wedge surface (8).

\[
p_b = R_v \times \frac{(\cos \alpha - \mu \times \sin \alpha)}{(\sin \alpha + \mu \times \cos \alpha)} \times \frac{\tan \alpha}{t} \tag{8}
\]

3.2 - Relative Displacement Between Concrete and Rock

In order to determine the vertical relative displacement between the concrete lining and the rock mass is first needed to determine the horizontal displacements of both.

In the case of the concrete horizontal displacement, it has the radial pressure \( p_b \) actuating on all the perimeter of the lining (Fig. 3).
5), caused by the rock. So admitting thin-wall hypothesis the displacement is given by (9).

\[ \delta_c = \frac{p_b \times (R_c + e)}{e \times E_c} \times (R_c + e) \]  \hspace{1cm} (9)

Figure 5 – Representation of the concrete under pressure

In the case of fractured rock region, the action of an inner pressure \( (p_b) \) applied by concrete, and outside pressure \( (p_b') \) applied by the non-fractured rock zone (Fig. 6). Both these pressures cause displacements on the inner and outer faces of the fractured rock ring, respectively, that are given by (10) and (11) (Timoshenko, 1940).

\[ \delta_c = \frac{1 - \nu_{rf}}{E_{rf}} \times \frac{(R_c + e)^2 \times p_b - R_d^2 \times p_b'}{R_d^2 - (R_c + e)^2} \times (R_c + e) \]  \hspace{1cm} (10)

\[ \delta_{r1} = \frac{1 - \nu_{rf}}{E_{rf}} \times \frac{(R_c + e)^2 \times p_b - R_d^2 \times p_b'}{R_d^2 - (R_c + e)^2} \times R_d + \frac{1 + \nu_{rf}}{E_{rf}} \]  \hspace{1cm} (11)

Figure 6 – Representation of the fractured rock ring and actuating pressures

The non-fractured rock zone is actuated only by the pressure \( (p_b') \) exerted by the fractured rock ring (Fig. 7).

Figure 7 – Representation of the non-fractured rock ring and actuating pressures

Considering Timoshenko (1940) it is possible to determine the displacement of the non-fractured rock ring inner face (12).

\[ \delta_{r1} = \frac{R_d \times p_b'}{E_r} \times \left[ \frac{101}{99 + \nu_r} \right] \]  \hspace{1cm} (12)

To set the pressure \( (p_b') \) (13) it is necessary to equalize (11) and (12).

\[ p_b' = p_b \times m \]  \hspace{1cm} (13)
In which \( m \) is given by (14).
\[
m = \frac{2 \times (R_c + e)^2 \times R_d}{c + a}
\]
(14)

In which \( c \) and \( a \) are given by (15) and (16), respectively.
\[
c = \frac{E_{rf}}{E_r} \times R_d \times \left( R_d^2 - (R_c + e)^2 \right) \times \left[ 101 \frac{99}{99} + \nu_r \right]
\]
(15)

\[
a = \left[ (1 - \nu_{rf}) \times R_d^3 + (1 + \nu_{rf}) \times (R_c + e)^2 \times R_d \right]
\]
(16)

Finally the vertical relative displacement is given by (17).
\[
\delta_{v_{r1}} = (\delta_c + \delta_r) \times \cot a
\]
(17)

### 3.3 Vertical Deformation of the Rock Mass

For the determination of the rock wall vertical displacement it is considered that there is a conical dispersion of the actuating tensions (Fig. 8).

It is necessary to distinguish two zones of the rock in depth: the zone which lies between the top and bottom of the shaft (zone 1); and the zone immediately below the first, and that supports the shaft (zone 2).

For the zone 1 the diameter of the tension dispersion and the area that receives tensions, are given by (18) and (19), respectively.
\[
D(x) = 2 \times (x \times \cot \theta + R_c + e + t)
\]
(18)

\[
A'(x) = \pi \times (x \times \cot \theta + R_c + e + t)^2 - \pi \times (R_c + e)^2
\]
(19)

This makes it easy to determine the vertical displacement of the zone 1 (20).
\[
\delta_{v_{r1}} = \frac{P}{\pi E_{rv}} \left[ \frac{\ln(x \times \cot \theta + t)}{2 \times \cot \theta \times (R_c + e)} - \frac{\ln(x \times \cot \theta + 2(R_c + e) + t)}{2 \times \cot \theta \times (R_c + e)} - \frac{\ln t - \ln(2(R_c + e) + t)}{2 \times \cot \theta \times (R_c + e)} \right]
\]
(20)

Analyzing the tension dispersion in the zone 2, with a depth of \( L_m \) meters below the bottom of the shaft, it is necessary to define the upper and lower receiving tensions areas \( A_1 \) and \( A_2 \), respectively.
\[
A_1 = \pi \times (x \times \cot \theta + R_c + e + t)^2 - \pi \times (R_c + e)^2
\]
(21)

\[
A_2 = \frac{\pi \times [D(x) + 2 \times L_m \times \cot \theta]^2}{4}
\]
(22)

So the vertical displacement of the zone 2 is given by (23).
\[
\delta_{v_{r2}} = \frac{P \times L_m}{E_{rv} \times (A_2 - A_1)} \times \ln \frac{A_2}{A_1}
\]
(23)

### 3.4 Stiffness of the Wedge-Rock Set

Considering Hooke’s law (Timoshenko & Goodier, 1951), the stiffness of one isolated wedge is given by (24).
\[
K = \frac{P}{\delta_{v_{tot}}}
\]
(24)
In which \( \delta_{V,\text{tot}} \) is the addition of the displacements previously determined (25).

\[
\delta_{V,\text{tot}} = (\delta_c + \delta_r) \times \cot \alpha + \delta_{V,r1} + \delta_{V,r2} \tag{25}
\]

It is possible to eliminate the force \( P \) out of the equation (24) through the adoption of the following simplifications of the displacements.

\[
C_1 = \frac{\delta_c}{P} \tag{26}
\]

\[
C_2 = \frac{\delta_r}{P} \tag{27}
\]

\[
R_1 = \frac{\delta_{V,r1}}{P} \tag{28}
\]

\[
R_2 = \frac{\delta_{V,r2}}{P} \tag{29}
\]

Considering also the number of wedges by meter vertically \( (n) \) it is obtain the stiffness of the wedge-rock set of the shaft (30).

\[
K_{\text{spring}} = \frac{n}{(C_1 + C_2) \times \cot \alpha + R_1 + R_2} \tag{30}
\]

### 3.5 Axial Load Distribution

At this point are defined the terms needed to obtain the axial forces both in solid rock and in concrete ring. Assuming that on the structure acts only a distributed load \( (p) \) and the possibility that the displacement field varies linearly along depth (Fig. 9), it is given the equation (31) by equilibrium.

\[
N(x) + \int_0^x f(x) \, dx = p \times x \tag{31}
\]

It is now conceivable the expression for the distributed force on the rock (32).

\[
f(x) = K_{\text{spring}} \times \frac{\delta_{\text{top}}}{L} \times (L - x) \tag{32}
\]

Integrating the equation above, we get the expression for the total force on the rock for a determined depth (33).

\[
\int_0^x f(x) \, dx = K_{\text{spring}} \times \frac{\delta_{\text{top}}}{L} \times \left(L \times x - \frac{x^2}{2}\right) \tag{33}
\]

Replacing (33) in (31) it is obtained the expression for the total force on the concrete lining for a determined depth (34).

\[
N(x) = p \times x + K_{\text{spring}} \times \frac{\delta_{\text{top}}}{L} \times \left(\frac{x^2}{2} - L \times x\right) \tag{34}
\]

It is also important to define the displacement on the top of the shaft (35).

\[
\delta_{\text{topo}} = \int_0^L \frac{N(x)}{E_c \times A} \, dx = \frac{3}{2} \times \frac{p \times L^2}{(K_{\text{mola}} \times L^2 + 3 \times E_c \times A)} \tag{35}
\]
In short, it is thus possible to predict what the amount of load charges transferred to the rock mass (33), and also the axial force in the concrete ring (34), depending on the depth.

4. **Analytic Model Sensitivity Analysis**

At this point are tested the results of the analytic model through the application to a shaft with the physic and geometrical characteristics represented in tables 1, 2, 3 and 4.

**Table 1 – Characteristics of the concrete lining**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td>300</td>
</tr>
<tr>
<td>Rc (m)</td>
<td>2.95</td>
</tr>
<tr>
<td>e (m)</td>
<td>0.3</td>
</tr>
<tr>
<td>Ec (Mpa)</td>
<td>20000</td>
</tr>
<tr>
<td>q (KN/m)</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 2 – Characteristics of the rock mass**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rd (m)</td>
<td>15</td>
</tr>
<tr>
<td>Er (Mpa)</td>
<td>10000</td>
</tr>
<tr>
<td>Erf (Mpa)</td>
<td>10000</td>
</tr>
<tr>
<td>Erv (Mpa)</td>
<td>10000</td>
</tr>
<tr>
<td>µ</td>
<td>0.7</td>
</tr>
<tr>
<td>νr</td>
<td>0.3</td>
</tr>
<tr>
<td>νrf</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 3 – Characteristics of the wedge**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>α (°)</td>
<td>30</td>
</tr>
<tr>
<td>t (m)</td>
<td>0.2</td>
</tr>
<tr>
<td>n (wedge/m)</td>
<td>0.1</td>
</tr>
<tr>
<td>av</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 4 – Values for the tension dispersion**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>θ (°)</td>
<td>45</td>
</tr>
<tr>
<td>Lm (m)</td>
<td>100</td>
</tr>
</tbody>
</table>

Now using the values of the tables above and equations (33) and (34) it is obtain the load distribution (Fig. 10).

**Figure 10 – Load distribution in function of z**

It is observed that there is tension in the initial part of the shaft and compression closer to the bottom, this due to the fact of the consideration of the spring-type effect of wedges, while the rock mass is always under compression.

In figure 11 are shown the displacements in function of z, which are liner as expected.

**Figure 11 – Vertical displacements in function of z**
5. Comparison of the Analytic Model with a Computational Model

At this point, it proceeds to a shaft modeling through the SAP2000 finite elements method (FEM) software, using shell elements (Fig. 12) so as to approximate the model to the geometric characteristics of the shaft of reinforced concrete circular cross-section presented before.

![Figure 12 - Circular cross-section modeled through SAP2000](image)

### 5.1 Modeling Without Springs

It begins then to analyze the model consists of shell elements without any spring along its length, which corresponds to the situation that there are no wedges throughout the depth of the shaft ($K_{molae} = 0$) in which total load is supported only by reinforced concrete.

Regarding the displacements (Fig. 13) note that, in fact, the admitted hypothesis for the analytical model in which there is a linear variation of the displacement field is not exact, since the variation in displacement obtained by FEM appears to have a parabolic form. Still, the admitted hypothesis is a reasonable approximation, notwithstanding the existence of smaller displacements along the depth but with the same displacement at the top of the shaft compared to the results supplied by the calculation program.

![Figure 13 - Vertical displacements in function of z (without springs)](image)

### 5.2 Modeling With Springs

In order to simulate now a shaft in which there are wedges on the rock mass along its length, were added springs on the FEM model, simulating the stiffness of $K_{spring} = 385011.4 \, KN/m/m$, given by equation (30). Here, the total load imposed in the shaft, shall be notionally allocated between the rock mass and concrete.

It is possible to observe the load distribution comparison of the analytical model and the SAP2000 model in the figures 14 and 15.
It is noted that the results given by the analytic model are not exact, this is due to the inaccuracy of the hypothesis for the displacement field. Still these results are a good approximation of the real ones.

As anticipated, the displacements (Fig. 16) given by the analytic model are different from the displacements given by the SAP2000 model.

6. Discussion of the Displacement Field

Due to the non-accuracy of the hypothesis adopted, in which the displacement field varies linearly in depth, it is now considered a new hypothesis, in which the displacement field varies parabolically in depth (Fig. 17), in order to make an improved approximation of the actual result of forces and displacements of the shaft in relation to the first one.

Having said that it is necessary to redefine the equations corresponding to the force distributed along the shaft (32), the axial force
on the rock mass (33), the axial force in the concrete (34) and displacement at the top of the shaft (35).

\[ f(x) = K_{spring} \times \frac{\delta_{top}}{L^2} \times (L^2 - x^2) \]  
(36)

\[ \int_0^x f(x)dx = K_{spring} \times \frac{\delta_{top}}{L^2} \times \left( L^2 \times x - \frac{x^3}{3} \right) \]  
(37)

\[ N(x) = p \times x + K_{spring} \times \frac{\delta_{top}}{L^2} \times \left( \frac{x^3}{3} - L^2 \times x \right) \]  
(38)

\[ \delta_{topo} = \int_0^L \frac{N(x)}{E_c \times A} dx \]
\[ = 6 \times \frac{p \times L^2}{12 \times E_c \times A + 5 \times K_{mola} \times L^2} \]  
(39)

Comparing the results of this new hypothesis with the results from the previous one and with the results of the FEM model (Figs. 18 and 19) it is viable to say that although these last results are also not exact, they represent a better approximation to the SAP2000 results than the first hypothesis.

The displacements approximation is also improved with the adoption of the new hypothesis relatively to the previous one (Fig. 20).

7. Structural Design of the Shaft

At this moment it is proceeded to the structural design of the shaft, which includes checking the concrete compression on the bottom of the
shaft, because that is where there is located the highest compressive strength, and also the steel reinforcement needed to accommodate the tensile strength.

In order to ensure that the concrete does not come into compression failure, it is needed to check condition (40) (Eurocode 2), which provides the maximum limit for the compressive stress acting on the concrete.

\[ \sigma_c = \frac{N_t^{\text{max}} \times \gamma}{A_c} \leq f_{ca} = \frac{\alpha_c \times f_{ck}}{\gamma_c} \quad (40) \]

As the concrete considered is a C30/37 (a value of \( f_{ck} = 30 \, MPa \)), the design value of the concrete compression failure is given by (41).

\[ f_{ca} = \frac{0.05 \times 3.0}{1.5} = 17 \, MPa \quad (41) \]

Comparing table 5 and equation (41) it is possible to see that the concrete compression is verified for all the three models (\( \gamma = 1.5 \)).

### Table 5 - Values of the compression stress

<table>
<thead>
<tr>
<th>( N_t^{\text{max}} , (KN) )</th>
<th>( A_c ) ( (m^2) )</th>
<th>( \sigma_c ) ( (MPa) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td>2829,28</td>
<td>5,843</td>
</tr>
<tr>
<td>Analytic Model - Linear Hypothesis</td>
<td>12058,08</td>
<td>5,843</td>
</tr>
<tr>
<td>Analytic Model - Parabolic Hypothesis</td>
<td>9665,83</td>
<td>5,843</td>
</tr>
</tbody>
</table>

Regarding the longitudinal reinforcement design, since it is a circular section vertical shaft, it can be calculated as if it were a reinforced concrete structural wall according to Eurocode 2 (42).

\[ A_s = \frac{F_t}{f_{yd}} \quad (42) \]

Once there are no moments applied on the structure the tensile force it is given by \( F_t = N_t^{\text{max}} \times \gamma \). So the needed reinforcement for each model is presented in table 6 (\( f_{yd} = 435 \, MPa \)).

### Table 6 - Needed Reinforcement for each model

| | FEM | Analytic Model - Linear Hypothesis | Analytic Model - Parabolic Hypothesis |
|-----------------|-----------------|-----------------|
| \( N_t^{\text{max}} \, (KN) \) | 780,76 | 3706,67 | 2376,45 |
| \( F_t \, (KN) \) | 1171,1 | 5560,0 | 3564,7 |
| \( A_s \) \( (cm^2) \) | 26,9 | 127,8 | 81,9 |
| \( A_s \) \( (cm^2/m) \) | 1,32 | 6,26 | 4,01 |

Reinforcement on each side of the ring: \( \phi 6//0.30 \), \( \phi 10//0.20 \), \( \phi 8//0.20 \)

It is also essential to determine the minimum reinforcement for the shaft according to Eurocode 2 (43).

\[ A_{s,\text{min}} = \frac{f_{ctm}}{f_{yd}} \times A_c \quad (43) \]

On table 7 is given the minimum reinforcement adopted (\( f_{ctm} = 2,9 \, MPa \)).

### Table 7 –Minimum Reinforcement

| \( A_{s,\text{min}} \) \( (cm^2) \) | 338,9 |
| \( A_{s,\text{min}} \) \( (cm^2/m) \) | 16,60 |

Reinforcement on each side of the ring: \( \phi 16//0.20 \)

8. Conclusions

The primary goal of this work, that was obtaining an analytic model for the to obtain the load distribution between the rock mass and the reinforced concrete structure, was fulfilled, even though the results given by this model are an approximation to the real ones, because of the fact that the admitted hypothesis regarding
the displacement field is not accurate; nevertheless it is a plausible approximation.

Concerning the result of the load distribution obtained by the analytical model developed, it is noted that for the axial force in the concrete, that traction is observed in large part of the shaft length, and compression near the bottom of the shaft, recognizing that this result is due to the spring-type effect conferred by the wedges of the rock mass, since the axial force in the rock increases greatly in the first meters of depth and tends to stabilize in the last meters.

Regarding the structural design of the shaft is concluded that either for analytical model, either to the SAP2000 model are verified the safety of concrete in compression and the adopted steel reinforcement to correspond to the minimum reinforcement imposed by regulations.

9. References

American Society of Civil Engineers (1989), Civil Engineering Guidelines for Planning and Designing Hydroelectric Developments, Volume 2 (Waterways), New York, U.S.A.


