An Underwater Target Localization System using Single Range Measurements

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Abstract — This paper describes the design of an underwater positioning system for an Autonomous Underwater Vehicle. The methodology used for systems design fall in the class of single-range (also called single-beacon) navigation, whereby a surface vehicle equipped with GPS tracks the motion of an underwater target by measuring successive distance to it using a simple acoustic ranging device. The paper describes the design of a single-range based underwater target position system using Extended Kalman Filtering theory. The design model incorporates the motion of the surface vehicle (tracker). The performance of the systems developed is assessed in simulation.

Keywords: Autonomous Underwater Vehicles, Acoustic range measurements, Single Beacon, Extended Kalman Filter.

I. INTRODUCTION

There has been an increasing rise of interest in underwater exploration. It is the least explored ecosystem and a very difficult one to explore. There are many issues involved in its exploration. The enormous pressure, the lack of air supply and its massive extension have been very difficult to tackle.

Recently, however this interest has been pushing the industry into a new way of exploring the underwater ecosystem. The introduction of Autonomous Underwater Vehicles (AUVs) has made it possible to explore the underwater world in safe manner, without jeopardizing human lives. Furthermore, AUVs allow for increased efficiency in the collection of relevant ocean data.

With AUVs leading the way in the exploration of underwater ecosystems, it is now crucial to determine their positions while executing scientific or commercial missions. In such situations, GPS positioning is not a possibility because electromagnetic waves suffer considerable attenuation in the water [1]. For this reason, common positioning systems rely on the use of acoustic devices to compute the vehicle’s distance to a number of transponders fixed on the seafloor. The range measurements are then used to determine the vehicle’s position by trilateration. These systems are expensive, costly to deploy and calibrate, and not easily transportable from one area to another. For these reasons, a new technique has come to the fore, that is normally referred to as single-beacon navigation. In the context of this thesis, the term refers to the problem of positioning an underwater target by using a (companion) surface vehicle that knows its position accurately, measures the successive distances (ranges) to the target, and uses its spatial diversity to compute the position of the target [2].

The design of the underwater target positioning system is rooted in estimation theory, which allows for an elegant solution to the problem of target positioning when the range measurements are corrupted with noise, a characteristic of every real system including electronic ones [3]. In order to deal with measurement noise and temporarily missing data, a model of the underlying physical system must be taken into account and incorporated directly in the design process. In this paper, this is done by exploiting the use of an Extended Kalman Filtering (EKF) structure in order to deal with the fact that the design model is nonlinear [4], [5].

The paper details the steps involved in the design of an EKF for the problem at hand using range measurements. To this effect, the design model is derived, after which an EKF structure is mechanized.

The efficacy of the positioning system designed is evaluated in simulation. Special attention is given to the problem of imparting sufficient spatial diversity to the motion of the surface (tracker) vehicle, so as to yield good observability conditions for the tracker/target ensemble.

II. STATE OF THE ART

What follows is a very brief account of the state of the art of acoustic-based positioning, followed by a cursory exposition of the key concepts at the core of Extended Kalman Filtering techniques.

A. Acoustic Navigation Systems and Single Beacon

There are several acoustic navigation systems capable of determining the position of an underwater vehicle, often referred to as a target. One of the most common is the Long Baseline (LBL) system [6], [7] e [8]. It consists of at least four transponders moored on the ocean floor (at well-known positions) that are interrogated by the underwater vehicle. This allows the latter to compute the distances to the transponders and, by trilateralation, its own position in inertial coordinates. Notice in this case the absence of a companion surface vehicles. One major disadvantage of this system is the cost of the operations needed to anchor the sensors (transponders) at the bottom of the ocean. Another system is the so called GIB (GPS Intelligent Buoy’s) [9] e [10], also known as inverted LBL. With this system, instead of anchoring the sensors at the bottom of the ocean, they will be positioned at the ocean’s surface. Using this solution, the
sensors can move around according to the needs of the operation. However, there are still enormous advantages related to the number of sensors used.

Considering the disadvantages of such systems, there is the need for a new breed underwater target positioning system. One possible solution falls in the scope of a body of methods called Single Beacon Navigation [11]. In the scope of the present paper, we explore the methodology whereby a single vehicle operating at the surface (observer) maneuvers and, while in motion, measures its successive ranges to the underwater target that is being positioned. Under certain conditions, it is indeed possible for the observer to get good estimates of the motion of the target. To this effect, we rely on the use of Extended Kalman Filtering techniques.

Underlying the work done is the assumption that it is possible to compute the position of the target using range measurements only. This problem is quite challenging, for it requires the use of advanced nonlinear observability tools, as well as computing observer trajectories that will (in a well defined sense) maximize the information available for target positioning. The dual of this problem was tackled in [11].

B. Filtering using Extended Kalman Filter

One common problem in every electronic system is the presence of noise. The noise will affect the system and its measurements. Filtering can be considered as separating the information we seek to obtain from all the rest of the information that is not desired, called noise [3].

With this in mind it is important to study the theory of filtering. One of the most employed filters is the Kalman Filter. In our context, the Kalman Filter is a linear, discrete-time, time-varying, optimal parameter estimator. It obtains the minimum mean-square state error estimate.

Most systems that will be studied in underwater scenarios are non-linear. For this kind of systems, instead of the Kalman Filter one must use the Extended Kalman Filter [4]. In simple terms, the Extended Kalman Filter linearizes at each cycle the system and measurement dynamics, around the last estimate of the state. Then, a Kalman Filter is applied to the linearized model. The dynamic and measurement model of a generic systems can be written in the form (see (1), [4] and [5])

\[
\begin{align*}
X_{k+1} &= f_k(X_k, U_k) + W_k \\
Y_k &= h_k(X_k) + V_k
\end{align*}
\]

where \(X_k\) represents the state vector, \(U_k\) the vector of control inputs, \(W_k\) is process white noise, \(V_k\) is measurement white noise, \(Y_k\) is the measurement, \(f_k\) captures the system dynamics, and \(h_k\) is the measurement dynamics. For (1), the EKF equations are given by

\[
\begin{align*}
\hat{X}(k+1) &= f_k(\hat{X}(k)|k), U_k) \\
\hat{P}(k+1|k) &= F(k)P(k|k)F^T(k) + Q(k)
\end{align*}
\]

and

\[
\begin{align*}
\hat{X}(k+1) &= \hat{X}(k+1|k) - \hat{K}k(k+1) \left[ Y(k+1) - h_k(\hat{X}(k+1|k)) \right] \\
\hat{P}(k+1|k) &= \left(I - \hat{K}k(k+1)P(k+1|k)F^T(k)\right) \hat{P}(k+1|k)
\end{align*}
\]

where \(F\) and \(H\) are the linearized dynamics of \(f_k\) and \(h_k\), respectively. Where we assume, \(w_k\) and \(v_k\) are sequences of white, zero mean, Gaussian noise with zero mean. From (3), a new term arises, \(K\), which is the Kalman Gain that acts on the obtained measurement and the predicted measurement.

From (2) and (3) it is possible to see that this is a recursive filter. Equation (2) is called the prediction step and (3) the filtering step. In the prediction step, during the previous time instant this step gives some information about the state system in the next instant. Same goes for the covariance. In the filtering step, the filter takes into account the measurement and corrects the state estimate according to it. Using this filter it will be possible to compute the target’s position using the range measurements.

III. RANGE ONLY LOCALIZATION PROBLEM

One way to compute the position of the underwater target is to use range measurements to a certain set of reference points [2]. The position to each reference points is known. So, to use the range measurements and get the position of the target we need to have at least three different reference points in a two-dimensional scenario. The ranges are computed with acoustic signals. It is possible to get the range knowing the speed of the acoustic signal in the medium and the time travel of the signal.

One can define the range only localization problem as the way to compute the target’s position estimate with a set of given ranges, which are corrupted by noise [2] e [11].

It is show in [11] that a method can be derived to compute the position of the target using only one reference point, a sensor at the surface. This solution is the Single Beacon method. This solution will be combined with the Extended Kalman Filter. Using the dynamic of the system that the EKF will know it will be possible to obtain the position of the target with one sensor.

A. System model: straight line

In this section, a system for the model will be applied so it is possible to yield a solution for the estimate of the position.
Models for both the observer and the target will be derived. The observer will be free to move, following the target and describes circles around it. In the meanwhile, it will be measuring the range to the target. Initially, the target moves in a straight line.

The observer has the following dynamics,

\[
\begin{align*}
\dot{v}_o &= v \begin{bmatrix} \cos(\omega t + \phi) \\ \sin(\omega t + \phi) \end{bmatrix} + v_t \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \end{bmatrix} \\
\dot{p}_o &= v_o
\end{align*}
\]  

where \(v_o\) and \(p_o\) are the observer velocity and position, \(v_t\) is the target linear velocity and, \(\psi\), it’s heading angle and \(v = 0.5 \, \text{m/s} \omega = 0.1 \, \text{rad/s} \phi = 0 \, \text{rad}\).

As for the target its dynamics are

\[
\dot{\varphi} = v_t \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \end{bmatrix}
\]  

For the system dynamics that will be applied to the EKF, it will use the relative position between the target and the observer, represented in discrete time in (8)

\[
f_e(x_e,\xi_e, k) = \begin{bmatrix} \ddot{p}_x(k) + h[v(k) \cos(\psi(k)) - v_{ex}(k)] + \xi_x \\
\ddot{p}_y(k) + h[v(k) \sin(\psi(k)) - v_{ey}(k)] + \xi_y \\
v(k) \\
\psi(k) \end{bmatrix}
\]  

in which, \(\ddot{p}_x\) and \(\ddot{p}_y\), are the relative position, or the difference, between the target and the observer; and \(v\) is the linear target velocity.

The measurement is simply the range between the observer and the target.

Using (2) to (5) from the EKF, it is possible to get some results from simulation. Table 1 shows the simulation parameters.

<table>
<thead>
<tr>
<th>Table 1 - Simulation parameters.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(h)</td>
<td>0.2 \text{s}</td>
</tr>
<tr>
<td>(\Sigma(0))</td>
<td>\begin{bmatrix} 0 &amp; 0.1 &amp; \pi/4 \end{bmatrix}'</td>
</tr>
<tr>
<td>(P(0))</td>
<td>\text{diag}([5^2, 5^2, 0.2^2, \pi^2])</td>
</tr>
<tr>
<td>(\sigma_{\text{px}} = \sigma_{\text{py}})</td>
<td>0.001</td>
</tr>
<tr>
<td>(\sigma_{\text{range}})</td>
<td>0.001</td>
</tr>
<tr>
<td>(Q)</td>
<td>\text{diag}([0.001^2, 0.001^2, 0.001^2, 0.001^2])</td>
</tr>
<tr>
<td>(R)</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The following results were obtained with the simulation.

Figure 1 shows the resulting estimated error position for the target. As it can be seen, the resulting error is smaller than 0.2 m. It gives a good indication that the filter is indeed estimating the target position correctly. Moreover, Figure 1 shows the trajectory for the observer and estimated position for the target. It shows that the observer is following the target and describing circles around it while doing so.

Using the filtering techniques from the EKF that take into consideration the dynamics of the system it is possible to obtain an estimate for the position.

B. System model: straight line with arcs

The next model will take into consideration the fact that the target may describe some curves during its mission. The same dynamic for the observer (6) and for the target (7) is used. The only difference is the heading of the target that is now given by

\[
\psi(k + 1) = \psi(k) + hr
\]  

where \(r\) is the rate at which the heading is changing.

The new system dynamics is given as following
The same simulation parameters from Table 1 are considered, except the initial state vector estimation that is now:

\[
\begin{pmatrix}
7^2 & 5^2 & 2^2 \\
\end{pmatrix}
\]

The results were as following.

\[
f_k(x_k, u_k, k) = \begin{bmatrix}
\bar{x}_k(k) + h[v(k) \cos(\psi(k)) - v_{xy}(k)] + \xi_k \\
\bar{x}_k(k) + h[v(k) \sin(\psi(k)) - v_{xy}(k)] + \xi_k \\
\psi(k) - hr(k)
\end{bmatrix}
\]

\[(10)\]

The results were as following.

Figure 3 – Observer and estimated target trajectories.

Initially, there is a mismatch in the estimate of the target, Figure 3. But soon after, the filter was capable of correctly estimating the position for the target. It is possible to see the target describing the curves and the observer continues to follow it.

Finally, Figure 4 shows the target’s position error estimate.

Figure 4 – Position error estimate.

Once again, the error while estimating the target position is small. So it is possible to get a proper target position estimate, using the EKF and range measurements only.

IV. OBSERVER STRATEGY

The previous chapter provided the tools to estimate a target’s position using an observer that would measure the range. The observer is supposed to follow the target as it moves during missions. But to do so, the observer had to know the target dynamics and how he is moving.

Some problems arise in that scenario. One problem is that it might not be desired to know the target’s position in the first hand. Other problem is if there is an offset in the observer and target initial positions, that offset will remain until the end of the mission. If that happens, the observer may not be describing circles around the target. Another issue to take in consideration is that the target’s dynamics may suffer some unexpected change. If that ever happens, the observer will not be following the target and problems may arise with that.

All those problems motivate the need for a strategy one in which the observer can keep up with the target’s movement, without directly knowing its trajectory.

A. Driving the observer with target’s estimates

The main goal is to find the target position. That was achieved in the last chapter. If we now seek to give some strategy to the observer so he can follow the target, without previously knowing its dynamics, one could use the estimates to drive the observer.

The goal is now to use the position estimate from the filter and use them for the observer. In that way, the observer will follow the target no matter what type of movement he describes.

This new strategy will allow the observer to be used in different scenarios without knowing what movement or trajectory the target will take. In addition to that, the observer and target initial position offset can be corrected.

B. Controlling the observer position

To achieve the new strategy for the observer it is required that his position is somehow controlled.

We shall consider that the filter and target behaviors are the same. For the observer, his dynamics is similar as before,

\[
\begin{align*}
v_o &= v \left[ \cos(\omega t + \phi) \right] + v_A \\
\hat{p}_o &= \nu_0
\end{align*}
\]

\[(11)\]

As it can be seen, the observer still experiences the same circle movement around the target but there is a component to his velocity, \(v_A\), that will be controlled so it uses the target’s estimate. That \(v_A\) and its corresponding position, \(p_A\), represent the hypothetical central velocity and position of the circumference the observer describes around the target. We seek a way to control \(v_A\) and \(p_A\), so that \(p_A\) will go to the target position, that will be refereed as \(p_T\). With the EKF it is
possible to get an estimate of $p_T$ and those will be used in the observer.

Let us define the position error,

$$e_t = p_A - p_T,$$  \hspace{1cm} (12)

as the error between the observer’s “central” position and the target’s position. This error has to go to zero in order for the observer to follow the target.

Taking the derivative of (12), we get

$$\dot{e}_t = \ddot{p}_A - \ddot{p}_T.$$  \hspace{1cm} (13)

Applying a simple control,

$$\ddot{p}_A = \ddot{p}_T - ke_t,$$  \hspace{1cm} (14)

in which, $k$, is a control gain that corrects the error, so the observer’s position goes to the target’s position.

Combining (13) with (14), the derivative of the error is the following,

$$\dot{e}_t = -ke_t.$$  \hspace{1cm} (15)

Solving (15), we get,

$$e(t) = e(t_0)e^{-(t-t_0)}$$  \hspace{1cm} (16)

and it can be shown that the error goes in fact to zero, as desired. With this approach it is now possible for the observer to follow the target using the target’s position estimates.

Figure 5 – Target’s position estimation error.

Yet again, the filter can achieve a good performance, giving a good estimate of the targets position, Figure 5. The initial error from the filter is compensated and it converges to a value below 0.4 m.

Figure 6 – Observer and estimated target’s trajectory.

In Figure 6, it is possible to see the observer adjusting his trajectory in order to follow the target. It is using the estimates given by the filter. This means the strategy used is working properly.

C. Straight Line: results and simulation

Let’s consider again the case where the target only moves in a straight line. We shall consider two distinct cases: one where the observer’s and target’s position are close and another where their further away.

In the first case, observer and target are approximately 15 meters apart from each other.

| Table 2 – Simulation parameters. |
|---------------------|------------------|
| $h$                | 0.2 s            |
| $X(0)$             | $[0 - 5.15^2 0^2]$ |
| $P(0)$             | diag([5^2 5^2 0.2^2 π^2]) |
| $\sigma_{px} = \sigma_{py}$ | 0.001 |
| $\sigma_{range}$  | 0.01             |
| $Q$                | diag([0.001^2 0.001^2 0.001^2 0.001^2]) |
| $R$                | 0.05             |
| $k$                | 0.01             |

We choose as control gain, $k$, the value of 0.01. With this, we obtained the following results.
In Figure 7, the error between the observer and the target is shown to be going to zero, this means the observer is correctly following the target.

Now if the observer and target are initially separated further more from each other, with a difference of approximately 65 meters we get the following results.

As Figure 9 shows, the observer is changing its trajectory to follow the target. Although, it appears that is doing so in a very abrupt way. This trajectory simply looks impossible. In the first moments, the observer is correcting its position so fast that looks like it is not even describing a circumference, the radius seems to be zero, which is very difficult to reproduce in practice.

With the same control gain, we can still properly estimate the target position, Figure 8. So the filter is capable of giving a good estimation for the target position if the observer and target are close or even if they are further away. The difference in estimating the position in both situations is very small.

The error between the initial positions is again, going to zero. So the strategy adopted is working fine.

Now we should take some considerations regarding the control gain, k. What happens when that value changes? Now we shall consider what happens to the observer when the control gain is reduced to 0.001.
From Figure 11, it is clear that the observer is changing its trajectory to follow the target, but it is doing so very slowly. That is shown also in Figure 12.

Figure 11 – Observer and estimated target’s trajectory, \( k = 0.001 \).

Figure 12 - Error between observer “central” position and target’s position, \( k = 0.001 \).

Figure 13, shows the state error estimation. Still the error is small. But when compared with a higher value of control gain, it can be seen that the error in estimating the position is higher.

So there is the need to properly choose the control gain for the system. If it is small it will take longer for the observer to converge to the target’s position. If it is a big value, it may experience some difficulties to replicate trajectories in real life scenarios.

As comparison, Figure 14 shows different trajectories for the observer with different values for the control gain. It is clear that, \( k \), has a direct influence in the observer’s trajectory.

Figure 14 – Different trajectories for the observer with different values of \( k \).

D. Straight line and arcs: results and simulations

The results for this scenario are similar to the straight line. Let’s simply assume that the observer and target are separated by a relatively long distance. Simulating this scenario, the results are as following.
With a small value of $k$, the observer takes too long to get to the target’s trajectory, Figure 15. It is the same result as before. So a small value of $k$ is not appropriate to use.

From Figure 16, it is possible to see that the error between the observer and target’s position is going to zero. But it is not doing so within a proper time period. Not even after 1500 s it was possible to achieve a zero error value.

Raising the gain, $k$, to 0.01, the following results were obtained.

The same conclusions drawn from the straight line are valid in this scenario. It is crucial to get a proper value for $k$. If it is too small, it will take too long to follow the target and initial estimates for the position present higher errors. With a high value of $k$, it will get less time to follow the target and it is possible to get better estimates for its position, but it may raise some maneuverability issues in the observer.

V. CONCLUSIONS

It was possible to see during the course of this work that it is fundamental to know the position of an underwater vehicle while exploring the bottom of the ocean. The typical approaches one can use in land to obtain the position are not valid in the underwater world. There is the need for another solution. Many solutions have been studied, and most of them rely of navigation acoustic systems. Between the different acoustic systems one that is of interest because of its advantages is the Single Beacon. Using acoustic range measurements it is possible to get a target’s position with that system.

Combining the Single Beacon with the theory of the Extended Kalman Filter it was possible to draw a solution for the localization of an underwater target that is free to move.

Another goal was to think of a strategy for the observer to follow the target. The goal was to use the estimates previously obtain for the target’s position and use them so the observer was capable of following it. There was the need to control the observer’s position according to those estimates. It was possible to achieve that goal as well.

As future work, it would be desirable that it could be implemented in real life scenario. The work presented here would serve as a guideline for a real life scenario.

REFERENCES


