Abstract - This work consists on the investigation and analysis of some aspects related to electromagnetic wave propagation in radars. To accomplish that, MATLAB® simulations are developed to allow a better comprehension of the different radio wave propagation phenomena that occur in certain environments. The simulations are based on the following features: representation of the electric field and power interference pattern due to ground reflection, representation of the receiver power in radars with a particular case of low altitude radar, surface and volume clutter effects, visualization of the effects caused by the inversion of the refraction index, trapping in ducts. Finally the work addresses the study of absorption and depolarization caused by rain and wind.

I. INTRODUCTION

Since the Second World War in 1940s, the radar became a key system in the military environment. As result, several persons started the study of radars. Nowadays this system is installed all over the world with several purposes (civil and military), namely surveillance, meteorological forecast and traffic monitoring and control. An important aspect to understand is the radio wave propagation in radars. As a form of electromagnetic radiation, radio waves are affected by different phenomena when they are transmitted or propagated from one point on the Earth to another, such as reflection, refraction, diffraction, absorption, polarization and scattering. The studies of all these phenomena are of great relevance for the understanding and development of radars systems.

To support the continuously increasing number of air traffic, the radars needs to be as accurate as possible because is a technology which deals with human life. In the other hand, the meteoroidal forecast, for human needs, should be more reliable.

A theoretical approach with simulations developed in MATLAB® is presented in this paper. These simulations are based on radio wave propagation in radars to allow the visualization and analysis of many factors related with meteorological aspects in the propagation and lead to a better comprehension of reflection, diffraction, refraction, scatter, clutter, absorption and depolarization. This paper is composed by four topics. The first consists in the radio wave propagation in radars, with simulations illustrating the interference of free space and reflected wave in the target and from the eco, the radar equation, with the case of low altitude radar, radar cross section and radar clutter (surface and volume). The second topic is about refraction, with emphasis in ducts. Finally, the third topic relates to absorption and depolarization in radars, with a simulation of a typical scenario of wind and rain in radars.

II. RADIO WAVE PROPAGATION IN RADARS

Due to the presence of the Earth, several phenomena might occur during radio wave propagation. Reflection is one of them. The first point in this chapter is the effect of the reflected ray in the received electric field and power. The interference between the direct ray and the reflected ray produces an amplitude oscillation in the resulting electric field around a mean value, which is the free-space field.

The scenario is shown in Fig. 1.

The electric field due to interference of the direct and reflected rays is given by

$$E = E_d [1 + |\Gamma| \exp (j\Delta\phi)]$$

Where $E_d = \sqrt{60P_c G_r / d}$ represents the maximum amplitude of far field in free-space, $\Gamma$ the reflection coefficient of the ground and $\Delta\phi$ the phase difference between both rays. The reflection coefficient depends on the polarization, which can be horizontal or vertical and are given by:

$$\Gamma_h = \frac{\sin \Psi - \sqrt{n^2 - \cos^2 \Psi}}{\sin \Psi + \sqrt{n^2 - \cos^2 \Psi}}$$

for horizontal polarization and

$$\Gamma_v = \frac{n \sin \Psi - \sqrt{n^2 - \cos^2 \Psi}}{n \sin \Psi + \sqrt{n^2 - \cos^2 \Psi}}$$

for vertical polarization. The phase difference is related with the trajectory difference between the direct and the reflected ray, and is represented by:
\[ \Delta \phi = \text{arg}\{\Gamma}\} - 2\pi \frac{\Delta \nu}{\lambda} \]

where \( \Delta \nu = r_d - r_d \).

The electric field oscillates between maximums and minimums. A maximum occurs when \( e^{i\Delta \phi} = 1 \) and a minimum for \( e^{i\Delta \phi} = -1 \). From the electric field equation, these are given by:

\[
\left( \frac{E}{E_d} \right)_{\text{max}} = 1 + |\Gamma|, \quad \text{even } n
\]

\[
\left( \frac{E}{E_d} \right)_{\text{min}} = 1 - |\Gamma|, \quad \text{odd } n
\]

and they occur when

\[ \frac{2\pi \Delta \nu}{\lambda} - \text{arg}\{\Gamma}\} = n\pi \]

The Fig. 2 represents the simulation result for the electric variation in a given distance, using vertical polarization and fixed antenna height.

[Graph showing electric field variation]

Proceeding to the analysis of the Fig. 2, it is possible to distinguish the maximums and minimums of the electric field (blue line) around the free-space field (red line), due to the interference between the direct and the reflected rays. The Brewster angle, which represents the incidence angle where \( |\Gamma| \) is approximately zero, is visible. At this distance, the electric field coincides with the red line, i.e., it is approximately equal to the free-space field.

Using the fact that the power relates with electric field as \( P \propto E^2 \) is possible to normalize the received power with the transmitted power by:

\[
P_r = \left| \frac{E}{E_d} \right|^2 = |f_d(\theta_d)F_d(\theta_d)\Gamma_{r,\nu}\left(2e^{i\nu(\Delta \nu - \nu\theta_d)}\right)|^2
\]

where \( f_d \) is the antenna factor and \( F_D \) is the antenna array factor.

The radar equation is given by

\[
\frac{P_r}{P_e} = \frac{1}{4\pi d^2} G(e) \cdot \sigma \cdot \frac{1}{4\pi d^2} \cdot A(e) = \frac{G^2 \sigma \lambda^2}{4\pi d^4}
\]

with \( G \) related with the gain of the antenna, \( d \) the distance, \( \lambda \) the wavelength, \( \sigma \) the target cross section and \( A_e = (\lambda^2 / 4\pi) \) the effective aperture.

The Fig. 3 represents the received power for a radar at 2.8 GHz. The height of the radar antenna is 10 m and the target height is 20 m. The propagation is over the sea.

[Graph showing received power]

With this simulation, the interference between the reflected and direct rays will affect the received power in the radar. As we can see, there are maximums and minimums caused by the interference but they are much more spaced than the case of received power in the target. This is justified by the fact that there are two ways for the signal propagation.

Another interesting case to analyze is the low altitude radar. For low altitude radar, the height of the antennas is very low compared to the distance of the target. For example, in nautical radars, the height of the antennas is negligible when compared to the distance from the radar antenna to the target. This allows the approximation of assume \( \Gamma_{\nu,\nu} \approx -1 \).

This approximation allows:

\[
E \approx E_d e^{-i\Delta \nu} 2 j \sin(jk\Delta \nu)
\]

Then

\[
\frac{P_r}{P_e} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^2 d^4} \frac{1}{16} \left( \frac{k h \lambda}{d} \right)^4
\]

In the Fig. 4 is possible to see that after certain distance, the two cases follow the same variation. This is justified by the fact that after the referred distance, the power decay changes from \( 1/d^4 \) to \( 1/d^3 \).
In the next section of the chapter, a key concept is explored, which is the radar cross section (RCS) and is defined by the symbol \( \sigma \). The RCS is a key concept because it describes the characteristic of the target. It is defined by the scattered/reflected wave from the target.

\[
\sigma = \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_s}{E_i} \right|^2
\]

where \( E_s \) is the scattered field and \( E_i \) is the incident field and \( R \) is the distance to the target. It is possible to describe the RCS in a matrix, which defines the RCS for co-polarization and cross-polarization.

\[
\begin{bmatrix}
E_{sh} \\
E_{sv}
\end{bmatrix} = \begin{bmatrix}
\sigma_{HH} & \sigma_{VH} \\
\sigma_{HV} & \sigma_{VV}
\end{bmatrix} \begin{bmatrix}
E_{il} \\
E_{iv}
\end{bmatrix}
\]

The RCS can be classified based on the target dimension \( (l) \) related to the wavelength \( (\lambda) \) as:

- Rayleigh \( (l \ll \lambda) \)
- Resonance \( (l \approx \lambda) \)
- Optics \( (l \gg \lambda) \)

For the surface clutter, the relation between the clutter and the signal is defined as:

\[
(SCR)_{sc} = \frac{2\sigma_i \cos \Psi_c}{\sigma^2 \theta_{3\text{dB}} R c \tau}
\]

where \( \sigma_i \) is the target cross section, \( \Psi_c \) is the incident angle, \( \sigma^0 \) the clutter scatter coefficient, \( \theta_{3\text{dB}} \) the half power beam width, \( R \) the distance to the target, \( c \) the speed of light in vacuum and \( \tau \) is the pulse width. The Fig. 6 shows the terms defined for the surface clutter.

For the volume clutter, the relation between the clutter and the signal is defined as:

\[
(SCR)_{v} = \frac{8\sigma_i}{\pi \theta_a \theta_e c R^2 \sum_{i=1}^{N} \sigma_i}
\]

where \( \sigma_i \) is the target cross section, \( \theta_a \) the radar aperture in azimuth, \( \theta_e \) the aperture in elevation, \( R \) the distance to the target, \( c \) the speed of light in vacuum, \( \tau \) is the pulse width and \( \sum_{i=1}^{N} \sigma_i \) is the sum of the individual elements for the clutter. The Fig. 7 shows the terms defined for the volume clutter.
III. REFRACTION IN RADARS

Another very important factor that influences actively the trajectory of electromagnetic waves is the refraction index variation in the atmosphere. By using the modified refractivity to characterize different atmosphere layers, it was possible to simulate the trajectory of the rays in different situations and to demonstrate the influence of the index in them.

The modified refractivity is given by

\[ M = N + 10^6 \frac{h}{a} \]

where \( N \) is the refractivity, \( h \) is the height in the atmosphere and \( a \) the Earth radius. The refractivity is given by

\[ N = N(0) + \left( \frac{dM}{dh} - 157 \right) h \]

with \( N(0) = 315 \) being the refractivity at the surface.

With these equations it is possible to define the nature of the atmosphere for the following ray tracing simulations. The simulations are based on two cases. The case of a standard atmosphere and the case where rays are trapped inside a duct, due to special conditions that occur in the atmosphere. A standard atmosphere is characterized by a linear variation of the modified refractivity and that profile doesn’t change in height.

The analytical model used to represent the trajectory is given by

\[ z = \frac{2}{\pm A \mu} \left( \sqrt{\mu h + b} - \sqrt{\mu h_0 + b} \right) \]

where \( A = \sqrt{2} \times 10^{-3} \), \( b = \left( 10^6 / 2 \right) \alpha_0^2 - \mu h_0 \), \( h_0 \) represents the height of the transmitter and \( \alpha_0 \) is the departure angle of each ray relative to the horizontal.

The next figure represents the behaviour of the rays in a standard atmosphere, with different departure angles.

As we can see, when \( dM/dh > 0 \) the rays tend to rise and move away from the surface of the Earth. On the other hand, if \( dM/dh \) is negative the rays tend to fall to the surface (Fig. 9).

Under special meteorology conditions favorable to the formation of inverted layers where \( dM/dh \) becomes negative, a duct is generated. In these circumstances rays travel longer distances and may cause interference in other systems. This behaviour is due to the fact that ducts act like waveguides where propagation losses are minimized. The duct takes place when the atmosphere is structured by two or three layers. Each layer presents a different modified refractivity and the signals of \( dM/dh \) in two consecutive layers are opposed.

The Fig. 10 and Fig. 11 demonstrate the effects of surface ducts and raised ducts, in a two and three layer atmosphere, respectively.

From the analysis of the figures we can verify that the rays are contained in the layer where \( dM/dh \) is negative. As a consequence, rays can travel longer distances as opposed to normal conditions.
For better interpretation of the relation between this meteorological phenomena and radars, a simple and objective scenario is analyzed.

For a case without duct we have:

$$P_{\text{no-duct}} = \frac{P \cdot G^2 \cdot \sigma \lambda^2}{(4 \pi)^3 d^4} A,$$

And for a case with duct:

$$P_{\text{duct}} = \frac{P \cdot G^2 \cdot \sigma \lambda^2}{16 \pi^3 (dh)^4}$$

For surveillance radar, working at 2.8 GHz, with transmitting antenna at 200 m a typical sensitivity for this system is $P_{\text{min}} = -104\,\text{dBm}$. The Fig. 12 represents the received power for the two cases: Duct and normal propagation. We can clearly see the effect of the duct which allows the radar receives signal for a higher distance when compared to the standard atmosphere.

There are several signal processing mechanism to avoid the detection of targets out of the radar range, but they are not presented in this project.

IV. ABSORPTION AND DEPOLARISATION IN RADARS

The chapter of absorption and depolarization in radars refers to three main atmospheric aspects. These aspects are atmospheric gases, rain and wind. Gases and rain will affect the propagation because they will attenuate the signal, in other words, they will absorb the power travelling in the atmosphere. Rain and wind may change the polarization of the signal.

As said, the attenuation is caused by gases and/or by rain. The two most relevant gases that attenuate the radio wave signal are water vapor and oxygen.

The gases will affect the radio wave propagation with a supplementary attenuation defined by:

$$A = \int_{\text{ray}} \alpha(s) \, ds$$

where $\alpha = \alpha_o + \alpha_w$.

So in a simple way we can define:

$$A = \alpha_o r_e = \alpha_{o_0} r_{e_0} + \alpha_{o_w} r_{e_w}$$

Where $\alpha_{o_0}$ represents the attenuation coefficient for the oxygen is measured in mean sea level and $\alpha_{o_w}$ is the attenuation coefficient for water vapor for the same condition.

In the Fig. 13 and Fig. 14 we can see the variation the attenuation coefficient for water vapor and oxygen with frequency.
Due to the fact that the attenuation coefficient is defined for mean sea level, there is another parameter to be defined which is the effective distance of the travelling ray ($r_e$). The Fig.15 and Fig.16 represents the effective distance for two cases: $\Psi = 0$rad and $\Psi = 0.1$rad

To study the attenuation caused by rain, there is one very well-known law called Ryde’s law.

$$\alpha = KR^\gamma \ [\text{dB/km}]$$

where $K$ and $\gamma$ are functions of frequency and $R$ is rain rate.

The Fig. 17 represents two graphs which allow to obtain $K$ and $\gamma$ when we have the frequency. So, to obtain the attenuation in a cell with length $d$:

$$A = \alpha.d$$

The results for Ryde’s law are very satisfactory. This theory tells that attenuation caused by rain takes for starting point the interaction between a plane wave and a sphere (Mye theory), an empirical statistic distribution for rain droplet size and speed distribution for the droplet in the borders of the rain cell. In that way, rain droplets are similar to a statistic dielectric distribution with values of $\varepsilon_r$ and $\sigma$ close to the values for water. The individual drops are considered independent (no shadow effect).
The fact of the ray is travelling a rain cell will affect the propagation in other perspective, depolarization. The Fig. 18 represents a typical scenario of propagation in windy and rainy condition.

The rain droplets, after certain rate (\( \approx 10\text{mm/h} \)), will take a flattened shape. This will create a differential attenuation between the vertical and the horizontal axis. In other words, the attenuation for horizontal polarization will be higher than the vertical polarization. The wind will introduce a rotation in the droplets, but the rotation will make effect in polarization only if the rain rate is higher than \( \approx 10\text{mm/h} \). This is due to the fact that if the rain rate is low, the rain droplets are considered spheres, so a rotation will not affect the polarization.

To understand the case in study, a simulation with typical radar parameters will be made. Let’s assume the radar operating at 2.8GHz with \( P_e = 20\text{kW} \), the rain length \( d = 2\text{km} \), \( R = 50\text{mm/h} \) and a rain droplet inclination defined by \( \phi \).

The Fig. 19 represents the received power for the transmitted polarization (vertical), and the horizontal polarization. Is possible to see that the fact of the rotation introduced by rain and wind, will generate some power in the cross channel (horizontal).

To eliminate this issue, two solutions are more obvious; Avoid rain and wind or use another type of polarization. Obviously, the first solution is not possible. The second solution is the plausible one (for example, circular polarization).

With this simple simulation, is possible to see how the atmospheric aspects can affect the radio wave propagation in radar systems.

V. CONCLUSIONS

The last few years have witnessed a phenomenal growth in air traffic flow, which addresses to the need of more accurate radar systems. The main goal of this work is to investigate and analyze the influence that certain environments can have in wave propagation in radars. To provide the needed results a series of simulations of propagation models studied in the course were made. The addressed aspects are the reflection, refraction, diffraction and depolarization.

In the reflection topic, the interference of the reflected ray on the ground with the direct ray was demonstrated through the existence of maximums and minimums of the electric field and received power. The influence of an array of antennas was shown as well. The electric field with ground reflection is influenced by the frequency, by the distance and height of the receiving antennas and by polarization, all causing variations of the field around a mean value.

In the refraction chapter the ray tracing was demonstrated with a standard atmosphere and for atmospheres with special conditions. When the modified refractivity gradient is negative the ray goes down, otherwise, the ray goes up. Under special conditions in a stratified atmosphere a duct is formed and the trapped rays can travel longer distances in the layers where the gradient is negative.

The last topic is about depolarization in radars, where a simulation with a typical scenario of propagation in rainy and windy conditions. The simulation allows the understanding of the effect of the flattered shape of the droplets, a differential attenuation and phase between vertical and horizontal axis and the effect of wind, introducing a rotation in the rain drops. The differential attenuation and phase aggregate with the rotation will cause power decay in the polarization in use while allowing an increasing power in the cross polarization channel.
VI. REFERENCES


